Week 2 Predicate Logic

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Negate "If a user is active, at least one network link will be available."

p : "A user is Active"

q : "A network link will be available"

Starting with:

$$p \to q$$

and $p \to q \equiv \neg p \vee q(x)$

negation:

$$\neg(p \to q)$$

$$\neg(p \to q) \equiv p \wedge \neg q(x)$$

$\therefore$ if a user is active, and all newtwork links are not available.

Why $\forall x \exists y P(x, y) \neq \exists x \forall y P(x, y)$?

$\forall x \exists y (x + y = 0)$ is true

All x's have an additive inverse

$\exists x \forall y (x + y = 0)$ is false!

All y's are not the additive inverses of a single x

The norm/convention

- Negations only occur and are written immediately before predicates, and not before quantifiers. Quanitifers: $\forall$ *or* $\exists$.

- They have the highest priority over anything else (they affect the proposition that comes right after).

Negate at the boundaries first:

$$\neg(\forall x \exists y (x \cdot y = 1)) \equiv \exists x \neg(\exists y (x \cdot y = 1))$$
$$\equiv \exists x \forall y \neg(x \cdot y = 1)$$
$$\equiv \exists x \forall y (x \cdot y \neq 1)$$

Bound/Free Variables

"A bound variable is restricted by a quantifier ($\forall$ or $\exists$) within an expression, making its value determined by the context (e.g., $\forall x P(x)$). A free variable is not constrained by a quantifier, allowing it to take any value within its domain. Bound variables act as placeholders, while free variables often depend on external definitions. " Fiveable 2025