Lecture 2 Examples

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Jan 22 2026

Negate "If a user is active, at least one network link will be available."

p : "A user is Active"

q : "A network link will be available"

Starting with:

$$p \to q$$

and $p \to q \equiv \neg p \vee q(x)$

negation:

$$\neg(p \to q)$$

$$\neg(p \to q) \equiv p \wedge \neg q(x)$$

$\therefore$ if a user is active, and all newtwork links are not available.

Why $\forall x \exists y P(x, y) \neq \exists x \forall y P(x, y)$?

$\forall x \exists y (x + y = 0)$ is true

All x's have an additive inverse

$\exists x \forall y (x + y = 0)$ is false!

All y's are not the additive inverses of a single x

The norm/convention

Negations only occur and are written immediately before predicates, and not before quantifiers. Quanitifers: $\forall \, or \, \exists$

Negate at the boundaries first:

$$\begin{aligned} \neg(\forall x \exists y (x \cdot y = 1)) &\equiv \exists x \neg(\exists y (x \cdot y = 1)) \\ &\equiv \exists x \forall y \neg(x \cdot y = 1) \\ &\equiv \exists x \forall y (x \cdot y \neq 1) \end{aligned}$$