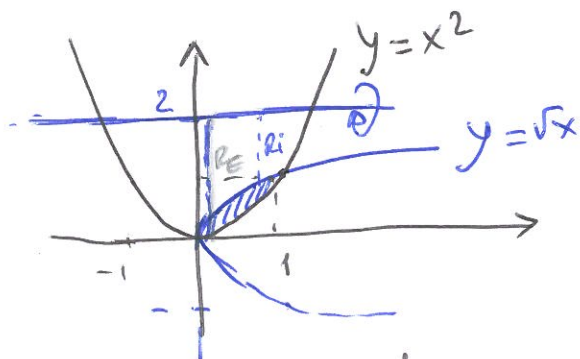


## Extra

## Ficha 6

exercício 1 da Ficha 6, considerando:

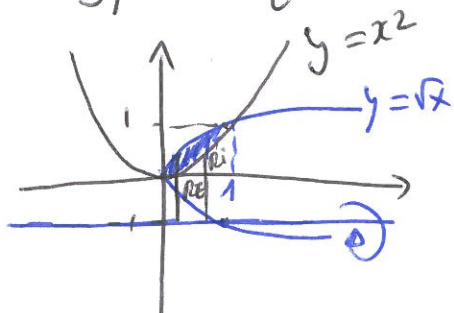
a) rotação em torno da reta  $x=2$



$R_E \rightarrow$  raio externo  
 $R_i \rightarrow$  raio interno

$$\begin{aligned} \text{Vol}(S) &= \pi \int_0^1 (R_E^2 - R_i^2) dx = \pi \int_0^1 (2-x^2)^2 - (2-\sqrt{x})^2 dx \\ &= \pi \int_0^1 (4 - 4x^2 + x^4) - (4 - 4\sqrt{x} + x) dx = \pi \int_0^1 \cancel{4} - 4x^2 + x^4 - \cancel{4} + 4\sqrt{x} - x dx \\ &= \pi \int_0^1 x^4 - 4x^2 + 4\sqrt{x} - x dx \stackrel{(R2)}{=} \pi \left[ \frac{x^5}{5} - 4\frac{x^3}{3} + 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2} \right]_0^1 \\ &= \pi \left[ \frac{1}{5} - \frac{4}{3} + \frac{8}{3} - \frac{1}{2} - 0 \right] = \pi \left[ \frac{1}{5} + \frac{4}{3} - \frac{1}{2} \right] \stackrel{(2)}{\stackrel{(5)}}{=} \pi \left[ \frac{2}{10} + \frac{4}{3} - \frac{5}{10} \right] \\ &= \pi \left[ -\frac{3}{10} + \frac{4}{3} \right] \stackrel{(3)}{\stackrel{(10)}}{=} \pi \left[ -\frac{9+40}{30} \right] = \frac{31\pi}{30} \text{ unidades cúbicas} \end{aligned}$$

b) rotação em torno da reta  $y=-1$



$$\begin{aligned} \text{Vol}(S) &= \pi \int_0^1 R_E^2 - R_i^2 dx = \pi \int_0^1 \frac{(\sqrt{x}-(-1))^2}{R_E} - (x^2-(-1))^2 dx \\ &= \pi \int_0^1 (\sqrt{x}+1)^2 - (x^2+1)^2 dx \\ &= \pi \int_0^1 x + 2\sqrt{x} + 1 - (x^4 + 2x^2 + 1) dx = \pi \int_0^1 x + 2\sqrt{x} - x^4 - 2x^2 dx \\ &= \pi \left[ \frac{x^2}{2} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^5}{5} - 2\frac{x^3}{3} \right]_0^1 = \pi \left[ \frac{1}{2} + \frac{4}{3} - \frac{1}{5} - \frac{2}{3} - 0 \right] \\ &= \frac{29\pi}{30} \text{ unidades cúbicas} \end{aligned}$$