Week 11 Assignment and solution

1. Interpolation is a process for
   a) extracting feasible data set from a given set of data.
   b) finding a value between two points on a line or curve.
   c) removing unnecessary points from a curve.
   d) all of the mentioned

   Solution: (b) Interpolation is the process of finding a value between two points on a line or curve.

2. Linear interpolation is used to find
   a) a curve that touches all data points
   b) a straight line that touches maximum number of points
   c) a straight line that gives least sum error from a given set of points
   d) a mathematical model of that passes through all the given data points

   Solution: (c) linear interpolation is a method of curve fitting using linear polynomials to construct new data points within the range of a discrete set of known data points. Linear interpolant is the straight line between data points such that the sum error of all the given points are minimized.

3. Given two data points $(a, f(a))$ and $(b, f(b))$, the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by

   a) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$

   b) $f_1(x) = \frac{x}{a-b} f(a) + \frac{x}{b-a} f(b)$

   c) $f_1(x) = f(a) + \frac{f(b)-f(a)}{b-a}(b-a)$

   d) $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$

   Solution: (d)

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

$$\begin{aligned} f_1(x) &= \sum_{i=0}^{1} L_i(x) f(x_i) \\ &= L_0(x) f(x_0) + L_1(x) f(x_1) \\ &= L_0(x) f(a) + L_1(x) f(b) \end{aligned}$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$\begin{aligned} L_0(x) &= \prod_{\substack{j=0 \\ j \neq 0}}^{1} \frac{x - x_j}{x_0 - x_j} \\ &= \frac{x - x_1}{x_0 - x_1} \\ &= \frac{x - b}{a - b} \end{aligned}$$

$$L_1(x) = \prod_{\substack{j=0 \\ j \neq 1}}^{1} \frac{x - x_j}{x_1 - x_j}$$

$$= \frac{x - x_0}{x_1 - x_0}$$

$$= \frac{x - a}{b - a}$$

$$f_1(x) = L_0(x) f(a) + L_1(x) f(b)$$

$$= \frac{x - b}{a - b} f(a) + \frac{x - a}{b - a} f(b)$$

4. Efficiency of the trapezoidal rule increases when
   a) integration is carried out for sufficiently large range
   b) instead of trapezoid, we take rectangular approximation function
   c) number of segments are increased
   d) integration is performed for only integer range

Solution: (c) Approximation increases with the increase of the number of segments between the lower and upper limit.

5. The following function is an example of recursion in C. If the argument passed to the function (i.e. n) is 5, find the output.

```
int fun2(int n)
{
  if(n == 0)
    return 0;
  fun2(n/2);
  printf("%d", n%2);
 return 0;
}
```

   a) 2.5
   b) 101
   c) 2.0000
   d) 1

Solution: (b) The function finds the binary equivalent of the number n. Thus, the output is 101.

6. What is the output of the following C code?

```
#include <stdio.h>
main()
{
    static int x = 3;
    x++;
    if (x <= 5)
    {
        printf("hi ");
        main();
    }
}
```

   a) hi
   b) hi hi
   c) infinite times hi
   d) compiler error

Solution: (b) As x is declared as static integer, hi is printed for x=4 and x=5. Re-declaring the statement “static int x=3” does not change the present value of x. Thus, two times hi will be printed.

7. Recursion uses which queuing structure?
   a) LIFO

b) FIFO
c) Weighted Fair Queuing
d) Class Based Weighted Fair Queuing

Solution: (a) Recursion uses LIFO queuing structure.

8. How many stars will be printed if the following program is called from the main function with argument n=5?

```
void func(int n)
{
   int i = 0;
   if (n > 1)
    func(n-1);
  for (i = 0; i < n; i++)
    printf(" * ");
}
```

Solution: The func is called 5 times and each times '*' is printed n number of times. Thus, n(n+1)/2 i.e. 15 times '*' will be printed.

9. How is error measured in case of interpolation?
   a) Distance between the data points
   b) Square of the distance between the data points
   c) Half the distance between the data points
   d) None of the mentioned

   Solution: (b) Square of the distance between the data points

10. Lagrange polynomial are used for
    a) Linear Interpolation
    b) Spline Interpolation
    c) Polynomial Interpolation
    d) None

Solution: (c) Polynomial Interpolation

11. Using Bisection method, negative root of $x^3 - 4x + 9 = 0$ correct to three decimal places is
    a) -2.506
    b) -2.706
    c) -2.406
    d) None

Solution: (b) -2.706

12. The value of $\int_{2.5}^{4} \ln(x)\,dx$ calculated using the Trapezoidal rule with five subintervals is (* range is given in output rather than single value to avoid approximation error)
    a) 1.45 to 1.47
    b) 1.74 to 1.76
    c) 1.54 to 1.56
    d) 1.63 to 1.65

Solution: (b) 1.74 to 1.76

13. The real root of the equation $5x - 2\cos(x) - 1 = 0$ (up to two decimal accuracy) is [You can use any method known to you. A range is given in output rather than single value to

avoid approximation error]

a) 0.53 to 0.56
b) 0.45 to 0.47
c) 0.35 to 0.37
d) 0.41 to 0.43

Solution: (a) 0.53 to 0.56

14. What will be the output of the following code segment?

```
void function(int n)
{
  if(n == 0)
  return;
  printf("%d ",n);
  function(n-1);
}
int main()
{
  function(10);
  return 0;
}
```

a) 10
b) 1
c) 10 9 8.....1
d) 10 9 8....0

Solution: (c)
It will print from 10 to 1.

15. Consider the same recursive C function that takes two arguments

```
unsigned int func(unsigned int n, unsigned int r)
{
 if (n  > 0) return (n%r +  func(n/r, r));
 else return 0;
}
```

What is the return value of the function func when it is called as func(513, 2)?

Solution: 2

func(513, 2) will return 1 + func(256, 2). All subsequent recursive calls (including func(256, 2)) will return 0 + func(n/2, 2) except the last call func(1, 2) . The last call func(1, 2) returns 1. So, the value returned by func(513, 2) is 1 + 0 + 0.... + 0 + 1=2.