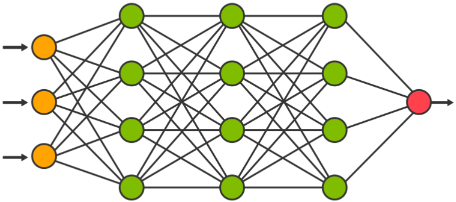
Introduction to Neural Networks

ML Instruction Team, Fall 2022

CE Department
Sharif University of Technology



Biological Analogy

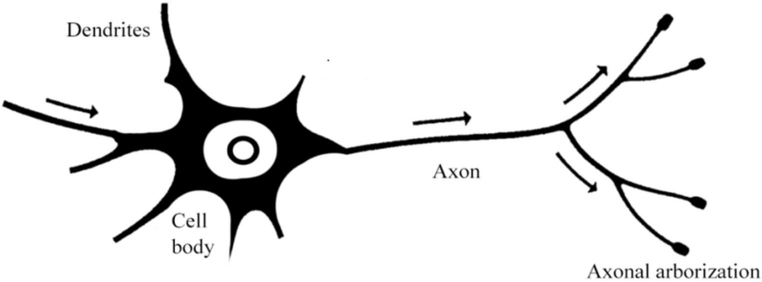


Figure: Anatomy of a biological neuron [1].

Activation Functions

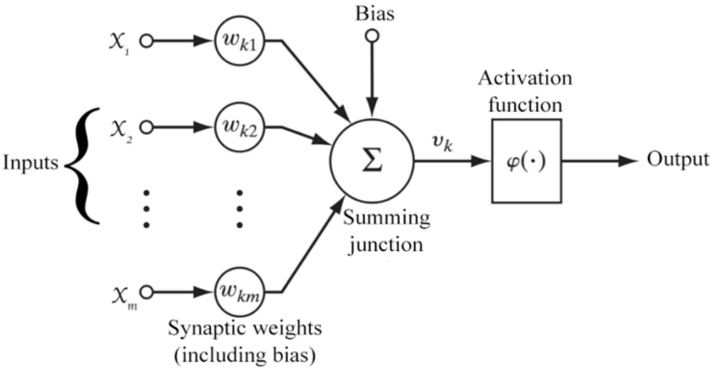


Figure: Neural network neuron [1].

Activation Functions

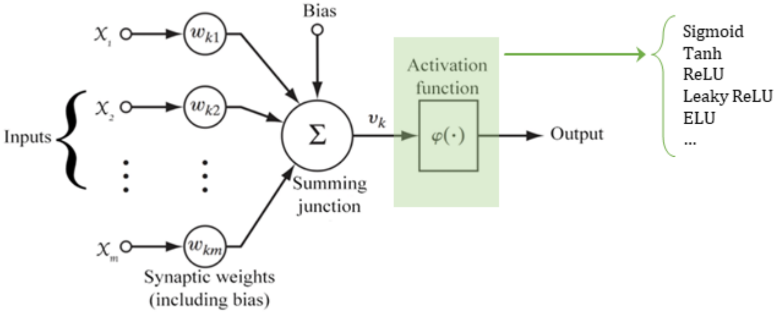
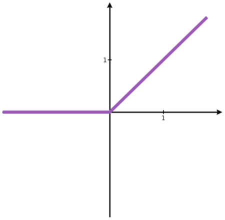
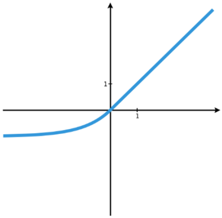
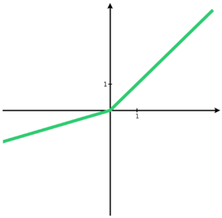


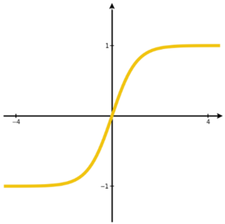
Figure: Activation function

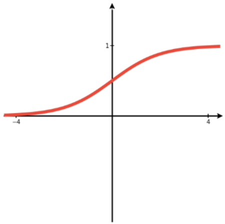
Activation Functions

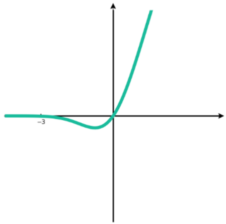
ReLU	ELU	Leaky ReLU
$f(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$	$f(x) = \begin{cases} x & x > 0 \\ \alpha(e^{x} - 1) & x \leq 0 \end{cases}$	$f(x) = \begin{cases} x & x \geq 0 \\ 0.01x & x < 0 \end{cases}$
		

Activation Functions

Tanh	Sigmoid	GELU
$f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$	$\sigma(x) = \dfrac{1}{1 + e^{-x}}$	$f(x) = \dfrac{1}{2}x\left(1 + \text{erf}\left(\dfrac{x}{\sqrt{2}}\right)\right)$







<u>Softmax</u>

$$f(x) = \frac{e^{x_i}}{\sum_{j=1}^{J} e^{x_j}} \quad i = 1, \cdots, J$$

Gradient Descent

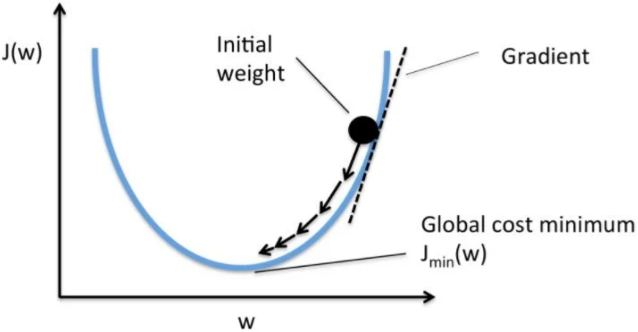


Figure: Gradient descent [3].

Gradient Descent

- Let's define our problem:
  - We have dataset $\mathcal{D} = \{x^i, y^{(i)}\}_{i=1}^n$.
  - f is a single layer perceptron.
  - Define $\hat{y}^{(i)} = f(x^{(i)})$.
- We want to minimize following cost function:

$$\mathcal{J}(\boldsymbol{w}) = \frac{1}{2}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$

- We are going to use gradient descent algorithm. $\boldsymbol{w}$ will be updated as follows:

$$\boldsymbol{w}^{t+1} = \boldsymbol{w}^t - \eta\nabla_{\boldsymbol{w}}\mathcal{J}$$

Gradient Descent

- Let's find $\nabla_{\boldsymbol{w}} \mathcal{J}$:

$$
\begin{aligned}
\frac{\partial J}{\partial w_j} &= \frac{\partial}{\partial w_j} \frac{1}{2} \sum_i (y^{(i)} - \hat{y}^{(i)})^2 \\
&= \frac{1}{2} \sum_i \frac{\partial}{\partial w_j} (y^{(i)} - \hat{y}^{(i)})^2 \\
&= \frac{1}{2} \sum_i 2(y^{(i)} - \hat{y}^{(i)}) \frac{\partial}{\partial w_j} (y^{(i)} - \hat{y}^{(i)}) \\
&= \sum_i (y^{(i)} - \hat{y}^{(i)}) \frac{\partial}{\partial w_j} \left(y^{(i)} - \sum_j w_j x_j^{(i)} \right) \\
&= \sum_i (y^{(i)} - \hat{y}^{(i)}) (-x_j^{(i)})
\end{aligned}
$$

$$
\Delta w_j = -\eta \frac{\partial J}{\partial w_j} = -\eta \sum_i (y^{(i)} - \hat{y}^{(i)}) (-x_j^{(i)}) = \eta \sum_i (y^{(i)} - \hat{y}^{(i)}) x_j^{(i)}
$$

$$
\mathbf{w} := \mathbf{w} + \Delta \mathbf{w}
$$

Limitations of SLP

- What are the limitations of SLP?
- Can we learn all functions with SLP?

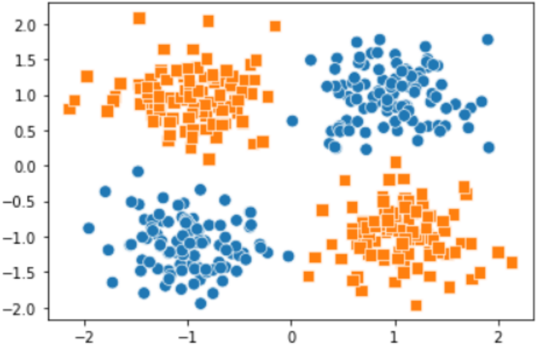


Figure: The XOR function is not linear separable.

Limitations of SLP

- As we saw in the XOR case, nonlinear separable functions can not be learned by SPLs.

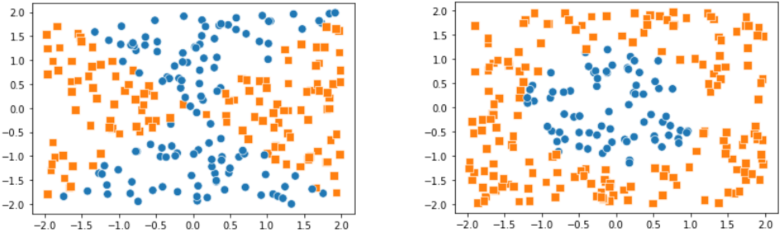


Figure: Examples of nonlinear separable functions.

- How to solve this?

Limitations of SLP

- What if we knew some feature space which our data is linear separable in?!

(x, y) $\Longrightarrow$ (r, θ)

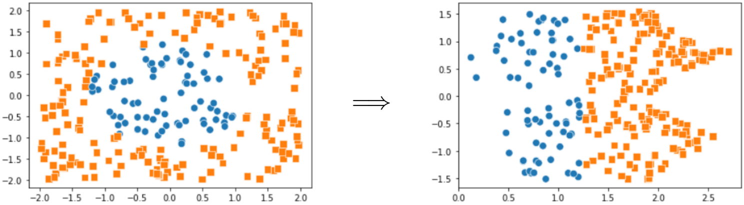


Figure: Data is linear separable after transformation.

Multi-Layer Perceptron

- So if we know some $f_1, \cdots, f_4$ we can use SLP to solve problem.

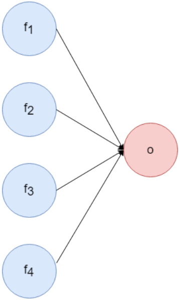


Figure: Using feature space f to solve problem.

- How to learn this f_is? Use SLP!

Multi-Layer Perceptron

- We can use inputs (x_i) to learn features (f_i)

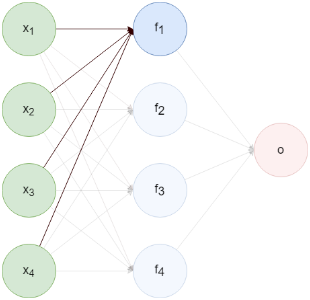


Figure: Using inputs to learn features.

- What if f_is are not sufficient? We can add more layers!

Multi-Layer Perceptron

- Adding more layers we will have a bigger network.

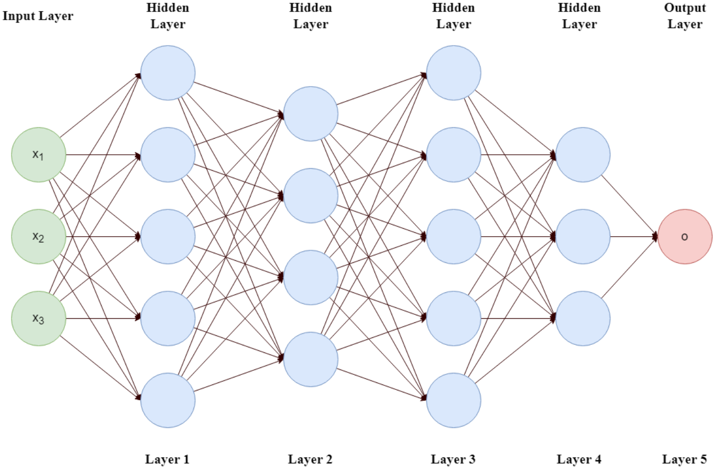


Figure: A 5 layer MLP.

Architecture of MLPs

- Important questions:
  - How many hidden layer should we have?
  - In each hidden layer, how many neuron should we have?

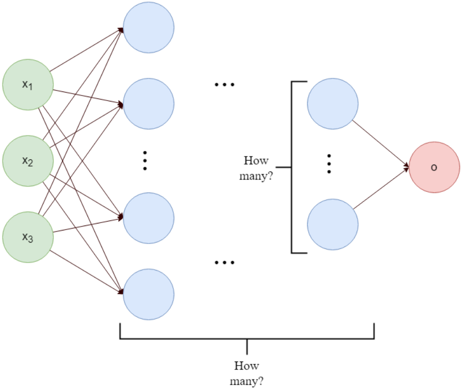


Figure: How many layers and neurons should we have?

Architecture of MLPs

- In practice:
  - You have limited resources
  - There is no universal rule to choose this hyperparameters
  - Need to experiment for different number of layers and neurons in each layer

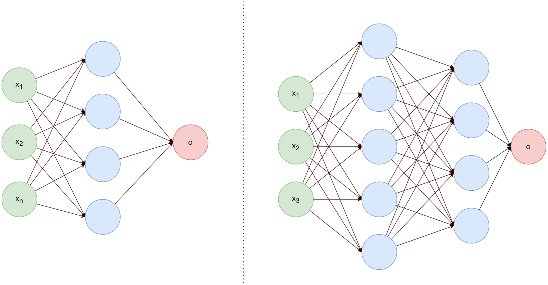


Figure: Experiment for different architecture and choose the best model.

Activation Function of Hidden Layers

- One can use any activation function for each hidden units
- Usually people use the same activation function for all neurons in one layer
- The important point is to use nonlinear activation functions

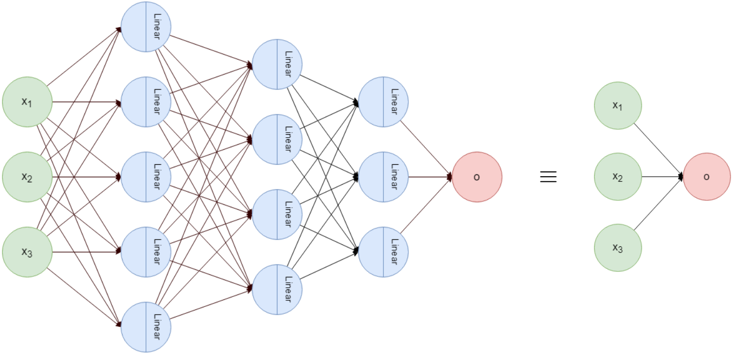


Figure: MLP with linear activation functions is equivalent to simple SLP.

XOR problem

- Now let's solve XOR problem with MLPs.
- We have two binary inputs, build an MLP to calculate their **XOR**.

- First let's build logical **AND** and **OR** functions.

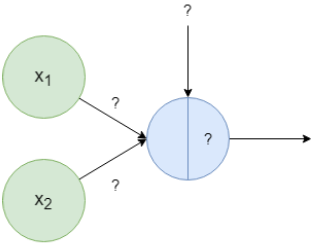
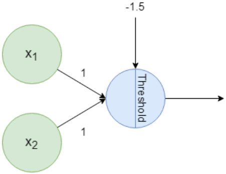
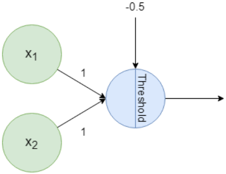


Figure: We need to find weights, biases and activation function.

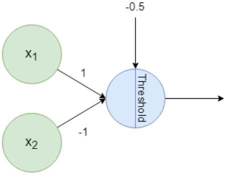
XOR problem



(a) $x_1 \wedge x_2$

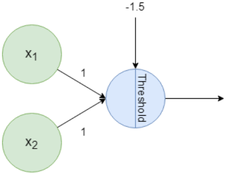


(b) $x_1 \vee x_2$

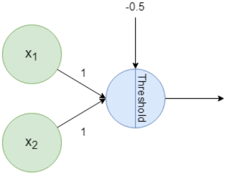


(c) $x_1 \wedge \overline{x_2}$

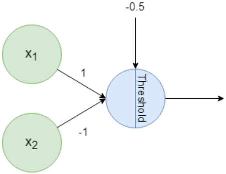
XOR problem



(a) $x_1 \land x_2$



(b) $x_1 \lor x_2$



(c) $x_1 \land \overline{x_2}$

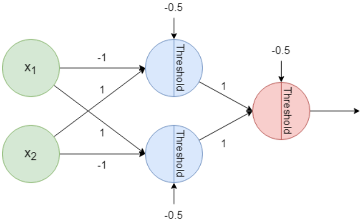
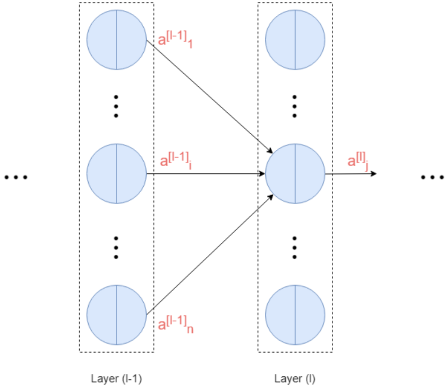


Figure: MLP for XOR function.

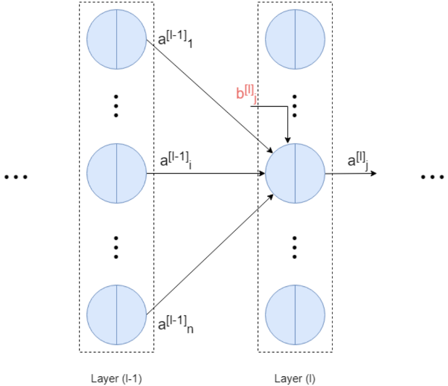
MLP notation

- $a_i^{[l]}$: i-th neuron outpu in layer l
- $\boldsymbol{a}^{[l]}$: layer l output in vector form



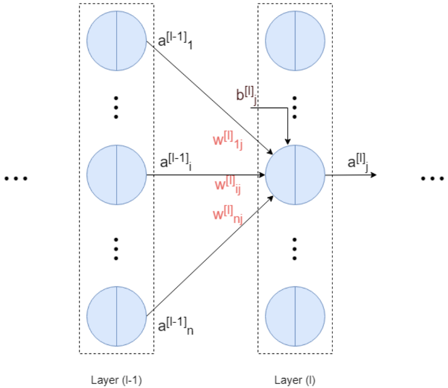
MLP notation

- $b_i^{[l]}$: i-th neuron biase in layer l
- $\boldsymbol{b}^{[l]}$: layer l biases in vector form



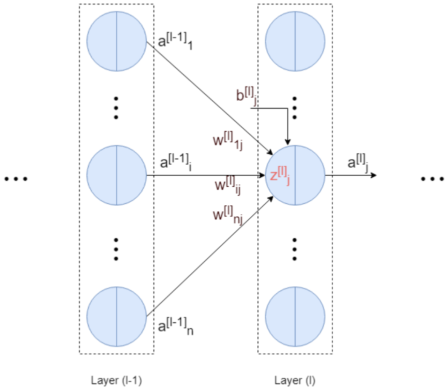
MLP notation

- $W_{ij}^{[l]}$: weight of the edge between i-th nuron in layer $l-1$ and j-th neuron in layer l



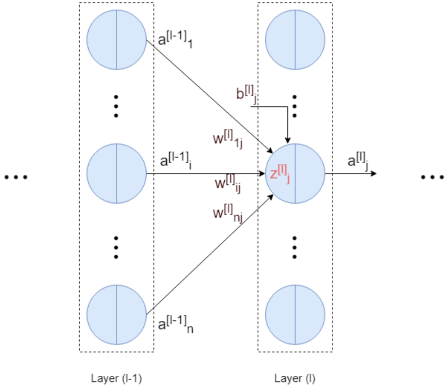
MLP notation

- $z_j^{[l]}$: j-th neuron input in layer l
- $z_j^{[l]} = b_j^{[l]} + \sum_{i=1}^{n} W_{ij}^{[l]} a_i^{[l-1]}$



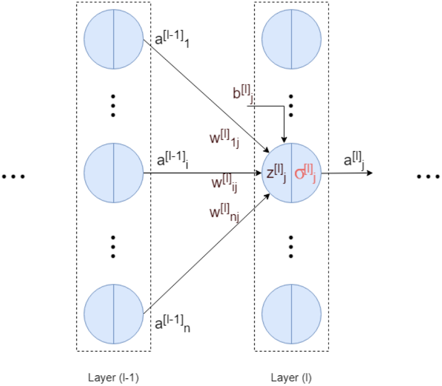
MLP notation

- $\boldsymbol{z}^{[l]}$: input of layer l in vector form
- $\boldsymbol{z}^{[l]} = \boldsymbol{b}^{[l]} + (W^{[l]})^T \boldsymbol{a}^{[l-1]}$



MLP notation

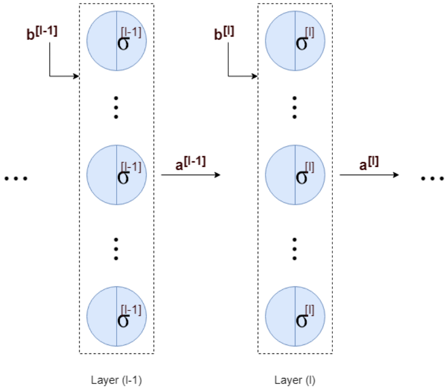
- $\sigma_j^{[l]}$: j-th neuron activation function in layer l



MLP notation

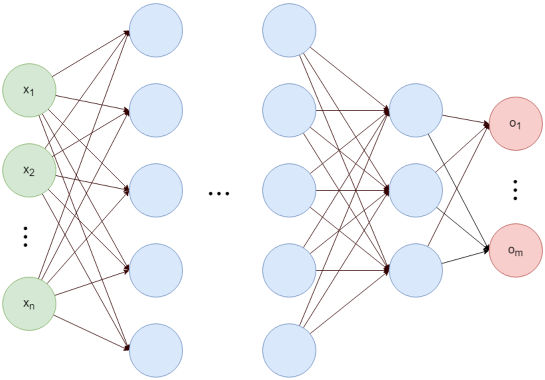
- If we assume all neurons in one layer have the same activation function then:

$$\boldsymbol{a}^{[l]} = \sigma^{[l]}\left(\boldsymbol{b}^{[l]} + (W^{[l]})^T\boldsymbol{a}^{[l-1]}\right)$$



MLP notation

- So for a network with L layer, and $\boldsymbol{x}$ as its input we will have:

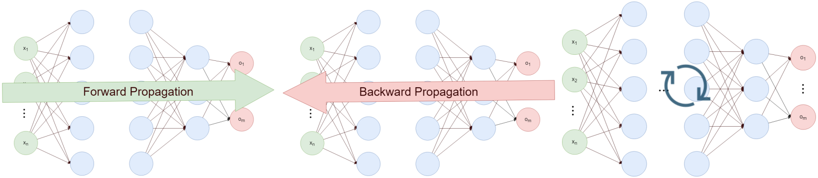


$$\boldsymbol{o} = \boldsymbol{a}^{[L]} = \sigma^{[L]}\left(\boldsymbol{b}^{[L]} + (W^{[L]})^T \sigma^{[L-1]}\left(\cdots \sigma^{[1]}\left(\boldsymbol{b}^{[1]} + (W^{[1]})^T \boldsymbol{x}\right)\cdots\right)\right)$$

Learning MLPs

- Till here we have used networks with predefined weights and biases.
- How to learn these parameters?

- The idea is to use gradient descent



(a) Forward pass (b) Backward pass (c) Update parameters

Thank You!

Any Question?

References

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