An Agent Based Model for Competitive Equilibrium in Electricity Markets

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Abstract

The research presented herein aims to computationally model participants in an electric utilities market. Utilizing an agent based (AB) approach, players are treated as uniquely intelligent, independent agents who seek to maximize utility via selective bidding in a marketplace. Agents must decide the optimal supply based on expectations of their opponent's quantities supplied, the carbon make up of said supply, as well as expectations of future carbon tax rates. In making their decisions, agents are able to look back at historical quantity supplied, as well as look into their own cost structures in order to predict future outcomes. Agents are also able to invest and divest selectively from assets, giving them additional agency in affecting their outcomes. **The overarching goal of the model is to provide a framework in which economic planners can predict optimal resource allocation given market participants and production variables that are unequally distributed across a given geography**

Fundamental basis of the model come from various economic models, including oligopolistic competition– specifically Carnout competition– and Walrasian auctioning. The former denotes the market structure where firms decide independently on the output they will supply, while the later describes the mechanism by which competitors find equilibrium via repeated bidding.

The model is accomplished via a mix of computational tools, specifically the python programming language and its associated libraries. The prediction mechanism is achieved through multiple regression, while all optimization is performed via linear algebra toolkits. In order to increase efficiency in calculations, the model can be performed in parallel across many cores, assigning each agent and its associated properties to a single core for computation.

The competitive equilibrium model can be augmented to include spatial optimization, where agents are aware of their location as well as that of their competitors. This geospatial optimization introduces an added layer of production considerations and decisions for each player.

Results show that the speed in which agents reach a steady state is inversely proportional to the number of market participants, the decrease in consumer price per megawatt as the number of market participants increases, the the rate at which marginal taxes on carbon effect production on participants of different scales. These results are inline with the expectations put forth by optimization theory, Cornout competition, and Pigovian taxes. Spatially, agents distribute themselves in regions with the most favorable tax regimes, with special consideration paid to the distance to major demand nodes as predicted.

Acknowledgments

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1 Competitive Equilibrium in Electricity Markets

The change from government-regulated to competitive electric markets created a market structure that shares more similarities with a oligopoly than a truly competitive market. In 1989, the United Kingdom passed the Electricity Act of 1989 which provided for the privatization of the electricity industry in Great Britain. Sixteen years later, California passes the Electric Utility Industry Reconstruction Act with the purported goal of increasing competition with dire effects. Since then, much work has been dedicated to understanding and modeling electric markets.

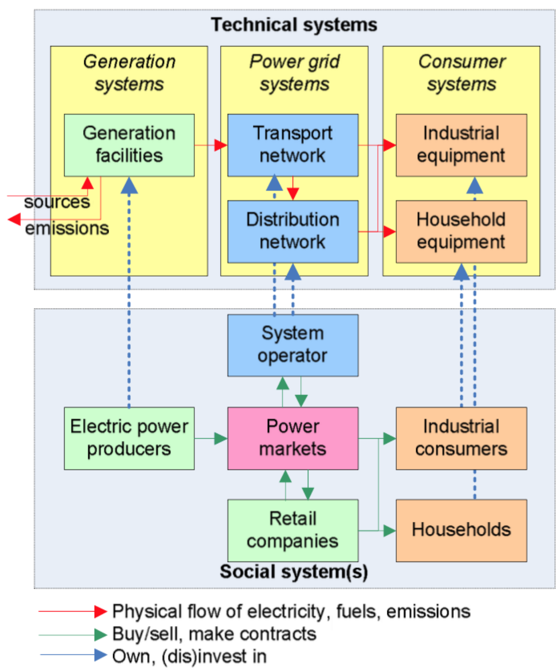


Figure 1: Electric Markets and Their Constituent Systems [4]

Figure 1 shows the relationship between various systems within the electric market. The flow of electricity, and therefore emissions, travels from the industrial producers of electricity– who primarily rely on fossil fuels for their heat source– through the distribution networks to the homes and offices of the consumer. The generation facilities are the focus of this research, specifically their decision to scale and preferred fuel source in a competitive environment.

1.1 Market Equilibrium

Competitive equilibrium in the model is defined as the state in which there are no changes in production levels between iterations. At each iteration, all sellers in the market decide on a level of production that will produce the highest utility given a known demand curve and an *expected* aggregate electricity supply. At the end of each iteration, all bids are summed, the price is calculated, and agents decide if they over or underproduced based on their individual production costs. Computationally, the algorithm that settles production acts as a market manager, more specifically a Walrasian auctioneer.

Each agent is treated as uniquely intelligent, using the output of previous rounds to adjust their expectations until a steady-state competitive equilibrium is reached. This process is a mirror of a Walrasian auction: instead of each agent calculating demand at all possible prices, all agents simultaneously submit bids for how much they are willing to produce at each possible price point. If after all bids have been submitted, and an agent's market expectations are above the actualized quantities, it will subsequently lower its productions proportional to the difference between the two amounts. In the computational model, this process is performed via a negative feedback loop.[1]

1.1.1 The Asymmetric Information Problem

One of the critical problems with modeling electric markets is the dependence on all parties of perfect information regarding their rivals cost structures. Since electricity

[1] See fig. 6

markets are primarily a margins-driven business, producers can only operate when the cost per-megawatt is above their production costs. However, since production is sent to an "electric pool", where multiple producer's output is amalgamated, the optimal production bundle of each agent is dependent on the summation of all bids in the market, and therefore the marginal costs of all producers. If agents cannot perfectly know all other participant's cost structure, they will not be able to accurately predict the aggregate supply, and suboptimal production will result.

1.2 Market Structure: Competitive or Oligopolistic

It has been argued that the competitive equilibrium model is only applicable in markets in which there are a large number of sellers. In small markets– like they one modeled here– each player's production dramatically influences the market price. This market power to alter the price means that the market functions similar to an oligopoly.

The market modeled has properties similar to a Cournot oligopoly, namely: output is homogenous and chosen simultaneously across all firms, firms act independently, firms are price setters, and firms seek to maximize utility given opponent's actions. Critically, the underlying assumption of the Cournot model is the "not" conjecture, i.e. that the firm takes the output of its competitors as given and that its own production decision will not effect its competitor's production outcomes.[2]

1.3 Potential Models

Previous work into oligopolistic electricity markets have utilized the Cournot, Bertrand, Stackelberg, supply function equilibrium (SFE) and collusion models. Similar to the

[2]Figure 6 shows how this assumption is implemented in the computational model.

model presented herein, Hogan and Cardel *et al* use a Cournot quantity approach to a single period of market trading [8]. Building off this approach, Chen, Wong *et al* created a coevolutionary computational (CCEM) model in which two players evolve in the power "ecosystem" based on their fitness, which in this case is a profit function[9].

Agent-based computational economics (ACE) is a particularly well-suited platform for capturing the dynamic interaction between many different agents. In ACE, each market participant is modeled as an agent who is given a utility function that it seeks to maximize at each interval. One downside of this model however, is that that it its most basic form, the players cannot form any strategic 'thinking' based on previous turns. ACE can be combined with other forms of computational modeling, specifically genetic algorithms (GA) to produce "smart" models.

1.3.1 Argonne National Laboratory EMCAS

Researchers at Argonne National Laboratory (ANL) Decision and Information Sciences developed a software tool that models participants in an electric market. The Electricity Market Complex Adaptive System (EMACAS) models "heterogeneous, decentralized decision structures"[10] as agents, each with their own set of objectives and decision making rules. The expanded model can use both historical data (e.g. previous prices) as well as predict future prices. EMCAS is a complete software suite, complete with a GUI and programmable agent characteristics. ECMAS is a commercially available and has been used by 1300+ users working across the gamut of enterprises, including the Illinois Commerce Commission.

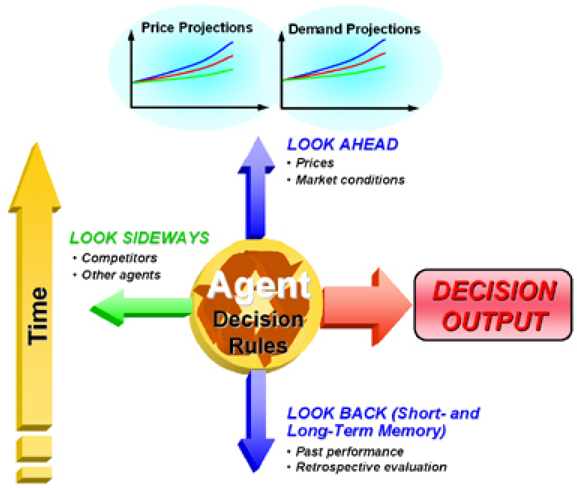


Figure 2: EMCAS Flow Diagram [10]

The work contained herein follows a similar approach to that of EMCAS, utilizing an agent-based approach to the power markets. The overlap between the two projects can be used to validate the techniques and approach used in the model presented in this paper.

2 Carbon Dioxide Production

One of the goals of this model is to show how a marginal carbon tax can effect the pollution outcomes of the market, with the specific aim of discovering how policymakers can introduce carbon taxes with minimal distortions to the electricity market.

In the model, agents predict total carbon emissions and use this information to plan their production. Since the tax rate is dependent on the total supply, agents are incentivized to work together and keep carbon output at or below the predefined tax threshold.[3]

[3]See section 3.2.2 for a detailed explanation of agent's CO_2 expectations

2.1 The Carbon Problem

Over the past 100 years the global temperature has risen 1.53 °F. However, since ocean temperature tends to rise slower than land, the overall effect is more pronounced for Earth's landmasses.

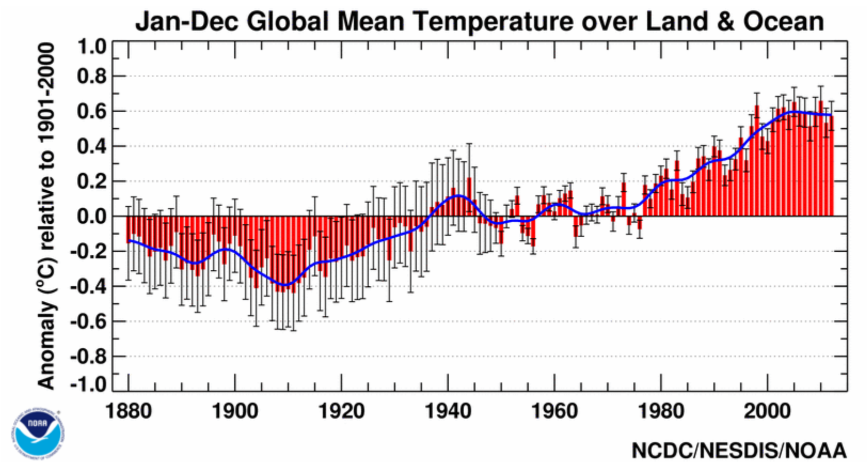


Figure 3: Rise in Global Temperatures Since 1880 [5]

Climatologists attribute this sustained rise in global temperatures to the increased use of fossil fuels for transportation and power. In the US, the largest source of these CO_2 emissions come from the generation of electric power followed by transportation. [4] Since the decision to construct a new power plant is more strictly an economic decision than automobiles which have longstanding cultural considerations, it is the opinion of the author that electric production represents the low-hanging fruit of carbon reduction.

[4] The global average of CO_2 emissions from electric generation is roughly $\frac{1}{3}$)

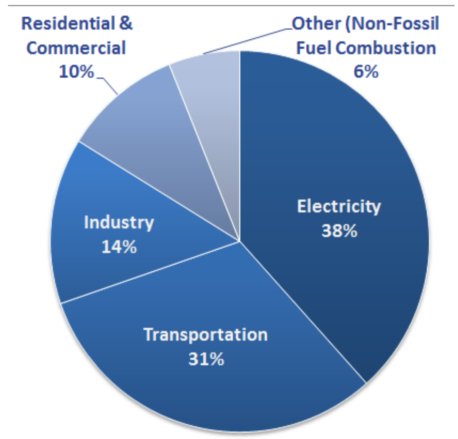


Figure 4: Breakdown of Greenhouse Gas Emissions by Source [1]

2.2 Efforts Towards a Carbon Tax

A carbon tax is a mechanism which aims to disincentivize the production of greenhouse gases (GHGs) via raising the cost of pollution. GHGs represent a negative externality, and therefore taxation can be used to more accurately reflect the social cost of carbon– a form of taxation known as a Pigovian tax after economist Arthur Pigou[3].

In the United States, there is no nationwide tax on carbon, however some municipalities have enacted a true carbon tax. In 2008, the Bay Area, California passed a 0.044 USD tax per tonne of carbon, however this fee is well below the suggested price of 40-60 USD per tonne[6].

3 Model Formulation

An agent based model is created in which multiple power providers independently select output based on expectations of future market conditions: including CO2 emissions, aggregate supply, and the regulatory environment. The market price per megawatt and effective tax rate are set via the value and composition of aggregate supply– meaning each producer must at every turn accurately predict the combined output and its associated carbon emissions in order to choose its own optimal production bundle. If the combined carbon output is greater than an exogenous, known threshold, then a marginal tax on carbon is enacted.

In the system, agents are aware of the market clearing price (MCP) and use their own cost structure coupled with historical bids to estimate their opponent's future bidding behavior. In this sense, agents are able to "look back" and "look in" in order to formulate their optimal bids for the next period[5]. Agents seek to optimize their utility for time period $t+1$ by placing bids such that their utility– a combination of revenue, utilization rate and emissions–is maximized in the market place.

3.1 Agent Characteristics

Each member of the class *agent* has the following unique attributes:

1. Quantity of various production assets
2. Cost of production of each asset
3. Utility Coefficients
4. Available liquidity for investment
5. Damping coefficient

The characteristics listed above are variable for each agent and are initialized at the beginning of the simulation. All of the characteristics, except the utility

[5] Or, to "look forward"

coefficients, are dynamic and are updated as the simulation progresses. In the case where the above are identical for all agents, the quantities produced are the same for all agents and the steady state solution is immediately achieved [6].

3.2 Optimal Production

At each turn, agents find the optimal production via a simple utility function given their expectations of future output. Where U is the total utility, R is the revenue, C is the carbon emissions, u is the utilization rate, and α, β, γ are the utility coefficients for each:

$$\max_{\forall q \in Q} U_i(R, C, u) = \alpha_i R_i + \beta_i C_i + \gamma_i u_i \tag{1}$$

3.2.1 Expected Revenue

Expected revenue for each player, i, is calculated as a function of the cost per megawatt-hour for each production technology j, the quantity of each production technology supplied by player i, and the expected market price, $P^{exp.}$, for electricity–a function of aggregate supply, S_{agg}. In the model, a generic, linear demand curve is given, from which the market price is determined.

$$P_i^{exp.} = 10 + .2 S_{agg}^{exp.} \tag{2}$$

$$R_i^{exp.} = P_i^{exp.} Q_i - \sum_{j=0}^{N} c_i^j * q_i^j - \tau C_i \tag{3}$$

Where Q_i is the total production for agent i, c_i is the marginal cost of production

[6] This is derived from the fact that producers use their own cost structure to as their initial guess of opponent's production.

for generation technology j, τ is the marginal tax rate[7], C_i is the agent's carbon emissions, and q is the quantity of each generation technology produced by the agent.

In the original and most simplified version of the model, the demand curve is exogenous, known, and static.

3.2.2 Expected Carbon Emissions

Similar to expected revenue, agents also calculate the expected carbon emissions for each turn. Since agents face a marginal tax rate if and only if total carbon emissions are above a predefined threshold, the optimal production bundle will be influenced by expected carbon emissions.

if $C_{agg.} \geq C_{max}$ **then**

  $tax = C_i^{CO2} * \tau$

else

  $tax = 0$

end if

Of the three different production technologies, only natural gas produces CO2 [8]. This implies that should an agent predict carbon emissions to be above the threshold, they will shift production towards clean technologies. Where κ is the amount of CO2 produced per megawatt-hour by burning natural gas:

$$CO2_i^{exp.} = CO2_{opp}^{exp.} + q_i^{CO2} * \kappa \tag{4}$$

In the expanded game, agents also predict whether the tax rate and threshold will change in the future, as they must plan investment into different power generation

[7] The relative costs of each table can be found in appendix B

[8] Natural gas produces 117 lbs of CO2 per million BTU [1]

facilities. As the expectations that the carbon tax will increase, agents move capital away from natural gas facilities and towards renewable energies. Where τ is the carbon tax rate, T is the maximum amount of carbon allowable, p_1 is the probability that the tax rate increases, p_2 is the probability that the tax rate is less than or equal to its current level, p_a is probability that the threshold decreases, and p_b is the probability that the threshold is greater than or equal to its current levels:

$$\begin{aligned} E_{gas}[\tau, T] = P^{future} * Q_{gas} - [p_1\tau(Q_{gas} - p_a T) + p_2\tau(Q_{gas} - p_b T) \\ + p_1\tau(Q_{gas} - p_b T) + p_2\tau(Q_{gas} - p_a T)] \end{aligned} \tag{5}$$

Eq. 5 is the expected value operator for the agent's investment decision. Thus, the future regulatory environment and the accuracy of which agents can know it have dramatic effects on the outcome of the game.

3.2.3 Utilization Rate

The final component of agent's utility is the utilization rate, defined as:

$$u_i = \sum_{j=1}^{N} 1 - \frac{A_j - q_j}{A_j} \tag{6}$$

Where A_j is the total amount of asset j that the agent owns. The utilization rate is incorporated to ensure that agents are less inclined to disregard one asset type all together. In this sense, the utilization rate can be thought of as a minimum production diversification metric. Adding the utilization rate into the utility function also serves to keep agents from foregoing using their existing assets.

3.3 Cost Structure

The costs of production vary with the type of generation technology used. However, in order to mimic the benefits of scale, the marginal cost for all types decreases with the total amount of assets the producer has. Ergo the more gas plants an agent has, the lower the marginal cost of producing electricity from natural gas.

3.4 Investment

In the expanded model, agents may divest themselves from certain assets as well as "research" improved methods for generation. Based on the the existing portfolio of power generation and the benefits of shifting towards various types of production, agents may sell off assets at depreciated values or reinvest profits into lowering the cost per megawatt of a certain technology– effectively investing in more capacity.

The addition of future investment alters the utility calculations of agents. Since investment or divestment will effect optimal bundles for remainder of the game, agents must calculate the total utility gained or lost for the next five periods [9]. Agents choose to invest or divest in technologies based on their current and previous utilization rates of technologies, as well as their expectations of the carbon tax rate and the amount of carbon produced in the future. To look forward, agents use a multiple regression algorithm to predict utility based on the expectations of the aforementioned variables. Thus at every turn, each agent calculates the utility gained over five periods for all permutations of buying, selling, or maintaining each asset.

[9] In actuality, agents would need to calculate the utility for an infinite time horizon. However, for simplicity, agents predict that the game will last another five turns, regardless of the actual number of turns remaining

```
for asset in Assets do
    ∑_{t=1}^{5} U_i^{sell}
    for asset in Remaining Assets do
        ∑_{t=1}^{5} U_i^{invest}
        for asset in Remaining Assets do
            ∑_{t=1}^{5} U_i^{maintain}
        end for
    end for
end for
Find max permutation
```

3.5 Modifying Expectations

Players are unaware of their opponent's utility coefficients and their cost structure, thus they are unable to accurately predict at what quantity their opponents will produce at (section 1.1.1). This imperfect information causes players to under or over produce which causes less than expected utility. In order to minimize this differential, after each turn players compare their expected aggregate supply to the actualized value. Players will dampen or amplify their production proportional to the delta they experience.

When delta is negative and the agent has overproduced, the agent will modify their future expectations by $\zeta > 1$. In the opposite situation, and the agent has underproduced, they will modify their future expectations by $\zeta < 1$. As the solution approaches steady state, δ will converge to zero and ζ converges to unity (fig. 7).

Figure 6 details how the negative feedback loop is implemented. The optimization routine takes expected supply as an input and finds the optimal production

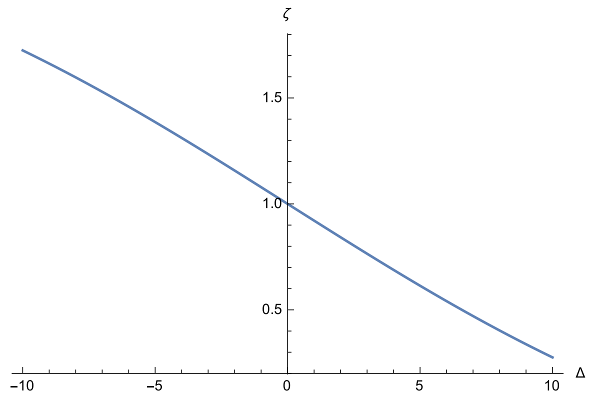


Figure 5: Expected Supply Damping Function

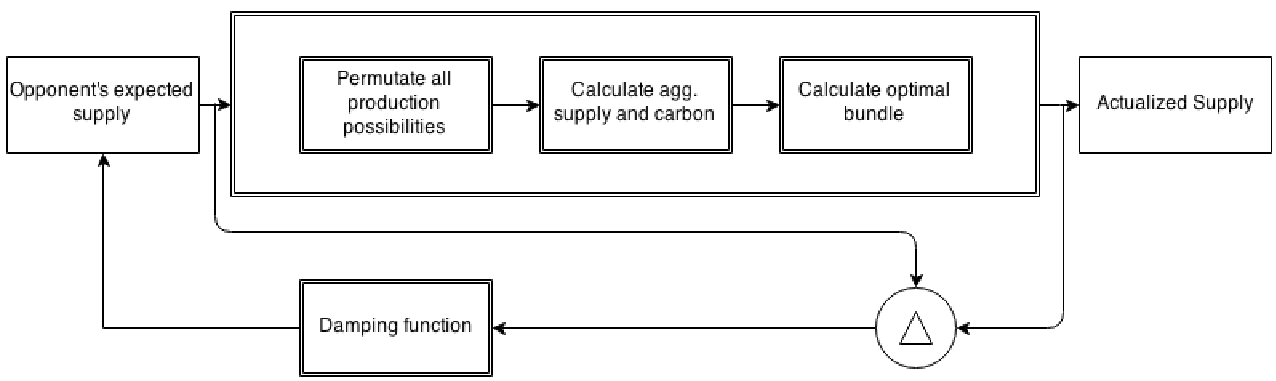


Figure 6: Block Diagram of Feedback Loop

bundle based on these expectations. After bid submission, the comparator finds the difference in expectation vs. reality and sends the result to the damping function. The damping function in-turn, creates a suitable damping ratio and multiplies it to the previous expectations to create future expectations.

$$Q_{t+1}^{exp} = \zeta Q_t^{exp} \tag{7}$$

4 Competetive Equilibrium: Achieved Results

The first series of tests consisted of two agents, each with different utility coefficients, available assets, and cost structures. Agent *1*, show in green, is the larger of the two producers[10], and has the majority of its production capacity in natural gas. There are differences in the utility coefficients, however they are not extreme.[11]

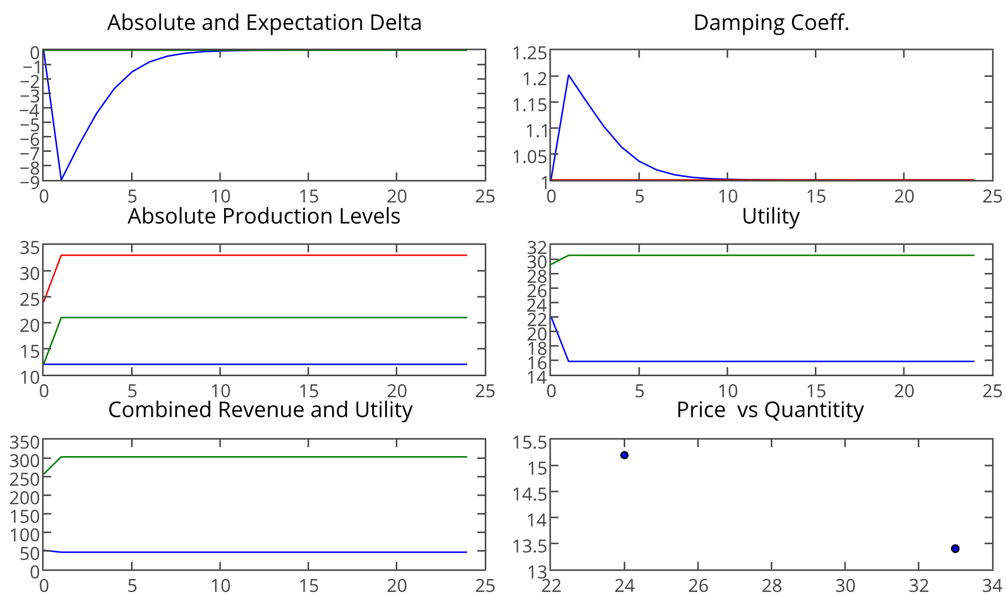


Figure 7: Summary of Results, Case 1

Figure 7 shows the results of the baseline simulation. Here, you can see that α quickly converges to unity as δ converges to nil. Agent *1*, the larger of the two producers, becomes the dominate player in the market with approximately 70% market share. In the model, fast convergence is indicative of two agents being able to quickly

[10]Agent *2* is shown in blue

[11]Recall that the closer the costs and coefficients are, the faster steady state will be reached

'feel out'[12] the opponent's optimal production bundle via the auction mechanism. Since there are only two agents, both players are able to identify their opponent's best production bundle with certainty by simply attributing all residual supply to the opponent. As the market expands, it can be shown that this methodology fails (section 4.1).

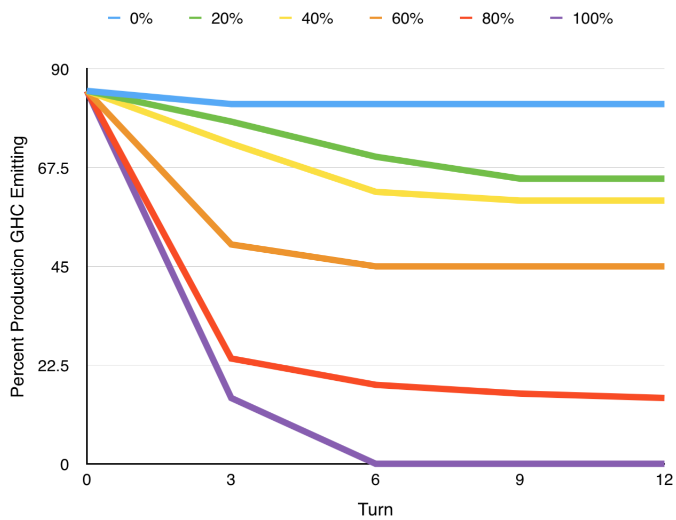


Figure 8: Percent Polluting in Portfolios for Various Tax Rates, Case 1

Figure 8 shows the percentage of polluting technologies in the agent's portfolio as a function of the turn for various tax rates. As expected, when the tax rate is equal to the cost of natural gas, using gas becomes unprofitable and agents quickly divest from it. The sharp decline in emissions seen for an eighty percent tax rate suggest that the optimal tax rate is somewhere between 60% and 80% of the price of natural gas.

[12]In Walras' words, 'tatonnement', or 'groping'

4.1 Handling Market Expansion

The model is expandable and can handle an arbitrary number of players in the market [13]. However, as the number of agents increases, the amount of iterations required to reach a steady state increases too, as the damping function must now incorporate multiple variable outputs. Numerically, the model becomes more sensitive to initial conditions and less stable.[14] and as a result, the model requires the all agent's initial conditions to be closer together.

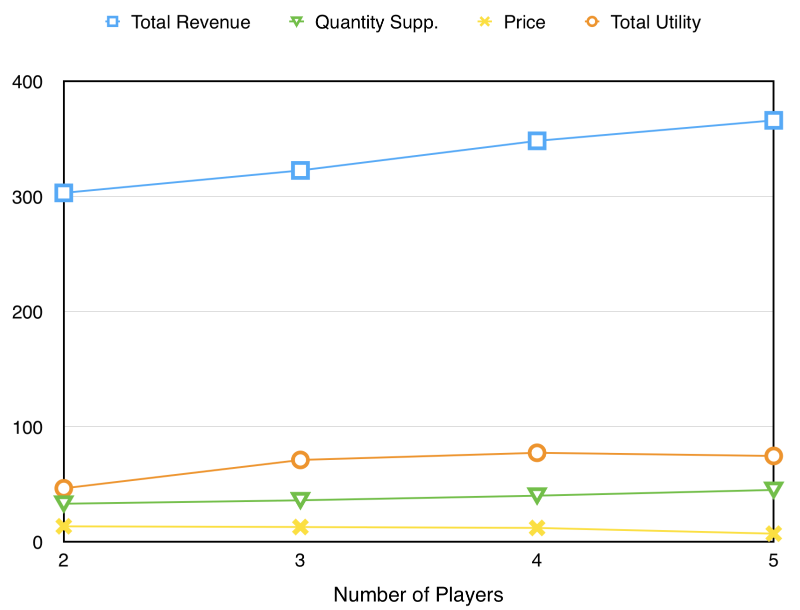


Figure 9: Model Quantities as a Function of the Number of Players

Figure 9 shows how various quantities in the model react as the number of players increases. As expected, the price falls as N, the number of players, increases. This price decrease is accompanied by a subsequent increase in both the combined revenue of all agents and the total utility. This is inline with expectations of how oligopolies

[13] However, past four players the algorithm is no longer critically damped, and continues to oscillate around a fixed average; see appendix F

[14] Here stability refers the ability for the algorithm to converge to its limit

act: they supply an artificially low quantity of goods such that they may maximize there own individual profit. As the number of players increases, competition brings the prices down until the competitive equilibrium price and quantity is reached. The Cournot theorem validates this outcome, stating that as the number of firms in the market goes to infinity, the quantity supplied goes to the competitive level and the price will converge to marginal cost.

$$\lim_{N \to \infty} Q^{oligopoly} = Q^{competitive} \tag{8}$$

While the total utility and revenue is higher as the number of players increases, no one agent has is able to significantly increase its own share of the gains. Thus, while the end consumer would win in the form of lower utility prices and higher power availability, the power producers will generate less revenue as the number of power providers increases.

Another unwanted side effect of decreased competition is an increase in the amount of carbon produced. As mentioned before, as the number of agents decreases, players are able to guess their opponent's moves with higher and higher certainty. At each iteration, agents chose their own carbon output based on what they expect their opponent's to produce (section 3.2.2). The inverse relationship between players and certainty of outcomes forces agents to become more cautious with their carbon output with a larger number of players, since it is harder to account for how much carbon will be produced. It follows then that, for $N > 2$ the total carbon output was zero, as firms were forced to be more cautious with CO_2 production. For $N \leq 2$, the amount of carbon production is always equal to the maximum allowable before the marginal tax is enacted.

The reduction of carbon achieved by competition is counter-intuitive, especially since the overall size of the electricity market grows. However, the results indicate that the asymmetric information presented to producers will cause them to be overly cautious and produce emission free so long as the price differential between emitting and non-emitting technologies is small. It can also be argued that the increased number of players made it more difficult for the participants to collude in their carbon production.

4.2 Effects of Taxation on Producers of Different Scales

The second experiment that the model was used for was to see how a carbon tax would effect producers of different sizes. Two agents were created, a smaller producer and a much larger one. In order to isolate the effects of scale, both producers had the same percentage of each generation type in their portfolio. The hypothesis was that large producers would be still be able to produce using polluting technologies as they would be able to leverage their scale into lower input prices. Smaller producers, would be forced into using non-emitting generation, and subsequently face smaller profit margins. This anticipated result would be counterproductive to the policy's spirit: large polluters could keep polluting at a higher rate since other market competitors would be priced out, and smaller, more environmentally-friendly players would be disincentivized since they would have smaller profit margins using more expensive technology and pooled pricing.

The hypothesis was not validated by the model, however. As the carbon tax was ramped up, both small and large producer's production were equally effected. The size of the overall market and the shape of the demand curve were the deciding factors in the production bundles. The large producer could supply the majority of the market using non-emitting technologies, while the smaller counter party couldn't.

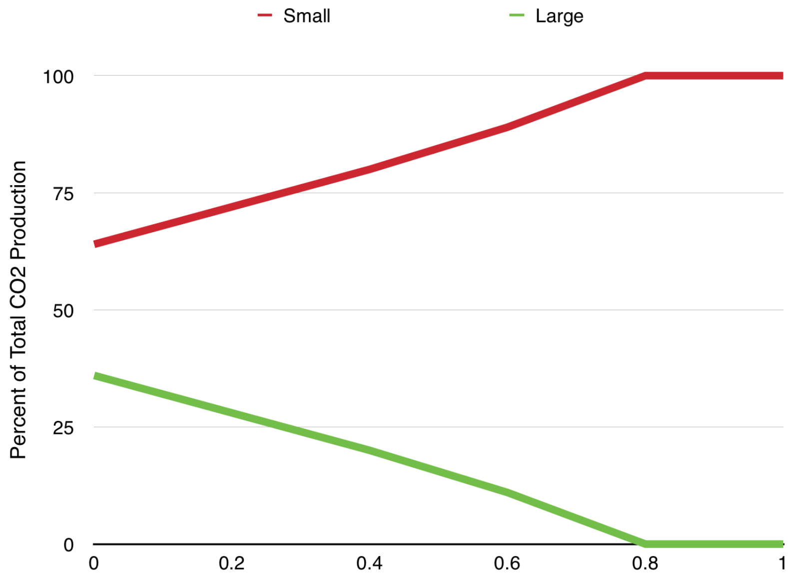


Figure 10: Percent of Carbon Produced as Function of Marginal Tax Rate

This lead to the smaller player producing the entirety of the carbon content, while the larger producer was able to make up for the smaller margins by increased volume.

The results of this experiment suggest that policy actions such as a marginal tax on production will be effective independent of the size of the firms affected.

5 Spatial Considerations

Previous research developed an agent based model in which individual players find their optimal location based on known environmental variables. At each turn, agents are able to compute their cost functions at various geographic locations, effectively searching for their geographically optimal location with consideration to other market participants.

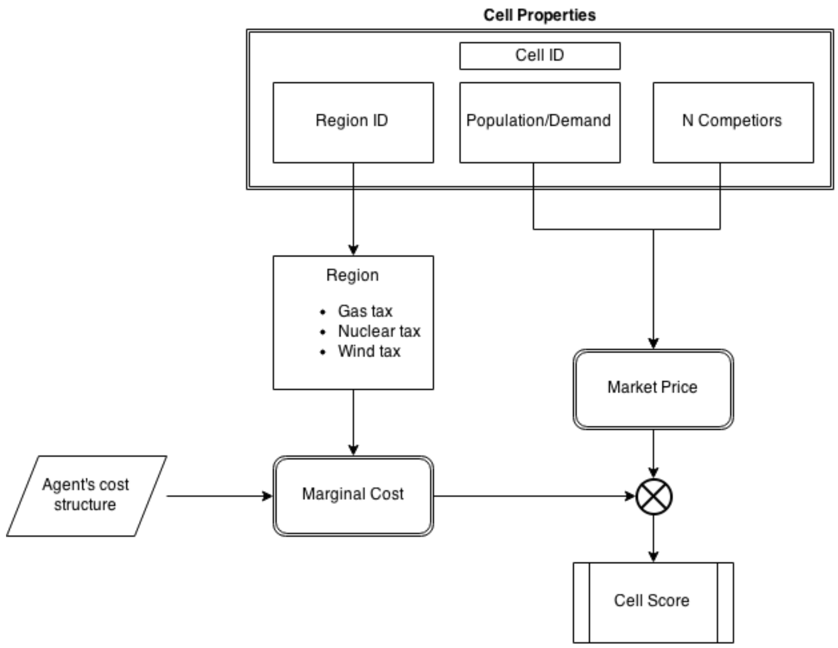


Figure 11: Flowchart for spatial analysis

5.1 Model Formulation

Underlying the model is a superposition of generalized parameter layers, which are, in ascending order: geographic terrain, population, regulatory environment, and finally the agents themselves. Each planer layer contains information corresponding to a set of arbitrary supplied parameter values. These parameter values could include population data, energy usage, average wind speeds etc. on a per unit area basis. In this paper specifically, a .svg [15] file of Western Europe is used. Each cell in the array is assigned a cell ID which is linked to tables containing *occupied*, *region ID*, and *centroid GPS*. Cells are thus aware whether or not agents are occupying them, to which regulatory jurisdiction they are subject to, and how far away they are from major demand nodes.

[15] Scalable Vector Graphic

5.1.1 Geography

The geography is a raster graphic which produces a discrete array from an imported image. The the size of the mesh can be adjusted depending on the resolution required by the user or what is available via the parameter data. GIS data is overlaid and linked to the cell, enabling the use of GPS coordinates to calculate distance. Geography is particularly important in this model as utilities suffer a transmission loss factor per mile transported, thus creating strong incentives to be geographically near demand nodes.

5.1.2 Cell Properties

In this model each cell is mapped to a corresponding European nation and their respective energy policies. The numerical values for the location's taxes and/or subsidies are inputted into the cost functions of agents operating within their boundaries. For example, the cells comprising France could have tax coefficients of $\alpha = 1$, $\beta = 1.5$, and $\gamma = -1$ for wind, natural gas, and nuclear respectively [16]. These values are then combined with the agent's intrinsic production cost to evaluate the performance at that cell.

These regional parameters can be abstracted to contain any arbitrary set of user-defined "regions" or clusters. These regions can be anything from national boundaries to residential or commercial zoning.

The second cell property described in this model is population. Population figures for the area in question are laid over the region ID and used to estimate demand across the service area.

[16] Here negative values represent a subsidy for the specified technology

In its the current formulation, each agent is given an adjustable service area parameter which dictates how far the agents can distribute their goods. Agents then take the population for all cells within their radius to calculate the shape of the demand curve. Mathematically, this operation can be represented as the convolution of the sub matrix R centered at the agent's location with the size determined by the aforementioned range parameter.

5.1.3 Market

The final component of the spatial framework is agent's ability to discover the number of players competing in their geographic market. Agents are aware of other players within their service range and internalize this information in their supply side calculations. Intuitively, this will provide incentive for players to be uniformly distributed around major demand nodes such that they may act as the monopoly power provider.

5.2 Results

At the time of submission, the competitive equilibrium model has not been fully integrated with the spatial model. However, each model is fully functional independently, and results for the geospatial optimization module are described below.

The generalized algorithm for the turn-based geospatial optimization is as follows:

while turn != maxTurn **do**
    for plant in numPlants **do**
        for tile in vision **do**
            find total population

calculate demand

find tax code

calculate production costs

end for

move to lowest cost cell

end for

turn = turn + 1

end while

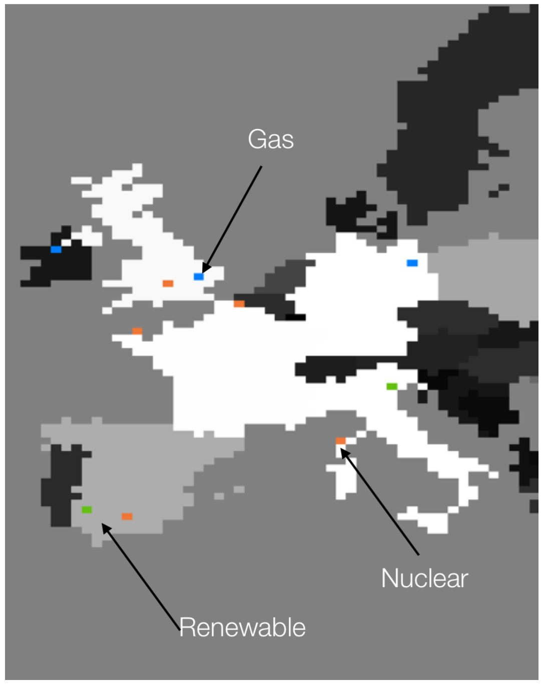
The results are inline with user expectations: the agents with move to the areas in which have the most favorable tax codes and distribute themselves uniformly with a noticeable bias towards regions of high demand as seen in Figure 12.

In the simulation depicted in Figure 12, France has the most favorable tax laws, including heavy subsidies for nuclear and wind technologies. The model also uses point demand nodes instead of distributed population parameters, i.e. all of a nation's energy [17] use is centered in at the capital. The effect of the point demand nodes can explain why many power plants leverage France's central location for production and distribution. It is worth noting that the transmission loss is the same for water and land based transmission, explaining why a number of power plants supplying London are located across the English Channel in France.

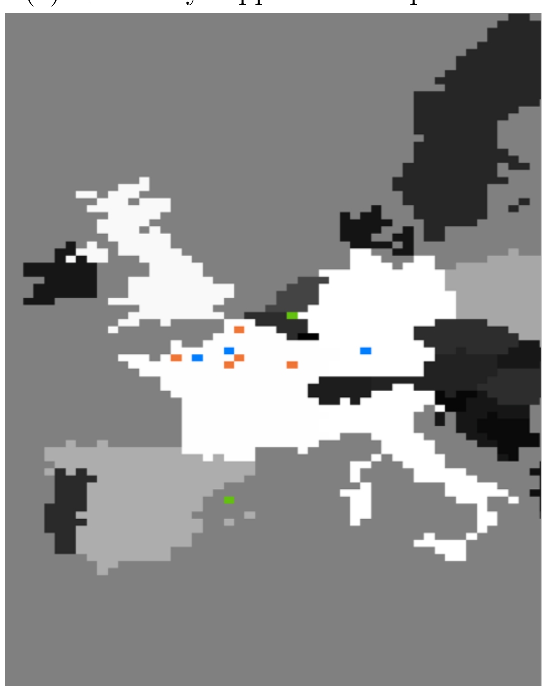
The results generated are self-consistent in so far that if the simulation is ran with the same layer parameters and enough iterations [18], the agents will settle at the same locations. Thus the model is completely dependent on the values of the parameters used, a problem that is discussed in further detail in Section 7.1.

[17] The countries modeled include: Great Britain, France, Germany, Spain, Italy and Portugal

[18] i.e. as the number of iterations approaches infinity



(a) Randomly supplied initial positions



(b) Final positions

Figure 12: Distribution of Power Producing Resources

6 Computational Techniques

One of the major limitations of this type of agent based models is the time required to run a complete simulation. For the competitive equilibrium module, the number of turns required to reach steady state is around five (Section 4.1), with each turn consisting of the calculation of both expected and actualized cost and production values for each player. For the geospatial model, the number of calculations depends on the number of squares in an agent's vision as well as the number of agents [19]. Thus, a nontrivial amount of time would be needed to compute a more in-depth model consisting of more players, parameter layers, and a higher resolution grid.

To limit the time required to run a full simulation, the algorithm has been modified to run in parallel across many compute cores. Instead of the calculations for each agent having to queue serially, each agent can be assigned its own core, allowing all agent calculations to be performed simultaneously, drastically reducing time required. Specifically, the CUDA programming language developed by Nvidia is leveraged to harness the power of modern graphics processing units (GPUs), many of which are equipped with upwards of 500 independent cores. This time reduction becomes invaluable when multiple parameter values are used (Section 7.2)

7 Conclusions

The model can be used effectively to show the benefits of competition in a market (section 4.1), and to a lesser extent, how a carbon effects different players in market (section 3.2.2).

As the number of players increases, the price decreases, thus falling in line with

[19] In the cited simulation over 5000 turns (around 10 minutes) were run to ensure that a steady state solution was found

predefined expectations on how competition works in the marketplace. This result not only serves to validate the model and its methodology, but also illustrates the difficulties of high dimensional optimization problems in general. In the case of the model, five[20] is the maximum number of producers that can lead to a Pareto Optimal outcome. Figure 9 shows that at five market participants, the marginal gain in *Total Utility* is negative, going from 77 to 74.

7.1 Problems with the Model

While to model can be used to demonstrate the effects of competition in oligopolistic energy markets and the associated changes in carbon production, a number of problems exist. Agents in the model see the output of all competitors as a homogenous sum, and cannot account for individual production. The grouping of output as stated above is a limitation since changes in an individual production cannot be accounted for, i.e. only a single damping function exists, as opposed to a personalized function for all market participants. Multiple damping functions adds increased complexity, but also introduces increased numerical instability. Multiple damping functions could, however, allow for agents to identify the cost and utility structures of the other players, eventually leading to globally optimal solution. Such an algorithm could utilize changes in opponent's production to fit the cost and utility coefficients via multivariate regression. Agent's would therefore be increasing the accuracy of their predictions at each iteration, thereby making the model a truly "smart" model.

As mentioned in Section 4.1, the model breaks down when the number of agents is greater than four. This failure arises from the added market dynamic brought

[20] Which is to say 10, since both the production and the composition are to be predicted for each player

about by adding players [21]. As the number of players increases, the number of production decisions in an agent's forecasting increases, causing the algorithm's model of the system to become undefined. This problem is more a limitation of the damping algorithm than any valuable economic insight, although it does serve as a nice example of the real-world difficulty of accurately forecasting any type of outcome in a true market [22]. A method to bypass this player limit include adding individual damping functions for each player. Alternatively, agents could be given a set of initial expectations regarding opponent production; the entire game could be re-simulated using a different set of initial conditions, thereby increasing the chances of reaching a convergent solution.

7.2 Future Work

Future iterations of the model will include the suggestions mentioned in Section 7.1, specifically the creation of multiple initial predictions. Currently, all agent's use their own cost and utility coefficients to create their initial guess, i.e. "based on my preferences, how much supply should the market have?", and then either or decrease expectations based of this starting point. If a player is significantly off the actual, they will dramatically alter production and will tend to get trapped around that production value. This implementation not only makes the convergence incredibly sensitive to initial conditions, but also suggests that currently agent's are not to finding the globally optimal solution because their initial expectations are misguided.

Multiple initial conditions and parameter values can also be used to create a Monte Carlo wrapper around the entirety of the simulation. One of the major

[21] Recall that each player has three outputs that must be predicted

[22] See "The Curse of Dimensionality"

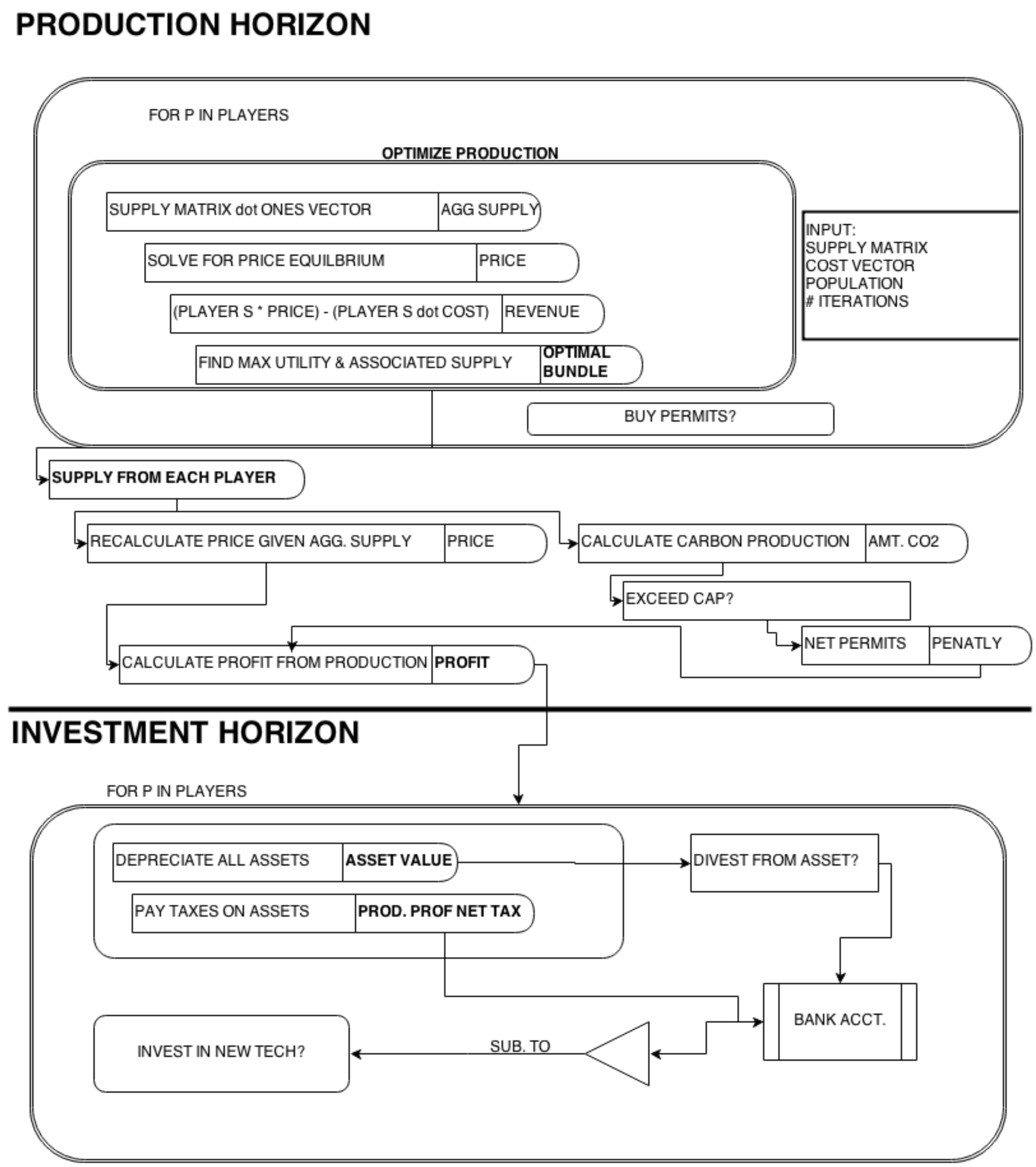
concerns with any computational model is the accuracy of parameter values [23]– making the model is more or less deterministic [24] after the parameter values have been set. By wrapping the parameter selection in a Monte Carlo process, a massive range of initial conditions and parameters can be tested and the outcomes judged based on the frequency at which they appear [25].

[23] Frequently cited as "garbage in, garbage out"

[24] There is randomness introduced by the selection of the order in which agents move

[25] A similar method is used in forecasting option prices

A WireFrame


PRODUCTION HORIZON
FOR P IN PLAYERS
OPTIMIZE PRODUCTION
SUPPLY MATRIX dot ONES VECTOR
AGG SUPPLY
SOLVE FOR PRICE EQUILBRIUM
PRICE
(PLAYER S * PRICE) - (PLAYER S dot COST)
REVENUE
FIND MAX UTILITY & ASSOCIATED SUPPLY
OPTIMAL BUNDLE
INPUT:
SUPPLY MATRIX
COST VECTOR
POPULATION
ITERATIONS
BUY PERMITS?
SUPPLY FROM EACH PLAYER
RECALCULATE PRICE GIVEN AGG. SUPPLY
PRICE
CALCULATE CARBON PRODUCTION
AMT. CO2
EXCEED CAP?
NET PERMITS
PENATLY
CALCULATE PROFIT FROM PRODUCTION
PROFIT
INVESTMENT HORIZON
FOR P IN PLAYERS
DEPRECIATE ALL ASSETS
ASSET VALUE
DIVEST FROM ASSET?
PAY TAXES ON ASSETS
PROD. PROF NET TAX
BANK ACCT.
INVEST IN NEW TECH?
SUB. TO

B Levelized Costs for Power Production

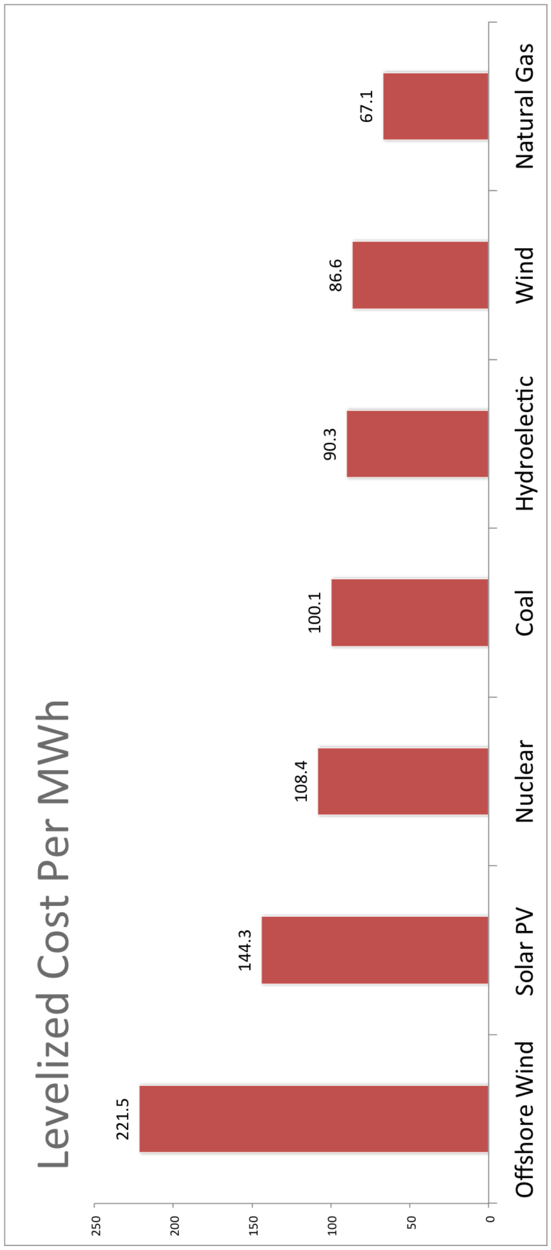
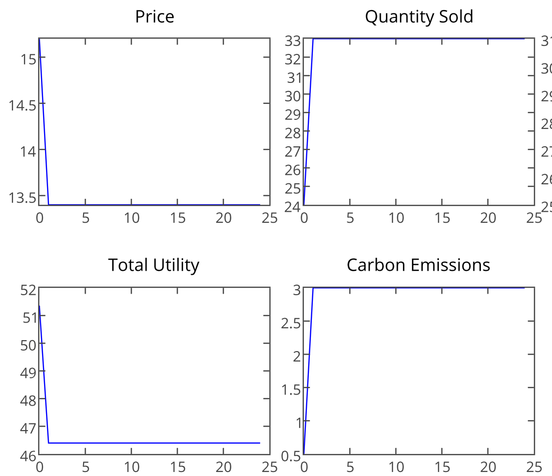
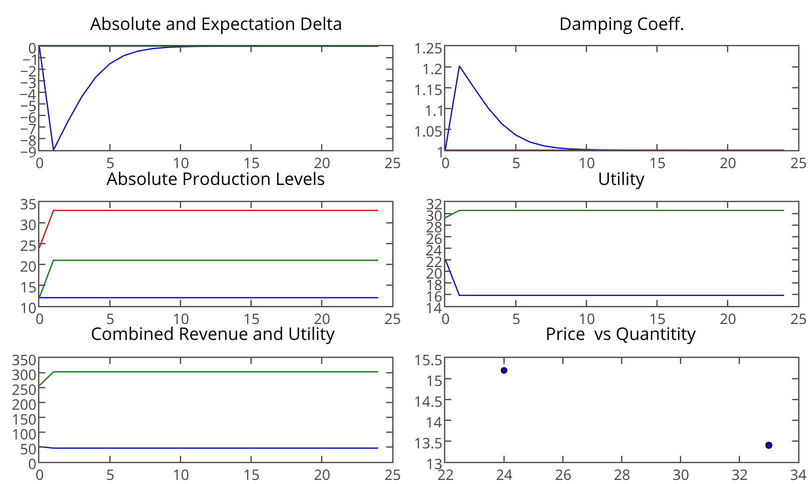


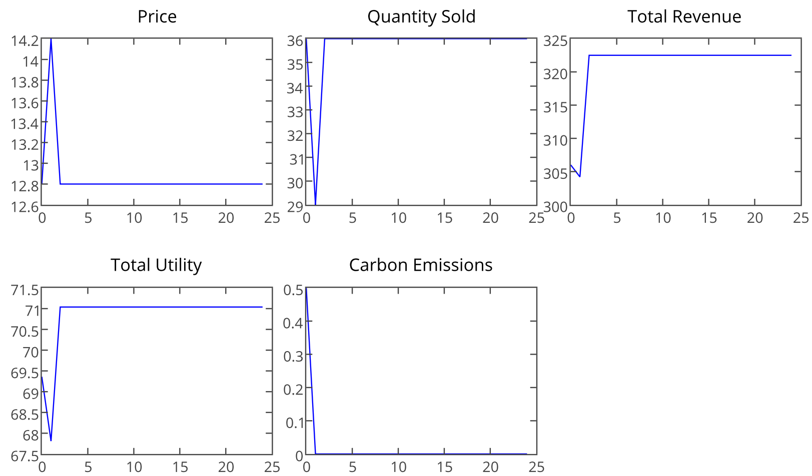
Figure 13: EPA.gov, 2012 [11]

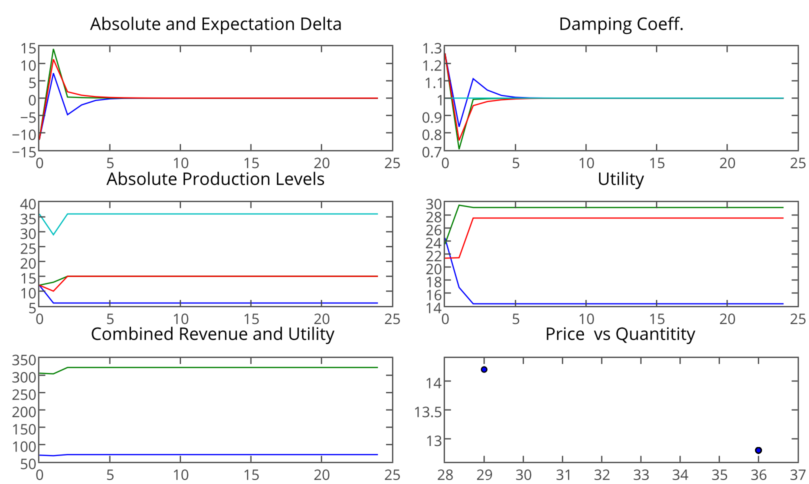
C Two Players



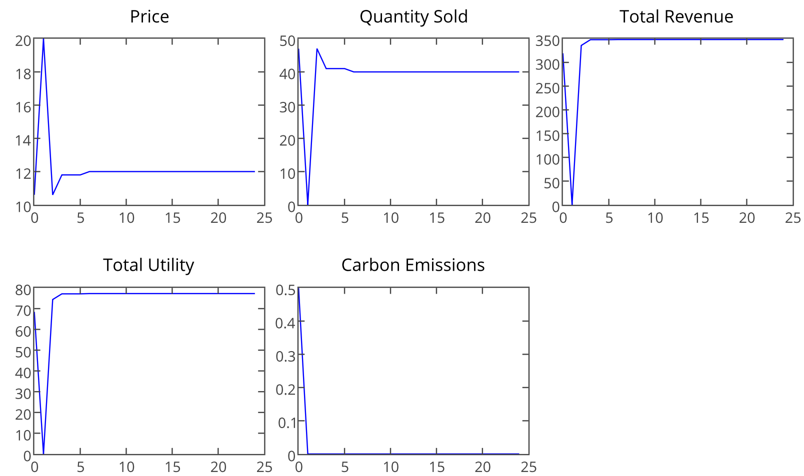


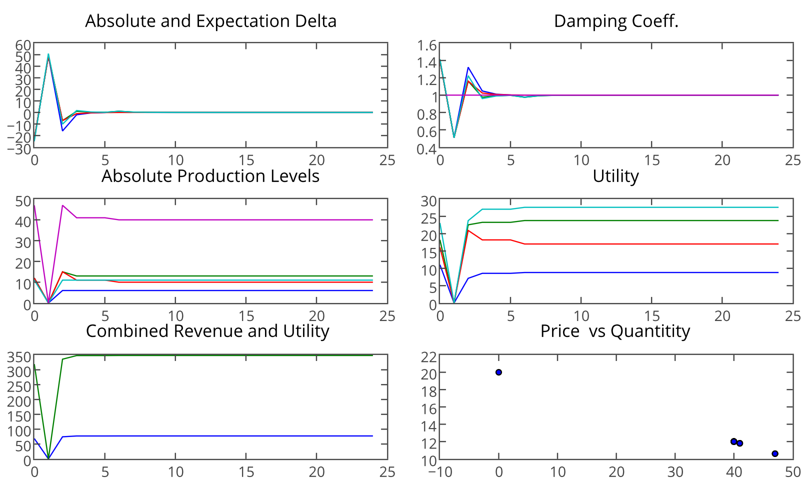
D Three Players



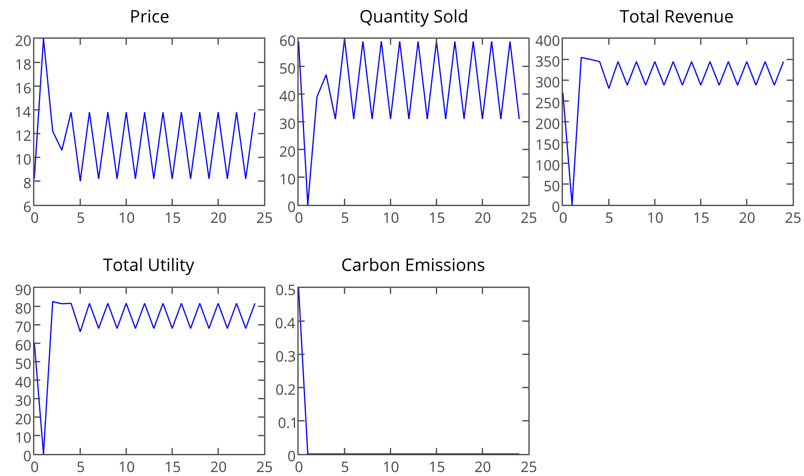


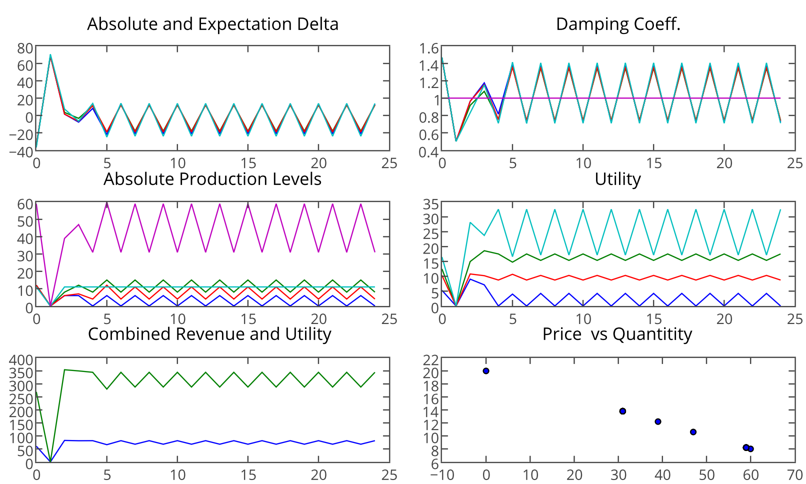
E Four Players





F Five Players





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