

Week 3 Proofs

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Inference

Rules of Inference deduce new true statements and conclusions from premises.

one standard rule: show premise, show how premises connect, and show how premises connect to conclusion

Modus Ponens - method of approving

Modus Tollens - method of denying

Hypothetical syllogism - chain together two conditional statements

Disjunctive syllogism - conclude one alternative is true when you know at least one of two things must be true and one of them is false.

Critical Row - $p \rightarrow q$ is T and q is T

Definitions

- Prime Numbers

A prime integer cannot be factored by an integer that is not itself or one

if n is prime \iff

$$(n > 1) \wedge (\forall r, s \in \mathbb{Z}^+, (n = rs) \rightarrow (r = 1 \wedge s = n) \vee (s = 1 \wedge r = n))$$

Thus, a prime integer is greater than one, and is factorized by 2 variables; and for all instances those are 1 and itself.

- Composite Numbers

A composite number is factored by 2 integers that are not 1 and itself

if n is composite \iff

$(n > 1) \wedge (\forall r, s \in \mathbb{Z}^+, (n = rs) \rightarrow (1 < r < n) \wedge (1 < s < n))$ Thus, a composite number is factored by 2 integers that are both $\in (1, n)$

- Even Numbers

n is even $\iff \exists k \in \mathbb{Z} | (n = 2k)$

Thus, there exists an integer k that is half of n

- Odd Numbers

n is odd $\iff \exists k \in \mathbb{Z} | (n = 2k + 1)$

Thus, there exists a k integer such that it is -1 of an even integer