

# Loss

ML Instruction Team, Fall 2022

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# Classification Loss

- Both the **cross-entropy** and the **Kullback–Leibler** (KL) divergence measures the distance between two probability distributions  $P$  and  $Q$ .

$$H(P, Q) = - \sum_x P(x) \log Q(x)$$

$$KL(P \mid Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)} = H(P, Q) - H(P)$$

- In the above formula,  $H(P) = H(P, P)$  is the entropy of the distribution  $P$ , which is a constant term.
- Hence, it turns out that the minimization of KL divergence is equivalent to the minimization of cross-entropy.

# Classification Loss

- Logarithmic loss indicates how close a distribution of prediction probability comes to the real probability distribution of the data in [binary classification](#).

$$H(p, q) = -\frac{1}{m} \sum_{i=1}^m y_i \log(p(y_i)) + (1 - y_i) \log(1 - p(y_i))$$

where  $y_i \in \{0, 1\}$

- Cross-entropy loss indicates how close a distribution of prediction probability comes to the real probability distribution of the data in [multilabel classification](#).

$$H(p, q) = -\frac{1}{m} \sum_{i=1}^m \sum_{j=1}^k y_j^{(i)} \log(p(y_j^{(i)}))$$

where  $y_j^{(i)}$  and  $p(y_j^{(i)})$  is respectively the true and predicted probability of the class  $i$ -th of the sample  $j$ -th.

# Classification Loss

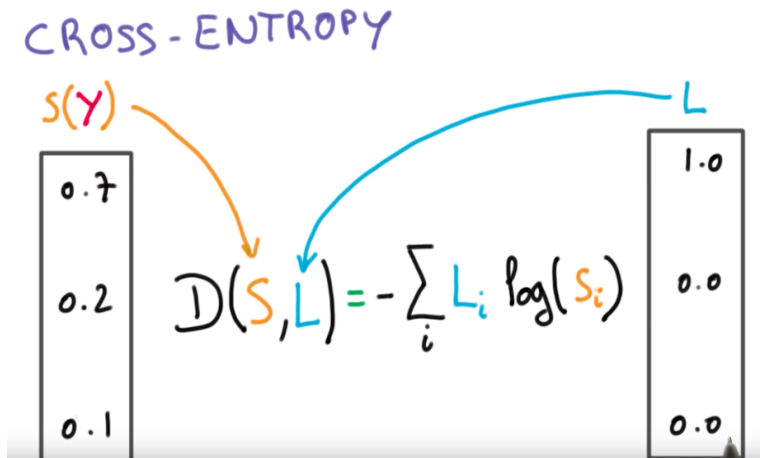


Figure: Cross Entropy Evaluation, [Source](#)

# Regression Loss

$$\text{MSE}(y, \hat{y}) = \frac{1}{m} \sum_{i=1}^m (y_i - \hat{y}_i)^2$$

$$\text{MAE}(y, \hat{y}) = \frac{1}{m} \sum_{i=1}^m |(y_i - \hat{y}_i)|$$

$$\text{MAPE}(y, \hat{y}) = \frac{1}{m} \sum_{i=1}^m \left| \frac{(y_i - \hat{y}_i)}{y_i} \right|$$

$$\text{Logcosh}(y, \hat{y}) = \frac{1}{m} \sum_{i=1}^m \cosh(y_i - \hat{y}_i)$$

$$L_{\delta}(y, \hat{y}) = \begin{cases} \frac{1}{2}(y - \hat{y})^2 & \text{for } |y - \hat{y}| \leq \delta \\ \delta \cdot (|y - \hat{y}| - \frac{1}{2}\delta), & \text{otherwise} \end{cases}$$

# Regression Loss

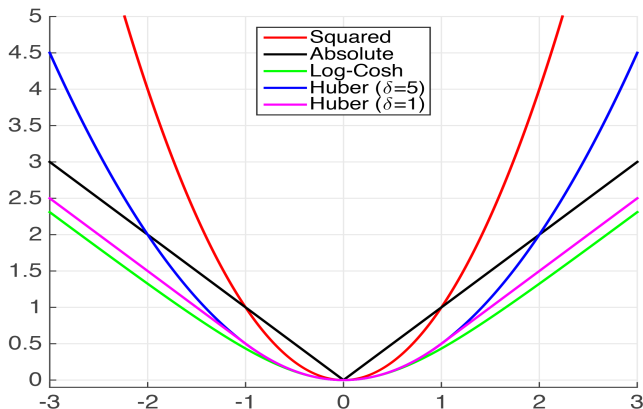


Figure: Various Regression Losses, [Source](#)

**Thank You!**

**Any Question?**