

IXO Modelling

Objective

Determine if there is a feasible mechanism to tie the reserve of the market maker mechanism with the reserve of the brokerage mechanism, to maintain the peg stability during the period needed to bootstrap the system. Acknowledging the discussed assumptions, incentive mechanisms, and actors, in the system - the ultimate objective is to ensure actors are incentivized to supply their assets into the system in exchange for utility so that the system can be bootstrapped effectively.

Sprint 1 Scope

1. Customize and parameterize the existing augmented bonding curve model developed by BlockScience
2. Determine the most suitable mechanism for tying the two reserves together (using the Kyber network and Uniswap mechanisms as inspiration)
3. Determine what parameters we have available to stabilize the market maker peg (initial assumption - use the reserve ratio, and possible exit taxation to prime this mechanism)
4. Determine if the mechanism is theoretically feasible

Resources

1. Augmented bonding curve hatch_sim notebook: https://github.com/BlockScience/conviction/blob/master/hatch_sim.ipynb
2. Kyber network reserve types: <https://developer.kyber.network/docs/Reserves-Types/>

IXO Liquidity Mechanism

Uniswap notes:

Uniswap mechanism for liquidity provider / delegators brokerage:

- Uniswap liquidity pools are autonomous and use the Constant Product Market Maker ($x * y = k$)
- A small liquidity provider fee (0.3%) is taken out of each trade and added to the reserves
- Formal verification: <https://github.com/runtimeverification/verified-smart-contracts/blob/uniswap/uniswap/x-y-k.pdf>
- The invariant is that the ratio of $e:t:l$ is preserved and $k = e * t$ increases (e is ether, t is exchange tokens, and l is total liquidity / supply of uni tokens)
- By supplying and burning liquidity (uni tokens), the investor can exchange for their share of ether and tokens

Uni-token === IXO staking token If you stake you're delegating, and delegators receive a proportion of the networks fees, block rewards/inflation, service fees from oracles

Terminology:

- c_{-} - IXO-Cosmos equivalent of Ethereum tokens
- native - native to IXO-Cosmos network
- $cIXOS$ - staking tokens
- $cIXO$ - utility tokens

Actor flow:

1. Participant buys into market maker with Dai, and is given Ethereum based IXO tokens.
2. Participant stakes IXO into the brokerage mechanism in exchange for IXOS. The IXO stake is backed by a proportion of the DAI collateral pool.
3. The IXO and IXOS pools are mirrored on the IXO-Cosmos network, and backed by DAI - post IBC these assets can be made native to the IXO-Cosmos network.
4. $cIXO$ become utility tokens that can be spent in IXO-Cosmos network, and minted through block rewards to be rewarded to actors in the system. The utility comes from the asset backing of the $cIXO$ tokens in the brokerage.

Working example:

1. Participant purchases 100 IXO tokens @ 0.1 DAI per IXO for a total of 10 DAI.

2. Participant provides liquidity in brokerage contract by depositing 50 IXO, retaining 50 IXO for the time being. The asset backing is provided by the bonding curve Dai collateral pool.
3. Participant is rewarded with 50 IXOS tokens to represent the proportion of the liquidity provided.
4. These IXOS tokens are mirrored and can be used within the IXO-Cosmos network as equivalent cIXOS tokens, these are low velocity and are essentially staked for IXO-Cosmos incentive rewards.
5. Participants within the IXO-Cosmos network, who are incentivized/rewarded with cIXO tokens for supporting the network roles, may atomic swap/mirror their cIXO tokens for IXO tokens and in effect DAI via the bonding curve.

Observations:

- Based on last two actions - what is the best way to couple the cIXO token value to the DAI reserve to maintain the cross blockchain peg?
- Objective - make the cIXO utility token stable.
- cIXOS staked on IXO-Cosmos side for various incentive rewards.

To reduce invariance:

- Introduce arbitrage rebalancing
- Introduce algorithmic binding
- Introduce capacitor like smoothing of binding - treat the pool like a capacitor and the variance as AC electricity, determine the RC ratio necessary to filter out noise. The smoothing capacitor acts smooths the ripple current, to an appropriate level of stability. It does this by storing charge (perhaps tax), and discharging when necessary.

Dependencies

```
import sys
sys.path.append("../..venv/lib/python3.7/site-packages" )

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import seaborn as sns
import networkx as nx

from cadCAD.configuration.utils import config_sim #env_trigger, var_substep_trigger, config_sim, psub_list
from cadCAD.engine import ExecutionMode, ExecutionContext, Executor
from cadCAD.configuration import append_configs

%matplotlib inline

from helpers import *
from bonding_curve_eq import *
```

Liquidity Mechanisms and Actors Overview

```
from enum import Enum, auto
class Nodes(Enum):
    # Actors
    SERVICE_PROVIDER = auto() # makes claims about contributions and state of project
    EVALUATOR = auto() # validates claims
    DELEGATOR = auto() # stakes into the brokerage to provide liquidity
    PARTICIPANT = auto() # generic type for any user of IXO-Cosmos utility tokens
    # Mechanisms
    MARKET_MAKER = auto() # Issues IXO tokens in exchange for DAI
    BROKERAGE = auto() # Issues IXOS / staking tokens to delegators

# IBC Ethereum Peg-Zone:
# 1. IXO from AMM and DAI staked into reserve pool by delegators in exchange for IXOS
```

2. cIXO and cIXOS matched on IXO-Cosmos network
3. This mechanism can be spun down, while maintaining the liquidity created, when the IBC mechanism is o

Global Parameters

```
liquidity_provider_fee = .03 # percent
tax = 0 # exit tax - introduces friction and possibly funds to prime stability mechanisms
theta = 0 #.35 # Funding pool taxation - 0 implies all contributions allocated to market

# From scenario spreadsheet
initial_ixo_supply = 2e9
initial_ixo_price = .02
initial_ixo_reserve = initial_ixo_supply * initial_ixo_price

founders_stake = .1 # percent
initial_ixos_supply = initial_ixo_supply * founders_stake # initial supply of staking vouchers

initial_ixo_price = .10 # average IXO price from ixo.world Euro cents
initial_ixos_price = .20 # staking voucher price from AMM in Euro cents

ixo_delegation_distribution_reserve = 1e6
ixo_delegation_distribution_supply = 1e6/initial_ixo_price #200e6 # 5 strategic delegations worth Euro 1 M

# Augmented bonding curve parameters
# Hatch state
d0 = initial_ixo_reserve/1e6 # million DAI
p0 = initial_ixo_price # DAI per tokens

R0 = d0*(1-theta) # million DAI
S0 = d0/p0

kappa = 1 # Bonding curve curvature - using an initial kappa value of 1 we can keep a constant token price

# Brokerage - Kyber Fed Price Reserve
# Policy to bound proposed conversion rate
sanity_rate = 10 # 1 DAI = 10 IXO
reasonable_difference = .1 # percent

# Target portfolio
# 50% DAI / 50% IXO

# Rebalancing
# Or: auto_instant - rebalance performed after every trade
time_spacing = 1 # time period after which to perform another rebalance
price_spacing_diff = .5 # percentage difference from last rebalance after which to perform another rebalan

# Pricing
maximum_spread = 0.02 # dictated by for example Kyber
minimum_spread = 0.005
spread = 0.01 # dependant on liquidity
quoted_price = 0 # TODO

# cadCAD configuration
time_steps = 24 # months, use days for more granularity

amm_initial_reserve, amm_initial_invariant, amm_initial_spot_price = initialize_bonding_curve(initial_ixo_
# cadCAD Parameter Sets
```

```
#####
```

```
# amm == automated market maker  
# brk == brokerage
```

```
# params = {  
#     'brokerage': {  
#         'sanity_rate': 10,  
#         'max_spread': .02,  
#     }  
# }
```

```
params = {  
    'sweep': [1], # for selecting parameter sweep from results  
    # Automated Market Maker parameters  
    'amm.kappa': [kappa],  
    'amm.invariant': [amm_initial_invariant],  
    'amm.tax_rate': [0],  
    # Brokerage  
    'brk.sanity_rate': [10], # 1 DAI = 10 IXX  
    'brk.reasonable_diff': [.5], # percent  
    'brk.rebalance_diff': [False], # percentage difference from last rebalance after which to perform another  
    'brk.max_spread': [.02], # price  
    'brk.min_spread': [.005],  
    'max_delegators': [20],  
    'max_validators': [20],  
    'max_service_providers': [20],  
    'max_participants': [60], # Sum of above  
}
```

Sigmoidal Participant Base Growth

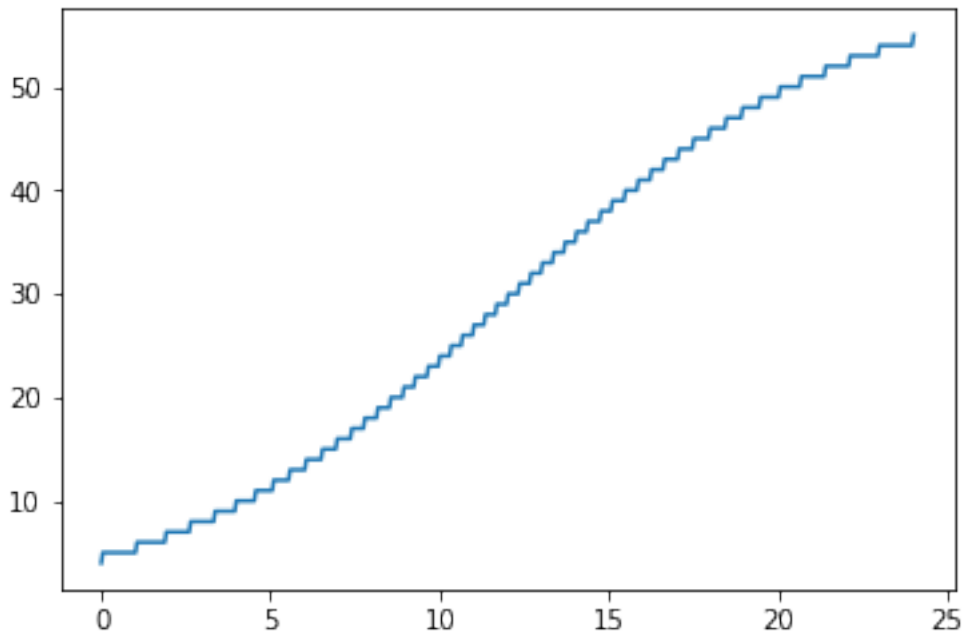
```
# Number of actors  
max_delegators = 20  
max_validators = 20  
max_service_providers = 20  
max_participants = max_delegators + max_validators + max_service_providers
```

```
# Sigmoidal growth  
default_inflection = time_steps/2  
default_slope = 1/5
```

```
def sigmoidal_growth(step, max_value, inflection=default_inflection, slope=default_slope):  
    y = np.divide(max_value, (1 + np.power(np.e, -np.multiply(np.subtract(x, inflection), slope))))  
    y = np.floor(y)  
    y[y < 0] = 0  
    return y
```

```
x = np.linspace(0, time_steps, 500)  
y = sigmoidal_growth(x, max_participants)
```

```
plt.plot(x, y)  
plt.show()
```



Initialize System and Network

```
# Node generators
def gen_new_brokerage(
    network,
    index,
    _params=params,
    pair=('IXO', 'DAI'),
    initial_supply=(0,0)):

    brokerage = {}
    brokerage['pair'] = pair
    brokerage['target'] = .5 # Target portfolio pair[0]:pair[1]
    brokerage['rate'] = _params['brk.sanity_rate'] #TODO
    brokerage['supply'] = {
        pair[0]: 0,
        pair[1]: 0,
    }
    brokerage['sanity_rate'] = _params['brk.sanity_rate']
    brokerage['reasonable_diff'] = _params['brk.reasonable_diff']
    brokerage['rebalance_diff'] = _params['brk.rebalance_diff']
    brokerage['max_spread'] = _params['brk.max_spread']
    brokerage['min_spread'] = _params['brk.min_spread']

    return brokerage

def initialize_network():
    network = nx.Graph()
    color_map = []

    brokers = 1
    for index in range(brokers):
        color_map.append(index)
        network.add_node(index, _type=Nodes.BROKERAGE)
        broker = gen_new_brokerage(network, index)
```

```

        network.node[index].update(broker)

    return (network, color_map)

# Policies
def driving_process(_params, step, sL, s):
    participants = get_node_ids_of_type(network, Nodes.PARTICIPANT)

    participant_growth = sigmoidal_growth(step, _params['max_participants'])
    new_participants = participant_growth - len(participants)

    if new_participants:
        new_participant_holdings = 100 #TODO
    else:
        new_participant_holdings = 0

    return({'new_participants': new_participants,
            'new_participant_holdings': new_participant_holdings})

def participants_decisions(params, step, sL, s):

    network = s['network']
    participants = get_nodes_by_type(network, 'participant')
    proposals = get_nodes_by_type(network, 'proposal')
    candidates = [j for j in proposals if network.nodes[j]['status']=='candidate']
    sensitivity = params['sensitivity']

    gain = .01
    delta_holdings={}
    proposals_supported={}
    for i in participants:

        engagement_rate = .3*network.nodes[i]['sentiment']
        if np.random.rand()<engagement_rate:

            force = network.nodes[i]['sentiment']-sensitivity
            delta_holdings[i] = network.nodes[i]['holdings']*gain*force

            support = []
            for j in candidates:
                booster = social_affinity_booster(network, j, i)
                #print(booster)
                affinity = network.edges[(i, j)]['affinity']+booster
                cutoff = sensitivity*np.max([network.edges[(i,p)]['affinity'] for p in candidates])
                if cutoff < .5:
                    cutoff = .5

                if affinity > cutoff:
                    support.append(j)

            proposals_supported[i] = support
        else:
            delta_holdings[i] = 0
            proposals_supported[i] = [j for j in candidates if network.edges[(i,j)]['tokens']>0 ]

    return({'delta_holdings':delta_holdings, 'proposals_supported':proposals_supported})

```

```

#Mechanisms for updating the state based on driving processes
def update_network(params, step, sL, s, _input):

    network = s['network']
    #    supply = s['supply']

    new_participants = _input['new_participants'] # Number of

    #    if new_participant:
    #        new_participant_holdings = _input['new_participant_holdings']
    #        network = gen_new_participant(network, new_participant_holdings)

    #    if new_proposal:
    #        network= gen_new_proposal(network,funds,supply,trigger_func )

    #    #update age of the existing proposals
    #    proposals = get_nodes_by_type(network, 'proposal')

    #    for j in proposals:
    #        network.nodes[j]['age'] = network.nodes[j]['age']+1
    #        if network.nodes[j]['status'] == 'candidate':
    #            requested = network.nodes[j]['funds_requested']
    #            network.nodes[j]['trigger'] = trigger_func(requested, funds, supply)
    #        else:
    #            network.nodes[j]['trigger'] = np.nan

    key = 'network'
    value = network

    return (key, value)

def increment_supply(params, step, sL, s, _input):

    supply = s['amm.supply']
    supply_arrival = _input['new_participant_holdings']

    #increment funds
    supply = supply + supply_arrival

    key = 'amm.supply'
    value = supply

    return (key, value)

def increment_reserve(params, step, sL, s, _input):

    supply = s['amm.supply']
    supply_arrival = _input['new_participant_holdings']

    #increment funds
    supply = supply + supply_arrival

    kappa = params['amm.kappa']
    V0 = params['amm.invariant']

```

```

R = reserve(supply, V0, kappa)

key = 'amm.reserve'
value = R

return (key, value)

def update_tokens(params, step, sL, s, _input):

    network = s['network']
    delta_holdings = _input['delta_holdings']
    proposals = get_nodes_by_type(network, 'proposal')
    candidates = [j for j in proposals if network.nodes[j]['status']=='candidate']
    proposals_supported = _input['proposals_supported']
    participants = get_nodes_by_type(network, 'participant')
    alpha = params['alpha']
    min_support = params['min_supp']

    for i in participants:
        network.nodes[i]['holdings'] = network.nodes[i]['holdings']+delta_holdings[i]
        supported = proposals_supported[i]
        total_affinity = np.sum([ network.edges[(i, j)]['affinity'] for j in supported])
        for j in candidates:
            if j in supported:
                normalized_affinity = network.edges[(i, j)]['affinity']/total_affinity
                network.edges[(i, j)]['tokens'] = normalized_affinity*network.nodes[i]['holdings']
            else:
                network.edges[(i, j)]['tokens'] = 0

            prior_conviction = network.edges[(i, j)]['conviction']
            current_tokens = network.edges[(i, j)]['tokens']
            network.edges[(i, j)]['conviction'] =current_tokens+alpha*prior_conviction

    for j in candidates:
        network.nodes[j]['conviction'] = np.sum([ network.edges[(i, j)]['conviction'] for i in participants ])
        total_tokens = np.sum([network.edges[(i, j)]['tokens'] for i in participants ])
        if total_tokens < min_support:
            network.nodes[j]['status'] = 'killed'

    key = 'network'
    value = network

    return (key, value)

def update_supply(params, step, sL, s, _input):

    supply = s['amm.supply']
    delta_holdings = _input['delta_holdings']
    delta_supply = np.sum([v for v in delta_holdings.values()])

    supply = supply + delta_supply

    key = 'amm.supply'
    value = supply

    return (key, value)

```



```

def update_reserve(params, step, sL, s, _input):

    supply = s['amm.supply']
    delta_holdings = _input['delta_holdings']
    delta_supply = np.sum([v for v in delta_holdings.values()])
    supply = supply + delta_supply

    kappa = params['amm.kappa']
    V0 = params['amm.invariant']

    #print("kappa="+str(kappa))
    R = reserve(supply, V0, kappa)

    key = 'amm.reserve'
    value = R

    return (key, value)

def update_price(params, step, sL, s, _input):

    supply = s['amm.supply']
    delta_holdings = _input['delta_holdings']
    delta_supply = np.sum([v for v in delta_holdings.values()])
    supply = supply + delta_supply

    kappa = params['amm.kappa']
    V0 = params['amm.invariant']

    R = reserve(supply, V0, kappa)
    price = spot_price(R, V0, kappa)

    key = 'amm.spot_price'
    value = price

    return (key, value)

```

Configure cadCAD

```

# cadCAD Initial State
#####
network, color_map = initialize_network()
nx.draw_kamada_kawai(network, node_color=color_map, font_weight='bold', with_labels=True)

initial_state = {
    'network': network,
    'amm.supply': initial_ixo_supply,
    'amm.reserve': amm_initial_reserve,
    'amm.spot_price': amm_initial_spot_price,
    # 'timestamp': '2019-01-01 00:00:00'
}

```



```
runs = 1 # Monte Carlo runs

#####
# Settings of general simulation parameters, unrelated to the system itself
# `T` is a range with the number of discrete units of time the simulation will run for;
# `N` is the number of times the simulation will be run (Monte Carlo runs)
# In this example, we'll run the simulation once (N=1) and its duration will be of 10 timesteps

simulation_parameters = config_sim({
    'T': range(time_steps),
    'N': runs,
    'M': params
})
simulation_parameters

[{'N': 1,
  'T': range(0, 24),
  'M': {'sweep': 1,
        'amm.kappa': 1,
        'amm.invariant': 10.0,
        'amm.tax_rate': 0,
        'brk.sanity_rate': 10,
        'brk.reasonable_diff': 0.5,
        'brk.rebalance_diff': False,
        'brk.max_spread': 0.02,
        'brk.min_spread': 0.005,
        'max_delegators': 20,
        'max_validators': 20,
        'max_service_providers': 20,
        'max_participants': 60}}]
```

```

# Brokerage framework (see Uniswap/Kyber)
# Liquidity provision
def add_liquidity():
    return False

```

```

def remove_liquidity():
    return False

```

```

def get_input_price():
    return False

```

```

def get_output_price():
    return False

```

```

# Token trading
def collateralToToken():
    return False

```

```

def tokenToCollateral():
    return False

```

```

def tokenToToken():
    return False

```

Perform Simulation

```

# ts_format = '%Y-%m-%d %H:%M:%S'
# t_delta = timedelta(days=30, minutes=0, seconds=0)
# def time_model(_g, step, sL, s, _input):
#     y = 'time'
#     x = ep_time_step(s, dt_str=s['time'], fromat_str=ts_format, _timedelta=t_delta)
#     return (y, x)

```

```

# exogenous_states = {
#     'time': time_model
# }
# env_processes = {}

```

```

partial_state_update_blocks = [
    { # Generate new participants / new bonds
      'policies': {
        'generate': driving_process,
      },
      'variables': {
        'network': update_network,
        'amm.supply': increment_supply,
        'amm.reserve': increment_reserve,
      }
    },
    { # Participants provide liquidity via brokerage
      'policies': {
        'provide': provide_liquidity,
      },
      'variables': {
        'network': update_brokerage,
      }
    },
]

```

```

{ # Brokerage rebalance
  'policies': {
    'rebalance': brokerage_rebalance,
  },
  'variables': {
    'network': update_brokerage,
  }
},
{ # Accounting
  'policies': {
    'participants_act': participants_decisions,
  },
  'variables': {
    'network': update_holdings,
    'amm.supply': update_supply,
    'amm.reserve': update_reserve,
    'amm.spot_price': update_price,
  }
}
]

append_configs(
  initial_state=initial_state, #dict containing variable names and initial values
  partial_state_update_blocks=partial_state_update_blocks, #dict containing state update functions
  sim_configs=simulation_parameters, #dict containing simulation parameters
  # raw_exogenous_states=exogenous_states,
  # env_processes=env_processes
)

# sys.setrecursionlimit(5000)
from cadCAD import configs

exec_mode = ExecutionMode()
exec_context = ExecutionContext(context=exec_mode.multi_proc)
run = Executor(exec_context=exec_context, configs=configs)

i = 0
verbose = False
results = {}
for raw_result, tensor_field in run.execute():
  result = pd.DataFrame(raw_result)
  if verbose:
    print()
    print(f"Tensor Field: {type(tensor_field)}")
    print(tabulate(tensor_field, headers='keys', tablefmt='psql'))
    print(f"Output: {type(result)}")
    print(tabulate(result, headers='keys', tablefmt='psql'))
    print()
  results[i] = {}
  results[i]['result'] = result
  results[i]['simulation_parameters'] = simulation_parameters[i]
  i += 1

```

Augumented bonding curve Mathematics

Hatch Sale Equations

- Initial Raise d_0 Dai

- Initial Reserve is $R_0 = (1 - \theta)d_0$
- Hatch sale Price p_0 (dai per token) determines the initial supply: $S_0 = d_0/p_0$
- Power Function Invariant shape: $V(R, S) = \frac{S^\kappa}{R}$
- Price function (DAI per Token): $P(R) = \frac{\kappa R^{(\kappa-1)/\kappa}}{V_0^{1/\kappa}}$
- Supply function (tokens): $S(R) = \sqrt[\kappa]{V_0 R}$
- Reserve function (xDAI): $\$R(S) = \frac{S^\kappa}{V_0}$
- The invariant coef: $V_0 = V(R_0, S_0) = \frac{S_0^\kappa}{R_0} = \left(\frac{1}{p_0(1-\theta)}\right)^\kappa R_0^{\kappa-1}$
- The post hatch price: $p_1 = P(R_0) = \frac{\kappa R_0^{(\kappa-1)/\kappa}}{V_0^{1/\kappa}} = \kappa R_0^{(\kappa-1)/\kappa} \cdot (1-\theta)p_0 \cdot R_0^{-(\kappa-1)/\kappa} = \kappa(1-\theta)p_0$
- The Return factor: $\frac{p_1}{p_0} = \kappa(1-\theta)$

Invariant Preserving Deposit-to-Mint

- Deposit ΔR x dai
- Conservation equation: $V(R + \Delta R, S + \Delta S) = \frac{(S + \Delta S)^\kappa}{R + \Delta R} = V_0$
- Derived Mint equation: $\Delta S = \text{mint}(\Delta R; (R, S)) = \sqrt[\kappa]{V_0(R + \Delta R)} - S$
- Realized Price is: $\bar{P}(\Delta R) = \frac{\Delta R}{\Delta S} = \frac{\Delta R}{\sqrt[\kappa]{V_0(R + \Delta R)} - \sqrt[\kappa]{V_0(R)}} \rightarrow \left(\frac{\partial S(R)}{\partial R}\right)^{-1}$ as $\Delta R \rightarrow 0$
- The limiting price is the spot price: $\lim_{\Delta R \rightarrow 0} \bar{P}(\Delta R) = \left(\frac{\partial S(R)}{\partial R}\right)^{-1} = \left(\frac{V_0^{1/\kappa} \cdot R^{1/\kappa-1}}{\kappa}\right)^{-1} = \frac{\kappa R^{1-1/\kappa}}{V_0^{1/\kappa}} = \frac{\kappa R^{(\kappa-1)/\kappa}}{V_0^{1/\kappa}} = P(R)$

Invariant Preserving Burn-to-Withdraw

- Burn ΔS tokens
- Conservation equation: $V(R - \Delta R, S - \Delta S) = \frac{(S - \Delta S)^\kappa}{R - \Delta R} = V_0$
- Derived Withdraw equation: $\Delta R = \text{withdraw}(\Delta S; (R, S)) = R - \frac{(S - \Delta S)^\kappa}{V_0}$
- Realized Price is: $\bar{P}(\Delta S) = \frac{\Delta R}{\Delta S} = \frac{R - \frac{(S - \Delta S)^\kappa}{V_0}}{\Delta S} = \frac{R}{\Delta S} - \frac{(S - \Delta S)^\kappa}{V_0 \Delta S} \rightarrow \frac{\partial R(S)}{\partial S}$ as $\Delta S \rightarrow 0$
- The limiting price is the spot price: $\lim_{\Delta S \rightarrow 0} \bar{P}(\Delta S) = \frac{\partial R(S)}{\partial S} = \frac{\kappa S^{\kappa-1}}{V_0} = \frac{\kappa \cdot (\sqrt[\kappa]{V_0 R})^{\kappa-1}}{V_0} = \frac{\kappa R^{(\kappa-1)/\kappa}}{V_0^{1/\kappa}} = P(R)$
- Given friction coef ϕ
 - sent to burning agent address: $\Delta R_{\text{agent}} = (1 - \phi)\Delta R$
 - sent to the funding pool address: $\Delta R_{\text{pool}} = \phi\Delta R$
 - due to the friction the true realized price for the agent is $(1 - \phi) \cdot \bar{P}(\Delta S)$
 - due to the friction the true return factor post withdraw is: $\kappa(1 - \theta)(1 - \phi)$

Bonding curve parameter analysis

`V0 = invariant(R0,S0,kappa)`

`reserve = np.arange(0,100,.01)`

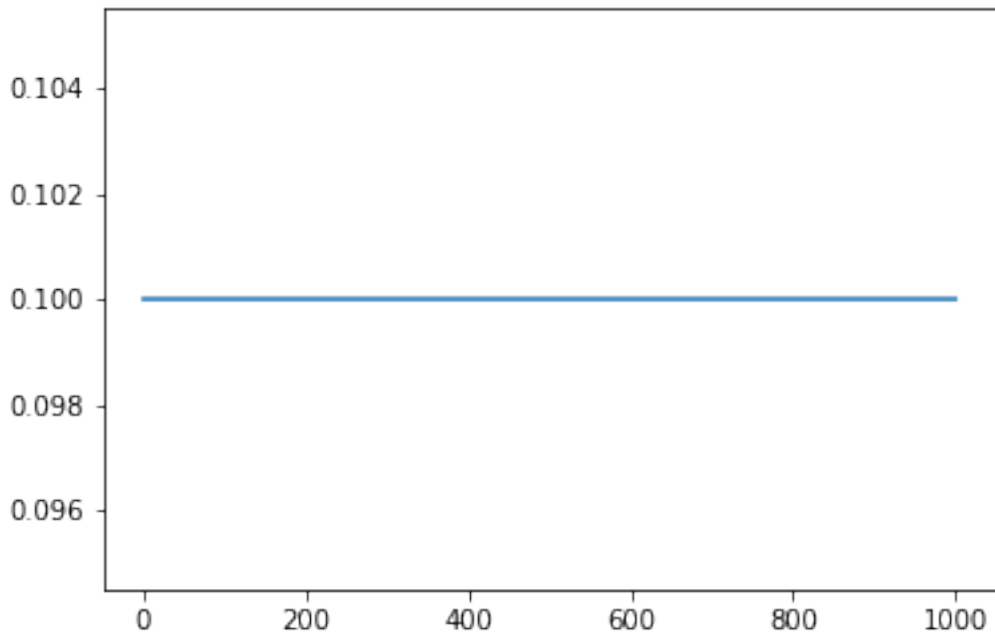
`supp = np.array([supply(r, V0, kappa) for r in reserve])`

`price = np.array([spot_price(r, V0, kappa) for r in reserve])`

`fig, axs = plt.subplots()`

`axs.plot(supp, price)`

`[<matplotlib.lines.Line2D at 0x7fa0c3b15780>]`



```

V0 = invariant(R0,S0,kappa)

reserve = np.arange(0,100,.01)
supp = np.array([supply(r, V0, kappa) for r in reserve])
price = np.array([spot_price(r, V0, kappa) for r in reserve])

fig, ax1 = plt.subplots()

color = 'tab:red'
ax1.set_xlabel('Reserve (Millions of xDAI)')
ax1.set_ylabel('Supply (Millions of Tokens)', color=color)
ax1.plot(reserve, supp, '--', color=color)
ax1.tick_params(axis='y', labelcolor=color)

ax2 = ax1.twinx() # instantiate a second axes that shares the same x-axis

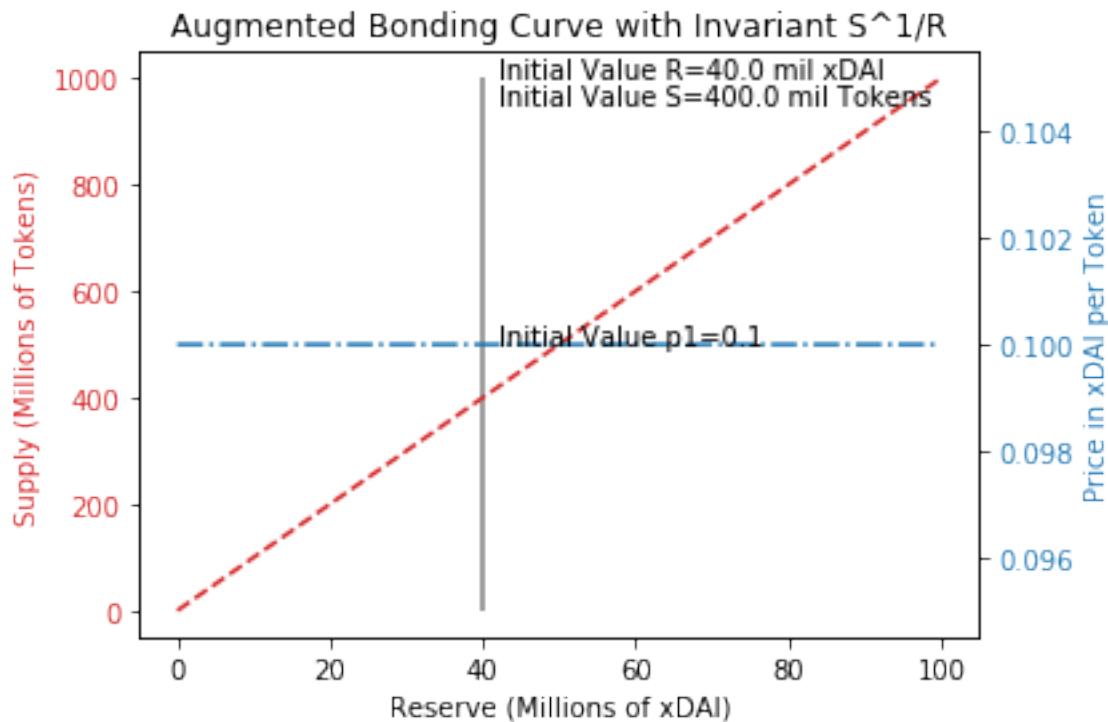
color = 'tab:blue'
ax2.set_ylabel('Price in xDAI per Token', color=color) # we already handled the x-label with ax1
ax2.plot(reserve, price, '-.', color=color)
ax2.tick_params(axis='y', labelcolor=color)

ax1.vlines(R0,0,supp[-1], alpha=.5)
ax1.text(R0+.02*reserve[-1], supp[-1], "Initial Value R="+str(int(100*R0)/100)+" mil xDAI")
ax1.text(R0+.02*reserve[-1], .95*supp[-1], "Initial Value S="+str(S0)+" mil Tokens")
#ax1.hlines(S0,0,R0)

ax2.text(R0+.02*reserve[-1], price[3], "Initial Value p1="+str(int(100*spot_price(R0,V0,kappa))/100))

plt.title('Augmented Bonding Curve with Invariant S~'+str(kappa)+'/'R')
fig.tight_layout() # otherwise the right y-label is slightly clipped
plt.show()

```



```
fig, ax1 = plt.subplots()

cp = 100
color = 'tab:red'
ax1.set_xlabel('Supply (Millions of Tokens)')
ax1.set_ylabel('Reserve (Millions of xDAI)', color=color)
ax1.plot(supp[cp:], reserve[cp:], '--', color=color)
ax1.tick_params(axis='y', labelcolor=color)

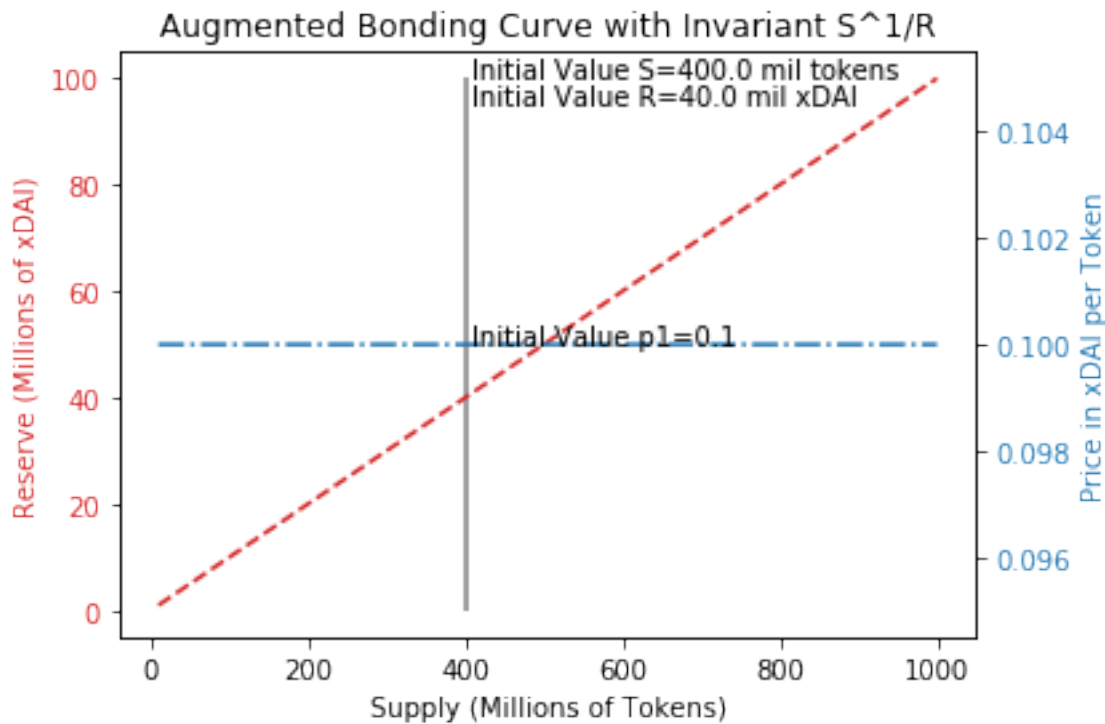
ax2 = ax1.twinx() # instantiate a second axes that shares the same x-axis

color = 'tab:blue'
ax2.set_ylabel('Price in xDAI per Token', color=color) # we already handled the x-label with ax1
ax2.plot(supp[cp:], price[cp:], '-.', color=color)
ax2.tick_params(axis='y', labelcolor=color)

ax1.vlines(S0,0,reserve[-1], alpha=.5)
ax1.text(S0*1.02, reserve[-1], "Initial Value S="+str(int(100*S0)/100)+" mil tokens")
ax1.text(S0*1.02, .95*reserve[-1], "Initial Value R="+str(R0)+" mil xDAI")
#ax1.hlines(S0,0,R0)

ax2.text(S0*1.02, price[3], "Initial Value p1="+str(int(100*spot_price(R0,V0,kappa))/100))

plt.title('Augmented Bonding Curve with Invariant S1/R')
fig.tight_layout() # otherwise the right y-label is slightly clipped
plt.show()
```



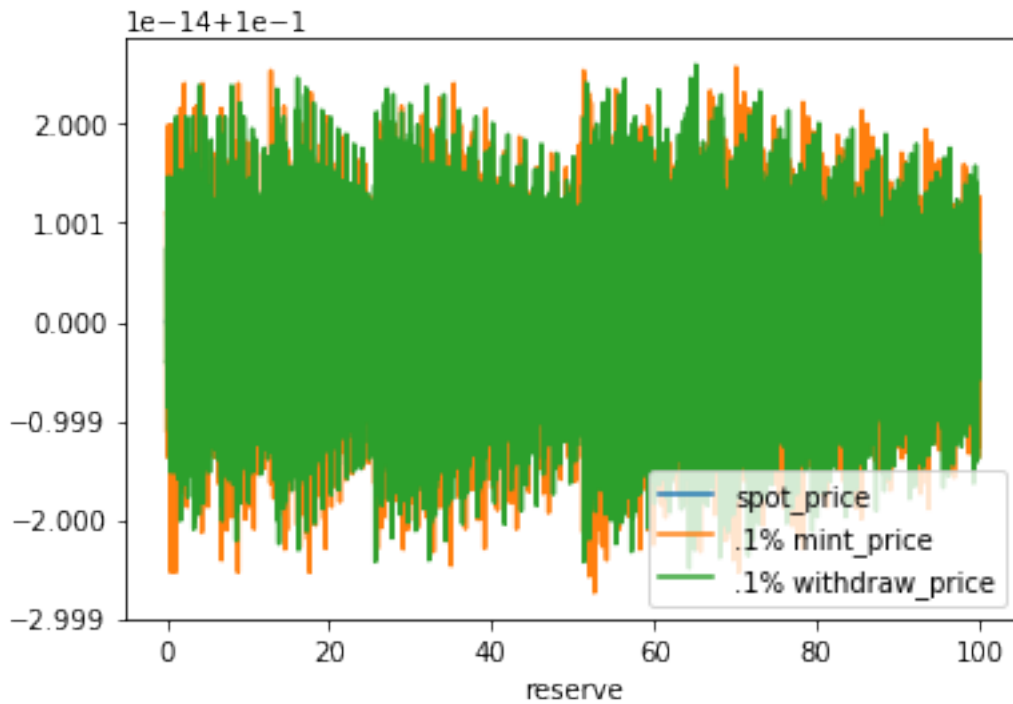
```
#given V0 and kappa
#sweep the reserve
reserve = None
reserve = np.arange(.01,100,.01)
price = np.array([spot_price(r,V0,kappa) for r in reserve])

#realized price for withdrawing burning .1% of tokens
withdraw_price=[withdraw(supply(r,V0,kappa)/1000, r,supply(r,V0,kappa), V0, kappa)[1] for r in reserve]

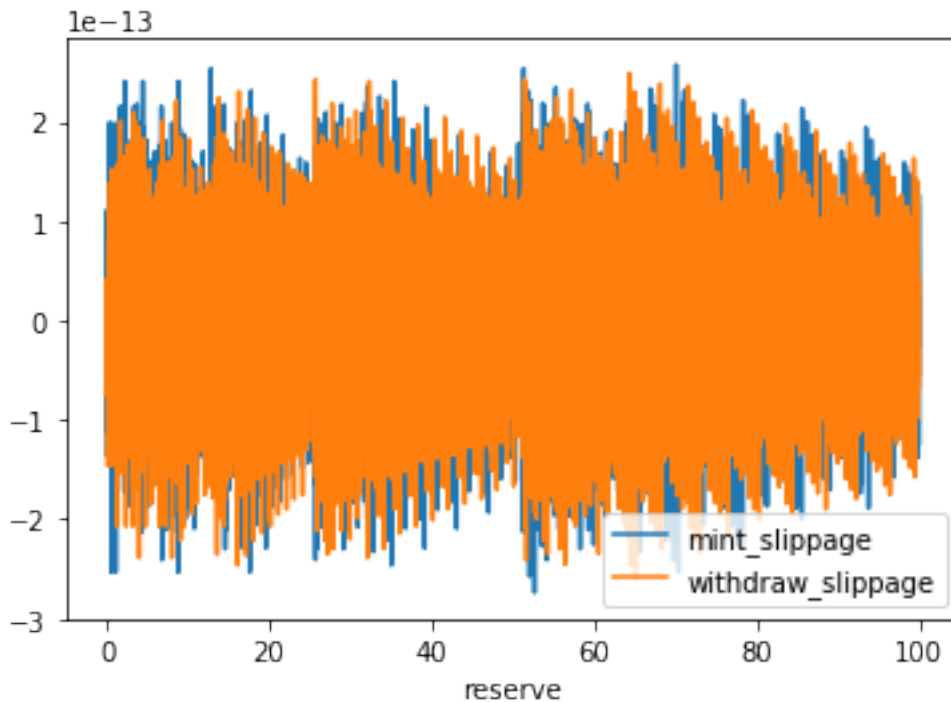
#realized price for depositing .1% more Xdai into the reserve
mint_price=[mint(r/1000, r, supply(r,V0,kappa), V0, kappa)[1] for r in reserve]

pdf = pd.DataFrame({'reserve':reserve, 'spot_price':price, '.1% mint_price':mint_price, '.1% withdraw_price':withdraw_price})
pdf.plot(x='reserve')
```

<matplotlib.axes._subplots.AxesSubplot at 0x7fa0c3abd9b0>



```
pdf['mint_slippage'] = (pdf['.1% mint_price']-pdf['spot_price'])/pdf['spot_price']
pdf['withdraw_slippage'] = (pdf['spot_price']-pdf['.1% withdraw_price'])/pdf['spot_price']
pdf.plot(x='reserve', y = ['mint_slippage', 'withdraw_slippage'])#, logy=True)
<matplotlib.axes._subplots.AxesSubplot at 0x7fa0c3c5e5f8>
```



```
#given V0 and kappa
R = 20
S = supply(R,V0,kappa)
p = spot_price(R,V0,kappa)
```

```

#sweep the transaction fraction
TXF = np.logspace(-6, 0, num=1000)

#realized price for withdrawing burning .1% of tokens
withdraw_price2=[withdraw(S*txf, R,S, V0, kappa)[1] for txf in TXF]

#realized price for depositing .1% more Xdai into the reserve
mint_price2=[mint(R*txf, R,S, V0, kappa)[1] for txf in TXF]

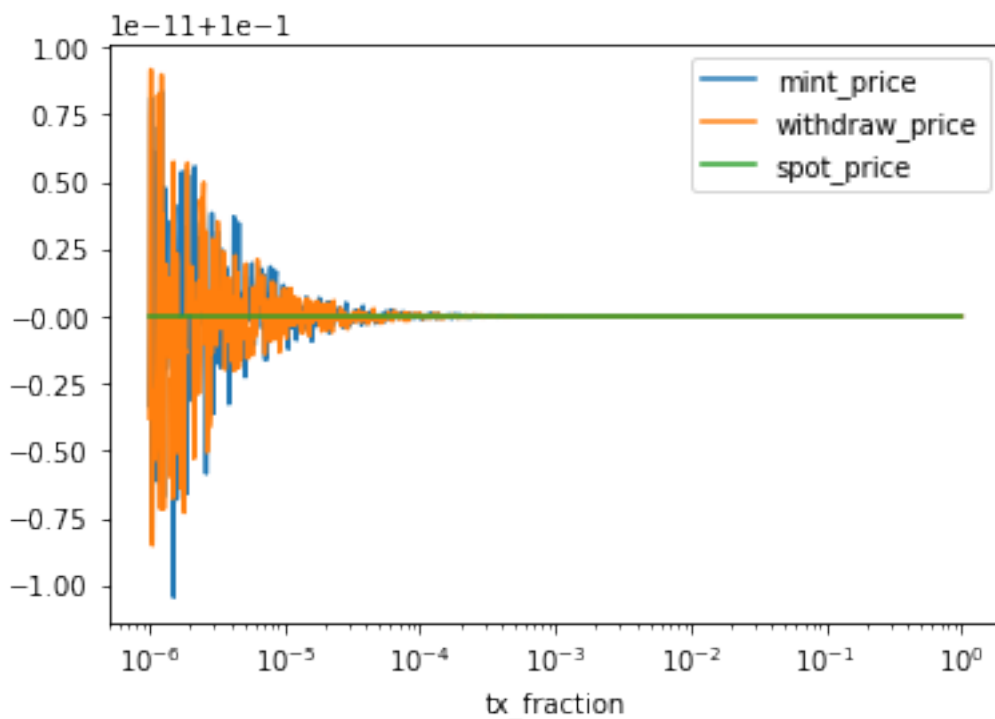
print(S)

200.0

pdf2 = pd.DataFrame({'tx_fraction':TXF, 'spot_price':p*np.ones(len(TXF)), 'mint_price':mint_price2,'withdraw_price':withdraw_price2})
pdf2.plot(x='tx_fraction',y=['mint_price','withdraw_price','spot_price'], logx=True)

<matplotlib.axes._subplots.AxesSubplot at 0x7fa0c3b82b38>

```



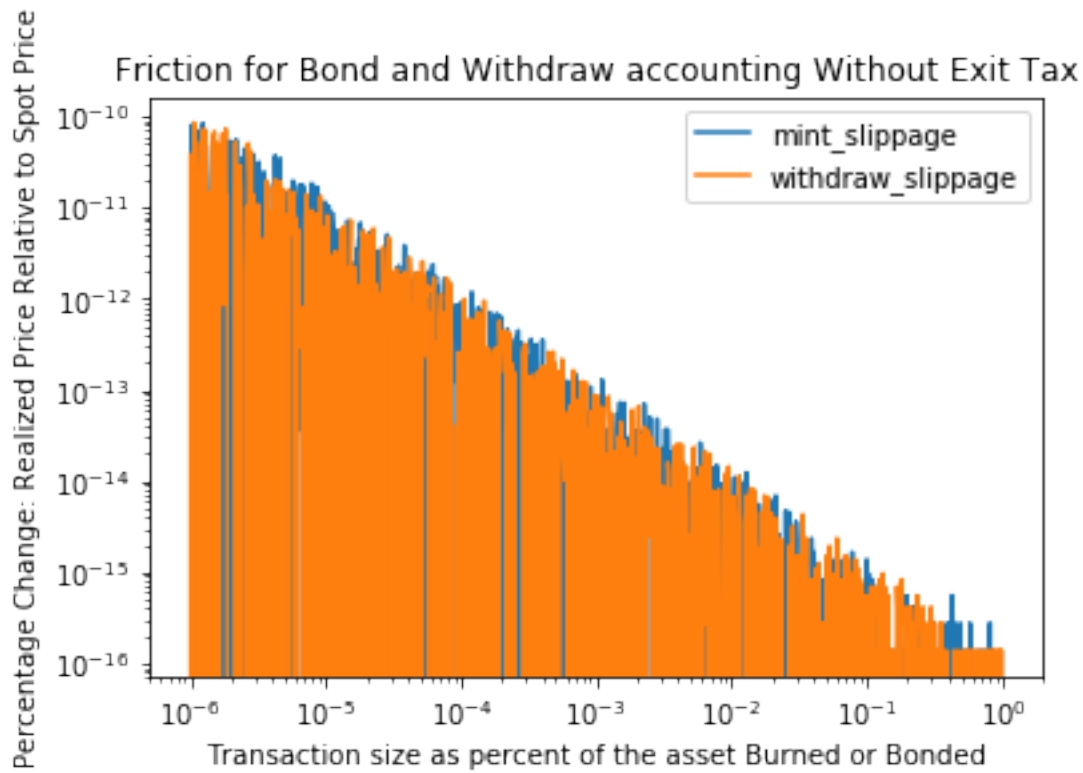
```

pdf2['mint_slippage'] = (pdf2['mint_price']-pdf2['spot_price'])/pdf2['spot_price']
pdf2['withdraw_slippage'] = (pdf2['spot_price']-pdf2['withdraw_price'])/pdf2['spot_price']

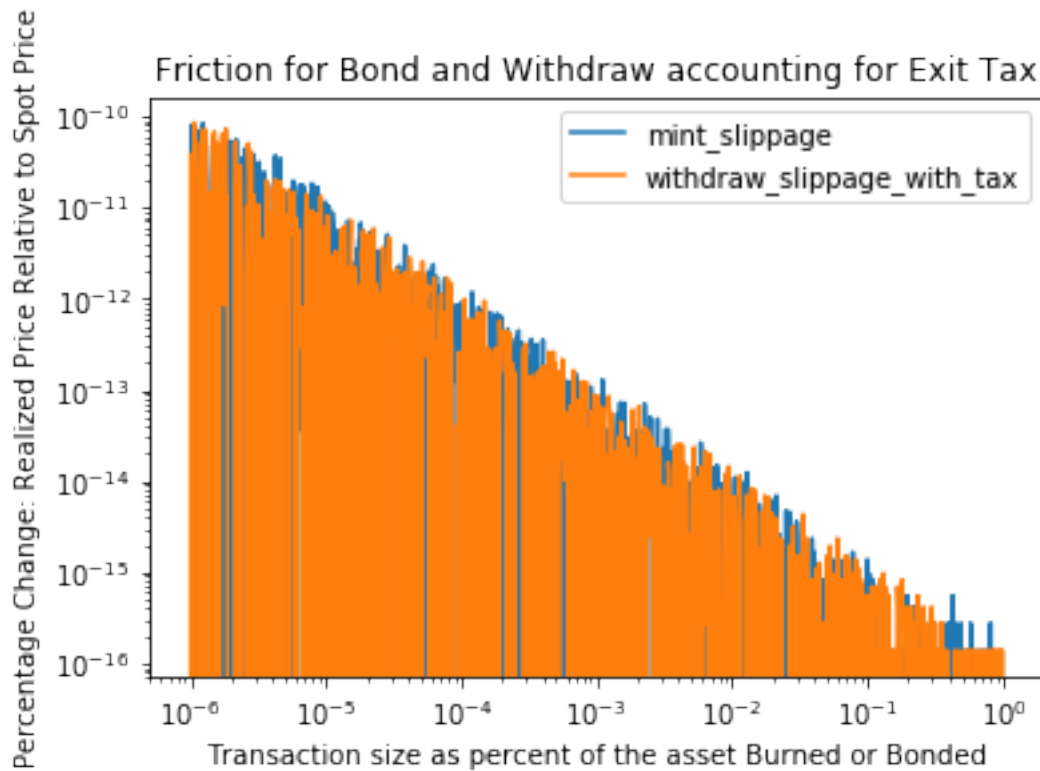
pdf2.plot(x='tx_fraction', y = ['mint_slippage', 'withdraw_slippage'], logx=True, logy=True)
plt.title("Friction for Bond and Withdraw accounting Without Exit Tax")
plt.ylabel("Percentage Change: Realized Price Relative to Spot Price")
plt.xlabel("Transaction size as percent of the asset Burned or Bonded")

Text(0.5, 0, 'Transaction size as percent of the asset Burned or Bonded')

```



```
pdf2['withdraw_slippage_with_tax'] = pdf2['withdraw_slippage']+tax
pdf2.plot(x='tx_fraction', y = ['mint_slippage', 'withdraw_slippage_with_tax'], logx=True, logy=True)
plt.title("Friction for Bond and Withdraw accounting for Exit Tax")
plt.ylabel("Percentage Change: Realized Price Relative to Spot Price")
plt.xlabel("Transaction size as percent of the asset Burned or Bonded")
Text(0.5, 0, 'Transaction size as percent of the asset Burned or Bonded')
```



```

Kappa_List = [1,2,4,6,8]
for kappa in Kappa_List:

    V0 = invariant(R0,S0,kappa)

    reserve = np.arange(0,100,.01)
    supp = np.array([supply(r, V0, kappa) for r in reserve])
    price = np.array([spot_price(r, V0, kappa) for r in reserve])

    fig, ax1 = plt.subplots()

    color = 'tab:red'
    ax1.set_xlabel('Reserve (Millions of xDAI)')
    ax1.set_ylabel('Supply (Millions of Tokens)', color=color)
    ax1.plot(reserve, supp, '--', color=color)
    ax1.tick_params(axis='y', labelcolor=color)

    ax2 = ax1.twinx() # instantiate a second axes that shares the same x-axis

    color = 'tab:blue'
    ax2.set_ylabel('Price in xDAI per Token', color=color) # we already handled the x-label with ax1
    ax2.plot(reserve, price, '-', color=color)
    ax2.tick_params(axis='y', labelcolor=color)

    ax1.vlines(R0,0,supp[-1], alpha=.5)
    ax1.text(R0+.02*reserve[-1], supp[-1], "Initial Value R="+str(int(100*R0)/100)+" mil xDAI")
    ax1.text(R0+.02*reserve[-1], .95*supp[-1], "Initial Value S="+str(int(100*S0)/100)+" mil Tokens")
    #ax1.hlines(S0,0,R0)

    ax2.text(R0+.02*reserve[-1], price[3], "Initial Value p1="+str(int(1000*spot_price(R0,V0,kappa))/1000)

```

```
plt.title('Augmented Bonding Curve with Invariant  $S^{1/R}$ ')
fig.tight_layout() # otherwise the right y-label is slightly clipped
plt.show()
```

