Deree - American College of Greece

INVESTMENTS & PORTFOLIO MANAGEMENT

Project

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Introduction

Question 1 & 2 - Asset Allocation & Risk Tolerance

The portfolio management begins determining the allocation to asset classes considered for investment through the **Strategic Asset Allocation** . The existing asset classes in the investing world, which enclose **different risk σ & return r characteristics**, are the following: *Money Market Instruments (cash), Equities(stocks), Fixed Income (bonds), Derivatives (Real Estate, Commodities, Futures)*

Risk Tolerance

Assessing our risk tolerance regularly is ensured by our investment horizon and the balancing between lower expose to higher risk σ and higher expose to higher returns r. **General Rule** : a lower σ is commonly associated with lower potential r and the converse.

So, an average risk tolerance assessment was conducted through the completion of the **Questionnaire** attached in the appendix.The quiz is not regarded as *investment advice*, but just as initial risk indication. The answers are below:

Q	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**score**
A	**b-3**	**b-2**	**c-3**	**b-2**	**b-2**	**b-2**	**c-3**	**b-2**	**b-3**	**b-3**	**b-2**	**b-2**	**b-2**	*31*

The investment profile that corresponds to the score is that of an *Above average risk tolerance investor*.

Investment Policy Statement

- **Objectives**
  The portfolio objective is financing professional future needs. Investment goals and objectives are **long term growth** and **preservation of capital**, in an above *average risky manner*
  - **Return profile** : Long-term Growth & Capital Preservation
  - **Risk profile** : Average/moderate risk tolerance.
- **Constraints**
  A step before starting a professional career, with 30% knowledge of investment products and 50% inclination to learn, the goal is to commit portfolio management program for > 5 years with **low liquidity** requirements and **medium** sensitivity to tax savings. However, for the purpose of this project, the IPS should be adapted to a **short-term** horizon.
  - **Liquidity** : Low
  - **Time Horizon** : 2 years
  - **Tax Concerns** : Low
  - **Legal Issues** : -
  - **Unique Circumstances** : Securities concerning healthcare domain should be avoided because of the *coronavirus* pandemic financial impact on the prices.

Asset Allocation

According to the above **IPS** the strategy adopted for the asset allocation problem is the following :

- **Portfolio Assets**
  The portfolio should be consisted of **at least 6 stocks**, **2 corporate bonds** and **2 commodities**. No cash option is available.

$$r_{commodities} > r_{stock} > r_{bonds}$$
$$\sigma_{commodities} < \sigma_{stock} < \sigma_{bonds}$$

- **Portfolio Assets Allocation**
  The strategy shares characteristics of a **moderately aggressive portfolio**. In other terms, the asset composition favors the **equities (the 6 stocks)** rather than **fixed-income securities (the 2 bonds)**. Both *risk-free (T-bills)* and *risky (bonds, stocks, commodities)* assets are utilized pragmatically. So, the assets included are :

- **Risk-Free Assets**
  - **T-bills** : The *1 Month Treasury Bill* of the US government is chosen. Accurate pricing data is required, so portfolio, so portfolio monitoring is conducted in shorter time periods [1]
- **Risky Assets**
  - **Stocks** : Large-cap stocks from well-established companies and the risk of failure is minimal in terms of growth prospects. (moderate amount of capital gains distribution)
  - **Bonds** : Some bonds to balance the risk of the portfolio.
  - **Commodities** : Despite being riskier (risk we can take) than stocks and bonds, they protect against inflation and can enhance the diversification of the portfolio.

- **Index**

  This portfolio will be compared to *S&P 500* to monitor its performance. Whens this index goes up, it should be up a little more than a moderate investment portfolio and the converse. We seek to achieve returns greater than taxes and inflation.

- **Portfolio Asset Weights**

Stocks	*Bonds*	*Commodities*	*T-bills*
45%	35 %	10 %	10%

Question 3 - Effects of the Short-Term Investment

Short-Term Investment & Asset Allocation

Most portfolios with long term horizons are linked to increased exposure to equities, generating greater potential rewards. Moreover, the crucial risk entailed with equities is mitigated because of the time available to recover from losses.

However, the investing policy is applied for a short-term horizon of 2 years. Moreover, due to the special **constraints**, a larger amount of capital is allocated in **fixed income securities (bonds)** which are less exposed to short term price changes and provide stability to the portfolio. Since the portfolio is consisted of only 2 less riskier assets (bonds) and of at least 8 riskier (commodities, stocks), holding more bond securities is the most appropriate strategy to limit the risk.

Short-Term Investment & Diversification

Diversifying through asset classes consists a means of defence against a financial crisis and can also prevent potential losses. However, reducing the exposure to particular assets may limit the potential profit in the short run. Diversification strategy is commonly reflected to a selection of negatively correlated assets, yet, the particular risk aversion profile implies a less variable allocation of distinct asset classes in order to avoid a mitigate performance. Moreover there are a few specific factors which affect the performance of assets in the short-run:

- *Economy Growth* : Stocks perform well when economy grows, bonds when economy slows.
- *Supply & Demand* : Commodities prices are subject to sharp swings in volatility.
- *Asset Bubbles* : A progressive increase in an assets price.
- *Business Cycle* : Small businesses perform better in early stage of recovery, while large-cap in the latter part.
- *Investing in June* : This is the third-worst month for the SP 500 and the second-worst month for the Dow Jones industrial average.

In this direction, the portfolio should be properly diversified based on the IPS. Hens, a variety of stocks, bonds and commodities should be examined and those that move fairly in sync with one another should be selected. Moreover, the final selections of the three asset classes should be adjusted to any unexpected circumstances such as coronavirus pandemic, for instance. Finally, the current phase of the business cycle is the expansion, therefore, investing in large-cap stocks could be a wise option for a short term investment horizon.

[1] The main reason for choosing T-bill of 1 month maturity over T-bills of 3-month, 6-month or 1-year

Question 4 - Descriptive Statistics of Instruments

The securities were selected for a historic time period of more than 5 years from *1 January 2015* until *24 March 2020*. For *Bitcoin* there is no weekend gap, while for *Palladium* there are some extra trading days, so the total numbers of trading days varies.

- **Stocks, Bonds** : 1327
- **Commodities** : 1925 for Bitcoin, 1375 for Palladium

Historic data of commodities are then filtered, by keeping only those values of dates existing in all instruments. The values retrieved for each instrument are *High, Low, Open, Close, Volume, Adj Close*

<table>
<tr><th>Stocks</th><th>Bonds</th><th>Commodities</th></tr>
<tr>
<td>
<table>
<tr><th>Name</th><th>Ticker</th></tr>
<tr><td>Apple</td><td>AAPL</td></tr>
<tr><td>Microsoft</td><td>MSFT</td></tr>
<tr><td>Amazon</td><td>AMZN</td></tr>
<tr><td>Google</td><td>GOOG</td></tr>
<tr><td>Facebook</td><td>FB</td></tr>
<tr><td>Netflix</td><td>NFLX</td></tr>
<tr><td>NVIDIA</td><td>NVDA</td></tr>
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<tr><th>Name</th><th>Ticker</th></tr>
<tr><td>HCA Heatlhcare</td><td>HCA</td></tr>
<tr><td>Allegiant</td><td>ALGT</td></tr>
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<table>
<tr><th>Name</th><th>Ticker</th></tr>
<tr><td>Bitcoin</td><td>BTC-USD</td></tr>
<tr><td>Palladium</td><td>PA=F</td></tr>
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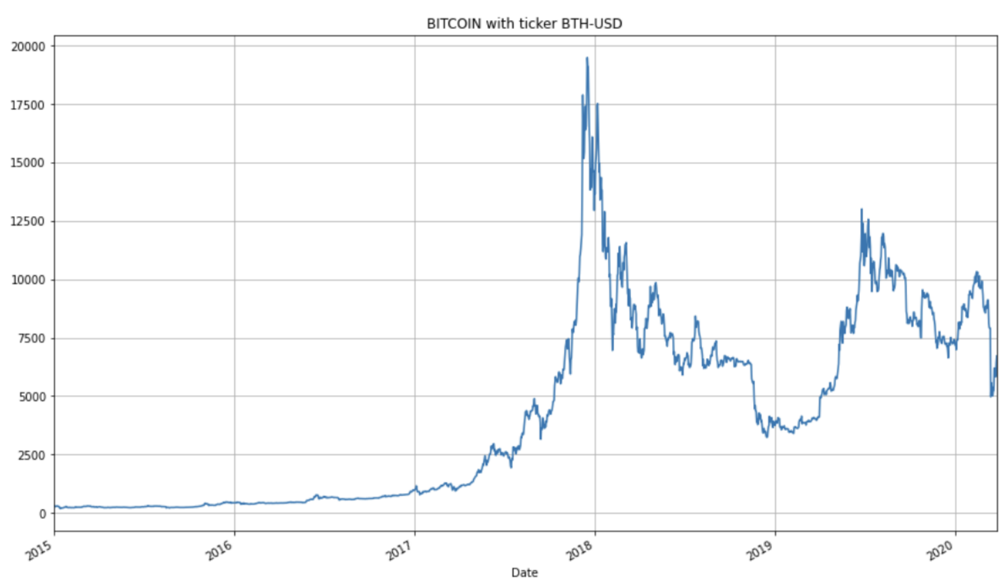
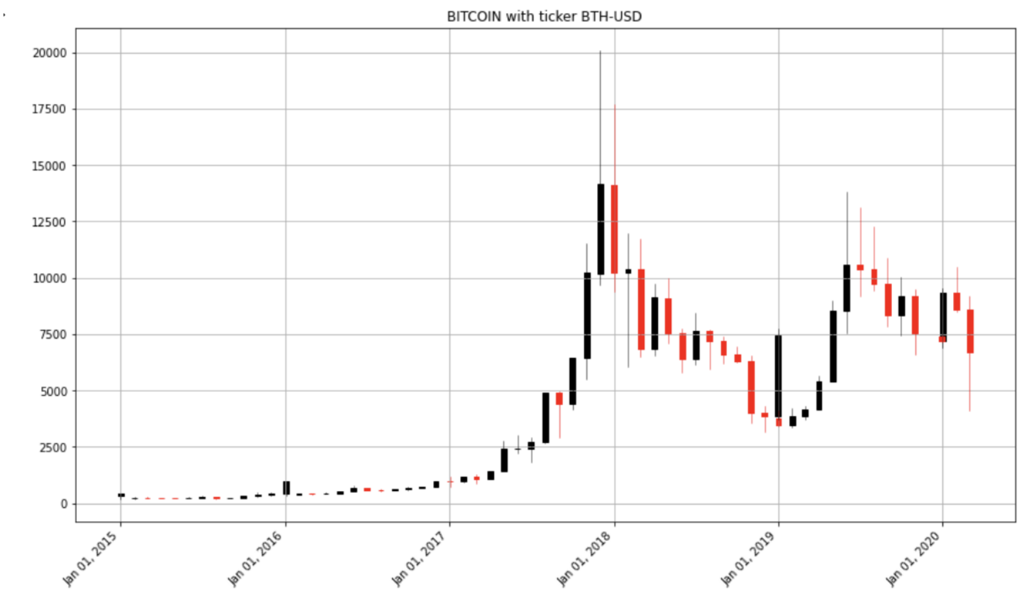
Commodities Visualization Example

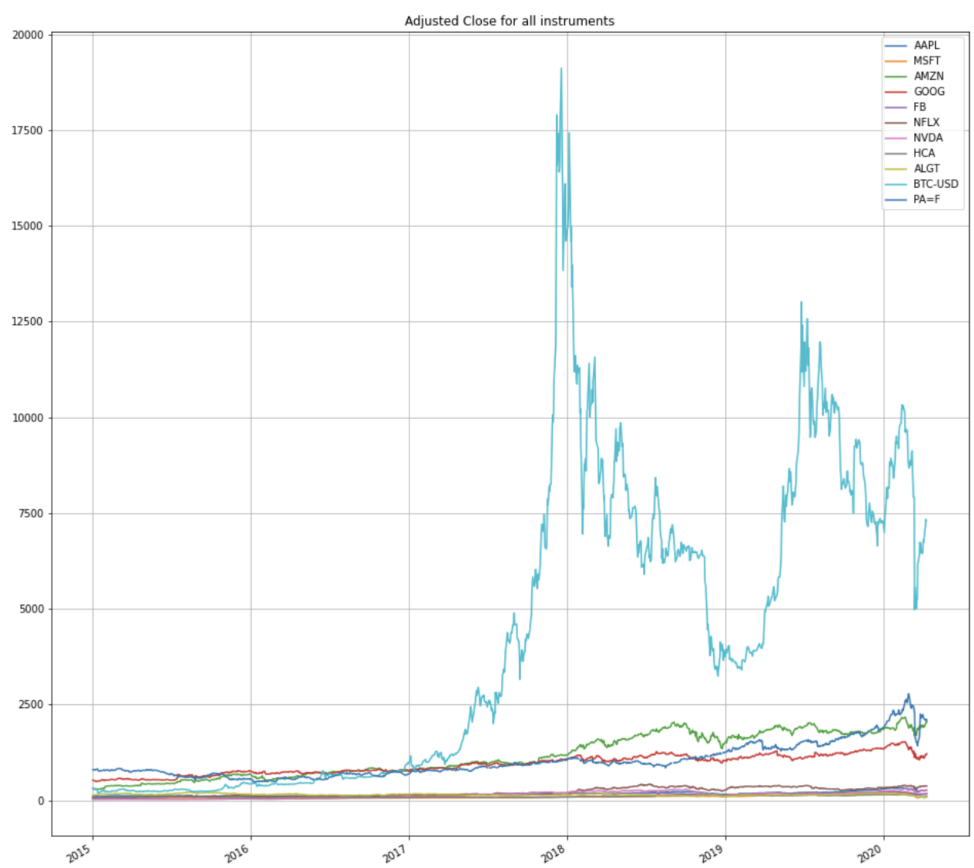
In order to calculate **risk** and **return**, *Adjusted Closed* prices are used to accurately reflect instruments' value after having accounted for any corporate actions. Data interpretation is of vital importance. *Bitcoin* is an interesting example to plot. Furthermore, a **candlestick chart** is visualized. A black candlestick indicates days where closing prices were higher than open (*gain*), whereas a red candlestick indicates days where prices were lower (*loss*)





Instruments Visualization

In this subsection we plot the *Adjusted Close* price for all instruments. Last year, *Bitcoin* faced a bubble, where value dropped almost 8 times.



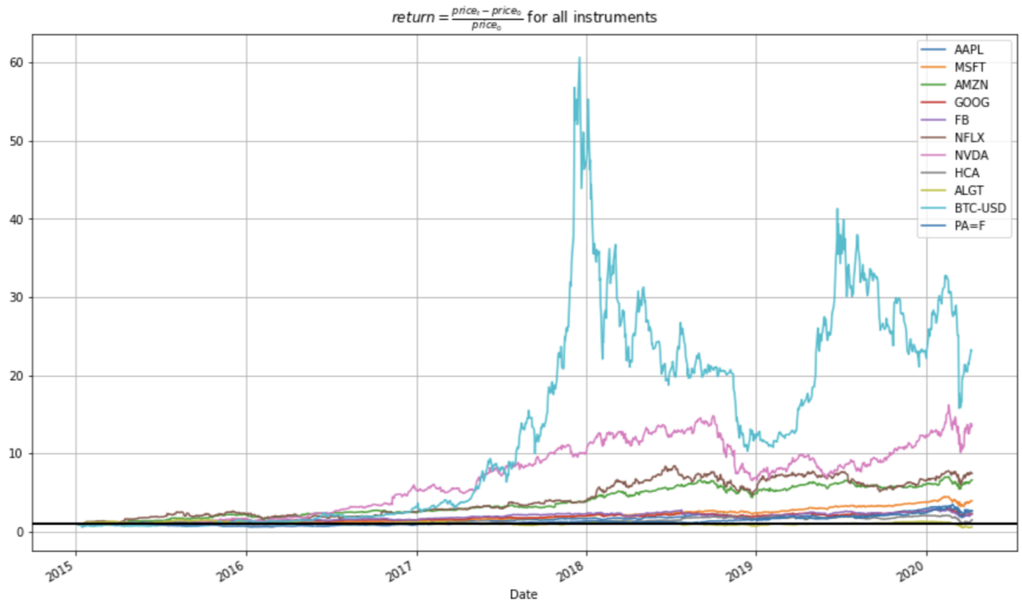
Descriptive Statistics

Let's introduce some statistics calculated with *Adjusted Close* prices.

1. **Average Daily Return** [2]

[2] The formulae are in the appendix section

This useful plot of returns indicates how profitable each instrument was since the beginning of the period. Most of them are correlated in terms of **trend** (not seasonality).



The following average **5-year daily return** will measure the past returns for each asset, indicating the average profitability of each instrument.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
r	55.82	96.78	274.16	78.49	85.33	318.68	579.42	32.77	7.02	1219.36	27.14

BTC-USD may have the greatest average return, however its value is much more volatile.

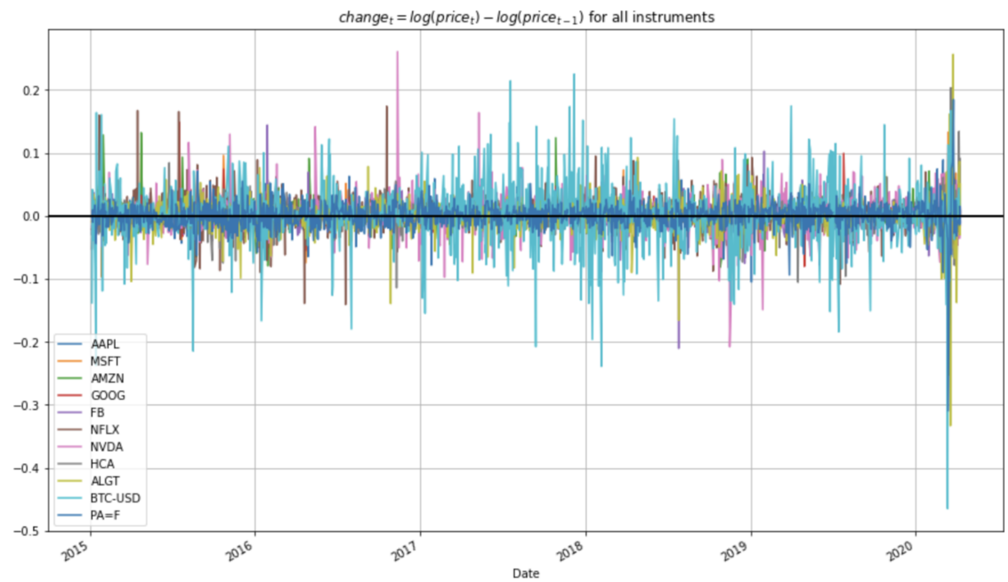
BTC-USD may have the greatest average return, however its value is much more volatile (*ups and downs*). The instrument is chosen for **diversification** since it is not significantly correlated to any investable asset class.

2. **Annual Percentage Rate (APR)**

However, annualizing the returns via the **annual percentage rate (APR)** will maintain the returns on common scale. There are T = 252 trading days. The returns will be the following: *returns*:

- **Simple Returns** which is the percentage difference of of the instrument's initial price (5 years ago). This was calculated before. This is : $return_t$
- **Changes** which are the log differences between the instrument's price for 2 consecutive periods. This metric **does not** depend on the denominator of a fraction, while clearly corresponds to the modelling of the instrument prices in continuous time. This is : $change_t$

So, the **first** approach makes trend apparent, while the **second** is a more advanced method when modelling the the selected instrument's behavior.



According to the previous visualization, the returns are quite volatile for most of these instruments, which can move +/- 10% just on any given day.

Furthermore, having found our **log changes** we can therefore calculate the APR for each year separately, and the **average APR return** as the mean of all year's APR.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
2015	1.45	23.72	83.95	41.36	32.12	95.49	56.79	-4.35	18.91	53.98	-30.56
2016	14.64	16.81	14.95	3.75	13.57	18.03	129.02	12.47	7.22	93.35	24.27
2017	41.57	35.68	47.14	32.00	44.75	48.26	68.62	19.42	-1.02	319.79	48.51
2018	-1.70	23.41	32.42	3.06	-23.25	45.01	-25.46	40.62	-35.36	-107.95	19.32
2019	67.96	48.03	23.62	28.79	49.29	25.25	66.20	22.33	60.88	88.71	46.78
2020	-2.97	11.14 7	2.70	-5.75	-10.87	17.92	21.28	-19.93	-53.28	10.96	19.04
APR	19.92	26.13	35.44	17.06	17.52	41.21	52.37	11.41	-0.36	76.12	20.87

Using log returns helps creating a common scale and leads to comparable results. The average performance for 5 years is a good indicator for an investor. This calculation is based on equal year weighting $w_{2015} = w_{2016} = ... = w_{2020} = \frac{1}{6} = 0.166$, however year 2020 should be weighted less especially since COVID-2019 virus led to tremendous fall of all instrument's prices.

3. **Annual Percentage Yield (APY)**
In order to fully capture useful insights through historical data, the effects of intra-year compounding is used. So, the results are the following for the *cummulative_ return* :

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
5YTD	167.85	218.87	196.45	57.12	64.93	217.64	704.01	62.30	-47.62	1509.27	281.01
3YTD	61.44	80.98	65.91	25.49	14.58	85.89	60.16	40.77	-27.70	-5.12	83.67
YTD	-2.24	11.21	12.75	-5.68	-10.46	18.00	22.56	-19.75	-54.01	9.60	20.58

while the final APY for the 5 and more years historical data is :

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
APY	15.97	17.51	17.04	9.38	10.93	19.23	27.65	10.94	-5.07	36.44	20.00

Here *cummulative_ return* is used as an YTD indicator. Furthermore the **APR** return results are in accordance with **APY** return results, however the latter offers more concrete insights for an investor.

4. **Standard Deviation**

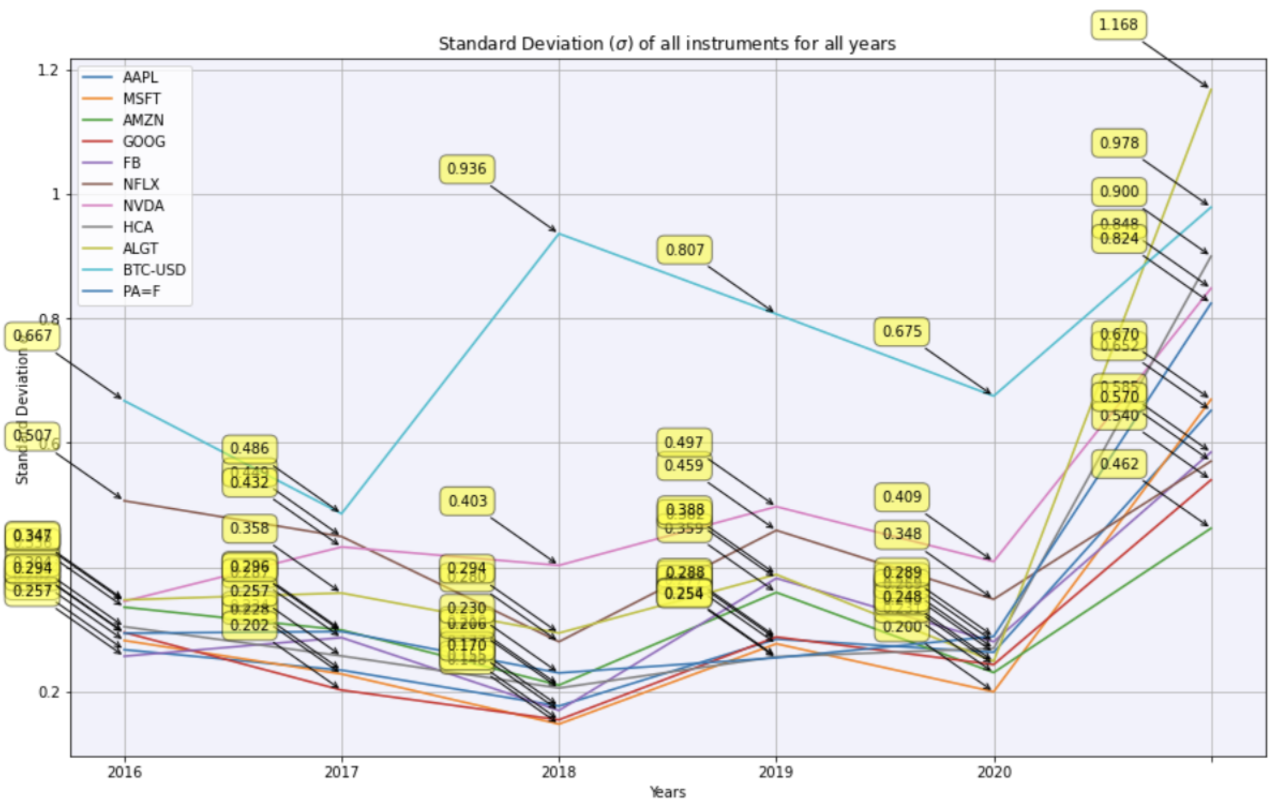
Calculating the **standard deviation** will measure each price's volatility and predict each instrument's risk performance. Volatile prices are linked to higher standard deviation while low prices are not subject to great swings. In order to capture better spectrum of instrument information $change_t$ instead of $return_t$ will be used for the calculation changes.

	$AAPL$	$MSFT$	$AMZN$	$GOOG$	FB	$NFLX$	$NVDA$	HCA	$ALGT$	$BTC\text{-}USD$	$PA{=}F$
σ	0.313	0.301	0.316	0.287	0.327	0.436	0.489	0.365	0.467	0.758	0.364

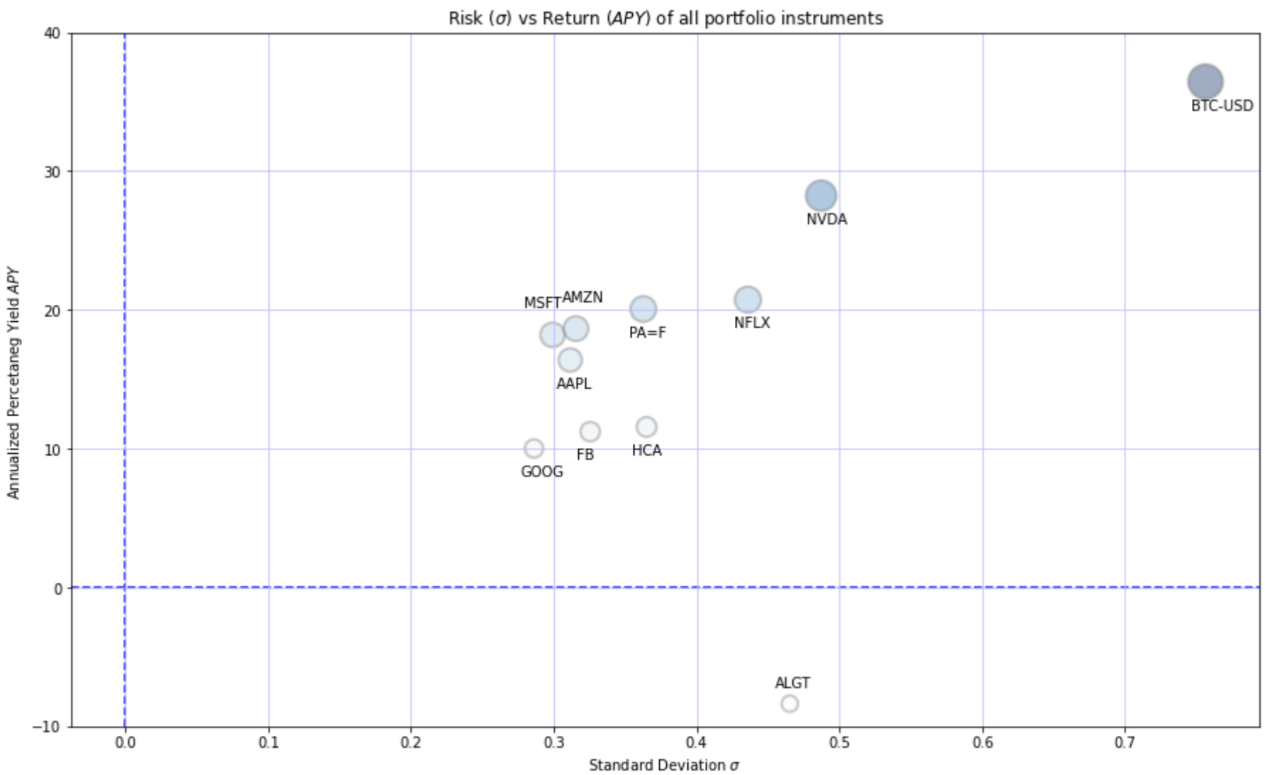
5. **Variance**

The relationship between return and volatility for each investment will be optimized through calculating the variance of the assets. The higher the variance of the price, the higher risk for each asset.

	$AAPL$	$MSFT$	$AMZN$	$GOOG$	FB	$NFLX$	$NVDA$	HCA	$ALGT$	$BTC\text{-}USD$	$PA{=}F$
σ^2	0.122	0.120	0.107	0.097	0.124	0.199	0.267	0.191	0.319	0.604	0.176



Although, the volatility of bonds and stocks is roughly at the same levels for each asset class, an unexpected high volatility value occurred due to COVID-2019. Almost all securities nowadays are prone to price swings, so our portfolio analysis should be reconsidered when the market is stabilized.



Bitcoin **BTC-USD** has the greatest return but also the highest volatility as expected. Despite having selected bonds **HCA** and **ALGT** to lower portfolio's risk, this year's performance is significantly bad, especially for **ALGT** (**negative average return**). *Allegiant Travel* was a potentially favourable company to invest, however COVID-19 crumbled world economies.

Question 5 - Additional Risk Metrics & CAPM

An expanded version of CAPM (Single-Index Factor) model is applied for every instrument to calculate the following specific risk metrics of the instruments against the market (§P 500):

- Covariance Matrix Cov_I , Correlation Matrix ρ_I, Correlation with the Market $\rho_{I,M}$,
- Alpha α_I
- Beta β_I
- Sharpe Ratio SR_I
- R squared R_I^2

The results are the following:

Covariance Matrix

All the values are scaled correctly when multiplied by the factor $\cdot 10^{-6}$.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
AAPL	323	210	179	185	191	194	267	147	175	58	24
MSFT	210	299	210	210	200	216	281	151	186	79	24
AMZN	179	210	367	209	215	263	241	106	121	45	22
GOOG	185	210	209	283	216	221	240	142	155	62	17
FB	191	200	215	216	374	225	252	138	195	63	20
NFLX	194	216	263	221	225	723	301	137	152	90	32
NVDA	267	281	241	240	252	301	817	207	277	104	54
HCA	147	151	106	142	138	137	207	434	204	120	36
ALGT	175	186	121	155	195	152	277	204	712	36	22
BTC-USD	58	79	45	62	63	90	104	120	36	2221	5
PA=F	24	24	22	17	20	32	54	36	22	5	429

	Stocks	*Bonds*	*Commodities*
Stocks	258	163	49
Bonds	163	388	53
Commodities	49	53	665

Correlation Matrix

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
AAPL	1.000	0.674	0.519	0.613	0.550	0.401	0.520	0.392	0.364	0.067	0.064
MSFT	0.674	1.000	0.633	0.722	0.599	0.464	0.567	0.419	0.403	0.097	0.065
AMZN	0.519	0.633	1.000	0.649	0.579	0.509	0.440	0.264	0.236	0.049	0.054
GOOG	0.613	0.722	0.649	1.000	0.665	0.487	0.499	0.405	0.345	0.077	0.050
FB	0.550	0.599	0.579	0.665	1.000	0.433	0.455	0.343	0.377	0.069	0.048
NFLX	0.401	0.464	0.509	0.487	0.433	1.000	0.391	0.244	0.212	0.070	0.057
NVDA	0.520	0.567	0.440	0.499	0.455	0.391	1.000	0.347	0.363	0.077	0.091
HCA	0.392	0.419	0.264	0.405	0.343	0.244	0.347	1.000	0.366	0.122	0.082
ALGT	0.364	0.403	0.236	0.345	0.377	0.212	0.363	0.366	1.000	0.028	0.038
BTC-USD	0.067	0.097	0.049	0.077	0.069	0.070	0.077	0.122	0.028	1.000	0.005
PA=F	0.064	0.065	0.054	0.050	0.048	0.057	0.091	0.082	0.038	0.005	1.000

	Stocks	*Bonds*	*Commodities*
Stocks	0.607387	0.337162	0.067311
Bonds	0.337162	0.683255	0.068142
Commodities	0.067311	0.068142	0.502758

- The instruments of each asset class have a strong positive correlation among to each other as expected.
- There is a significant correlation between *Stocks* and *Bonds*, indicating that their mean-variance performance moves closely. In terms of diversification we could select other bonds more negatively correlated.
- Moreover, **Commodities** are a bet on unexpected inflation, and have an almost-zero correlation to the rest 2 asset classes. Their mean-variance performance is characterized by high returns but in grounds of high volatility.

Correlation with the Market $\rho_{I,M}$

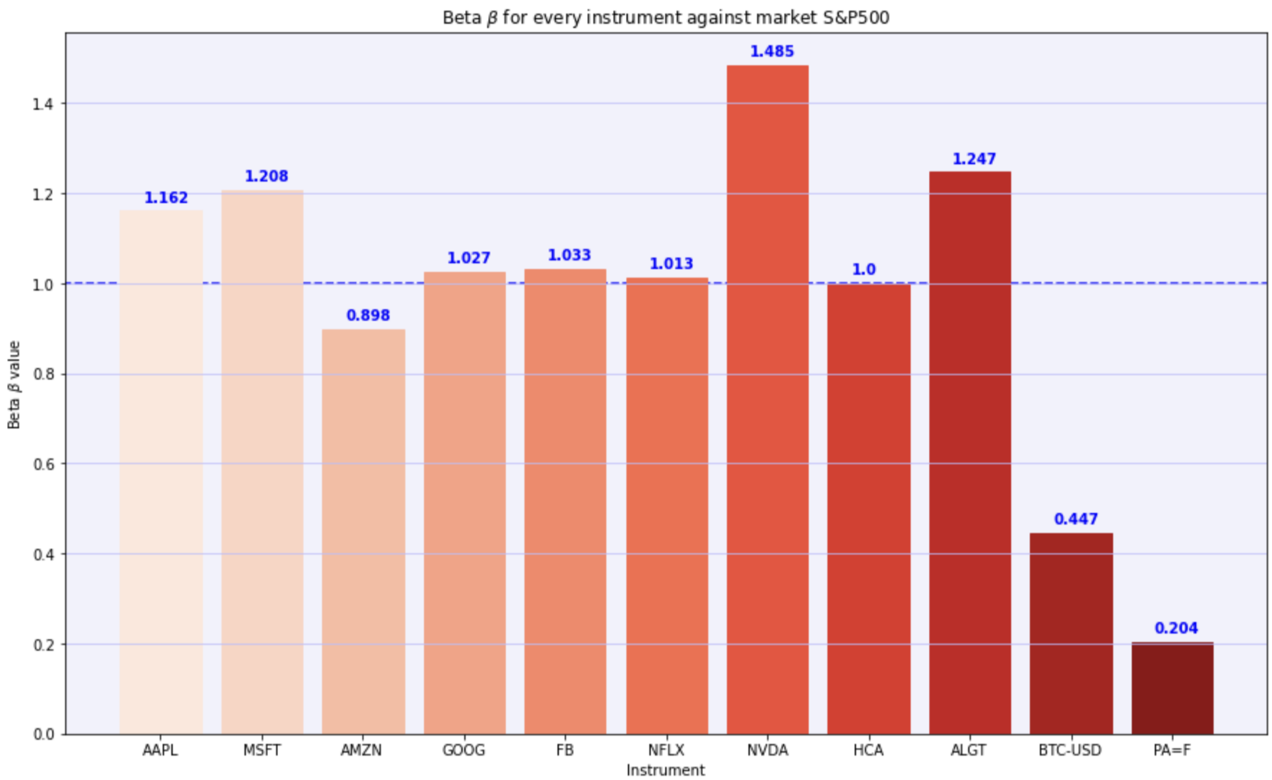
	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
ρ	74.61 %	80.79 %	57.00 %	71.75 %	63.58 %	46.52 %	61.12 %	55.08 %	53.69%	11.88 %	11.26%

R squared R^2

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
R^2	55.67 %	65.28 %	32.50 %	51.48 %	40.42 %	21.64 %	37.35 %	30.34 %	28.83%	1.41 %	1.26%

Beta β

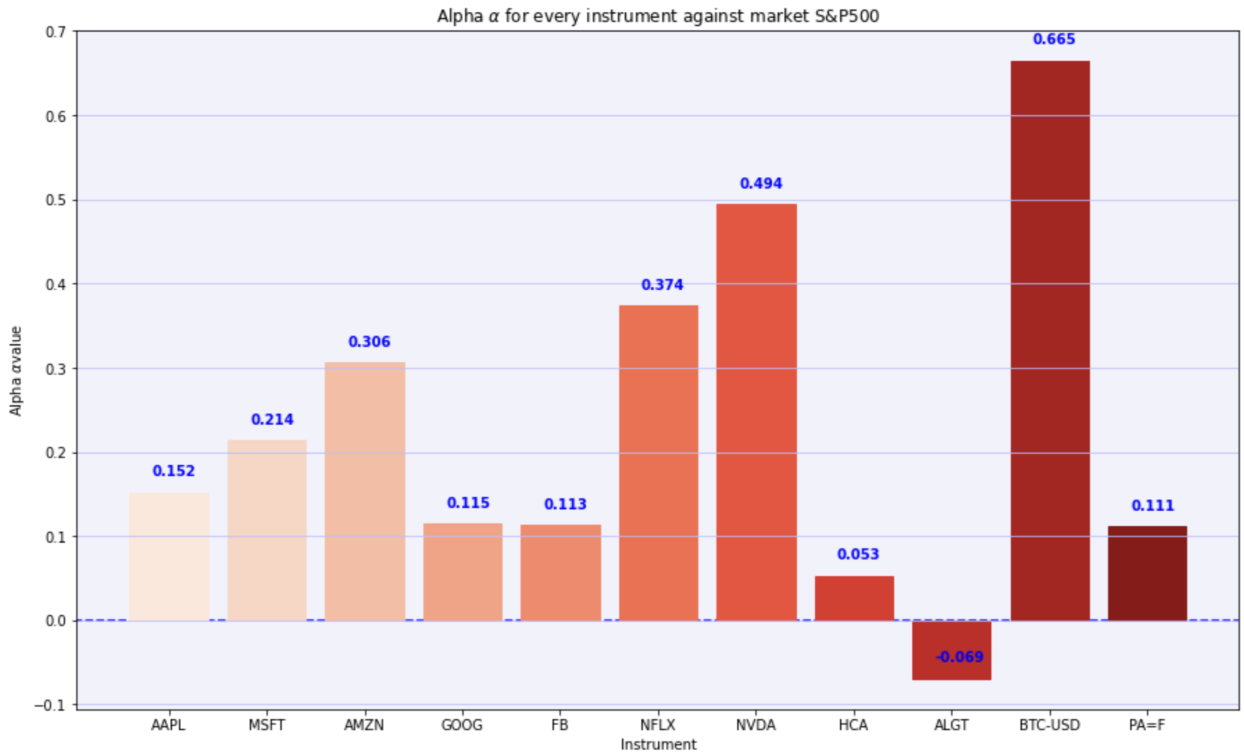
	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
β	1.162	1.208	0.898	1.027	1.033	1.013	1.485	1.000	1.247	0.447	0.204



- All the beta values are > 0 implying positive correlations with the volatility of the market **S&P 500**
- Instruments **AAPL, MSFT, GOOG, FB, NFLX, NVDA, ALGT** have beta values . > 1. Hence they are theoretically more volatile than market benchmark **S&P 500**. For example $\beta_{AAPL} = 1.162$, implying a volatility 16.2% greater than **S&P 500**
- Instrument **AMZN** has $beta_{AMZN} < 1$ implying that is not as risky as the rest instruments. More over instruments beta $beta_{BTC-USD}\beta_{PA=F}$ is really low close to 0, implying that indeed these commodities are less risky than market, but this is not a useful insight, since they are almost uncorrelated ($\sim 11\%$ correlation) so we cannot trust the number for its general volatility.

Alpha α

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
α	0.152	0.214	0.306	0.115	0.113	0.374	0.494	0.053	-0.069	0.665	0.111



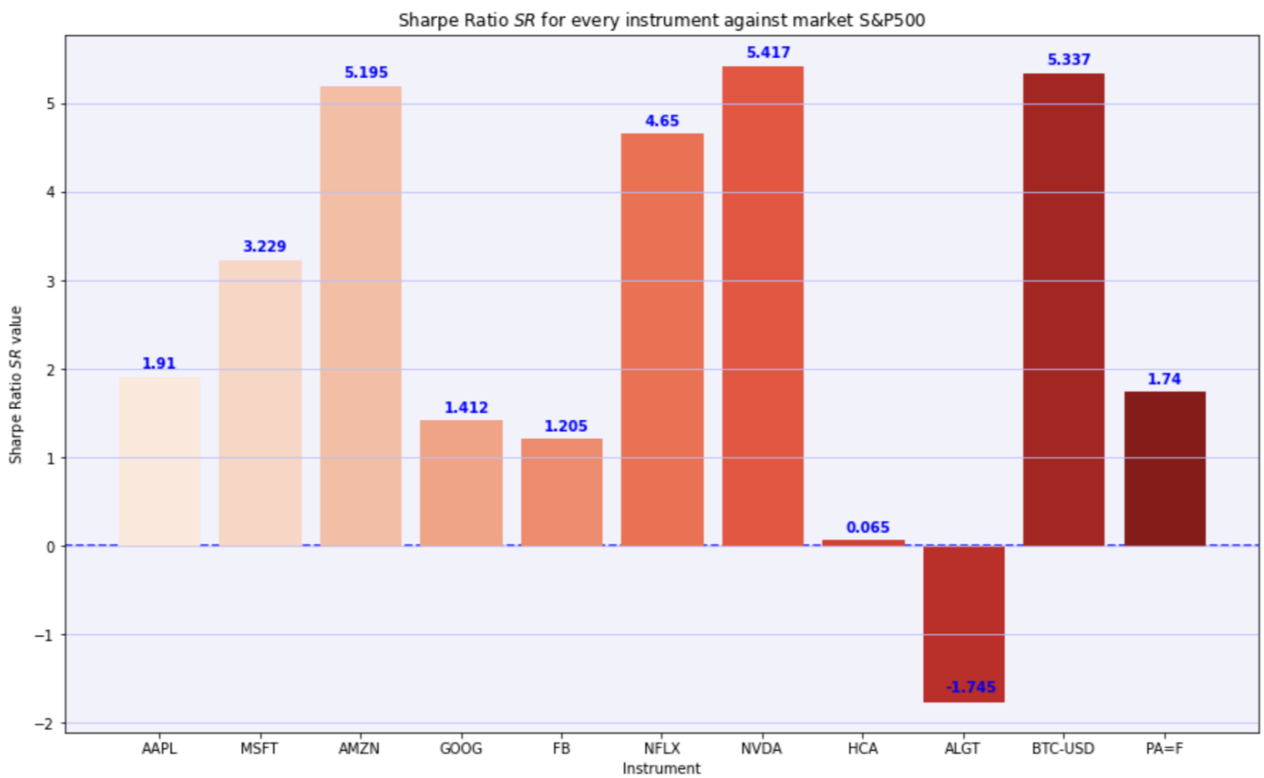
- all instruments except **ALGT** have alpha > 0. **BTC-USD** has the largest alpha value of 0.664, followed by **NVDA** with 0.494, generating 66.4% and 49.4% excess return over **S&P 500** having already adjusted for the inherited market risk (beta).

- All the *alpha* results are partially reliable since $R^2 \sim 35\%$, and so reliable are the excess generated, except from instruments $\alpha_{BTC-USD}$ and $\alpha_{PA=F}$ for which $R^2 < 2\%$ and so alpha is **not meaningful**

- Instrument **ALGT** has $\alpha_{ALGT} < 0$ and the result is reliable $R^2_{ALGT} = 53.69\%$, so it is not the most efficient instrument for someone to invest. We see once more that its performance is badly affected by CONVID-19.

Sharpe Ratio SR

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*ALGT*	*BTC-USD*	*PA=F*
SR	1.910	3.229	5.195	1.412	1.205	4.650	5.417	0.065	-1.745	5.336	1.739



Concluding Remarks

- Sharpe ratio is always meaningful, by taking into consideration both **return** and **risk**. The Market does not affect the result. So, instruments **AMZN, NVDA, BTC-USD, NFLX** are the best in terms of total *risk-return performance*
- Instrument **ALGT** is the worst by far. It will not be used in the final portfolio. We used instead *Vertex Pharmaceuticals Inc* **VRTX** bond which has a better mean-variance performance and belongs to the health-care domain along with **HCA**. Everything calculated in excel for portfolio optimization concerns the use of this bond.

Question 6 - Portfolio Risk & Return

Portfolio = Risk Free + Risky Porftolio

Our portfolio consists of :

- **risk-free instrument** (1 instrument)
- **risky portfolio** (11 instruments)

So if w denotes the share of wealth invested in the **risky portfolio** , then $1 - w$ denotes the share of wealth invested in the **risky-free asset**. So, the **return** and the **risk** of the entire portfolio will be :

- **Return**

$$r_P = \sum_{i=1}^{N} w_i \cdot r_i = (1-w) \cdot r_{riskfree} + w \cdot r_{risky}$$

- **Expected Return**

$$\mathbb{E}[r_P] = \sum_{i=1}^{N} w_i \cdot \mathbb{E}[r_i] = (1-w) \cdot r_{riskfree} + w \cdot \mathbb{E}[r_{risky}]$$

- **Risk**

$$\sigma_P^2 = (1-w)^2 \cdot \sigma_{riskfree}^2 + w^2 \cdot \sigma_{risky}^2 + 2 \cdot w \cdot (1-w) \cdot \sigma_{risky} \cdot \sigma_{riskfree} \cdot \rho_{risky,riskfree} \Longrightarrow$$

$$\sigma_P^2 = w^2 \cdot \sigma_{risky}^2$$

Risky Porftolio

Let us consider now the 11-instrument risky portfolio. Let R_i denote the return on instrument i ($i = \{$AAPL, MSFT, ...$\}$). Assuming that **Constant Expected Return** (CER) holds, then all returns are *normally distributed* while this distribution is totally characterized by the means μ_i, variances σ_i^2 and covariances of the returns. [3]

$$R_i \sim \mathcal{N}(\mu_i, \sigma_i^2) = \mathcal{N}(\mathbb{E}[R_i], \sigma_i^2) = \mathcal{N}(APR_i, \sigma_i^2)$$

$$cov(R_i, R_j) = \sigma_{i,j}$$

We can now define the following vectors for the **returns** $\boldsymbol{R}$ and the **weights** $\boldsymbol{w}$.

$$\boldsymbol{R} = \begin{pmatrix} r_{APPL} \\ r_{MSFT} \\ ... \\ r_{BTC-USD} \end{pmatrix}, \qquad \boldsymbol{w} = \begin{pmatrix} w_{APPL} \\ w_{MSFT} \\ ... \\ w_{BTC-USD} \end{pmatrix}$$

So the **expected returns** $\mathbb{E}[\boldsymbol{R}]$ will be

$$\mathbb{E}[\boldsymbol{R}] = \mathbb{E}\left[\begin{pmatrix} r_{APPL} \\ r_{MSFT} \\ ... \\ r_{BTC-USD} \end{pmatrix}\right] = \begin{pmatrix} \mathbb{E}[r_{APPL}] \\ \mathbb{E}[r_{MSFT}] \\ ... \\ \mathbb{E}[r_{BTC-USD}] \end{pmatrix} = \begin{pmatrix} APR_{APPL} \\ APR_{MSFT} \\ ... \\ APR_{BTC-USD} \end{pmatrix} = \begin{pmatrix} \mu_{APPL} \\ \mu_{MSFT} \\ ... \\ \mu_{BTC-USD} \end{pmatrix} = \boldsymbol{\mu}$$

[3] Calculations of these are in the appendix

Using matrix notations we have the following :

- **Return**

$$r_{risky} = \boldsymbol{w}^T \cdot \boldsymbol{R} = \begin{pmatrix} w_{AAPL}, & ... & w_{BTC-USD} \end{pmatrix} \cdot \begin{pmatrix} r_{AAPL} \\ r_{MSFT} \\ ... \\ r_{BTC-USD} \end{pmatrix} = \sum_{i=1}^{11} w_i \cdot r_i$$

- **Expected Return**

$$\mathbb{E}[r_{risky}] = \mathbb{E}[\boldsymbol{w}^T \cdot \boldsymbol{R}] = \boldsymbol{w}^T \cdot \mathbb{E}[\boldsymbol{R}] = \begin{pmatrix} w_{AAPL}, & ... & w_{BTC-USD} \end{pmatrix} \cdot \begin{pmatrix} \mathbb{E}[r_{AAPL}] \\ \mathbb{E}[r_{MSFT}] \\ ... \\ \mathbb{E}[r_{BTC-USD}] \end{pmatrix} = \sum_{i=1}^{11} w_i \cdot \mathbb{E}[r_i]$$

- **Risk**

$$\sigma^2_{risk} = var(\boldsymbol{w}^T \cdot \boldsymbol{R}) = \boldsymbol{w}^T \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{w} =$$

$$= \begin{pmatrix} w_{AAPL}, & ... & w_{BTC-USD} \end{pmatrix} \cdot \begin{pmatrix} \sigma^2_{AAPL} & ... & \sigma_{AAPL,BTC-USD} \\ ... & & \\ \sigma_{BTC-USD,AAPL} & ... & \sigma^2_{BTC-USD} \end{pmatrix} \cdot \begin{pmatrix} w_{AAPL} \\ w_{MSFT} \\ w_{BTC-USD} \end{pmatrix} =$$

$$= w^2_{AAPL} \cdot \sigma^2_{AAPL} + ... + w^2_{BTC-USD} \cdot \sigma^2_{BTC-USD} + 2 \cdot w_{AAPL} \cdot w_{MSFT} \sigma_{AAPL,MSFT} + ...$$

- **Weight Constraint**

$$\boldsymbol{x}^T \mathbf{1} = \begin{pmatrix} w_{AAPL}, & ... & w_{BTC-USD} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ ... \\ 1 \end{pmatrix} = w_{AAPL} + ... + w_{BTC-USD} = \sum_{i=1}^{11} w_i = 1$$

Initial Portfolio Return & Risk

Since **returns** and **risks** are only affected by the historical price data, they remain **constant** for a specific period of time. In particular, a 5-year-and-more period was selected. Therefore, in order to achieve better **mean-variance ratio (sharpe ratio)** we should definitely consider the simulation of different weight vectors while taking into consideration the capital allocation constraints.

The initial weights based on our *above average risk aversion* strategy were:

Stocks	*Bonds*	*Commodities*	*T-bills*
45%	35 %	10 %	10%

Each instrument will be initially weighted equally so

Total Portfolio			
Risky Portfolio			
Stocks	*Bonds*	*Commodities*	*Tbills*

Stocks

Ticker	*Weight*
AAPL	0.0643
MSFT	0.0643
AMZN	0.0643
GOOG	0.0643
FB	0.0643
NFLX	0.0643
NVDA	0.0643
Total	**0.45**

Bonds

Ticker	*Weight*
HCA	0.1166
VRTX	0.1166
Total	**0.35**

Commodities

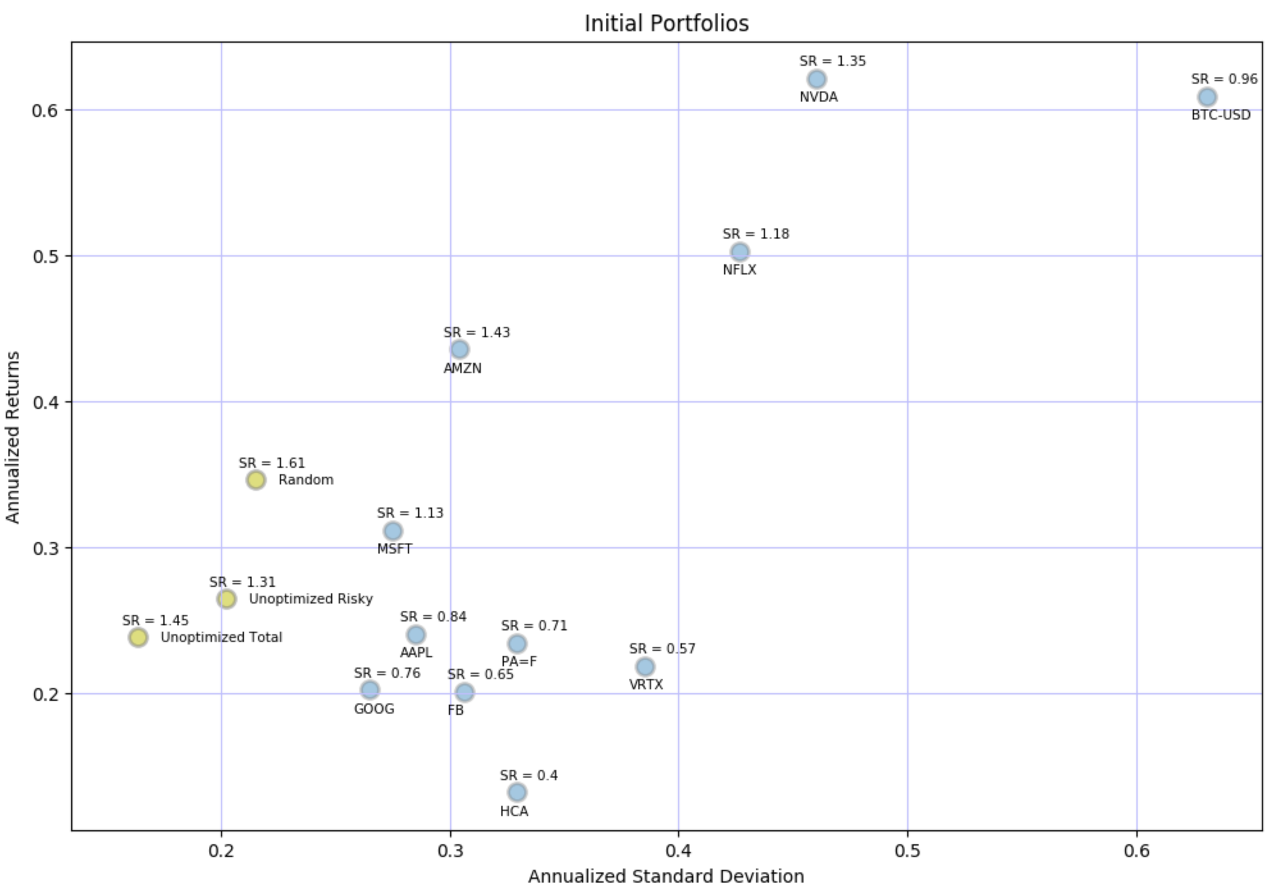
Ticker	*Weight*
BTC-USD	0.005
PA=F	0.005
Total	**0.10**

Tbills

Ticker	*Weight*
IRX	0.10
Total	**0.10**

So the results are the following:

	Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio* SR
Risky Portfolio	26.484 %	0.202	1.308
Total Portfolio	23.837 %	0.164	1.453



- The *Unoptimized Risky Portfolio* has higher return than the *Unoptimized Total Portfolio* but also a higher risk. However, the higher a Sharpe ratio the better the investment. So by investing 10% also in the risk-free instrument we get a better portfolio.
- This was a sample approach based on our *investing horizon* and our *risk-aversion*. However the analysis is expanded to find the optimal weights that maximizes (or minimizes) the objective functions.

Question 7 - Optimal Portfolio & Efficient Frontier

Diversifying through different weights and type of assets (*bonds, stocks, commodities*), which enclose different sources of factor risk, provides a lower volatility for the portfolio. Thus any portfolio generated will likely be better than any instrument individually. The assumption also holds for the graph of the previous question.

The portfolio is consisted of 11 instruments. The initial *mean-variance performance* was already calculated for the initial weight distribution. In terms of optimization however, we seek the the best risk-adjusted return (optimal weights) to conclude upon the 2 efficient portfolios. *Maximum Sharpe Ratio Portfolio* (or *Tangency Portfolio*) and *Minimum Volatility Portfolio*.

The steps are the following:

1. **Optimizing the risky portfolio**
   In this section, every combination of weights should be tested. Since the weights are real numbers, this is impossible. So, a *uniform sampling* is performed to reach a fair consensus upon the results. There are 2 approaches:

   (a) **Generating weights for each asset class**
   There are 3 asset classes, so if the weight increment step is set to 10%, this leads to 55 different equal combinations.

Stocks Weight	*Bonds Weight*	*Commodities Weight*	*Mean*	*Standard Deviation*	*Sharpe Ratio*	*Variance*
0,00%	0,00%	100,00%	56,29%	40,99%	1,271	16,80%
0,00%	10,00%	90,00%	52,43%	37,41%	1,289	14,00%
0,00%	20,00%	80,00%	48,56%	34,11%	1,301	11,63%
0,00%	30,00%	70,00%	44,70%	31,14%	1,301	9,70%
0,00%	40,00%	60,00%	40,83%	28,64%	1,280	8,20%
0,00%	50,00%	50,00%	36,97%	26,72%	1,227	7,14%
0,00%	60,00%	40,00%	33,10%	25,51%	1,133	6,51%
0,00%	70,00%	30,00%	29,24%	25,12%	0,997	6,31%
0,00%	80,00%	20,00%	25,37%	25,59%	0,828	6,55%
0,00%	90,00%	10,00%	21,51%	26,87%	0,645	7,22%
0,00%	100,00%	0,00%	17,64%	28,86%	0,466	8,33%
10,00%	0,00%	90,00%	54,29%	37,28%	1,344	13,90%
10,00%	10,00%	80,00%	50,42%	33,85%	1,366	11,46%
10,00%	20,00%	70,00%	46,56%	30,75%	1,378	9,46%
10,00%	30,00%	60,00%	42,69%	28,09%	1,371	7,89%
10,00%	40,00%	50,00%	38,83%	25,99%	1,333	6,75%
10,00%	50,00%	40,00%	34,96%	24,60%	1,251	6,05%
10,00%	60,00%	30,00%	31,10%	24,06%	1,119	5,79%
10,00%	70,00%	20,00%	27,23%	24,40%	0,944	5,95%
10,00%	80,00%	10,00%	23,37%	25,61%	0,749	6,56%
10,00%	90,00%	0,00%	19,50%	27,55%	0,556	7,59%
20,00%	0,00%	80,00%	52,28%	33,78%	1,424	11,41%
20,00%	10,00%	70,00%	48,42%	30,56%	1,447	9,34%
20,00%	20,00%	60,00%	44,55%	27,75%	1,455	7,70%
20,00%	30,00%	50,00%	40,69%	25,48%	1,432	6,49%
20,00%	40,00%	40,00%	36,82%	23,92%	1,364	5,72%
20,00%	50,00%	30,00%	32,96%	23,21%	1,240	5,39%
20,00%	60,00%	20,00%	29,09%	23,42%	1,064	5,48%
20,00%	70,00%	10,00%	25,23%	24,52%	0,858	6,01%
20,00%	80,00%	0,00%	21,36%	26,42%	0,650	6,98%
30,00%	0,00%	70,00%	50,28%	30,57%	1,508	9,34%
30,00%	10,00%	60,00%	46,41%	27,63%	1,528	7,63%
30,00%	20,00%	50,00%	42,55%	25,21%	1,521	6,36%
30,00%	30,00%	40,00%	38,68%	23,49%	1,469	5,52%
30,00%	40,00%	30,00%	34,82%	22,60%	1,355	5,11%
30,00%	50,00%	20,00%	30,95%	22,66%	1,181	5,13%
30,00%	60,00%	10,00%	27,09%	23,65%	0,968	5,60%
30,00%	70,00%	0,00%	23,22%	25,47%	0,747	6,49%
40,00%	0,00%	60,00%	48,27%	27,73%	1,590	7,69%
40,00%	10,00%	50,00%	44,41%	25,19%	1,597	6,34%
40,00%	20,00%	40,00%	40,54%	23,31%	1,560	5,43%
40,00%	30,00%	30,00%	36,68%	22,26%	1,460	4,95%
40,00%	40,00%	20,00%	32,81%	22,16%	1,292	4,91%
40,00%	50,00%	10,00%	28,95%	23,02%	1,076	5,30%
40,00%	60,00%	0,00%	25,08%	24,75%	0,844	6,12%
50,00%	0,00%	50,00%	46,27%	25,40%	1,656	6,45%
50,00%	10,00%	40,00%	42,40%	23,39%	1,634	5,47%
50,00%	20,00%	30,00%	38,54%	22,19%	1,548	4,92%
50,00%	**30,00%**	**20,00%**	**34,67%**	**21,93%**	**1,390**	**4,81%**
50,00%	40,00%	10,00%	30,81%	22,64%	1,176	5,13%
50,00%	50,00%	0,00%	26,94%	24,25%	0,938	5,88%
60,00%	**0,00%**	**40,00%**	**44,26%**	**23,73%**	**1,689**	**5,63%**
60,00%	10,00%	30,00%	40,40%	22,39%	1,617	5,01%
60,00%	20,00%	20,00%	36,53%	21,97%	1,472	4,83%
60,00%	30,00%	10,00%	32,67%	22,53%	1,264	5,08%
60,00%	40,00%	0,00%	28,80%	24,00%	1,026	5,76%
70,00%	0,00%	30,00%	42,26%	22,87%	1,665	5,23%
70,00%	10,00%	20,00%	38,39%	22,30%	1,534	4,97%
70,00%	20,00%	10,00%	34,53%	22,70%	1,337	5,15%
70,00%	30,00%	0,00%	30,66%	24,01%	1,103	5,76%
80,00%	0,00%	20,00%	40,25%	22,89%	1,575	5,24%
80,00%	10,00%	10,00%	36,39%	23,13%	1,392	5,35%
80,00%	20,00%	0,00%	32,52%	24,27%	1,168	5,89%
90,00%	0,00%	10,00%	38,25%	23,81%	1,431	5,67%
90,00%	10,00%	0,00%	34,38%	24,78%	1,219	6,14%
100,00%	0,00%	0,00%	36,24%	25,52%	1,256	6,51%

Concluding Remarks

- From this analysis, a considerably larger sharpe ratio is achieved in the diversified portfolio than from using either stocks, bonds or commodities individually.
- *Efficient Portfolios*
  - **Maximum Sharpe Ratio Portfolio**
    It is recommended that the highest reward-to-risk ratio is from investing 60% of our portfolio into **Stocks**, 0% into **Bonds** and 40% into **Commodities**. Holding a portfolio with these weights give us the best Sharpe Ratio of $SR_{max} = 1.689$.

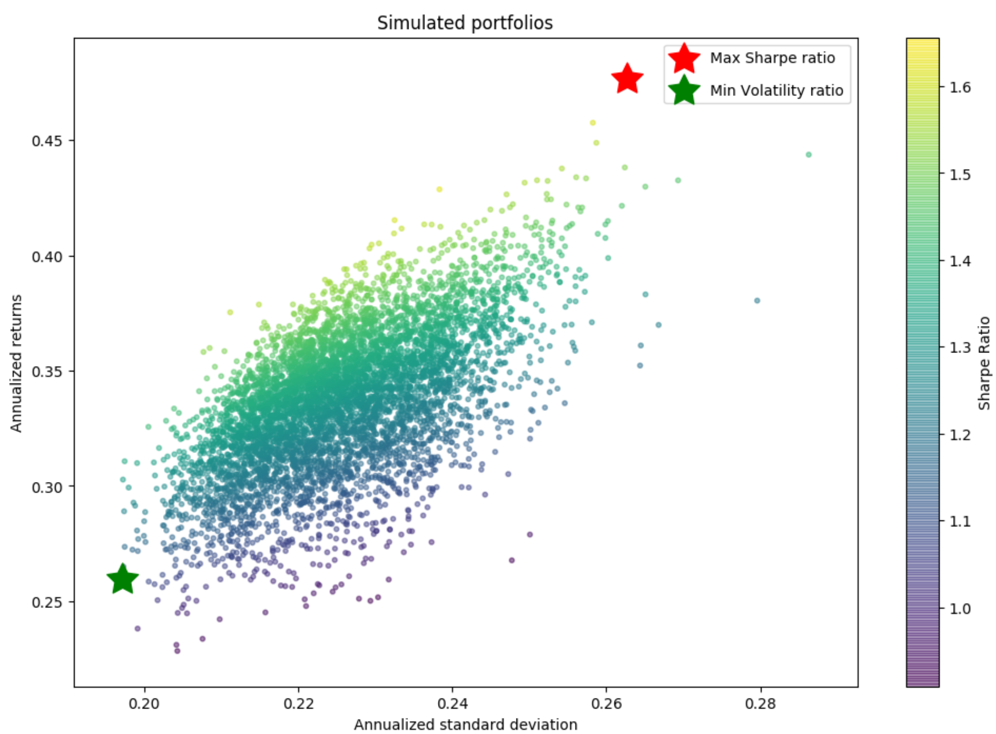
Expected Return $\mathbb{E}[r]$	*Risky* σ	*Sharpe Ratio* SR
44.26 %	0.2373	1.689

  - *Minimum Volatility Portfolio*
    It is recommended that the minimum risk ratio is from investing 50% of our portfolio into **Stocks**, 30% into **Bonds** and 20% into **Commodities**. Holding a portfolio with these weights give us the best Sharpe Ratio of $SR_{max} = 1.390$. Since the goal was to minimize the risk, lower return were expected. Diversification here is far clearer than before.

Expected Return $\mathbb{E}[r]$	*Risky* σ	*Sharpe Ratio* SR
34.67 %	0.2193	1.390

- This approach was implemented in Excel and searches the optimal weights among the 3 different asset classes. The function *MMULT()* was used in order to calculate the *standard deviation*. An important prerequisite was the covariance matrix (check the appendix for the mathematical analysis) which was firstly calculated for all the 11 instruments (11×11) and afterwards compressed to the 3 asset classes by averaging the proper values (3×3). This assumption can be translated into **equal weighting for every instrument inside an asset class.**

(b) **Generating weights for each instrument**
This approach was implemented in Python and searched the optimal weights among all the 11 instruments, without any equal weighting assumption. The simulation provided 6000 portfolios with random weights generated by using the function *numpy.random.random()*. After calculating *return, standard deviation, sharpe ratio* in the same way, we plot all these random portfolios along with the 2 efficient ones.



Concluding Remarks

- *Efficient Portfolios*
  - **Maximum Sharpe Ratio Portfolio**

    It is recommended that the highest reward-to-risk ratio is from investing 71.91% of our portfolio into **Stocks**, 7.80% into **Bonds** and 20.29% into **Commodities**.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*VRTX*	*BTC-USD*	*PA=F*
w	1.25	1.85	28.12	0.48	3.30	13.45	23.48	2.32	5.48	16.63	3.66

Expected Return $\mathbb{E}[r]$	*Risky* σ	*Sharpe Ratio SR*
52.61 %	0.269	1.797

  - *Minimum Volatility Portfolio*

    It is recommended that the minimum risk ratio is from investing 54.56% of our portfolio into **Stocks**, 17.75% into **Bonds** and 25.34% into **Commodities**.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*VRTX*	*BTC-USD*	*PA=F*
w	17.77	8.17	3.33	20.25	3.15	1.84	2.41	16.54	1.21	5.84	19.5

Expected Return $\mathbb{E}[r]$	*Risky* σ	*Sharpe Ratio SR*
27.92 %	0.198	1.198

- For the *Minimum Volatility Portfolio*, weighting of bonds is increased from 7.80% to 17.75%, while the *stock* and *commodities* weighting preserved at same levels by weighting more this time the instruments that **had lower standard deviation**. Allocating larger percentages to these makes intuitive sense, since risk minimization is required.
- This instrument-based simulation generated 6000 portfolios, whereas the asset-class-based simulation generated only 55 portfolios. The first discovered more portfolios in the feasible set and reached a much more optimal solution in which the *Maximum Sharpe Ratio Portfolio* has greater *return* and *sharpe ratio* and the *Minimum Volatility Portfolio* has lower *risk*

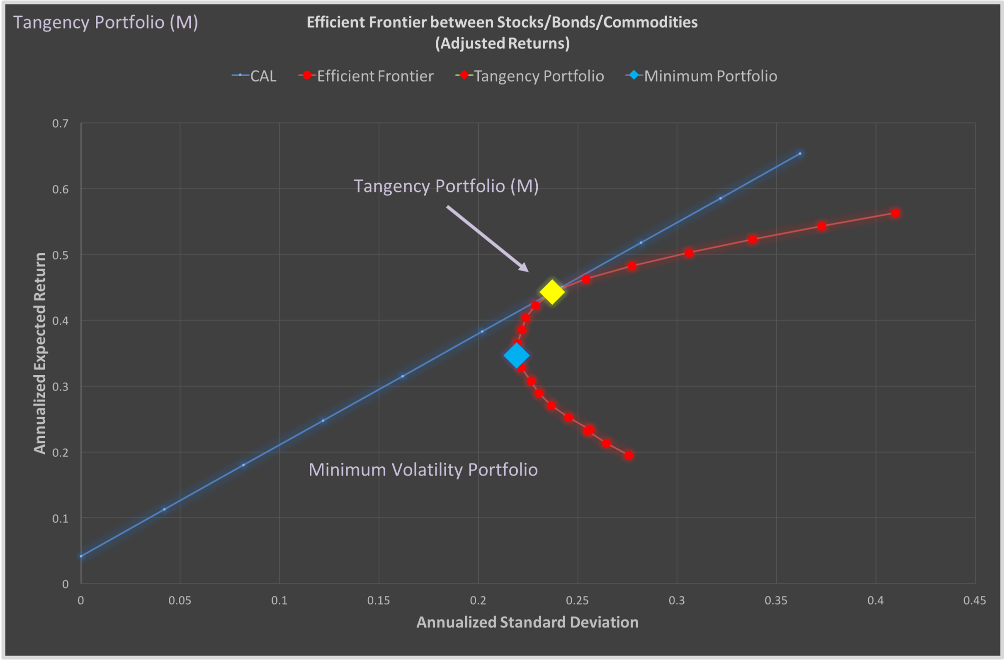
2. **Optimizing the Total Portfolio - Efficient Frontier & Capital Allocation Line**

Efficient Frontier

We proceeded with the first asset-class-based approach. The collection of all 55 data points $(x, y) = (standarddeviation, return)$ constitute the feasible region, which is convex to the left. The efficient frontier will only include some of these data points, the portfolios that have the lowest risk for every level of return met (the left convert bound of the curve).

Capital Allocation Line (CAL)

The portfolio is formed using the calculated optimal weights for the *risky portfolio*, and taking into consideration the inclusion of the risk-free instrument (T-bills). The average yearly risk-free rate is roughly 4.188%. Therefore, the *Capital Allocation Line* has slope of 1.689, irrespective of the weighting between the risk-free instrument and the risky portfolio.



Concluding Remarks

- This graph can help investors make their preferred investment decisions. The "M" marker represents the points where the rational investor would hold the pool of the risky instruments in the same proportion as theoretically efficient market portfolio. Under CAPM, all rational and non infinitely risk-averse investors should hold a position in the Capital Allocation Line, since *return is maximized for any given level of risk.*

Efficient Frontier Data

Stocks Weight	Bonds Weight	Commodities Weight	Mean	Standard Deviation	Sharpe Ratio
0,00%	0,00%	100,00%	56,29%	40,99%	1,271
10,00%	0,00%	90,00%	54,29%	37,28%	1,344
20,00%	0,00%	80,00%	52,28%	33,78%	1,424
30,00%	0,00%	70,00%	50,28%	30,57%	1,508
40,00%	0,00%	60,00%	48,27%	27,73%	1,590
50,00%	0,00%	50,00%	46,27%	25,40%	1,657
60,00%	0,00%	40,00%	44,26%	23,73%	168,87%
70,00%	0,00%	30,00%	42,26%	22,87%	1,665
60,00%	10,00%	30,00%	40,40%	22,39%	1,617
50,00%	20,00%	30,00%	38,54%	22,19%	1,548
60,00%	20,00%	20,00%	36,53%	21,97%	1,472
50,00%	30,00%	20,00%	34,67%	21,93%	139,00%
40,00%	40,00%	20,00%	32,81%	22,16%	1,292
50,00%	40,00%	10,00%	30,81%	22,64%	1,176
40,00%	50,00%	10,00%	28,95%	23,02%	1,076
30,00%	60,00%	10,00%	27,09%	23,65%	0,968
20,00%	70,00%	10,00%	25,23%	24,52%	0,858
10,00%	80,00%	10,00%	23,37%	25,61%	0,749
30,00%	70,00%	0,00%	23,22%	25,47%	0,747
20,00%	80,00%	0,00%	21,36%	26,42%	0,650
10,00%	90,00%	0,00%	19,50%	27,55%	0,556
0,00%	100,00%	0,00%	17,64%	28,86%	0,466

- The performance of the optimal portfolios after allocating 10% in the risk-free instrument is :
  - *Maximum Sharpe Ratio Total Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio* SR
47.77 %	0.2425	1.797

– *Minimum Volatility Total Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio SR*
27.92 %	0.1981	1.198

- We finalize our decision by investing in the *Maximum Sharpe Ratio Total Portfolio* which holds a great *mean-variance performance*, while its risk is acceptable according to our risk-aversion strategy (utility analysis in the next question).

Question 8 - Macroeconomic & Microeconomic events

In general, surveys have found the significance - through statistical analysis - for different Macroeconomic and *Microeconomic* events on instrument prices.

	Insignificant	*Significant*
Microeconomic	Capital Adequacy, Liquidity	Earnings, Size
Macroeconomic	Industrial Production, Exchange Rate, Inflation	Money Supply, Interest Rates, Crises

Applying a *Principal Component Analysis (PCA)* would prove survey's findings and eventually keep the few most important factors that could be used as extra features since they capture important price data information. So, the analysis is now expanded from a *mean-variance perspective* of :

$$Data = \{sample_1, ..., sample_N\} = \{(return_1, risk_1), ..., (return_N, risk_N)\}$$

to a *multi-factor perspective* of :

$$Data = \{(return_1, risk_1, size_1, money\_supply_1, ...), ..., (return_N, risk_N, size_N, money\_supply_N, ...)\}$$

Crude Oil War (March 2020 -)

After Russia had declined to join the *OPEC*, Saudi Arabia increased the oil prices in most the last 2 decades, provoking a war between these largest oil powers.

- **Stock, Bond , Tbill Market**
  The war has worsened things, by injecting volatility in many *stock instruments*. More specifically, *SP 500* was contracted by 7.6%. Because of the **Corona Virus**, 3 billion people are in lockdown, so global oil requirements could drop by 20%.
- **Commodities Market**
  This war could tear also the commodities markets which fanned the flammes of a volatile stock market, leading to a greater economic downturn

Corona Virus (Q1 2020 -

This virus operates as a **macroeconomic factor**. The impact of this is significant in *equity* and *commodities* market.

- **Stock, Bond, Tbill Market**
  The impact of the virus in stock prices is clear, especially when blue stock chips like Apple and Amazon. **AAPL** faced a 12% fall and **AMZN** 15% fall. Some of them will recover soon especially freight companies like Amazon since delivery demands are increased.
- **Commodities Market**
  Prices of significant commodities are still intact. Furthermore, future markets and gold prices are increasing. Speculators try to make quick money, by increasing the prices of various essential commodities like oilseeds and pulses, running on thin volumes.

As we see, the corona virus brought 2 dynamic potentials, some instruments prices are plummeting, some other are growing. The biggest price drops and rises since Febrouary 20th, due to COVID-19.

	AAPL	*MSFT*	*AMZN*	*GOOG*	*FB*	*NFLX*	*NVDA*	*HCA*	*VRTX*	*BTC-USD*	*PA=F*
Price drop	29.9%	26.6%	22.1%	30.4%	31.2%	22.6%	36.3%	53.9%	18.5%	48.2%	45.3%
Price Rise	-	-	10.7%	-	-	18.2%	-	- 8	28.%	48.2%	-

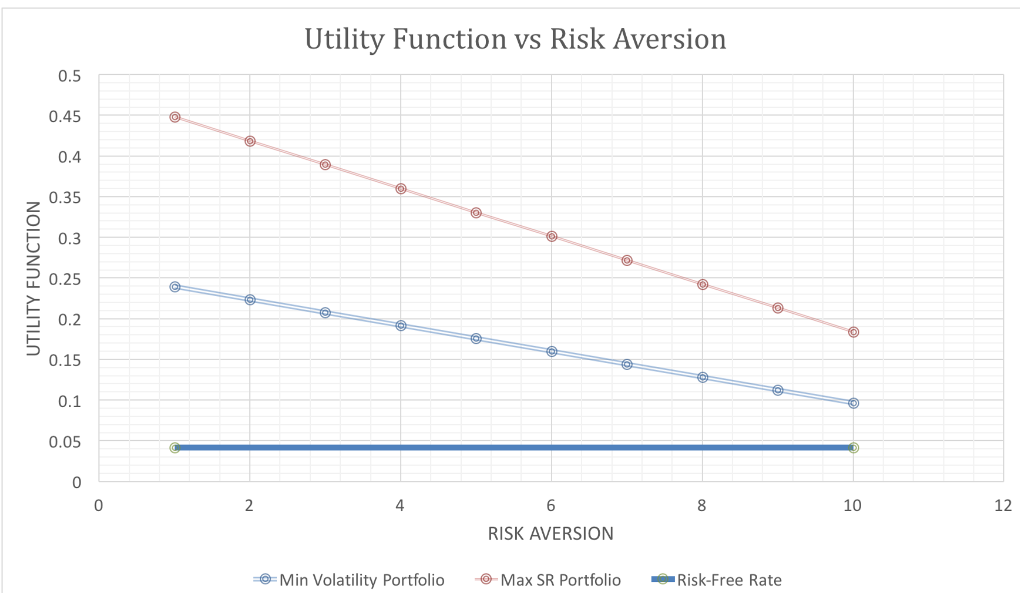
Instrument-Based Analysis

Instrument	Events
AAPL	On 16/04/2019, investors sued the company for artificially inflated prices for the date range of 02/11/2018 - 02/01/2019, seeking compensation of the losses. Stock price decreased by 9.96% in one day (from $157.92 to $142.19)
MSFT	
AMZN	
GOOG	
FB	On 20/11/2018 Facebook shares dropped nearly 40 percent from its peak in July due to Cambridge Analytica scandal.
NFLX	18/07/19 Netflix shares dropped more than 10%, on heavy trading volume, as Wall Street analysts and investors took full measure of the streaming giant's second-quarter subscriber buffering.
NVDA	
VRTX	
BTC-USD	A sell-off of most cryptocurrencies in January 2018 led to the bitcoin crash. After an unprecedented boom in 2017, the price of bitcoin fell by about 65% during the month from 06/01/2018 to 06/02/2018
PA=F	

Question 9 - Evaluate Portfolio's performance

Utility Function

Next, we consider the utility function of our portfolio against multiple risk aversions. We use *Certainty Equivalence Test*. The result is the following :



Concluding Remarks

- Even with the highest risk aversion of 10, our utility exceeds a lot the risk-free rate of 4.19%. The guaranteed cash from investing only in **T-bill** yields less than the expected utility as a given risky portfolio with absolute certainty.
  - *Maximum Sharpe Ratio Total Portfolio*

Utility for Maximum SR Portfolio	
Risk Aversion	Utility
1	44,83%
2	41,89%
3	38,95%
4	36,01%
5	33,07%
6	30,13%
7	27,19%
8	24,25%
9	21,31%
10	18,37%

  - *Minimum Volatility Total Portfolio*

Utility for Minimum Volatility Portfolio	
Risk Aversion	Utility
1	23,96%
2	22,37%
3	20,78%
4	19,19%
5	17,60%
6	16,01%
7	14,42%
8	12,83%
9	11,24%
10	9,65%

- If each instrument is considered individually, then yes it would be favorable to invest in *T-bills*. However, investing in this *Total Portfolio* yields more than investing solely to *T-bills*.

Excel SOLVER

In this section, tge weight vectors of both optimal risky portfolios is initialized to 0 in order for the Solver to find the optimal weights. The *return, standard deviation, variance, sharpe ratio* are calculated by properly using the functions MMULT(), TRANSPOSE() and the data *Mean Returns (AE6:AG6), Risk Free Rate (AP6), Covariance Matrix (AF159:AP169)*. The only constraint used was that the sum of the weights should be equal to 1:

$$\sum_{i=1}^{N} w_i = 1$$

The result is the following for the optimal portfolios :

- *Maximum Sharpe Ratio Risky Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio* SR
52.47 %	0.2519	1.917

- *Maximum Sharpe Ratio Risky Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio* SR
27.04 %	0.1906	1.199

Concluding Remarks

- The feasible set of weight vector combinations examined by the Excel is far larger than that of the 6000 portfolios so achieving better results is expected. For the Maximum SR Risky Portfolio SOLVER found a portfolio with $SR_{risky} = 1.917$, so 0.12 higher than our best solution. For the Minimum Volatility Risky Portfolio found a portfolio with $\sigma_{risky} = 0.1906$, so 0.0075 or 0.75% lower than our best solution.

- It is worth mentioning that for both optimization problems, SOLVER did allocate 0% in some instruments, which means we should exclude them from the optimal portfolio. However if we need to invest in all of them we can set the following constraint which implies the allocation of at least 1% to each instrument

$$\sum_{i=1}^{N} w_i = 1$$
$$w_i >= 0.01 \qquad \forall i \in (0, N)$$

The result is the following :

  - *Maximum Sharpe Ratio Risky Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio SR*
50.91 %	0.2474	1.889

  - *Maximum Sharpe Ratio Risky Portfolio*

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio SR*
27.59 %	0.1913	1.224

The result is still favorable. The $SR_{risky} = 1.889$ is still higher by 0.092 and $\sigma_{risky} = 0.1913$ is still lower by 0.0068 or 0.68%.

Passive Investment VS Active Portfolio Optimization

1. Since **passive management strategy** replicates an already diversified portfolio with predetermined risk and higher long-term results. An equivalent passive strategy would be investing capital in *technology, healthcare, cryptocurrency mutual funds or ETFs* as indicated by portfolio instruments' nature. Passive management offers :

   - **low risk**, because of an already diversification in such huge amount of instruments included in the fund.
   - **low cost fee** which means more of investors' money could be invested.
   - **fewer trading decisions**. Our healthcare-related bonds *HCA, VRTX* are just an example. Instead, investing in the whole sector is offered through this passive strategy on the *Ogorek Index Fund.*

2. On the other hand, the **semi-aggresive active management** strategy adopted in this project indicates the diversification through riskier instruments with higher turnover and return rates to outperform *S&P 500.* We consider large-caps as more efficient and able to potentially favor our choice by reflecting companies' events and new information at their prices. In *Q10 : Future Work* section further steps of portfolio management are proposed to ensure this approach.

3. **Concluding Remark** : **Active Strategy** was chosen. The volatility of this falling market (because of COVID-19) offers great tax-loss harvesting opportunities. The prices do not reflect what they do not know in this pandemic crisis, so skills in active management will be considered necessary. Moreover, this crisis has led to tremendous **ETFs trading**, questioning the reliability of these funds. Their rise the last decade was due to bank policies and to the 2008 financial crisis which provoked many failures in active managing, leading investors to less volatile solutions. However, this competition forced clarification of metrics and measures in portfolio construction in active management, making it more attractive.

Question 10 - Final Critical Evaluation

Evaluation

- In conclusion we find that investing in a diversified portfolio is a more favorable and more "useful" option than investing individually in the risk-free rate or in individual instruments. Even with risk aversion of 10 we achieve much better utility than the risk-free rate for both portfolios (18.37% and 9.65%). Especially for our *above average risk-aversion* strategy ($A = 3$), our mean-variance performance performance is actually really good achieving :

Expected Return $\mathbb{E}[r]$	*Risk* σ	*Sharpe Ratio* SR	*Utility*
47.77 %	0.2425	1.797	38.95%

- Even though the investment horizon may be wide, investing in the short run for various reasons my become a constraint in the investment process and affect the IPS. So, the portfolio needs to be adjusted to such circumstances and the security selection may not be just related to our risk aversion
- The diversified portfolio give us the ideal "reward-to-risk" ratio that minimizes risk for any given level of Expected Return. We also conclude that including a risk-free instrument into the portfolio helps in the construction of the *Capital Allocation Line* which is necessary for constructing a portfolio of 11 instruments and 1 risk-free instrument, where investors can choose their ideal position on line according to the degree of risk aversion.
- Through this project of **Strategic Capital Allocation**, we expanded our knowledge and used useful tools to extract instrument insights that can be proven helpful in future investing. However we need to point out :
  - **Return** : Estimating expected returns using historical data is not advisable. Since portfolio weights are too sensitive to very small changes in these returns, this can lead to a whole new optimal weight vectors.
  - **Risk** : Estimating risk through covariance matrices is easier, however not-entirely normally distributed returns can be also the cause for huge errors.
- Apart from diversifying through assets types differently responding to the market, the portfolio construction and management should follow any policy or company events or even any international *macro* or *microeconomic effects* on the international economy, such as the coronavirus outbreak.
- we select not to sell anything https://www.forbes.com/sites/davidrae/2020/04/03/coronavirus-stock-market-volatility/4e1ce31c7060
-

Future Work

- **General**
  - A major improvement to be made is to consider a longer time period. The period used, 2015- 2020, is largely affected by factors such as the *2020 corona virus*. We could expand the time period way back to 2000's but also taking into consideration macroeconomic factors such as the *2002 downturn* and the *2008 crash crisis*
  - If short sales were permitted the feasible set of solutions would be expanded in the right side containing more portfolios that have the same low levels of return but with low volatility. Mitigating though the problem of extreme portfolios, neither short-sales nor borrowing can be allowed.
- **Mean-Variance framework**
  - We also may wish to consider our data on a rolling basis. Our calculations are all done from historical means, but considering rolling returns may provide a more accurate view on our investments.
  - The modelling of expected returns was through the means of changes. We could test other metrics like APR_i, APY_i to represent them and build a feasible set of efficient frontiers based on all approaches.
- **Black-Litterman framework**
  - In order to better estimate *return* and *risk* compared to *mean-variance analysis*, we could use the Bayesian technique *Black-Litterman*

Appendix A Risk Tolerance Questionnaire

Investment Risk Tolerance Quiz[1]

Do you want to improve your personal finances? Start by taking this quiz to get an idea of your investment risk tolerance – one of the fundamental issues to consider when planning your investment strategy, either alone or in consultation with a financial services professional.

Choose the response that best describes you – there are no “right” or “wrong” answers. Just have fun!

When you're done, check the scoring grid to interpret your quiz score.

Note: By taking this quiz you will be contributing to a study on measuring financial risk tolerance. Your results will be recorded anonymously. We are not collecting any identifying information.

1. In general, how would your best friend describe you as a risk taker?
   a. A real gambler
   b. Willing to take risks after completing adequate research
   c. Cautious
   d. A real risk avoider

2. You are on a TV game show and can choose one of the following. Which would you take?
   a. $1,000 in cash
   b. A 50% chance at winning $5,000
   c. A 25% chance at winning $10,000
   d. A 5% chance at winning $100,000

3. You have just finished saving for a “once-in-a-lifetime” vacation. Three weeks before you plan to leave, you lose your job. You would:
   a. Cancel the vacation
   b. Take a much more modest vacation
   c. Go as scheduled, reasoning that you need the time to prepare for a job search
   d. Extend your vacation, because this might be your last chance to go first-class

4. If you unexpectedly received $20,000 to *invest*, what would you do?
   a. Deposit it in a bank account, money market account, or an insured CD
   b. Invest it in safe high quality bonds or bond mutual funds
   c. Invest it in stocks or stock mutual funds

5. In terms of experience, how comfortable are you investing in stocks or stock mutual funds?
   a. Not at all comfortable
   b. Somewhat comfortable
   c. Very comfortable

6. When you think of the word “risk” which of the following words comes to mind first?
   a. Loss
   b. Uncertainty
   c. Opportunity
   d. Thrill

7. Some experts are predicting prices of assets such as gold, jewels, collectibles, and real estate (hard assets) to increase in value; bond prices may fall, however, experts tend to agree that government bonds are relatively safe. Most of your investment assets are now in high interest government bonds. What would you do?
   a. Hold the bonds
   b. Sell the bonds, put half the proceeds into money market accounts, and the other half into hard assets
   c. Sell the bonds and put the total proceeds into hard assets
   d. Sell the bonds, put all the money into hard assets, and borrow additional money to buy more

8. Given the best and worst case returns of the four investment choices below, which would you prefer?
   a. $200 gain best case; $0 gain/loss worst case
   b. $800 gain best case; $200 loss worst case
   c. $2,600 gain best case; $800 loss worst case
   d. $4,800 gain best case; $2,400 loss worst case

9. In addition to whatever you own, you have been given $1,000. You are now asked to choose between:
   a. A sure gain of $500
   b. A 50% chance to gain $1,000 and a 50% chance to gain nothing

10. In addition to whatever you own, you have been given $2,000. You are now asked to choose between:
    a. A sure loss of $500
    b. A 50% chance to lose $1,000 and a 50% chance to lose nothing

11. Suppose a relative left you an inheritance of $100,000, stipulating in the will that you invest **ALL** the money in **ONE** of the following choices. Which one would you select?
    a. A savings account or money market mutual fund
    b. A mutual fund that owns stocks and bonds
    c. A portfolio of 15 common stocks
    d. Commodities like gold, silver, and oil

12. If you had to invest $20,000, which of the following investment choices would you find most appealing?
    a. 60% in low-risk investments 30% in medium-risk investments 10% in high-risk investments
    b. 30% in low-risk investments 40% in medium-risk investments 30% in high-risk investments
    c. 10% in low-risk investments 40% in medium-risk investments 50% in high-risk investments

13. Your trusted friend and neighbor, an experienced geologist, is putting together a group of investors to fund an exploratory gold mining venture. The venture could pay back 50 to 100 times the investment if successful. If the mine is a bust, the entire investment is worthless. Your friend estimates the chance of success is only 20%. If you had the money, how much would you invest?
    a. Nothing
    b. One month’s salary
    c. Three month’s salary
    d. Six month’s salary

---

[1] Risk Tolerance Quiz Source:
Grable, J. E., & Lytton, R. H. (1999). Financial risk tolerance revisited: The development of a risk assessment instrument. *Financial Services Review*, 8, 163-181.

Investment Risk Tolerance Quiz Scoring Grid

The scoring for the risk tolerance quiz questions is as follows:

1. a=4; b=3; c=2; d=1
2. a=1; b=2; c=3; d=4
3. a=1; b=2; c=3; d=4
4. a=1; b=2; c=3
5. a=1; b=2; c=3
6. a=1; b=2; c=3; d=4
7. a=1; b=2; c=3; d=4
8. a=1; b=2; c=3; d=4
9. a=1; b=3
10. a=1; b=3
11. a=1; b=2; c=3; d=4
12. a=1; b=2; c=3
13. a=1; b=2; c=3; d=4

In general, the score that you receive on the *Investment Risk Tolerance Quiz* can be interpreted as follows:

18 or below = Low risk tolerance (i.e., conservative investor)

19 to 22 = Below-average risk tolerance

23 to 28 = Average/moderate risk tolerance

29 to 32 = Above-average risk tolerance

33 and above = High risk tolerance (i.e., aggressive investor)

Appendix B Descriptive Metrics

Per instrument we calculate the following , considering that the total trading days are $N = 1327$, when per year there are $T = 252$ trading days

1. **Average Daily Return**
   Instruments's return since the beginning of the period of interest is calculated as following. Furthermore, the average return would be just the average of all these N returns.

$$return_t = \frac{price_t - price_0}{price_0}$$

$$average\_return = \frac{\sum_{t=1}^{t=N} return_t}{N}$$

2. **Annual Percentage Rate (APR)** vs **Annual Percentage Yield**

   (a) Alternatively we could use calculate the change of each instrument **per day**. There are 2 different approaches:

   - Calculate **the percentage increase (or decrease)** of a instrument when comparing day t to day $t+1$:

$$growth_t = \frac{price_{t+1} - price_t}{price_t}$$

   - However we can also model the growth with **log differences**

$$change_t = log(price_t) - log(price_{t-1})$$

   (b) In order to properly define the annualized formulae for APR, APY we should define the cumulative return :

$$return_t = \frac{price_t - price_0}{price_0} = \frac{price_t}{price_0} - 1 \Rightarrow 1 + return_t = \frac{price_t}{price_0} \qquad \textbf{(1)}$$

   However the price of the asset is chaning every second, so the period of compounding n approaches infinite (*continuous compounding*). So :

$$lim_{n+\infty}(1 + \frac{return_t}{n})^n = e^{return_t} \qquad \textbf{(2)}$$

   So, from **(2)**, assuming continuous compounding we get :

$$e^{return_t} = 1 + return_t \qquad \textbf{(3)}$$

   Therefore, from **(1)** and **(3)** we get :

$$e^{return_t} = \frac{price_t}{price_0}$$

   while by applying *logarithmic function* we finally get :

$$return_t = log\left(\frac{price_t}{price_0}\right) = log(price_t) - log(price_0)$$

   (c) Finally, we are ready to calculate the $cumulative\_return_{year}$ which represent the total return until that year from 2015, while the $return_{year}$ stands for the total return of that year and only.

$$cumulative\_return_{year} = log\left(\frac{price_t}{price_0}\right)$$

$$= log\left(\frac{price_t}{price_{t-1}} + +... + \frac{price_1}{price_0}\right)$$

$$= (log(price_t) - log(price_{t-1})) + ... + (log(price_1) - log(price_0))$$

$$= change_t + change_{t-1} + ... + change_0$$

So,

$$cumulative\_return_{year} = \sum_{t=1}^{(year-2015+1)\cdot T} change_t$$

$$return_{year} = \sum_{t=(year-2015)\cdot T}^{(year-2015+1)\cdot T} change_t$$

(d) So, the APR, which is the annual rate of interest that is paid on an investment, without taking into account the compounding of interest within that year , is calculated as following :

$$APR_{year} = 100\% \cdot return_{year}$$

So, the APY, which takes into consideration the frequency with which the interest is applied—the effects of intra-year compounding, is calculated as following: [4]

$$APY_{year} = 100\% \cdot (1 + cummulative\_return_{year})^{\frac{T}{x}} - 1$$

(e) Finally the APR by taking into consideration all the years is :

$$APR = 100\% \cdot \sum_{year=2015}^{2020} return_{year}$$

year	period	x
2020	01-01-2020 to 10-04-2020 (today)	67 days
2019	01-01-2019 to 31-12-2019	T + 70 = 319 days
2018	01-01-2018 to 31-12-2018	2T + 70 = 571 days
2017	01-01-2017 to 31-12-2017	3T + 70 = 823 days
2016	01-01-2016 to 31-12-2016	4T + 70 = 1075 days
2015	01-01-2015 to 31-12-2015	5T + 70 = 1327 days

3. **Variance** This is a risk metric. In order to be annualized we simply multiply by the total trading days of the year T

$$\sigma^2_{year} = \frac{\sum_{t=(year-2015)\cdot T}^{(year-2015+1)\cdot T} \left(change_t - \overline{change}\right)^2}{T} \cdot T$$

So, the standard deviation σ^2 for all the years is just the average of the yearly σ^2_{year}

$$\sigma^2 = \frac{\sum_{year=2015}^{2020} \sigma^2_{year}}{6}$$

4. **Standard Deviation** This is a risk metric.

$$\sigma_{year} = \sqrt{\sigma^2} = \sqrt{\frac{\sum_{t=(year-2015)\cdot T}^{(year-2015+1)\cdot T} \left(change_t - \overline{change}\right)^2}{T}} \cdot \sqrt{T}$$

So, the variance σ for all the years is just the average of the yearly σ_{year}

$$\sigma = \frac{\sum_{year=2015}^{2020} \sigma_{year}}{6}$$

5

[4]If $year = 2017$ for example when we compute the average change from 2017 until today, taking into consideration $3 * 252 + x$ trading days

[5]For both **variance, standard deviation** separate yearly calculations are omitted.

Appendix C Additional Risk Metrics & Capital Asset Pricing Model (CAPM)

- **Risk-Free Asset**
  3-month T-Bill is chosen as a risk-free rate. We expect the lowest return with the lowest risk.
- **Market**
  the SP 500 index is chosen as the **Market**
- **Linear Regression Model in Finance**
  A linear regression model has the following form :

$$y = \alpha + \beta x + \epsilon$$

where

  - ϵ = error factor
  - α = **intercept** of the model
  - β = **slope** of the model

In finance we use a special type of linear regression model which is CAPM modelling, which describes the relationship between systematic risk and expected return for assets. We can assume the following:

(1) All investors have the same information, forming the same expectations, so a **mean-variance approach** holds for the same optimum portfolio, which constitute the *market portfolio*. Moreover, the **separation theorem** stands for identifying different investor preferences.

(2) Furthermore, every asset's risk is part of its **standard deviation** influenced by the market risk.

(3) Finally, **equilibrium** will be achieved when the adjusted rate of return for every asset is equal

$$\frac{E[r_{APPL}] - r_{Tbill}}{\beta_{APPL}\sigma_M} = \frac{E[r_{AMZN}] - r_{Tbill}}{\beta_{AMZN}\sigma_M} = ... = \frac{E[r_M] - r_{Tbill}}{\beta_M\sigma_M}$$

So, since :

  - σ_M cancels out in every fractal.
  - $\beta_M = 1$ since the markets moves $1-1$ with the market.

$$\frac{E[r_{APPL}] - r_{Tbill}}{\beta_{APPL}} = \frac{E[r_{AMZN}] - r_{Tbill}}{\beta_{AMZN}} = ... = E[r_M] - r_{Tbill}$$

So, for every instrument the equilibrium can get the following form :

$$E[r_{APPL}] = r_{Tbill} + \beta_{APPL}(E[r_M] - r_{Tbill})$$
$$E[r_{AMZN}] = r_{Tbill} + \beta_{AMZN}(E[r_M] - r_{Tbill})$$
$$...$$

Therefore, the expected rate of return of every asset exceeds the risk-free rate of return by the product of the systematic risk (beta) and the risk premium of the market as whole. So if we generalize this :

$$E[r_I] - r_{RF} = \beta_I(E[r_M] - r_{RF})$$

Formula	Explanation
CAPM $E[r_I] - r_{RF} = \beta_I(E[r_M] - r_{RF})$	The expected rate of return of every asset exceeds the risk-free rate of return by : – by the product of the **systematic risk** β and the risk premium of the market as whole
Expanded CAPM(Single-Index Model) $E[r_I] - r_{RF} = \alpha + \beta_I(E[r_M] - r_{RF})$	The expected rate of return of every asset exceeds the risk-free rate of return by : – by the product of the **systematic risk** β and the risk premium of the market as whole – by the excess return that stands for the unexplained **unsystematic risk** α. This alpha stands for **Jensen's alpha**
Expanded CAPM (Single-Index Model) & error $E[r_I] - r_{RF} = \alpha + \beta_I(E[r_M] - r_{RF}) + \epsilon_I$	The expected rate of return of every asset exceeds the risk-free rate of return by : – **systematic risk** β and the risk premium of the market as whole – by the excess return that stands for the unexplained **unsystematic risk** α. This alpha stands for **Jensen's alpha** – the error ϵ is an error term measuring the vertical distance between the return of the asset I predicted by the equation and the real result.

- **Risk Metrics** α, β So, α, β is caclulated as following:

$$\beta_I = \frac{covariance_{I,M}}{variance_I} = \frac{covariance_{I,M}}{\sigma_I \cdot \sigma_I} = \frac{Cov_{I,M}}{\sigma_I \cdot \sigma_I} = \rho_{I,M} \cdot \frac{\sigma_I}{\sigma_M}$$

where

$$\rho_{I,M} = \frac{Cov_{I,M}}{\sigma_I \sigma_M} \in (0,1)$$

is the correlation between the **Market S&P 500** M and the corresponding instrument I

Finally by simply replacing $x = APR_I - r_{RF}$ and $y = APR_M - r_{RF}$ in the equation $y = \alpha + \beta \cdot x$ we get

$$\alpha_I = (APR_I - r_{RF}) - \beta \cdot (APR_M - r_{RF})$$

- **Risk Metrics Share Ratio** SR**, R-squared** R^2

$$SR_I = \frac{r_I - r_{RF}}{\sigma_I} = \frac{APR_I - r_{RF}}{\sigma_I}$$

$$R_I^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum_{t=1}^{N}((r_I - r_{RF}) - (\overline{r_I} - \overline{r_{RF}}))^2}{\sum_{t=1}^{N}((E[r_I] - r_{RF}) - (\overline{E[r_I]} - \overline{r_{RF}}))^2} = 1 - \frac{\sum_{t=1}^{N}(r_I - \overline{r_I})^2}{\sum_{t=1}^{N}(E[r_I] - \overline{r_I})^2} = 1 - \frac{\sum_{t=1}^{N}(APR_I - \overline{APR_I})^2}{\sum_{t=1}^{N}(E[r_I] - \overline{APR_I})^2}$$

The way of interpreting the coefficient of determination R^2 is as the percentage variation of the dependent variable $E[r_I]$ that is explained by this linear regression model. Based on the OLS theory and general statistics :

$$R_I^2 = \rho_{I,M}^2$$

We do not forget :

- r_I the actual value of return for the instrument I, which is the APR calculated previously as the average value of APRs for all the years.
- $E[r_I]$ the predicted value of return for the instrument I based on the *Single Index Model* as $E[r_I] = r_{RF} + \alpha + \beta \cdot (E[r_M] - r_{RF})$. So it is a prediction for average APR of the instrument.

Parameter	Explanation
$\boldsymbol{\alpha}$	– $\alpha < 0$: investment in asset I was too risky for the return – $\alpha = 0$: investment in asset I earned adequate return for the risk taken – $\alpha > 0$: investment in asset I earned excess return for the risk taken $\alpha > 0$ in order to "beat" the market and earn excess return
$\boldsymbol{\beta}$	– $\beta > 1$: investment in asset I is more volatile than the market – $0 < \beta < 1$: investment in asset I is less volatile than the market – $\beta = 0$: uncorrelated to the market – $\beta < 0$: negatively uncorrelated to the market $\beta < 1$ so that instruments included in the portfolio are less volatile than the market
$\boldsymbol{SR}$	It is important to achieve **large sharpe ratio**, since it indicates that instrument's returns (APR) are large relative to its volatility ($sigma$). Furthermore, the larger the ratio the higher the earnings on average than the risk-free rate (Tbill)
$\boldsymbol{R^2}$	For optimal models (under squared-error loss, shift and scale invariance), R is the square of the correlation between the true and predicted outcomes. This relationship is not true for general f and y. Here linear regression was applied (CAPM) so $R^2 = \rho^2$. – $R^2 = 1$, the market **S&P500** completely explains the instrument returns – $R^2 = 0$, the market **S&P500** does not explain the instrument returns at all.

Appendix D Efficient Frontier & Capital Allocation Line (CAL)

Optimal Weights in the Risky Portfolio

The methodology followed is the following:

1. Find the *return* and risk characteristics of all 11 instruments. (means, variances, covariances)
2. Construct the **risky optimal portfolio**:
   - Calculate the weights of the optimal portfolio
   - Compute *return* and risk of this portfolio

   We solved the following *optimization problem*:

Let $N = \#instruments$ then resolve the following :

$$maximize_{\mathbb{E}[r_{risky}],\sigma_{risky}} \left(SR_{risky} = \frac{\mathbb{E}[r_{risky}] - r_{RF}}{\sigma_{risky}} \right)$$

subject to :

$$\mathbb{E}[r_{risky}] = \sum_{i=1}^{N} w_i \cdot \mathbb{E}[r_i]$$

$$\sigma_{risky} = \boldsymbol{w}^T \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{w} = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_i w_j Cov_{i,j}}$$

3. Allocate capital between the **optimal risky portfolio** and the **risk-free instrument**
   - Calculate the fraction (weights) of the total portfolio allocated to the **tangency portfolio** and the **risk-free instrument**

   We solve the following problem :

> The final *return* and *risk* of the portfolio will be, when $\sigma_{RF} \sim 0$ and $Cov_{risky,RF} \sim 0$. So we need to find the w weight allocated in the risky portfolio and the obvious $1 - w$ allocated in the risk-free instrument
>
> $$\mathbb{E}[r_P] = w \cdot \mathbb{E}[r_{risky}] + (1 - w) \cdot r_{RF}$$
>
> $$\begin{aligned} \sigma_P &= \sqrt{w^2 \cdot \sigma_{risky}^2 + (1 - w)^2 \cdot \sigma_{RF}^2 + 2 \cdot w \cdot (1 - w) \cdot Cov_{risky,RF}} \\ &= w \cdot \sigma_{risky} \end{aligned}$$

4. Calculate the *utility* for different risk aversions to check if the utility always exceeds the risk-free instrument.

> The utility function is defined as following. The 0.5 scales the marginal utility (first derivative) and here reflects use of fractional returns.
>
> $$\mathbb{E}[U_P] = \mathbb{E}[r_P] - \frac{1}{2} \cdot A \cdot \sigma_P^2$$