Image Enhancement (Spatial Filtering)

Image Enhancement Revisited

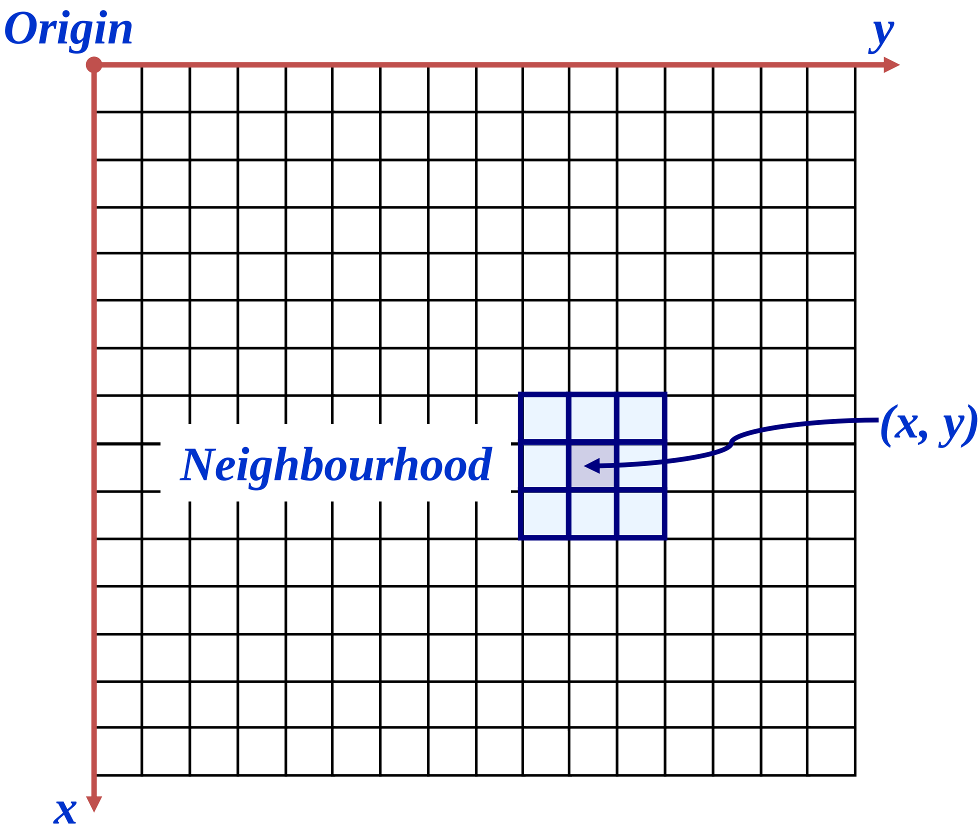
- Spatial domain methods:

  Operate directly on pixels
- Frequency domain methods:

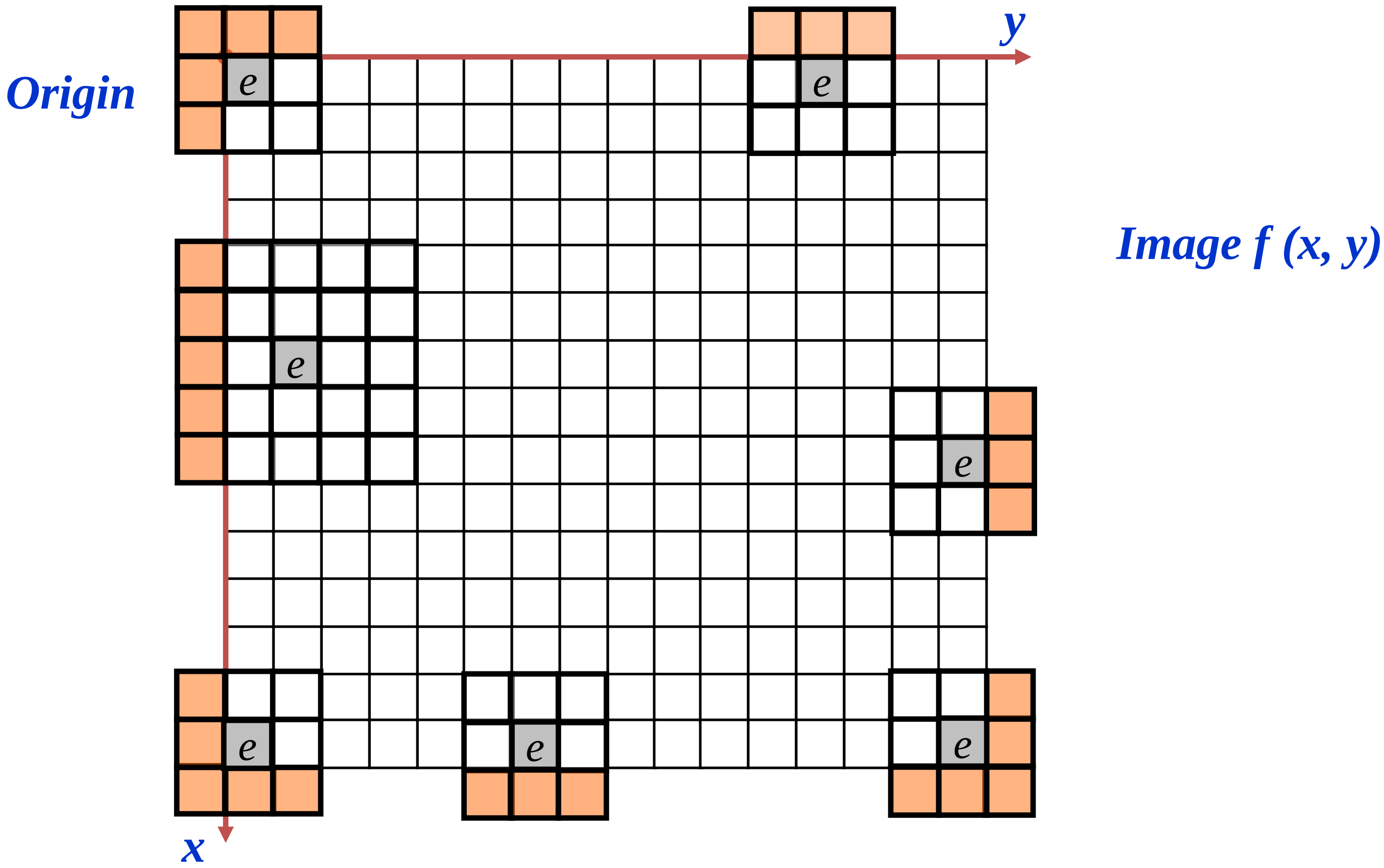
  Operate on the transform of an image

Neighbourhood Operations

- Operate on a neighbourhood of pixels



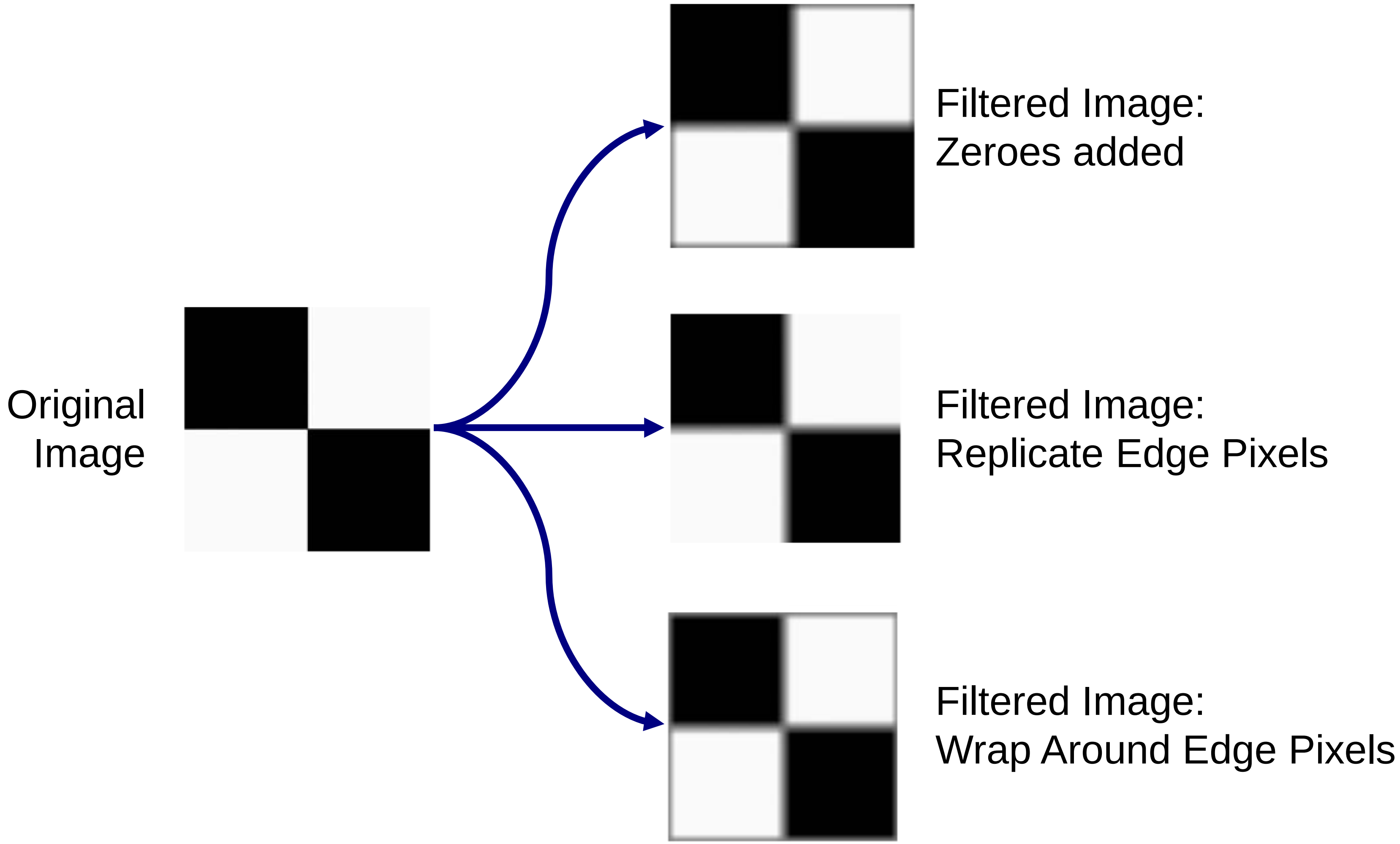
Processing of pixels at the edges



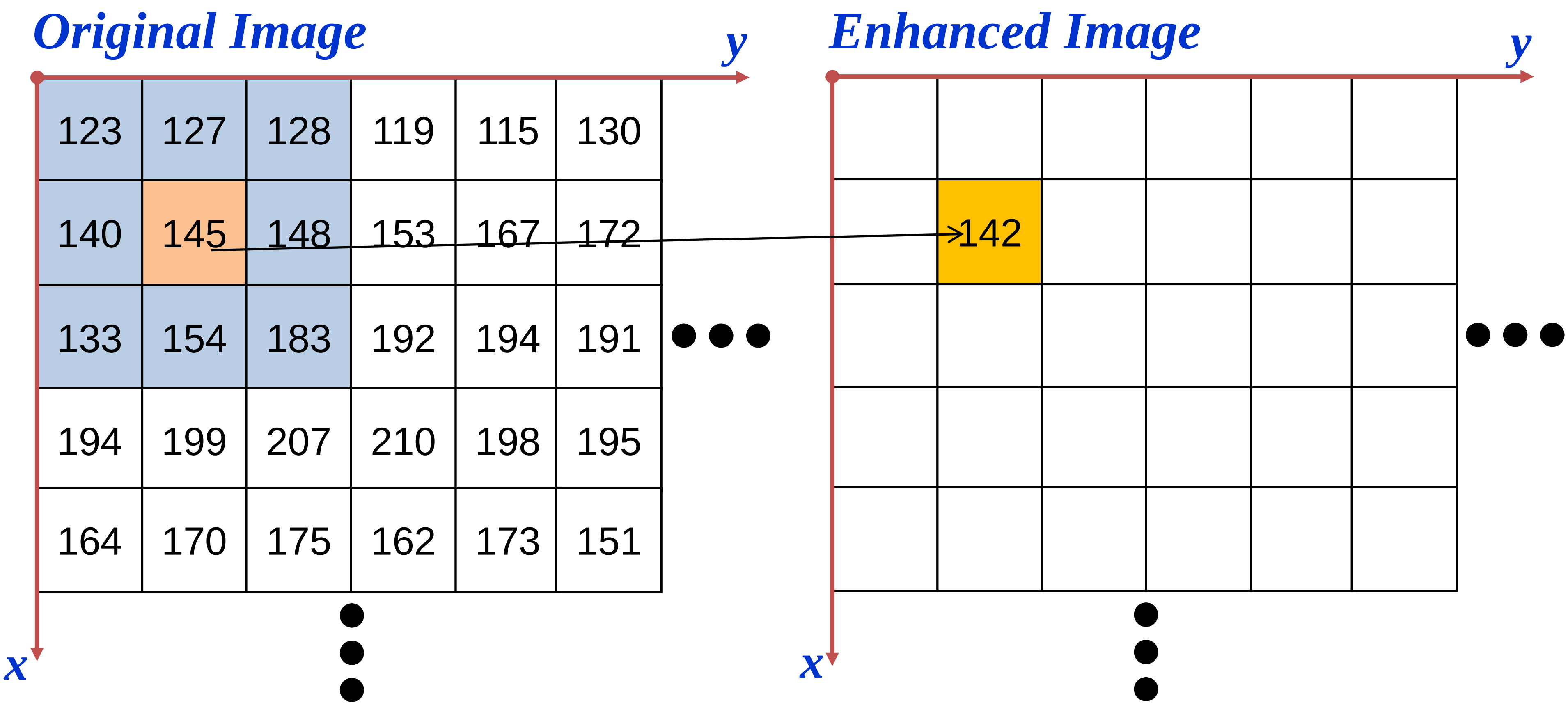
Approaches to process pixels at the edges

- Add pixels at corners with either all white or all black pixels
- Replicate border pixels
- Truncate the image
- Allow pixels wrap around the image

Examples: modified image at the edges



Neighbourhood Operations Example



2-D Convolution

a	b	c
d	e	e
f	g	h

Original Image Pixels

r	s	t
u	v	w
x	y	z

Filter

z	y	x
w	v	u
t	s	r

Swapped Filter

a	b	c
d	e	e
f	g	h

Original Image Pixels

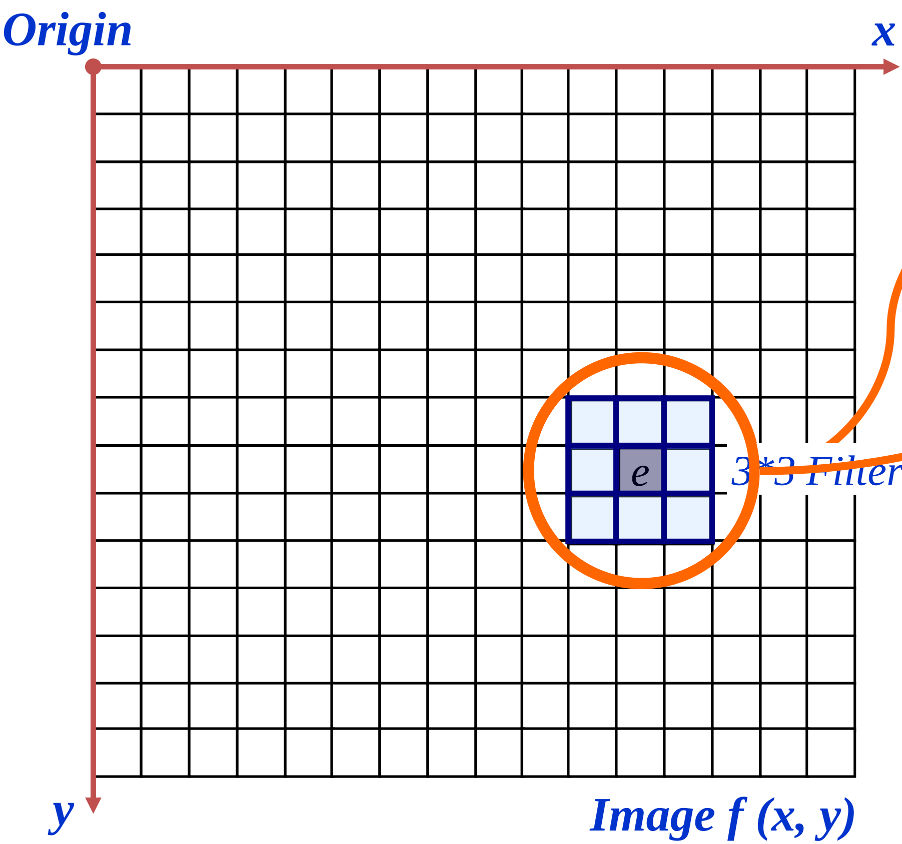
$*$

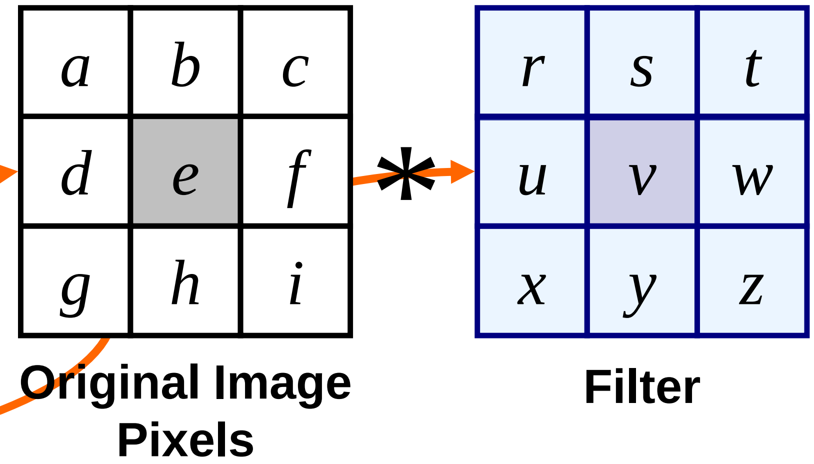
z	y	x
w	v	u
t	s	r

Swapped Filter

$$
\begin{aligned}
e_{convolution} &= v \times e + z \times a + y \times b + x \times c + \\
&\quad w \times d + u \times e + t \times f + s \times g + r \times h
\end{aligned}
$$

The Spatial Filtering Process





$$e_{processed} = v*e + r*a + s*b + t*c + u*d + w*f + x*g + y*h + z*i$$

Procedure is repeated for every pixel in the original image to generate the filtered image

Smoothing Spatial Filters

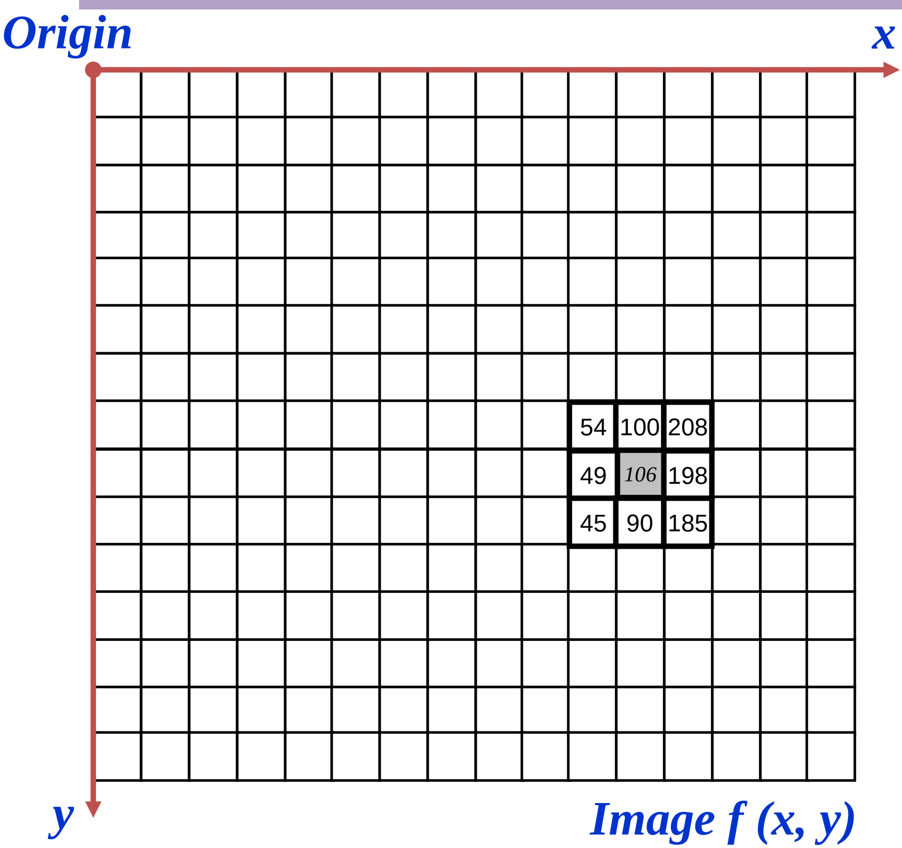
- Averages all of the pixels in a neighbourhood around a central value
- Useful in reducing noise from images and for highlighting gross detail

Averaging filter

A=(1/9)

1	1	1
1	1	1
1	1	1

Smoothing Spatial Filtering



54	100	208
49	106	198
45	90	185

Original Image Pixels

$*$ 1/9

1	1	1
1	1	1
1	1	1

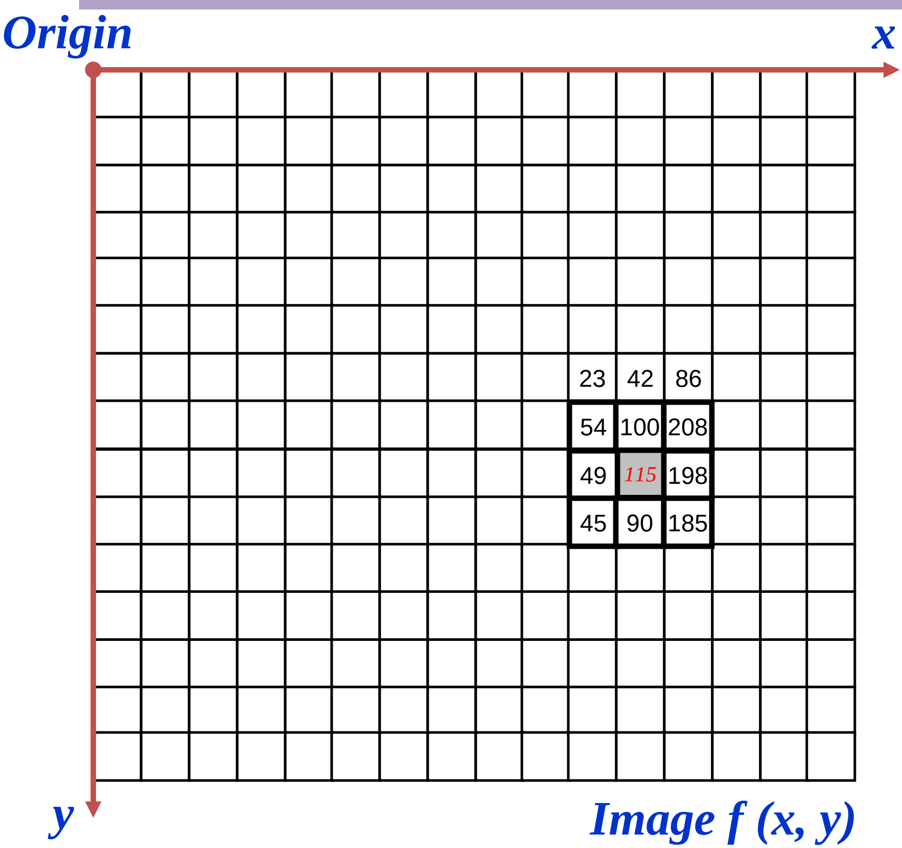
Filter

*3*3 Smoothing Filter*

$$e = {}^{1}/_{9}[106 + 54 + 100 + 208 + 49 + 198 + 45 + 90 + 185]$$

$$= 115$$

Smoothing Spatial Filtering



54	*100*	*208*
49	*106*	*198*
45	*90*	*185*

Original Image Pixels

$*$ 1/9

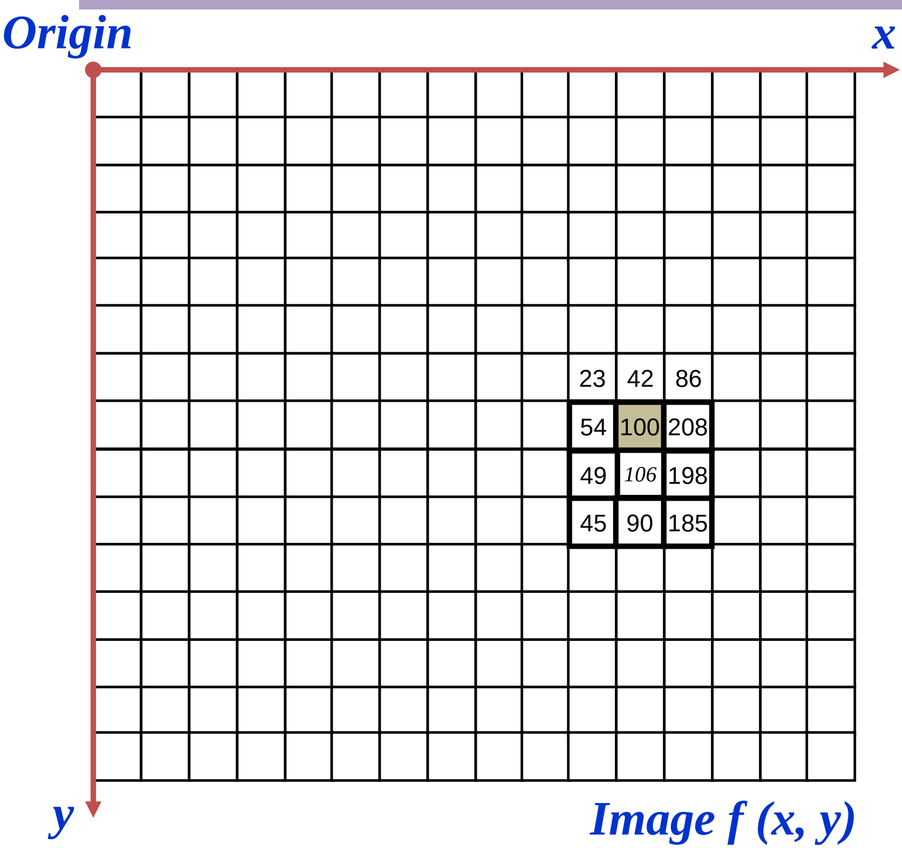
1	1	1
1	1	1
1	1	1

Filter

*3*3 Smoothing Filter*

$$
\begin{aligned}
e = {} & {}^1/_9*106 + \\
& {}^1/_9*54 + {}^1/_9*100 + {}^1/_9*208 + \\
& {}^1/_9*49 + {}^1/_9*198 + \\
& {}^1/_9*45 + {}^1/_9*90 + {}^1/_9*185 \\
= {} & 115
\end{aligned}
$$

Smoothing Spatial Filtering

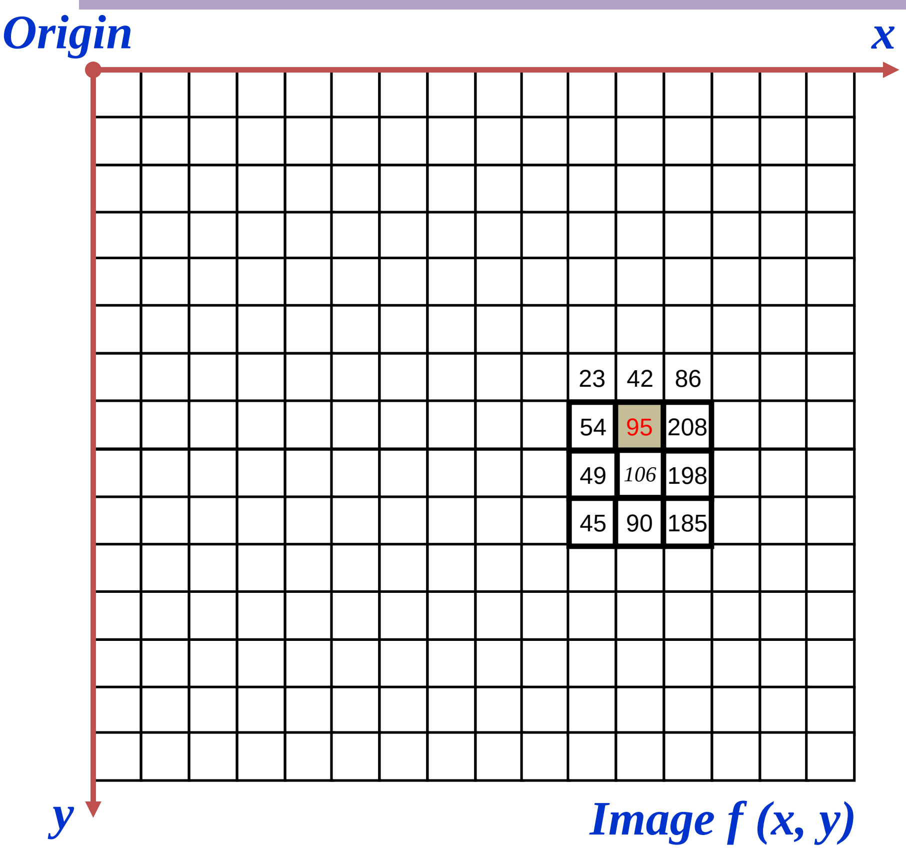


$$\begin{bmatrix} 23 & 42 & 86 \\ 54 & 100 & 208 \\ 49 & 106 & 185 \end{bmatrix} * \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

*3*3 Smoothing Filter*

$$e = 94.77 \rightarrow 95$$

Smoothing Spatial Filtering

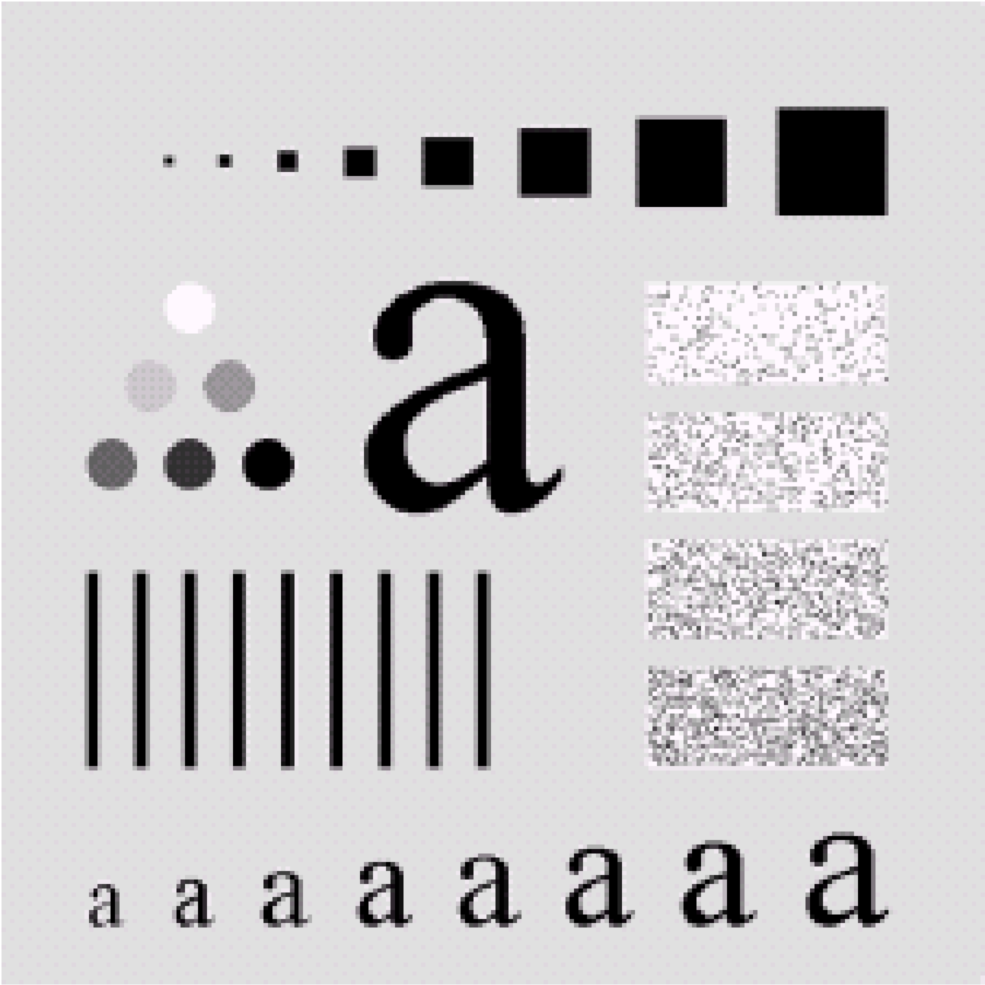


$$\begin{bmatrix} 23 & 42 & 86 \\ 54 & 100 & 208 \\ 49 & 106 & 185 \end{bmatrix} * \begin{bmatrix} 1/9 & {}^1/_9 & {}^1/_9 \\ {}^1/_9 & {}^1/_9 & {}^1/_9 \\ {}^1/_9 & {}^1/_9 & {}^1/_9 \end{bmatrix}$$

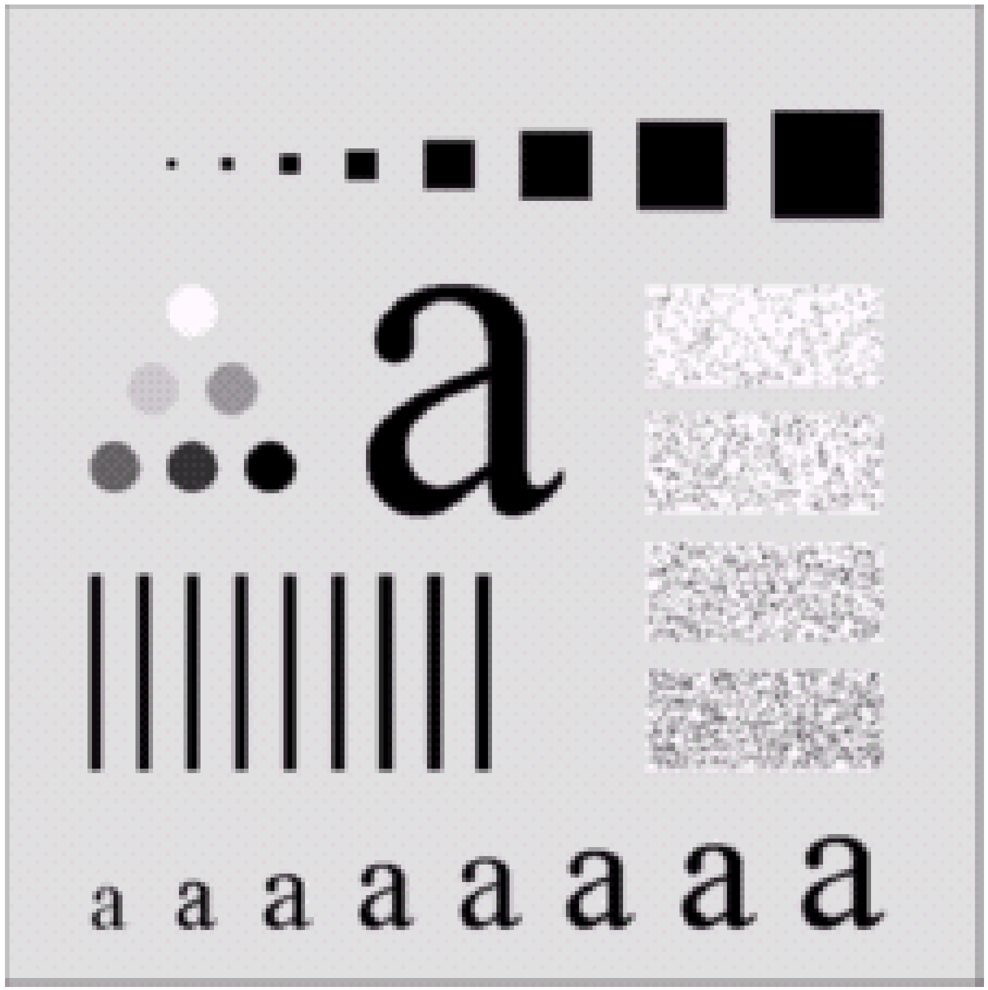
*3*3 Smoothing Filter*

$$e = 94.77 \rightarrow 95$$

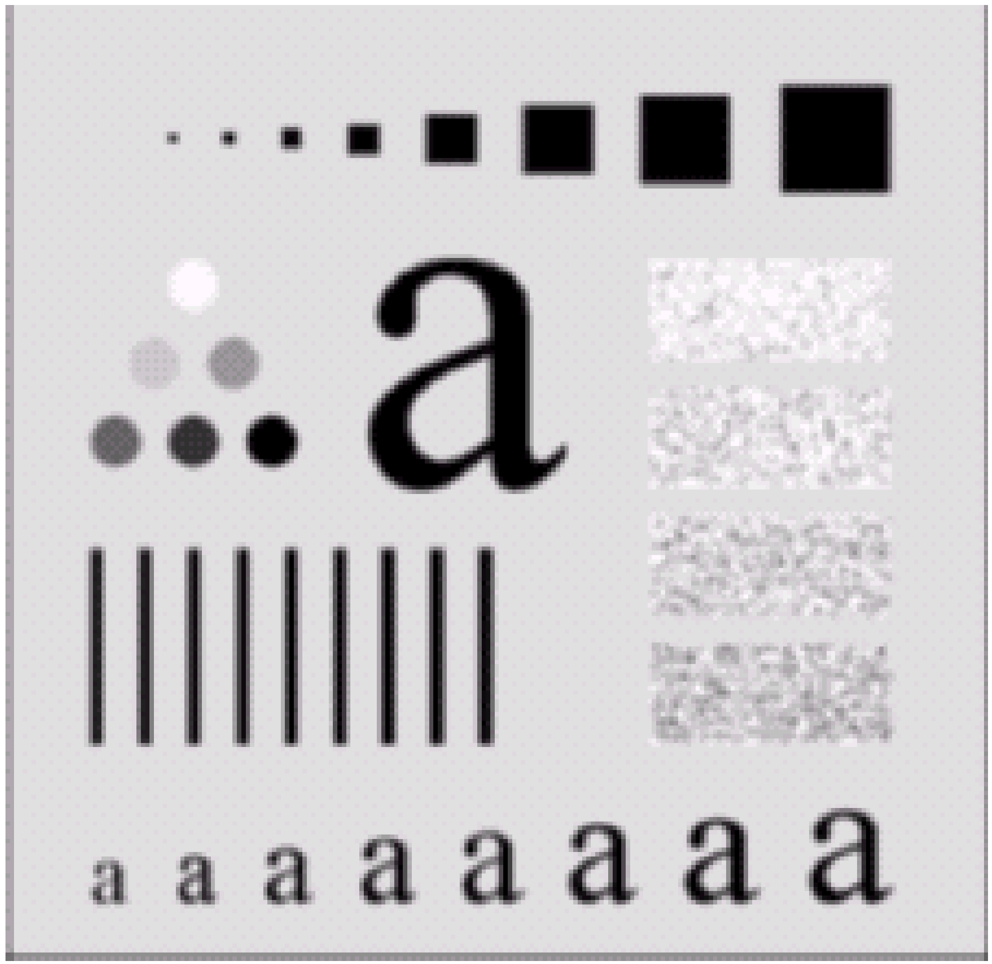
Original Image



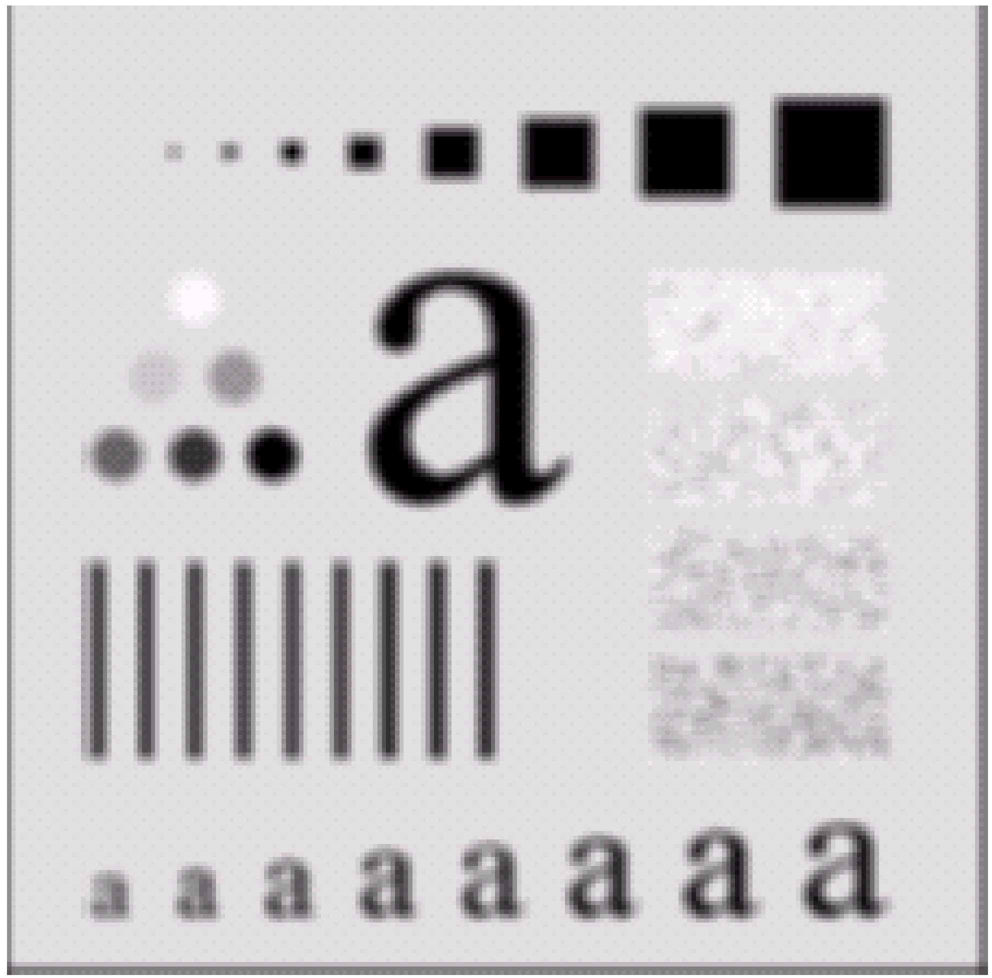
3x3 mask



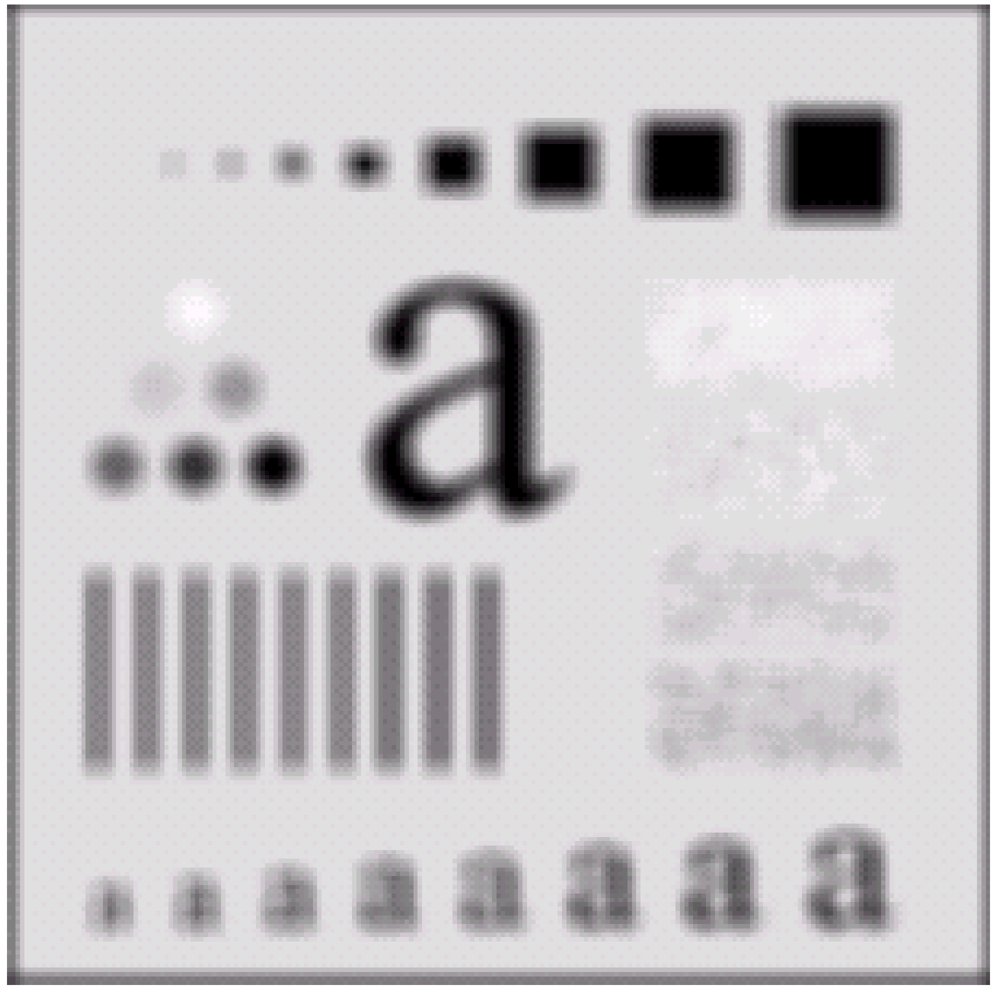
5x5 mask



9x9 mask



15x15 mask



35x35 mask

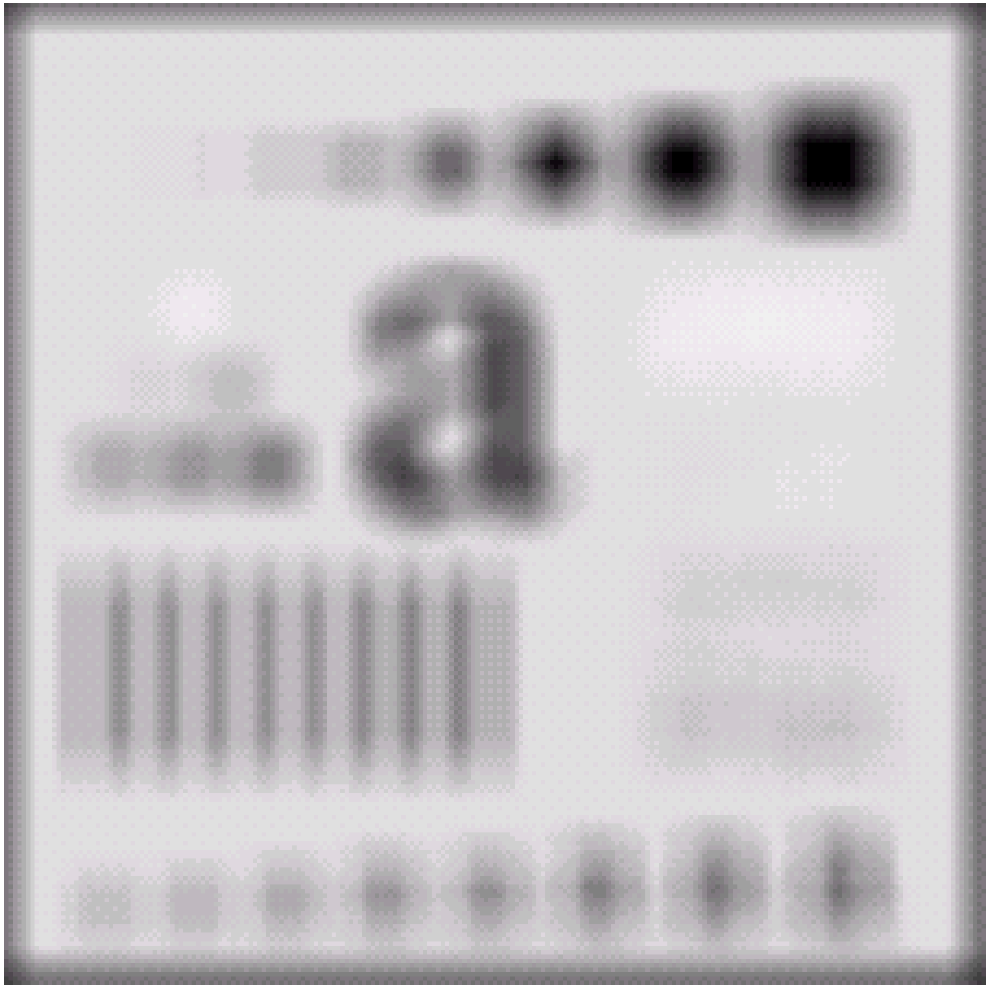
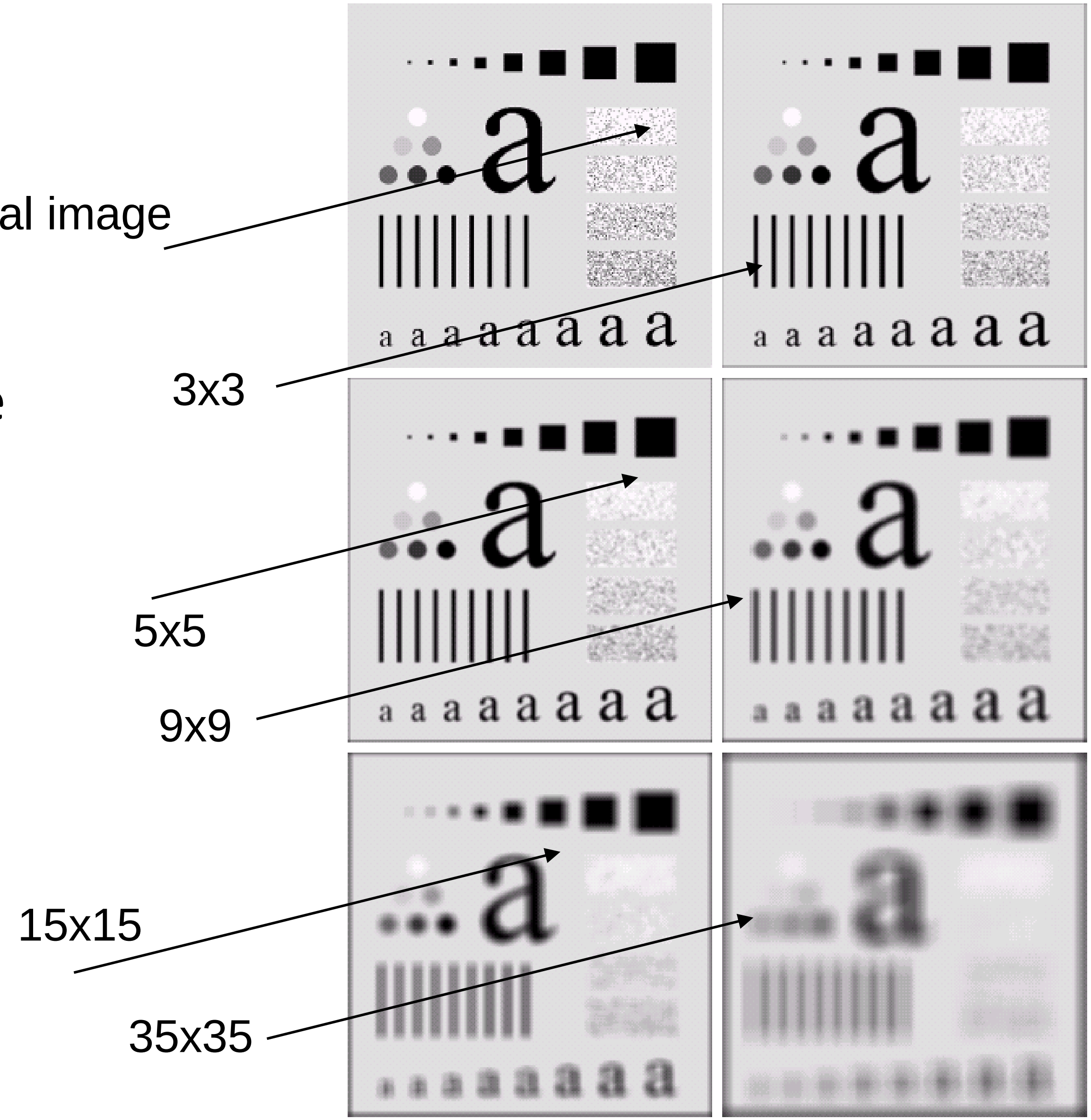


Image Smoothing Example

Sharpness/detail begins to disappear with increase in size of mask



Limitations of averaging filter

- Leads to blurring of image
- Attenuates impulse noise
- Does not remove impulse noise

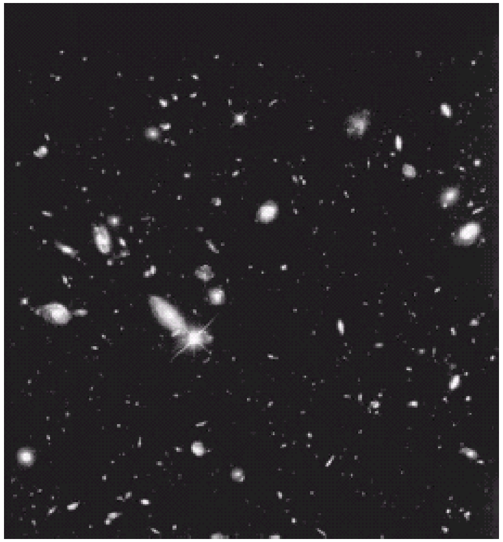
Weighted Smoothing Filters

- Elements of mask have different weights.
- Provides more effective smoothing
- Pixels closer to the central pixel are more important
- Often referred to as a weighted averaging

$$A = (1/16)\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

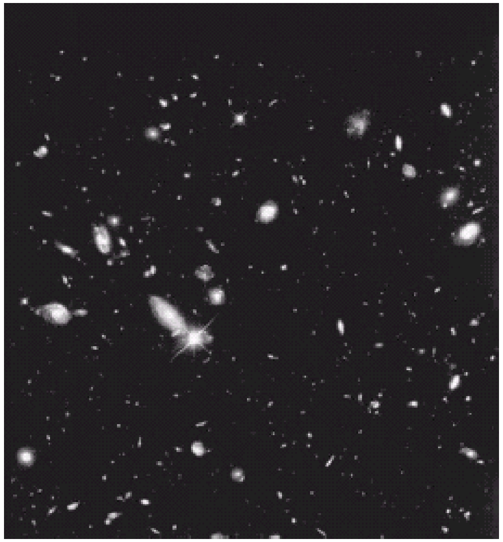
Weighted averaging filter

Application of average filter (smoothening & thresholding)

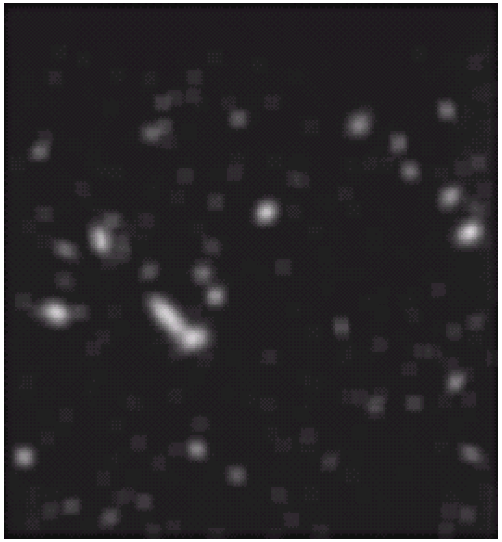


Original Image

Application of average filter (smoothening & thresholding)

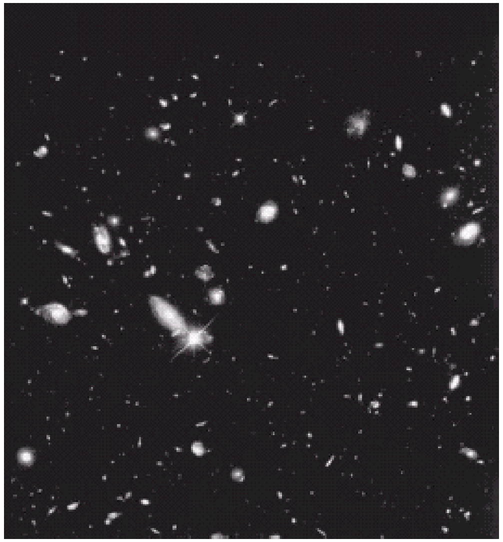


Original Image

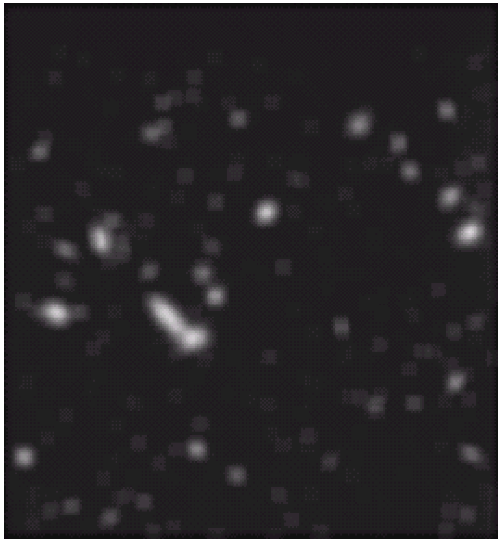


Smoothed Image

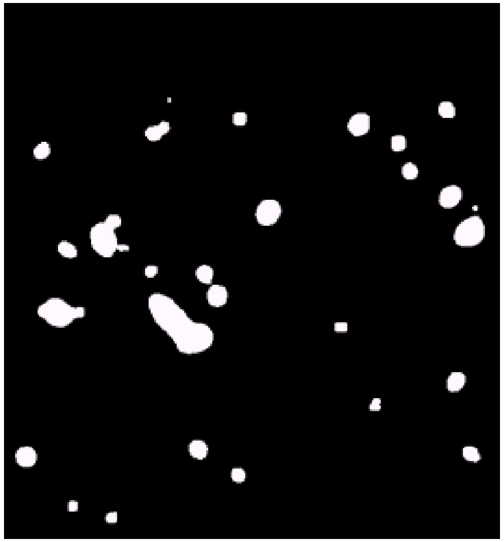
Application of average filter (smoothening & thresholding)



Original Image



Smoothed Image
5 × 5 mask



Thresholded Image
G(x,y) =255, if f(x,y)>120
=0, otherwise

- Removes finer details
- Thresholding separates gross details

Simple Neighbourhood Operations

- **Min:** minimum in the neighbourhood
- **Max:** maximum in the neighbourhood
- **Median:** midpoint value of the set

Minimum and Maximum Filter

2	3	6
1	2	8
7	4	5

Image Before filter

→

2	3	6
1	1	8
7	4	5

Image after minimum filter

2	3	6
1	8	8
7	4	5

Image after Maximum filter

Median filter

2	3	6
1	2	8
7	4	5

Image Before filter

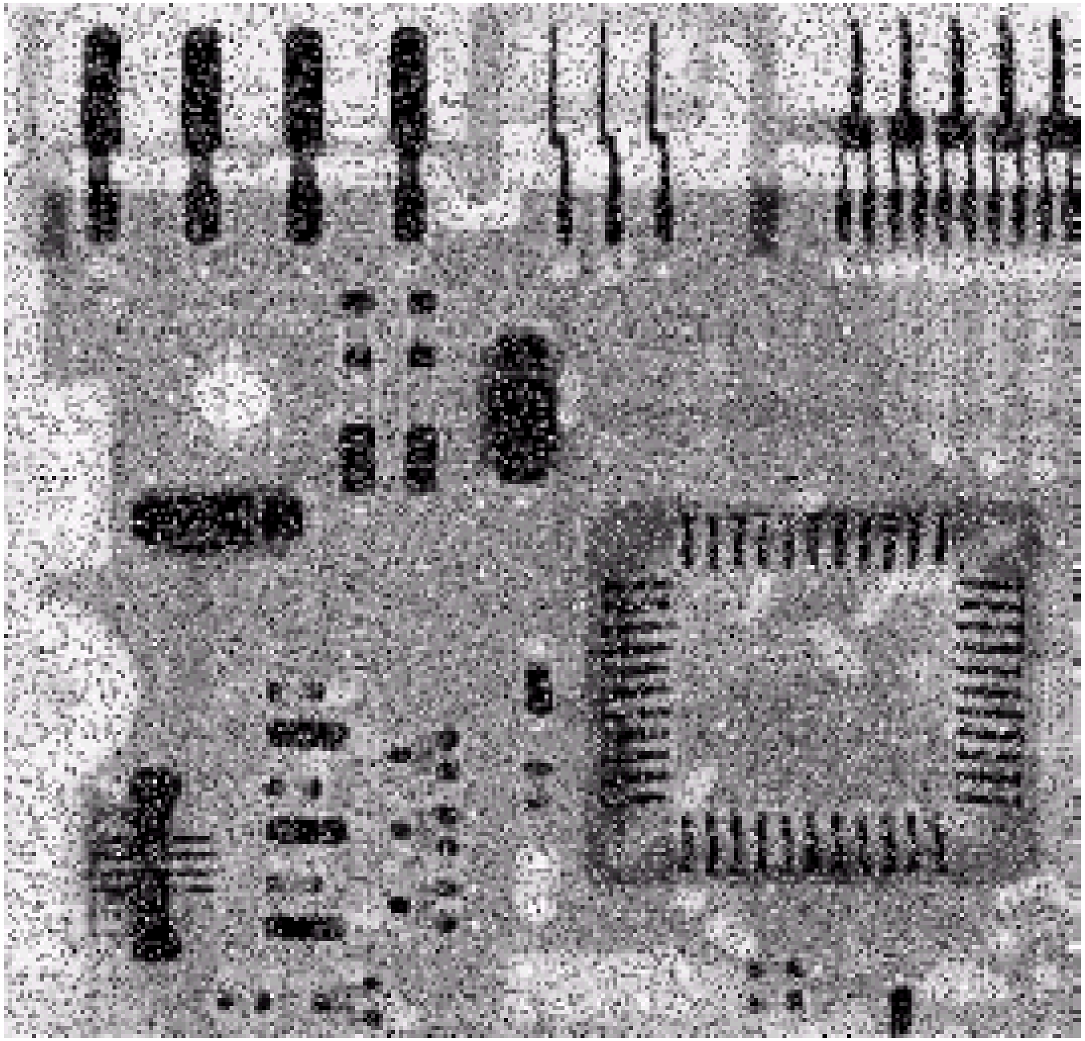
→ [2 3 6 1 2 8 7 4 5] → sort in ascending

[1 2 2 3 4 5 6 7 8]

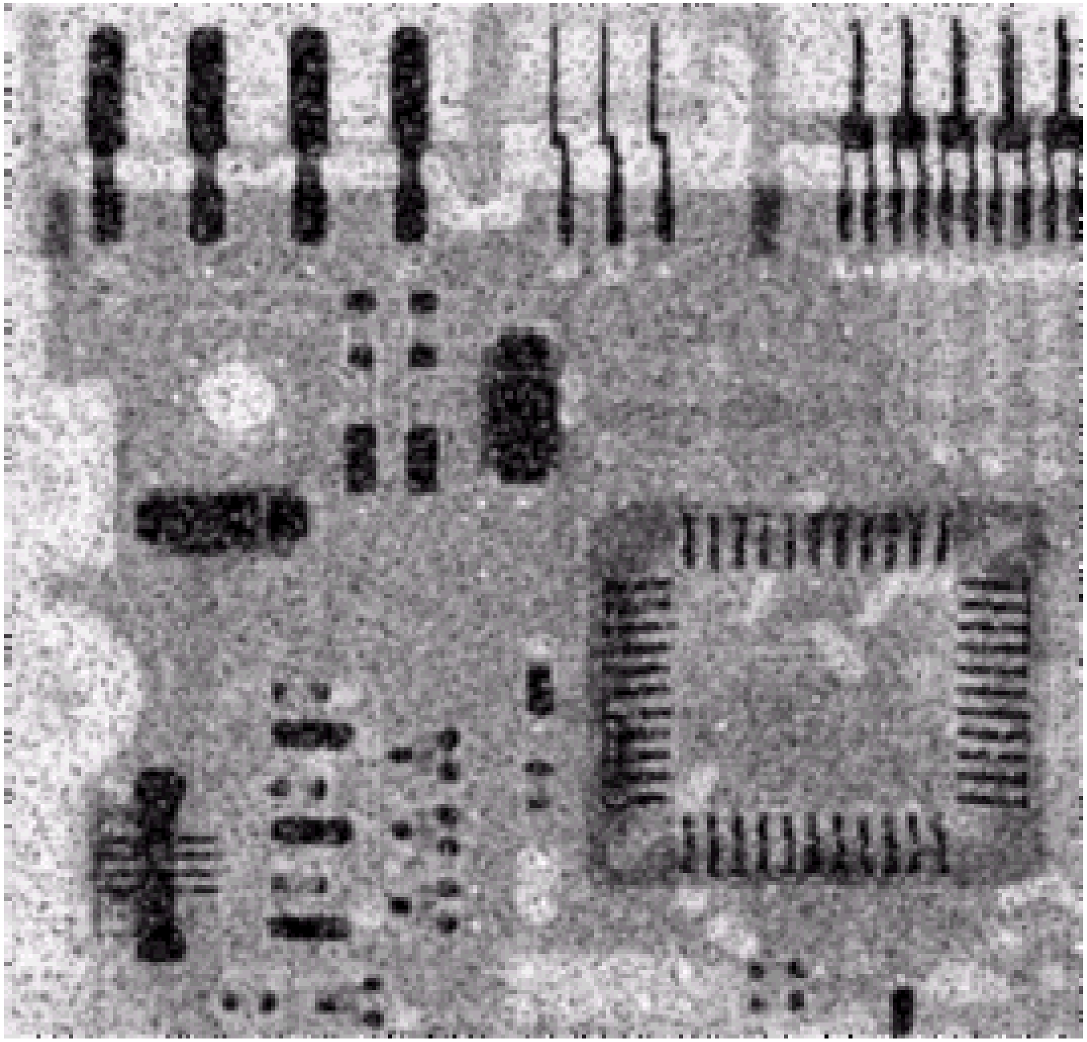
2	3	6
1	4	8
7	4	5

After filter

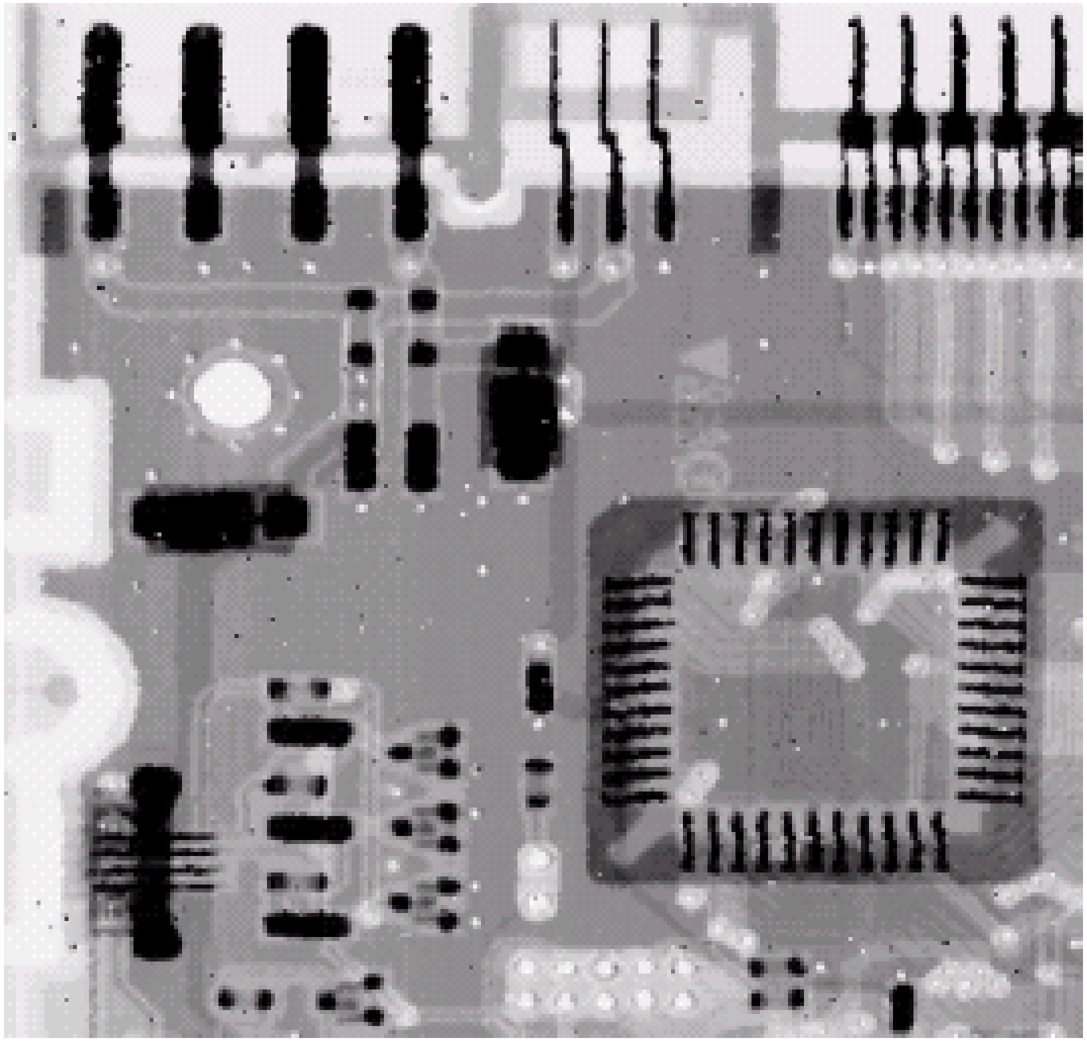
Image with salt and pepper /impulse noise



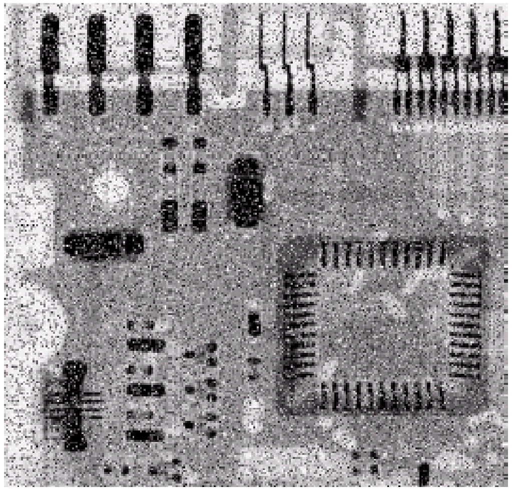
After Averaging Filter (blurs)



After Median Filter



Averaging Filter Vs. Median Filter



Original Image With Noise

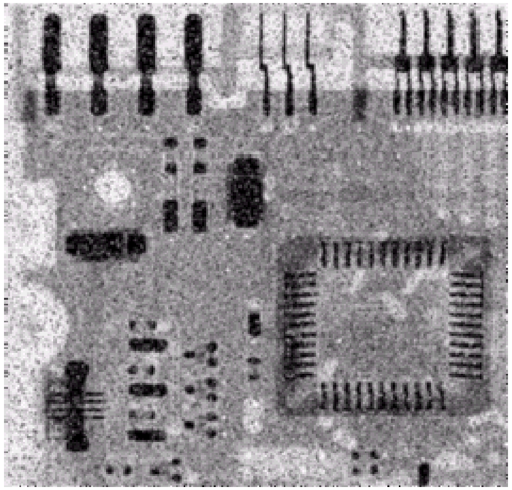


Image After Averaging Filter

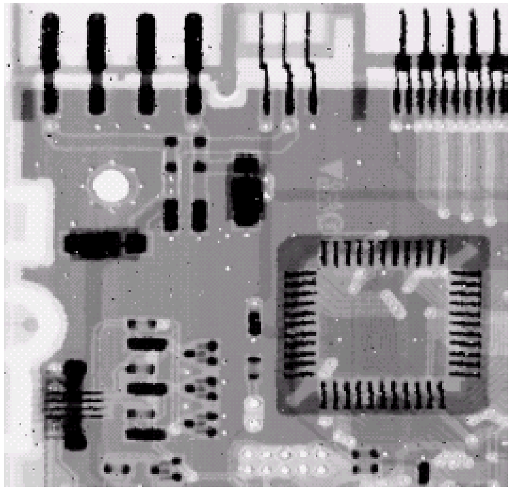


Image After Median Filter

Median filter works better than an averaging filter for salt and pepper noise

Example: Spatial filters

Image matrix is given below. Determine the effect of

1. 3x3 and 5x5 averaging filters
2. 3x3 weighted averaging filter
3. 3x3 Minimum, maximum and median filters

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

Example: Spatial filters

Image matrix is given below. Determine the effect of

1. 3x3 and 5x5 averaging filters
2. 3x3 weighted averaging filter
3. 3x3 Minimum, maximum and median filters

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

First location of filter for 3x3 filter

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

Next location of mask for 3x3 filter

Example: Spatial filters

Image matrix is given below. Determine the effect of

1. 3x3 and 5x5 averaging filters and minimum filter

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

*(1/9)

1	1	1
1	1	1
1	1	1

=

45	56	42	63	54
20	46	46	45	53
63	44	41	42	47
67	44	42	43	51
43	36	42	65	43

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

*(1/25)

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

=

45	56	42	63	54
20	47	56	28	53
63	59	46	38	47
67	36	27	48	51
43	36	42	65	43

Example: Spatial filters

Image matrix is given below. Determine the effect of

1. 3x3 weighted averaging filters

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

$* (1/16)$

1	2	1
2	4	2
1	2	1

=

45	56	42	63	54
20	47	45	44	53
63	47	39	40	47
67	43	38	44	51
43	36	42	65	43

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

$\rightarrow$min(3x3) =

45	56	42	63	54
20	20	26	26	53
63	20	26	26	47
67	26	26	26	51
43	36	42	65	43

Example: Spatial filters

Image matrix is given below. Determine the effect of

1. 3x3 weighted averaging filters

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

$\rightarrow$max(3x3) =

45	56	42	63	54
20	63	63	63	53
63	67	59	56	47
67	67	65	65	51
43	36	42	65	43

45	56	42	63	54
20	47	56	28	53
63	59	26	38	47
67	36	27	48	51
43	36	42	65	43

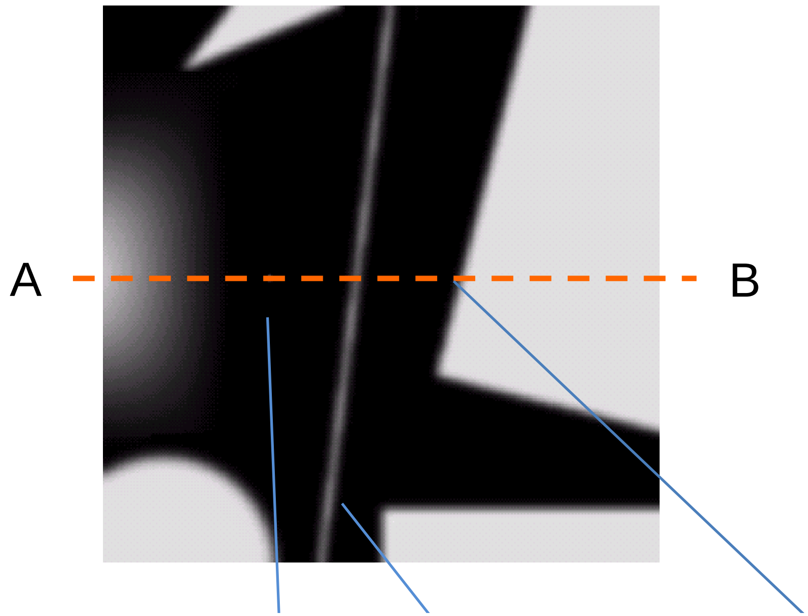
$\rightarrow$median(3x3) =

45	56	42	63	54
20	47	47	47	53
63	47	38	47	47
67	42	38	43	51
43	36	42	65	43

Sharpening Spatial Filters

- Remove blurring in images
- Highlight transition in intensity (edges)
- Used in electronic printing, medical imaging, industrial inspection etc.
- Uses spatial differentiation
- Differentiation measures the rate of change of a function

Spatial Differentiation



Spatial Differentiation

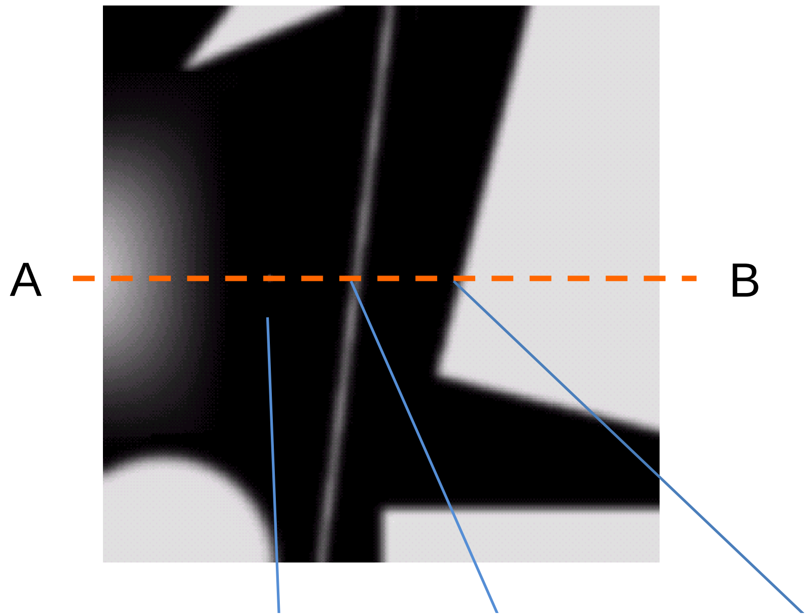
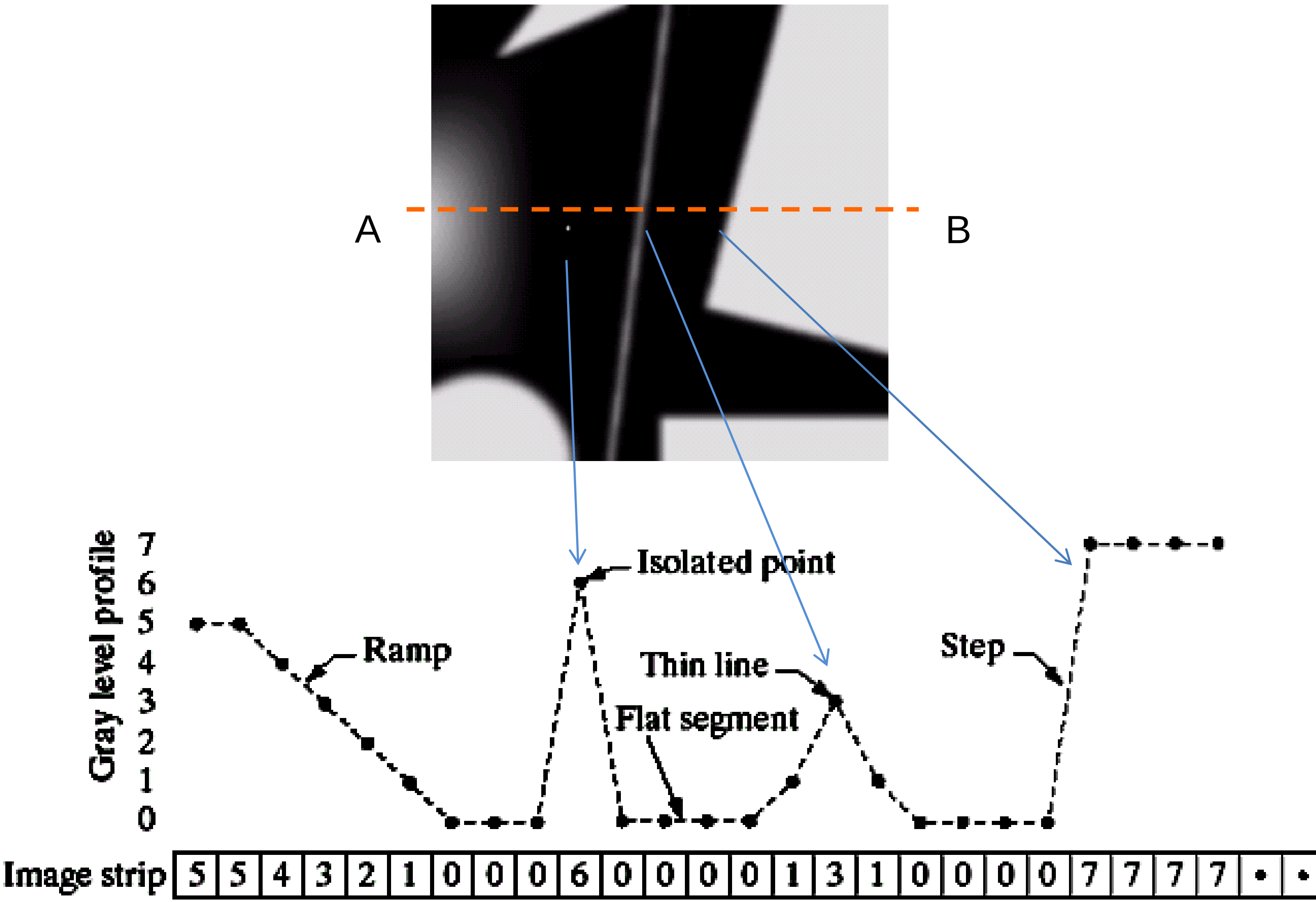


Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•

Spatial Differentiation



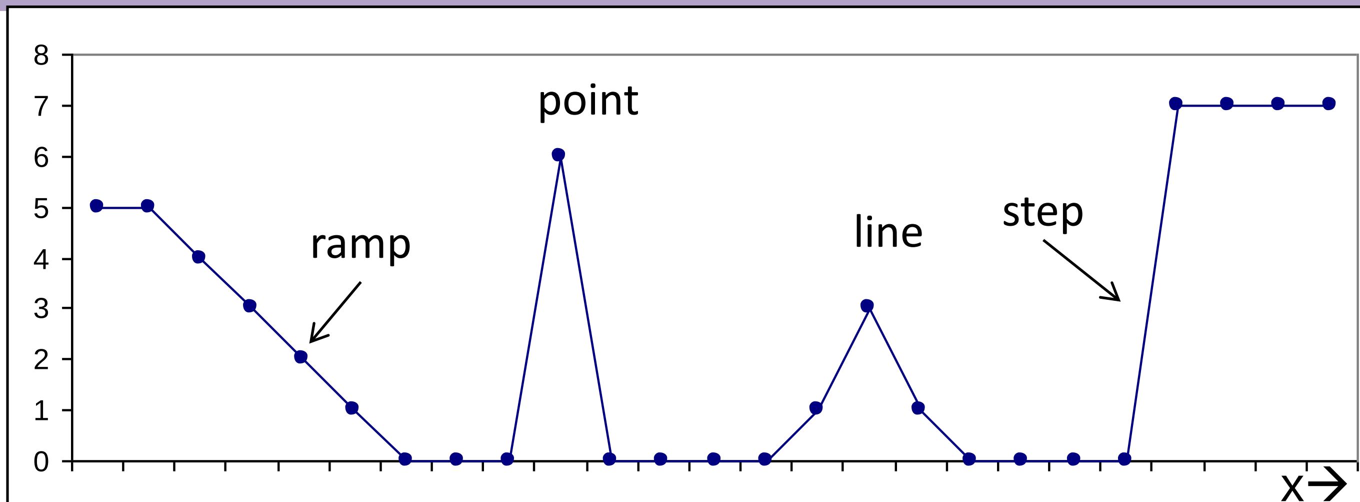
1st Derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- f(x) is pixel value at location, x
- Difference between consecutive values
- Measures the rate of change of the function

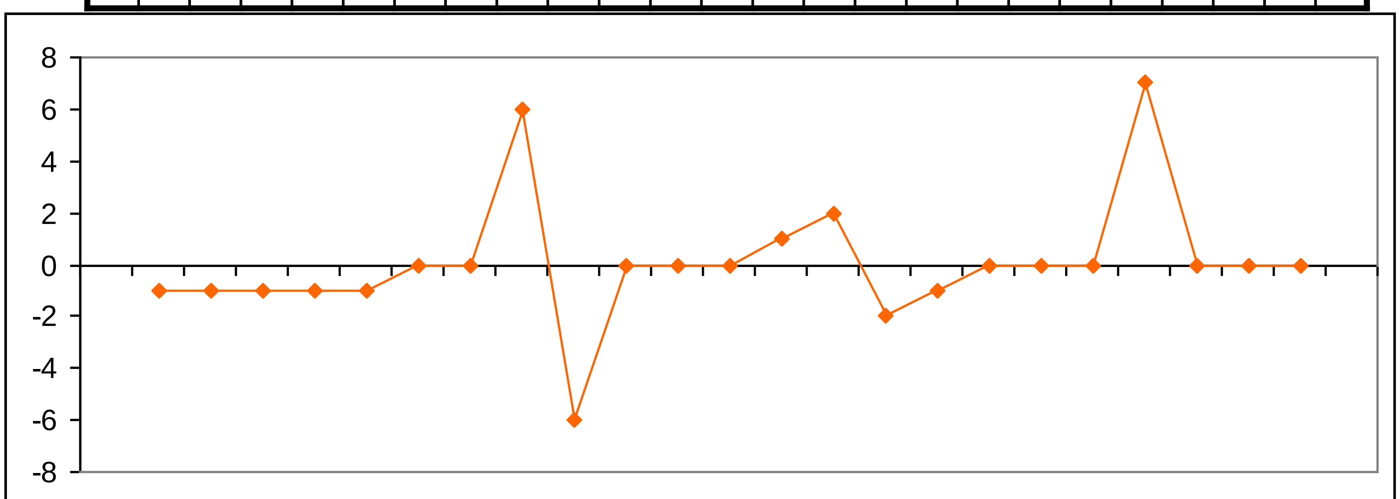
1st Derivative (cont…)

Intensity profile, f(x)



f(x)	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
f’(x)	0	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	

First derivative, f’(x)



- Non zero at the start and during ramp
- Non zero at the start and zero during the step

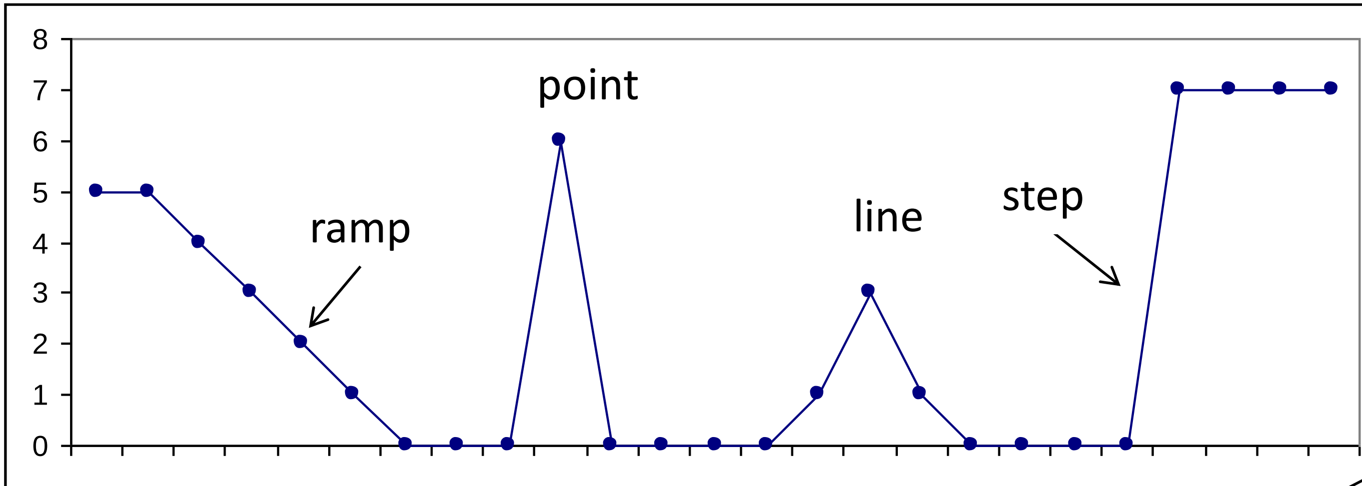
2^{nd} Derivative

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

Considers values both before and after the current value

2^{nd} Derivative (cont…)

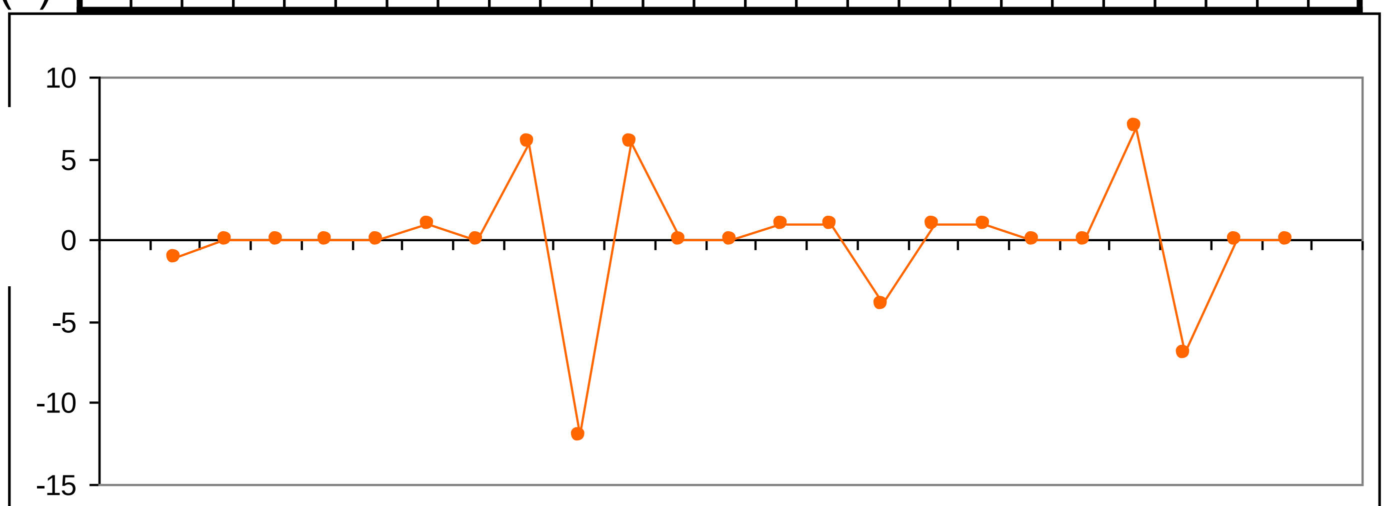
Intensity profile, f(x)



Sign changes at onset of step

f(x)	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
f''(x)		-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	

Second derivative, f''(x)

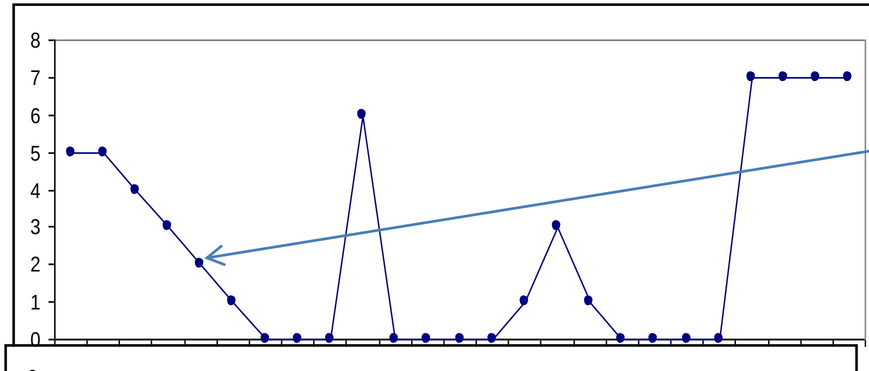


Non zero at the onset and end of ramp and step

Zero in the middle

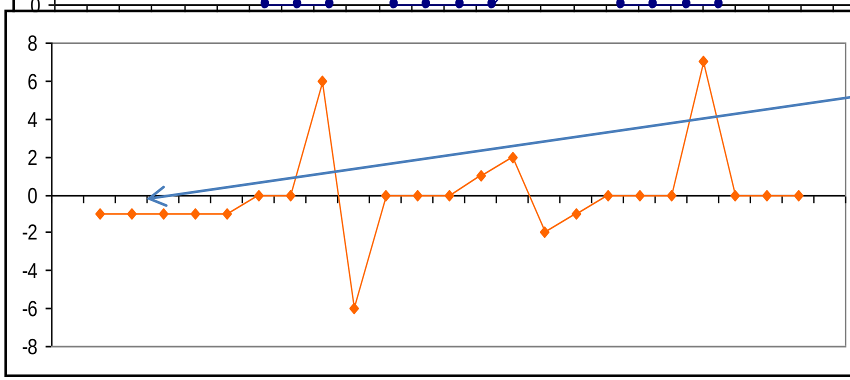
1st and 2nd Derivative

f(x)



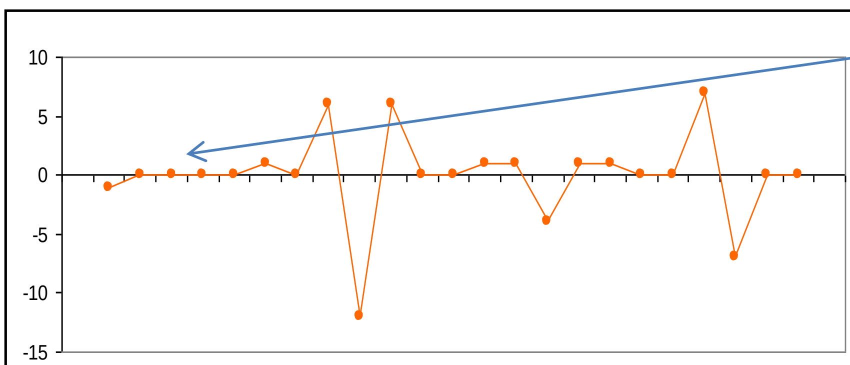
ramp like transition

f’(x)



first derivative results in thick edges because of nonzero value along a ramp

f’’(x)



Second derivative produces a double edge one pixel thick, separated by zeros. Therefore sharpens fine details better than first derivative

Example derivatives

- Intensity of pixels along a line in an image is

  f(x)=[1 4 1 0 0 7 7 7 7 0 0 0 1 2 3 4 5 4 3 2 1]

  Compute first order derivative
- f'(x) = f(x+1)-f(x)
- f'(x) = [3,-3,-1,0,7,0,0,0,-7,0,0,1,1,1,1,1,-1,-1,-1,-1]
- Compute second order derivative
- f''(x) = f(x+1)+f(x-1)-2f(x)
- f''(x)= [2,-6,2,1,7,-7,0,0,-7,7,0,1,0,0,0,-2,0,0,0,0]

The Laplacian operator

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 2nd order derivative in the x and y direction is defined as

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

The Laplacian operator

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

Called positive operator

or

$$\nabla^2 f = [-f(x+1,y) - f(x-1,y) - f(x,y+1) - f(x,y-1) + 4f(x,y)]$$

Called negative operator

- If we apply positive Laplacian operator on the image then we subtract the resultant image from the original image to get the sharpened image
- Similarly if we apply negative Laplacian operator then we have to add the resultant image onto original image to get the sharpened image

The Laplacian operator

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

or

$$\nabla^2 f = -[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Filter based on Laplacian operator is

image

a	b	d
e	f	g
h	i	j

The Laplacian operator

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

or

$$\nabla^2 f = -[f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

Filter/mask based on Laplacian operator is

image

a	b	d
e	f	g
h	i	j

positive Laplacian filter

0	1	0
1	-4	1
0	1	0

The Laplacian operator

Or $$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

$$\nabla^2 f = [-f(x+1,y) - f(x-1,y) - f(x,y+1) - f(x,y-1) + 4f(x,y)]$$

Filter based on Laplacian operator is

image

a	b	d
e	f	g
h	i	j

Positive Laplacian

0	1	0
1	-4	1
0	1	0

or

Negative Laplacian

0	-1	0
-1	4	-1
0	-1	0

The Laplacian

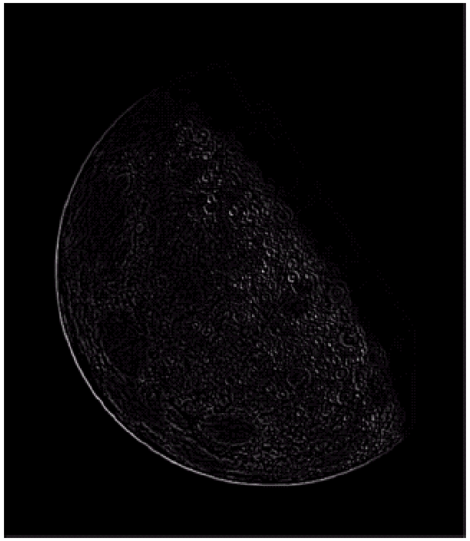
Highlights edges and other discontinuities

Original
Image

The Laplacian

Highlights edges and other discontinuities

Original
Image

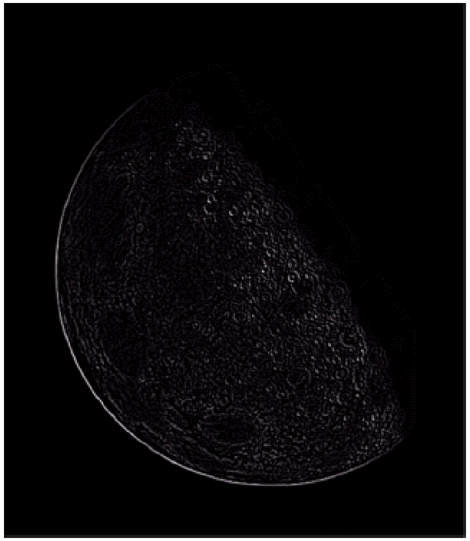


Laplacian
Filtered Image

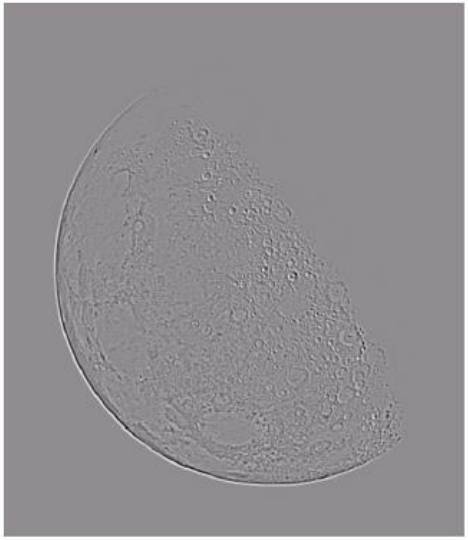
The Laplacian

Highlights edges and other discontinuities

Original
Image



Laplacian
Filtered Image



Laplacian
Filtered Image
Scaled for Display

Image Enhancement using Laplacian

Original
Image

-

Positive Laplacian
Filtered Image

=

Sharpened
Image

Sharpened image has enhanced edges and fine detail

Image Enhancement using Laplacian

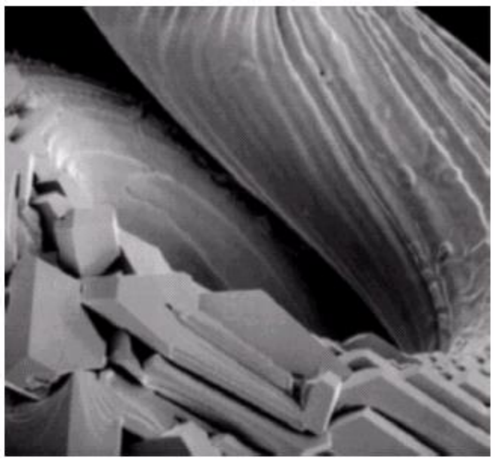
Simplified Image Enhancement

The entire enhancement can be combined into a single filtering operation

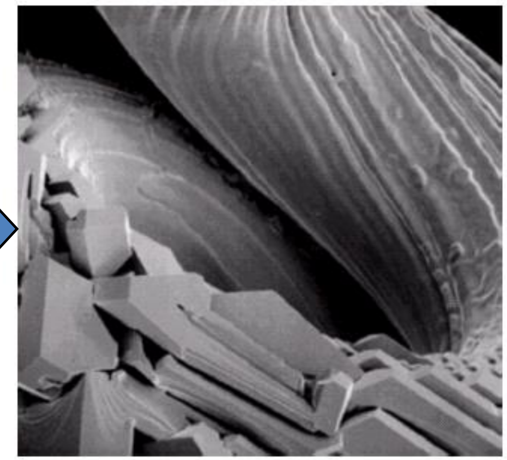
$$
\begin{aligned}
g(x, y) &= f(x, y) - \nabla^2 f \\
&= f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)] \\
&= 5f(x, y) - [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)]
\end{aligned}
$$

Image Enhancement using the Lapacian

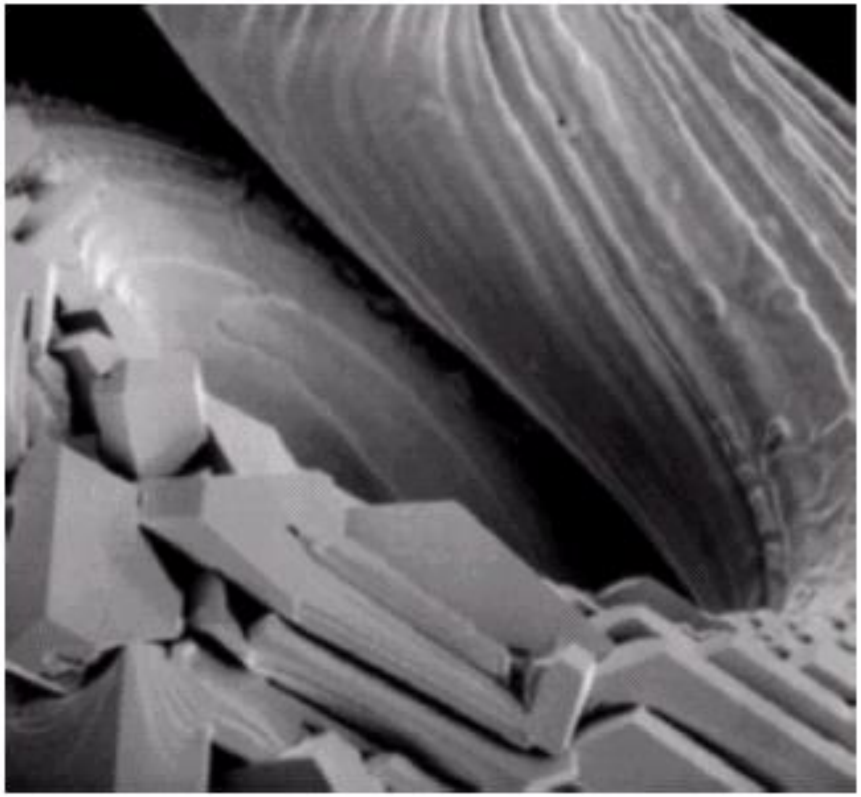
$$g(x,y) = 5f(x,y) - f(x+1,y) - f(x-1,y) - f(x,y-1) - f(x,y+1)$$



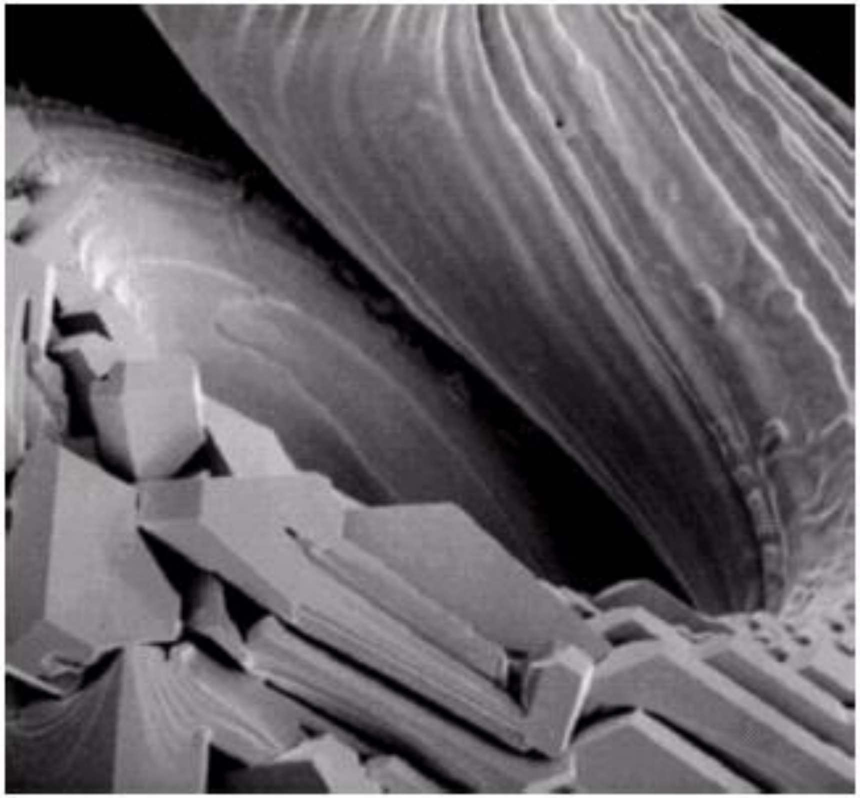
0	-1	0
-1	5	-1
0	-1	0



Simplified Image Enhancement



original



enhanced

Variants of Laplacian

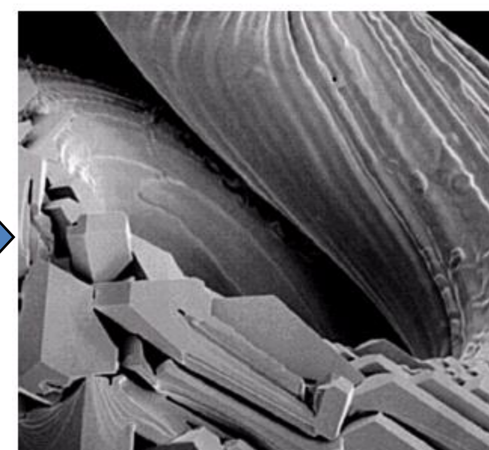
0	1	0
1	-4	1
0	1	0

Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian

-1	-1	-1
-1	8	-1
-1	-1	-1



Example Variant of Laplacian

1	4	5	2	7
0	4	0	6	2
3	2	1	0	2
7	5	2	3	1
4	3	2	5	1

*

-1	-1	-1
-1	8	-1
-1	-1	-1

=

1	4	5	2	7
0	16	-24	29	2
3	-6	-14	-17	2
7	-16	-5	10	1
4	3	2	5	1

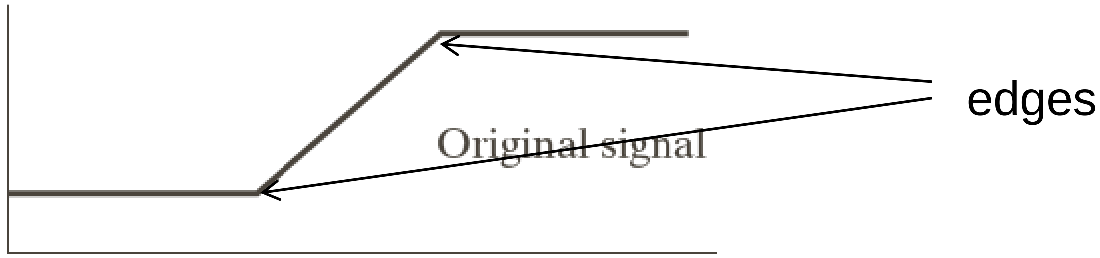
Unsharp (smoothed) Masking and Highboost Filtering

- Apply averaging mask to blur the original image
- Subtract the blurred image from the original image
- Difference is called the mask
- Add weighted mask to the original

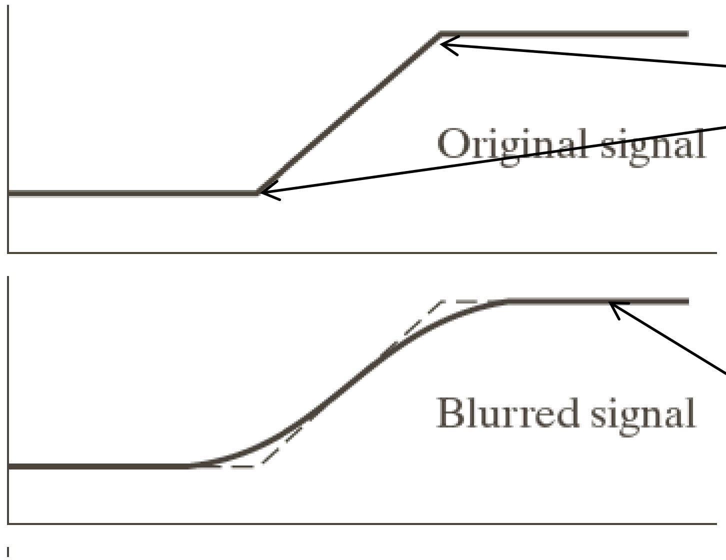
Steps for image sharpening

- $g_{mask}(x,y) = f(x,y) - f_b(x,y)$
  f_b represents blurring
- $g(x,y) = f(x,y) + k*g_{mask}(x,y)$, where, $k \geq 0$
  $k=1$, unsharp masking
  $k>1$, highboost filtering
  $k<1$, reduces effect of unsharp mask
- $g(x,y)$ can be <0 or >255
- Scale it accordingly

Highboost Filtering



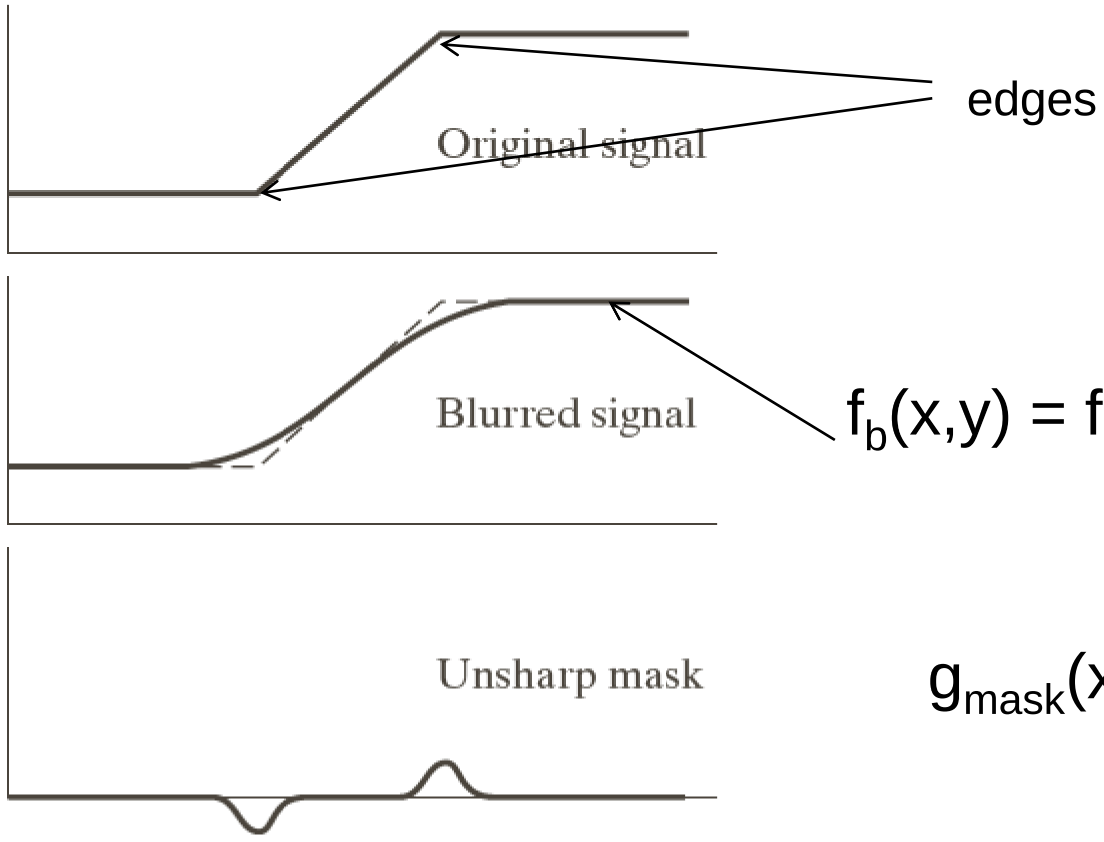
Highboost Filtering



edges

$f_b(x,y) = f(x,y) * \text{averaging filter}$

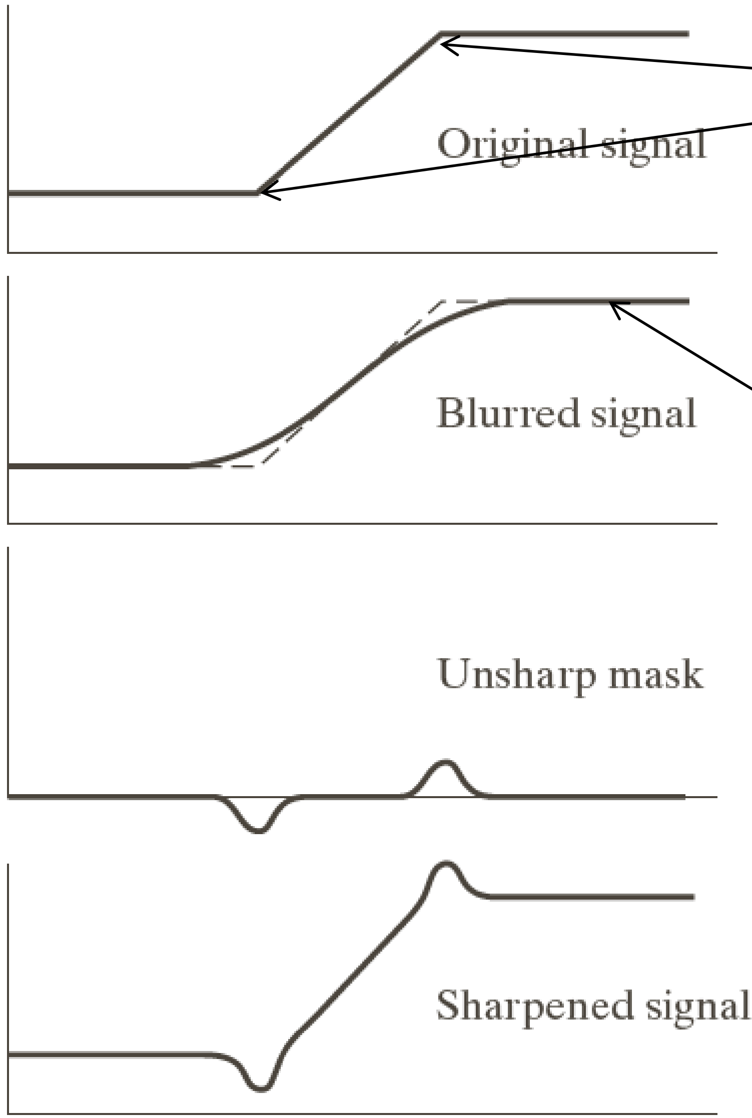
Highboost Filtering



$f_b(x,y) = f(x,y) * \text{averaging filter}$

$g_{mask}(x,y) = f(x,y) - f_b(x,y)$

Highboost Filtering



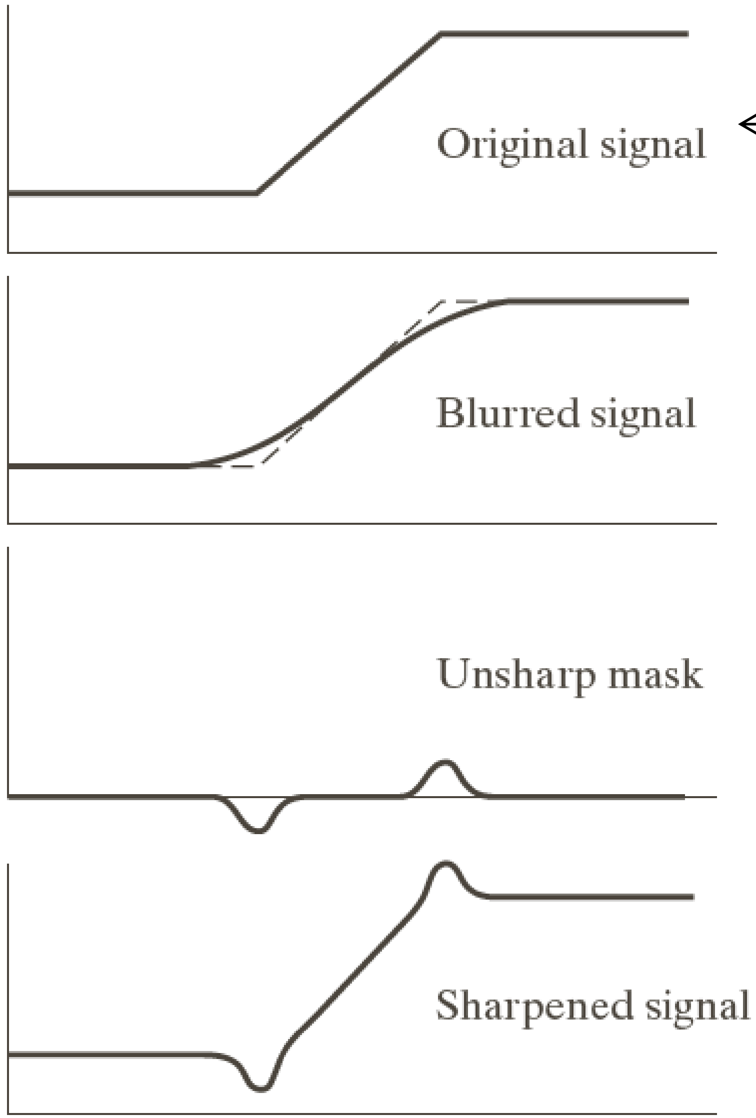
edges

$f_b(x,y) = f(x,y) * \text{averaging filter}$

$g_{mask}(x,y) = f(x,y) - f_b(x,y)$

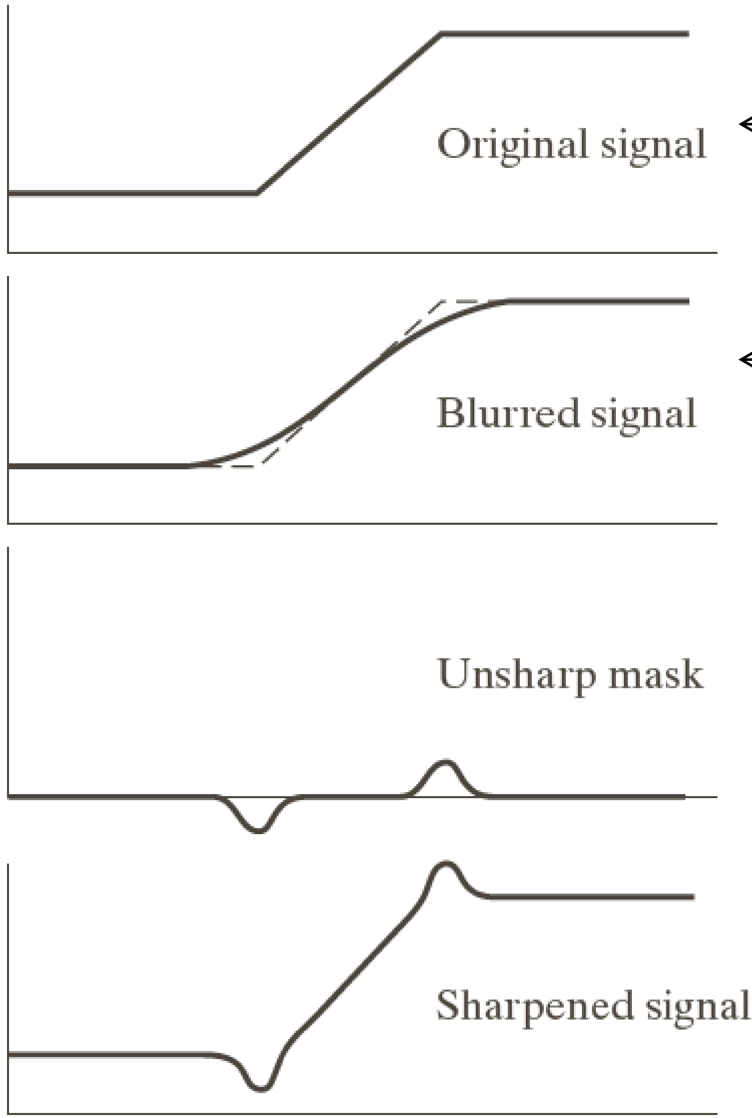
$g(x,y) = f(x,y) + k*g_{mask}(x,y)$, where, $k \geq 0$

Highboost Filtering



f(x,y)

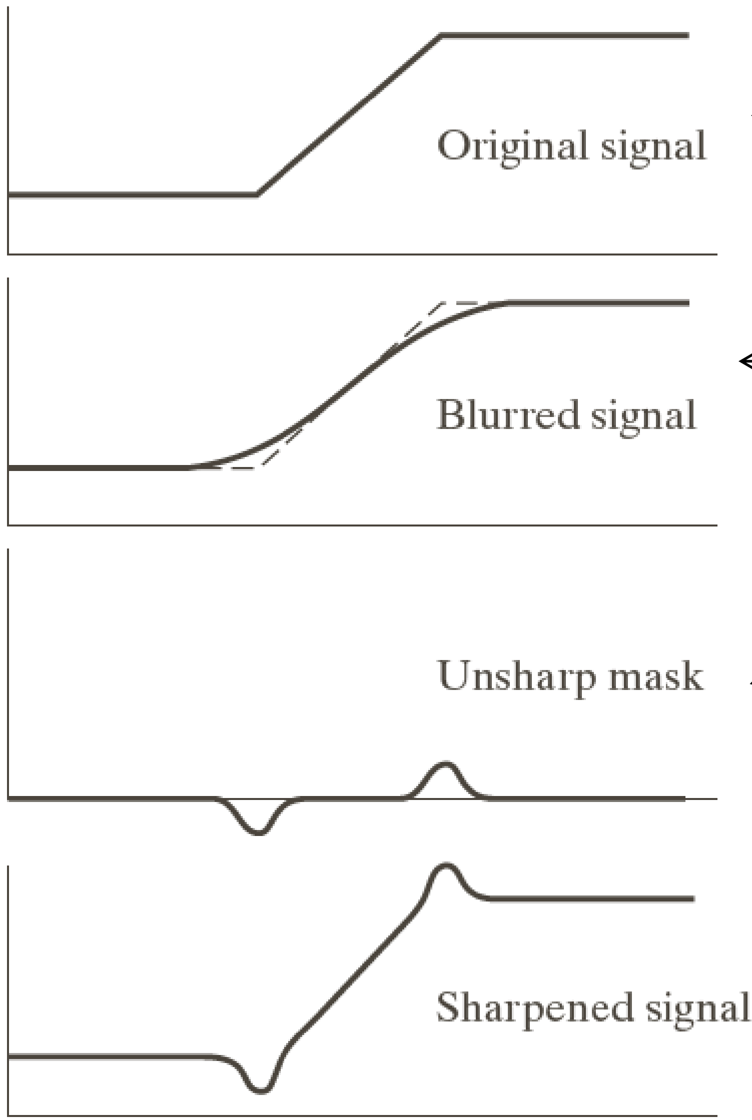
Highboost Filtering



$f(x,y)$

$f_b(x,y) = f(x,y) * h(x,y)$

Highboost Filtering

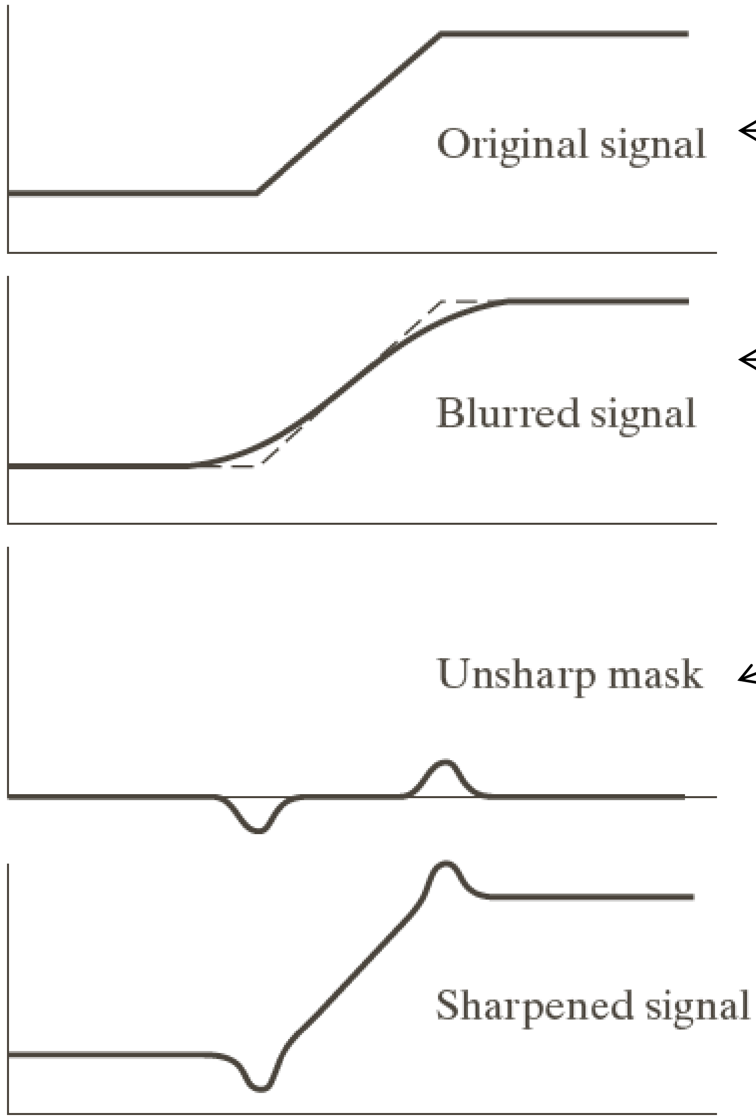


$f(x,y)$

$f_b(x,y) = f(x,y) * h(x,y)$

$g_{mask}(x,y) = f(x,y) - f_b(x,y)$

Highboost Filtering



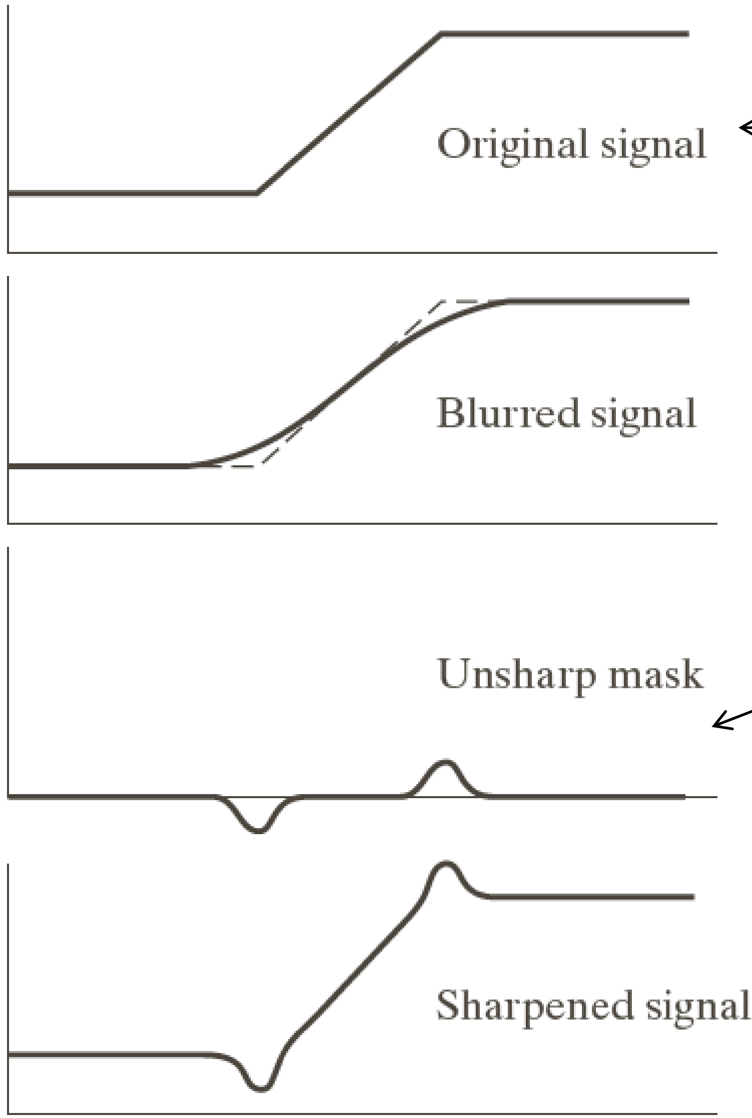
$f(x,y)$

$f_b(x,y) = f(x,y) * h(x,y)$

$g_{mask}(x,y) = f(x,y) - f_b(x,y)$

$f(x,y) + k * g_{mask}(x,y)$, $k=1$

Highboost Filtering



$f(x,y)$

$f_b(x,y) = f(x,y) * h(x,y)$

$f(x,y) - f_b(x,y)$

$f(x,y) + k * g_{mask}(x,y)$, $k=1$

$f(x,y) + k * g_{mask}(x,y)$, $k=4.5$

Gradient of image

- Gradient is first order derivative
- Gradient of *f(x,y)*

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

Gradient

- Magnitude of gradient (gradient image)

$$M(x, y) = mag(\nabla \mathrm{f})$$

$$= \left[G_x^2 + G_y^2\right]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

- Size of *M(x,y)* is same as image.

$$M(x, y) \approx |G_x| + |G_y|$$

Gradient using 3×3 filter

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

G_x

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)|$$

$$+|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

G_y

Sobel Operators

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

→

G_x

-1	-2	-1
0	0	0
1	2	1

+

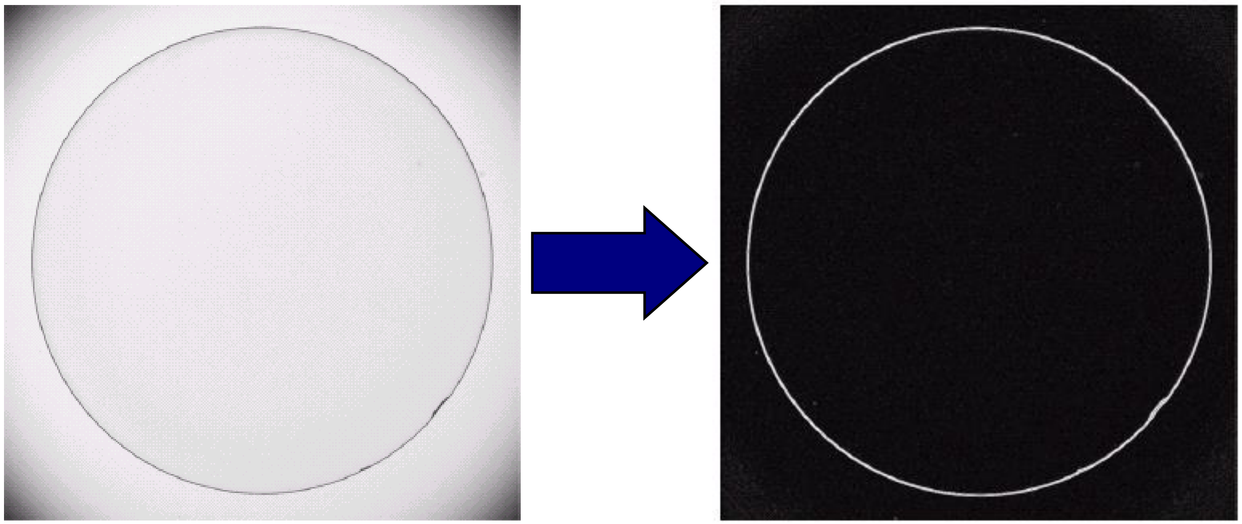
G_y

-1	0	1
-2	0	2
-1	0	1

$$M(x, y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)|$$

$$+|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Sobel Example



Sobel filters are typically used for edge detection

An image of a contact lens which is enhanced in order to make defects more obvious