Partial Equilibrium in Mathematica

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Much of microeconomic analysis is based on the concept of a partial equilibrium. By using computational tools– such as Mathematica– we can quickly derive, plot and adapt analytical models of consumer and firm behaviour, and how the intersection of the two fix market equilibrium.

1 Utility and Production Functions

The basis of all economic analysis is the consumer and their goal of utility-maximization. The most common numerical repersentation of two-input production and consumer utility are the Cobb-Douglas and Leontief functions. These forms satisfy the following properties depending on if they are used to repersent production or utility, respectively:

1. monotonicly increasing/decreasing
2. concave/convex
3. nonintersecting

The general Cobb-Douglas function takes the form

$$Y = L^{\beta} K^{\alpha}$$

Where Y is production, L is labor, K is capital, and α, β are the elasticities of labor and capital respectively. The Cobb-Douglass (**Figure 1**) function assumes that continous substitution between goods x_1 and x_2 in production/utility.

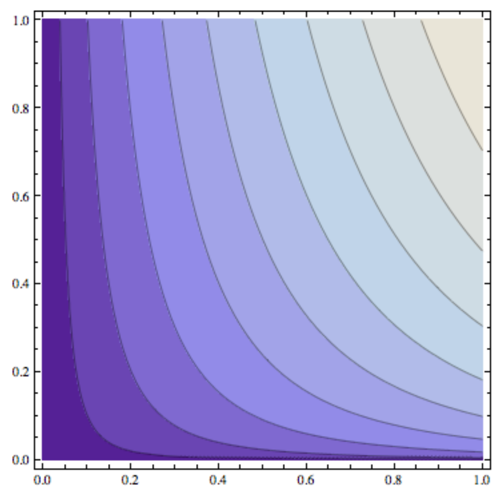

Figure 1: A Cobb-Douglas Production Function, $\rho = .7$

$$\alpha + \beta = 1$$

The above equation implies a special conditon when the production function experinces constant returns to scale.
Cobb-Douglas production is a special case of the constant elasticity of substitution (CES) production function as described by Solow (Solow, 1956) when $\lim \gamma \to 0$. The two-factor CES function (**Figure 2**) is of the form:

$$Y = \alpha K^{\gamma} + (1 - \alpha) L^{\gamma})^{\frac{1}{\gamma}}$$

A Leontief function is a special case of a Cobb-Douglas function in which there is no substitutability between goods x_1 and x_2 and thus there exists a predefined proportion of goods x_1 and x_2 used in the production of Y.

$$Y(x_1, x_2) = min(c_1 x_1, c_2 x_2)$$

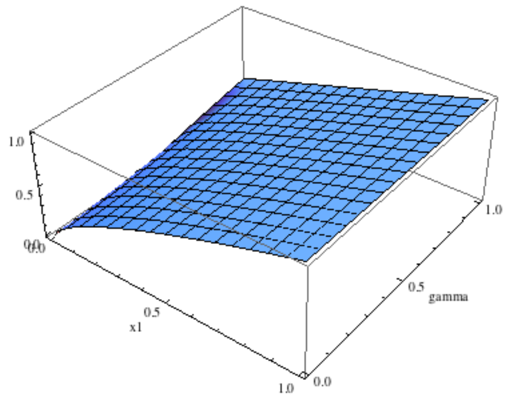

Figure 2: CES Function, $\gamma = [.01, 1], x_1 = [0, 1], x_2 = 1$

When the proportion

$$\frac{c_1 x_1}{c_2 x_2} = 1$$

the Leontief has a 45°line through the contour plot as seen in **Figure 3**:

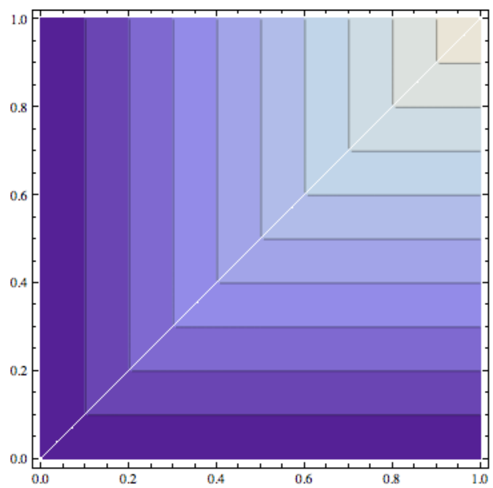

Figure 3: A Leontief Production Function

2 Consumer Theory

Traditionally, consumer theory is modeled as agents maximizing their utility (strictly a function of consumption, C) subject to their budget constraint, (m). This implies that given some bundle of goods x_1, x_2 at prices p_1, p_2, the rational consumer will purchase the bundle that maximizes their total utility, U. The optimal bundle will allways include an nonzero amount of both goods since utility is subject to the principle of diminishing marginal returns. A Cobb-Douglas utlilty function with elasticities α will take the form:

$$maxU = x_1^{\alpha} x_2^{1-\alpha} subjectto : m = p_1 x_1 + p_2 x_2$$

Via logarithims, this equation is equivelent to:

$$log(U) = p_1 log(x_1) + p_2 log(x_2)$$