Library mk_theorems

mk_theorems

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Require Export mk_structure.

Theorem MKT4 : ∀ x y z, z ∈ x \/ z ∈ y <-> z ∈ (x ∪ y).

Theorem MKT4' : ∀ x y z, z ∈ x /\ z ∈ y <-> z ∈ (x ∩ y).

Ltac deHun :=
    match goal with
      | H: ?c ∈ ?a∪?b
          |- _ => apply MKT4 in H as [] ; deHun
      | _ => idtac
    end.

Ltac deGun :=
    match goal with
        | |- ?c ∈ ?a∪?b => apply MKT4 ; deGun
        | _ => idtac
    end.

Ltac deHin :=
    match goal with
      | H: ?c ∈ ?a∩?b
          |- _ => apply MKT4' in H as []; deHin
      | _ => idtac
    end.

Ltac deGin :=
    match goal with
        | |- ?c ∈ ?a∩?b => apply MKT4'; split; deGin
        | _ => idtac
    end.

Theorem MKT5 : ∀ x, x ∪ x = x.

Theorem MKT5' : ∀ x, x ∩ x = x.

Theorem MKT6 : ∀ x y, x ∪ y = y ∪ x.

Theorem MKT6' : ∀ x y, x ∩ y = y ∩ x.

Theorem MKT7 : ∀ x y z, (x ∪ y) ∪ z = x ∪ (y ∪ z).

Theorem MKT7' : ∀ x y z, (x ∩ y) ∩ z = x ∩ (y ∩ z).

Theorem MKT8 : ∀ x y z, x ∩ (y ∪ z) = (x ∩ y) ∪ (x ∩ z).

Theorem MKT8' : ∀ x y z, x ∪ (y ∩ z) = (x ∪ y) ∩ (x ∪ z).

Theorem MKT11: ∀ x, ¬ (¬ x) = x.

Theorem MKT12 : ∀ x y, ¬ (x ∪ y) = (¬ x) ∩ (¬ y).

Theorem MKT12' : ∀ x y, ¬ (x ∩ y) = (¬ x) ∪ (¬ y).

Fact setminP : ∀ z x y, z ∈ x -> ~ z ∈ y -> z ∈ (x ~ y).

Global Hint Resolve setminP : core.

Fact setminp : ∀ z x y, z ∈ (x ~ y) -> z ∈ x /\ ~ z ∈ y.

Theorem MKT14 : ∀ x y z, x ∩ (y ~ z) = (x ∩ y) ~ z.

Theorem MKT16 : ∀ {x}, x ∉ Φ.
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Ltac emf :=
    match goal with
        H: ?a ∈ Φ
        |- _ => destruct (MKT16 H)
    end.

Ltac eqE := eqext; try emf; auto.

Ltac feine z := destruct (@ MKT16 z).

Theorem MKT17 : ∀ x, Φ ∪ x = x.

Theorem MKT17' : ∀ x, Φ ∩ x = Φ.

Theorem MKT19 : ∀ x, x ∈ μ <-> Ensemble x.

Theorem MKT19a : ∀ x, x ∈ μ -> Ensemble x.

Theorem MKT19b : ∀ x, Ensemble x -> x ∈ μ.

Global Hint Resolve MKT19a MKT19b : core.

Theorem MKT20 : ∀ x, x ∪ μ = μ.

Theorem MKT20' : ∀ x, x ∩ μ = x.

Theorem MKT21 : ¬ Φ = μ.

Theorem MKT21' : ¬ μ = Φ.

Ltac deHex1 :=
    match goal with
        H: ∃ x, ?P
        |- _ => destruct H as []
    end.

Ltac rdeHex := repeat deHex1; deand.

Theorem MKT24 : ∩Φ = μ.

Theorem MKT24' : ∪Φ = Φ.

Theorem MKT26 : ∀ x, Φ ⊂ x.

Theorem MKT26' : ∀ x, x ⊂ μ.

Theorem MKT26a : ∀ x, x ⊂ x.

Global Hint Resolve MKT26 MKT26' MKT26a : core.

Fact ssubs : ∀ {a b z}, z ⊂ (a ~ b) -> z ⊂ a.

Global Hint Immediate ssubs : core.

Fact esube : ∀ {z}, z ⊂ Φ -> z = Φ.

Theorem MKT27 : ∀ x y, (x ⊂ y /\ y ⊂ x) <-> x = y.

Theorem MKT28 : ∀ {x y z}, x ⊂ y -> y ⊂ z -> x ⊂ z.

Theorem MKT29 : ∀ x y, x ∪ y = y <-> x ⊂ y.

Theorem MKT30 : ∀ x y, x ∩ y = x <-> x ⊂ y.

Theorem MKT31 : ∀ x y, x ⊂ y -> (∪x ⊂ ∪y) /\ (∩y ⊂ ∩x).

Theorem MKT32 : ∀ x y, x ∈ y -> (x ⊂ ∪y) /\ (∩y ⊂ x).


Theorem MKT33 : ∀ x z, Ensemble x -> z ⊂ x -> Ensemble z.

Theorem MKT34 : Φ = ∩μ.
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Theorem MKT34' : μ = ∪μ.

Lemma NEexE : ∀ x, x ≠ Φ <-> ∃ z, z∈x.

Ltac NEele H := apply NEexE in H as [].

Theorem MKT35 : ∀ x, x ≠ Φ -> Ensemble (∩x).

Theorem MKT37 : μ = pow(μ).

Theorem MKT38a : ∀ {x}, Ensemble x -> Ensemble pow(x).

Theorem MKT38b : ∀ {x}, Ensemble x -> (∀ y, y ⊂ x <-> y ∈ pow(x)).


Lemma Lemma_N : ~ Ensemble \{ λ x, x ∉ x \}.

Theorem MKT39 : ~ Ensemble μ.

Fact singlex : ∀ x, Ensemble x -> x ∈ [x].

Global Hint Resolve singlex : core.

Theorem MKT41 : ∀ x, Ensemble x -> (∀ y, y ∈ [x] <-> y = x).

Ltac eins H := apply MKT41 in H; subst; eauto.

Theorem MKT42 : ∀ x, Ensemble x -> Ensemble ([x]).

Global Hint Resolve MKT42 : core.

Theorem MKT43 : ∀ x, [x] = μ <-> ~ Ensemble x.

Theorem MKT42' : ∀ x, Ensemble ([x]) -> Ensemble x.

Theorem MKT44 : ∀ {x}, Ensemble x -> ∩[x] = x /\ ∪[x] = x.

Theorem MKT44' : ∀ x, ~ Ensemble x -> ∩[x] = Φ /\ ∪[x] = μ.

Corollary AxiomIV': ∀ x y, Ensemble (x ∪ y)
    -> Ensemble x /\ Ensemble y.

Theorem MKT46a : ∀ {x y}, Ensemble x -> Ensemble y
    -> Ensemble ([x|y]).

Global Hint Resolve MKT46a : core.

Theorem MKT46b : ∀ {x y}, Ensemble x -> Ensemble y
    -> (∀ z, z ∈ [x|y] <-> (z = x \/ z = y)).

Theorem MKT46' : ∀ x y, [x|y] = μ <-> ~ Ensemble x \/ ~ Ensemble y.

Theorem MKT47a : ∀ x y, Ensemble x -> Ensemble y -> ∩[x|y] = x ∩ y.

Theorem MKT47b : ∀ x y, Ensemble x -> Ensemble y
    -> ∪[x|y] = x ∪ y.

Theorem MKT47' : ∀ x y, ~ Ensemble x \/ ~ Ensemble y
    -> (∩[x|y] = Φ) /\ (∪[x|y] = μ).


Theorem MKT49a : ∀ {x y}, Ensemble x -> Ensemble y
    -> Ensemble ([x,y]).

Global Hint Resolve MKT49a : core.

Theorem MKT49b : ∀ x y, Ensemble ([x,y]) -> Ensemble x /\ Ensemble y.

Theorem MKT49c1 : ∀ {x y}, Ensemble ([x,y]) -> Ensemble x.

Theorem MKT49c2 : ∀ {x y}, Ensemble ([x,y]) -> Ensemble y.
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Ltac ope1 :=
    match goal with
        H: Ensemble ([?x,?y])
        |- Ensemble ?x => eapply MKT49c1; eauto
    end.

Ltac ope2 :=
    match goal with
        H: Ensemble ([?x,?y])
        |- Ensemble ?y => eapply MKT49c2; eauto
    end.

Ltac ope3 :=
    match goal with
        H: [?x,?y] ∈ ?z
        |- Ensemble ?x => eapply MKT49c1; eauto
    end.

Ltac ope4 :=
    match goal with
        H: [?x,?y] ∈ ?z
        |- Ensemble ?y => eapply MKT49c2; eauto
    end.

Ltac ope := try ope1; try ope2; try ope3; try ope4.

Theorem MKT49' : ∀ x y, ~ Ensemble ([x,y]) -> [x,y] = μ.

Fact subcp1 : ∀ x y, x ⊂ (x ∪ y).

Global Hint Resolve subcp1 : core.

Lemma Lemma50a : ∀ x y, Ensemble x -> Ensemble y -> ∪[x,y] = [x|y].

Lemma Lemma50b : ∀ x y, Ensemble x -> Ensemble y -> ∩[x,y] = [x].

Theorem MKT50 : ∀ {x y}, Ensemble x -> Ensemble y
    -> (∪[x,y] = [x|y]) /\ (∩[x,y] = [x]) /\ (∪(∩[x,y]) = x)
        /\ (∩(∩[x,y]) = x) /\ (∪(∪[x,y]) = x∪y) /\ (∩(∪[x,y]) = x∩y).

Lemma Lemma50' : ∀ (x y: Class), ~ Ensemble x \/ ~ Ensemble y
    -> ~ Ensemble ([x]) \/ ~ Ensemble ([x | y]).

Theorem MKT50' : ∀ {x y}, ~ Ensemble x \/ ~ Ensemble y
    -> (∪(∩[x,y]) = Φ) /\ (∩(∩[x,y]) = μ) /\ (∪(∪[x,y]) = μ)
        /\ (∩(∪[x,y]) = Φ).

Definition First z := ∩(∩z).

Definition Second z := (∩ ∪ z) ∪ ((∪(∪z)) ~ (∪(∩z))).

Theorem MKT53 : Second μ = μ.

Theorem MKT54a : ∀ x y, Ensemble x -> Ensemble y
    -> First ([x,y]) = x.

Theorem MKT54b : ∀ x y, Ensemble x -> Ensemble y
    -> Second ([x,y]) = y.

Theorem MKT54' : ∀ x y, ~ Ensemble x \/ ~ Ensemble y
    -> First ([x,y]) = μ /\ Second ([x,y]) = μ.

Theorem MKT55 : ∀ x y u v, Ensemble x -> Ensemble y
    -> ([x,y] = [u,v] <-> x = u /\ y = v).

Fact Pins : ∀ a b c d, Ensemble c -> Ensemble d
    -> [a,b] ∈ [[c,d]] -> a = c /\ b = d.

Ltac pins H := apply Pins in H as []; subst; eauto.

Fact Pinfus : ∀ a b f x y, Ensemble x -> Ensemble y
    -> [a,b] ∈ (f ∪ [[x,y]]) -> [a,b] ∈ f \/ (a = x /\ b = y).
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Ltac pinfus H := apply Pinfus in H as [?|[]]; subst; eauto.

Ltac eincus H := apply AxiomII in H as [_ [H|H]]; try eins H; auto.

Ltac PP H a b := apply AxiomII in H as [? [a [b []]]]; subst.

Fact AxiomII' : ∀ a b P,
    [a,b] ∈ \{\ P \}\ <-> Ensemble ([a,b]) /\ (P a b).

Ltac appoA2G := apply AxiomII'; split; eauto.

Ltac appoA2H H := apply AxiomII' in H as [].

Theorem MKT58 : ∀ r s t, (r ∘ s) ∘ t = r ∘ (s ∘ t).

Theorem MKT59 : ∀ r s t, Relation r -> Relation s
    -> r ∘ (s ∪ t) = (r ∘ s) ∪ (r ∘ t)
       /\ r ∘ (s ∩ t) ⊂ (r ∘ s) ∩ (r ∘ t).

Fact invp1 : ∀ a b f, [b,a] ∈ f⁻¹ <-> [a,b] ∈ f.

Fact uiv : ∀ a b, (a ∪ b)⁻¹ = a⁻¹ ∪ b⁻¹.

Fact iiv : ∀ a b, (a ∩ b)⁻¹ = a⁻¹ ∩ b⁻¹.

Fact siv : ∀ a b, Ensemble a -> Ensemble b -> [[a,b]]⁻¹ = [[b,a]].

Theorem MKT61 : ∀ r, Relation r -> (r⁻¹)⁻¹ = r.

Theorem MKT62 : ∀ r s, (r ∘ s)⁻¹ = (s⁻¹) ∘ (r⁻¹).


Fact opisf : ∀ a b, Ensemble a -> Ensemble b -> Function ([[a,b]]).

Fact PisRel : ∀ P, Relation \{\ P \}\.

Global Hint Resolve PisRel : core.

Theorem MKT64 : ∀ f g, Function f -> Function g -> Function (f ∘ g).

Corollary Property_dom : ∀ {x y f}, [x,y] ∈ f -> x ∈ dom(f).

Corollary Property_ran : ∀ {x y f}, [x,y] ∈ f -> y ∈ ran(f).

Fact deqri : ∀ f, dom(f) = ran(f⁻¹).

Fact reqdi : ∀ f, ran(f) = dom(f⁻¹).

Fact subdom : ∀ {x y}, x ⊂ y -> dom(x) ⊂ dom(y).

Fact undom : ∀ f g, dom(f ∪ g) = dom(f) ∪ dom(g).

Fact unran : ∀ f g, ran(f ∪ g) = ran(f) ∪ ran(g).

Fact domor : ∀ u v, Ensemble u -> Ensemble v -> dom([[u,v]]) = [u].

Fact ranor : ∀ u v, Ensemble u -> Ensemble v -> ran([[u,v]]) = [v].

Fact fupf : ∀ f x y, Function f -> Ensemble x -> Ensemble y
    -> ~ x ∈ dom(f) -> Function (f ∪ [[x,y]]).

Fact dos1 : ∀ {f x} y, Function f -> [x,y] ∈ f
    -> dom(f ~ [[x,y]]) = dom(f) ~ [x].

Fact ros1 : ∀ {f x y}, Function f⁻¹ -> [x,y] ∈ f
    -> ran(f ~ [[x,y]]) = ran(f) ~ [y].

Theorem MKT67a: dom(μ) = μ.

Theorem MKT67b: ran(μ) = μ.

Theorem MKT69a : ∀ {x f}, x ∉ dom(f) -> f[x] = μ.
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Theorem MKT69b : ∀ {x f}, x ∈ dom(f) -> f[x] ∈ μ.

Theorem MKT69a' : ∀ {x f}, f[x] = μ -> x ∉ dom(f).

Theorem MKT69b' : ∀ {x f}, f[x] ∈ μ -> x ∈ dom(f).

Corollary Property_Fun : ∀ y f x, Function f
    -> [x,y] ∈ f -> y = f[x].

Lemma uvinf : ∀ z a b f, ~ a ∈ dom(f) -> Ensemble a -> Ensemble b
    -> (z ∈ dom(f) -> (f ∪ [[a,b]])[z] = f[z]).

Lemma uvinp : ∀ a b f, ~ a ∈ dom(f) -> Ensemble a -> Ensemble b
    -> (f ∪ [[a,b]])[a] = b.

Fact Einr : ∀ {f z}, Function f -> z ∈ ran(f)
    -> ∃ x, x ∈ dom(f) /\ z = f[x].

Ltac einr H := New H; apply Einr in H as [? []]; subst; auto.

Theorem MKT70 : ∀ f, Function f -> f = \{\ λ x y, y = f[x] \}\.

Corollary Property_Value : ∀ {f x}, Function f -> x ∈ dom(f)
    -> [x,f[x]] ∈ f.

Fact subval : ∀ {f g}, f ⊂ g -> Function f -> Function g
    -> ∀ u, u ∈ dom(f) -> f[u] = g[u].

Corollary Property_Value' : ∀ f x, Function f -> f[x] ∈ ran(f)
    -> [x,f[x]] ∈ f.

Corollary Property_dm : ∀ {f x}, Function f -> x ∈ dom(f)
    -> f[x] ∈ ran(f).

Theorem MKT71 : ∀ f g, Function f -> Function g
    -> (f = g <-> ∀ x, f[x] = g[x]).

Ltac xo :=
    match goal with
        |- Ensemble ([?a, ?b]) => try apply MKT49a
    end.

Ltac rxo := eauto; repeat xo; eauto.

Lemma Ex_Lemma73 : ∀ {u y}, Ensemble u -> Ensemble y
    -> let f:= \{\ λ w z, w ∈ y /\ z = [u,w] \}\ in
        Function f /\ dom(f) = y /\ ran(f) = [u] × y.

Theorem MKT73 : ∀ u y, Ensemble u -> Ensemble y
    -> Ensemble ([u] × y).

Lemma Ex_Lemma74 : ∀ {x y}, Ensemble x -> Ensemble y
    -> let f := \{\ λ u z, u ∈ x /\ z = [u] × y \}\ in
        Function f /\ dom(f) = x
        /\ ran(f) = \{ λ z, ∃ u, u ∈ x /\ z = [u] × y \}.

Lemma Lemma74 : ∀ {x y}, Ensemble x -> Ensemble y
    -> ∪(\{ λ z, ∃ u, u ∈ x /\ z = [u] × y \}) = x × y.

Theorem MKT74 : ∀ {x y}, Ensemble x -> Ensemble y
    -> Ensemble (x × y).

Theorem MKT75 : ∀ f, Function f -> Ensemble dom(f) -> Ensemble f.

Fact fdme : ∀ {f}, Function f -> Ensemble f -> Ensemble dom(f).

Fact frne : ∀ {f}, Function f -> Ensemble f -> Ensemble ran(f).

Theorem MKT77 : ∀ x y, Ensemble x -> Ensemble y
    -> Ensemble (Exponent y x).


Fact Property_Asy : ∀ {r x u}, Asymmetric r x -> u ∈ x
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    -> ~ Rrelation u r u.

Corollary wosub : ∀ x r y, WellOrdered r x -> y ⊂ x
    -> WellOrdered r y.

Theorem MKT88a : ∀ {r x}, WellOrdered r x -> Asymmetric r x.

Theorem MKT88b : ∀ r x, WellOrdered r x -> Transitive r x.

Theorem MKT90 : ∀ n x r, n ≠ Φ -> (∀ y, y ∈ n -> rSection y r x)
    -> rSection (∩n) r x /\ rSection (∪n) r x.

Theorem MKT91 : ∀ {x y r}, rSection y r x -> y <> x
    -> (∃ v, v ∈ x /\ y = \{ λ u, u ∈ x /\ Rrelation u r v \}).

Theorem MKT92 : ∀ {x y z r}, rSection x r z -> rSection y r z
    -> x ⊂ y \/ y ⊂ x.

Theorem MKT94 : ∀ {x r y f}, rSection x r y -> Order_Pr f r r
    -> On f x -> To f y -> (∀ u, u ∈ x -> ~ Rrelation f[u] r u).

Lemma f11vi : ∀ f u, Function f -> Function f⁻¹ -> u ∈ ran(f)
    -> f[(f⁻¹)[u]] = u.

Lemma f11inj : ∀ f a b, Function f -> Function f⁻¹
    -> a ∈ dom(f) -> b ∈ dom(f) -> f[a] = f[b] -> a = b.

Lemma f11iv : ∀ f u, Function f -> Function f⁻¹ -> u ∈ dom(f)
    -> (f⁻¹)[f[u]] = u.

Fact f11pa : ∀ {f x y}, Function1_1 f -> [x,y] ∈ f
    -> Function1_1 (f ~ [[x,y]]).

Fact f11pb : ∀ f x y, Function1_1 f -> Ensemble x -> Ensemble y
    -> ~ x ∈ dom(f) -> ~ y ∈ ran(f) -> Function1_1 (f ∪ [[x,y]]).

Theorem MKT96a : ∀ {f r s}, Order_Pr f r s -> Function1_1 f.

Theorem MKT96b : ∀ {f r s}, Order_Pr f r s -> Order_Pr (f⁻¹) s r.

Theorem MKT96 : ∀ f r s, Order_Pr f r s
    -> Function1_1 f /\ Order_Pr (f⁻¹) s r.

Lemma lem97a : ∀ f g u r s x y, Order_Pr f r s -> Order_Pr g r s
    -> rSection dom(f) r x -> rSection dom(g) r x
    -> rSection ran(f) s y -> rSection ran(g) s y
    -> FirstMember u r (\{ λ a, a ∈ (dom(f) ∩ dom(g))
        /\ f [a] <> g [a] \}) -> Rrelation f[u] s g[u] -> False.

Lemma le97 : ∀ f g, Function f -> Function g
    -> (∀ a, a ∈ (dom(f) ∩ dom(g)) -> f[a] = g[a])
    -> dom(f) ⊂ dom(g) -> f ⊂ g.

Theorem MKT97 : ∀ {f g r s x y}, Order_Pr f r s -> Order_Pr g r s
    -> rSection dom(f) r x -> rSection dom(g) r x
    -> rSection ran(f) s y -> rSection ran(g) s y -> f ⊂ g \/ g ⊂ f.

Lemma Lemma99c : ∀ y r x a b, rSection y r x -> a ∈ y -> ~ b ∈ y
    -> b ∈ x -> Rrelation a r b.

Ltac RN a b := rename a into b.

Theorem MKT99 : ∀ {r s x y}, WellOrdered r x -> WellOrdered s y
    -> ∃ f, Function f /\ Order_PXY f x y r s
        /\(dom(f) = x \/ ran(f) = y).

Theorem MKT100 : ∀ {r s x y}, WellOrdered r x -> WellOrdered s y
    -> Ensemble x -> ~ Ensemble y -> ∃ f, Function f
        /\ Order_PXY f x y r s /\ dom( f) = x.

Theorem MKT100' : ∀ r s x y, WellOrdered r x /\ WellOrdered s y
    -> Ensemble x -> ~ Ensemble y
    -> ∀ f, Function f /\ Order_PXY f x y r s /\ dom(f) = x
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    -> ∀ g, Function g /\ Order_PXY g x y r s /\ dom(g) = x
    -> f = g.


Theorem MKT101 : ∀ x, x ∉ x.

Theorem MKT102 : ∀ x y, x ∈ y -> y ∈ x -> False.

Lemma cirin3f : ∀ x y z, x ∈ y -> y ∈ z -> z ∈ x -> False.

Theorem MKT104 : ~ Ensemble E.

Theorem MKT107 : ∀ {x}, Ordinal x -> WellOrdered E x.

Theorem MKT108 : ∀ x y, Ordinal x -> y ⊂ x -> y <> x -> Full y
    -> y ∈ x.

Lemma Lemma109 : ∀ {x y}, Ordinal x -> Ordinal y
    -> ((x ∩ y) = x) \/ ((x ∩ y) ∈ x).

Theorem MKT109 : ∀ {x y}, Ordinal x -> Ordinal y
    -> x ⊂ y \/ y ⊂ x.

Theorem MKT110 : ∀ {x y}, Ordinal x -> Ordinal y
    -> x ∈ y \/ y ∈ x \/ x = y.

Corollary Th110ano : ∀ {x y}, Ordinal x -> Ordinal y
    -> x ∈ y \/ y ⊂ x.

Theorem MKT111 : ∀ x y, Ordinal x -> y ∈ x -> Ordinal y.

Lemma Lemma113 :∀ u v, Ensemble u -> Ensemble v -> Ordinal u
    -> Ordinal v -> (Rrelation u E v \/ Rrelation v E u \/ u = v).

Theorem MKT113a : Ordinal R.

Theorem MKT113b : ~ Ensemble R.

Global Hint Resolve MKT113a MKT113b : core.

Theorem MKT114 : ∀ x, rSection x E R -> Ordinal x.

Corollary Property114 : ∀ x, Ordinal x -> rSection x E R.

Theorem MKT118 : ∀ x y, Ordinal x -> Ordinal y
    -> (x ⊂ y <-> x ≼ y).

Theorem MKT119 : ∀ x, Ordinal x
    -> x = \{ λ y, (y ∈ R /\ Less y x) \}.

Theorem MKT120 : ∀ x, x ⊂ R -> Ordinal (∪x).

Lemma Lemma121 : ∀ x, x ⊂ R -> x <> Φ -> FirstMember (∩x) E x.

Theorem MKT121 : ∀ x, x ⊂ R -> x <> Φ -> (∩x) ∈ x.

Lemma Lem123 : ∀ x, x ∈ R -> (PlusOne x) ∈ R.

Global Hint Resolve Lem123 : core.

Theorem MKT123 : ∀ x, x ∈ R
    -> FirstMember (PlusOne x) E (\{ λ y, (y ∈ R /\ Less x y) \}).

Theorem MKT124 : ∀ x, x ∈ R -> ∪(PlusOne x) = x.

Theorem MKT126a : ∀ f x, Function f -> Function (f|(x)).

Theorem MKT126b : ∀ f x, Function f -> dom(f|(x)) = x ∩ dom(f).

Theorem MKT126c : ∀ f x, Function f
    -> (∀ y, y ∈ dom(f|(x)) -> (f|(x))[y] = f[y]).

Corollary frebig : ∀ f x, Function f -> dom(f) ⊂ x -> f|(x) = f.
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Corollary fresub : ∀ f h, Function f -> Function h -> h ⊂ f
    -> f|(dom(h)) = h.

Corollary fuprv : ∀ f x y z, Ensemble x -> Ensemble y
    -> ~ x ∈ z -> (f ∪ [[x,y]])|(z) = f|(z).

Theorem MKT127 : ∀ {f h g}, Function f -> Ordinal dom(f)
    -> (∀ u, u ∈ dom(f) -> f[u] = g[f|(u)]) -> Function h
    -> Ordinal dom(h) -> (∀ u, u ∈ dom(h) -> h[u] = g[h|(u)])
    -> h ⊂ f \/ f ⊂ h.

Theorem MKT128a : ∀ g, ∃ f, Function f /\ Ordinal dom(f)
    /\ (∀ x, Ordinal_Number x -> f[x] = g[f|(x)]).

Lemma lem128 : ∀ {f g h}, Function f -> Function h
    -> Ordinal dom(f) -> Ordinal dom(h)
    -> (∀ x, Ordinal_Number x -> f[x] = g [f|(x)])
    -> (∀ x, Ordinal_Number x -> h[x] = g [h|(x)])
    -> h ⊂ f -> h = f.

Theorem MKT128b : ∀ g, ∀ f, Function f /\ Ordinal dom(f)
        /\ (∀ x, Ordinal_Number x -> f[x] = g[f|(x)])
    -> ∀ h, Function h /\ Ordinal dom(h)
        /\ (∀ x, Ordinal_Number x -> h[x] = g[h|(x)]) -> f = h.


Fact EnEm : Ensemble Φ.

Global Hint Resolve EnEm : core.

Fact powEm : pow(Φ) = [Φ].

Theorem MKT132 : ∀ x y, Integer x -> y ∈ x -> Integer y.

Theorem MKT133 : ∀ {x y}, y ∈ R -> LastMember x E y
    -> y = PlusOne x.

Theorem MKT134 : ∀ {x}, x ∈ ω -> (PlusOne x) ∈ ω.

Global Hint Resolve MKT134 : core.

Theorem MKT135 : Φ ∈ ω /\ (∀ x, x ∈ ω -> Φ ≠ PlusOne x).

Theorem MKT135a : Φ ∈ ω.

Global Hint Resolve MKT135a : core.

Theorem MKT135b : ∀ x, x ∈ ω -> Φ ≠ PlusOne x.

Theorem MKT136 : ∀ x y, x ∈ ω -> y ∈ ω -> PlusOne x = PlusOne y
    -> x = y.

Corollary Property_W : Ordinal ω.

Global Hint Resolve Property_W : core.

Theorem MKT137 : ∀ x, x ⊂ ω -> Φ ∈ x
    -> (∀ u, u ∈ x -> (PlusOne u) ∈ x) -> x = ω.

Theorem MKT138 : ω ∈ R.

Theorem MiniMember_Principle : ∀ S, S ⊂ ω -> S ≠ Φ
    -> ∃ a, a ∈ S /\ (∀ c, c ∈ S -> a ≼ c).

Theorem Mathematical_Induction : ∀ (P :Class -> Prop), P Φ
    -> (∀ k, k ∈ ω -> P k -> P (PlusOne k)) -> (∀ n, n ∈ ω -> P n).

Ltac MI x := apply Mathematical_Induction with (n:=x); auto; intros.

Fact caseint : ∀ {x}, x ∈ ω
    -> x = Φ \/ (∃ v, v ∈ ω /\ x = PlusOne v).
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Theorem The_Second_Mathematical_Induction : ∀ (P: Class -> Prop),
    P Φ -> (∀ k, k ∈ ω -> (∀ m, m ≺ k -> P m) -> P k)
    -> (∀ n, n ∈ ω -> P n).


Fact f2Pf : ∀ {f} P, let g := \{\ λ u v, v = f[P u] \}\ in
    Function f -> (∀ h, Ensemble h -> g[h] = f[P h]).

Fact c2fp : ∀ {x c g P f}, Ensemble x -> dom(c) = μ ~ [Φ]
    -> (∀ x, x ∈ dom(c) -> c[x] ∈ x)
    -> (∀ h, Ensemble h -> g[h] = c[P h])
    -> (∀ x, Ordinal_Number x -> f[x] = g[f|(x)])
    -> Function f -> Ordinal dom(f)
    -> (∀ u, u ∈ dom(f) -> Ensemble (P (f|(u)))
        -> f[u] ∈ (P (f|(u)))).

Theorem MKT140 : ∀ x, Ensemble x
    -> ∃ f, Function1_1 f /\ ran(f) = x /\ Ordinal_Number dom(f).

Theorem MKT142 : ∀ n, Nest n -> (∀ m, m ∈ n -> Nest m)
    -> Nest (∪n).

Theorem MKT143 : ∀ x, Ensemble x -> ∃ n, (Nest n /\ n ⊂ x)
    /\ (∀ m, Nest m -> m ⊂ x -> n ⊂ m -> m = n).


Fact eqvp : ∀ {x y}, Ensemble y -> x ≈ y -> Ensemble x.

Theorem MKT145 : ∀ x, x ≈ x.

Global Hint Resolve MKT145 : core.

Theorem MKT146 : ∀ {x y}, x ≈ y -> y ≈ x.

Theorem MKT147 : ∀ y x z, x ≈ y -> y ≈ z -> x ≈ z.

Theorem MKT150 : WellOrdered E C.

Theorem MKT152a : Function P.

Global Hint Resolve MKT152a : core.

Theorem MKT152b : dom(P) = μ.

Theorem MKT152c : ran(P) = C.

Corollary Property_PClass : ∀ {x}, Ensemble x -> P [x] ∈ C.

Global Hint Resolve Property_PClass : core.

Theorem MKT153 : ∀ {x}, Ensemble x -> P[x] ≈ x.

Global Hint Resolve MKT153 : core.

Fact pveqv : ∀ x y, Ensemble y -> P[x] = y -> x ≈ y.

Fact carE : ∀ {x}, P[x] = Φ -> x = Φ.

Theorem MKT154 : ∀ x y, Ensemble x -> Ensemble y
    -> (P[x] = P[y] <-> x ≈ y).

Theorem MKT155 : ∀ x, P[P[x]] = P[x].

Theorem MKT156 : ∀ x, (Ensemble x /\ P[x] = x) <-> x∈C.

Theorem MKT157 : ∀ x y, y ∈ R -> x ⊂ y -> P[x] ≼ y.

Theorem MKT158 : ∀ {x y}, x ⊂ y -> P[x] ≼ P[y].

Theorem MKT159 : ∀ x y u v, Ensemble x -> Ensemble y
    -> u ⊂ x -> v ⊂ y -> x ≈ v -> y ≈ u -> x ≈ y.

Theorem MKT160 : ∀ {f}, Function f -> Ensemble f
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    -> P[ran(f)] ≼ P[dom(f)].

Theorem MKT161 : ∀ {x}, Ensemble x -> P[x] ≺ P[pow(x)].

Theorem MKT162 : ~ Ensemble C.



Lemma Lemma163a : ∀ {x y}, Ensemble x -> ~ x ∈ y
    -> y = (y ∪ [x]) ~ [x].

Lemma Lemma163b : ∀ {x y}, x ∈ y -> y = (y ~ [x]) ∪ [x].

Lemma Lemma163c : ∀ {x y z}, x ~ y ~ z = x ~ z ~ y.

Theorem MKT163 : ∀ x y, x∈ω -> y∈ω -> (PlusOne x) ≈ (PlusOne y)
    -> x ≈ y.

Theorem MKT164 : ω ⊂ C.
Theorem MKT165 : ω ∈ C.

Corollary Property_Finite : ∀ {x}, Finite x -> Ensemble x.

Lemma finsub : ∀ {A B}, Finite A -> B ⊂ A -> Finite B.

Lemma finsin : ∀ z, Ensemble z -> Finite ([z]).

Lemma finue : ∀ {x z}, Finite x -> Ensemble z -> ~ z ∈ x
    -> P[x ∪ [z]] = PlusOne P[x].

Fact finse : ∀ f {y u z}, P[y] = PlusOne u -> u ∈ ω -> Function f
    -> Function f⁻¹ -> dom(f) = y -> ran(f) = PlusOne u -> z ∈ y
    -> P[y ~ [z]] = u.

Lemma lem167a : ∀ r x f, WellOrdered r P[x] -> Function1_1 f
    -> dom(f) = x -> ran(f) = P[x]
    -> WellOrdered \{\ λ u v, Rrelation f[u] r f[v] \}\ x.

Lemma lem167b : ∀ {f r}, ω ⊂ ran(f) -> WellOrdered r⁻¹ dom(f)
    -> Order_Pr f r E -> False.

Theorem MKT167 : ∀ x, Finite x <-> ∃ r, WellOrdered r x
    /\ WellOrdered (r⁻¹) x.

Lemma lem168 : ∀ {x y r s}, WellOrdered r x -> WellOrdered s y
    -> WellOrdered \{\ λ u v, (u ∈ x /\ v ∈ x /\ Rrelation u r v)
        \/ (u ∈ (y ~ x) /\ v ∈ (y ~ x) /\ Rrelation u s v)
        \/ (u ∈ x /\ v ∈ (y ~ x)) \}\ (x ∪ y).

Theorem MKT168 : ∀ x y, Finite x -> Finite y -> Finite (x ∪ y).

Lemma Lemma169 : ∀ x y, ∪(x ∪ y) = (∪x) ∪ (∪y).

Theorem MKT169 : ∀ x, Finite x -> (∀ z, z ∈ x -> Finite z)
    -> Finite (∪ x).

Theorem MKT170 : ∀ x y, Finite x -> Finite y -> Finite (x × y).

Lemma lem171 : ∀ {x y}, y ∈ x
    -> pow(x) = pow(x ~ [y]) ∪ \{ λ z, z ⊂ x /\ y ∈ z \}.

Theorem MKT171 : ∀ x, Finite x -> Finite pow(x).

Theorem MKT172 : ∀ x y, Finite x -> y ⊂ x -> P[y] = P[x] -> x = y.

Theorem MKT173 : ∀ x, Ensemble x -> ~ Finite x
    -> ∃ y, y ⊂ x /\ y <> x /\ x ≈ y.

Theorem MKT174 : ∀ x, x ∈ (R ~ ω) -> P[PlusOne x] = P[x].

Lemma lem177a : ∀ {a b}, a ∈ R -> b ∈ R
    -> Max a b = a \/ Max a b = b.
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Lemma lem177b : ∀ {a b}, a ∈ R -> b ∈ R -> Max a b ∈ R.

Lemma lem177c : ∀ P a b c d, Ensemble ([a,b]) -> Ensemble ([c,d])
    -> Rrelation ([a,b]) \{\ λ a b, ∃ u v x y, a = [u,v]
       /\ b = [x,y] /\ P u v x y \}\ ([c,d]) <-> P a b c d.

Theorem MKT177 : WellOrdered ≪ (R × R).

Lemma lem178a : ∀ {a b}, a ∈ R -> b ∈ R
    -> a ∈ (PlusOne (Max a b)).

Lemma lem178b : ∀ u v x y, u ∈ R -> v ∈ R -> x ∈ R -> y ∈ R
    -> Max u v ≺ Max x y \/ Max u v = Max x y
    -> PlusOne (Max u v) ⊂ PlusOne (Max x y).

Theorem MKT178 : ∀ u v x y, Rrelation ([u,v]) ≪ ([x,y])
    -> [u,v] ∈ ((PlusOne (Max x y)) × (PlusOne (Max x y))).

Fact le179 : ∀ x, x ∈ R -> P[x] ∈ ω -> P[x] = x.

Fact t69r : ∀ f x, Function f -> Ensemble f[x] -> f[x] ∈ ran(f).

Fact CsubR : C ⊂ R.

Global Hint Resolve CsubR : core.

Fact plusoneEns : ∀ z, Ensemble z -> Ensemble (PlusOne z).

Global Hint Resolve plusoneEns : core.

Fact pclec : ∀ {x}, x ∈ R -> P[x] ≼ x.

Lemma lem179a : ∀ x y, Ensemble x -> Ensemble y
    -> P[x×y] = P[(P[x]) × (P[y])].

Lemma lem179b : ∀ z x, x ∈ C -> z ∈ R -> P[z] ∈ x -> z ∈ x.

Theorem MKT179 : ∀ {x}, x ∈ (C ~ ω) -> P[x × x] = x.

Fact wh1 : ∀ {x y}, Ensemble x -> y <> Φ -> P[y] ⊂ P[x]
    -> P[x] ≼ P[y×x].

Fact wh2 : ∀ x y, x ⊂ y -> (x × y) ⊂ (y × y).

Fact wh3 : ∀ x y, Ensemble x -> Ensemble y -> P[x × y] = P[y × x].

Theorem MKT180a : ∀ x y, x ∈ C -> y ∈ C -> y ∉ ω -> x ≠ Φ
    -> P[x] ⊂ P[y] -> P[x × y] = Max P[x] P[y].

Theorem MKT180b : ∀ x y, x ∈ C -> y ∈ C -> y ∉ ω -> x ≠ Φ -> y ≠ Φ
    -> P[x × y] = Max P[x] P[y].

Theorem MKT180 : ∀ x y, x ∈ C -> y ∈ C -> x ∉ ω \/ y ∉ ω -> x ≠ Φ
    -> y ≠ Φ -> P[x × y] = Max P[x] P[y].

Fact wh4 : ∀ x y, x ⊂ y -> (y ~ x) ∪ x = y.

Theorem MKT181a : ∃ f, Order_Pr f E E /\ dom(f) = R
    /\ ran(f) = C ~ ω.

Theorem MKT181b : ∀ f g, Order_Pr f E E -> Order_Pr g E E
    -> dom(f) = R -> dom(g) = R -> ran(f) = C ~ ω
    -> ran(g) = C ~ ω -> f = g.
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