CSCE 222

Homework 2

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Contents

1 Section 1.6

Question 14

a)

p = ”In the class”

q = ”Owns a red convertible”

r = ”Got a speeding ticket”

$\forall x(q(x) \rightarrow r(x))$ premise

$p(x) \wedge q(x)$ Premise

$q(x)$ Simplification

$r(x)$ Modus ponens

Due to linda, there is at least one person who has gotten a speeding ticket in this class

b)

p = "Taken a course in discrete mathematics"

q = "Can take a course in algorithms"

$\forall x(p(x) \to q(x))$

$p(M) \land p(A) \land p(R) \land p(V) \land p(K)$ Premise

$q(M) \land q(A) \land q(R) \land q(V) \land q(K)$ Modus Ponens

Because all 5 took discrete math, this implies all 5 are eligible for algorithms

c)

p = "produced by John Sayles"

q = "Is wonderful"

$\forall x(p(x) \to q(x))$

$p(Coal)$ Premise

$q(Coal)$ Modus Ponens

Because the movie about coal miners is produced by John Sayles, the movie is wonderful; thus, there is a wonderful movie about coal miners.

d)

p = "Been to France"

q = "Visited the Lovre"

$\forall x(p(x) \to q(x))$

$p(x)$ Premise

$q(x)$ Modus Ponens

Question 24

The error is on step 3, because you cannot simplify $P(c) \vee Q(c)$ to $P(c)$

FRQ

p = "There is an undeclared variable"

q = "There is a syntax error in the first five lines"

r = "There is a missing semicolon"

s = "Variable name is misspelled"

$q \to (r \vee s)$

$p \vee q$

$\neg r$ Premise

$\neg s$ Premise

$\neg(r \vee s) \to \neg q = (\neg r \wedge \neg s) \to \neg q$ Modus Tollens

$\neg q$

Given there is no missing semicolon or misspelled variable name, there is not a syntax error in the first 5 lines.

2 Section 1.7

Question 18

$\forall m, n, k \in \mathbb{Z}, mn = 2k \iff$ mn is even. By definition.

$(m = 2k \wedge n = 2k+1) \vee (m = 2k+1 \wedge n = 2k) \to mn = (2(k(2k+1))) \vee (2(k(2k)))$

Through association, mn is even by definition.

$mn \in \mathbb{Z}$ under multiplication since $k \in \mathbb{Z}$.

Therefore, if m or n are even integers, mn is even.

Question 20

p = "n is an integer" q = "is even"

a) By contraposition

$p(n) \wedge q(3n+2) \to q(n)$ Premise

$\neg q(n) \to \neg p(n) \vee \neg q(3n+2)$ Contraposition

(3n + 2) = (3n + 2(1)) Identity

2(1) is even by definition. Even + Even = Even, Odd + Even = Odd

Since: 2k + 2k = 2(2k) and (2k + 1) + (2k) = 2(2k) + 1

By the same reduction, 3n + 2 = 2(n+1) + n = n

$\neg q(n) \equiv \neg q(3n+2)$ By definition of even

This makes the condition true, and since the contrapositive is true, the original is true.

b) By contradiction

$p(n) \wedge q(3n+2) \to q(n)$ Premise

Assume $\neg q(n)$ By Contradiction

3n+2 = 3(2k+1) + 2 = 6k + 5 = 2(3k + 2) + 1

This implies $\neg q(3n+2)$ which contradicts the premise

Question 34

if rational x was equivalent to rational x/2, it could be written as $x = \frac{p}{q}$ and $x/2 = \frac{p}{q/2}$, where q is just another integer.

Then, 3x-1 can also be written as $\frac{3p}{q} - 1 = \frac{3p-q}{q}$

x can take on any real number, so with different combinations of p and q,

x is rational ≡ x/2 is rational ≡ 3x-1 is rational

FRQ

Prove: $x \in \mathbb{Z}^+ \rightarrow (x = 2k \iff 7x + 4 = 2k | k \in \mathbb{Z}^+)$

7x+4 = 2(3x+2) + x

This implies 7x+4 is only a multiple of 2 if x is also a multiple of 2, where x is a positive integer.

FRQ

R = "The square root of any irrational number is irrational."

a)

$\neg R$ = "The square root of any irrational number is rational."

b)

prove: $\sqrt{irrational} = irrational$

Assume opposite: $\sqrt{irrational} = rational$

$\sqrt{irrational} = \frac{p}{q}$ Definition of a rational number

$irrational = (\frac{p}{q})^2$

$p, q \in \mathbb{Z}$, so $(\frac{p}{q})^2$ is likely to be rational, which is a contradiction.

By contradiction R is true, "The square root of any irrational number is irrational."

3 Section 2.1

Question 46

Use proofs?

Multiplication of sets is not commutative: If A = (a_1, a_2) and B = (b_1, b_2)

AxB is a set of a_1 and a_2 with all combinations of B, in that order. Therefore the order of multiplication matters, and the resulting set is not necessarily equivalent if switched. (AxB) $\times$(C x D) and A x (B x C) $\times$D are not the same.

Question 48

Prove $A \times B = A \times C \rightarrow B = C$

A, B, and C are nonempty sets | Premise

$A \times B = \{(a, b)\}|a \in A \wedge b \in B$ | Cartesian Product

$A \times C = \{(a, c)\}|a \in A \wedge c \in C$ | Cartesian Product

$\forall a, b, c((a, b) = (a, c) \rightarrow b = c) \rightarrow B = C$ | Definition of Set Equality

4 Section 2.2

Question 20

a)

p = "element in A" q = "element in B" r = "element in C"

$(A \cup B \cup C) = (A \cup B) \cup C$ | Association Law

$(p \vee q) \vee r$ | Written as propositions

$(p \vee q) \rightarrow (p \vee q) \vee r$ | Tautological

Therefore $(A \cup B) \equiv (A \cup B \cup C)$

b)

$A \cap B \cap C \equiv (A \cap B) \cap C$ | Association Law

$x \in (A \cap B \cap C)) \iff x \in A \wedge x \in B \wedge x \in C$ | Meaning of Intersection

$x \in A \wedge x \in B \wedge x \in C \rightarrow x \in A \wedge x \in B$ | Simplification

$\therefore A \cap B \cap C \subseteq A \cap B$

c)

$(A-B)-C \equiv (A-C)-B$ | Commutative Law

$(A-C)-B \equiv \{\forall x(x \in (A-C) \wedge (A-C)-B)\}$ | Equivalence of a set intersection

$\forall x(x \in (A-C) \wedge (A-C)-B) \rightarrow \forall x(x \in (A-C))$

All elements of this set have to be in A-C, thus it is a subset of A-C, therefore

$(A-B)-C \subseteq A-C$

d)

$\forall x(x \in (A-C) \rightarrow x \notin C)$ | Definition of set subtraction

$\forall x(x \in (C-B) \rightarrow x \in C)$ | Definition of set subtraction

$\forall x \neg((x \in (A-C)) \wedge (x \in (C-B))) \rightarrow (A-C) \cap (C-B) = \emptyset$ | Since by definition of interesection, for all elements there are no elements existing in both sets, the resulting set is empty by definition of an empty set.

e)

$(B-A) \cup (C-A) \equiv \{x \in B | x \notin A\} \cup \{x \in C | x \notin A\}$ |Definition of Set Difference

$\equiv \{x \in B \cup C | x \notin A\}$ | Definition of Set Union

$\equiv (B \cup C) \cap A^c$ | Definition of a Complement

$\equiv (B \cup C) - A$ | Definition of Set Difference

$\therefore (B-A) \cup (C-A) \equiv (B \cup C) - A$