Week 3 Proofs

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Inference

Rules of Inference deduce new true statements and conclusions from premises.

one standard rule: show premise, show how premises connect, and show how premises connect to conclusion

Modus Ponens - method of approving

Modus Tollens - method of denying

Hypothetical sylloogism - chain together two conditional statements

Disjunctive syllogism - conclude one alternative is true when you know at least one of two things must be true and one of them is false.

Critical Row - $p \to q \; is \; T \; and \; q \; is \; T$

Definitions

- Prime Numbers

  A prime integer cannot be factored by an integer that is not itself or one

$$\text{if n is prime} \iff$$
$$(n > 1) \land (\forall r, s \in \mathbb{Z}^+, (n = rs) \to (r = 1 \land s = n) \lor (s = 1 \land r = n))$$

Thus, a prime integer is greater than one, and is factorized by 2 variables; and for all instances those are 1 and itself.

- Composite Numbers

  A composite number is factored by 2 integers that are not 1 and itself

  $$\text{if n is composite} \iff$$
  $$(n > 1) \wedge (\forall r, s \in \mathbb{Z}^+, (n = rs) \rightarrow (1 < r < n) \wedge (1 < s < n))$$ Thus, a composite number is facotred by 2 integers that are both $\in$ (1,n)

- Even Numbers

  $$\text{n is even} \iff \exists k \in \mathbb{Z} | (n = 2k)$$
  Thus, there exists an integer k that is half of n

- Odd Numbers

  $$\text{n is odd} \iff \exists k \in \mathbb{Z} | (n = 2k + 1)$$
  Thus, there exists a k integer such that it is -1 of an even integer

  This will be used in proofs later on next week. Remember for exam and homework. Homework needs to be a pdf uploaded.