

Homework 2

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1 Section 1.6

Question 14

a)

p = "In the class"

q = "Owns a red convertible"

r = "Got a speeding ticket"

$\forall x(q(x) \rightarrow r(x))$ premise

$p(x) \wedge q(x)$ Premise

$q(x)$ Simplification

$r(x)$ Modus ponens

Due to linda, there is at least one person who has gotten a speeding ticket in this class

b)

p = "Taken a course in discrete mathematics"

q = "Can take a course in algorithms"

$\forall x(p(x) \rightarrow q(x))$

$p(M) \wedge p(A) \wedge p(R) \wedge p(V) \wedge p(K)$ Premise

$q(M) \wedge q(A) \wedge q(R) \wedge q(V) \wedge q(K)$ Modus Ponens

Because all 5 took discrete math, this implies all 5 are eligible for algorithms

c)

p = "produced by John Sayles"

q = "Is wonderful"

$\forall x(p(x) \rightarrow q(x))$

$p(Coal)$ Premise

$q(Coal)$ Modus Ponens

Because the movie about coal miners is produced by John Sayles, the movie is wonderful; thus, there is a wonderful movie about coal miners.

d)

p = "Been to France"

q = "Visited the Louvre"

$\forall x(p(x) \rightarrow q(x))$

$p(x)$ Premise

$q(x)$ Modus Ponens

Question 24

The error is on step 3, because you cannot simplify $P(c) \vee Q(c)$ to $P(c)$

FRQ

p = "There is an undeclared variable"

q = "There is a syntax error in the first five lines"

r = "There is a missing semicolon"

s = "Variable name is misspelled"

$$q \rightarrow (r \vee s)$$

$$p \vee q$$

$\neg r$ Premise

$\neg s$ Premise

$$\neg(r \vee s) \rightarrow \neg q = (\neg r \wedge \neg s) \rightarrow \neg q \text{ Modus Tollens}$$

$$\neg q$$

Given there is no missing semicolon or misspelled variable name, there is not a syntax error in the first 5 lines.

2 Section 1.7

Question 18

$\forall m, n, k \in \mathbb{Z}, mn = 2k \iff mn \text{ is even.}$ By definition.

$$(m = 2k \wedge n = 2k+1) \vee (m = 2k+1 \wedge n = 2k) \rightarrow mn = (2(k(2k+1))) \vee (2(k(2k)))$$

Through association, mn is even by definition.

$mn \in \mathbb{Z}$ under multiplication since $k \in \mathbb{Z}$.

Therefore, if m or n are even integers, mn is even.

Question 20

p = "n is an integer" q = "is even"

a) By contraposition

$p(n) \wedge q(3n + 2) \rightarrow q(n)$ Premise

$\neg q(n) \rightarrow \neg p(n) \vee \neg q(3n + 2)$ Contraposition

$(3n + 2) = (3n + 2(1))$ Identity

$2(1)$ is even by definition. Even + Even = Even, Odd + Even = Odd

Since: $2k + 2k = 2(2k)$ and $(2k + 1) + (2k) = 2(2k) + 1$

By the same reduction, $3n + 2 = 2(n+1) + n = n$

$\neg q(n) \equiv \neg q(3n + 2)$ By definition of even

This makes the condition true, and since the contrapositive is true, the original is true.

b) By contradiction

$p(n) \wedge q(3n + 2) \rightarrow q(n)$ Premise

Assume $\neg q(n)$ By Contradiction

$3n+2 = 3(2k+1) + 2 = 6k + 5 = 2(3k + 2) + 1$

This implies $\neg q(3n + 2)$ which contradicts the premise

Question 34

if rational x was equivalent to rational x/2, it could be written as $x = \frac{p}{q}$ and $x/2 = \frac{p}{q/2}$, where q is just another integer.

Then, $3x-1$ can also be written as $\frac{3p}{q} - 1 = \frac{3p-q}{q}$

x can take on any real number, so with different combinations of p and q,
x is rational \equiv x/2 is rational \equiv 3x-1 is rational

FRQ

Prove: $x \in \mathbb{Z}^+ \rightarrow (x = 2k \iff 7x + 4 = 2k | k \in \mathbb{Z}^+)$

$$7x + 4 = 2(3x + 2) + x$$

This implies $7x + 4$ is only a multiple of 2 if x is also a multiple of 2, where x is a positive integer.

FRQ

R = "The square root of any irrational number is irrational."

a)

$\neg R$ = "The square root of any irrational number is rational."

b)

prove: $\sqrt{\text{irrational}} = \text{irrational}$

Assume opposite: $\sqrt{\text{irrational}} = \text{rational}$

$\sqrt{\text{irrational}} = \frac{p}{q}$ Definition of a rational number

$$\text{irrational} = \left(\frac{p}{q}\right)^2$$

$p, q \in \mathbb{Z}$, so $\left(\frac{p}{q}\right)^2$ is likely to be rational, which is a contradiction.

By contradiction R is true, "The square root of any irrational number is irrational."

3 Section 2.1

Question 46

Use proofs?

Multiplication of sets is not commutative: If $A = (a_1, a_2)$ and $B = (b_1, b_2)$

$A \times B$ is a set of a_1 and a_2 with all combinations of B , in that order. Therefore the order of multiplication matters, and the resulting set is not necessarily equivalent if switched. $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same.

Question 48

Prove or disprove that if A , B , and C are nonempty sets and $A \times B = A \times C$, then $B = C$.

A , B , and C are nonempty sets — Given

Prove $A \times B = A \times C \rightarrow B = C$

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

$$A \times C = \{(a, c) \mid a \in A \wedge c \in C\}$$

$$\forall a, b, c ((a, b) = (a, c) \rightarrow b = c) \rightarrow B = C$$

4 Section 2.2

Question 20