Homework Assignment 0
Machine Learning for Bioinformatics,
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1 Probability & Statistics

1.1

Suppose X_1, X_2, ... are independent normal random variables with mean 0 and variance 9. N is also an integer random variable, independent of X_is, with mean 2 and variance 1. We define $S \triangleq \sum_{i=1}^{N} X_i$.

- Prove: $Var(X) = \mathbb{E}(Var(X|Y)) + Var(\mathbb{E}(X|Y))$
- Find the variance of S.
- Derive the correlation coefficient between the random variables S and N.

1.2

Suppose the data samples of an experiment are drawn from a normal distribution $\mathcal{N}(\mu,\, \sigma^2)$. We assume that the variance of this population σ^2, is known, but its mean μ, is unknown, and we want to estimate it from N independent observations X_1,...,X_N.

- Find the ML estimate for the population mean.
- Suppose that the prior distribution of the parameter μ is the normal distribution $\mathcal{N}(\alpha,\, \beta^2)$. Find the MAP estimate for the population mean. What is the effect of choosing this prior on the posterior distribution?
- Explain the relationship between these two estimates when the number of samples is increased.

1.3

Suppose $X_1, X_2, ..., X_n$ are iid random variables. Find the probability density function of $Y_1 = max[X_1, X_2, ..., X_n]$ and $Y_2 = min[X_1, X_2, ..., X_N]$.

2 Linear Algebra

2.1

Suppose $A, B \in M_{m\times n}(\mathbb{R})$. Show that $\langle A, B\rangle = tr(B^T A)$ is an inner product.

2.2

Assume that x is a vector and A is a square matrix. Show that:

- $\dfrac{\partial x^T A x}{\partial x} = 2x^T A$
- $\dfrac{\partial trace(x^T A x)}{\partial x} = x^T(A + A^T)$

2.3

Suppose $\lambda_1, \lambda_2, ..., \lambda_n$ are the eignvalues of matrix A, Prove:

- $\lambda_1 + \lambda_2 + ... + \lambda_n = trace(A)$
- $\lambda_1\lambda_2...\lambda_n = det(A)$
- AB and BA have the same set of eigenvalues.
- A and A^T have the same set of eigenvalues.