Material Selections and Shaft Designs for a Wind Turbine

Executive Summary

The materials best suited to minimizing costs and the total mass of the specified wind turbine, while maintaining given safety requirements, are:

- Aluminum, 7249, wrought, T76511
- Magnesium, ZK60A-T5, wrought
- Low alloy steel, AISI 5160, tempered at 315°C & oil quenched

After optimizing wind turbine designs for each material, the following calculated material costs and total masses were as follows:

	Total Cost (USD)	Total Mass (kg)
Aluminum, 7249	217.29	69.2
Magnesium, ZK60A-T5	213.84	56.2
Low alloy steel, AISI 5160	64.96	110.1

- To save the most money and have the highest relative strength and durability, the low alloy steel should be used. However, the wind turbine will be relatively heavy and might cause problems due to its weight.
- To have the lightest wind turbine, the magnesium alloy should be used. However, the magnesium alloy is more brittle than and not as durable or strong as the other two metals.
- To have a combination of good strength and durability and low weight, the aluminum alloy should be used. However, the wind turbine will have a slightly higher cost. If weight is a major factor, the aluminum alloy should be chosen over the magnesium alloy.

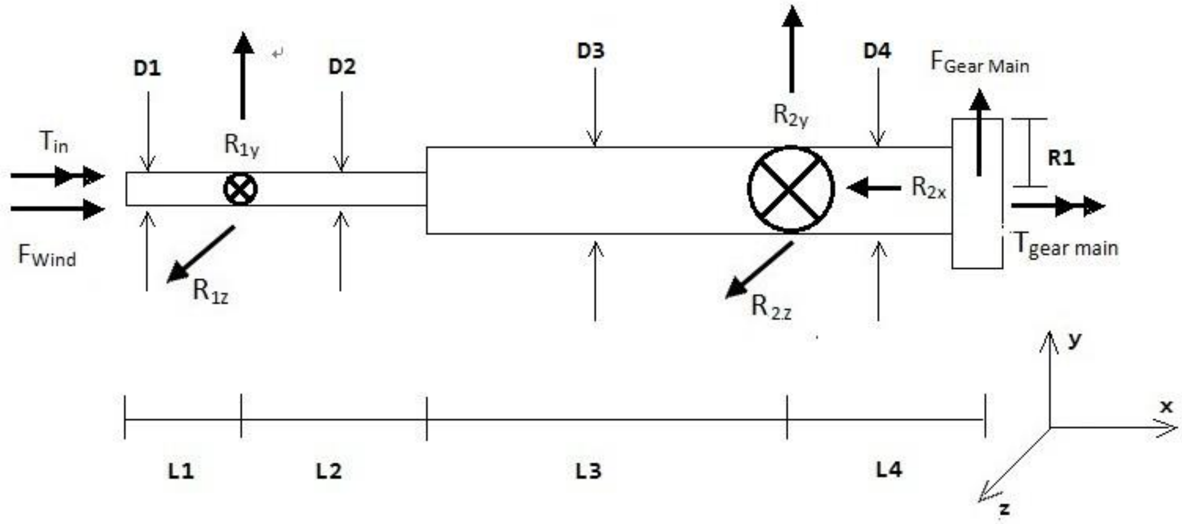
Introduction

The goal of this project was to determine appropriate materials to be used in the design of a wind turbine consisting of a main shaft and a crankshaft. The materials selected are to support loads and moments along these shafts for a given set of wind speeds and durations. Specifically, along each section of the shafts, the materials selected must be strong enough to maintain a given factor of safety against yielding, maintain a given factor of safety against stress for stress-based fatigue, and avoid exceeding a given maximum allowable angle of twist. These constraints are non-negotiable and must be met while satisfying objectives of minimizing material costs and total mass of the wind turbine.

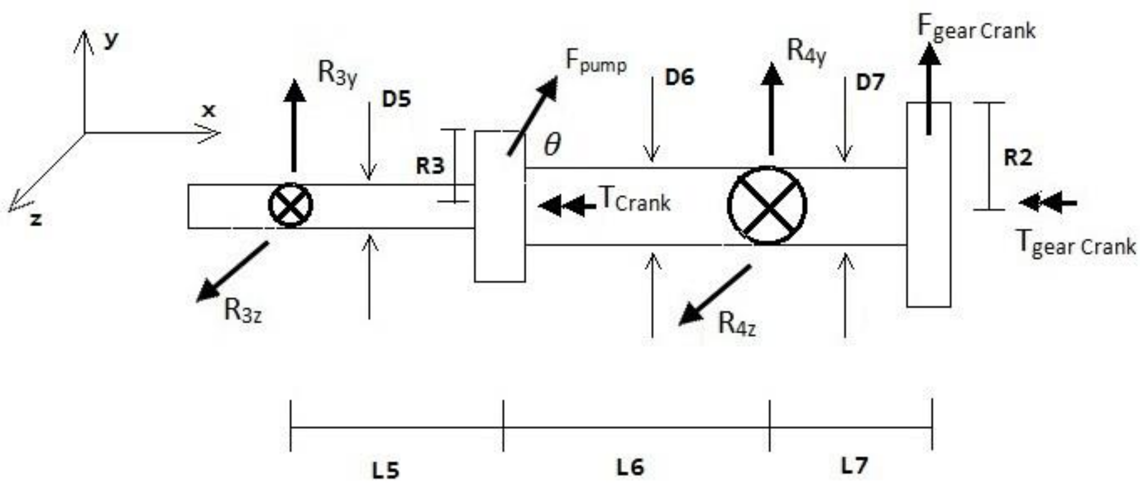
The process of material selection involved using the given constraints and objectives to calculate relevant material indices. These indices were then used in conjunction with CES EduPack 2012 and its coupling capabilities to aid in selecting the best materials for the wind turbine.

The following diagrams show the general outlines for the main shaft and crankshaft of the wind turbine.

Main Shaft



Crankshaft



For this wind turbine, all lengths are given and cannot be altered. The shaft sections all have circular cross-sections.

The applied forces and torques are determined by wind speeds. These forces then determine reaction forces and bending moments.

The wind turns a propeller on the main shaft. Gears and a chain connect and transmit power from the main shaft to the crankshaft. The crankshaft contains the mechanism that pumps water or generates electricity, etc.

Approach

To begin, material indices for each objective and constraint were calculated.

Objective: Minimize mass of $\pi r^2 l\rho$, where r is the radius, l is the length, ρ is the density.

Constraint: $\phi < \phi_{allowable}$, where ϕ represents the angle of twist.

$\phi = \frac{Tl}{JG} < \phi_{allowable} \Rightarrow \frac{Tl}{(\frac{\pi}{2})(r^4)(G)} < \phi_{allowable} \Rightarrow r^2 > \sqrt{\frac{2Tl}{\pi G\phi_{allowable}}}$

Thus, $mass > \pi l\rho\sqrt{\frac{2Tl}{\pi G\phi_{allowable}}}$

The material index to maximize is then $\boxed{\frac{G^{\frac{1}{2}}}{\rho}}$

Constraint: $X > X_o$, where X represents the safety factor against yielding.

$X = \frac{\sigma_o}{\bar{\sigma}_H} > X_o \Rightarrow \bar{\sigma}_H < \frac{\sigma_o}{X_o}$

For the given geometries, loadings, and reactions, we have $\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{2\sigma_x{}^2 + 6\tau_{xy}{}^2}$

Because the main stress raisers come from bending and torsion, we can ignore stress caused by axial loading.

$\bar{\sigma}_H = \frac{1}{\sqrt{2}}\sqrt{2\sigma_x{}^2 + 6\tau_{xy}{}^2} = \frac{1}{\sqrt{2}}\sqrt{2(\frac{Mr}{I})^2 + 6(\frac{Tr}{J})^2} = \frac{1}{\pi r^3}\sqrt{8M^2 + 6T^2} < \frac{\sigma_o}{X_o}$

Letting $k = \sqrt{8M^2 + 6T^2} \Rightarrow \frac{k}{\pi r^3} < \frac{\sigma_o}{X_o} \Rightarrow r^2 > (\frac{kX_o}{\pi\sigma_o})^{2/3}$

Thus, mass $> \pi l\rho(\frac{kX_o}{\pi\sigma_o})^{2/3}$

The material index to maximize is then $\boxed{\frac{\sigma_o{}^{2/3}}{\rho}}$

Constraint: $X > X_s$, where X represents the safety factor against stress for stress-based fatigue.

$X = X_N{}^{-b} > X_s$ (Note: X_N is against life for stress-based fatigue)

$X_N{}^{-b} = (\frac{N_f}{N})^{-b}$

Knowing there is a mean stress of zero in both shafts, we have $(\frac{N_f}{N})^{-b} = (\frac{\sigma_a}{N\sigma'_f})^{-1}$

$(\frac{\sigma_a}{N\sigma'_f})^{-1} = (\frac{Mr}{N\sigma'_f I})^{-1} = \frac{N\sigma'_f\pi r^3}{4M} > X_s \Rightarrow r^2 > (\frac{kX_o}{\pi\sigma'_f})^{2/3}$

Thus, mass > $\pi l\rho(\frac{kX_o}{\pi\sigma'_f})^{2/3}$

The material index to maximize is then $\boxed{\frac{\sigma'_f{}^{2/3}}{\rho}}$

Objective: Minimize the cost of $C_m \pi r^2 l \rho$, where C_m is the cost per unit mass. Material indices are the same as the ones found before, except they are each divided by C_m.

Constraint: $\phi < \phi_{allowable}$

The material index to maximize is $\boxed{\frac{G^{\frac{1}{2}}}{C_m \rho}}$

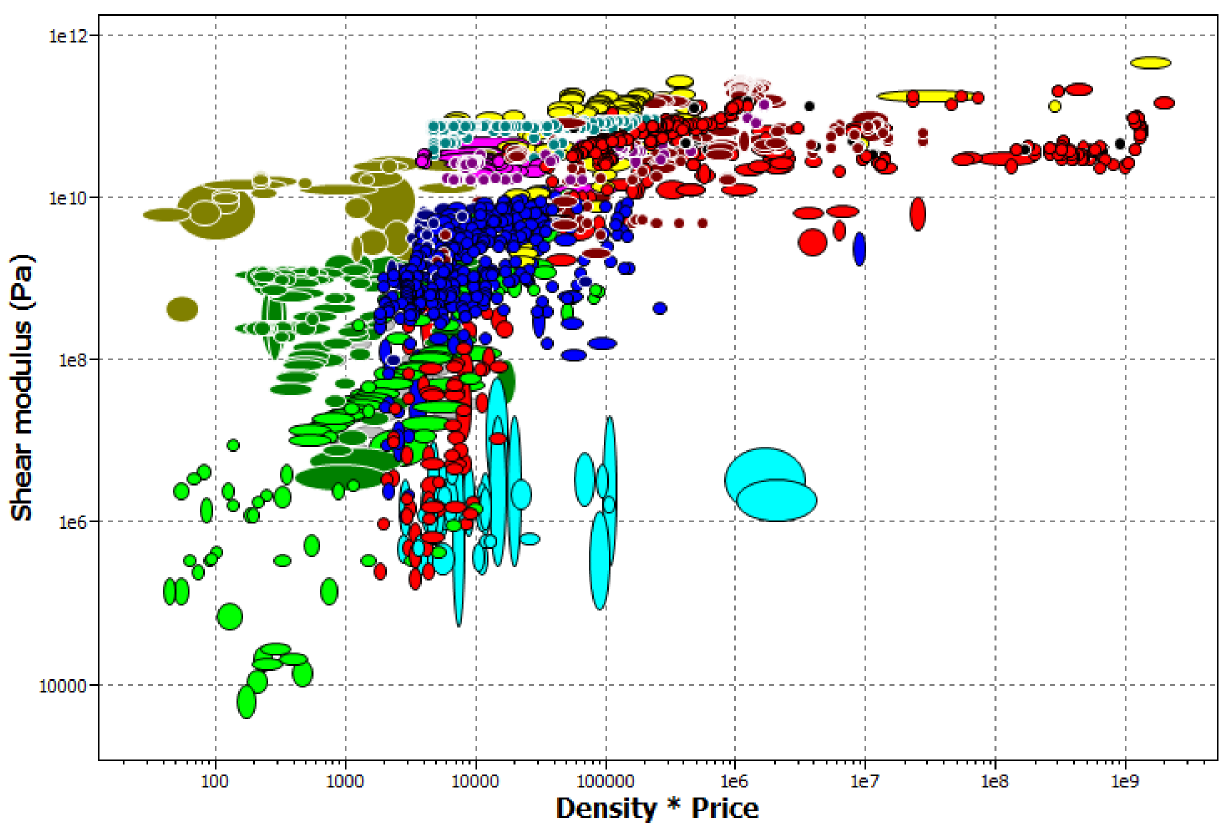
Constraint: $X > X_o$

The material index to maximize is $\boxed{\frac{{\sigma_o}^{2/3}}{C_m \rho}}$

Constraint: $X > X_s$

The material index to maximize is $\boxed{\frac{{\sigma'_f}^{2/3}}{C_m \rho}}$

These material indices were then used to plot materials against their properties in the CES EduPack 2012 software, which houses information on a large variety of engineering materials. An example of one of these plots is shown below. The scales are logarithmic. Values are in SI units.



For each material index, the numerator and denominator determine the plot's *y-axis* and *x-axis*, respectively. Each plot has a gradient line plotted corresponding to its material index (MI). The slopes of these lines were calculated using the condition $MI = C$, a constant.

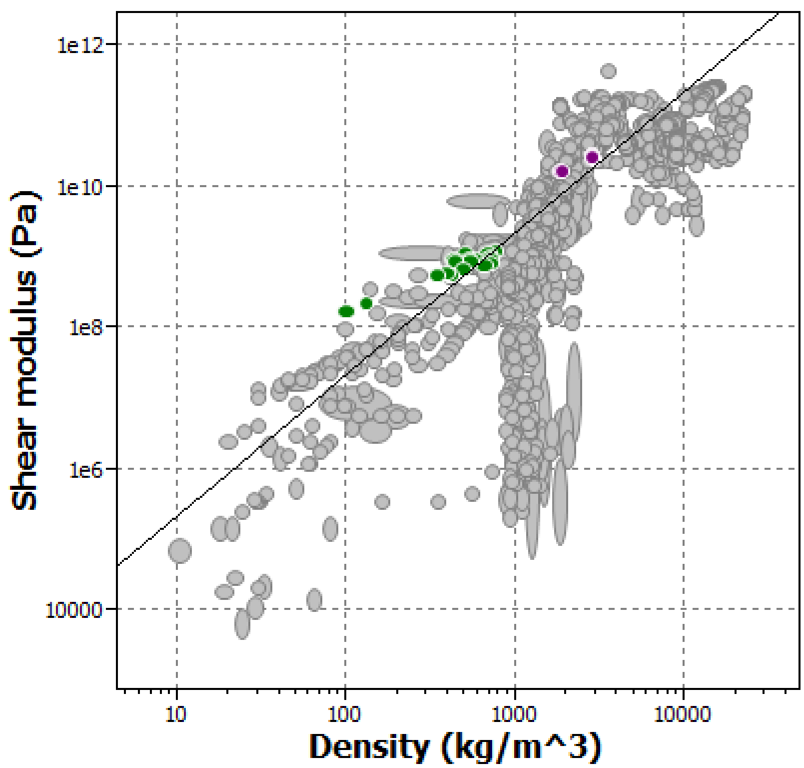
Consider minimizing mass with the constraint $\phi < \phi_{allowable}$. Setting the material index

$$\frac{G^{\frac{1}{2}}}{\rho} = C$$

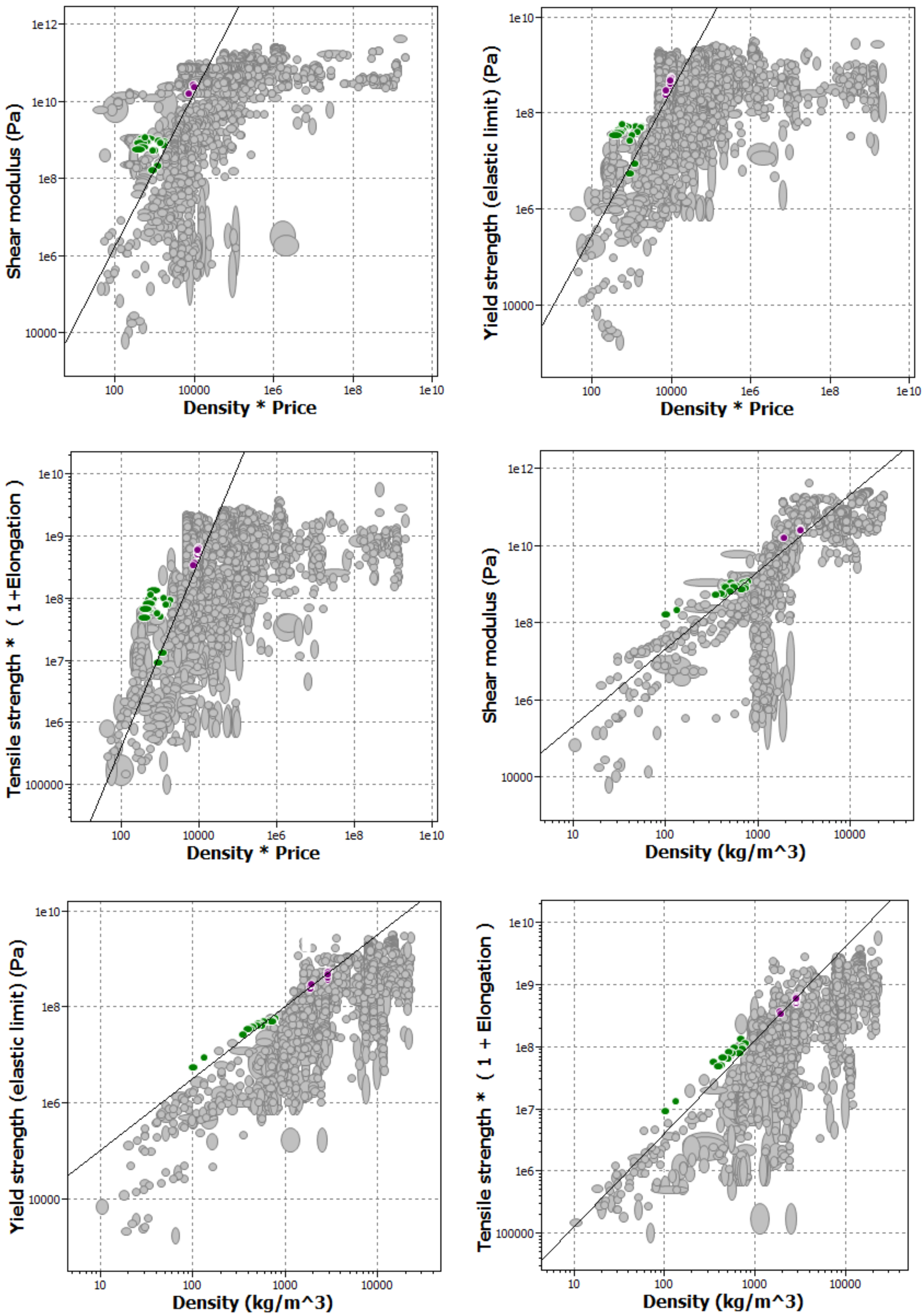
and taking logs gives

$$\log G = \log C + 2 \log \rho$$

The slope of the line is thus 2. The *y-axis* represents G, the shear modulus, whereas the *x-axis* represents ρ, the density. We want to consider materials to the left and above these lines in order to maximize the material index, thereby minimizing mass. The lines themselves can be moved up or down when considering materials. A plot for this example is shown below.



Six plots were made for the six material indices. Their corresponding gradient lines were also plotted. Using the coupling function of the software, optimal materials were found by moving all six gradient lines upward and towards the left. This eliminated materials that were not suitable for minimizing both mass and cost while considering constraints. An example of the six plots is shown below. All values are in SI units.



While using the CES software, it was found that the best materials given the objectives and constraints were types of wood such as oak, birch, fir, and redwood. However, these materials would be poor choices for the design of a wind turbine due to their high flammability, water absorption, ability to rot, and chances of attracting insects or animals.

The next best materials of different variety found were:

- Aluminum, 7249, wrought, T76511 (material 1)
- Magnesium, ZK60A-T5, wrought (material 2)
- Low alloy steel, AISI 5160, tempered at 315°C & oil quenched (material 3)

A summary of relevant properties is tabulated below.

	Aluminum, 7249	**Magnesium, ZK60A-T5**	**Low alloy steel, AISI 5160**
Density (kg/m^3)	2820	1830	7850
Price (USD/kg)	3.14	3.81	0.59
Young's Modulus (GPa)	76.5	45	209
Poisson's Ratio	0.34	0.31	0.29
Yield Strength (MPa)	530	292	1590
***Basquin Coefficient (MPa)**	645	376	2180
***Basquin Exponent**	-0.076	-0.047	-0.068

Other notable properties are tabulated below.

	Aluminum, 7249	**Magnesium, ZK60A-T5**	**Low alloy steel, AISI 5160**
Tensile Strength (MPa)	574	358	2000
Elongation at Fracture (strain)	0.124	0.05	0.09
Fatigue Strength at 10^7 cycles (MPa)	180	170	695
Service Temperature Range (°C)	-273 to 170	-273 to 170	-48 to 305
Flammable?	no	no	no

*These values were calculated in the following way:

σ'_f	Basquin Coefficient	b	Basquin Exponent
σ_{TS}	Tensile Strength	σ_{FS}	Fatigue Strength at 10^7 cycles
ϵ_n	Elongation at Fracture		

$$\sigma'_f = \sigma_{TS}(1 + \epsilon_n)$$

$$b = \frac{\log \frac{\sigma_{FS}}{\sigma'_f}}{\log(2\times 10^7)}$$

For each material, a MATLAB program that analyzed the wind turbine was run, requiring input text files containing wind history, wind turbine design, load factors, and the material's properties (see user manual for formatting). The program recorded and plotted shaft geometries along with the shafts' safety factors against yielding, safety factors against stress for stress-based fatigue, days until the shaft fails, and maximum angles of twist.

In order to optimize the wind turbine, diameters were minimized to minimize mass and cost, while making sure not to violate any constraints. Iterations were performed until a final design was reached. This final design was recorded and outputted in a text file. The final designs using each material are shown in the plots in the section *Results and Recommendations*.

Fun Side Note

If cost were not a factor, diamond would be the strongest, most durable, and lightest material to use; however, its use would not be practical due to brittleness. If weight were not a factor (a situation in which there were no gravity), aerated concrete would be the cheapest material to use. Relevant information for the two is shown below.

	Diamond	**Aerated Concrete**
Density (kg/m^3)	3510	700
Price (USD/kg)	46700	0.07
Young's Modulus (GPa)	1050	12
Poisson's Ratio	0.20	0.19
Yield Strength (MPa)	2800	0.75
***Basquin Coefficient (MPa)**	2867	0.75
***Basquin Exponent**	-0.011	-0.024

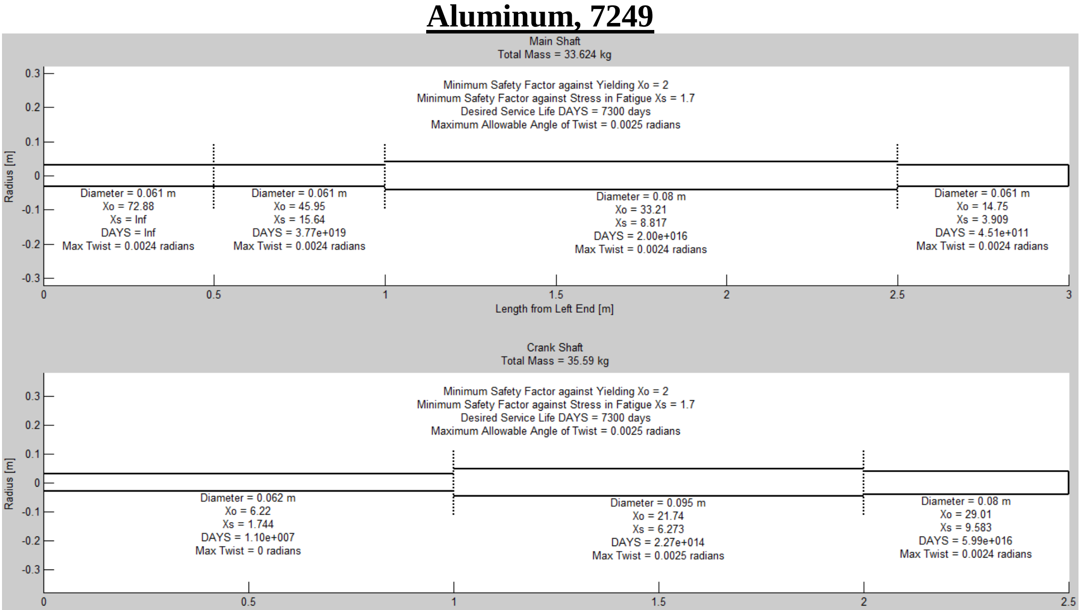
Below is a table summarizing the total cost and mass of the wind turbine when using either diamond or aerated concrete.

	Total Cost (USD)	**Total Mass (kg)**
Diamond	994710	21.3
Aerated Concrete	24.15	345

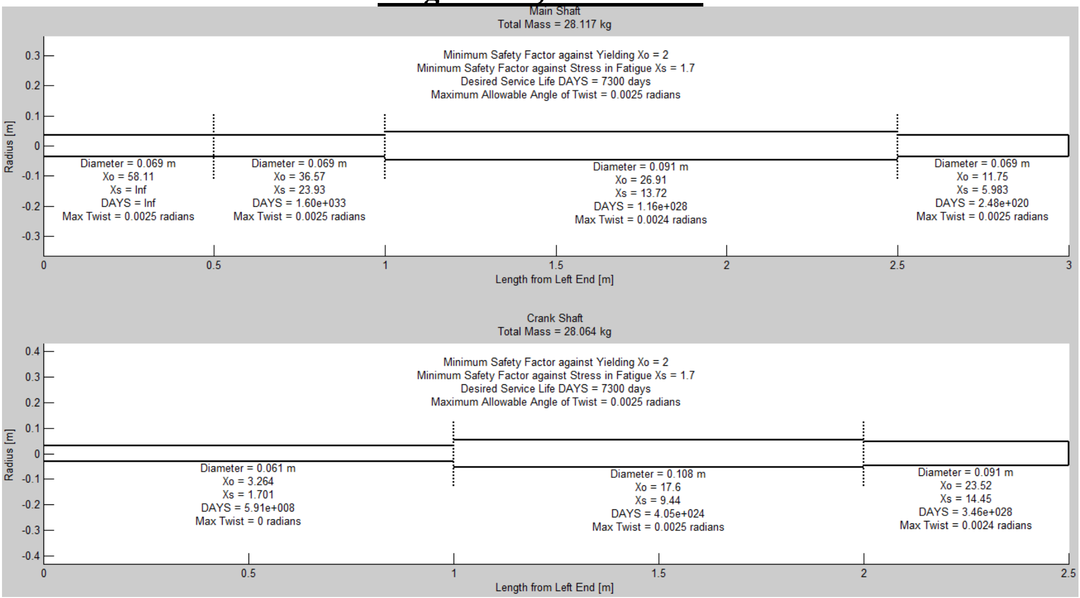
Results and Recommendations

The following plots show the final designs of the wind turbine for each selected material.

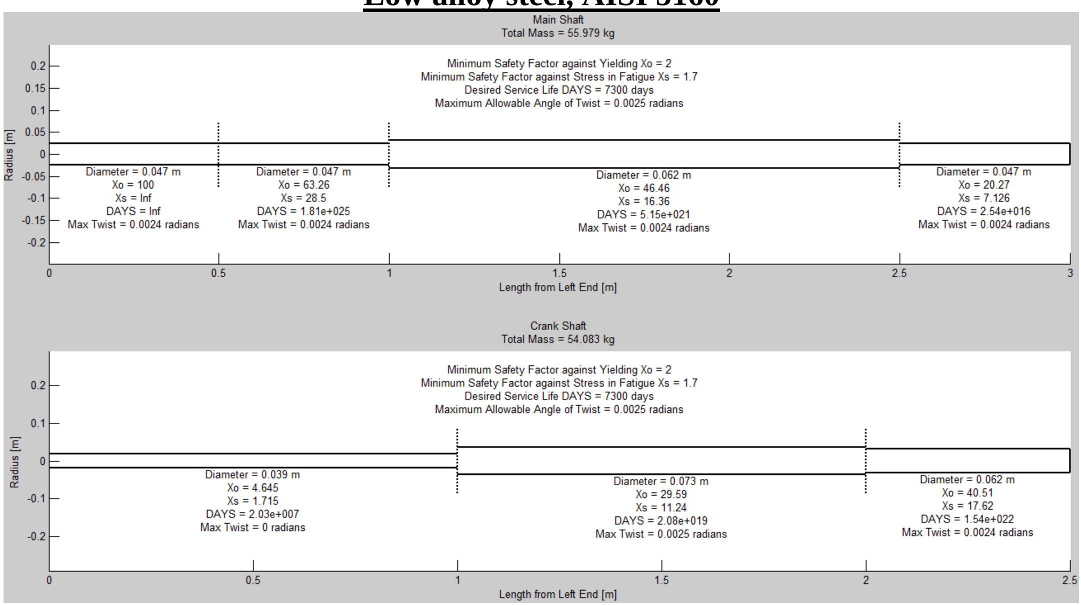
Aluminum, 7249



Magnesium, ZK60A-T5



Low alloy steel, AISI 5160



Zoom-in for clearer views.

The table below shows how well the materials selected met the objectives of minimizing cost and mass. One can see that the heaviest but cheapest material to use is the steel alloy. The lightest material to use is the magnesium alloy. Note that, although the magnesium alloy had the highest relative cost per mass, it turned out to be cheaper to use than the aluminum alloy due to the lower mass required for the wind turbine. Below is a table showing the total costs and masses for the wind turbine for each selected material.

	Total Cost (USD)	**Total Mass (kg)**
Aluminum, 7249	217.29	69.2
Magnesium, ZK60A-T5	213.84	56.2
Low alloy steel, AISI 5160	64.96	110.1

Below is a ranking of each of the three materials selected for several properties.

	Most	**Intermediate**	**Least**
Strong	Steel	Aluminum	Magnesium
Durable (against fluids and sunlight)	Steel	Aluminum	Magnesium
Durable (against fracture)	Aluminum	Steel	Magnesium
Longest Lasting	Steel	Magnesium	Aluminum
Hard	Steel	Aluminum	Magnesium
Environmentally Friendly (low CO2 footprint)	Steel	Magnesium	Aluminum
Inexpensive	Steel	Magnesium	Aluminum
Lightweight	Magnesium	Aluminum	Steel

To summarize in words:

- To save the most money and have the highest relative strength and durability, the low alloy steel should be used. However, the wind turbine will be relatively heavy, possibly increasing loads and reactions due to its own weight—this might lower the actual provided strength and durability of the steel.
- To have the lightest wind turbine, the magnesium alloy should be used. However, the magnesium alloy is more brittle than and not as durable or strong as the other two metals.
- To have a combination of good strength and durability and low weight, the aluminum alloy should be used. However, the wind turbine will have a slightly higher cost and leaves a larger CO2 footprint. Unless weight is a major factor, the aluminum alloy should be chosen over the magnesium alloy.