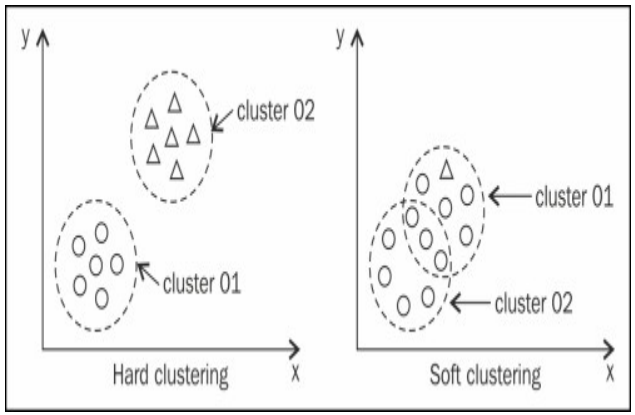
Unsupervised Learning: Clustering

ML Instruction Team, Fall 2022

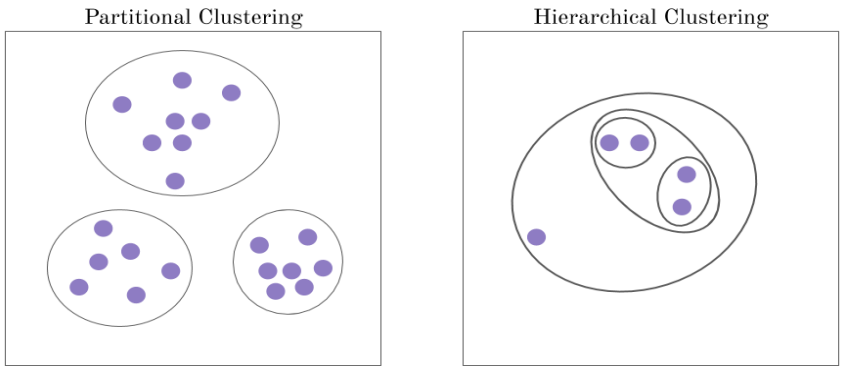
CE Department
Sharif University of Technology

Categories of Clustering

- Hard: Where each point is assigned to exactly one cluster.
- Soft: Where each point can be assigned to several clusters with certain probabilities that add up to 1.
- Partitional: Where all clusters are on the same level.
- Hierarchical Where the clustering is done from fine to coarse by merging points successively to larger and larger clusters (agglomerative hierarchical clustering).



(a) Hard vs Soft Clustering, Source



(b) Partitional vs Hierarchical Clustering, Source

Figure: Different clustering techniques

Hard Partitional Clustering: K-Means

- The idea is to represent each cluster k by a center point c_k and assign each data point x_n to one of the clusters k.
- The center points and the assignment are then chosen such that the mean squared distance between data points and center points

$$J = \sum_{n=1}^{N} \sum_{n \in c_k} r_{nk} \|x_n - c_k\|^2$$

  is minimized.
- Two step minimization:
  - First we keep the assignment fixed and optimize the position of the center points.
  - Next, we keep the center points fixed and optimize the assignment.

Hard Partitional Clustering: K-Means

- If the assignment is fixed, it is easy to show that the optimal choice of the center positions is given by:

$$c_k = \frac{1}{N_k} \sum_{n \in c_k} x_n$$

- If the center points are fixed, it is obvious that each point should be assigned to the nearest center position.
- The K-means algorithm now consists of applying these two optimizations in turn until convergence.

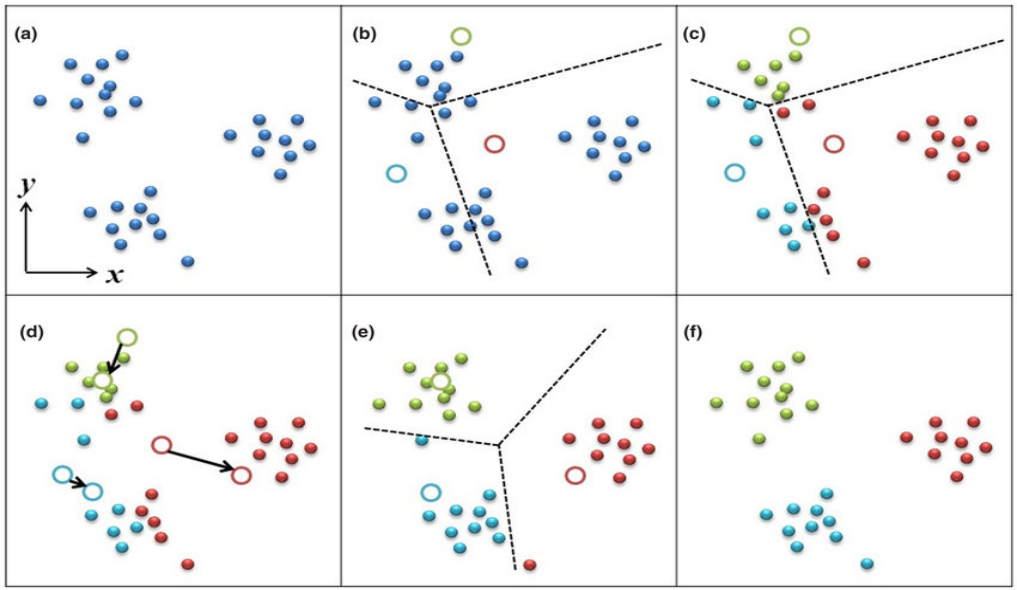


Figure: K-means in practice, Source

Hard Partitional Clustering: K-Means

- The initial center locations?
- Convergence Guaranty?
- Drawbacks:
  - A paramount drawback of this and many other clustering algorithms is that the number of clusters is not determined.
  - The result of the algorithm is not necessarily a global optimum of the error function.
- Solutions
  - Any solution for the first drawback?
  - Run the algorithm several times with different initial center locations and pick the best result.

Hard Partitional Clustering: K-Means

- To evaluate the quality of a clustering a plethora of validity indices have been proposed, one of which is Davies-Bouldin (DB) index.
- Cluster Dispersion: Which can be interpreted as a generalized standard deviation.

$$\delta_k := \sqrt{\frac{1}{N_k} \sum_{n \in c_k} \|x_n - c_k\|^2}$$

- Cluster Similarity: Is defined such that two clusters are considered similar if they have large dispersion relative to their distance.

$$S_{kl} := \frac{\delta_k + \delta_l}{\|c_k - c_l\|}$$

- Considering aforementioned definitions, an overall validation of the clustering can be done by the DB index:

$$V_{DB} := \frac{1}{k} \sum_{k=1}^{K} \max_{l \neq k} S_{kl}$$

Hard Partitional Clustering: K-Means

- The DB index does not systematically depend on K and is therefore suitable to find the best optimal number of clusters.

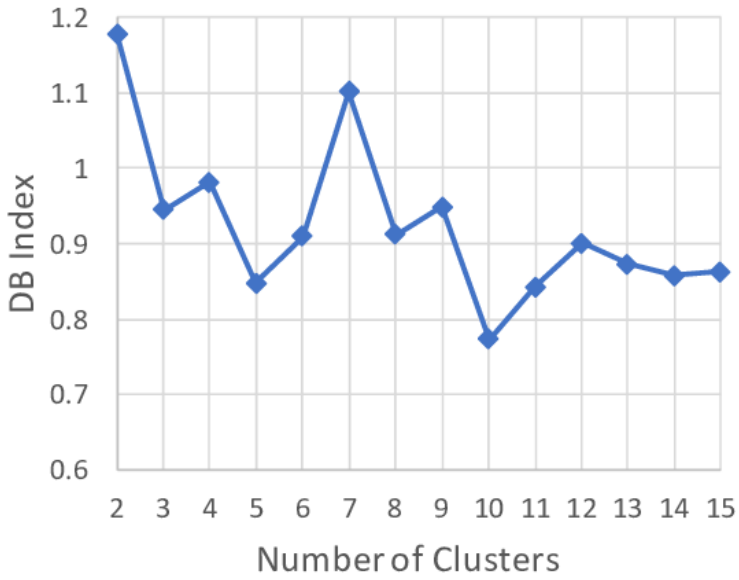


Figure: DB Index, Source

Thank You!

Any Question?