GPU Programming for Video Games

3D to 2D Projection

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(Based on slides by Prof. Hsien-Hsin Sean Lee)
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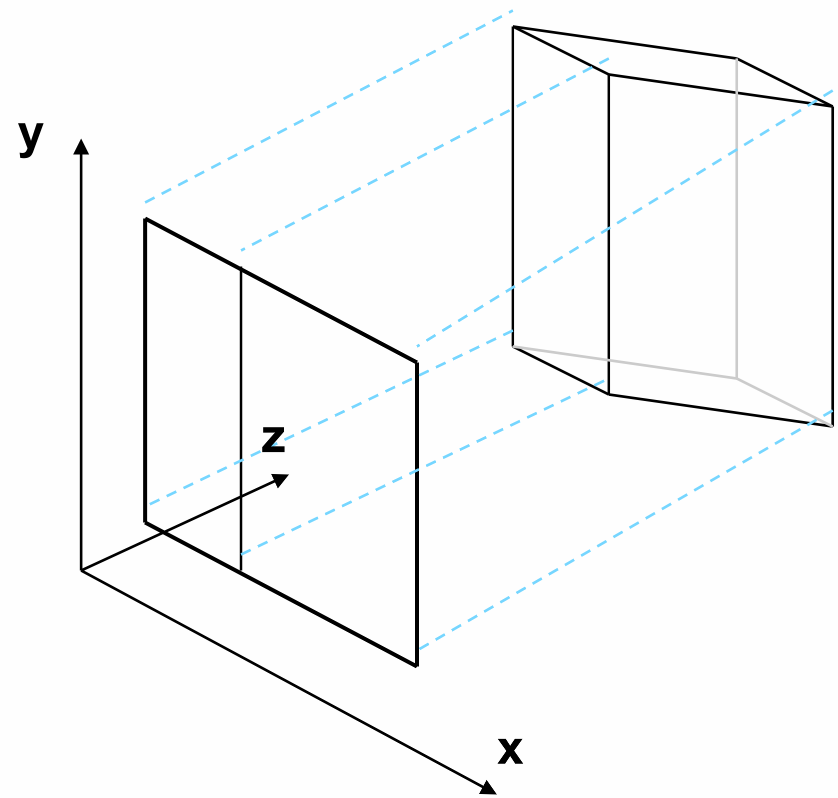
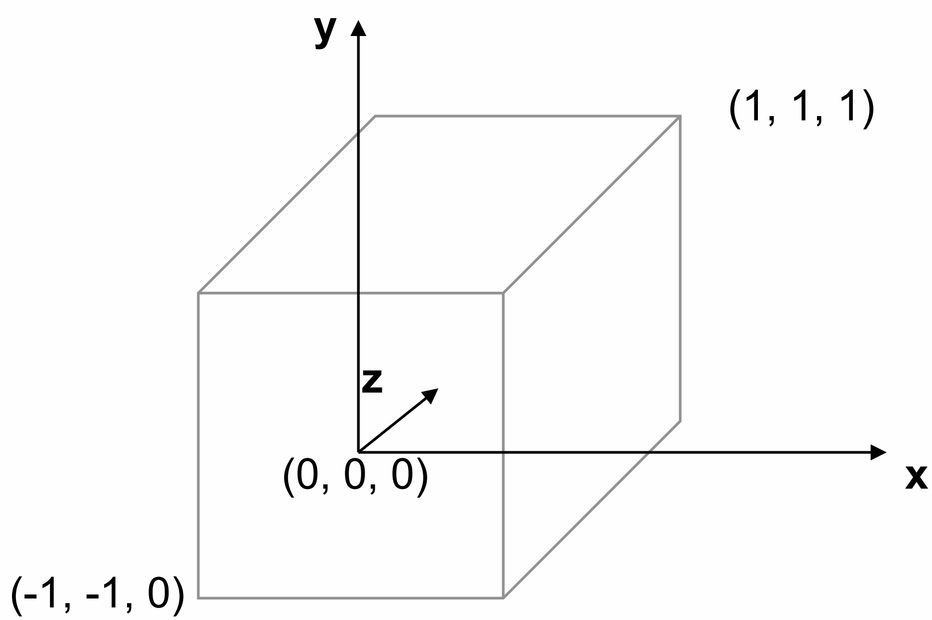
Projection from 3D space



Much discussion adapted from Joe Farrell's article:
http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__1/)

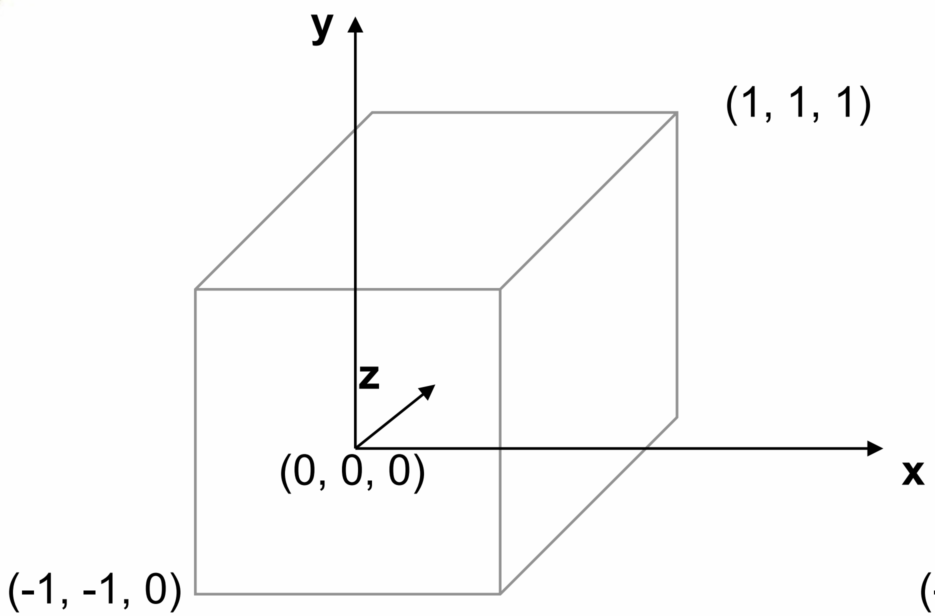
Canonical view volume

- Projection transforms your geometry into a canonical view volume in *normalized device coordinates ("clip space")*
- Only X- and Y-coordinates will be mapped onto the screen
- Z will be almost useless, but used for depth test



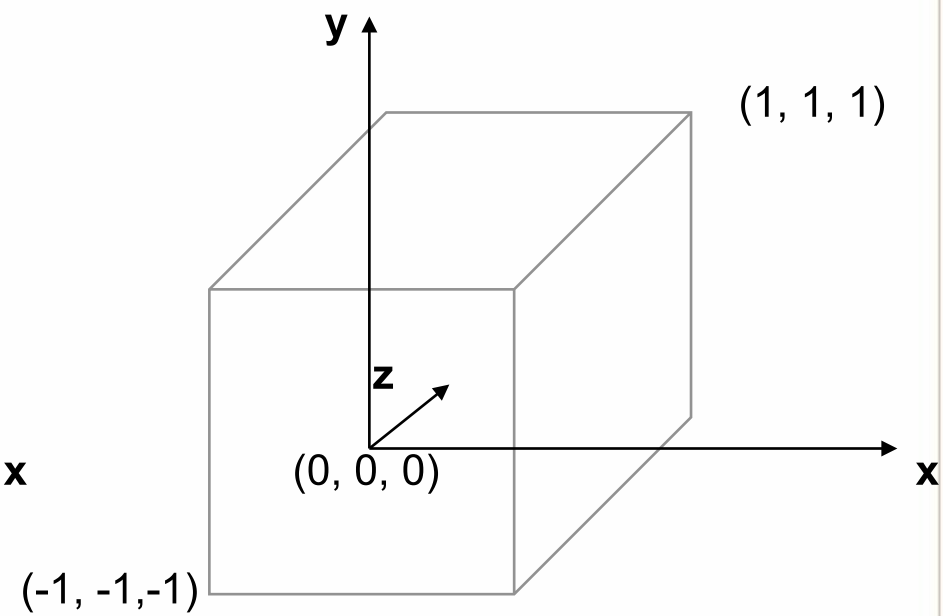
Canonical view volume (LHS)
(-1, -1, 0) to (1,1,1) used by Direct3D

Strange "conventions"



Canonical "clips space" view volume (LHS)
(-1, -1, 0) to (1,1,1) used by
D3D/XNA

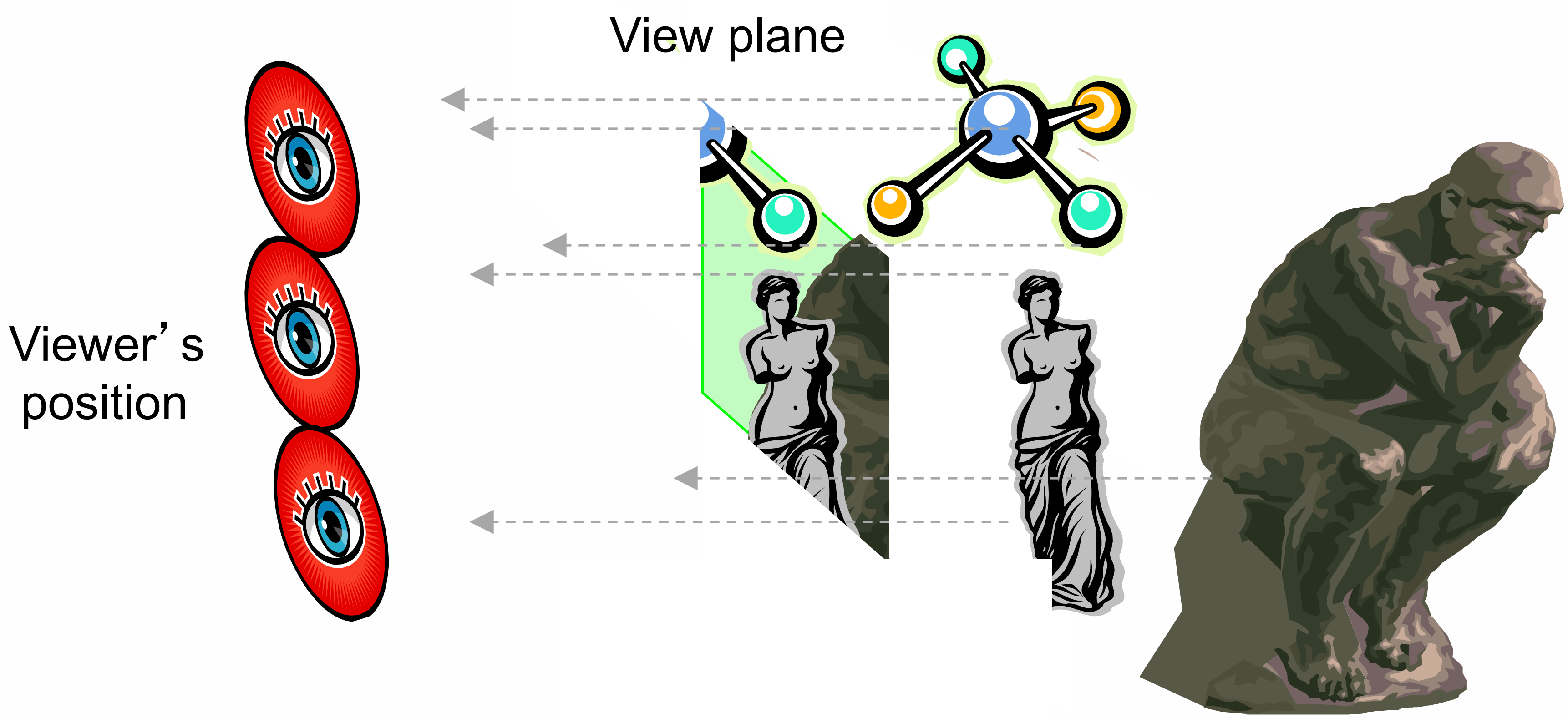
(remember unnormalized
eye-space coordinates in
Direct3D are in a LHS,
but in XNA are in a RHS!!!)



Canonical "clip space" view volume (LHS)
(-1, -1, 1) to (1,1,1) used by
OpenGL/Unity

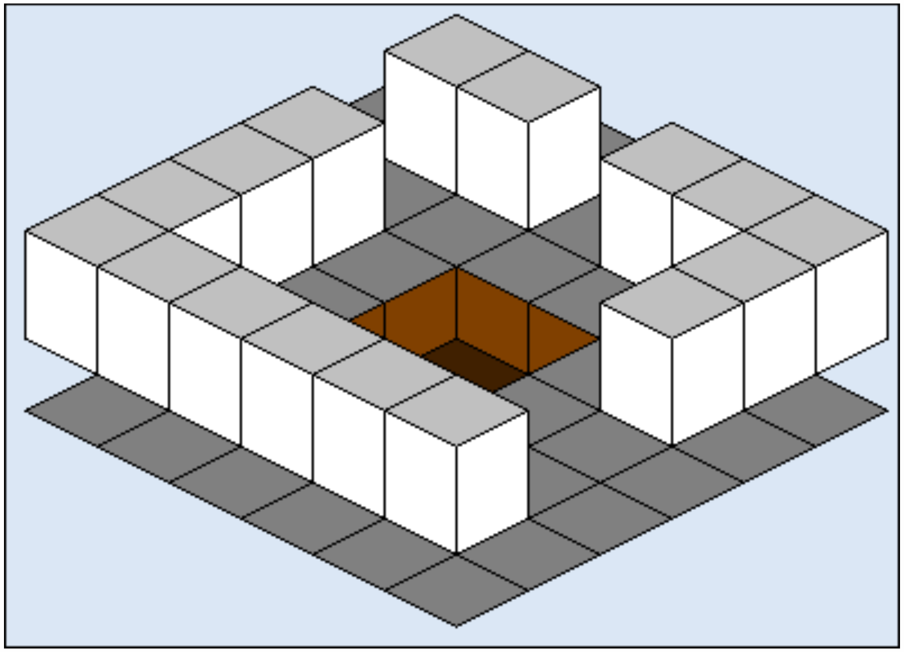
(remember unnormalized
eye-space coordinates in
OpenGL are in a RHS,
but in Unity are in a LHS!)

Orthographic (or parallel) projection



- Project from 3D space to the viewer's 2D space

Style of orthographic projection



- Same size in 2D and 3D
- No sense of distance
- Parallel lines remain parallel
- Good for tile-based games where camera is in fixed location (e.g., Mahjong or 3D Tetris)

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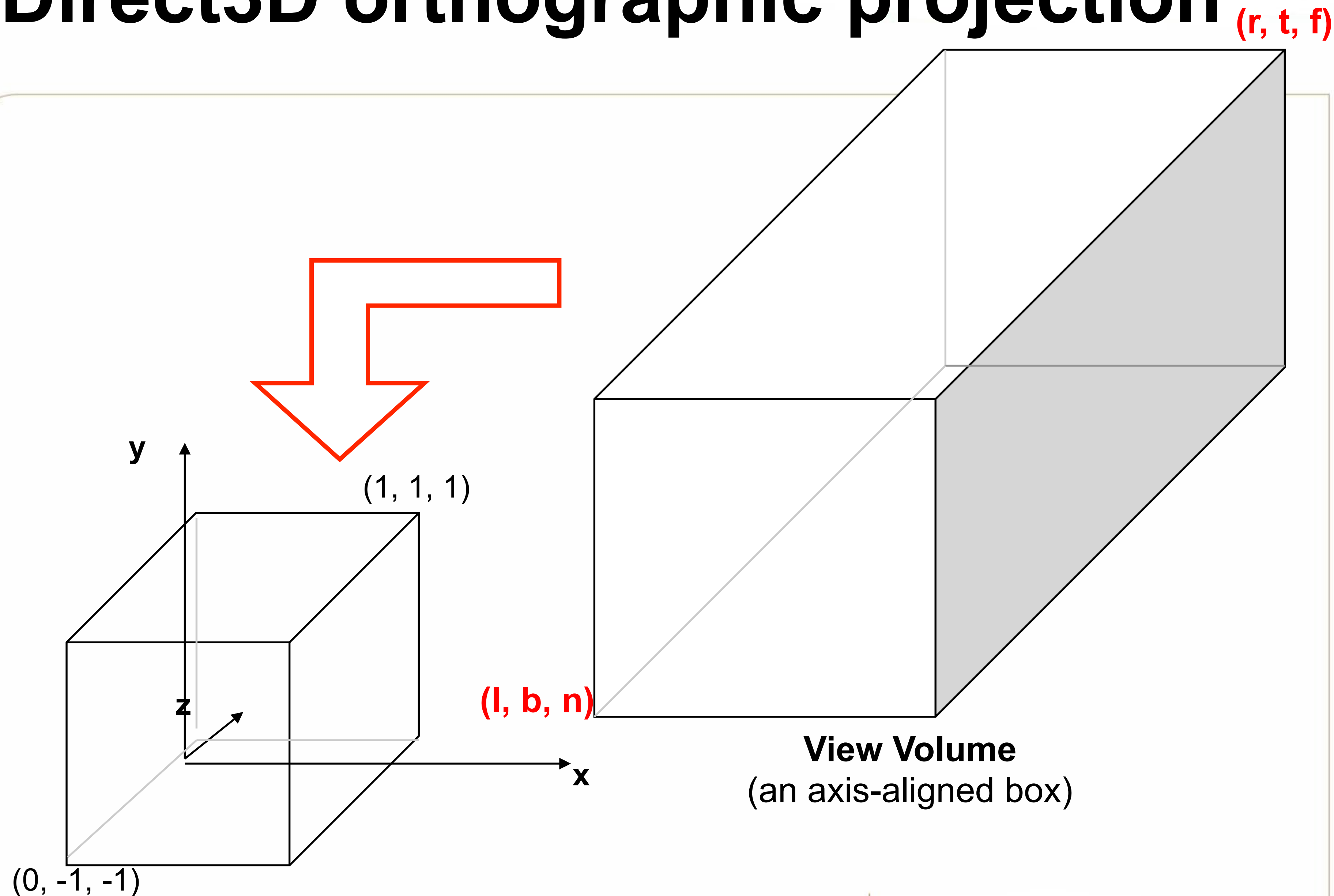
See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

Direct3D orthographic projection



Canonical view volume (Direct3D)

View Volume
(an axis-aligned box)

General orthographic math

- Derive x’ and y’

$$x \in [l, r] \quad x' \in [-1, 1]$$

$$l \le x \le r$$

$$0 \le x - l \le r - l$$

$$0 \le \frac{x-l}{r-l} \le 1$$

$$0 \le \frac{2(x-l)}{r-l} \le 2$$

$$-1 \le \frac{2(x-l)}{r-l} - 1 \le 1$$

$$-1 \le \frac{2x - 2l - r + l}{r-l} \le 1$$

$$-1 \le \frac{2x}{r-l} - \frac{r+l}{r-l} \le 1$$

$$\boxed{\therefore \quad x' = \frac{2x}{r-l} - \frac{r+l}{r-l}}$$

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

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D3D orthographic math for Z (LHS default)

- Derive z'

$$z \in [n, f] \qquad z' \in [0, 1]$$

$$n \le z \le f$$

$$0 \le z - n \le f - n$$

$$0 \le \frac{z - n}{f - n} \le 1$$

$$0 \le \frac{z}{f - n} - \frac{n}{f - n} \le 1$$

$$\boxed{\therefore \quad z' = \frac{z}{f - n} - \frac{n}{f - n}}$$

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

D3D orthographic results (LHS)

$$\boxed{x' = \frac{2x}{r-l} - \frac{r+l}{r-l}}$$

$$\boxed{y' = \frac{2y}{t-b} - \frac{t+b}{t-b}}$$

$$\boxed{z' = \frac{z}{f-n} - \frac{n}{f-n}}$$

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D3D orthographic matrix (LHS default) ✓

- Direct3D primarily uses LHS, z from 0 to 1, row vectors

$$[x',y',z',1]=[x,y,z,1]P \quad \text{where } P=\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{1}{f-n} & 0 \\ \frac{l+r}{l-r} & \frac{t+b}{b-t} & \frac{n}{n-f} & 1 \end{bmatrix}$$

- In Direct3D: D3DXMatrixOrthoOffCenterLH(*o,l,r,b,t,n,f)

http://msdn.microsoft.com/en-us/library/bb205347(VS.85).aspx

D3D orthographic math for Z (RHS weird)

- For RHS, in most API calls z clip parameters are positive, and clip space switches to using a LHS
- Derive z’

$$-z \in [n, f] \qquad z' \in [0,\ 1]$$

$$n \le -z \le f$$

$$0 \le -z - n \le f - n$$

$$0 \le \frac{-z - n}{f - n} \le 1$$

$$0 \le \frac{-z}{f-n} - \frac{n}{f-n} \le 1$$

$$\boxed{\therefore\ z' = \frac{-z}{f-n} - \frac{n}{f-n}}$$

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

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D3D orthographic matrix (RHS weird) ✓

$$[x', y', z', 1] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \boxed{\frac{1}{n-f}} & 0 \\ \frac{l+r}{l-r} & \frac{t+b}{b-t} & \frac{n}{n-f} & 1 \end{bmatrix}$$

- In Direct3D: D3DXMatrixOrthoOffCenterRH(*o,l,r,b,t,n,f)
- In XNA: Matrix.CreateOrthographicOffCenter(l,r,b,t,n,f)

http://msdn.microsoft.com/en-us/library/bb205348(VS.85).aspx
http://www.cs.utk.edu/~vose/c-stuff/opengl/glOrtho.html

Simpler D3D ortho matrix (LHS default) ✓

- Most orthographic projection setups
  - Z-axis passes through the center of your view volume
  - Field of view (FOV) extends equally far
    - To the *left* as to the *right* **(i.e., r = -l)**
    - To the *top* as to the *below* **(i.e., t=-b)**

$$[x', y', z', 1] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{2}{w} & 0 & 0 & 0 \\ 0 & \frac{2}{h} & 0 & 0 \\ 0 & 0 & \frac{1}{f-n} & 0 \\ 0 & 0 & \frac{n}{n-f} & 1 \end{bmatrix}$$

- In Direct3D: D3DXMatrixOrthoLH(*o,w,h,n,f)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

Simpler D3D ortho matrix (RHS weird)

✓

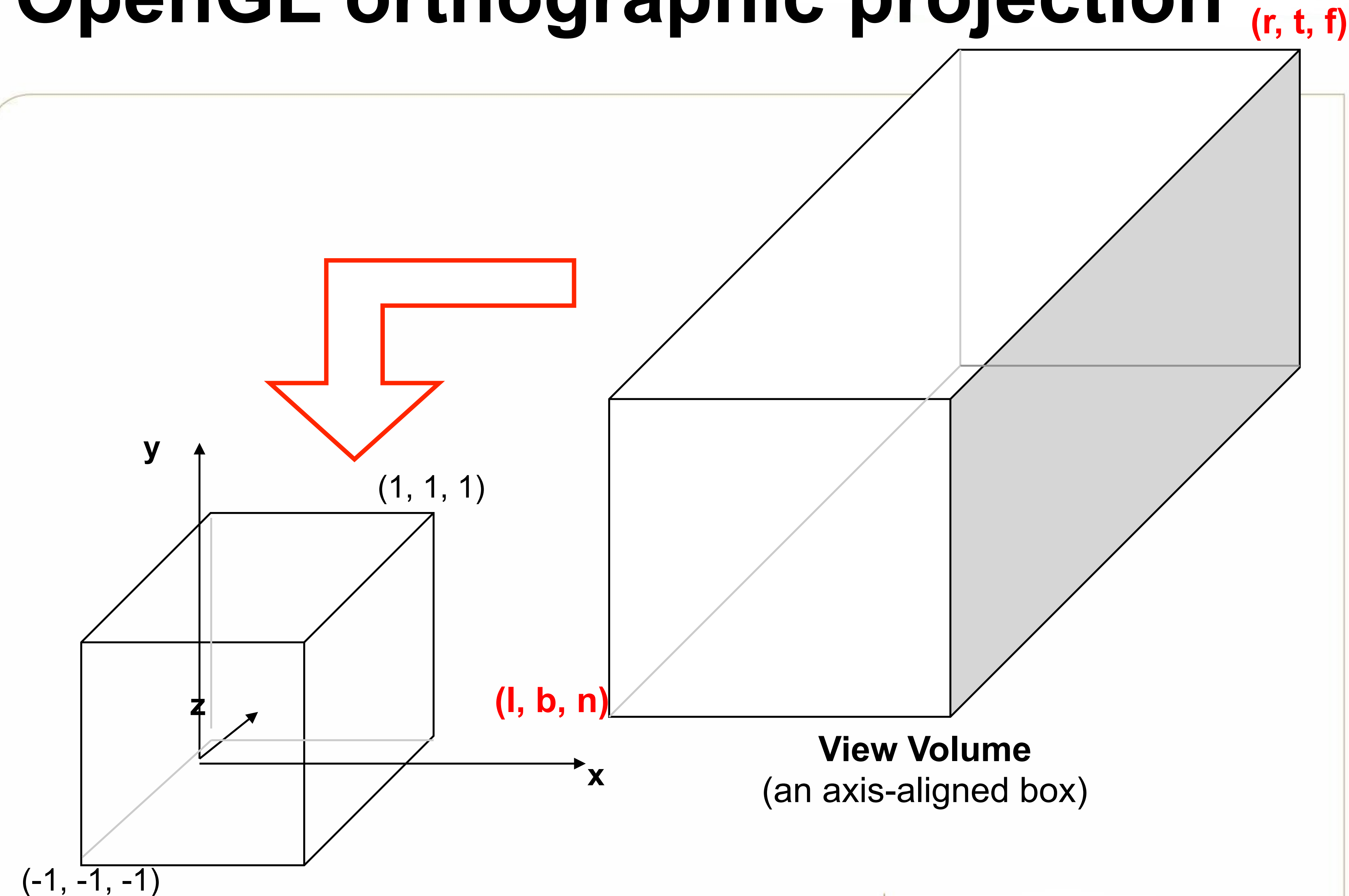
- For RHS, in most API calls z input parameters are positive, and clip space switches to using a LHS

$$[x',y',z',1]=[x,y,z,1]P \quad \text{where } P=\begin{bmatrix} \frac{2}{w} & 0 & 0 & 0 \\ 0 & \frac{2}{h} & 0 & 0 \\ 0 & 0 & \boxed{\frac{1}{n-f}} & 0 \\ 0 & 0 & \frac{n}{n-f} & 1 \end{bmatrix}$$

- In Direct3D: D3DXMatrixOrthoRH(*o,w,h,n,f)
- In XNA: Matrix.CreateOrthographic(w,h,n,f)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

OpenGL orthographic projection



Canonical view volume (OpenGL)

View Volume
(an axis-aligned box)

OpenGL orthographic math for Z (RHS)

- Derive z'

$$-z \in [n, f] \qquad z' \in [-1,\ 1]$$

$$n \le -z \le f$$

$$0 \le -z - n \le f - n$$

$$0 \le \frac{-z-n}{f-n} \le 1$$

$$0 \le \frac{2(-z-n)}{f-n} \le 2$$

$$-1 \le \frac{2(-z-n)}{f-n} - 1 \le 1$$

$$-1 \le \frac{-2z-2n-f+n}{f-n} \le 1$$

$$-1 \le \frac{-2z}{f-n} - \frac{f+n}{f-n} \le 1$$

$$\boxed{\therefore \quad z' = \frac{-2z}{f-n} - \frac{f+n}{f-n}}$$

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__2/Deriving-Projection-Matrices.htm

OpenGL orthographic results

$$\boxed{x' = \frac{2x}{r-l} - \frac{r+l}{r-l}}$$

$$\boxed{y' = \frac{2y}{t-b} - \frac{t+b}{t-b}}$$

$$\boxed{z' = \frac{-2z}{f-n} - \frac{f+n}{f-n}}$$

Ortho proj matrix (OpenGL/Unity) ✓

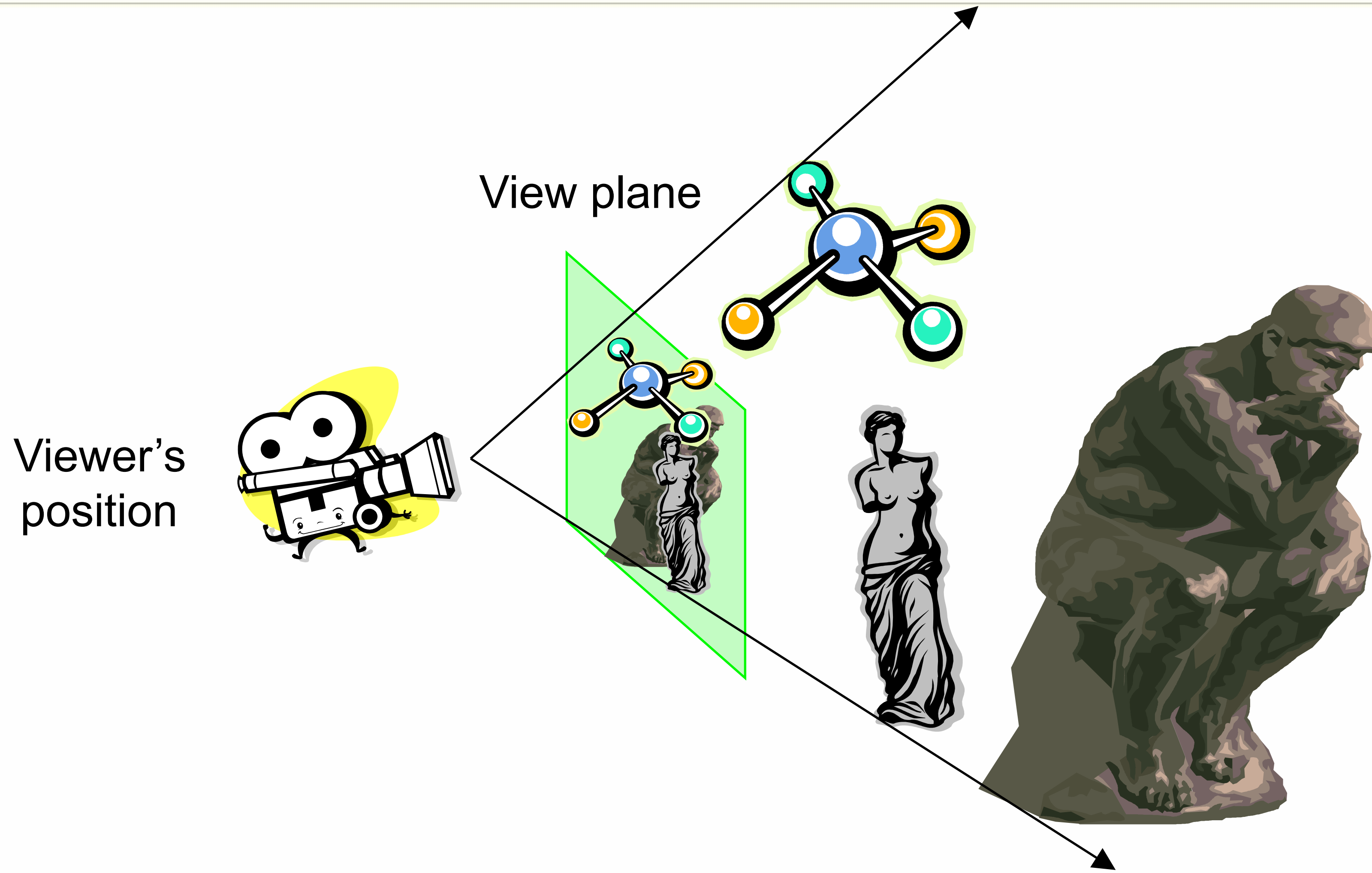
- For RHS, in most API calls z input parameters are positive, and clip space switches to using a LHS

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1' \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \text{ where } P = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \boxed{\frac{-2}{f-n}} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- In OpenGL: glOrtho(l,r,b,t,n,f)
- In Unity: Matrix4x4.Ortho(l,r,b,t,n,f) ????????????

See http://www.songho.ca/opengl/gl_projectionmatrix.html#ortho

Perspective projection



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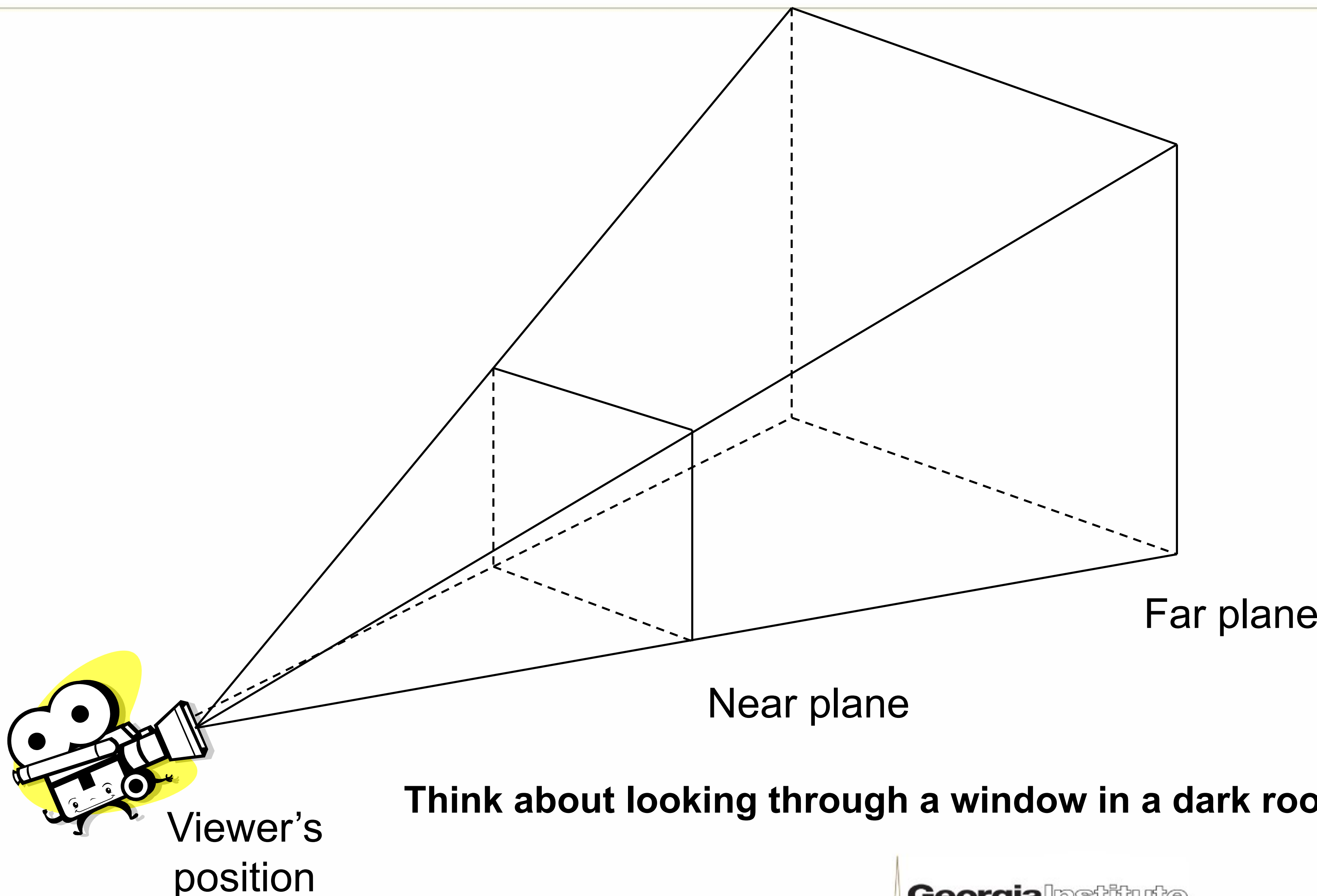
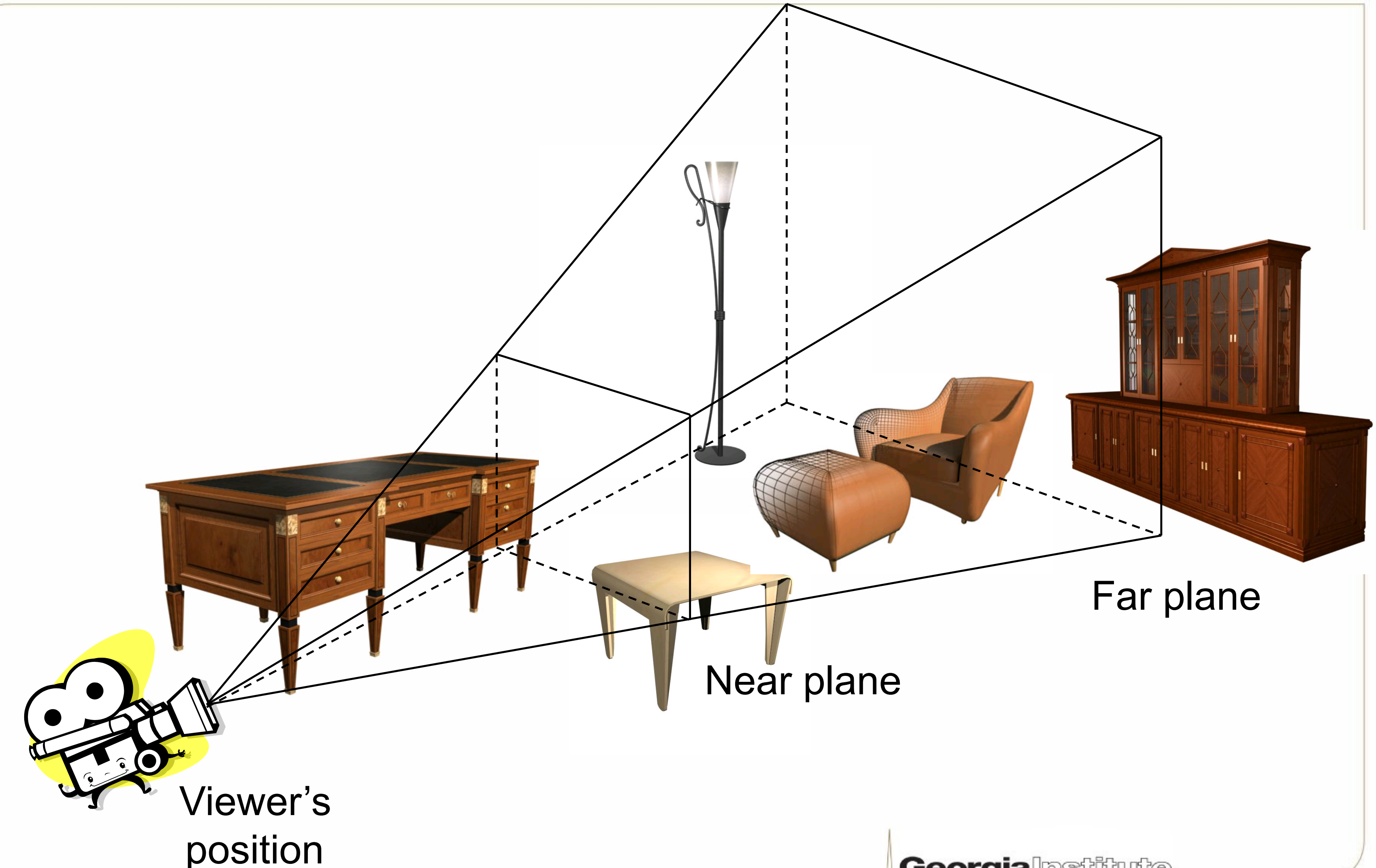
Viewing frustum



Think about looking through a window in a dark room

Viewing frustum with furniture



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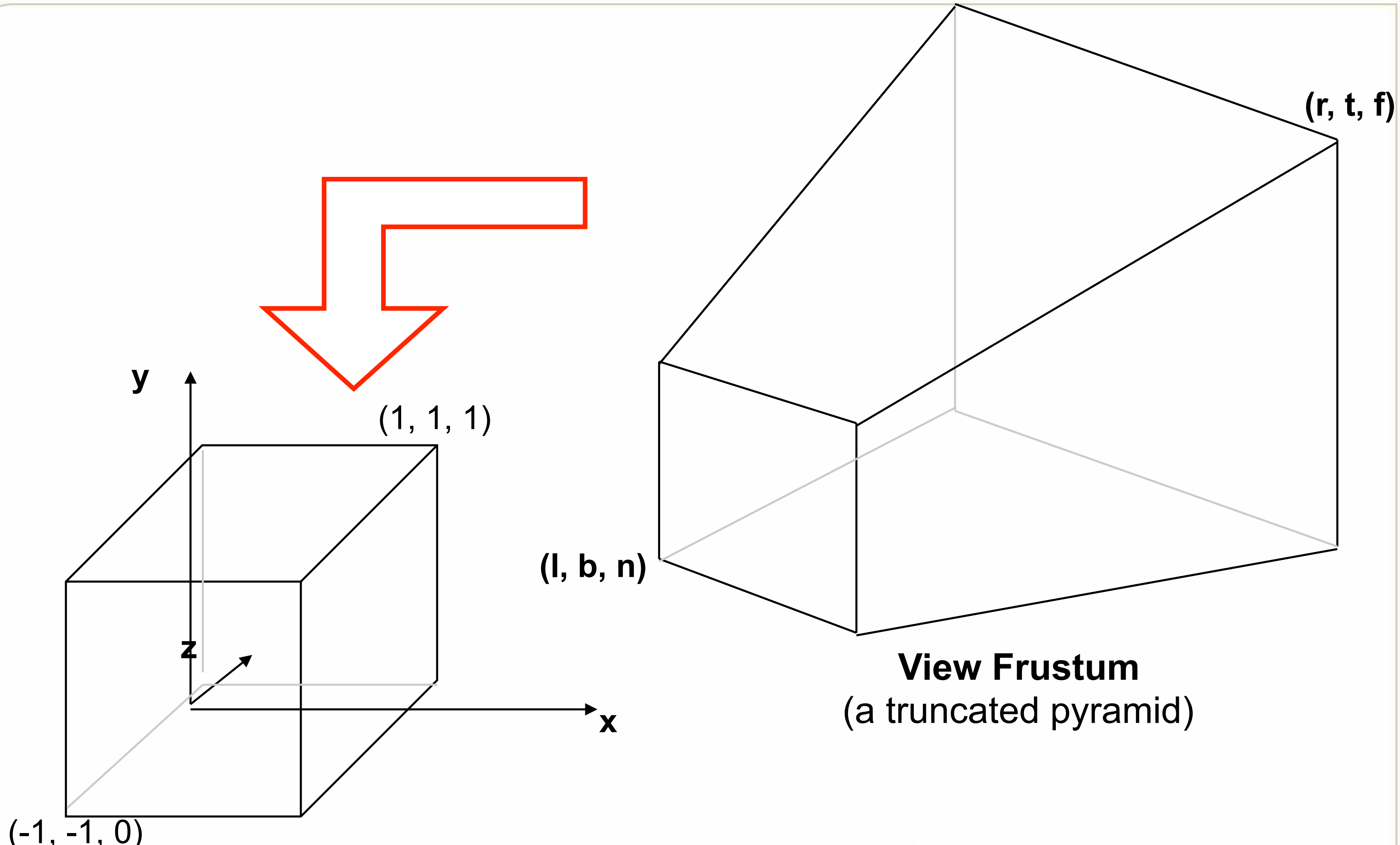
Direct3D perspective projection

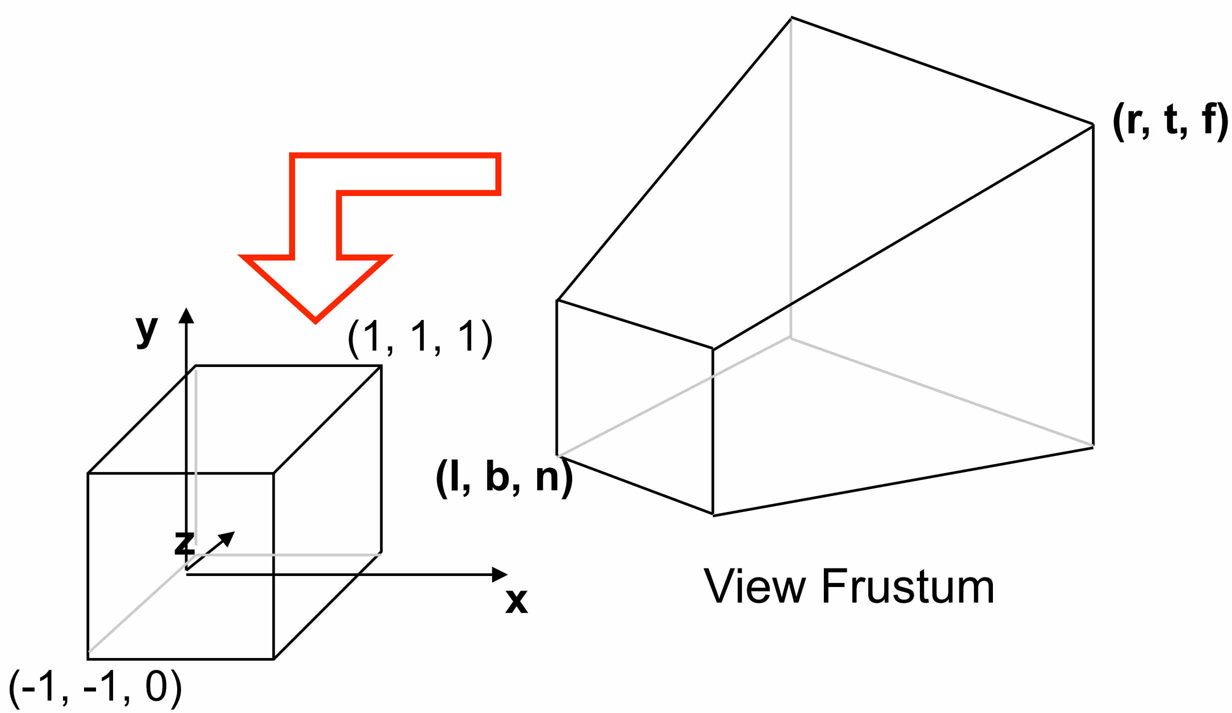


Canonical view volume (Direct3D)

View Frustum
(a truncated pyramid)

D3D perspective proj mapping

- Given a point (x,y,z) within the view frustum, project it onto the near plane z=n
- We will map x from [l,r] to [-1,1] and y from [b,t] to [-1,1]



Canonical view volume (Direct3D)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

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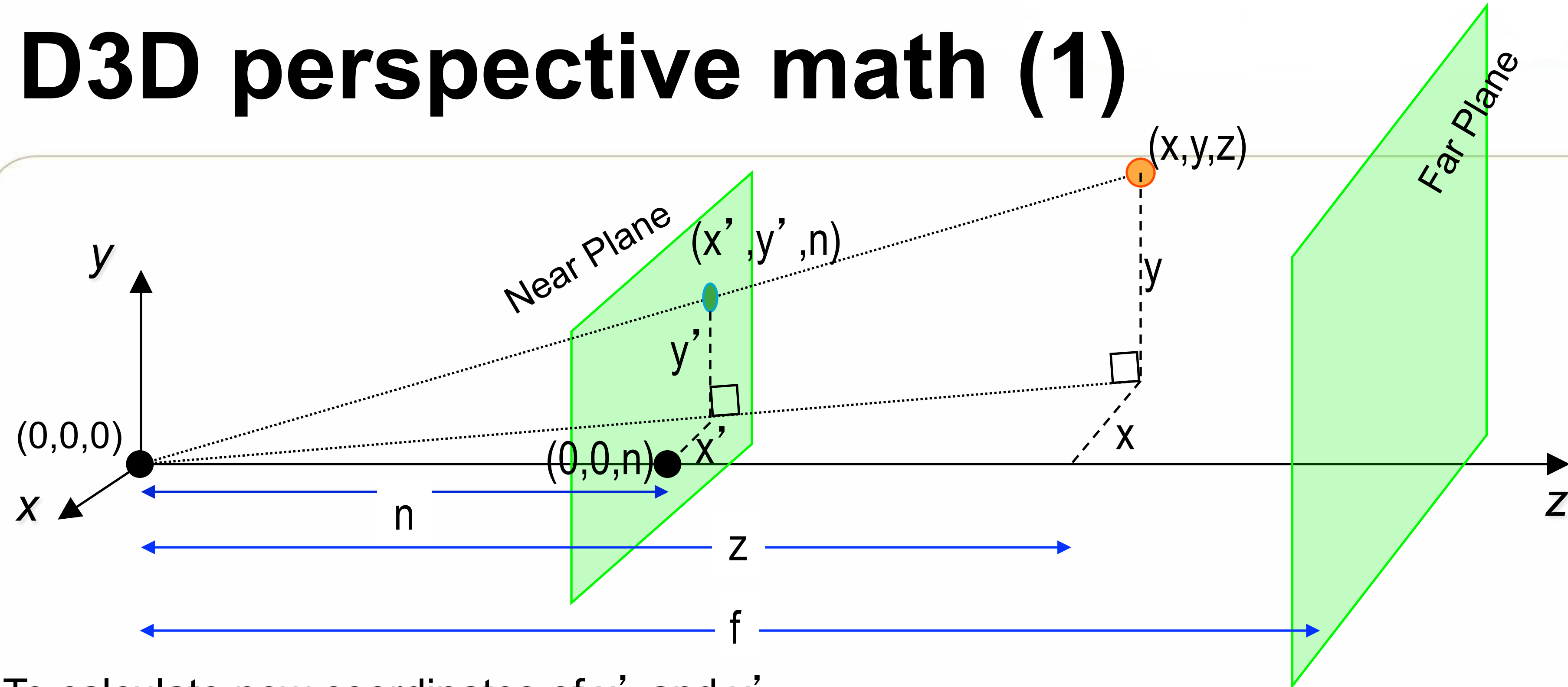
D3D perspective math (1)



To calculate new coordinates of x' and y'

$$\frac{x'}{x} = \frac{n}{z} \Rightarrow x' = \frac{nx}{z}$$

$$\frac{y'}{x'} = \frac{y}{x} \Rightarrow y' = \frac{yx'}{x} = \frac{y}{x} \cdot \frac{nx}{z} = \frac{ny}{z}$$

Next apply our orthographic projection formulas

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

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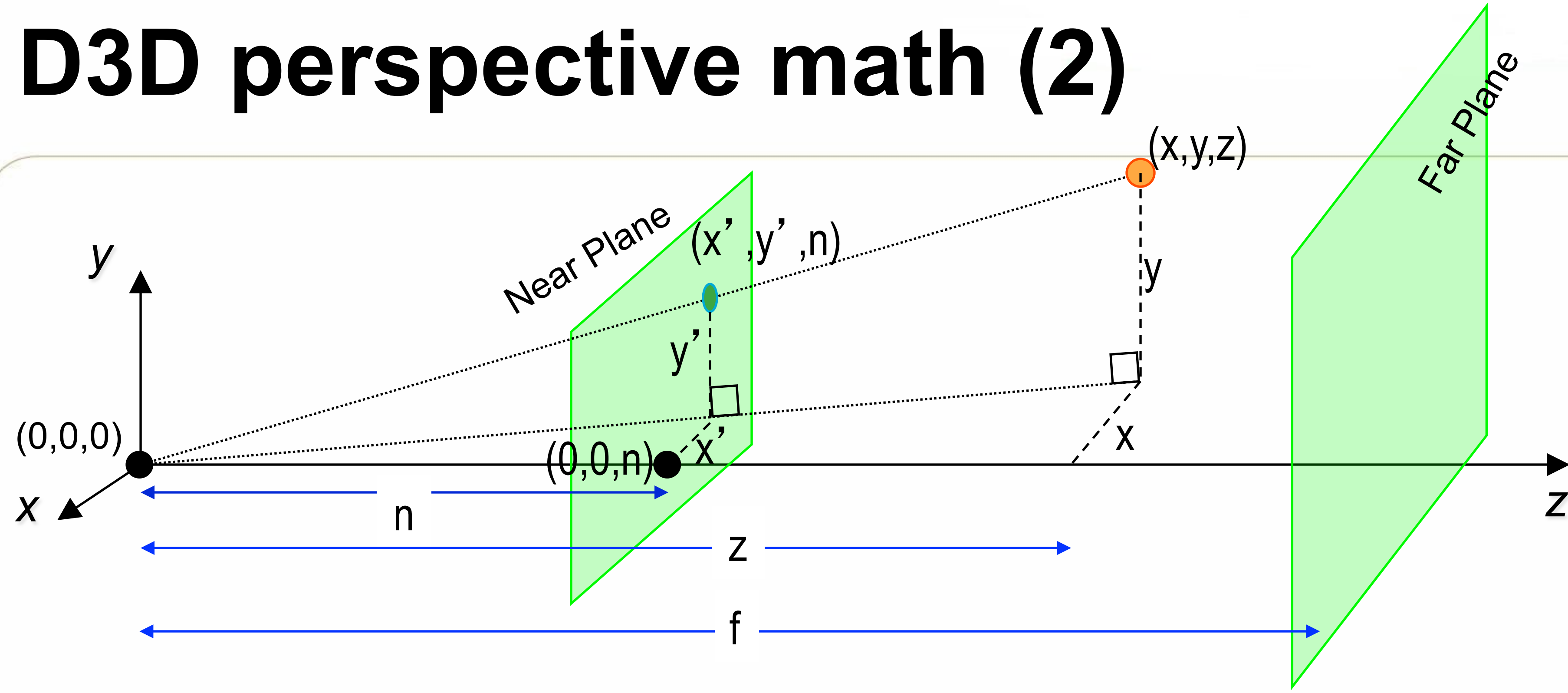
D3D perspective math (2)



$$x' = \frac{2}{r-l}\cdot\frac{nx}{z} - \frac{r+l}{r-l} \qquad y' = \frac{2n}{t-b}\cdot\frac{ny}{z} - \frac{t+b}{t-b}$$

$$x'z = \frac{2n}{r-l}x - \frac{r+l}{r-l}z \qquad y'z = \frac{2n}{t-b}y - \frac{t+b}{t-b}z$$

Now let's tackle the z' component

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

D3D perspective math (3)

$$x'z = \frac{2n}{r-l}x - \frac{r+l}{r-l}z$$

$$y'z = \frac{2n}{t-b}y - \frac{t+b}{t-b}z$$

$$z'z = pz + q \quad \text{where } p \text{ and } q \text{ are constants}$$

- We know z (depth) transformation has nothing to do with x and y

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

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D3D perspective math (4)

$z'z = pz + q$ where p and q are constants

$$\boxed{\begin{aligned} 0 &= pn + q \\ f &= pf + q \end{aligned}} \qquad \therefore p = \frac{f}{f-n} \quad \text{and} \quad q = -\frac{fn}{f-n}$$

$$z'z = \frac{f}{f-n} z - \frac{fn}{f-n}$$

- We know (boxed equations above)
  - z' = 0 when z=n (near plane)
  - z' = 1 when z=f (far plane)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

General D3D perspective matrix

$$x'z = \frac{2n}{r-l}x - \frac{r+l}{r-l}z$$

$$y'z = \frac{2n}{t-b}y - \frac{t+b}{t-b}z$$

$$z'z = \frac{f}{f-n}z - \frac{fn}{f-n}$$

$$w'z = z$$

$$[x'z, y'z, z'z, w'z] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{2n}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2n}{t-b} & 0 & 0 \\ -\frac{r+l}{r-l} & -\frac{t+b}{t-b} & \frac{f}{f-n} & 1 \\ 0 & 0 & -\frac{fn}{f-n} & 0 \end{bmatrix}$$

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See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

Simpler D3D perspective matrix

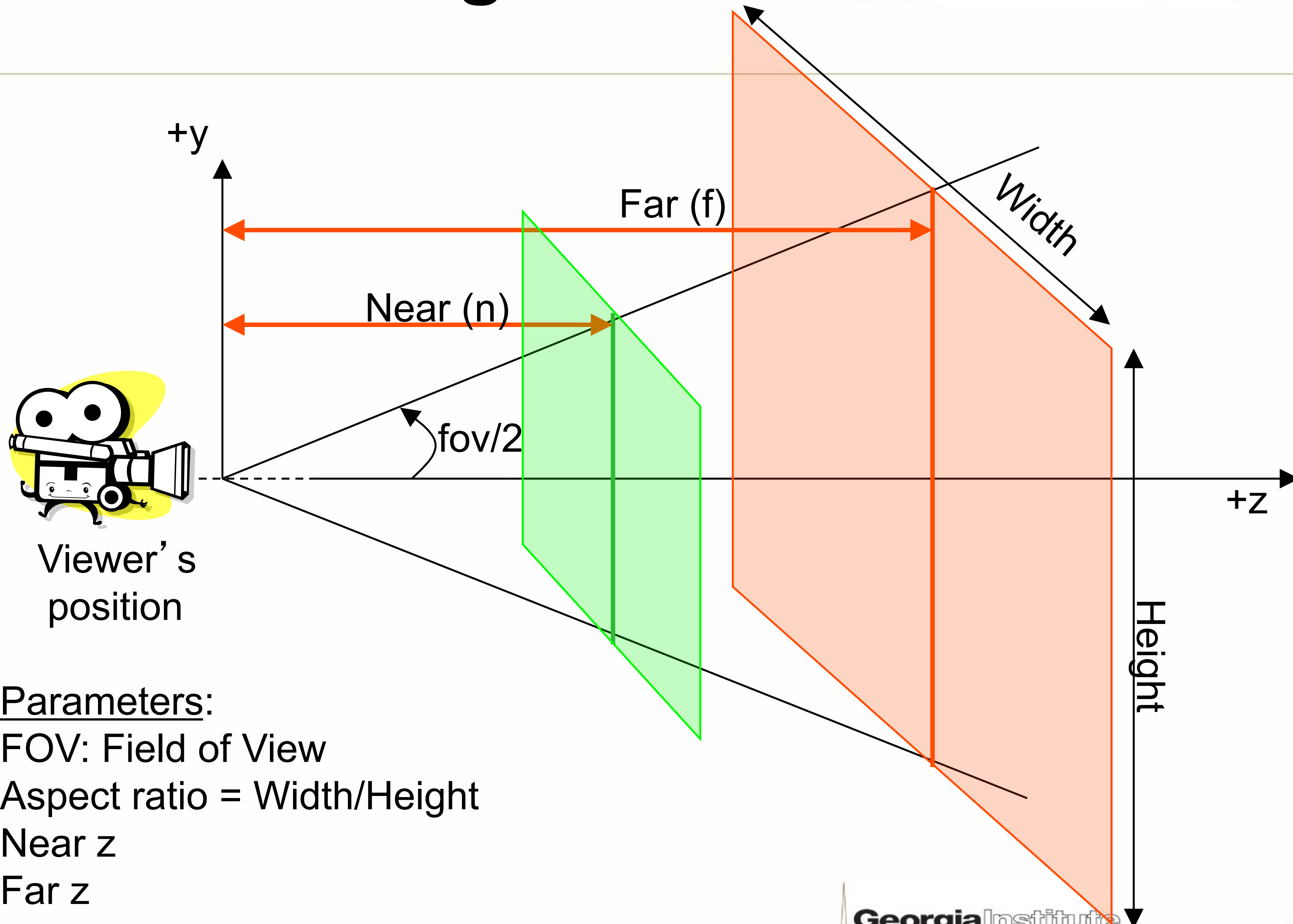
- Similar to orthographic projection, if *l=-r and t=-b,* we can simplify to

$$[x'z, y'z, z'z, w'z] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{2n}{w} & 0 & 0 & 0 \\ 0 & \frac{2n}{h} & 0 & 0 \\ 0 & 0 & \frac{f}{f-n} & 1 \\ 0 & 0 & -\frac{fn}{f-n} & 0 \end{bmatrix}$$

- In any case, we will have to divide by z to obtain [x',y', z', w']
  - Implemented by dividing by the fourth (w'z) coordinate

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

Define viewing frustum

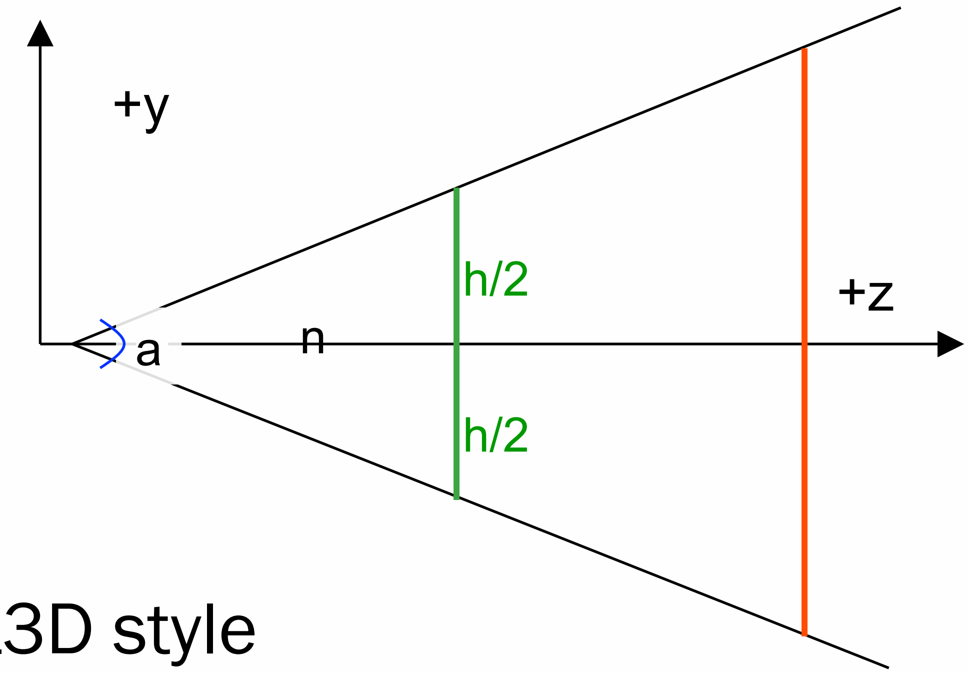


<u>Parameters</u>:
FOV: Field of View
Aspect ratio = Width/Height
Near z
Far z

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

Reparameterized D3D matrix

$$P = \begin{bmatrix} \boxed{\frac{2n}{w}} & 0 & 0 & 0 \\ 0 & \boxed{\frac{2n}{h}} & 0 & 0 \\ 0 & 0 & \frac{f}{f-n} & 1 \\ 0 & 0 & -\frac{fn}{f-n} & 0 \end{bmatrix}$$



$$\cot(\frac{a}{2}) = \frac{2n}{h}$$

Direct3D style

$$r = \frac{w}{h}$$

$$\frac{2n}{w} = \frac{2n}{rh} = \frac{2n}{r\frac{2n}{\cot(\frac{a}{2})}} = \frac{1}{r}\cot(\frac{a}{2})$$

Need to replace w and h with FOV and aspect ratio

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

D3D perspective matrix (LHS default)

✓

$$[x'z, y'z, z'z, w'z] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{1}{r}\cdot\cot(\frac{a}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{a}{2}) & 0 & 0 \\ 0 & 0 & \frac{f}{f-n} & 1 \\ 0 & 0 & -\frac{fn}{f-n} & 0 \end{bmatrix}$$

a: Field of View (FOV) r: aspect ratio $= \frac{\text{width}}{\text{height}}$ n: near plan f: far plane

- In Direct3D: D3DXMatrixPerspectiveFovLH(*o,a,r,n,f)

D3D perspective matrix (RHS weird) ✓

$$[x'z, y'z, z'z, w'z] = [x, y, z, 1]P \quad \text{where } P = \begin{bmatrix} \frac{1}{r}\cot(\frac{a}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{a}{2}) & 0 & 0 \\ 0 & 0 & \frac{f}{n-f} & -1 \\ 0 & 0 & \frac{fn}{n-f} & 0 \end{bmatrix}$$

a: Field of View (FOV) r: aspect ratio $= \dfrac{\text{width}}{\text{height}}$ n: near plan f: far plane

- In Direct3D: D3DXMatrixPerspectiveFovRH(*o,a,r,n,f)
- In XNA: Matrix.CreatePerspectiveFieldOfView(a,r,n,f)

http://msdn.microsoft.com/en-us/library/bb205351(VS.85).aspx

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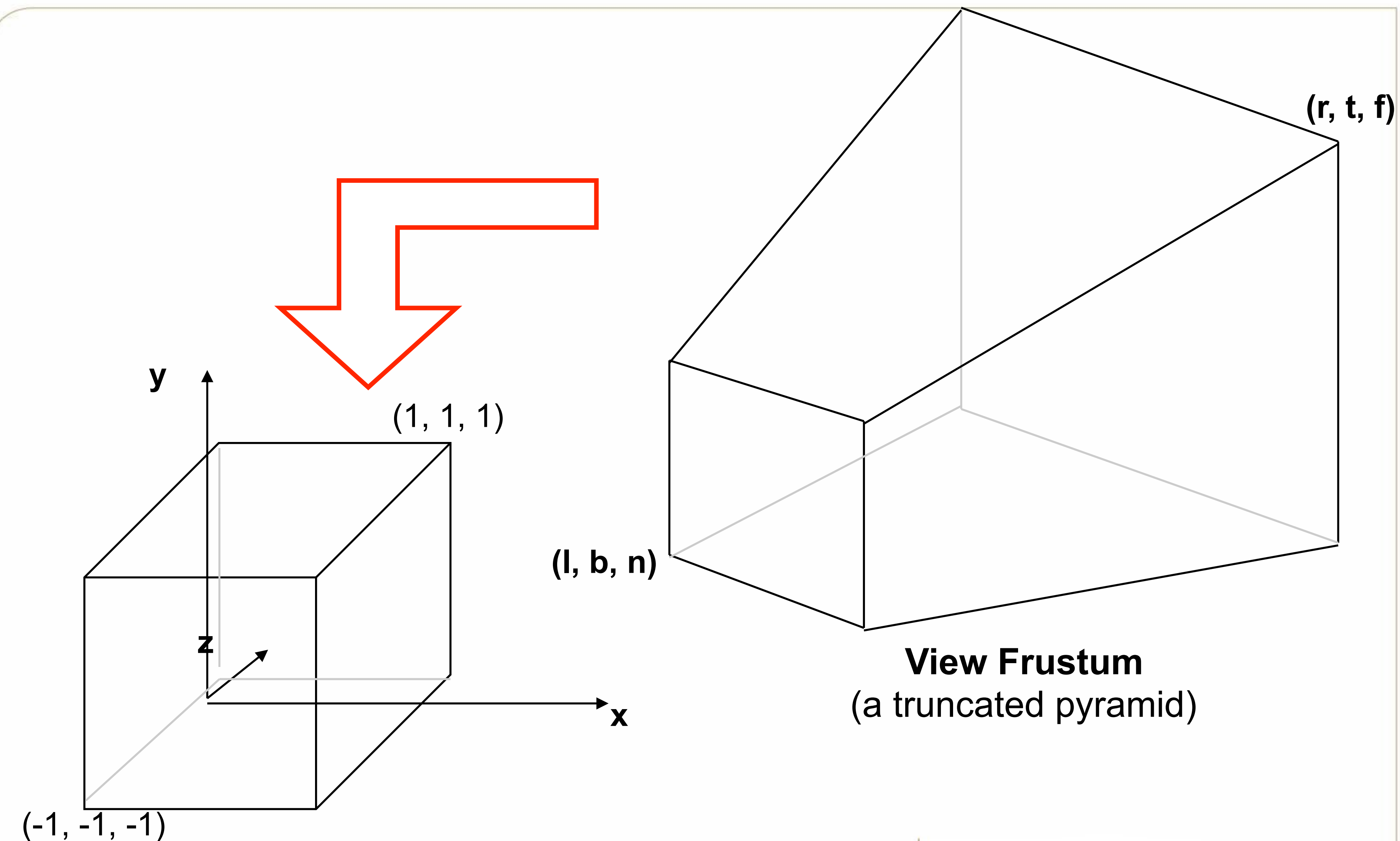
OpenGL/LHS perspective projection



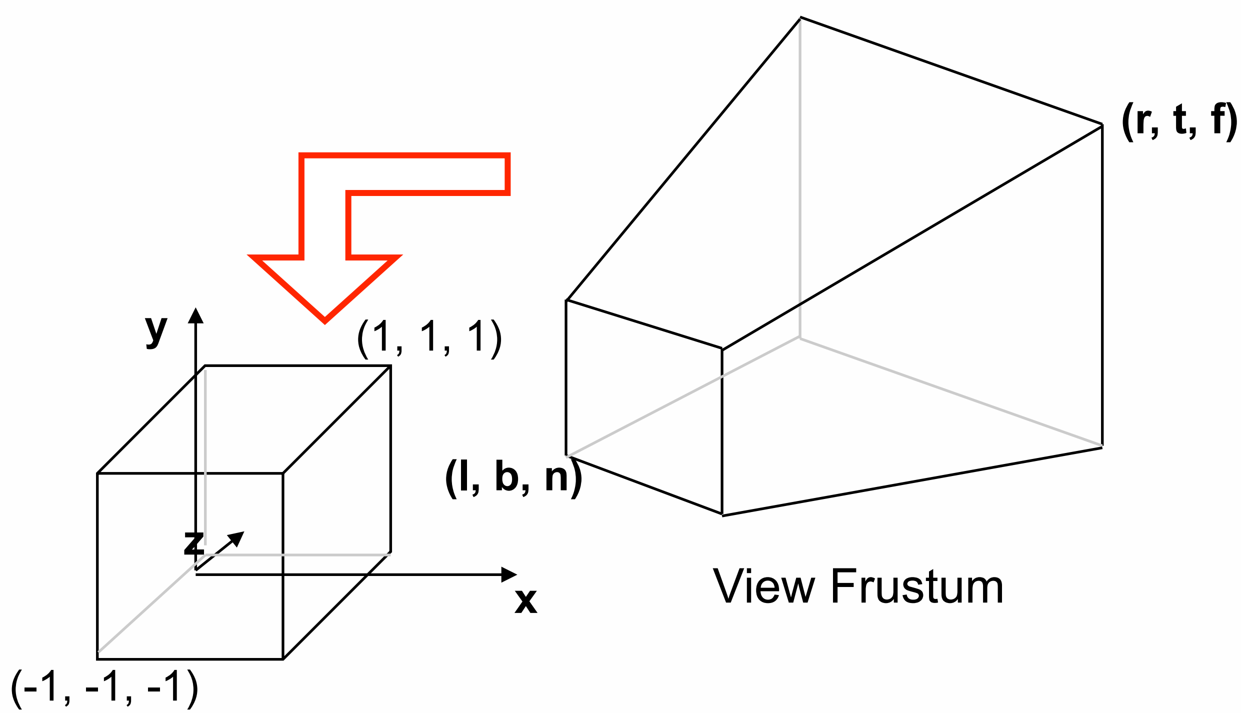
Canonical view volume (Unity)

View Frustum
(a truncated pyramid)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

OpenGL/LHS perspective proj mapping

- Given a point (x,y,z) within the view frustum, project it onto the near plane z=n
- We will map x from [l,r] to [-1,1] and y from [b,t] to [-1,1]



Canonical view volume (Unity)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

OpenGL/LHS perspective projection math

$z'z = pz + q$ where p and q are constants

$$\boxed{\begin{aligned} -n &= pn + q \\ f &= pf + q \end{aligned}} \quad ? \quad \therefore p = \frac{f+n}{f-n} \quad \text{and} \quad q = -\frac{2fn}{f-n}$$

$$z'z = \frac{f+n}{f-n}z - \frac{2fn}{f-n}$$

- We know (boxed equations above)
  - z' = -1 when z=n (near plane)
  - z' = 1 when z=f (far plane)

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

General OpenGL/LHS perspective matrix

$$x'z = \frac{2n}{r-l}x - \frac{r+l}{r-l}z$$

$$y'z = \frac{2n}{t-b}y - \frac{t+b}{t-b}z$$

$$z'z = \frac{f+n}{f-n}z - \frac{2fn}{f-n}$$

$$w'z = z$$

$$\begin{bmatrix} x'z \\ y'z \\ z'z \\ w'z \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{where } P = \begin{bmatrix} \frac{2n}{r-l} & 0 & -\frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & -\frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

Simpler OpenGL/LHS perspective matrix

- Similar to orthographic projection, if *l=-r and t=-b,* we can simplify to

$$\begin{bmatrix} x'z \\ y'z \\ z'z \\ w'z \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{where } P = \begin{bmatrix} \frac{2n}{w} & 0 & 0 & 0 \\ 0 & \frac{2n}{h} & 0 & 0 \\ 0 & 0 & \frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- In any case, we will have to divide by z to obtain [x',y', z', w']
  - Implemented by dividing by the fourth (w'z) coordinate

See http://www.codeguru.com/cpp/misc/misc/math/article.php/c10123__3/Deriving-Projection-Matrices.htm

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Simpler OpenGL/RHS perspective matrix ✓

$$\begin{bmatrix} x'z \\ y'z \\ z'z \\ w'z \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{where } P = \begin{bmatrix} \frac{1}{r}\cot(\frac{a}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{a}{2}) & 0 & 0 \\ 0 & 0 & \boxed{\frac{f+n}{n-f}} & \frac{2fn}{n-f} \\ 0 & 0 & \boxed{-1} & 0 \end{bmatrix}$$

a: Field of View (FOV) r: aspect ratio $= \dfrac{\text{width}}{\text{height}}$ n: near plan f: far plane

- In OpenGL: gluPerspective(a,r,n,f)

http://msdn.microsoft.com/en-us/library/bb205350(VS.85).aspx

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Unity perspective matrix ✓

$$\begin{bmatrix} x'z \\ y'z \\ z'z \\ w'z \end{bmatrix} = P \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad \text{where } P = \begin{bmatrix} \frac{1}{r}\cot(\frac{a}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{a}{2}) & 0 & 0 \\ 0 & 0 & \frac{f+n}{f-n} & \boxed{\frac{2fn}{f-n}} \\ 0 & 0 & \boxed{-1} & 0 \end{bmatrix}$$

a: Field of View (FOV) r: aspect ratio $= \dfrac{\text{width}}{\text{height}}$ n: near plan f: far plane

- In Unity: Matrix4x4.Perspective(a,r,n,f) ?????????????

http://msdn.microsoft.com/en-us/library/bb205350(VS.85).aspx

Custom projections in Unity

- From Camera.projectionMatrix documentation:

"Use a custom projection only if you really need a non-standard projection.

This property is used by Unity's water rendering to setup an *oblique projection* matrix.

Using custom projections requires good knowledge of transformation and projection matrices."

Unity's 2-D coordinate systems

- Viewport space:
  - (0,0) is bottom-left
  - (1,1) is top-right
- Screen space coordinates:
  - z "is in world units from the camera"
  - (0,0) is bottom-left
  - (Camera.pixelWidth,Camera.pixelHeight) is top-right
- GUI space coordinates:
  - (0,0) is upper-left
  - (Camera.pixelWidth,Camera.pixelHeight) is bottom-right

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Viewport transformation

- The actual 2D projection to the viewer
- Copy to your back buffer (frame buffer)
- Can be programmed, scaled, ...

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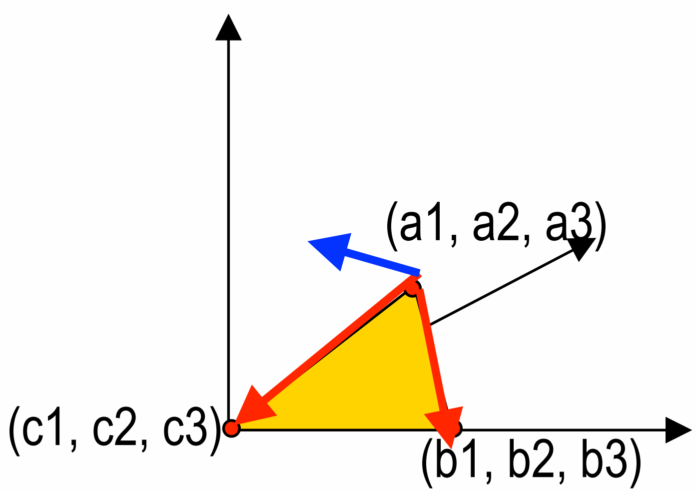
Backface culling

- Determine "facing direction"
- Triangle order matters
- How to compute a normal vector for 2 given vectors?
  - Using **cross product** of 2 given vectors



2 Vectors

$$\overrightarrow{V1} = (b1 - a1)\mathbf{i} + (b2 - a2)\mathbf{j} + (b3 - a3)\mathbf{k}$$

$$\overrightarrow{V2} = (c1 - a1)\mathbf{i} + (c2 - a2)\mathbf{j} + (c3 - a3)\mathbf{k}$$

Cross product

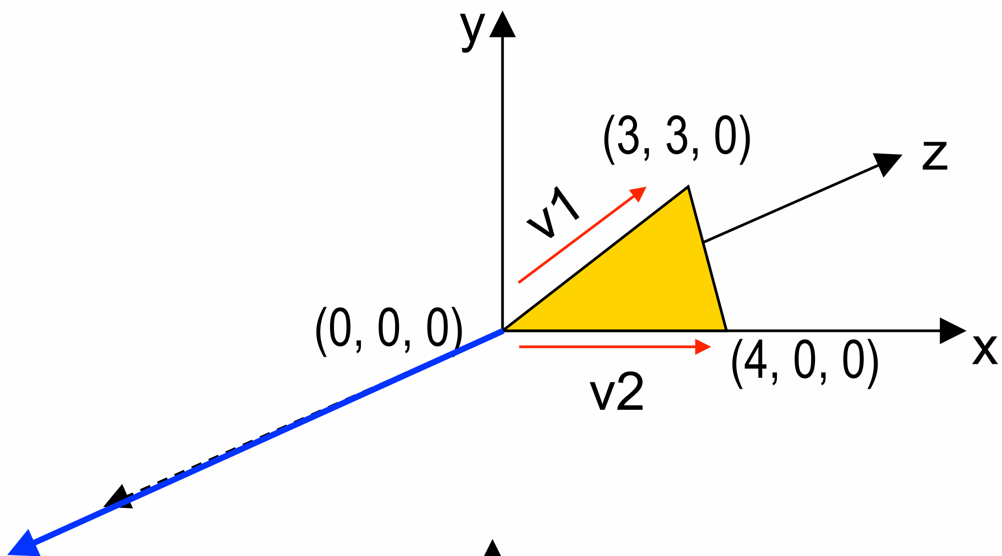
$$\overrightarrow{V1} = x_1\mathbf{i} + x_2\mathbf{j} + x_3\mathbf{k}$$

$$\overrightarrow{V2} = y_1\mathbf{i} + y_2\mathbf{j} + y_3\mathbf{k}$$

$$\overrightarrow{V1} \times \overrightarrow{V2} = (x_2 y_3 - x_3 y_2)\mathbf{i} + (x_3 y_1 - x_1 y_3)\mathbf{j} + (x_1 y_2 - x_2 y_1)\mathbf{k}$$

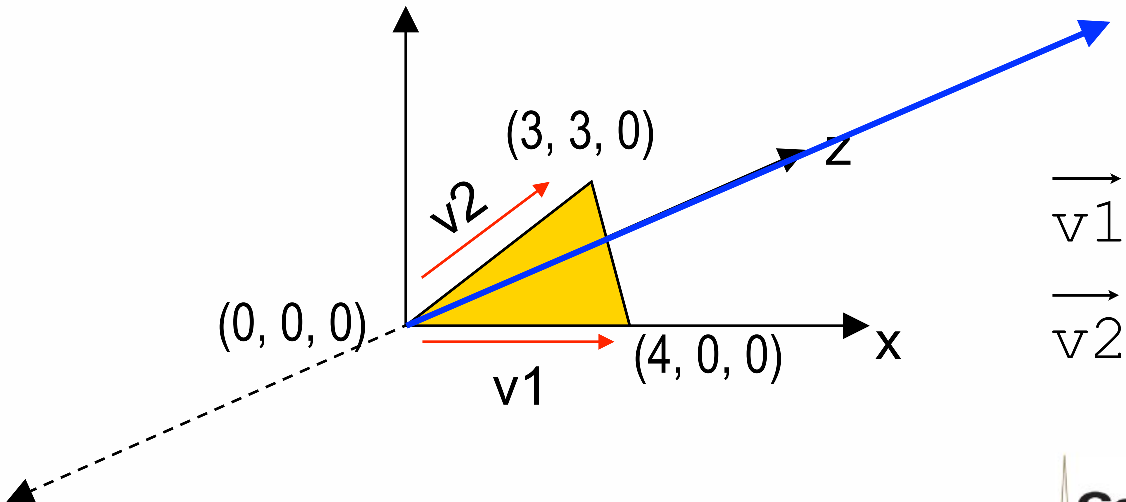
Compute the surface normal for a triangle

- Clockwise normals, LHS



$$\overrightarrow{v1} = 3\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$$

$$\overrightarrow{v2} = 4\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$



$$\overrightarrow{v1} = 4\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$\overrightarrow{v2} = 3\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}$$

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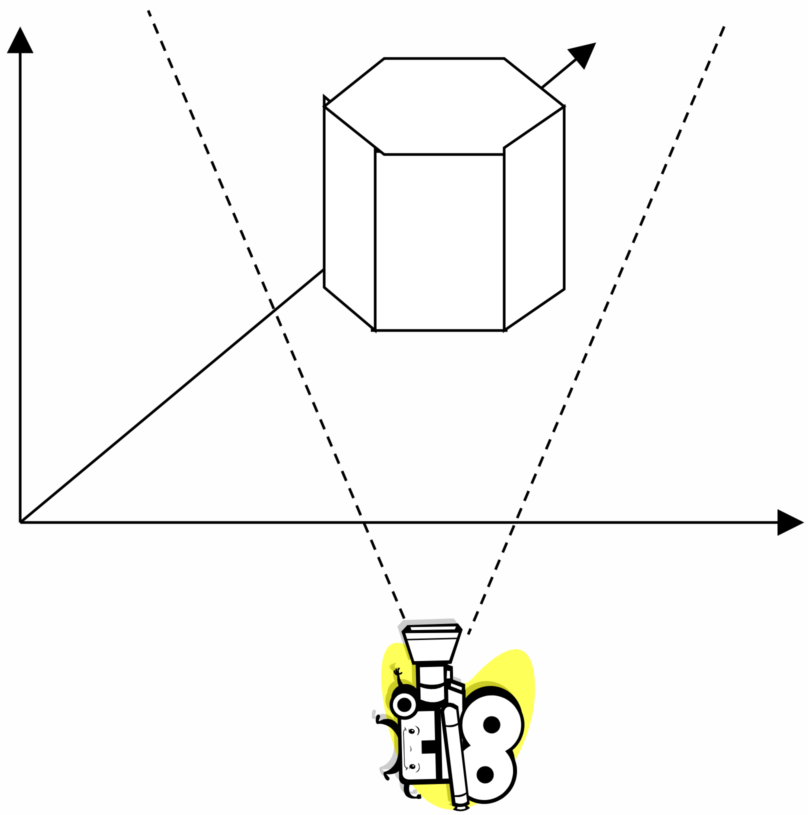
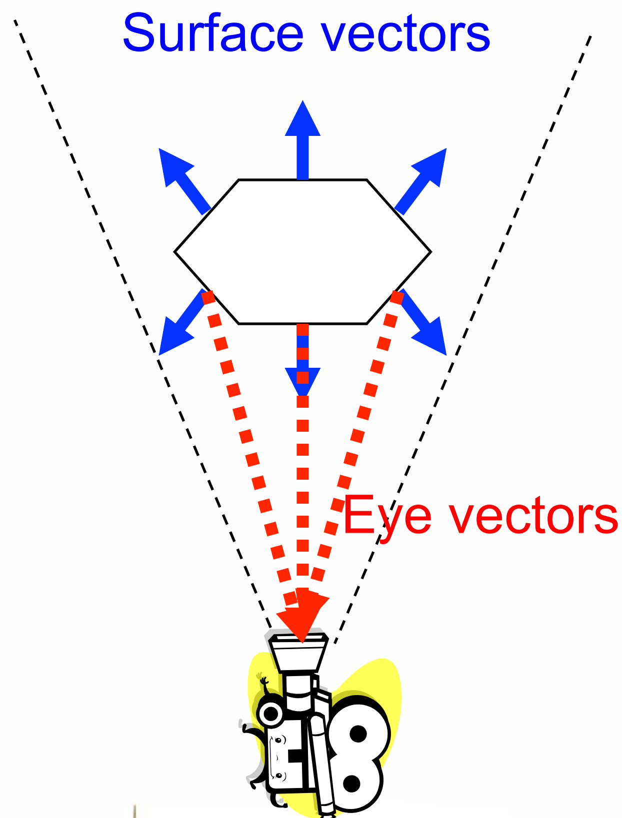
Backface culling method (1)

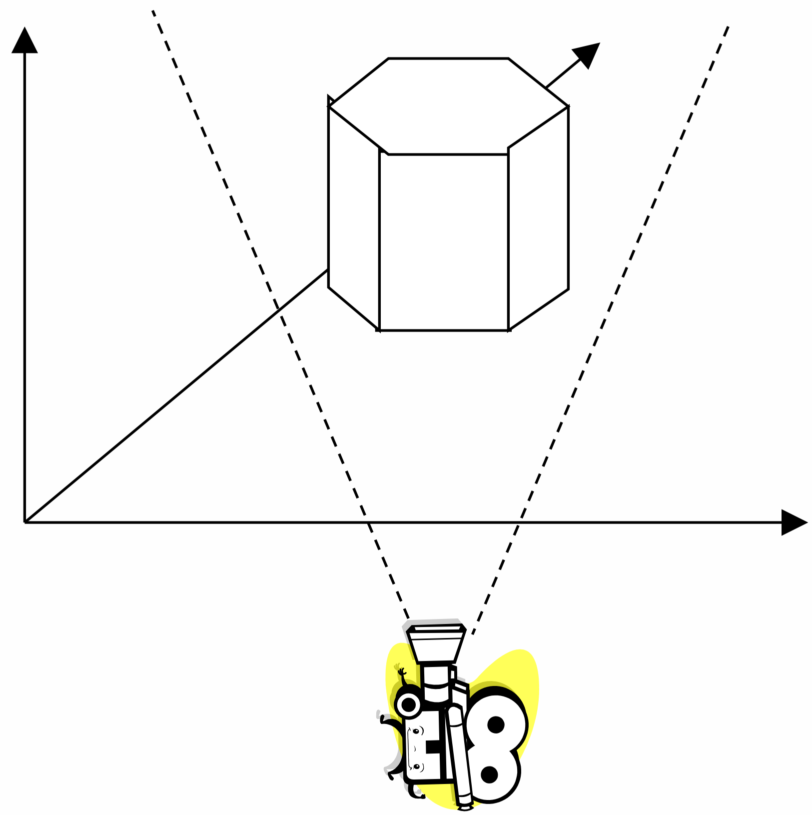
- Check if the normal is facing the camera
- How to determine that?
  - Use Dot Product

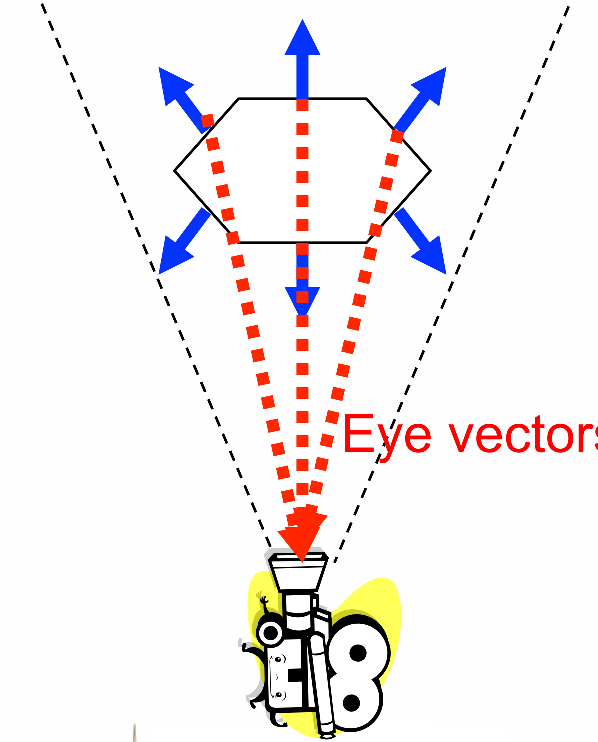




Backface culling method (2)

- Check if the normal is facing the camera
- How to determine that?
  - Use Dot Product





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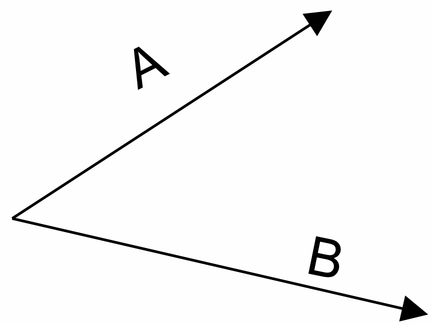
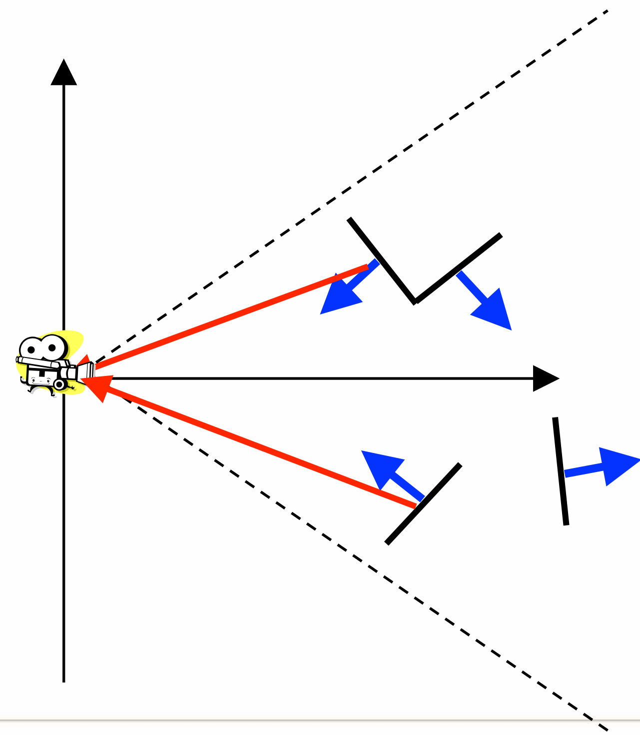
Dot product method (1)

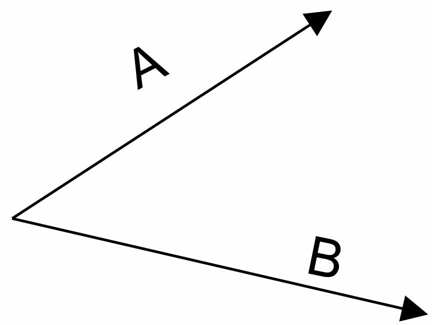


$$\vec{A} \bullet \vec{B} = |A||B|\cos\theta$$

$$\vec{A} \bullet \vec{B} > 0 \;\; \Rightarrow \;\; -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

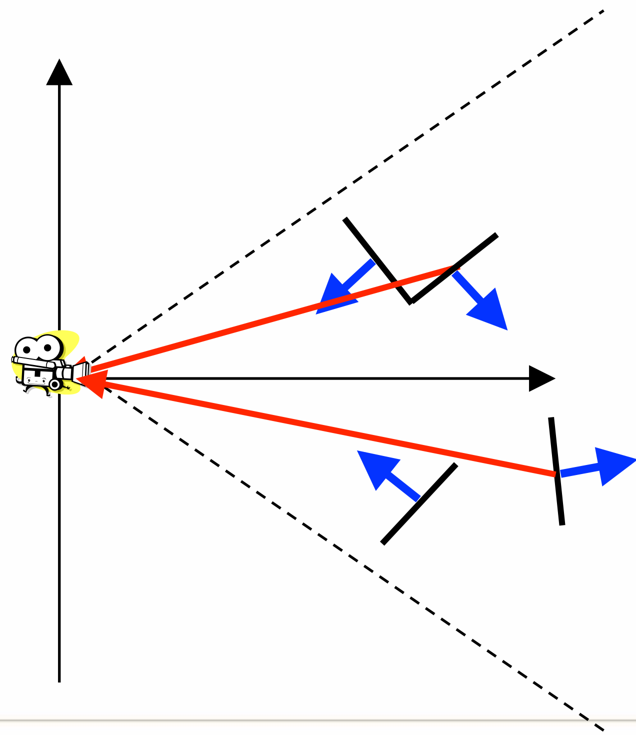


Dot product method (2)

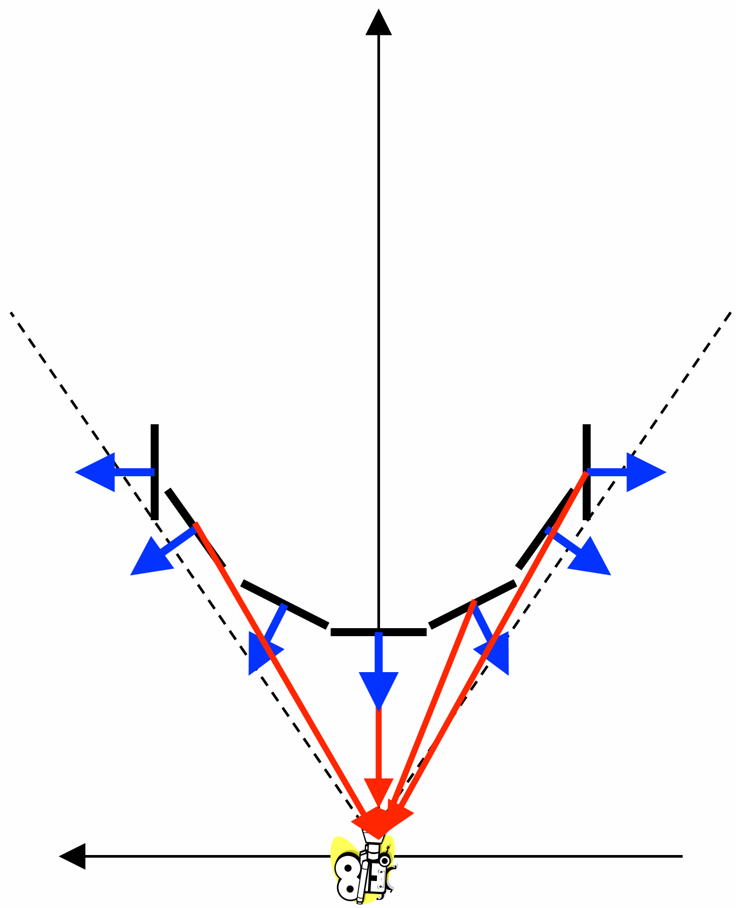


$$\vec{A} \bullet \vec{B} = |A||B|\cos\theta$$

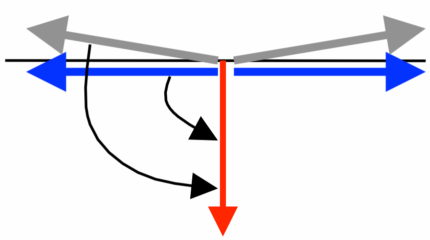
$$\vec{A} \bullet \vec{B} > 0 \;\Rightarrow\; -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



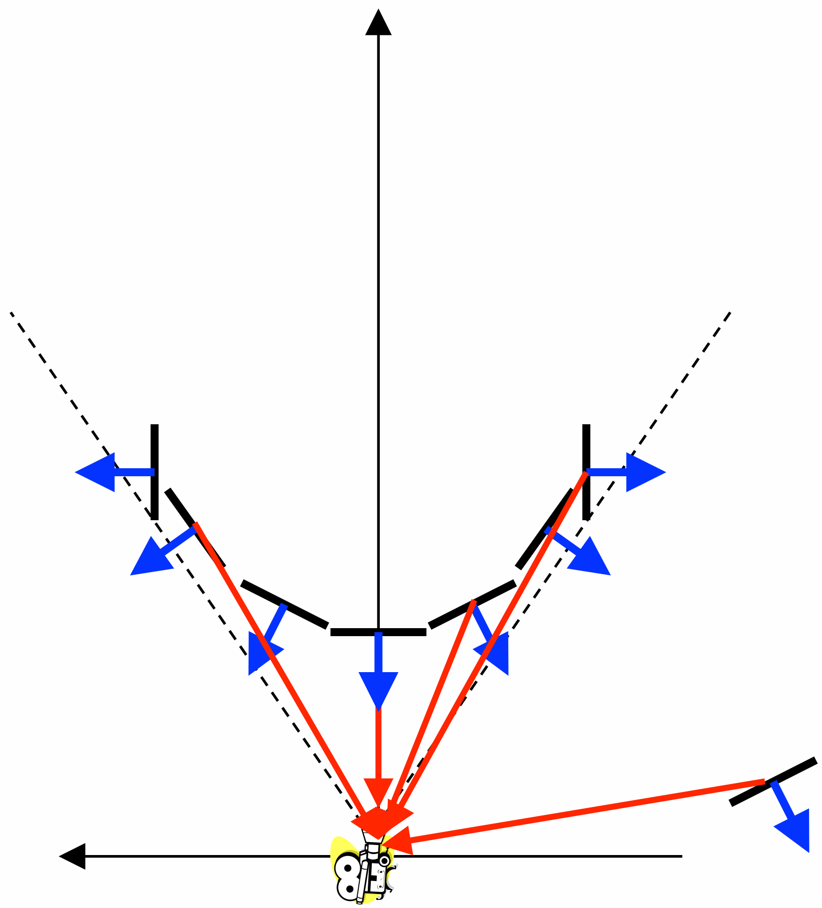
Dot product method (3)



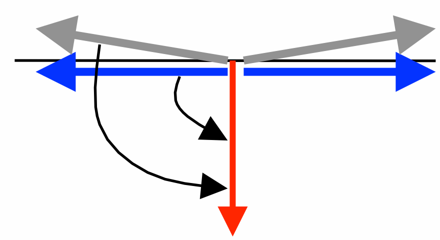
$$\vec{A} \bullet \vec{B} > 0 \;\Rightarrow\; -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$



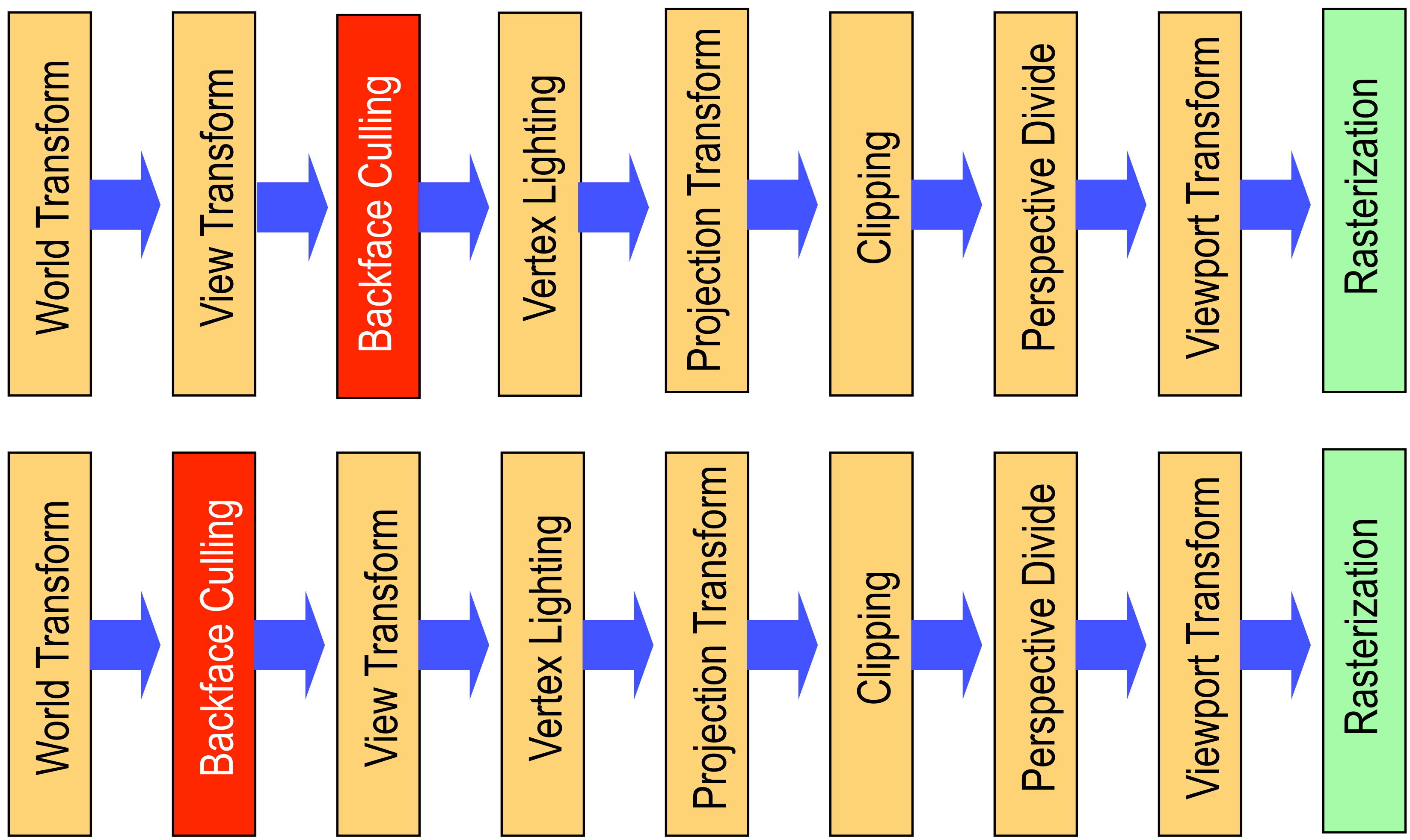
Caution!



$$\vec{A} \bullet \vec{B} > 0 \;\Rightarrow\; -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

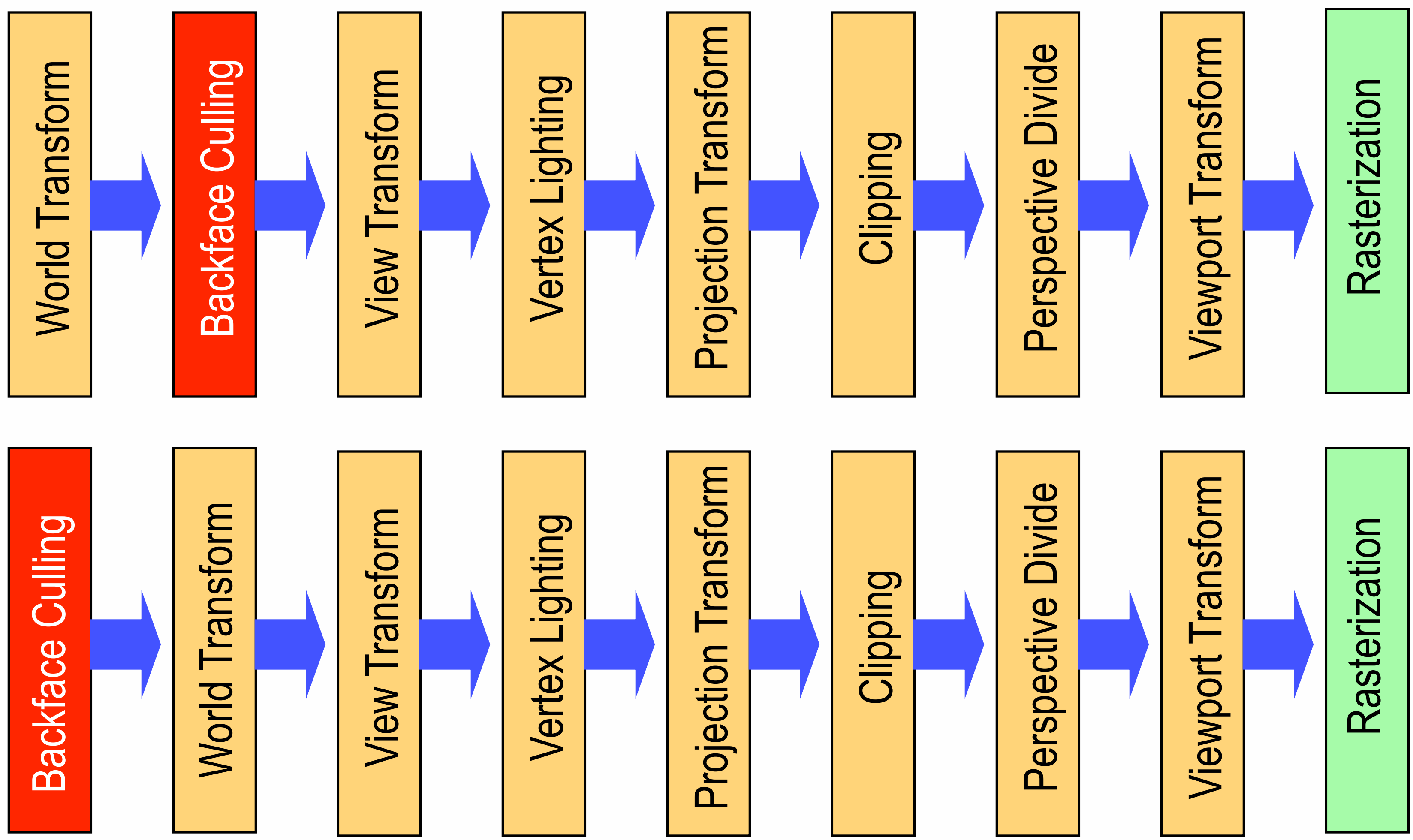


When to perform backface culling?



Make sure camera is in correct coordinates!

How about before you even start?



Transform camera vectors into object spaces

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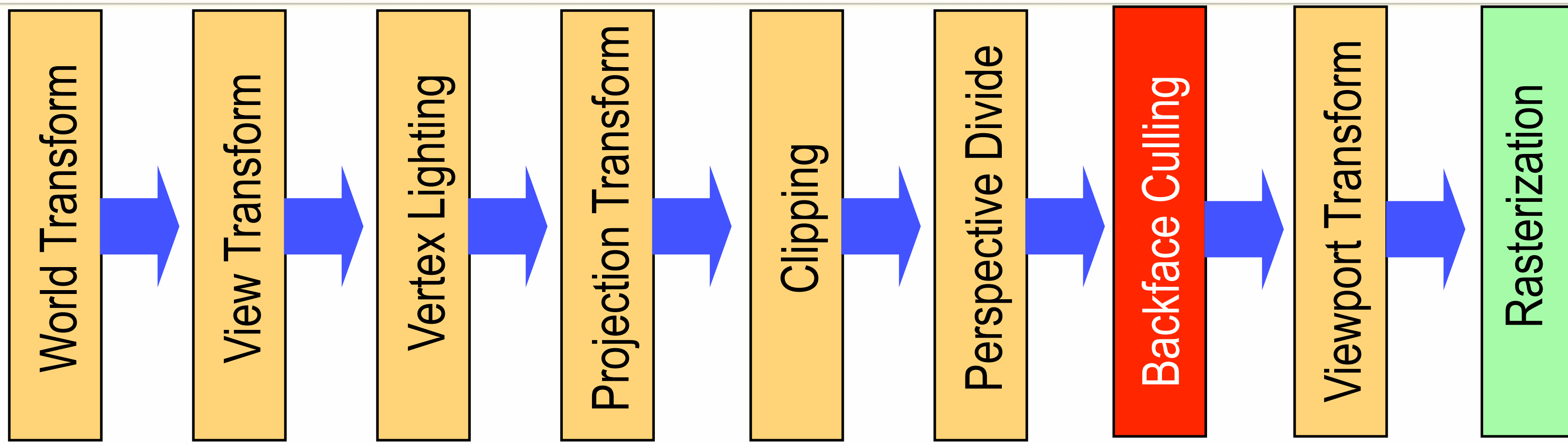
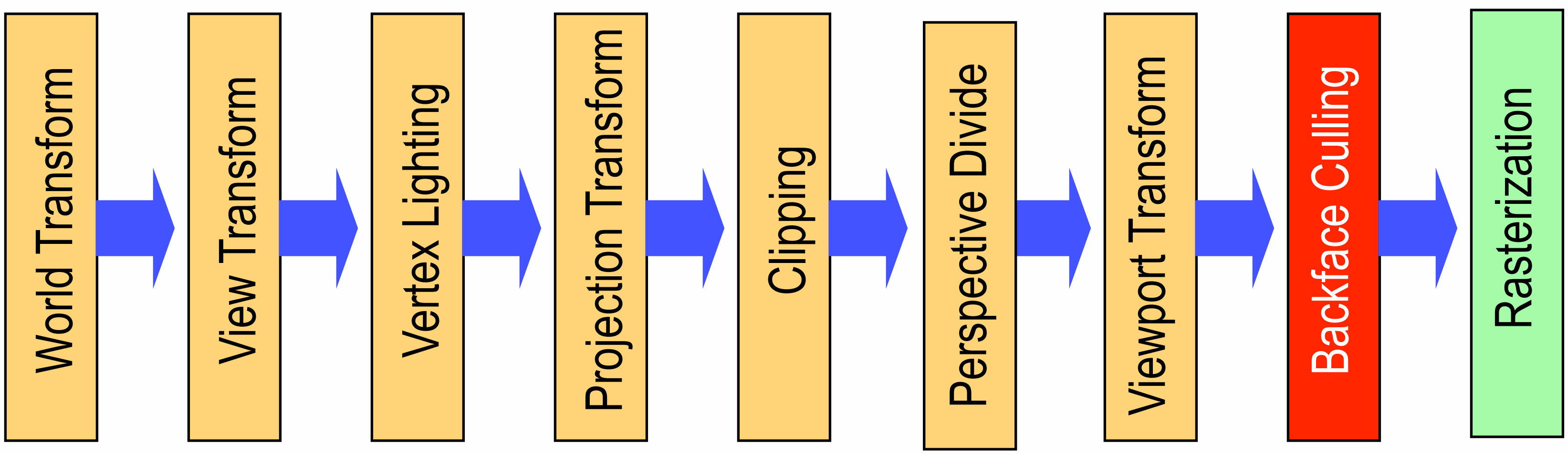
Or how about at the very end?



- Now you can just check "winding order" in 2D



- As "officially" done by OpenGL

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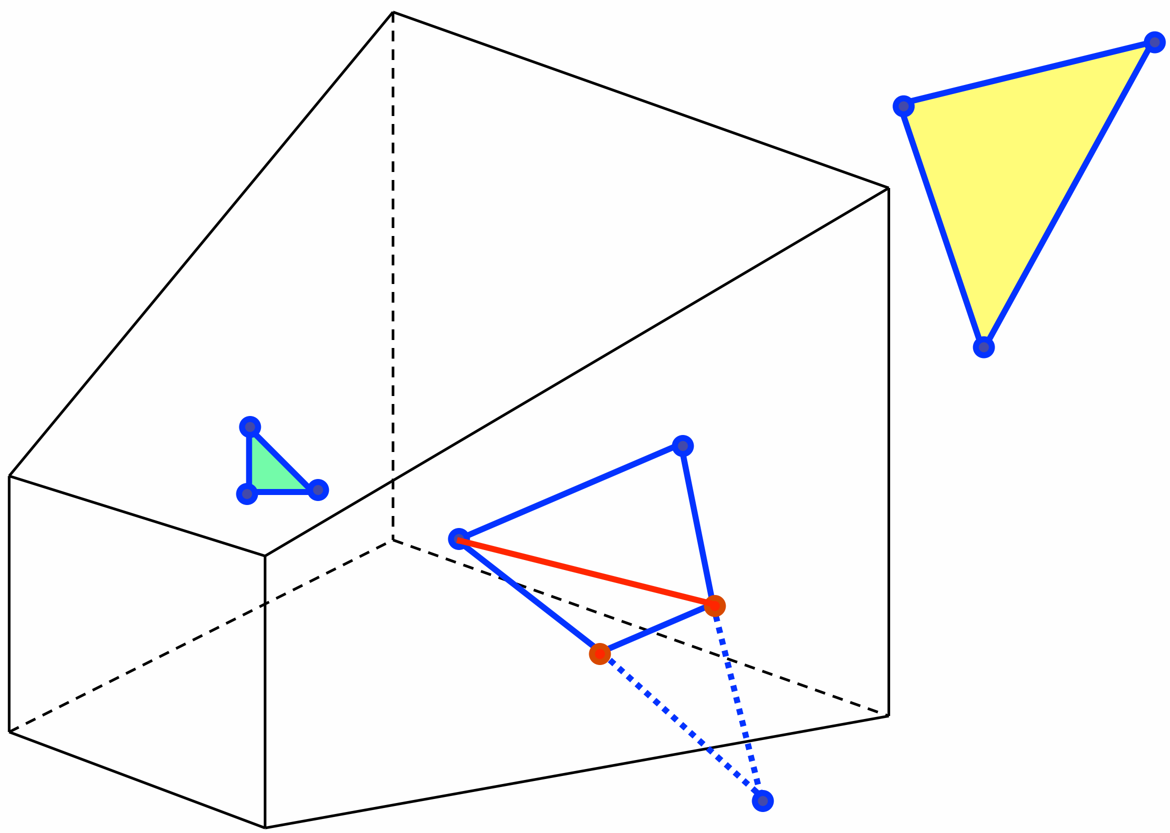
3D clipping



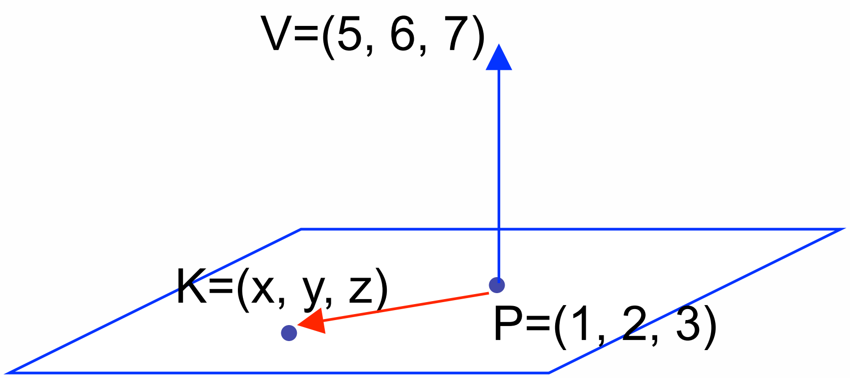
- Test 6 planes if a triangle is inside, outside, or partially inside the view frustum
- Clipping creates new triangles (triangulation)
  - Interpolate new vertices info

Appendix

Clipping against a plane

- Test each vertex of a triangle
  - Outside
  - Inside
  - Partially inside
- Incurred computation overhead
- Save unnecessary computation (and bandwidth) later
- Need to know how to determine a plane
- Need to know how to determine a vertex is inside or outside a plane

Specifying a plane

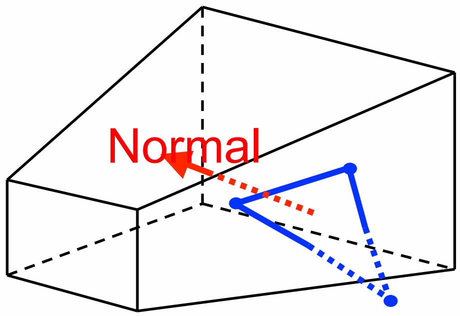


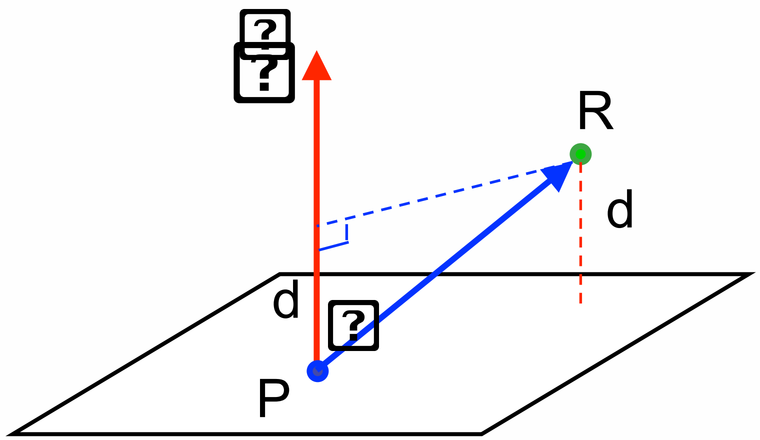
Plane equation

5*(x-1)+6*(y-2)+7*(z-3)=0

- You need two things to specify a plane
  - A point on the plane *(p0, p1, p2)*
  - A vector (normal) perpendicular to the plane *(a, b, c)*
  - Plane → $a*(x - p0) + b*(y - p1) + c*(z - p2) = 0$

Distance calculation from a plane (1)

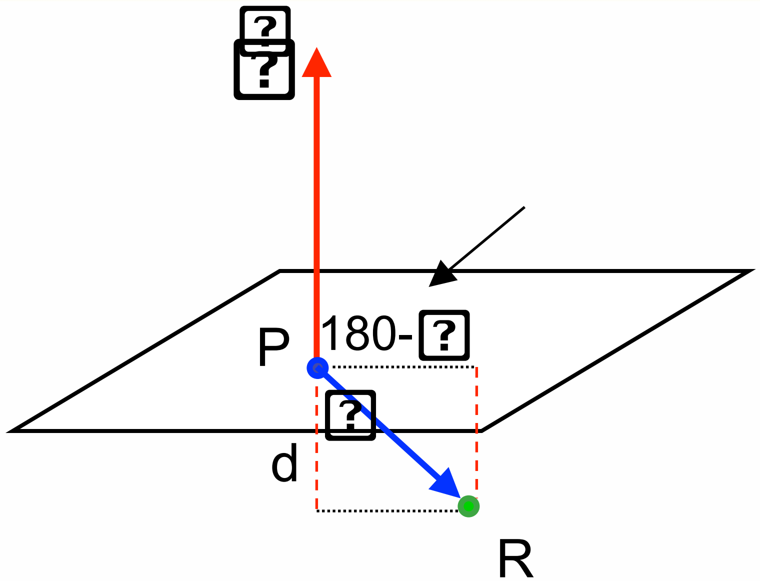




$$d = |R-P| \cdot \cos\theta = |R-P| \cdot \frac{\vec{v} \bullet (R-P)}{\left|\vec{v}\right| \cdot \left|R-P\right|} = \frac{\vec{v} \bullet (R-P)}{\left|\vec{v}\right|}$$

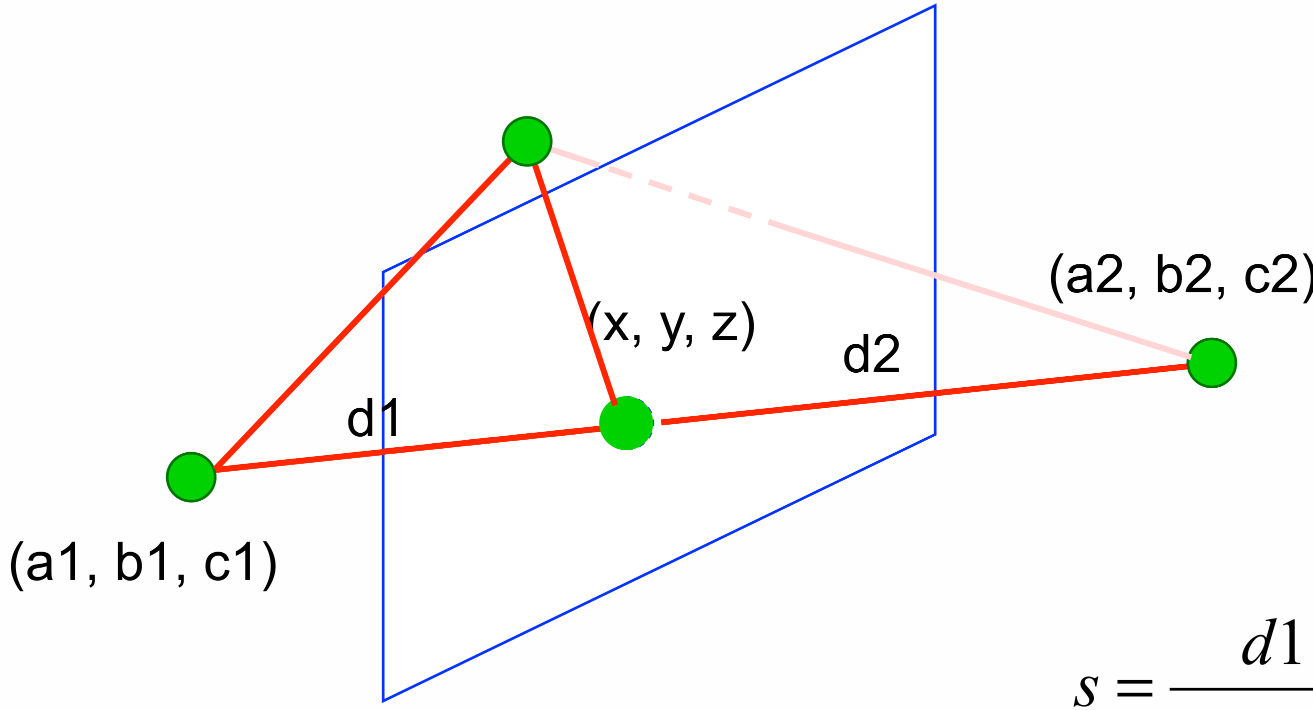
- Given a point R, calculate the distance
  - Distance > 0 inside the plane
  - Distance = 0 on the plane
  - Distance < 0 outside the plane

Distance calculation from a plane (2)



$$\mathrm{d} = |R - P| \cdot \cos(180 - \theta)$$

Triangulation using interpolation



$$s = \frac{d1}{d1 + d2}$$

$$x = a1 + s \cdot (a2 - a1)$$

$$y = b1 + s \cdot (b2 - b1)$$

$$z = c1 + s \cdot (c2 - c1)$$

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