Design and Analysis of Algorithms

L30: Knapsack problem
Dynamic Programming

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Resources

- Text book 1: Levitin
  - Sec **8 . 4**
- Text book 2: Horowitz
  - Sec 5 . 7

- R1: Introduction to Algorithms
  - Cormen et al.

Knapsack problem

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  - Note: All the weights w_i's and knapsack capacity W are integers, but values can be real numbers.
- Goal: solve the knapsack problem using dynamic programming.

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- Let $V[i,j]$ be the optimal solution to this instance

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  - i.e. the value of most valuable subsets of first i items that fit knapsack of capacity j.

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- Approach: divide first i items into two categories:

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- Approach: divide first i items into two categories:
  - Those that include i^{th} item, and

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  - Knapsack capacity j, $1 \leq j \leq w$
- Let $V[i, j]$ be the optimal solution to this instance
  - i.e. the value of most valuable subsets of first i items that fit knapsack of capacity j.
- Approach: divide first i items into two categories:
  - Those that include i^{th} item, and
  - Those that don't include i^{th} item

DP Approach: Knapsack

	0		j-w_i		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0						
i	0						
	0						
n	0						

DP Approach: Knapsack

	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0						
i	0						
	0						
n	0						

Table for solving knapsack problem using dynamic programming

DP Approach: Knapsack

	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0						
i	0				**V[i,j]**		
	0						
n	0						

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	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0						
i	0				**V[i,j]**		
	0						
n	0						

Table for solving knapsack problem using dynamic programming

- Category 1: subsets that do not include i^{th} item.

DP Approach: Knapsack

	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
$i-1$	0						
i	0				**V[i,j]**		
	0						
n	0						

Table for solving knapsack problem using dynamic programming

- Category 1: subsets that do not include i^{th} item.
  - Value of optimal subset is `V[i-1,j]`

DP Approach: Knapsack

	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0				**V[i-1,j]**		
i	0				**V[i,j]**		
	0						
n	0						

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	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0				**V[i-1,j]**		
i	0				**V[i,j]**		
	0						
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  - Value of optimal subset is V[i-1,j]
- Category 2: subsets that do include i^{th} item.

DP Approach: Knapsack

	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
i-1	0		**V[i-1,j-w_i]**		**V[i-1,j]**		
i	0				**V[i,j]**		
	0						
n	0						

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	0		j-w_i		j		W
0	0	0	0	0	0	0	0
	0						
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i	0				**V[i,j]**		
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  - Thus $j>w_i$ i.e. $j-w_i \geq 0$.

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	0		$j-w_i$		j		W
0	0	0	0	0	0	0	0
	0						
$i-1$	0		**V[i-1,j-w_i]**		**V[i-1,j]**		
i	0				**V[i,j]**		
	0						
n	0						

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- Category 1: subsets that do not include i^{th} item.
  - Value of optimal subset is $V[i-1,j]$
- Category 2: subsets that do include i^{th} item.
  - Thus $j>w_i$ i.e. $j-w_i \geq 0$.
  - Value of optimal subset is $v_i+V[i-1,j-w_i]$

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- Thus,

$$V[i,j] = \begin{cases} max\{V[i-1,j], v_i + V[i-1,\ j-w_i]\} & if\ j-w_i \geq 0 \\ V[i-1,\ j] & if\ j-w_i < 0 \end{cases} \quad (1)$$

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- Thus,

$$V[i, j] = \begin{cases} max\{V[i-1, j], v_i + V[i-1,\ j - w_i]\} & if\ j - w_i \geq 0 \\ V[i-1,\ j] & if\ j - w_i < 0 \end{cases} \qquad (1)$$

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  $V[i, 0] = 0$ for $i \geq 0$

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  $V[0, j] = 0$ for $j \geq 0$

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Example: Knapsack

$$V[i,j] = \begin{cases} max\{V[i-1,j], v_i + V[i-1,\ j-w_i]\} & if\ j-w_i \geq 0 \\ V[i-1,\ j] & if\ j-w_i < 0 \end{cases} \tag{1}$$

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  - Weights as
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  - (i.e. max weight it can hold is 5),
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    $w_1=2, w_2=1, w_3=3, w_4=2$
  - Values as
    $v_1=\$12, v_2=\$10, v_3=\$20, v_4=\$15$

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  - Need to compute V[4,5]

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  - Values as
    $v_1=\$12, v_2=\$10, v_3=\$20, v_4=\$15$
  - Need to compute V[4,5]
    - Max value with 4 items with knapsack capacity 5

Example Knapsack

Capacity→ wts, values↓		0	1	2	3	4	5
	0						
$w_1=2$ $v_1=12$	1						
$w_2=1$ $v_2=10$	2						
$w_3=3$ $v_3=20$	3						
$w_4=2$ $v_4=15$	4						

Example Knapsack

Capacity→ wts, values↓		0	1	2	3	4	5
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$V[0,j]=0$ **for** $0 \leq j \leq 5$

Example Knapsack

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
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V[0,j]=0 **for** $0 \le j \le 5$

V[i,0]=0 **for** $0 \le i \le 4$

Example Knapsack

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**					
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[0,j]=0$ **for** $0 \le j \le 5$
$V[i,0]=0$ **for** $0 \le i \le 4$

Example Knapsack

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
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Capacity→ wts, values↓		0	1	2	3	4	5
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$w_1=2$ $v_1=12$	1	**0**					
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Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**					
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**					
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

$=0$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**				
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

$=0$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**				
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$
$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**				
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

$=0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$

$=\max\{0,12+V[0,0]\} = 12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**			
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$
$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$
$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**			
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

V[1,1]=V[1−1,1] **since** $j=1<w_1=2$

=0

V[1,2]=max{V[0,2],12+V[0,2−2]};$j=2\geq w_1=2$

=max{0,12+V[0,0]} = 12

V[1,3]=max{V[0,3],12+V[0,3−2]};$j=3\geq w_1=2$

=max{0,12+V[0,1]}=12

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**		
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$

$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$
$=\max\{0,12+V[0,1]\}=12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**		
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$
$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$
$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$
$=\max\{0,12+V[0,1]\}=12$
$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**		
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$

$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$
$=\max\{0,12+V[0,1]\}=12$

$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$
$=\max\{0,12+V[0,2]\}=12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$

$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$
$=\max\{0,12+V[0,1]\}=12$

$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$
$=\max\{0,12+V[0,2]\}=12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

$\quad =0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$

$\quad =\max\{0,12+V[0,0]\} = 12$

$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$

$\quad =\max\{0,12+V[0,1]\}=12$

$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$

$\quad =\max\{0,12+V[0,2]\}=12$

$V[1,5]=\max\{V[0,5],12+V[0,5-2]\};j=5\geq w_1=2$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$
$=0$
$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$
$=\max\{0,12+V[0,0]\} = 12$
$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$
$=\max\{0,12+V[0,1]\}=12$
$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$
$=\max\{0,12+V[0,2]\}=12$
$V[1,5]=\max\{V[0,5],12+V[0,5-2]\};j=5\geq w_1=2$
$=\max\{0, 12+V[0,3]\}=12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[1,1]=V[1-1,1]$ **since** $j=1<w_1=2$

$=0$

$V[1,2]=\max\{V[0,2],12+V[0,2-2]\};j=2\geq w_1=2$

$=\max\{0,12+V[0,0]\} = 12$

$V[1,3]=\max\{V[0,3],12+V[0,3-2]\};j=3\geq w_1=2$

$=\max\{0,12+V[0,1]\}=12$

$V[1,4]=\max\{V[0,4],12+V[0,4-2]\};j=4\geq w_1=2$

$=\max\{0,12+V[0,2]\}=12$

$V[1,5]=\max\{V[0,5],12+V[0,5-2]\};j=5\geq w_1=2$

$=\max\{0, 12+V[0,3]\}=12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

V[2,1]=max{V[1,1],10+V[1,1-1]}; $j=1\geq w_2=1$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**					
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$

$=\max\{0,\ 10+V[1,0]\} = 10$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**				
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

V[2,1]=max{V[1,1],10+V[1,1-1]}; $j=1\geq w_2=1$

=max{0, 10+V[1,0]} = 10

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**				
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$

$=\max\{0, 10+V[1,0]\} = 10$

$V[2,2]=\max\{V[1,2], 10+V[1,2-1]\}$; $j=2\geq w_2=1$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
w_1=2 v_1=12	1	**0**	**0**	**12**	**12**	**12**	**12**
w_2=1 v_2=10	2	**0**	**10**				
w_3=3 v_3=20	3	**0**					
w_4=2 v_4=15	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1 \geq w_2=1$
$=\max\{0, 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2], 10+V[1,2-1]\}$; $j=2 \geq w_2=1$
$=\max\{12, 10+0\} = 12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**			
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$
$=\max\{0, 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2], 10+V[1,2-1]\}$; $j=2\geq w_2=1$
$=\max\{12, 10+0\} = 12$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
w_1=2 v_1=12	1	**0**	**0**	**12**	**12**	**12**	**12**
w_2=1 v_2=10	2	**0**	**10**	**12**			
w_3=3 v_3=20	3	**0**					
w_4=2 v_4=15	4	**0**					

V[2,1]=max{V[1,1],10+V[1,1-1]}; $j=1 \geq w_2=1$
=max{0, 10+V[1,0]} = 10
V[2,2]=max{V[1,2], 10+V[1,2-1]}; $j=2 \geq w_2=1$
=max{12, 10+0} = 12
V[2,3]=max{V[1,3], 10+V[1,3-1]}; $j=3 \geq w_2=1$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**			
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$

$=\max\{0,\ 10+V[1,0]\} = 10$

$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\}$; $j=2\geq w_2=1$

$=\max\{12,\ 10+0\} = 12$

$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\}$; $j=3\geq w_2=1$

$=\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**		
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$
$=\max\{0,\ 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\}$; $j=2\geq w_2=1$
$=\max\{12,\ 10+0\} = 12$
$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\}$; $j=3\geq w_2=1$
$=\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**		
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$

$=\max\{0,\ 10+V[1,0]\} = 10$

$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\}$; $j=2\geq w_2=1$

$=\max\{12,\ 10+0\} = 12$

$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\}$; $j=3\geq w_2=1$

$=\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$

$V[2,4]=\max\{V[1,4],\ 10+V[1,4-1]\}$; $j=4\geq w_2=1$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**		
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1\geq w_2=1$
$\quad =\max\{0,\ 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\}$; $j=2\geq w_2=1$
$\quad =\max\{12,\ 10+0\} = 12$
$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\}$; $j=3\geq w_2=1$
$\quad =\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$
$V[2,4]=\max\{V[1,4],\ 10+V[1,4-1]\}$; $j=4\geq w_2=1$
$\quad =\max\{12,\ 10+V[1,3]\}=\max\{12,22\}=22$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\}$; $j=1 \geq w_2=1$
$\quad =\max\{0,\ 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\}$; $j=2 \geq w_2=1$
$\quad =\max\{12,\ 10+0\} = 12$
$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\}$; $j=3 \geq w_2=1$
$\quad =\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$
$V[2,4]=\max\{V[1,4],\ 10+V[1,4-1]\}$; $j=4 \geq w_2=1$
$\quad =\max\{12,\ 10+V[1,3]\}=\max\{12,22\}=22$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[2,1]=\max\{V[1,1],10+V[1,1-1]\};\ j=1 \geq w_2=1$
$\quad =\max\{0,\ 10+V[1,0]\} = 10$
$V[2,2]=\max\{V[1,2],\ 10+V[1,2-1]\};\ j=2 \geq w_2=1$
$\quad =\max\{12,\ 10+0\} = 12$
$V[2,3]=\max\{V[1,3],\ 10+V[1,3-1]\};\ j=3 \geq w_2=1$
$\quad =\max\{12,\ 10+V[1,2]\}=\max\{12,22\}=22$
$V[2,4]=\max\{V[1,4],\ 10+V[1,4-1]\};\ j=4 \geq w_2=1$
$\quad =\max\{12,\ 10+V[1,3]\}=\max\{12,22\}=22$
$V[2,5]=\max\{V[1,5],\ 10+V[1,5-1]\};\ j=5 \geq w_2=1$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

V[2,1]=max{V[1,1],10+V[1,1-1]}; $j=1\geq w_2=1$
=max{0, 10+V[1,0]} = 10
V[2,2]=max{V[1,2], 10+V[1,2-1]}; $j=2\geq w_2=1$
=max{12, 10+0} = 12
V[2,3]=max{V[1,3], 10+V[1,3-1]}; $j=3\geq w_2=1$
=max{12, 10+V[1,2]}=max{12,22}=22
V[2,4]=max{V[1,4], 10+V[1,4-1]}; $j=4\geq w_2=1$
=max{12, 10+V[1,3]}=max{12,22}=22
V[2,5]=max{V[1,5], 10+V[1,5-1]}; $j=5\geq w_2=1$
=max{12, 10+V[1,4]}=max{12,22}=22

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
w_1=2 v_1=12	1	**0**	**0**	**12**	**12**	**12**	**12**
w_2=1 v_2=10	2	**0**	**10**	**12**	**22**	**22**	**22**
w_3=3 v_3=20	3	**0**					
w_4=2 v_4=15	4	**0**					

V[2,1]=max{V[1,1],10+V[1,1-1]}; $j=1 \geq w_2=1$
=max{0, 10+V[1,0]} = 10
V[2,2]=max{V[1,2], 10+V[1,2-1]}; $j=2 \geq w_2=1$
=max{12, 10+0} = 12
V[2,3]=max{V[1,3], 10+V[1,3-1]}; $j=3 \geq w_2=1$
=max{12, 10+V[1,2]}=max{12,22}=22
V[2,4]=max{V[1,4], 10+V[1,4-1]}; $j=4 \geq w_2=1$
=max{12, 10+V[1,3]}=max{12,22}=22
V[2,5]=max{V[1,5], 10+V[1,5-1]}; $j=5 \geq w_2=1$
=max{12, 10+V[1,4]}=max{12,22}=22

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**					
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**				
$w_4=2$ $v_4=15$	4	**0**					

V[3,1]=V[2,1] = 10; $(j=1<w_3=3)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**				
$w_4=2$ $v_4=15$	4	**0**					

V[3,1]=V[2,1] = 10; ($j=1<w_3=3$)
V[3,2]=V[2,2] = 12; ($j=2<w_3=3$)

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**			
$w_4=2$ $v_4=15$	4	**0**					

V[3,1]=V[2,1] = 10; ($j=1<w_3=3$)
V[3,2]=V[2,2] = 12; ($j=2<w_3=3$)

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**			
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$

$V[3,3]=\max\{V[2,3],\ 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**			
$w_4=2$ $v_4=15$	4	**0**					

V[3,1]=V[2,1] = 10; $(j=1<w_3=3)$
V[3,2]=V[2,2] = 12; $(j=2<w_3=3)$
V[3,3]=max{V[2,3], 20+V[2,3-3]}; $(j=3\geq w_3=3)$
=max{22, 20+0} = 22

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
w_1=2 v_1=12	1	**0**	**0**	**12**	**12**	**12**	**12**
w_2=1 v_2=10	2	**0**	**10**	**12**	**22**	**22**	**22**
w_3=3 v_3=20	3	**0**	**10**	**12**	**22**		
w_4=2 v_4=15	4	**0**					

V[3,1]=V[2,1] = 10; ($j=1<w_3=3$)
V[3,2]=V[2,2] = 12; ($j=2<w_3=3$)
V[3,3]=max{V[2,3], 20+V[2,3-3]}; ($j=3\geq w_3=3$)
=max{22, 20+0} = 22

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**		
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$

$V[3,3]=\max\{V[2,3], 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$

$=\max\{22, 20+0\} = 22$

$V[3,4]=\max\{V[2,4], 20+V[2,4-3]\};$ $(j=4\geq w_3=3)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**		
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$

$V[3,3]=\max\{V[2,3],\ 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$

$=\max\{22,\ 20+0\} = 22$

$V[3,4]=\max\{V[2,4],\ 20+V[2,4-3]\};$ $(j=4\geq w_3=3)$

$=\max\{22,\ 20+V[2,1]\}=\max\{22,30\}=30$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	
$w_4=2$ $v_4=15$	4	**0**					

V[3,1]=V[2,1] = 10; ($j=1<w_3=3$)
V[3,2]=V[2,2] = 12; ($j=2<w_3=3$)
V[3,3]=max{V[2,3], 20+V[2,3-3]}; ($j=3\geq w_3=3$)
=max{22, 20+0} = 22
V[3,4]=max{V[2,4], 20+V[2,4-3]}; ($j=4\geq w_3=3$)
=max{22, 20+V[2,1]}=max{22,30}=30

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$
$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$
$V[3,3]=\max\{V[2,3], 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$
$=\max\{22, 20+0\} = 22$
$V[3,4]=\max\{V[2,4], 20+V[2,4-3]\};$ $(j=4\geq w_3=3)$
$=\max\{22, 20+V[2,1]\}=\max\{22,30\}=30$
$V[3,5]=\max\{V[2,5], 20+V[2,5-3]\};$ $(j=5\geq w_3=3)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$

$V[3,3]=\max\{V[2,3],\ 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$

$=\max\{22,\ 20+0\} = 22$

$V[3,4]=\max\{V[2,4],\ 20+V[2,4-3]\};$ $(j=4\geq w_3=3)$

$=\max\{22,\ 20+V[2,1]\}=\max\{22,30\}=30$

$V[3,5]=\max\{V[2,5],\ 20+V[2,5-3]\};$ $(j=5\geq w_3=3)$

$=\max\{12,\ 20+V[2,2]\}=\max\{12,20+12\}=32$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**					

$V[3,1]=V[2,1] = 10;$ $(j=1<w_3=3)$

$V[3,2]=V[2,2] = 12;$ $(j=2<w_3=3)$

$V[3,3]=\max\{V[2,3], 20+V[2,3-3]\};$ $(j=3\geq w_3=3)$

$=\max\{22, 20+0\} = 22$

$V[3,4]=\max\{V[2,4], 20+V[2,4-3]\};$ $(j=4\geq w_3=3)$

$=\max\{22, 20+V[2,1]\}=\max\{22,30\}=30$

$V[3,5]=\max\{V[2,5], 20+V[2,5-3]\};$ $(j=5\geq w_3=3)$

$=\max\{12, 20+V[2,2]\}=\max\{12,20+12\}=32$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**					

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**					

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**				

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**				

$V[4,1]=V[3,1] = 10;$ $(j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\};$ $(j=2\geq w_4=2)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
w_1=2 v_1=12	1	**0**	**0**	**12**	**12**	**12**	**12**
w_2=1 v_2=10	2	**0**	**10**	**12**	**22**	**22**	**22**
w_3=3 v_3=20	3	**0**	**10**	**12**	**22**	**30**	**32**
w_4=2 v_4=15	4	**0**	**10**				

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)

=max{12,15+V[3,0]}=max{12,15+0}=15

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2\ \ v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1\ \ v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3\ \ v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2\ \ v_4=15$	4	**0**	**10**	**15**			

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)

=max{12,15+V[3,0]}=max{12,15+0}=15

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**			

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)

=max{12,15+V[3,0]}=max{12,15+0}=15

V[4,3]=max{V[3,3], 15+V[3,3-2]}; ($j=3\geq w_4=2$)

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**			

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)

=max{12,15+V[3,0]}=max{12,15+0}=15

V[4,3]=max{V[3,3], 15+V[3,3-2]}; ($j=3\geq w_4=2$)

=max{22,15+V[3,1]}=max{22,15+10}=25

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**		

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)

V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)

=max{12,15+V[3,0]}=max{12,15+0}=15

V[4,3]=max{V[3,3], 15+V[3,3-2]}; ($j=3\geq w_4=2$)

=max{22,15+V[3,1]}=max{22,15+10}=25

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**		

$V[4,1]=V[3,1] = 10;$ $(j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\};$ $(j=2\geq w_4=2)$

$=\max\{12,15+V[3,0]\}=\max\{12,15+0\}=15$

$V[4,3]=\max\{V[3,3], 15+V[3,3-2]\};$ $(j=3\geq w_4=2)$

$=\max\{22,15+V[3,1]\}=\max\{22,15+10\}=25$

$V[4,4]=\max\{V[3,4], 15+V[3,4-2]\};$ $(j=4\geq w_4=2)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**		

$V[4,1]=V[3,1] = 10;\ (j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\};\ (j=2\geq w_4=2)$

$=\max\{12,15+V[3,0]\}=\max\{12,15+0\}=15$

$V[4,3]=\max\{V[3,3],\ 15+V[3,3-2]\};\ (j=3\geq w_4=2)$

$=\max\{22,15+V[3,1]\}=\max\{22,15+10\}=25$

$V[4,4]=\max\{V[3,4],\ 15+V[3,4-2]\};\ (j=4\geq w_4=2)$

$=\max\{30,\ 15+V[3,2]\}=\max\{30,15+12\}=30$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	

$V[4,1]=V[3,1] = 10;$ $(j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\};$ $(j=2\geq w_4=2)$

$=\max\{12,15+V[3,0]\}=\max\{12,15+0\}=15$

$V[4,3]=\max\{V[3,3], 15+V[3,3-2]\};$ $(j=3\geq w_4=2)$

$=\max\{22,15+V[3,1]\}=\max\{22,15+10\}=25$

$V[4,4]=\max\{V[3,4], 15+V[3,4-2]\};$ $(j=4\geq w_4=2)$

$=\max\{30, 15+V[3,2]\}=\max\{30,15+12\}=30$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	

$V[4,1]=V[3,1] = 10;$ $(j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\};$ $(j=2\geq w_4=2)$

$\quad=\max\{12,15+V[3,0]\}=\max\{12,15+0\}=15$

$V[4,3]=\max\{V[3,3], 15+V[3,3-2]\};$ $(j=3\geq w_4=2)$

$\quad=\max\{22,15+V[3,1]\}=\max\{22,15+10\}=25$

$V[4,4]=\max\{V[3,4], 15+V[3,4-2]\};$ $(j=4\geq w_4=2)$

$\quad=\max\{30, 15+V[3,2]\}=\max\{30,15+12\}=30$

$V[4,5]=\max\{V[3,5], 15+V[3,5-2]\};$ $(j=5\geq w_4=2)$

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	

V[4,1]=V[3,1] = 10; ($j=1<w_4=2$)
V[4,2]=max{V[3,2],15+V[3,2-2]}; ($j=2\geq w_4=2$)
=max{12,15+V[3,0]}=max{12,15+0}=15
V[4,3]=max{V[3,3], 15+V[3,3-2]}; ($j=3\geq w_4=2$)
=max{22,15+V[3,1]}=max{22,15+10}=25
V[4,4]=max{V[3,4], 15+V[3,4-2]}; ($j=4\geq w_4=2$)
=max{30, 15+V[3,2]}=max{30,15+12}=30
V[4,5]=max{V[3,5], 15+V[3,5-2]}; ($j=5\geq w_4=2$)
=max{32, 15+V[3,3]}=max{32,15+22}=37

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

$V[4,1]=V[3,1] = 10$; $(j=1<w_4=2)$

$V[4,2]=\max\{V[3,2],15+V[3,2-2]\}$; $(j=2\geq w_4=2)$

$=\max\{12,15+V[3,0]\}=\max\{12,15+0\}=15$

$V[4,3]=\max\{V[3,3], 15+V[3,3-2]\}$; $(j=3\geq w_4=2)$

$=\max\{22,15+V[3,1]\}=\max\{22,15+10\}=25$

$V[4,4]=\max\{V[3,4], 15+V[3,4-2]\}$; $(j=4\geq w_4=2)$

$=\max\{30, 15+V[3,2]\}=\max\{30,15+12\}=30$

$V[4,5]=\max\{V[3,5], 15+V[3,5-2]\}$; $(j=5\geq w_4=2)$

$=\max\{32, 15+V[3,3]\}=\max\{32,15+22\}=37$

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value `V[4,5]` to prev. rows.

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value `V[4,5]` to prev. rows.
  - Thus, optimal subsets are

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

$V[4,5]=37\,(\neq V[3,5])$ implies $\underline{w_4=2}$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 ($\neq$V[3,5]) implies $\underline{w_4=2}$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value `V[4,5]` to prev. rows.
  - Thus, optimal subsets are

`V[4,5]=37(≠V[3,5])` implies $\underline{w_4=2}$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 (≠V[3,5]) implies $w_4=2$ is included

V[3,3]=22 (=V[2,3]) implies $w_3=3$ is not included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$\mathbf{w_3=3}$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value `V[4,5]` to prev. rows.
  - Thus, optimal subsets are

`V[4,5]=37(≠V[3,5])` implies $\underline{w_4=2}$ is included

`V[3,3]=22(=V[2,3])` implies $w_3=3$ is not included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$\mathbf{w_3=3}$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value `V[4,5]` to prev. rows.
  - Thus, optimal subsets are

`V[4,5]=37(≠V[3,5])` implies $\underline{w_4=2}$ is included

`V[3,3]=22(=V[2,3])` implies $w_3=3$ is not included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$\mathbf{w_3=3}$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37(≠V[3,5]) implies $\underline{w_4=2}$ is included

V[3,3]=22(=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22(≠V[1,3] implies $\underline{w_2=1}$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$\mathbf{w_2=1}$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$\mathbf{w_3=3}$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37(≠V[3,5]) implies $\underline{w_4=2}$ is included

V[3,3]=22(=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22(≠V[1,3] implies $\underline{w_2=1}$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$\mathbf{w_2=1}$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$\mathbf{w_3=3}$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$\mathbf{w_4=2}$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 (≠V[3,5]) implies $w_4=2$ is included

V[3,3]=22 (=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22 (≠V[1,3] implies $w_2=1$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 (≠V[3,5]) implies $w_4=2$ is included

V[3,3]=22 (=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22 (≠V[1,3] implies $w_2=1$ is included

V[1,2]=12 (≠V[0,2] implies $w_1=2$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	**0**	**0**	**0**	**0**	**0**	**0**
$w_1=2$ $v_1=12$	1	**0**	**0**	**12**	**12**	**12**	**12**
$w_2=1$ $v_2=10$	2	**0**	**10**	**12**	**22**	**22**	**22**
$w_3=3$ $v_3=20$	3	**0**	**10**	**12**	**22**	**30**	**32**
$w_4=2$ $v_4=15$	4	**0**	**10**	**15**	**25**	**30**	**37**

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 ($\neq$V[3,5]) implies $\underline{w_4=2}$ is included

V[3,3]=22 (=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22 ($\neq$V[1,3] implies $\underline{w_2=1}$ is included

V[1,2]=12 ($\neq$V[0,2] implies $w_1=2$ is included

Example Knapsack: Optimal Subset

Capacity→ wts, values↓		0	1	2	3	4	5
	0	0	0	0	0	0	0
$w_1=2$ $v_1=12$	1	0	0	12	12	12	12
$w_2=1$ $v_2=10$	2	0	10	12	22	22	22
$w_3=3$ $v_3=20$	3	0	10	12	22	30	32
$w_4=2$ $v_4=15$	4	0	10	15	25	30	37

- Optimal subset
  - Backtrack from maximal value V[4,5] to prev. rows.
  - Thus, optimal subsets are

V[4,5]=37 (≠V[3,5]) implies $\underline{w_4=2}$ is included

V[3,3]=22 (=V[2,3]) implies $w_3=3$ is not included

V[2,3]=22 (≠V[1,3] implies $\underline{w_2=1}$ is included

V[1,2]=12 (≠V[0,2] implies $w_1=2$ is included

Algorithm: Knapsack using DP

```
Algo DPKnapsack(w[1..n], v[1..n], W)
   int V[0..n,0..W],  P[1..n,1..W];
   for j=0 to W do
       V[0,j]=0
   for i=0 to n do
      V[i,0]= 0
   for i=1 to n do
      for j= 1 to W do
         if w[i]≤j and (v[i]+V[i-1,j-w[i]]) > V[i-1,j] then
            V[i,j]= v[i] + V[i-1,j-w[i]];
        else
            V[i,j]= V[i-1,j]
   return V[n,W] (and the optimal subset by backtracing)
```

Efficiency of Knapsack

Efficiency of Knapsack

- Time Efficiency: $\Theta(nW)$

Efficiency of Knapsack

- Time Efficiency: $\Theta(nW)$
- Space efficiency: $\Theta(nW)$

Summary

Summary

- Knapsack algorithm using dynamic programming

Summary

- Knapsack algorithm using dynamic programming
- Efficiency

Summary

- Knapsack algorithm using dynamic programming
- Efficiency
- Optimal subsets using backtracking