Unit 1 Evaluation

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Problem 1. *Let $f(x) = x^3 - 3x + 1$. Find all critical points of f and determine their nature.*

Solution: First, compute the derivative:

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

Set $f'(x) = 0$:

$$3(x^2 - 1) = 0 \implies x = \pm 1$$

Critical points are at $x = -1$ and $x = 1$. To determine nature:

$$f''(x) = 6x$$

- At $x = -1$: $f''(-1) = -6 < 0$ (local maximum) - At $x = 1$: $f''(1) = 6 > 0$ (local minimum)
■

Problem 2. *Prove that $\sqrt{2}$ is irrational.*

Solution: Assume for contradiction that $\sqrt{2} \in \mathbb{Q}$. Then we can write:

$$\sqrt{2} = \frac{p}{q}$$

where $p, q \in \mathbb{Z}$ with $\gcd(p, q) = 1$. Squaring both sides:

$$2 = \frac{p^2}{q^2} \implies p^2 = 2q^2$$

Thus p^2 is even, so p must be even. Write $p = 2k$:

$$(2k)^2 = 2q^2 \implies 4k^2 = 2q^2 \implies q^2 = 2k^2$$

Hence q^2 is even, so q is even. But this contradicts $\gcd(p, q) = 1$. ■

Problem 3. *Find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x-axis.*

Solution: Using the disk method:

$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx$$

$$= \pi \left[\frac{x^2}{2} \right]_0^4 = \pi \left(\frac{16}{2} - 0 \right) = 8\pi$$

$\blacksquare$