1.

• I = \x.x

• S = \x.\y.\z.x z(y z)

• l = S I

• 求 l m n

原式=\x.\y.\z.x z(y z) \x.x m n

=(\x.x) n(m n)

=n(m n)

2.

•ZERO = \f.\x.x

•SUCC = \n.\f.\x.f (n f x)

•求 SUCC (SUCC ZERO)

原式= \n.\f.\x.f (n f x) ( ( \n.\f.\x.f (n f x)) \f.\x.x)

=\n.\f.\x.f (n f x) (\f.\x.f( (\f.\x.x) f x) )

=\n.\f.\x.f (n f x)\f.\x f x

=\f.\x.f((\f.\x f x) f x)

\f.\x.f (f x)

3.

•POW = \b.\e.e b

•求POW TWO THREE

原式=(\b.\e.e b) TWO THREE

= (THREE)TWO

=(\f.\x.f(f(f x)))TWO

=\x. TWO(TWO(TWO x))

=\x. TWO(TWO((\a.\b.a(a b)) x))

=\x. TWO(TWO (\b.x(x b)))

=\x. TWO((\a.\b.a(ab)) (\b.x(x b)))

=\x. TWO(\b.x(x(x(x b))))

=\x. (\a.\b.a(ab)) (\b.x(x(x(x b))))

=\x.\b.x(x(x(x(x(x(x(x b)))))))

=\f.\x.f(f(f(f(f(f(f(f x)))))))

4.

• TRUE = \x.\y.x

• FALSE = \x.\y.y

• AND = \p.\q.p q p

• OR = \p.\q.p p q

• NOT = \p.\a.\b.p b a

• IF = \p.\a.\b.p a b

• 求NOT (NOT TRUE)

• 求IF (OR FALSE FALSE) a b

NOT (NOT TRUE)

=NOT((\p.\a.\b.p b a)\x.\y.x)

=NOT(\a.\b.( \x.\y.x) b a)

=NOT(\a.\b.b)

=( \p.\a.\b.p b a)(\x.\y.y)

=\a.\b.( \x.\y.y) b a

=\a.\b.a

=\x.\y.x

IF (OR FALSE FALSE) a b

= \p.\a.\b.p a b (OR FALSE FALSE) a b

= (OR FALSE FALSE) a b

= (\p.\q.p p q FALSE FALSE) a b

=( FALSE FALSE FALSE)a b

=(\x.\y.y FALSE FALSE)a b

=( FALSE)a b

=(\x.\y.y)a b

=b

5.

• LEQ = \m.\n.ISZERO (SUB m n)

• 求⼤大于等于GEQ

GEQ = OR (NOT LEQ) EQ

= (\p.\q.p p q) (( \p.\a.\b.p b a) LEQ) EQ

= (\m.\n.ISZERO (SUB n m)) ( \m.\n.ISZERO (SUB n m)) EQ

= ISZERO (SUB (\a.\b.ISZERO (SUB b a)) EQ)

= ISZERO (SUB (\b.ISZERO (SUB (\m.\n. AND (LEQ m n) (LEQ n m)) a)))

= ISZERO (SUB (\b.ISZERO (SUB (\m.\n. AND (\m.\n.ISZERO (SUB m n)( \n.\m.ISZERO (SUB n m)))))))

6.Lambda> FACT1 = \f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n)))

• Lambda> FACT = FACT1 FACT1

• 求 FACT THREE

FACT THREE = FACT1 FACT1 THREE

=(\f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n)))) FACT1 THREE

= IF (ISZERO THREE) ONE (MULT THREE (FACT1 FACT1 (PRED THREE)))

=IF (FALSE) ONE (MULT THREE (FACT1 FACT1 (TWO)))

=IF FALSE ONE (MULT THREE ((\f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n))))FACT1 TWO))

= IF FALSE ONE (MULT THREE (IF (ISZERO TWO) ONE (MULT TWO(FACT1 FACT1 (PRED TWO)))))

=IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO(FACT1 FACT1 (ONE)))))

=IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((\f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n))))FACT1 ONE))))

=IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF (ISZERO ONE) ONE (MULT ONE (FACT1 FACT1 (PRED ONE))))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (MULT ONE (FACT1 FACT1 (ZERO))))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (MULT ONE ((\f.\n.IF (ISZERO n) ONE (MULT n (f f (PRED n))))FACT1 (ZERO))))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (MULT ONE ((IF (ISZERO ZERO) ONE (MULT ZERO (FACT1 FACT1(PRED ZERO)))))))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (MULT ONE ((IF TURE ONE (MULT ZERO (FACT1 FACT1(ZERO)))))))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (MULT ONE ONE))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO((IF FALSE ONE (\f.\x.f x))))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE (MULT TWO (\f.\x.f x))))

= IF FALSE ONE(MULT THREE (IF FALSE ONE(\f.\x.f(f x))))

= IF FALSE ONE(MULT THREE (\f.\x.f(f x)))

= IF FALSE ONE(\f.\x.f(f(f(f(f(f x))))))

=( \p.\a.\b.p a b) FALSE ONE(\f.\x.f(f(f(f(f(f x))))))

=(\x.\y.y) ONE(\f.\x.f(f(f(f(f(f x))))))

=\f.\x.f(f(f(f(f(f x)))))

7.Lambda> ADD = W (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))

• 求 ADD TWO FOUR

ADD TWO FOUR = ( W (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)))) TWO FOUR

= (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) TWO FOUR

= IF (ISZERO FOUR) TWO ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (SUCC TWO) (PRED FOUR))

= IF FALSE TWO ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) THREE THREE)

= IF FALSE TWO ((IF (ISZERO THREE) THREE ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (SUCC THREE) (PRED THREE))))

= IF FALSE TWO ((IF FALSE THREE ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m))) FOUR TWO))))

= IF FALSE TWO (IF FALSE THREE (IF (ISZERO TWO) FOUR ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)( \f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)( SUCC FOUR) (PRED TWO)))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)( \f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) FIVE ONE))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR ((IF (ISZERO ONE) FIVE ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (SUCC FIVE) (PRED ONE)))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR ((IF FALSE FIVE ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (SIX)(ZERO)))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR ((IF FALSE FIVE ((IF (ISZERO ZERO) SIX ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (SUCC SIX) (PRED ZERO)))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR ((IF FALSE FIVE ((IF TRUE SIX ((\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m)) (\f.\n.\m.IF (ISZERO m) n (f f (SUCC n) (PRED m) (SUCC SIX) (PRED ZERO)))))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR (IF FALSE FIVE SIX)))

= IF FALSE TWO (IF FALSE THREE (IF FALSE FOUR SIX))

= IF FALSE TWO (IF FALSE THREE SIX)

= IF FALSE TWO SIX

=SIX

=\f.\x.f(f(f(f(f(f x)))))

8. Lambda> FACT2 = \f.\n.IF (ISZERO n) ONE (MULT n (f (PRED n)))

• Lambda> FACTY = Y FACT2

• 求FACTY THREE

FACTY THREE = Y FACT2 THREE

= ( \g.(\x.g (x x)) \x.g (x x))FACT2 THREE

=(\x.FACT2(x x))(\x.FACT2(x x)) THREE

= FACT2 (Y FACT2) THREE

= (\f.\n.IF (ISZERO n) ONE (MULT n (f (PRED n)))) (Y FACT2) THREE

= IF (ISZERO THREE) ONE (MULT THREE ((Y FACT2) (PRED THREE)))

= IF FALSE ONE (MULT THREE (FACT2(Y FACT2) TWO))

= IF FALSE ONE (MULT THREE (\f.\n.IF (ISZERO n) ONE (MULT n (f (PRED n)))( Y FACT2) TWO))

= IF FALSE ONE (MULT THREE (IF (ISZERO TWO) ONE (MULT TWO ((Y FACT2) (PRED TWO)))))

= IF FALSE ONE (MULT THREE (IF FALSE ONE (MULT TWO (FACT2 (Y FACT2) (ONE)))))

= IF FALSE ONE (MULT THREE (IF FALSE ONE (MULT TWO (IF (ISZERO ONE) ONE (MULT ONE ((Y FACT2) (PRED ONE)))))))

= IF FALSE ONE (MULT THREE (IF FALSE ONE (MULT TWO (IF FALSE ONE (MULT ONE (IF (ISZERO ZERO) ONE (MULT ZERO ((Y FACT2) (PRED ZERO)))))))))

= IF FALSE ONE (MULT THREE (IF FALSE ONE (MULT TWO (IF FALSE ONE (MULT ONE (IF TRUE ONE (MULT ZERO ((Y FACT2) (PRED ZERO)))))))))

= IF FALSE ONE (MULT THREE (IF FALSE ONE (MULT TWO (IF FALSE ONE (MULT ONE (ONE))))))

= MULT THREE (MULT TWO (MULT ONE (ONE)))

=SIX

=\f.\x.f(f(f(f(f(f x)))))

9. Lambda> CONS a (CONS b (CONS c NIL))

• 求CAR (CDR (CONS a (CONS b (CONS c NIL))))

原式= CAR (CDR (CONS a (CONS b ((\x.\y.\f. f x y) c NIL))))

= CAR (CDR (CONS a (CONS b (\f. f c NIL))))

= CAR (CDR (CONS a (\f. f b (\f. f c NIL))))

= CAR (CDR (\f. f a (\f. f b (\f. f c NIL))))

= CAR ((\p.p FALSE) (\f. f a (\f. f b (\f. f c NIL))))

= CAR ((\f. f a (\f. f b (\f. f c NIL))) FALSE)

= CAR ((FALSE a (\f. f b (\f. f c NIL))))

= CAR (\f. f b (\f. f c NIL))

= (\p.p TRUE) (\f. f b (\f. f c NIL))

= (\f. f b (\f. f c NIL))TRUE

= TRUE b (\f. f c NIL)

= b

• 10. 求 有序对的 LENGTH

LENGTH NIL= Y ((\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) ZERO) NIL

= (\g.\c.\x. NULL x c (g (SUCC c) (CDR x))) LENTH ZERO NIL

= NULL NIL ZERO (LENTH(SUCC ZERO)(CDR NIL))

= NULL NIL ZERO (LENTH((\n.\f.\x.f (n f x)) (\f.\x.x)))( (\p.p FALSE)( \x. TRUE)))

=NULL NIL ZERO(LENTH ONE TRUE)

= ( \p.p (\x.\y.FALSE)) NIL ZERO(LENTH ONE TRUE)

= NIL (\x.\y.FALSE) ZERO(LENTH ONE TRUE)

=TRUE ZERO(LENTH ONE TRUE)

= (\x.\y.x) ZERO(LENTH ONE TRUE)

=ZERO

=\f.\x.x