Ass 1 L.A Suraj Perdoor 1BM18CS114 Date...........

Expt. No. .... Page No. ....

1. Find the characteristic & minimal polynomial of i) $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & -2 \\ -1 & -1 & 3 \end{bmatrix}$

Sol

$$|A-\lambda I| = \begin{vmatrix} 3-\lambda & 1 & -1 \\ 2 & 4-\lambda & -2 \\ -1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - trace(A)\lambda^2 + (\sum_{i=j} |M_{ij}|)\lambda - |A| = 0$$

$trace(A) = 10$

$\sum_{i=j} |M_{ij}| = 28$ & $|A| = 24$

$$\lambda^3 - 10\lambda^2 + 28\lambda - 24 = 0$$

$(\lambda-2)^2(\lambda-6) \rightarrow$ Characteristic polynomial

$f(\lambda) = (\lambda-2)(\lambda-6)$ & $g(\lambda) = (\lambda-2)^2(\lambda-6)$

By Cayley Hamilton theorem, $g(\lambda) = g(A) = 0$ so ~~wea~~ need to test only for $f(\lambda)$

$f(A) = (A-2I)(A-6I) =$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & -1 \\ 2 & -2 & -2 \\ -1 & -1 & -3 \end{bmatrix} = 0$$

$\therefore$ $f(\lambda) = (\lambda-2)(\lambda-6)$ is the minimum polynomial

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ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$ $\quad trace(A) = 6$

$|A| = 6$

Sol $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$(\lambda-1)(\lambda-3)(\lambda-2) \rightarrow$ Characteristic polynomial

$g(\lambda) = (\lambda-1)(\lambda-3)(\lambda-2)$

By C.H.T $g(\lambda) = g(A) = 0$

$\therefore$ $(\lambda-1)(\lambda-3)(\lambda-2)$ is the min polynomial

iii) $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 4 & -2 \\ -1 & -1 & 3 \end{bmatrix}$ $\quad trace(A) = 10$

$|A| = 24$

sol $\lambda^3 - 10\lambda^2 + 28\lambda - 24 = 0$

$(\lambda-2)^2(\lambda-6) \Rightarrow$ characteristic polynomial

$f(\lambda) = (\lambda-2)(\lambda-6)$ & $g(\lambda) = (\lambda-2)^2(\lambda-6)$

$f(A) = (A-2I)(A-6I)$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & -1 \\ 2 & -2 & -2 \\ -1 & -1 & -3 \end{bmatrix} = 0$$

$\therefore$ $f(\lambda) = (\lambda-2)(\lambda-6)$ is min polynomial

Date ....

Expt. No. .... Page No. ....

iv) $\begin{bmatrix} 3 & 2 & -1 \\ 3 & 8 & -3 \\ 3 & 6 & -1 \end{bmatrix}$ $\quad trace(A) = 10$

$|A| = 24$

Sol $\lambda^3 - 10\lambda^2 + 28\lambda - 24 = 0$

$(\lambda-2)^2(\lambda-6) \Rightarrow$ characteristic polynomial

$f(\lambda) = (\lambda-2)(\lambda-6)$ & $g(\lambda) = (\lambda-2)^2(\lambda-6)$

By C.H.T, $g(\lambda) = g(A) = 0$

$f(\lambda) = f(A) = (A-2I)(A-6I)$

$$= \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & -1 \\ 2 & -2 & -2 \\ -1 & -1 & -3 \end{bmatrix} = 0$$

$\therefore$ $(\lambda-2)(\lambda-6)$ is the min polynomial

2. Find the characteristic & minimal polynomial of

$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & -4 \end{bmatrix}$ Sol $\begin{bmatrix} B & 0 \\ 0 & D \end{bmatrix}$ $\quad B = \begin{bmatrix} 4 & 0 \\ 1 & 4 \end{bmatrix}$

$D = \begin{bmatrix} -4 & 0 \\ 1 & -4 \end{bmatrix}$

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For B : $\lambda^2 - 8\lambda + 16 = 0$ trace $(B) = 8$; $|B| = 16$

$(\lambda-4)^2 \rightarrow$ Characteristic polynomial

$f(\lambda) = (\lambda - 4)$ & $g(\lambda) = (\lambda-4)^2$

$f(B) = (B - 4I) = \begin{bmatrix} 4 & 0 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq 0$

$\therefore$, $g(\lambda) = g(B) = 0$, Hence, $(\lambda-4)^2$ is the minimal polynomial of B

For D : $\lambda^2 + 8\lambda + 16 = 0$ trace $(D) = 8$; $|D| = 16$

$(\lambda+4)^2 \Rightarrow$ characteristic polynomial

$f(\lambda) = (\lambda + 4)$ & $g(\lambda) = (\lambda+4)^2$

$f(A) = (A + 4I) = \begin{bmatrix} -4 & 0 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq 0$

$\therefore$ $(\lambda+4)^2$ is the min polynomial

* Characteristic polynomial of A = product of characteristic polynomial of B & D

$= (\lambda-4)^2(\lambda+4)^2$

Also, the minimal of A $= \text{LCM}\,[m_1(t), m_2(t)]$

$= (\lambda-4)^2(\lambda+4)^2$

Date ....

Expt. No. .... Page No. ....

ii) $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ sol $\begin{bmatrix} B & 0 \\ 0 & D \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$

$D = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

For B: $\lambda^2 - 6\lambda + 9 = 0$ trace$(B) = 6$; $|B| = 9$

$(\lambda-3)^2 \Rightarrow$ Characteristic polynomial

$f(\lambda) = (\lambda - 3)$ & $g(\lambda) = (\lambda-3)^2$

$f(B) = (B - 3I) = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \neq 0$

By C.H.T, $g(\lambda) = g(B) = 0$ Hence, $(\lambda-3)^2$ is min polynomial of B

For D: $\lambda^2 - 6\lambda + 9 = 0$ trace$(D) = 6$; $|D| = 9$

$(\lambda-3))^2 \rightarrow$ characteristic polynomial

$f(\lambda) = (\lambda - 3)$ & $g(\lambda) = (\lambda-3)^2$

$f(D) = (D - 3I) = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 0$

$\therefore f(\lambda) = f(D) = 0$, Hence $(\lambda-3)$ is the minimal polynomial of D

* characteristic polynomial of A: $(\lambda-3)^2(\lambda-3)^2 = (\lambda-3)^4$

minimal polynomial of A $= LCM(\lambda-3, (\lambda-3)^2) = (\lambda-3)^2$

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iii) $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 3 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 5 \\ 0 & 6 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$

$\begin{bmatrix} B & C \\ 0 & D \end{bmatrix}$

For B: $\lambda^2 - 4\lambda + 3 = 0$ trace $(B) = 4$; $|B| = 3$

$(\lambda-3)(\lambda-1) \rightarrow$ Characteristic polynomial

Minimum polynomial of B: $(\lambda-3)(\lambda-1)$

For D: $\lambda^2 - \lambda + 30 = 0$ trace$(D) = 11$; $|D| = 30$

$(\lambda-6)(\lambda-5) \rightarrow$ Characteristic polynomial

Minimal polynomial of D : $(\lambda-6)(\lambda-5)$

* Characteristic polynomial of A :

$(\lambda-3)(\lambda-1)(\lambda-6)(\lambda-5)$

iv) $\begin{bmatrix} 9 & -1 & 5 & 7 \\ 8 & 3 & 2 & -4 \\ 0 & 0 & 3 & 6 \\ 0 & 0 & -1 & 8 \end{bmatrix}$ sol $D = \begin{bmatrix} 3 & 6 \\ -1 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 5 & 7 \\ 2 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 9 & -1 \\ 8 & 3 \end{bmatrix}$

For B: $\lambda^2 - 12\lambda + 35 = 0$ trace $(B) = 12$; $|B| = 35$

$(\lambda-7)(\lambda-5) \Rightarrow$ Characteristic polynomial

Minimal polynomial of B = $(\lambda-7)(\lambda-5)$

For D : $\lambda^2 - 11\lambda + 30 = 0$ trace $(D) = 11$; $|D| = 30$

$(\lambda-6)(\lambda-5) \rightarrow$ Characteristic polynomial

Minimal polynomial of D : $(\lambda-6)(\lambda-5)$

Characteristic polynomial of A :

$(\lambda-7)(\lambda-5)(\lambda-6)(\lambda-5)$

$= (\lambda-7)(\lambda-6)(\lambda-5)$

$\rightarrow$ Minimal polynomial of A :

$(\lambda-7)(\lambda-5)(\lambda-6)$

b. $\left[\begin{array}{cc|ccc} 4 & -1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ \hline 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{array}\right]$ Sol $B = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$

$D \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

For B : $\lambda^2 - 6\lambda + 9 = 0$ trace $(B) = 6$; $|B| = 9$

$(\lambda-3)^2 \rightarrow$ Characteristic polynomial

$f(\lambda) = (\lambda-3)$ & $g(\lambda) = (\lambda-3)^2$

$f(B) = (B - 3I) = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \neq 0$

$\therefore$ $g(\lambda) = g(B) = 0$. Hence $(\lambda-3)^2$ is minimal polynomial of B

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For D: $\lambda^3 - 9\lambda^2 + 27\lambda - 27 = 0$

$(\lambda-3)^3 \rightarrow$ Characteristic polynomial

$f(\lambda) = (\lambda-3)$ & $e(\lambda) = (\lambda-3)^2$ & $g(\lambda) = (\lambda-3)^3$

$$f(D) = (D-3I) = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \neq 0$$

$$e(D) = (D-3I)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq 0$$

$\therefore g(\lambda) = g(D) = 0$, hence minimal poly of $D = (\lambda-3)^3$

Characteristics of poly of A: $(\lambda-3)^3(\lambda-3)^2 = (\lambda-3)^5$

min poly of A: $LCM((\lambda-3)^2(\lambda-3)^2)$

vi) $$\begin{bmatrix} 2 & 7 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & +1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$ sol $B = \begin{bmatrix} 2 & 7 \\ 0 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

For B: $\lambda^2 - 4\lambda + 4 = 0$ trace$(B) = 4$ $\therefore |B| = 4$

$(\lambda-2)^2 \rightarrow$ characteristic polynomial

$f(\lambda) = (\lambda-2)$ & $g(\lambda) = (\lambda-2)^2$

$$f(B) = (B-2I) = \begin{bmatrix} 2 & 7 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 0 & 0 \end{bmatrix} \neq 0$$

$\therefore g(\lambda) = g(B) = 0$, Hence $(\lambda-2)^2$ is the minimal polynomial of B

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For D: $\lambda^2 - 5\lambda + 6 = 0$ trace$(D) = 5$; $|D| = 6$

$(\lambda-3)(\lambda-2) \rightarrow$ Characteristic polynomial

minimal polynomial of A : $(\lambda-2)^2(\lambda-3)(\lambda-2)$

$= (\lambda-3)(\lambda-2)^3$

Minimal polynomial of A $= (\lambda-3)(\lambda-2)^2$

3. Write Jordan Canonical form of blocks if $f(t) = (t-2)^2(t-3)$ is characteristic polynomial & $m(t) = (t-2)(t-3)$ is the minimal polynomial

Sol Given $f(t) = (t-2)^2(t-3)$ $m(t) = (t-2)(t-3)$

$$\text{diag}\left\{ [2][2][3] \right\}$$

4. Write Jordan canonical form of if $f(t) = (t-4)^4$ is characteristic polynomial & $m(t) = (t-4)^2$ is a minimal polynomial

Sol
$$\text{diag}\left\{ \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix} \right\}$$

5. Write Jordan canonical form ~~di~~ if $(t-7)^5$ [redacted] is characteristic polynomial & $m(t) = (t-7)^2$. In each case find minimal polynomial

$$\text{diag} = \left\{ \begin{bmatrix} 7 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ 0 & 7 \end{bmatrix} [7] \right\}$$

6. Write Jordan canonical form of blocks if $f(t) = (t-7)^7$ is Characteristic polynomial & $m(t) = (t-7)^3$ is minimal polynomial

Sol
$$\text{diag}\left\{ \begin{bmatrix} 7 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 7 & 1 & 0 \\ 0 & 7 & 1 \\ 0 & 0 & 7 \end{bmatrix} [7] \right\}$$

7. Find the Jordan Canonical form of $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$

trace$(A) = 6$

$|A| = 8$

Sol $\lambda^3 - 6\lambda^2 + 12\lambda - 8 = 0$

$(\lambda-2)^3 \rightarrow$ Char polynomial

$f(\lambda) = (\lambda-2)$ & $e(\lambda) = (\lambda-2)^2$ & $g(\lambda) = (\lambda-2)^3$

$$f(A) = (A-2I) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix} \neq 0$$

$$e(A) = (A-2I)^2 = \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $e(A) = 0$, Hence $(A-2I)^2 = (\lambda-2)^2$ is a minimal poly

$f(t) = (\lambda-2)^3$ & $m(t) = (\lambda-2)^2$

$$\therefore \text{diag}\left\{ \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} [2] \right\}$$