

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

“JnanaSangama”, Belgaum -590014, Karnataka.



LAB RECORD

Bio Inspired Systems (23CS5BSBIS)

Submitted by

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in partial fulfillment for the award of the degree of

**BACHELOR OF ENGINEERING
in
COMPUTER SCIENCE AND ENGINEERING**



**B.M.S. COLLEGE OF ENGINEERING
(Autonomous Institution under VTU)
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**B.M.S. College of Engineering,
Bull Temple Road, Bangalore 560019**
(Affiliated To Visvesvaraya Technological University, Belgaum)
Department of Computer Science and Engineering



CERTIFICATE

This is to certify that the Lab work entitled “ Bio Inspired Systems (23CS5BSBIS)” carried out by **Archita V (1BM23CS050)**, who is a bonafide student of **B.M.S. College of Engineering**. It is in partial fulfillment for the award of **Bachelor of Engineering in Computer Science and Engineering** of the Visvesvaraya Technological University, Belgaum. The Lab report has been approved as it satisfies the academic requirements of the above mentioned subject and the work prescribed for the said degree.

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Github Link:

<https://github.com/1BM23CS050/BIS-LA>

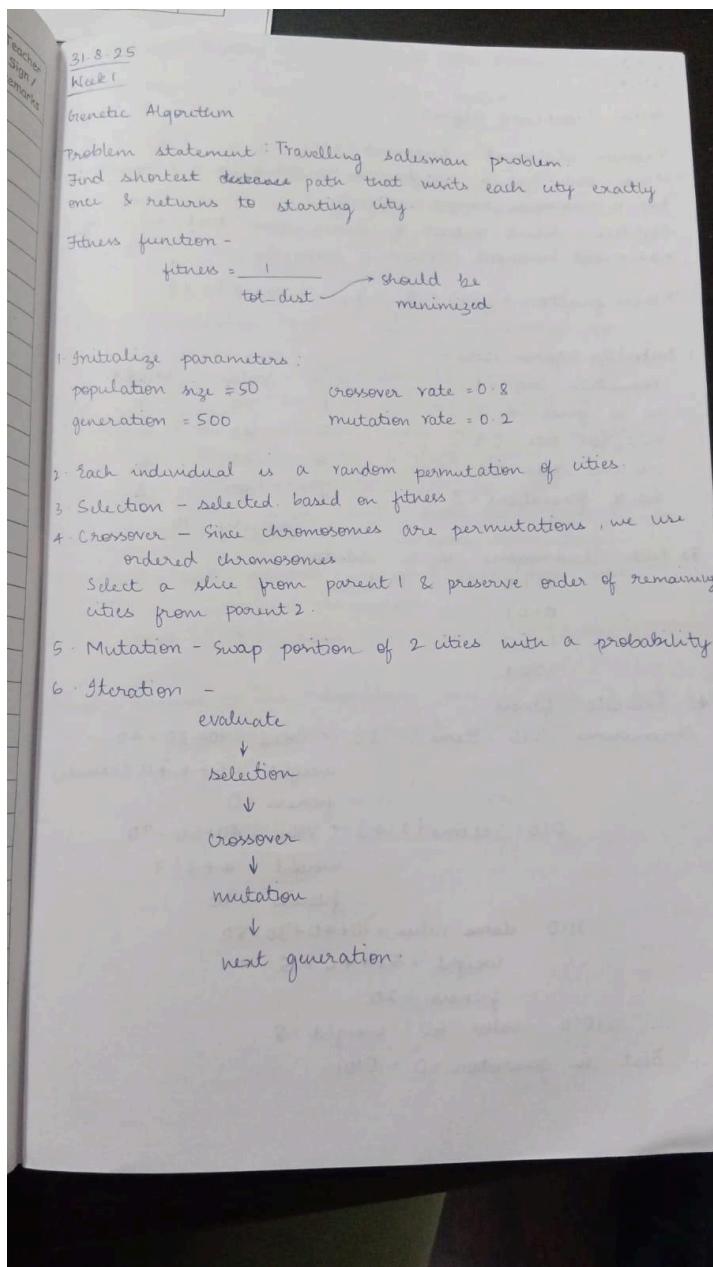
Program 1

Problem Statement:

Genetic Algorithm for Optimization Problems:

Find the best routes for delivery vehicles with limited capacity so that all customers are served once and total travel distance is minimized using a Genetic Algorithm.

Algorithm:



Code:

```
import math
import random
import copy
from typing import List, Tuple

random.seed(1)

# -----
# Utility functions
# -----


def euclidean(a: Tuple[float, float], b: Tuple[float, float]) -> float:
    return math.hypot(a[0] - b[0], a[1] - b[1])


def compute_distance_matrix(coords: List[Tuple[float, float]]) -> List[List[float]]:
    n = len(coords)
    dist = [[0.0]*n for _ in range(n)]
    for i in range(n):
        for j in range(n):
            dist[i][j] = euclidean(coords[i], coords[j])
    return dist


# -----
# Problem instance generator
# -----


def generate_instance(num_customers=15, grid_size=100, max_demand=10, depot=(50,50)):
    """
    Generates random customers with demands and coordinates.
    Index 0 is the depot, customers are 1..num_customers
    """
    coords = [depot]
    demands = [0]
    for _ in range(num_customers):
        coords.append((random.uniform(0, grid_size), random.uniform(0, grid_size)))
        demands.append(random.randint(1, max_demand))
    return coords, demands


# -----
# Chromosome -> Routes (split by capacity)
# -----


def split_routes_from_perm(perm: List[int], demands: List[int], capacity: int) -> List[List[int]]:
    """
    Greedy split: take customer sequence in perm, keep adding to current route
    """
```

```

while capacity is not exceeded. Start new route when needed.
"""

routes = []
cur_route = []
cur_load = 0
for cust in perm:
    d = demands[cust]
    if cur_load + d <= capacity:
        cur_route.append(cust)
        cur_load += d
    else:
        # close route and start new
        if cur_route:
            routes.append(cur_route)
            cur_route = [cust]
            cur_load = d
        if cur_route:
            routes.append(cur_route)
return routes

def route_cost(route: List[int], dist_matrix: List[List[float]]) -> float:
    if not route:
        return 0.0
    cost = 0.0
    prev = 0 # depot
    for cust in route:
        cost += dist_matrix[prev][cust]
        prev = cust
    cost += dist_matrix[prev][0] # return to depot
    return cost

def total_cost(routes: List[List[int]], dist_matrix: List[List[float]]) -> float:
    return sum(route_cost(r, dist_matrix) for r in routes)

# -----
# Genetic Algorithm components
# -----


def init_population(pop_size: int, customer_ids: List[int]) -> List[List[int]]:
    pop = []
    for _ in range(pop_size):
        perm = customer_ids[:]
        random.shuffle(perm)
        pop.append(perm)
    return pop

def tournament_selection(pop: List[List[int]], fitnesses: List[float], k=3) -> List[int]:

```

```

selected = random.sample(list(range(len(pop))), k)
best = min(selected, key=lambda i: fitnesses[i])
return copy.deepcopy(pop[best])

def order_crossover(parent1: List[int], parent2: List[int]) -> Tuple[List[int], List[int]]:
    """Order Crossover (OX) for permutations."""
    n = len(parent1)
    a, b = sorted(random.sample(range(n), 2))
    def ox(p1, p2):
        child = [None]*n
        # copy slice
        child[a:b+1] = p1[a:b+1]
        # fill remaining from p2 order
        pos = (b+1) % n
        p2_i = (b+1) % n
        while None in child:
            val = p2[p2_i]
            if val not in child:
                child[pos] = val
                pos = (pos+1) % n
                p2_i = (p2_i+1) % n
        return child
    return ox(parent1, parent2), ox(parent2, parent1)

def swap_mutation(perm: List[int], mut_rate=0.2) -> None:
    """In-place swap mutation (may perform multiple swaps based on mut_rate)."""
    n = len(perm)
    for i in range(n):
        if random.random() < mut_rate:
            j = random.randrange(n)
            perm[i], perm[j] = perm[j], perm[i]

# -----
# GA main loop
# -----


def evaluate_population(pop: List[List[int]], demands: List[int], capacity: int, dist_matrix: List[List[float]]):
    fitnesses = []
    for indiv in pop:
        routes = split_routes_from_perm(indiv, demands, capacity)
        fitnesses.append(total_cost(routes, dist_matrix))
    return fitnesses

def ga_vrp(
    coords: List[Tuple[float, float]],
    demands: List[int],

```

```

capacity: int,
pop_size=100,
gens=300,
crossover_rate=0.8,
mutation_rate=0.15,
elitism=2
):
    n = len(coords) - 1 # number of customers
    customer_ids = list(range(1, n+1))
    dist_matrix = compute_distance_matrix(coords)

    pop = init_population(pop_size, customer_ids)
    fitnesses = evaluate_population(pop, demands, capacity, dist_matrix)

    best_history = []
    for gen in range(gens):
        new_pop = []
        # Elitism: keep best individuals
        sorted_idx = sorted(range(len(pop)), key=lambda i: fitnesses[i])
        for i in range(elitism):
            new_pop.append(copy.deepcopy(pop[sorted_idx[i]]))

        while len(new_pop) < pop_size:
            # Selection
            p1 = tournament_selection(pop, fitnesses, k=3)
            p2 = tournament_selection(pop, fitnesses, k=3)
            # Crossover
            if random.random() < crossover_rate:
                c1, c2 = order_crossover(p1, p2)
            else:
                c1, c2 = p1, p2
            # Mutation
            swap_mutation(c1, mutation_rate)
            swap_mutation(c2, mutation_rate)
            new_pop.append(c1)
            if len(new_pop) < pop_size:
                new_pop.append(c2)

        pop = new_pop
        fitnesses = evaluate_population(pop, demands, capacity, dist_matrix)
        gen_best = min(fitnesses)
        best_history.append(gen_best)

        if (gen+1) % (max(1, gens//10)) == 0 or gen == 0:
            print(f'Gen {gen+1:4d} | best cost = {gen_best:.2f}')

    # final best

```

```

best_idx = min(range(len(pop)), key=lambda i: fitnesses[i])
best_perm = pop[best_idx]
best_routes = split_routes_from_perm(best_perm, demands, capacity)
best_cost = fitnesses[best_idx]
return {
    'best_perm': best_perm,
    'best_routes': best_routes,
    'best_cost': best_cost,
    'dist_matrix': dist_matrix,
    'history': best_history
}

# -----
# Example / Run
# -----


if __name__ == "__main__":
    # Example instance
    NUM_CUSTOMERS = 12
    VEHICLE_CAPACITY = 30
    coords, demands = generate_instance(num_customers=NUM_CUSTOMERS, grid_size=100,
max_demand=12, depot=(50,50))

    print("Depot coords:", coords[0])
    for i in range(1, len(coords)):
        print(f"Customer {i:2d} at {coords[i]} demand={demands[i]}")

    result = ga_vrp(
        coords=coords,
        demands=demands,
        capacity=VEHICLE_CAPACITY,
        pop_size=120,
        gens=300,
        crossover_rate=0.9,
        mutation_rate=0.12,
        elitism=4
    )

    print("\n==== BEST SOLUTION ====")
    print(f"Total cost: {result['best_cost']:.2f}")
    print("Routes (customers listed, depot implicit at start/end):")
    for idx, r in enumerate(result['best_routes'], start=1):
        load = sum(demands[c] for c in r)
        rcost = route_cost(r, result['dist_matrix'])
        print(f" Vehicle {idx}: {r} | load={load} | route_cost={rcost:.2f}")
    print("\nBest permutation (customer visit order):", result['best_perm'])

```

Output:

```
Depot coords: (50, 50)
Customer 1 at (13.436424411240122, 84.74337369372327) demand=2
Customer 2 at (25.50690257394217, 49.54350870919409) demand=8
Customer 3 at (47.224524357611664, 37.96152233237278) demand=4
Customer 4 at (9.385958677423488, 2.834747652200631) demand=7
Customer 5 at (43.27670679050534, 76.2280082457942) demand=1
Customer 6 at (69.58328667684435, 26.633056045725954) demand=4
Customer 7 at (59.11534350013039, 10.222715811004823) demand=6
Customer 8 at (3.0589983033553536, 2.54458609934608) demand=9
Customer 9 at (0.9204938554384978, 88.12338589221554) demand=11
Customer 10 at (21.659939713061338, 42.21165755827173) demand=1
Customer 11 at (52.76294143623982, 76.37009951314894) demand=8
Customer 12 at (55.28595762929651, 34.570041470875246) demand=11

Gen 1 | best cost = 483.53
Gen 30 | best cost = 393.75
Gen 60 | best cost = 381.97
Gen 90 | best cost = 381.97
Gen 120 | best cost = 381.97
Gen 150 | best cost = 381.97
Gen 180 | best cost = 381.97
Gen 210 | best cost = 381.97
Gen 240 | best cost = 381.97
Gen 270 | best cost = 381.97
Gen 300 | best cost = 381.97

== BEST SOLUTION ==
Total cost: 381.97
Routes (customers listed, depot implicit at start/end):
Vehicle 1: [9, 1, 5, 11] | load=22 | route_cost=142.14
Vehicle 2: [12, 6, 7, 3] | load=25 | route_cost=94.66
Vehicle 3: [2, 10, 8, 4] | load=25 | route_cost=145.16

Best permutation (customer visit order): [9, 1, 5, 11, 12, 6, 7, 3, 2, 10, 8, 4]
```

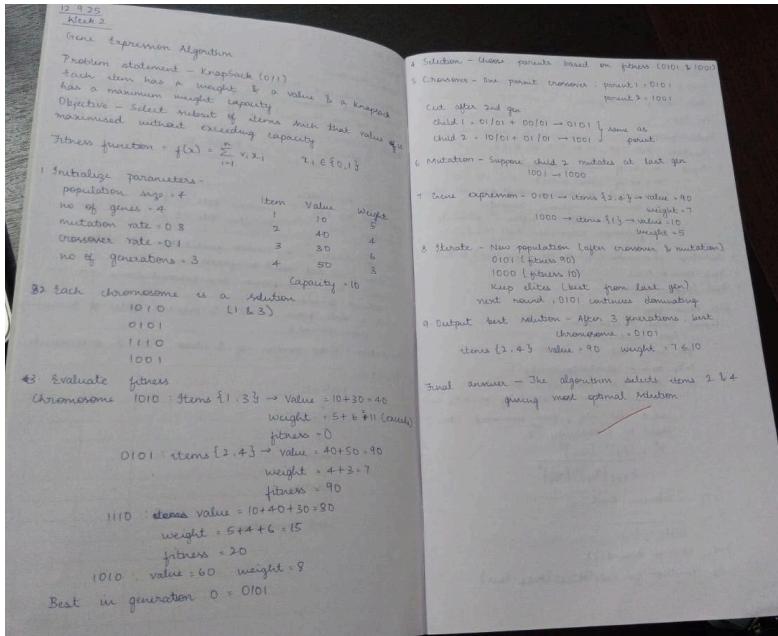
Program 2

Problem Statement:

Optimization via Gene Expression Algorithms:

Use Gene Expression Programming to find the best mathematical expression or variable values that optimize a given function.

Algorithm:



Code:

```

import random
import math

# Target function to approximate
def target_function(x):
    return x**2 + x + 1

# Terminal and function sets
TERMINALS = ['x']
FUNCTIONS = ['+', '-', '*', '/']

# Parameters
POP_SIZE = 100
MUTATION_RATE = 0.1
TOURNAMENT_SIZE = 3

```

```

GENERATIONS = 100

# -----
# Utility functions
# -----


def safe_div(a, b):
    """Safe division to avoid divide-by-zero."""
    try:
        return a / b if b != 0 else 1
    except:
        return 1

def random_gene(max_depth=3):
    """Randomly create an expression (gene) of limited depth."""
    if max_depth == 0 or (max_depth > 1 and random.random() < 0.3):
        return random.choice(TERMINALS)
    else:
        op = random.choice(FUNCTIONS)
        left = random_gene(max_depth - 1)
        right = random_gene(max_depth - 1)
        return f"({left}{op}{right})"

def evaluate_expression(expr, x):
    """Safely evaluate the evolved expression."""
    try:
        # Define the environment so eval can use math and safe_div properly
        env = {"x": x, "math": math, "safe_div": safe_div}
        # Replace '/' with safe_div(a,b)
        expr_safe = expr.replace("/", "safe_div")
        result = eval(expr_safe, env)
        if callable(result): # in case a function is accidentally returned
            return 0
        return float(result)
    except Exception:
        return 0

def fitness(expr):
    """Mean squared error against target function."""
    xs = [i for i in range(-10, 11)]
    errors = []
    for x in xs:
        y_true = target_function(x)
        y_pred = evaluate_expression(expr, x)
        errors.append((y_true - y_pred)**2)
    mse = sum(errors) / len(errors)
    return mse

```

```

# -----
# Genetic operators
# -----

def mutate(expr):
    """Randomly mutate part of the expression."""
    expr_list = list(expr)
    for i in range(len(expr_list)):
        if random.random() < MUTATION_RATE:
            expr_list[i] = random.choice(['x', '+', '-', '*', '/'])
    return ''.join(expr_list)

def crossover(parent1, parent2):
    """Single-point crossover."""
    if len(parent1) < 2 or len(parent2) < 2:
        return parent1, parent2
    cut1 = random.randint(1, len(parent1) - 1)
    cut2 = random.randint(1, len(parent2) - 1)
    child1 = parent1[:cut1] + parent2[cut2:]
    child2 = parent2[:cut2] + parent1[cut1:]
    return child1, child2

def tournament_selection(pop, fits):
    """Select the best of k random individuals."""
    chosen = random.sample(list(zip(pop, fits)), TOURNAMENT_SIZE)
    return min(chosen, key=lambda x: x[1])[0]

# -----
# Main GEP loop
# -----

def gene_expression_optimization():
    population = [random_gene() for _ in range(POP_SIZE)]

    for gen in range(GENERATIONS):
        fitnesses = [fitness(expr) for expr in population]
        best_idx = min(range(len(fitnesses)), key=lambda i: fitnesses[i])
        best_expr = population[best_idx]
        best_fit = fitnesses[best_idx]

        if gen % 10 == 0 or gen == GENERATIONS - 1:
            print(f"Generation {gen:3d} | Best Expr: {best_expr} | Fitness = {best_fit:.6f}")

        new_pop = [best_expr] # elitism
        while len(new_pop) < POP_SIZE:
            p1 = tournament_selection(population, fitnesses)

```

```

p2 = tournament_selection(population, fitnesses)
c1, c2 = crossover(p1, p2)
new_pop.extend([mutate(c1), mutate(c2)])
population = new_pop[:POP_SIZE]

print("\n✓ Optimization Complete!")
print(f"Best evolved expression: {best_expr}")
return best_expr

# -----
# Run
# -----
if __name__ == "__main__":
    best = gene_expression_optimization()

    print("\nSample comparison:")
    for x in [-3, 0, 2, 5]:
        print(f"x={x}>2} | Target={target_function(x)>8.3f} | Evolved={evaluate_expression(best,
x)>8.3f}")


```

Output:

```

Generation  0 | Best Expr: x | Fitness = 2487.000000
Generation 10 | Best Expr: x | Fitness = 2487.000000
Generation 20 | Best Expr: x | Fitness = 2487.000000
Generation 30 | Best Expr: x | Fitness = 2487.000000
Generation 40 | Best Expr: x | Fitness = 2487.000000
Generation 50 | Best Expr: x | Fitness = 2487.000000
Generation 60 | Best Expr: x | Fitness = 2487.000000
Generation 70 | Best Expr: x | Fitness = 2487.000000
Generation 80 | Best Expr: x | Fitness = 2487.000000
Generation 90 | Best Expr: x | Fitness = 2487.000000
Generation 99 | Best Expr: x | Fitness = 2487.000000

✓ Optimization Complete!
Best evolved expression: x

Sample comparison:
x=-3 | Target= 7.000 | Evolved= -3.000
x= 0 | Target= 1.000 | Evolved=  0.000
x= 2 | Target= 7.000 | Evolved=  2.000
x= 5 | Target= 31.000 | Evolved=  5.000

```

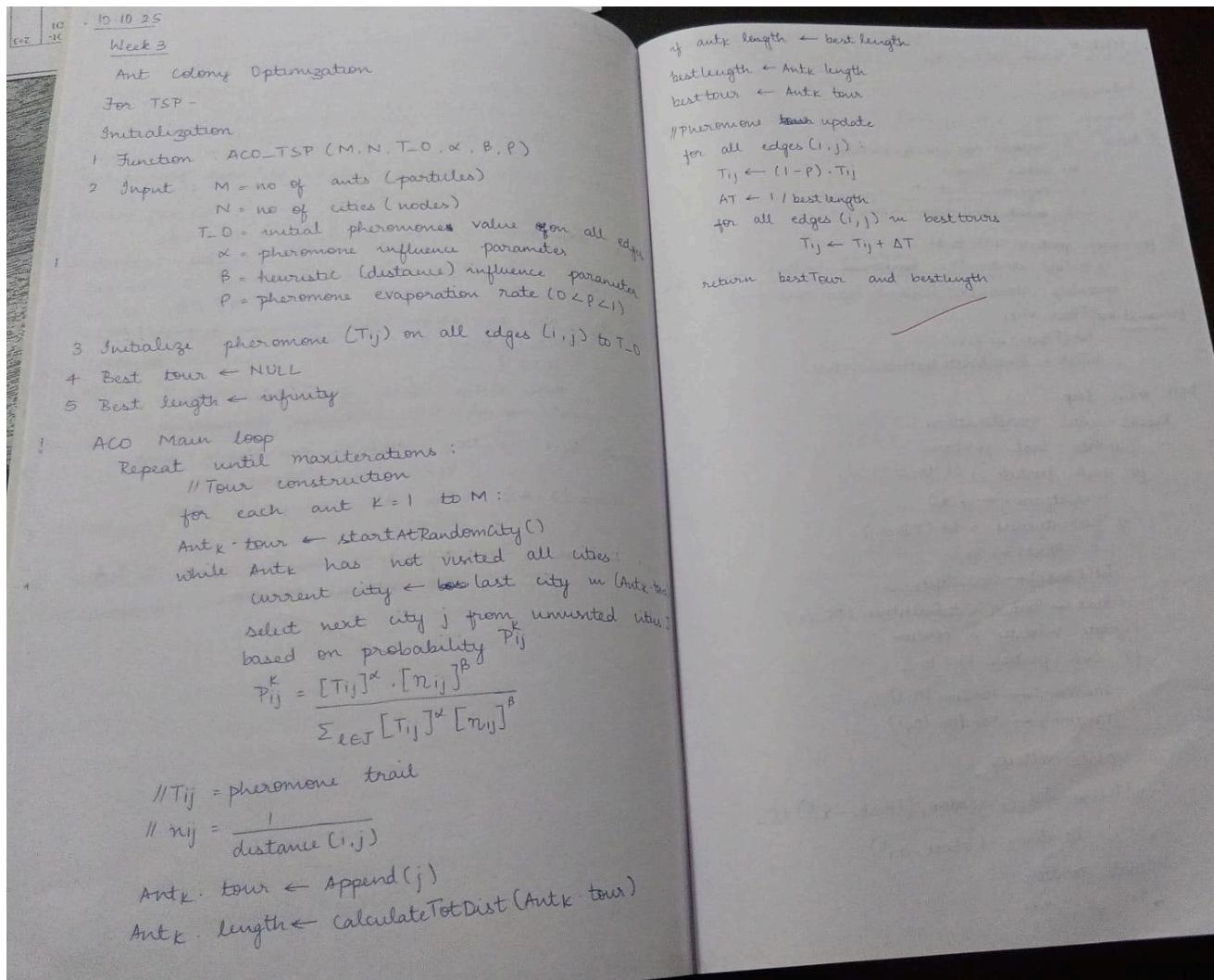
Program 3

Problem Statement:

Ant Colony Optimization for the Traveling Salesman Problem:

Use Ant Colony Optimization to find the shortest possible route that visits all cities exactly once and returns to the starting city.

Algorithm:



Code:

```
import random
import math
```

```
# -----
# PARAMETERS
# -----
```

```

NUM_CITIES = 10
NUM_ANTS = 20
ALPHA = 1.0      # pheromone importance
BETA = 5.0       # distance importance
RHO = 0.5        # evaporation rate
Q = 100          # pheromone deposit factor
ITERATIONS = 100

random.seed(1)

# -----
# CREATE TSP INSTANCE
# -----
def generate_cities(n, grid_size=100):
    """Randomly generate city coordinates."""
    return [(random.uniform(0, grid_size), random.uniform(0, grid_size)) for _ in range(n)]

def distance(a, b):
    return math.hypot(a[0] - b[0], a[1] - b[1])

def compute_distance_matrix(cities):
    n = len(cities)
    dist = [[0]*n for _ in range(n)]
    for i in range(n):
        for j in range(n):
            if i != j:
                dist[i][j] = distance(cities[i], cities[j])
    return dist

# -----
# ACO Core
# -----
def initialize_pheromones(n):
    """Initialize pheromone levels between cities."""
    return [[1.0 for _ in range(n)] for _ in range(n)]

def select_next_city(probabilities):
    """Roulette-wheel selection."""
    r = random.random()
    cumulative = 0.0
    for i, p in enumerate(probabilities):
        cumulative += p
        if r <= cumulative:
            return i
    return len(probabilities) - 1

def ant_tour(dist_matrix, pheromone):

```

```

"""Construct a complete tour for one ant."""
n = len(dist_matrix)
unvisited = list(range(n))
start = random.choice(unvisited)
tour = [start]
unvisited.remove(start)

while unvisited:
    current = tour[-1]
    probs = []
    denom = 0.0
    for j in unvisited:
        denom += (pheromone[current][j] ** ALPHA) * ((1.0 / dist_matrix[current][j]) ** BETA)
    for j in unvisited:
        num = (pheromone[current][j] ** ALPHA) * ((1.0 / dist_matrix[current][j]) ** BETA)
        probs.append(num / denom if denom > 0 else 0)
    next_city = unvisited[select_next_city(probs)]
    tour.append(next_city)
    unvisited.remove(next_city)
return tour

def tour_length(tour, dist_matrix):
    """Calculate total length of the tour."""
    total = 0.0
    for i in range(len(tour) - 1):
        total += dist_matrix[tour[i]][tour[i+1]]
    total += dist_matrix[tour[-1]][tour[0]] # return to start
    return total

def update_pheromones(pheromone, all_tours, all_lengths):
    """Evaporate and deposit pheromones based on ant tours."""
    n = len(pheromone)
    # Evaporation
    for i in range(n):
        for j in range(n):
            pheromone[i][j] *= (1 - RHO)
            if pheromone[i][j] < 1e-6:
                pheromone[i][j] = 1e-6

    # Deposit pheromones
    for tour, L in zip(all_tours, all_lengths):
        deposit = Q / L
        for i in range(len(tour) - 1):
            a, b = tour[i], tour[i+1]
            pheromone[a][b] += deposit
            pheromone[b][a] += deposit
        # return edge

```

```

pheromone[tour[-1]][tour[0]] += deposit
pheromone[tour[0]][tour[-1]] += deposit

# -----
# ACO MAIN LOOP
# -----
def aco_tsp(num_cities=NUM_CITIES):
    cities = generate_cities(num_cities)
    dist_matrix = compute_distance_matrix(cities)
    pheromone = initialize_pheromones(num_cities)

    best_tour = None
    best_length = float('inf')

    for it in range(ITERATIONS):
        all_tours = []
        all_lengths = []

        for ant in range(NUM_ANTS):
            tour = ant_tour(dist_matrix, pheromone)
            L = tour_length(tour, dist_matrix)
            all_tours.append(tour)
            all_lengths.append(L)

            if L < best_length:
                best_length = L
                best_tour = tour

        update_pheromones(pheromone, all_tours, all_lengths)

        if (it + 1) % 10 == 0 or it == 0:
            print(f'Iteration {it+1:3d} | Best Length: {best_length:.2f}')

    print("\n✓ Optimization Complete!")
    print(f'Best Tour Length: {best_length:.2f}')
    print(f'Best Tour: {best_tour}')
    return best_tour, best_length, cities

# -----
# RUN
# -----
if __name__ == "__main__":
    best_tour, best_length, cities = aco_tsp()

```

Output:

```

Starting ACO for 5 cities with 10 ants...
Iteration 1/500 Best Length = 9.89
Iteration 2/500 Best Length = 9.89
Iteration 3/500 Best Length = 9.89
Iteration 4/500 Best Length = 9.89
Iteration 5/500 Best Length = 9.89
Iteration 6/500 Best Length = 9.89
Iteration 7/500 Best Length = 9.89
Iteration 8/500 Best Length = 9.89
Iteration 9/500 Best Length = 9.89
Iteration 10/500 Best Length = 9.89
Iteration 11/500 Best Length = 9.89
Iteration 12/500 Best Length = 9.89
Iteration 13/500 Best Length = 9.89
Iteration 14/500 Best Length = 9.89
Iteration 15/500 Best Length = 9.89
Iteration 16/500 Best Length = 9.89
Iteration 17/500 Best Length = 9.89
Iteration 18/500 Best Length = 9.89
Iteration 19/500 Best Length = 9.89
Iteration 20/500 Best Length = 9.89
Iteration 21/500 Best Length = 9.89
Iteration 22/500 Best Length = 9.89
Iteration 23/500 Best Length = 9.89
Iteration 24/500 Best Length = 9.89
Iteration 25/500 Best Length = 9.89
Iteration 26/500 Best Length = 9.89
Iteration 27/500 Best Length = 9.89
Iteration 28/500 Best Length = 9.89
Iteration 29/500 Best Length = 9.89
Iteration 30/500 Best Length = 9.89
Iteration 31/500 Best Length = 9.89
Iteration 32/500 Best Length = 9.89
Iteration 33/500 Best Length = 9.89
Iteration 34/500 Best Length = 9.89
Iteration 35/500 Best Length = 9.89
Iteration 36/500 Best Length = 9.89
Iteration 37/500 Best Length = 9.89
Iteration 38/500 Best Length = 9.89
Iteration 39/500 Best Length = 9.89
Iteration 40/500 Best Length = 9.89
Iteration 41/500 Best Length = 9.89
Iteration 42/500 Best Length = 9.89
Iteration 43/500 Best Length = 9.89
Iteration 44/500 Best Length = 9.89
Iteration 45/500 Best Length = 9.89
Iteration 46/500 Best Length = 9.89
Iteration 47/500 Best Length = 9.89
Iteration 48/500 Best Length = 9.89
Iteration 49/500 Best Length = 9.89
Iteration 50/500 Best Length = 9.89

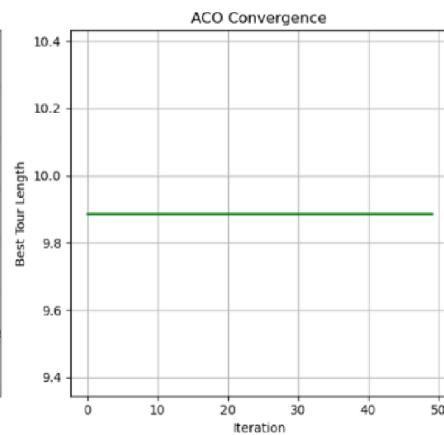
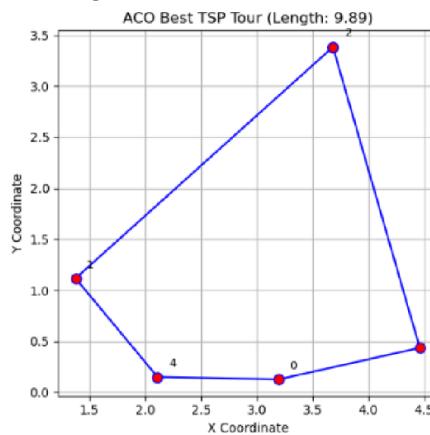
Results:
Cities: [(3.1971339922894186, 0.1250537711333468), (1.3751465918455963, 1.1160536907441139), (3.682356070820062, 3.3834974371145563), (4.46089783852422, 0.43469416314788076), (2.109649984263522, 0.1498609719035172)]
Best Tour (City Indices): [2, 3, 0, 4, 1]
Best Tour Length: 9.8880

```

```

--- Results ---
Cities: [(3.1971339922894186, 0.1250537711333468), (1.3751465918455963, 1.1160536907441139), (3.682356070820062, 3.3834974371145563), (4.46089783852422, 0.43469416314788076), (2.109649984263522, 0.1498609719035172)]
Best Tour (City Indices): [2, 3, 0, 4, 1]
Best Tour Length: 9.8880

```



Program 4

Problem Statement:

Particle Swarm Optimization for Function Optimization:

Apply Particle Swarm Optimization to find the optimal solution (minimum or maximum) of a given mathematical function.

Algorithm:

Week 4
Particle Swarm Optimization

Initialization

- 1 Function PSO (S, W, C-1, C-2)
- 2 Input : S = swarm size (no of particles)
- W = inertia weight
- C-1 = cognitive constant (personal)
- C-2 = social (global) constant
- 3 for each particle i = 1 to S
 - randomly initialize position (x_i) within search space
 - initialize velocity (v_i) randomly (often near zero)
- { personal best PBest_i ← x_i
- position Fit(PBest_i) ← f(x_i)
- GBest ← ParticleWithBestFitness (PBest)

PSO main loop

Repeat until max iterations :

// Update best positions

- for each particle i = 1 to S :
 - currentfitness ← f(x_i)
 - if currentfitness > Fit(PBest_i) :
 - PBest_i ← x_i
 - Fit(PBest_i) ← Currentfitness
- GBest ← ParticleWithBestFitness (PBest)

// Update velocity & position

- for each particle i = 1 to S :
 - random₁ ← Random (0, 1)
 - random₂ ← Random (0, 1)

// Update velocity

$$v_i^{t+1} = W \cdot v_i^t + C_1 \cdot random_1 \cdot (PBest_i - x_i^t) + C_2 \cdot random_2 \cdot (GBest - x_i^t)$$

// Update position

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$

Code:

```
import random
import math

# Define the mathematical function to optimize (minimize)
```

```

def objective_function(x, y):
    return x**2 + y**2 # Sphere function

# PSO parameters
num_particles = 30
max_iterations = 100
w = 0.7 # inertia weight
c1 = 1.5 # cognitive (particle) weight
c2 = 1.5 # social (swarm) weight

# Initialize particles
particles = []
for i in range(num_particles):
    x = random.uniform(-10, 10)
    y = random.uniform(-10, 10)
    velocity = [random.uniform(-1, 1), random.uniform(-1, 1)]
    fitness = objective_function(x, y)
    particles.append({
        "position": [x, y],
        "velocity": velocity,
        "best_pos": [x, y],
        "best_fit": fitness
    })

# Initialize global best
global_best = min(particles, key=lambda p: p["best_fit"])
global_best_pos = global_best["best_pos"]
global_best_fit = global_best["best_fit"]

# Main PSO loop
for t in range(max_iterations):
    for particle in particles:
        # Update velocity
        r1, r2 = random.random(), random.random()
        for i in range(2):
            cognitive = c1 * r1 * (particle["best_pos"][i] - particle["position"][i])
            social = c2 * r2 * (global_best_pos[i] - particle["position"][i])
            particle["velocity"][i] = w * particle["velocity"][i] + cognitive + social

        # Update position
        particle["position"][0] += particle["velocity"][0]
        particle["position"][1] += particle["velocity"][1]

    # Evaluate new fitness
    fit = objective_function(particle["position"][0], particle["position"][1])

    # Update personal best

```

```

if fit < particle["best_fit"]:
    particle["best_fit"] = fit
    particle["best_pos"] = particle["position"][:]

# Update global best
if fit < global_best_fit:
    global_best_fit = fit
    global_best_pos = particle["position"][:]

print(f"Iteration {t+1}/{max_iterations} - Best Fitness: {global_best_fit:.6f}")

print("\n✓ Optimization complete!")
print(f"Best position found: {global_best_pos}")
print(f"Minimum value of function: {global_best_fit:.6f}")

```

Output:

```

Starting PSO for 2 dimensions with 10 particles...
Initial GBest Score: 4.0825
Iteration 1/100: GBest Score = 4.0825e+00
Iteration 2/100: GBest Score = 2.3789e+00
Iteration 3/100: GBest Score = 2.1673e+00
Iteration 4/100: GBest Score = 2.1673e+00
Iteration 5/100: GBest Score = 1.4520e+00
Iteration 6/100: GBest Score = 1.2043e+00
Iteration 7/100: GBest Score = 9.8974e-01
Iteration 8/100: GBest Score = 5.5153e-02
Iteration 9/100: GBest Score = 9.0545e-03
Iteration 10/100: GBest Score = 9.0545e-03
Iteration 11/100: GBest Score = 9.0545e-03
Iteration 12/100: GBest Score = 9.0545e-03
Iteration 13/100: GBest Score = 9.0545e-03
Iteration 14/100: GBest Score = 8.0772e-03
Iteration 15/100: GBest Score = 8.0772e-03
Iteration 16/100: GBest Score = 8.0772e-03
Iteration 17/100: GBest Score = 7.3521e-03
Iteration 18/100: GBest Score = 7.3521e-03
Iteration 19/100: GBest Score = 4.1888e-03
Iteration 20/100: GBest Score = 4.1888e-03
Iteration 21/100: GBest Score = 1.0079e-03
Iteration 22/100: GBest Score = 1.0079e-03
Iteration 23/100: GBest Score = 1.0079e-03
Iteration 24/100: GBest Score = 1.0079e-03
Iteration 25/100: GBest Score = 1.0079e-03
Iteration 26/100: GBest Score = 3.8768e-04
Iteration 27/100: GBest Score = 1.8362e-04
Iteration 28/100: GBest Score = 1.8362e-04
Iteration 29/100: GBest Score = 7.5330e-05
Iteration 30/100: GBest Score = 7.5330e-05
Iteration 31/100: GBest Score = 7.5330e-05
Iteration 32/100: GBest Score = 7.5330e-05
Iteration 33/100: GBest Score = 7.5330e-05
Iteration 34/100: GBest Score = 7.5330e-05
Iteration 35/100: GBest Score = 7.5330e-05
Iteration 36/100: GBest Score = 7.5330e-05
Iteration 37/100: GBest Score = 7.5330e-05
Iteration 38/100: GBest Score = 7.5330e-05
Iteration 39/100: GBest Score = 7.5330e-05
Iteration 40/100: GBest Score = 7.5330e-05
Iteration 41/100: GBest Score = 7.5330e-05
Iteration 42/100: GBest Score = 7.5330e-05
Iteration 43/100: GBest Score = 4.9330e-05
Iteration 44/100: GBest Score = 4.9330e-05
Iteration 45/100: GBest Score = 3.2129e-05
Iteration 46/100: GBest Score = 3.2129e-05
Iteration 47/100: GBest Score = 3.2129e-05
Iteration 48/100: GBest Score = 8.4985e-06
Iteration 49/100: GBest Score = 1.3416e-06
Iteration 50/100: GBest Score = 1.3416e-06
Iteration 51/100: GBest Score = 1.3416e-06
Iteration 52/100: GBest Score = 1.3416e-06
Iteration 53/100: GBest Score = 1.3416e-06
Iteration 54/100: GBest Score = 1.3416e-06
Iteration 55/100: GBest Score = 1.3416e-06
Iteration 56/100: GBest Score = 1.3416e-06
Iteration 57/100: GBest Score = 1.3416e-06
Iteration 58/100: GBest Score = 1.3416e-06
Iteration 59/100: GBest Score = 3.1882e-07
Iteration 60/100: GBest Score = 3.1882e-07
Iteration 61/100: GBest Score = 3.1882e-07

```

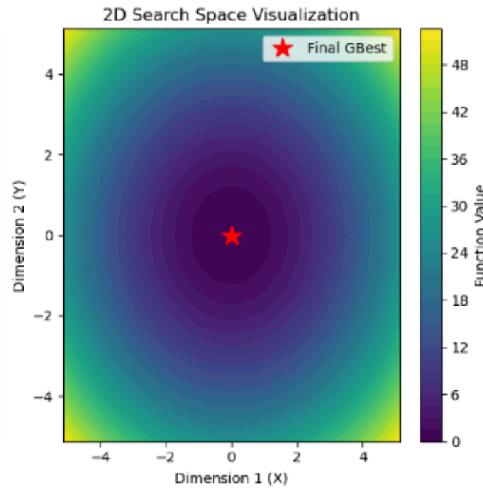
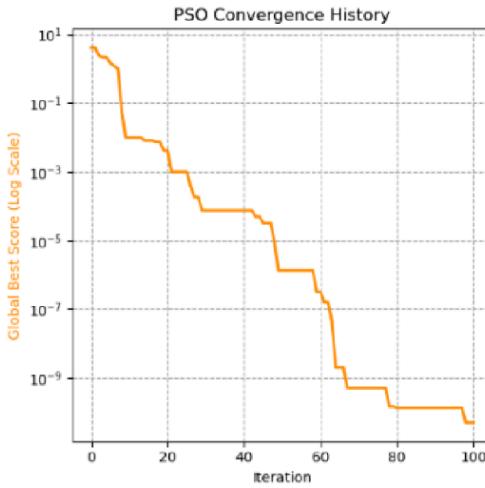
```

Iteration 72/100: GBest Score = 5.0642e-10
Iteration 73/100: GBest Score = 5.0642e-10
Iteration 74/100: GBest Score = 5.0642e-10
Iteration 75/100: GBest Score = 5.0642e-10
Iteration 76/100: GBest Score = 5.0642e-10
Iteration 77/100: GBest Score = 5.0642e-10
Iteration 78/100: GBest Score = 1.4724e-10
Iteration 79/100: GBest Score = 1.4724e-10
Iteration 80/100: GBest Score = 1.3409e-10
Iteration 81/100: GBest Score = 1.3409e-10
Iteration 82/100: GBest Score = 1.3409e-10
Iteration 83/100: GBest Score = 1.3409e-10
Iteration 84/100: GBest Score = 1.3409e-10
Iteration 85/100: GBest Score = 1.3409e-10
Iteration 86/100: GBest Score = 1.3409e-10
Iteration 87/100: GBest Score = 1.3409e-10
Iteration 88/100: GBest Score = 1.3409e-10
Iteration 89/100: GBest Score = 1.3409e-10
Iteration 90/100: GBest Score = 1.3409e-10
Iteration 91/100: GBest Score = 1.3409e-10
Iteration 92/100: GBest Score = 1.3409e-10
Iteration 93/100: GBest Score = 1.3409e-10
Iteration 94/100: GBest Score = 1.3409e-10
Iteration 95/100: GBest Score = 1.3409e-10
Iteration 96/100: GBest Score = 1.3409e-10
Iteration 97/100: GBest Score = 1.3409e-10
Iteration 98/100: GBest Score = 4.9838e-11
Iteration 99/100: GBest Score = 4.9838e-11
Iteration 100/100: GBest Score = 4.9838e-11

```

--- Results ---

Objective Function: Sphere Function
 Best Position Found: [5.786318486e-05 -1.94488851e-06]
 Minimum Score (Fitness): 4.983761e-11



Program 5

Problem Statement:

Cuckoo Search (CS) for travel salesman problem :

Use the Cuckoo Search Algorithm to determine the shortest route for a salesman to visit all cities and return to the starting point.

Algorithm:

17 10 25
Week 5
Cuckoo Search Algorithm

1. Set 1 Initialization
2. set parameters of your algorithm:
 n = no. of host nests
 x_i ($i = 1, 2, 3, \dots, n$)
 $P(a)$ = probability of discovering the cuckoo's egg
3. M_{ant} = maximum no. of iteration
4. Set $t = 0$ {counter initialization}
5. For ($i = 1 : i \leq n$) do
6. Generate initial population of n host x_i^t
7. Evaluate fitness function $f(x_i^t)$
8. End for
9. Generate a new solution (cuckoo) x_i^{t+1} randomly by Levy flight.
10. Evaluate fitness function $f(x_i^{t+1})$
11. Choose a nest x_j among n solutions ready
12. If $[f(x_i^{t+1}) > f(x_j^t)]$ then
13. Replace the solution x_j with the solution x_i^{t+1}
14. End if
15. Abandon a fraction P_a of worst nest
16. Build a new nest at new location Levy flight a fraction P_a of worst nest
17. Keep the best solution
18. Rank the solution and find current best solution
19. Set $t = t + 1$
20. Until ($t \geq M_{\text{ant}}$)
19. Produce best solution
Output = Best Nest x_i

Code:

```
import random
import math

# -----
# Helper functions
# -----


def distance(city1, city2):
    """Euclidean distance between two cities"""
    return math.sqrt((city1[0] - city2[0])**2 + (city1[1] - city2[1])**2)

def total_distance(path, cities):
    """Compute total distance of the path"""
    dist = 0
    for i in range(len(path)):
        city1 = cities[path[i]]
        city2 = cities[path[(i + 1) % len(path)]]
        dist += distance(city1, city2)
    return dist

def levy_flight(Lambda):
    """Generate step size using Lévy distribution"""
    sigma = (math.gamma(1 + Lambda) * math.sin(math.pi * Lambda / 2) /
             (math.gamma((1 + Lambda) / 2) * Lambda * 2**((Lambda - 1) / 2)))**(1 / Lambda)
    u = random.gauss(0, sigma)
    v = random.gauss(0, 1)
    step = u / abs(v)**(1 / Lambda)
    return step

def get_new_solution(path):
    """Generate a new random permutation by swapping two cities"""
    new_path = path[:]
    i, j = random.sample(range(len(new_path)), 2)
    new_path[i], new_path[j] = new_path[j], new_path[i]
    return new_path

# -----
# Cuckoo Search Algorithm
# -----


def cuckoo_search_tsp(cities, n_nests=15, pa=0.25, alpha=1, Lambda=1.5, max_iter=100):
    # Initialize nests (random permutations)
    nests = [random.sample(range(len(cities)), len(cities)) for _ in range(n_nests)]
    fitness = [total_distance(nest, cities) for nest in nests]

    best_nest = nests[fitness.index(min(fitness))]
```

```

best_fitness = min(fitness)

for iteration in range(max_iter):
    for i in range(n_nests):
        # Lévy flight (small random move)
        step_size = levy_flight(Lambda)
        new_nest = get_new_solution(nests[i])

        new_fitness = total_distance(new_nest, cities)

        # If better, replace
        if new_fitness < fitness[i]:
            nests[i] = new_nest
            fitness[i] = new_fitness

        # Update best
        if new_fitness < best_fitness:
            best_nest = new_nest[:]
            best_fitness = new_fitness

    # Abandon some nests with probability pa
    for i in range(n_nests):
        if random.random() < pa:
            nests[i] = random.sample(range(len(cities)), len(cities))
            fitness[i] = total_distance(nests[i], cities)

    print(f"Iteration {iteration+1}/{max_iter} - Best Distance: {best_fitness:.4f}")

return best_nest, best_fitness

# -----
# Example Run
# -----


if __name__ == "__main__":
    # Example: 10 cities with random coordinates
    cities = [(random.uniform(0, 100), random.uniform(0, 100)) for _ in range(10)]

    best_path, best_distance = cuckoo_search_tsp(cities, n_nests=20, max_iter=100)

    print("\n✓ Best Path Found:")
    print(best_path)
    print(f"Total Distance: {best_distance:.4f}")

```

Output:

Iteration 61/100 - Best Distance: 320.2649
Iteration 62/100 - Best Distance: 320.2649
Iteration 63/100 - Best Distance: 320.2649
Iteration 64/100 - Best Distance: 320.2649
Iteration 65/100 - Best Distance: 320.2649
Iteration 66/100 - Best Distance: 320.2649
Iteration 67/100 - Best Distance: 320.2649
Iteration 68/100 - Best Distance: 320.2649
Iteration 69/100 - Best Distance: 320.2649
Iteration 70/100 - Best Distance: 320.2649
Iteration 71/100 - Best Distance: 320.2649
Iteration 72/100 - Best Distance: 320.2649
Iteration 73/100 - Best Distance: 320.2649
Iteration 74/100 - Best Distance: 320.2649
Iteration 75/100 - Best Distance: 320.2649
Iteration 76/100 - Best Distance: 320.2649
Iteration 77/100 - Best Distance: 320.2649
Iteration 78/100 - Best Distance: 320.2649
Iteration 79/100 - Best Distance: 320.2649
Iteration 80/100 - Best Distance: 320.2649
Iteration 81/100 - Best Distance: 320.2649
Iteration 82/100 - Best Distance: 320.2649
Iteration 83/100 - Best Distance: 320.2649
Iteration 84/100 - Best Distance: 320.2649
Iteration 85/100 - Best Distance: 320.2649
Iteration 86/100 - Best Distance: 320.2649
Iteration 87/100 - Best Distance: 320.2649
Iteration 88/100 - Best Distance: 320.2649
Iteration 89/100 - Best Distance: 320.2649
Iteration 90/100 - Best Distance: 320.2649
Iteration 91/100 - Best Distance: 320.2649
Iteration 92/100 - Best Distance: 320.2649
Iteration 93/100 - Best Distance: 320.2649
Iteration 94/100 - Best Distance: 320.2649
Iteration 95/100 - Best Distance: 320.2649
Iteration 96/100 - Best Distance: 320.2649
Iteration 97/100 - Best Distance: 320.2649
Iteration 98/100 - Best Distance: 320.2649
Iteration 99/100 - Best Distance: 320.2649
Iteration 100/100 - Best Distance: 320.2649

Best Path Found:
[1, 8, 3, 5, 4, 0, 6, 9, 7, 2]
Total Distance: 320.2649

Program 6

Problem Statement:

Grey Wolf Optimizer (GWO) for Function Optimization:

Use Grey Wolf Optimization to find the best solution for a mathematical function by mimicking the hunting behavior of grey wolves.

Algorithm:

24 10 25
Week-6
11 17

C Grey Wolf Optimization Algorithm

1 Input $f(x)$ → objective function to be minimized
 n → no of wolves (population size)
 2 dim → dimension of the problem
 MaxIter → max no of iterations
 lb, ub → lower & upper bounds of search space
 Output : x_{alpha} → best (alpha) solution found

1. Initialize the population x_i ($i=1 \text{ to } n$) randomly within bounds [lb, ub]

2. Evaluate fitness (x_i) for each wolf

3. Identify :

- 6 $x_{\text{alpha}} = \text{best wolf}$ (lowest fitness)
- 7 $x_{\text{beta}} = \text{second best wolf}$
- 8 $x_{\text{delta}} = \text{third best wolf}$

4. For $t=1$ to MaxIter do:

a = $2 - (2 * t / \text{MaxIter})$

For each wolf i in population:

For each dimension j :

r1 = random(0, 1)
 r2 = random(0, 1)

A1 = $2 * a * r1 - a$
 C1 = $2 * r2$

$D_{\text{alpha}} = |C1 * x_{\text{alpha}}[j] - x_i[j]|$
 $x_i = x_{\text{alpha}}[j] - A1 * D_{\text{alpha}}$

r1 = random(0, 1)
 r2 = random(0, 1)

A2 = $2 * a * r1 - a$
 C2 = $2 * r2$

$D_{\text{beta}} = |C2 * x_{\text{beta}}[j] - x_i[j]|$
 $x_i = x_{\text{beta}}[j] - A2 * D_{\text{beta}}$

$r1 = \text{random}(0, 1)$
 $r2 = \text{random}(0, 1)$

A3 = $2 * a * r1 - a$
 C3 = $2 * r2$

$D_{\text{delta}} = |C3 * x_{\text{delta}}[j] - x_i[j]|$
 $x_i = x_{\text{delta}}[j] - A3 * D_{\text{delta}}$

$x_i_{\text{new}}[j] = (x_i + x_{\text{beta}} + x_{\text{delta}}) / 3$

→ Enforce boundary limits on x_i_{new}
 → Update each wolf's position $x_i = x_i_{\text{new}}$
 → Evaluate fitness $f(x_i)$ for all wolves
 → Update $x_{\text{alpha}}, x_{\text{beta}}, x_{\text{delta}}$ based on new fitness values

5. Return x_{alpha} as the best solution found

Code:

```
import random
import math
```

```

# -----
# Objective Function (to minimize)
# -----
def objective_function(position):
    # Sphere function: sum of squares
    return sum(x**2 for x in position)

# -----
# Grey Wolf Optimizer
# -----
def grey_wolf_optimizer(num_wolves=20, dim=2, max_iter=100, lower_bound=-10,
upper_bound=10):
    # Initialize the population of wolves randomly
    wolves = [[random.uniform(lower_bound, upper_bound) for _ in range(dim)] for _ in
range(num_wolves)]
    fitness = [objective_function(w) for w in wolves]

    # Initialize alpha, beta, delta (best 3 wolves)
    alpha, beta, delta = [0]*dim, [0]*dim, [0]*dim
    alpha_score, beta_score, delta_score = float("inf"), float("inf"), float("inf")

    # Main loop
    for t in range(max_iter):
        for i in range(num_wolves):
            score = fitness[i]

            # Update alpha, beta, delta
            if score < alpha_score:
                delta_score, delta = beta_score, beta[:]
                beta_score, beta = alpha_score, alpha[:]
                alpha_score, alpha = score, wolves[i][:]
            elif score < beta_score:
                delta_score, delta = beta_score, beta[:]
                beta_score, beta = score, wolves[i][:]
            elif score < delta_score:
                delta_score, delta = score, wolves[i][:]

        # Linearly decreasing a from 2 to 0
        a = 2 - t * (2 / max_iter)

        # Update each wolf's position
        for i in range(num_wolves):
            new_position = []
            for j in range(dim):
                r1, r2 = random.random(), random.random()
                A1 = 2 * a * r1 - a
                C1 = 2 * r2

```

```

D_alpha = abs(C1 * alpha[j] - wolves[i][j])
X1 = alpha[j] - A1 * D_alpha

r1, r2 = random.random(), random.random()
A2 = 2 * a * r1 - a
C2 = 2 * r2
D_beta = abs(C2 * beta[j] - wolves[i][j])
X2 = beta[j] - A2 * D_beta

r1, r2 = random.random(), random.random()
A3 = 2 * a * r1 - a
C3 = 2 * r2
D_delta = abs(C3 * delta[j] - wolves[i][j])
X3 = delta[j] - A3 * D_delta

new_x = (X1 + X2 + X3) / 3
# Keep within bounds
new_x = max(lower_bound, min(upper_bound, new_x))
new_position.append(new_x)

wolves[i] = new_position
fitness[i] = objective_function(new_position)

print(f"Iteration {t+1}/{max_iter} - Best fitness: {alpha_score:.6f}")

return alpha, alpha_score

# -----
# Example Run
# -----
if __name__ == "__main__":
    best_position, best_score = grey_wolf_optimizer(num_wolves=20, dim=5, max_iter=100)
    print("\n✓ Optimization complete!")
    print(f"Best position found: {best_position}")
    print(f"Minimum value of function: {best_score:.6f}")

```

Output:

```
Iteration 1/20 → Best fitness: 6.462624
Iteration 2/20 → Best fitness: 6.060991
▼   ...
Iteration 3/20 → Best fitness: 0.000334
Iteration 4/20 → Best fitness: 0.000334
Iteration 5/20 → Best fitness: 0.000334
Iteration 6/20 → Best fitness: 0.000334
Iteration 7/20 → Best fitness: 0.000334
Iteration 8/20 → Best fitness: 0.000334
Iteration 9/20 → Best fitness: 0.000334
Iteration 10/20 → Best fitness: 0.000334
Iteration 11/20 → Best fitness: 0.000000
Iteration 12/20 → Best fitness: 0.000000
Iteration 13/20 → Best fitness: 0.000000
Iteration 14/20 → Best fitness: 0.000000
Iteration 15/20 → Best fitness: 0.000000
Iteration 16/20 → Best fitness: 0.000000
Iteration 17/20 → Best fitness: 0.000000
Iteration 18/20 → Best fitness: 0.000000
Iteration 19/20 → Best fitness: 0.000000
Iteration 20/20 → Best fitness: 0.000000

Best Solution Found:
x = [1.99995979]
Fitness = 1.6169416916511636e-09
```

Program 7

Problem Statement:

Parallel Cellular Algorithm for Function Optimization:

Use a Parallel Cellular Algorithm to optimize a function efficiently by evolving solutions in a distributed and parallel manner.

Algorithm:

24.10.25
Week 7
Parallel Cellular Algorithm

Input: $f(x)$ → objective function to optimize
grid_size → size of grid
num_cells → total no of cells
max_iterations → max no of iterations
neighborhood → neighborhood structure
lb, ub → lower & upper bounds of search space

Output: best_solution → best solution found

1. Initialize : Create a 2D grid of cells
For each cell i in grid :
Assign a random value x_i within $[lb, ub]$

2. Evaluate fitness :
For each cell i :
 $\text{fitness}[i] = f(x_i)$

3. Identify best solution
best_solution = cell with minimum / maximum fitness

4. For iter = 1 to max_iterations do :
For each cell i in grid (in parallel) :
neighbors = cells in neighborhood of i
best_neighbor = neighbor with best fitness
 $x_{i_new} = (x_i + \text{best_neighbor}.value) / 2$
→ enforce boundaries : ensure $x_{i_new} \in [lb, ub]$
→ Update all cells simultaneously :
 $x_i = x_{i_new}$

S. Bhatti
24.10.25

Code:

```
import numpy as np
```

```

def parallel_cellular_algorithm(f, grid_size=(10, 10), max_iterations=100, lb=-10, ub=10):
    rows, cols = grid_size
    num_cells = rows * cols

    # 1. Initialize grid with random values
    X = np.random.uniform(lb, ub, size=(rows, cols))

    # Evaluate fitness
    fitness = f(X)

    # Track best global solution
    best_value = np.min(fitness)
    best_pos = np.unravel_index(np.argmin(fitness), fitness.shape)
    best_solution = X[best_pos]

    # Neighborhood offsets for 3x3 neighborhood
    neighbors_offset = [(-1,-1), (-1,0), (-1,1),
                        (0,-1), (0,0), (0,1),
                        (1,-1), (1,0), (1,1)]

    for iteration in range(max_iterations):
        X_new = np.copy(X)

        # 4. Update each cell in parallel logic
        for i in range(rows):
            for j in range(cols):
                # Collect neighbors
                neighbor_values = []
                for dx, dy in neighbors_offset:
                    ni, nj = i + dx, j + dy
                    if 0 <= ni < rows and 0 <= nj < cols:
                        neighbor_values.append((X[ni, nj], fitness[ni, nj]))

                # Get best neighbor (minimum fitness)
                best_neighbor_value, _ = min(neighbor_values, key=lambda x: x[1])

                # Update rule: average
                X_new[i, j] = (X[i, j] + best_neighbor_value) / 2

        # Boundary enforcement
        X_new = np.clip(X_new, lb, ub)

        # Replace old values
        X = X_new

        # Recalculate fitness
        fitness = f(X)

```

```

# Update global best
current_best = np.min(fitness)
if current_best < best_value:
    best_value = current_best
    best_pos = np.unravel_index(np.argmin(fitness), fitness.shape)
    best_solution = X[best_pos]

return best_solution, best_value

# -----
# Example: Optimize Sphere Function f(x) = x^2
# -----


def sphere_function(x):
    return x**2 # works element-wise

best_x, best_val = parallel_cellular_algorithm(
    f=sphere_function,
    grid_size=(10, 10),
    max_iterations=100,
    lb=-5,
    ub=5
)

print("Best solution:", best_x)
print("Best function value:", best_val)

```

Output:

```
[4] ✓ Os   new_cells = np.clip(new_cells, lb, ub)
          cells = new_cells
          fitness = obj_func(cells)
          current_best = np.min(fitness)
          if current_best < best_value:
              best_value = current_best
              best_position = np.unravel_index(np.argmin(fitness), fitness.shape)
          if (t + 1) % 10 == 0 or t == 0:
              print(f"Iteration {t+1}/{max_iter} → Best fitness: {best_value:.6f}")
          print("\nBest solution found:")
          print(f"x = {cells[best_position]:.6f}, fitness = {best_value:.6f}")
          return cells[best_position], best_value
      best_x, best_fit = parallel_cellular_algorithm(
          objective_function,
          grid_size=(10, 10),
          lb=-10,
          ub=10,
          max_iter=60
      )
...
... Iteration 1/60 → Best fitness: 0.001454
Iteration 10/60 → Best fitness: 0.000000
Iteration 20/60 → Best fitness: 0.000000
Iteration 30/60 → Best fitness: 0.000000
Iteration 40/60 → Best fitness: 0.000000
Iteration 50/60 → Best fitness: 0.000000
Iteration 60/60 → Best fitness: 0.000000

Best solution found:
x = 2.000000, fitness = 0.000000
```

