Ficha 5

1. $\left(\int_1^x \frac{\sqrt{1+t^4}}{t^2}\,dt\right)' = \frac{\sqrt{1+x^4}}{x^2}$.

2. $\left(\int_1^{\ln x} \operatorname{sen}(u+e^u)\,du\right)' = (\ln x)' \cdot \operatorname{sen}\left(\ln x + e^{\ln x}\right) =$

$= \frac{\operatorname{sen}(\ln x + x)}{x}$.

3. Já está feito na ficha 6.

4. $f(x) = \int_0^{x^3} e^{-t^2}\,dt$

Para estudar a monotonia de f, estuda-se o sinal de f':

$f'(x) = \left(\int_0^{x^3} e^{-t^2}\,dt\right)' = 3x^2 \cdot e^{-x^6}$

$f'(x) > 0 \Leftrightarrow 3x^2 e^{-x^6} > 0 \Leftrightarrow x \in \mathbb{R}\setminus\{0\}$

f é ~~sempre~~ crescente, $\forall x \in \mathbb{R}$.

5. Determinar f tal que $\int_0^{x^2} f(t)\,dt = x^3 e^x - x^4$

$\left(\int_0^{x^2} f(t)\,dt\right)' = \left(x^3 e^x - x^4\right)' \Leftrightarrow$

$2x \cdot f(x^2) = 3x^2 e^x + x^3 e^x - 4x^3$

$f(x^2) = \frac{3}{2} x\, e^x + \frac{x^2}{2} e^x - 2x^2$ para $x \neq 0$.

$f(x) = \frac{3}{2}\sqrt{x}\, e^{\sqrt{x}} + \frac{x}{2} e^{\sqrt{x}} - 2x$. para $x \neq 0$.

Ficha 2

Ficha 5

6. $\int_k^x f(t)\,dt = \operatorname{sen} x + \frac{1}{2}$

Tem-se que $\int_k^k f(t)\,dt = \operatorname{sen} k + \frac{1}{2} \Leftrightarrow 0 = \operatorname{sen} k + \frac{1}{2} \Leftrightarrow \operatorname{sen} k = -\frac{1}{2}$

$\Leftrightarrow k = -\frac{\pi}{6} + 2t\pi \ \vee \ k = \frac{7\pi}{2} + 2t\pi, \ t \in \mathbb{R}$.

e $\left(\int_k^x f(t)\,dt\right)' = \left(\operatorname{sen} x + \frac{1}{2}\right)' \Leftrightarrow f(x) = \cos x$.

7. Encontrar $P(x)$ de grau 2, $P(x) = ax^2 + bx + c$, é equivalente a determinar as constantes reais a, b, c.

$\boxed{P(0) = f(0)} \Leftrightarrow c = f(0) = \int_0^0 \frac{1 + \operatorname{sen} t}{2 + t^2}\,dt = 0 \Leftrightarrow \boxed{c = 0}$

$\boxed{P'(0) = f'(0)}$ onde $P'(x) = 2ax + b$ e $P'(0) = b$.

$f'(x) = \frac{1 + \operatorname{sen} x}{2 + x^2}$ e $f'(0) = \frac{1 + \operatorname{sen} 0}{2 + 0} = \frac{1}{2}$

logo $P'(0) = f'(0) \Leftrightarrow \boxed{b = \frac{1}{2}}$.

$\boxed{P''(0) = f''(0)}$ onde $P''(x) = 2a$ e $P''(0) = 2a$

$f''(x) = \frac{\cos x(2 + x^2) - 2x(1 + \operatorname{sen} x)}{(2 + x^2)^2}$ e $f''(0) = \frac{1(2+0) - 0}{4} = \frac{1}{2}$

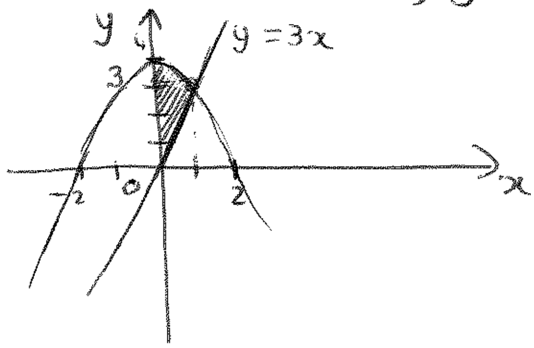
logo $P''(0) = f''(0) \Leftrightarrow 2a = \frac{1}{2} \Leftrightarrow \boxed{a = \frac{1}{4}}$

Assim, $\boxed{P(x) = \frac{1}{4}x^2 + \frac{1}{2}x}$.

Áreas planas

Ficha 5 (3)

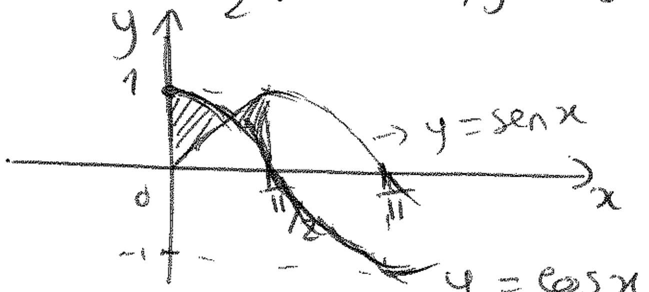
1.a) $x=0,\ x=1,\ y=3x,\ y=-x^2+4$



$$\int_0^1 (-x^2+4-3x)\,dx = \left[-\frac{x^3}{3}+4x-\frac{3x^2}{2}\right]_0^1 =$$

$$= -\frac{1}{3}+4-\frac{3}{2} = \frac{-2+24-9}{6} = \frac{13}{6}.$$

b) $x=0,\ x=\frac{\pi}{2},\ y=\operatorname{sen} x,\ y=\cos x.$



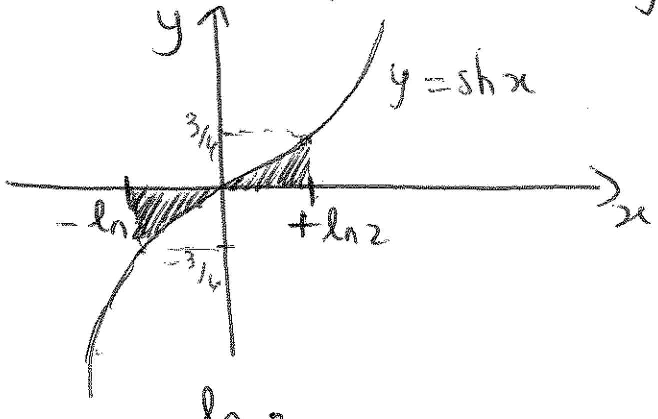
$$\int_0^{\pi/4} (\cos x-\operatorname{sen} x)\,dx + \int_{\pi/4}^{\pi/2} (\operatorname{sen} x-\cos x)\,dx = \left[\operatorname{sen} x+\cos x\right]_0^{\pi/4} +$$

$$+\left[-\cos x-\operatorname{sen} x\right]_{\pi/4}^{\pi/2} =$$

$$= \operatorname{sen}\frac{\pi}{4}+\cos\frac{\pi}{4}-\cos 0 \underbrace{-\cos\frac{\pi}{2}}_{=0}-\operatorname{sen}\frac{\pi}{2}+\cos\frac{\pi}{4}+\operatorname{sen}\frac{\pi}{4} =$$

$$= \sqrt{2}-1-1+\sqrt{2} = 2\sqrt{2}-2.$$

c) $y=0,\ x=-\ln 2,\ x=\ln 2 \qquad y=\operatorname{sh} x$



$$\operatorname{sh}(\ln 2) = \frac{e^{\ln 2}-e^{-\ln 2}}{2} = \frac{2-1/2}{2} = \frac{3}{4}$$

$$\operatorname{sh}(-\ln 2) = \frac{\frac{1}{2}-2}{2} = \frac{-3}{4}$$

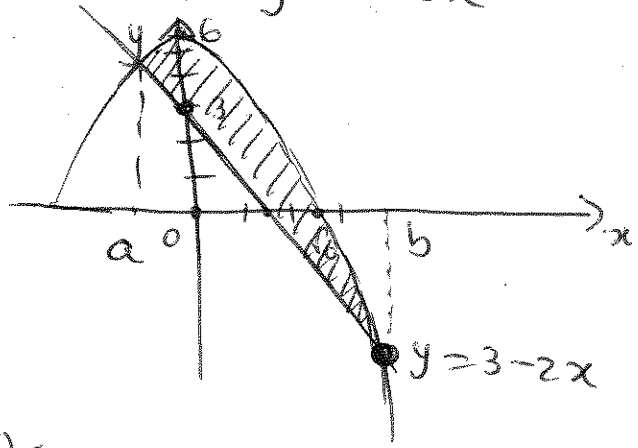
$$-\int_{-\ln 2}^{0} \operatorname{sh} x\,dx + \int_0^{\ln 2} \operatorname{sh} x = \left[-\operatorname{ch} x\right]_{-\ln 2}^{0} + \left[\operatorname{ch} x\right]_0^{\ln 2} =$$

$$= -\operatorname{ch} 0+\operatorname{ch}(-\ln 2)+\operatorname{ch}(\ln 2)-\operatorname{ch} 0 =$$

$$= -2+\frac{5}{4}+\frac{5}{4} = -2+\frac{5}{2} = 1/2.$$

d) $y + x^2 = 6$ e $y = 3 - 2x$ Ficha ④

Ficha 5



$$\int_a^b \left(6 - x^2 - (3-2x)\right) dx = \int_a^b \left(6 - x^2 - 3 + 2x\right) dx = \int_a^b (3 - x^2 + 2x)\, dx$$

$$= \left[3x - \frac{x^3}{3} + \frac{2x^2}{2}\right]_a^b .$$

Determina a e b, abcissas dos pt^{os} de intersecção das curvas

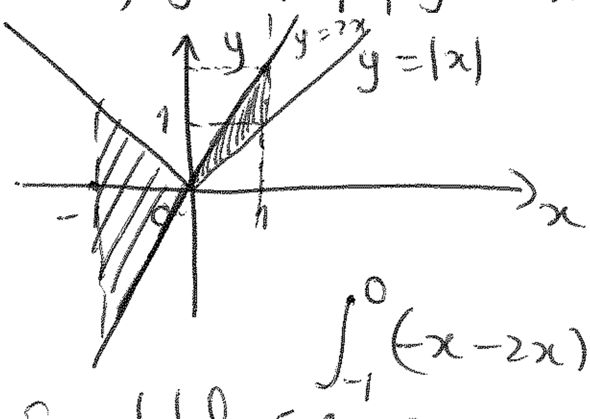
$$\begin{cases} y = 6 - x^2 \\ y = 3 - 2x \end{cases} \begin{cases} 3 - 2x = 6 - x^2 \\ \text{———} \end{cases} \begin{cases} x^2 - 2x + 3 - 6 = 0 \\ \end{cases} \begin{cases} x^2 - 2x - 3 = 0 \\ \end{cases}$$

$$\begin{cases} x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \Leftrightarrow x = 3 \vee x = -1 \end{cases}$$

Assim, $\displaystyle\int_{-1}^{3} (6 - x^2 - 3 + 2x)\, dx = \left[3x - \frac{x^3}{3} + x^2\right]_{-1}^{3} =$

$$= 9 - \frac{3^3}{3} + 3^2 - \left(-3 - \frac{(-1)^3}{3} + 1\right) = 9 - \cancel{3^2} + \cancel{3^2} + 3 - \frac{1}{3} + 1 = 13 - \frac{1}{3} = \frac{38}{3}.$$

e) $x = -1$, $y = |x|$, $y = 2x$, $x = 1$.



$$\int_0^1 (2x - x)\, dx = \int_0^1 x\, dx =$$

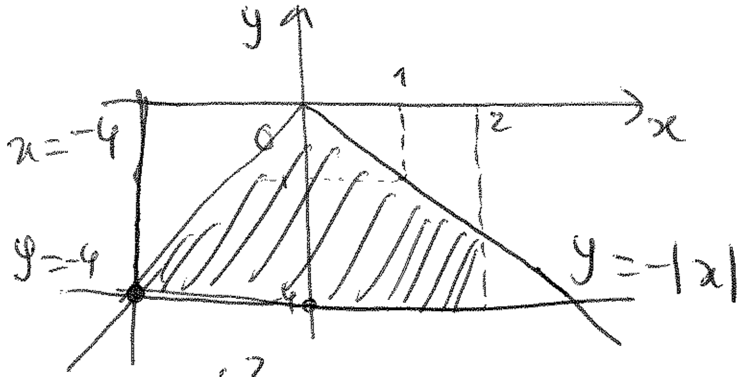
$$= \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}.$$

$$\int_{-1}^{0} (-x - 2x)\, dx = \int_{-1}^{0} -3x\, dx = \left[-\frac{3x^2}{2}\right]_{-1}^{0} = \frac{3}{2}$$

A área total é $\frac{1}{2} + \frac{3}{2} = 2$.

Ficha 5

f) $y=-|x|$, $y=-4$, $x=2$, $x=-4$



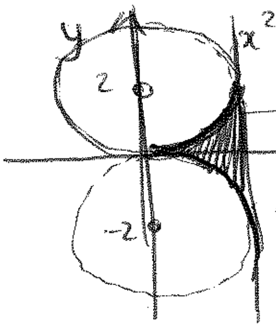
$$\int_{-4}^{2}\left(-|x|-(-4)\right)dx = \int_{-4}^{2}(-|x|+4)\,dx = \int_{-4}^{0}(-|x|+4)\,dx +$$

$$+\int_{0}^{2}(-|x|+4)\,dx = \int_{-4}^{0}(x+4)\,dx + \int_{0}^{2}(-x+4)\,dx =$$

$$=\left[\frac{x^2}{2}+4x\right]_{-4}^{0} + \left[-\frac{x^2}{2}+4x\right]_{0}^{2} = 0-\left(\frac{16}{2}-16\right)+\left(-\frac{4}{2}+8\right)-0 =$$

$$=-(-8)+6 = 14.$$

g) $x=0$, $x=2$, $x^2+(y-2)^2=4$, $x^2+(y+2)^2=4$.



$x^2+(y-2)^2=4$

$(y-2)^2=4-x^2 \Leftrightarrow y-2=\pm\sqrt{4-x^2} \Leftrightarrow y=2\pm\sqrt{4-x^2}$

$x^2+(y+2)^2=4 \quad \Leftrightarrow y=-2\pm\sqrt{4-x^2}$

$$\int_0^2 \left(2-\sqrt{4-x^2}\right)-\left(-2+\sqrt{4-x^2}\right)dx =$$

$$=\int_0^2\left(4-2\sqrt{4-x^2}\right)dx =$$

Fazendo a substituição $x=2\,\text{sen}\,t$

$dx = 2\cos t\,.dt$

$x=0 \Rightarrow t=0$

$x=2 \Rightarrow 2=2\,\text{sen}\,t \Leftrightarrow t=\pi/2$

$$=\int_0^{\pi/2}\left(4-2\sqrt{4-4\,\text{sen}^2t}\right)\cos t\,\cancel{2}\,dt = \int_0^{\pi/2}\left(4\cos t - 4\,.\cos^2 t\right)dt =$$

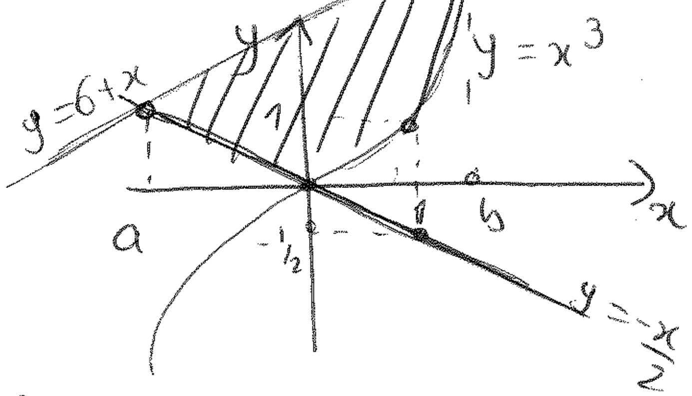
$$=4\int_0^{\pi/2}\cos t\,dt - 4\int_0^{\pi/2}\frac{\cos(2t)+1}{2}\,dt = 4\left[\text{sen}\,t\right]_0^{\pi/2} - 2\left[\frac{1}{2}\,\text{sen}(2t)+t\right]_0^{\pi/2} =$$

$$=4\left(\text{sen}\,\frac{\pi}{2}\right) - 2\left[\frac{1}{2}\,\text{sen}\,\pi + \frac{\pi}{2}\right] = 4-2\left(\frac{\pi}{2}\right) = 4-\pi.$$

1.h) $y-x=6$, $y-x^3=0$, $2y+x=0$ Fex1 6

Ficha 5

$y=6+x$, $y=x^3$, $y=-\frac{x}{2}$



$$\int_a^0\left(6+x-\left(-\frac{x}{2}\right)\right)dx+\int_0^b(6+x-x^3)\,dx=\int_a^0\left(6+\frac{3}{2}x\right)dx+\int_0^b(6+x-x^3)\,dx$$

$$=\left[6x+\frac{3x^2}{4}\right]_a^0+\left[6x+\frac{x^2}{2}-\frac{x^4}{4}\right]_0^b$$

Determinar a e b, abcissas dos pontos de intersecção.

a: $\begin{cases} y=6+x \\ y=-\frac{x}{2}\end{cases}$ $\begin{cases} -\frac{x}{2}=6+x \Leftrightarrow -\frac{3x}{2}=6 \Leftrightarrow x=\frac{-2\times 6}{3}=-4 \end{cases}$

$a=-4$

b: $\begin{cases} y=6+x \\ y=x^3\end{cases}$ $\begin{cases} x^3=x+6 \end{cases}$ $\begin{cases} x^3-x-6=0 \end{cases}$ $\begin{cases} x=2 \Rightarrow 2^3-2-6=0. \end{cases}$

$\underline{b=2}$

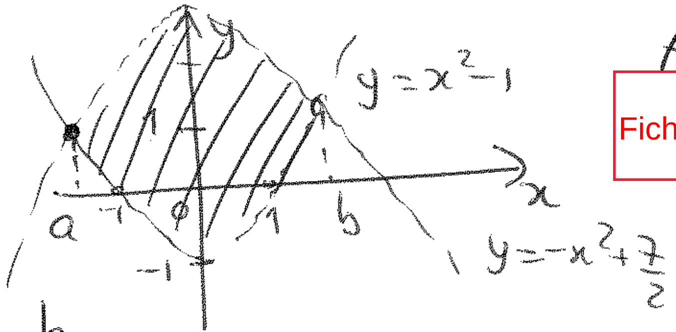
$$\int_{-4}^0\left(6+\frac{3x}{2}\right)dx+\int_0^2(6+x-x^3)\,dx=\left[6x+\frac{3x^2}{4}\right]_{-4}^0+\left[6x+\frac{x^2}{2}-\frac{x^4}{4}\right]_0^2=$$

$$=-\left[-24+\frac{3}{4}(-4)^2\right]+\left[12+2-\frac{2^4}{4}\right]=24-3\times4+14-2^2=24-12+14-4$$

$$=12+10=22.$$

Ficha 5

i) $y = -x^2 + \frac{7}{2}$, $y = x^2 - 1$



$$\int_a^b \left(-x^2 + \frac{7}{2}\right) - (x^2 - 1)\, dx = \int_a^b \left(-2x^2 + \frac{9}{2}\right) dx = \left[-\frac{2x^3}{3} + \frac{9}{2}x\right]_a^b$$

onde a e b são ~~pt de in~~ abcissas dos pt^{os} de intersecção dos curvos

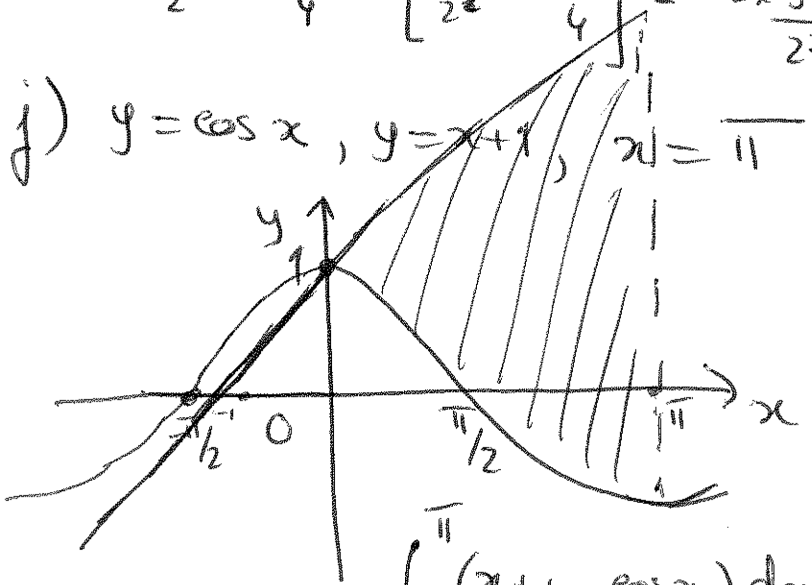
$$\begin{cases} y = x^2 - 1 \\ y = -x^2 + \frac{7}{2} \end{cases} \begin{cases} -x^2 + \frac{7}{2} = x^2 - 1 \end{cases} \begin{cases} 2x^2 - 1 - \frac{7}{2} = 0 \Leftrightarrow 2x^2 - \frac{9}{2} = 0 \end{cases}$$

$$x^2 = \frac{9}{4} \Leftrightarrow x = \pm \frac{3}{2}$$

$$\int_{-3/2}^{3/2} \left(-2x^2 + \frac{9}{2}\right) dx = \left[-\frac{2x^3}{3} + \frac{9}{2}x\right]_{-3/2}^{3/2} = -\frac{2}{3}\left(\frac{3}{2}\right)^3 + \frac{9}{2} \times \frac{3}{2} - \left[-\frac{2}{3}\left(-\frac{3}{2}\right)^3 - \frac{9}{2} \times \frac{3}{2}\right]$$

$$= -\frac{3^2}{2^2} + \frac{27}{4} - \left[\frac{3^2}{2^2} - \frac{27}{4}\right] = -2 \times \frac{3^2}{2^2} + 2 \times \frac{27}{4} = -\frac{3^2}{2} + \frac{27}{2} = \frac{18}{2} = 9.$$

j) $y = \cos x$, $y = x + 1$, $x = \pi$



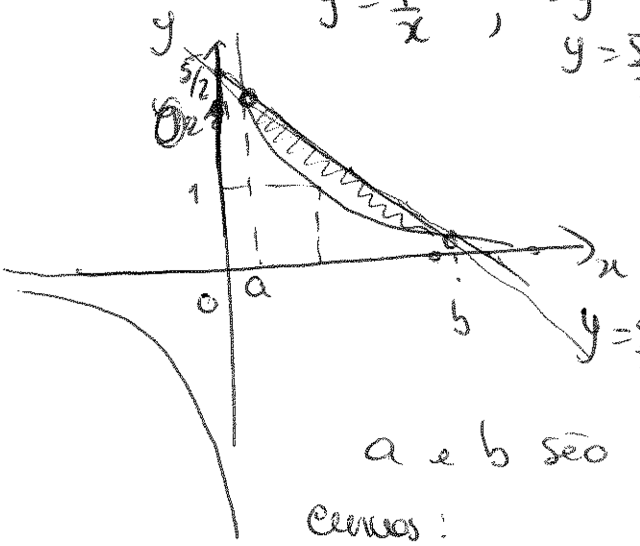
$$\int_0^{\pi} (x + 1 - \cos x)\, dx = \left[\frac{x^2}{2} + x - \operatorname{sen} x\right]_0^{\pi} =$$

$$= \frac{\pi^2}{2} + \pi - \operatorname{sen} \pi = \frac{\pi^2}{2} + \pi.$$

k) $y=\frac{1}{x}$, $2x+2y=5$

$y=\frac{1}{x}$; $2y=5-2x$

$y=\frac{5}{2}-x$



$$\int_a^b \left(\frac{5}{2}-x-\frac{1}{x}\right)dx = \left[\frac{5}{2}x-\frac{x^2}{2}-\ln|x|\right]_a^b$$

a e b são abcissas dos pt^{os} de intersecção das curvas:

$$\begin{cases} y=\frac{1}{x} \\ y=\frac{5}{2}-x \end{cases} \quad \frac{1}{x}=\frac{5}{2}-x \iff \frac{1}{x}+x-\frac{5}{2}=0 \iff \frac{2+2x^2-5x}{2x}=0$$

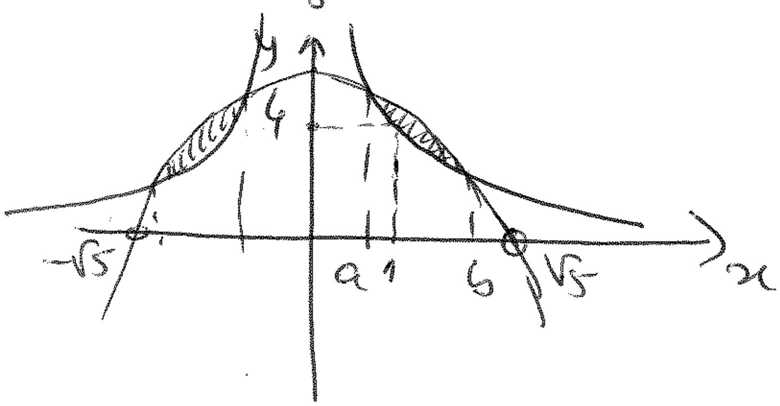
$\cancel{x=\frac{5\pm\sqrt{25-4}}{2}}$ $=\cancel{\frac{5\pm\sqrt{21}}{2}}$ $x=\frac{5\pm\sqrt{25-16}}{4}$

$x=\frac{5\pm3}{4} \iff x=2 \vee x=\frac{1}{2}$

$$\int_{1/2}^{2}\left(\frac{5}{2}-x-\frac{1}{x}\right)dx=\left[\frac{5x}{2}-\frac{x^2}{2}-\ln|x|\right]_{1/2}^{2}=5-2-\ln 2-\frac{5}{4}+\frac{1}{8}+\ln\frac{1}{2}=$$

$$=3-\ln 2-\frac{10+1}{8}-\ln 2=3-2\ln 2-\frac{9}{8}=\frac{15}{8}-2\ln 2.$$

l) $y=\frac{4}{x^2}$, $y=5-x^2$



$$2\int_a^b\left(5-x^2-\frac{4}{x^2}\right)dx$$

$$=2\left[5x-\frac{x^3}{3}+\frac{4}{x}\right]_a^b$$

onde a e b são as abcissas dos pt^{os} de intersecção das curvas

$$\begin{cases} y=5-x^2 \\ y=\frac{4}{x^2} \end{cases} \quad 5-x^2=\frac{4}{x^2} \iff x^2+\frac{4}{x^2}-5=0$$

$$\frac{x^4-5x^2-4}{x^2}=0$$

Folha Ficha 5 9

l) $x^4-5x^2-4=0 \Leftrightarrow x^2=\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm3}{2}$

$x^2=4 \vee x^2=1 \Leftrightarrow x=\pm2 \vee x=\pm1$

$\boxed{a=1;\ b=2}$

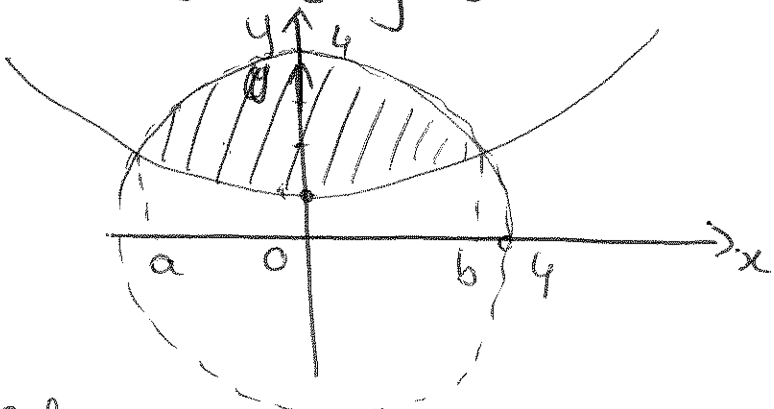
$2\int_1^2\left(5-x^2-\frac{4}{x^2}\right)dx = 2\left[5x-\frac{x^3}{3}+\frac{4}{x}\right]_1^2 = 2\left[10-\frac{2^3}{3}+2-5+\frac{1}{3}-4\right]$

$=2\left[10-\frac{8}{3}-3+\frac{1}{3}-4\right]=2\left[3-\frac{7}{3}\right]=2\left[\frac{2}{3}\right]=\frac{4}{3}$.

m) $x^2=12(y-1)$, $x^2+y^2=16$

$y-1=\frac{x^2}{12}$

$y=1+\frac{x^2}{12}$ $\qquad y=\pm\sqrt{16-x^2}$



$\int_a^b \sqrt{16-x^2}-\left(1+\frac{x^2}{2}\right)dx$ onde a e b são as abcissas dos pt^{os} de intersecção entre as curvas.

$\begin{cases}x^2+y^2=16\\ y=1+\frac{x^2}{12}\end{cases}$ $\begin{cases}x^2+\left(1+\frac{x^2}{12}\right)^2=16 \Leftrightarrow x^2+1+\frac{x^4}{144}+\frac{x^2}{6}=16 \Leftrightarrow\\ \frac{7}{6}x^2+\frac{x^4}{144}-15=0. \Leftrightarrow x^2=\frac{-7/6\pm\sqrt{\left(\frac{7}{6}\right)^2+4\times15\times\frac{1}{144}}}{\frac{2}{144}}\end{cases}$

$x^2=\frac{-\frac{7}{6}\pm\sqrt{\frac{49}{36}+\frac{15}{36}}}{\frac{2}{144}}=\frac{-\frac{7}{6}\pm\sqrt{\frac{64}{36}}}{\frac{2}{144}}=\frac{-\frac{7}{6}\pm\frac{8}{6}}{\frac{2}{144}}\Leftrightarrow$

$x^2=\frac{\frac{1}{6}}{\frac{2}{144}} \vee x^2=\frac{-\frac{15}{6}}{\frac{2}{144}} \Leftrightarrow x^2=12 \vee x^2=-15\times12$

$x=\pm\sqrt{12}$

$\int_{-\sqrt{2}}^{\sqrt{12}}\sqrt{16-x^2}-\left(1+\frac{x^2}{2}\right)dx=\int_{-\sqrt{12}}^{\sqrt{12}}\sqrt{16-x^2}\,dx-\int_{-\sqrt{12}}^{\sqrt{12}}\left(1+\frac{x^2}{2}\right)dx=$

No 1º integral, usa-se a substituição
$x = 4\,\text{sen}\,t$

$x=\sqrt{12} \Rightarrow \frac{\sqrt{12}}{4} = \text{sen}\,t \Leftrightarrow \text{sen}\,t = \frac{\sqrt{3}}{2} \Rightarrow t = \frac{\pi}{3}$

$x=-\sqrt{12} \Rightarrow t = -\frac{\pi}{3}$

$$\int_{-\sqrt{12}}^{\sqrt{12}} \sqrt{16-x^2}\,dx = \int_{-\frac{\pi}{3}}^{\pi/3} \sqrt{16-16\,\text{sen}^2 t}\,.\,4\cos t\,.dt = 16\int_{-\pi/3}^{\pi/3} \cos^2 t\,.dt =$$

$$=16\int_{-\pi/3}^{\pi/3} \frac{\cos(2t)+1}{2}\,dt = 8\int_{-\pi/3}^{\pi/3} (\cos(2t)+1)\,dt = 8\left[\frac{1}{2}\text{sen}(2t)+t\right]_{-\pi/3}^{\pi/3} =$$

$$= 8\left[\frac{1}{2}\text{sen}\,\frac{2\pi}{3} + \frac{\pi}{3} - \frac{1}{2}\text{sen}\left(-\frac{2\pi}{3}\right) + \frac{\pi}{3}\right] = 8\left[\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{2\pi}{3} - \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)\right] =$$

$$= 8\left[\frac{\sqrt{3}}{2} + \frac{2\pi}{3}\right] = 4\sqrt{3} + \frac{16\pi}{3}.$$

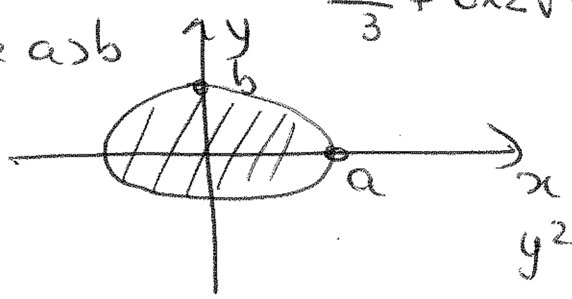
No 2º integral, $\int_{-\sqrt{12}}^{\sqrt{12}} \left(1+\frac{x^2}{2}\right) dx = \left[x + \frac{x^3}{6}\right]_{-\sqrt{12}}^{\sqrt{12}} = \sqrt{12} + \frac{12\sqrt{12}}{6} + \sqrt{12} + \frac{12\sqrt{12}}{6} =$

$$= 2\sqrt{12} + \frac{12\sqrt{12}}{3} = \frac{18\sqrt{12}}{3} = 6\sqrt{12}$$

Total

$$4\sqrt{3} + \frac{16\pi}{3} + 6\sqrt{12} = 4\sqrt{3} + \frac{16\pi}{3} + 6\times 2\sqrt{3} = 16\sqrt{3} + \frac{16\pi}{3}.$$

n) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ se $a>b$



$y^2 = b^2\left(1-\frac{x^2}{a^2}\right)$

$y = \pm b\sqrt{1-\frac{x^2}{a^2}}$

$4b\int_0^a \sqrt{1-\frac{x^2}{a^2}}\,dx$

Substituição $\frac{x}{a} = \text{sen}\,t. \Rightarrow dx = a\cos t\,.dt$

$x=0 \Rightarrow \text{sen}\,t = 0 \Rightarrow t=0$

$x=a \Rightarrow \text{sen}\,t = 1 \Rightarrow t = \pi/2.$

n) $4b\int_0^{\pi/2} \sqrt{1-\text{sen}^2 t} \cdot a\cos t \, dt =$

$= 4ba\int_0^{\pi/2} \cos^2 t \, dt = \frac{4ab}{2}\int_0^{\pi/2} (\cos(2t)+1)\, dt =$

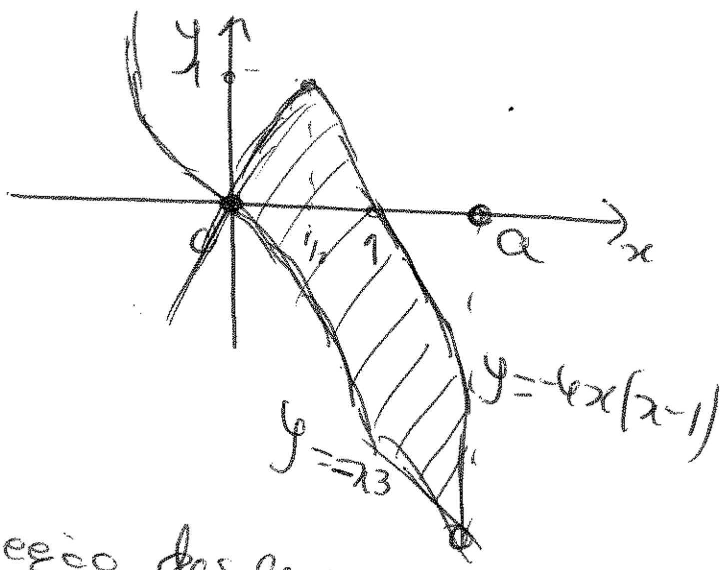
$= 2ab\left[\frac{1}{2}\text{sen}(2t)+t\right]_0^{\pi/2} = 2ab\left[\frac{1}{2}\text{sen}\,\pi + \frac{\pi}{2}\right] = ab\pi.$

Ficha 5

11

o) $y=-x^3$, $y=-(4x^2-4x)$

$y=-4x(x-1)$



$\int_0^a -(4x^2-4x)+x^3 \, dx$

$=\left[-\frac{4x^3}{3}+\frac{4x^2}{2}+\frac{x^4}{4}\right]_0^a$ onde

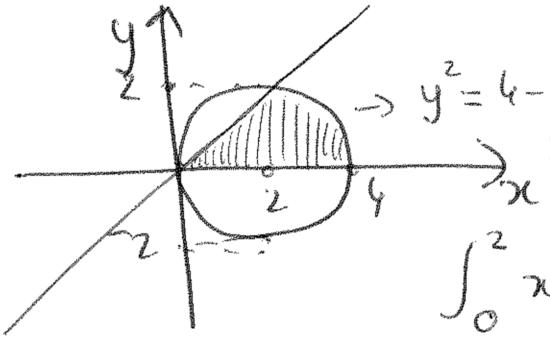
a é abcissa do ptº de intersecção das curvas.

$\begin{cases} y=-(4x^2-4x) \\ y=-x^3 \end{cases}$ $\begin{cases} x^3=4x^2-4x \Leftrightarrow x^3-4x^2+4x=0 \Leftrightarrow \\ \Leftrightarrow x(x^2-4x+4)=0 \Leftrightarrow \\ x(x-2)^2=0 \Leftrightarrow x=0 \vee x=2. \end{cases}$

$\int_0^2 -(4x^2-4x)+x^3 \, dx = \left[-\frac{4x^3}{3}+2x^2+\frac{x^4}{4}\right]_0^2 = -\frac{4\cdot 2^3}{3}+2\times 4+\frac{2^4}{4} =$

$= \frac{-4\times 8}{3}+8+4 = -\frac{32}{3}+12 = \frac{4}{3}.$

2-a) $\{(x,y)\in\mathbb{R}^2 : (x-2)^2+y^2\leq 4 \text{ e } 0<y\leq x\}$



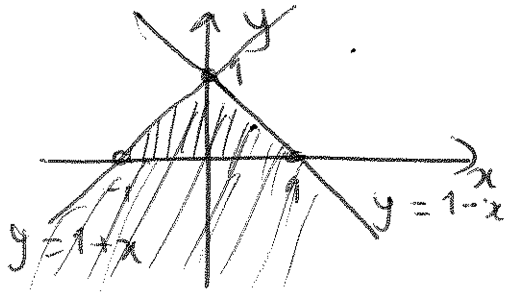
$\rightarrow y^2 = 4-(x-2)^2$

$y = \pm\sqrt{4-(x-2)^2}$

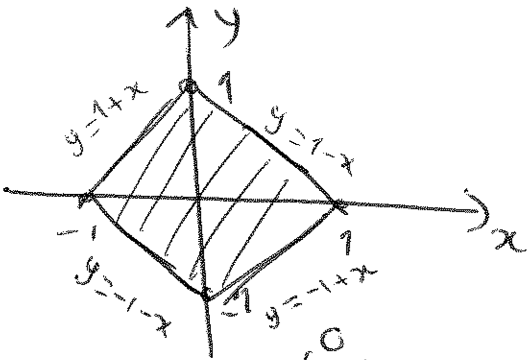
$$\int_0^2 x\,dx + \int_2^4 \sqrt{4-(x-2)^2}\,dx$$

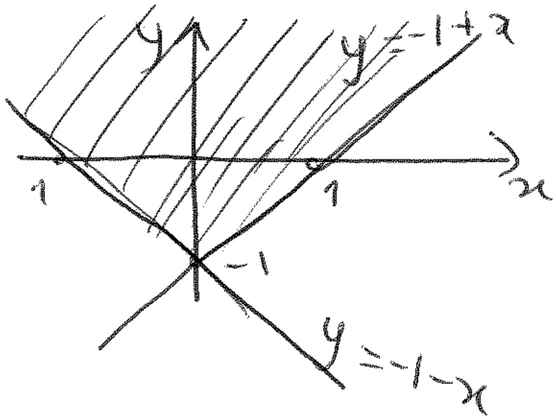
b) $\{(x,y)\in\mathbb{R}^2 : |x|+|y|\leq 1\}$

- $y\geq 0 \Rightarrow |x|+y\leq 1 \Leftrightarrow y\leq 1-|x|$



- $y\leq 0 \Leftrightarrow |x|-y\leq 1 \Leftrightarrow y\geq |x|-1$





$$\int_{-1}^0 (1+x)-(-1-x)\,dx + \int_0^1 (1-x)-(-1+x)\,dx =$$

$$= \int_{-1}^0 (2+2x)\,dx + \int_0^1 (2-2x)\,dx.$$

c) $\{(x,y)\in\mathbb{R}^2 : x\leq 3 \wedge y\geq x^2-4x+3 \text{ e } y\leq -x^2+5x-4\}$

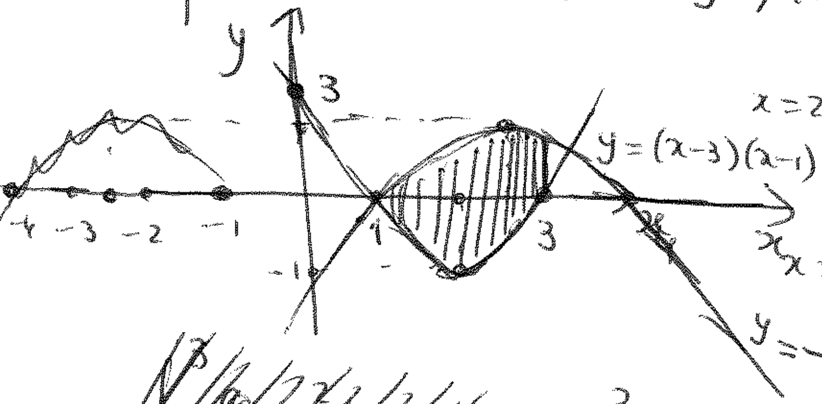
$0 = x^2-4x+3 \Leftrightarrow x = \frac{4\pm\sqrt{16-12}}{2} = \frac{4\pm 2}{2} = 2\pm 1 \Leftrightarrow x=3 \vee x=1.$

$y\geq x^2-4x+3 \Leftrightarrow y\geq (x-3)(x-1)$

$-x^2+5x-4=0 \Leftrightarrow x = \frac{-5\pm\sqrt{25-16}}{-2} \Leftrightarrow x = \frac{-5\pm 3}{-2} \Leftrightarrow x=+4 \vee x=+1$

$y\leq -x^2+5x-4 \Leftrightarrow y\leq -(x-4)(x-1)$

c) $\{(x,y)\in\mathbb{R}^2 : x\leq 3 \ e \ y\geq (x-3)(x-1) \ e \ y\leq -(x-4)(x-1)\}$



$y=(x-3)(x-1)$

$x=2 \Rightarrow y=(-1)(1)=-1$

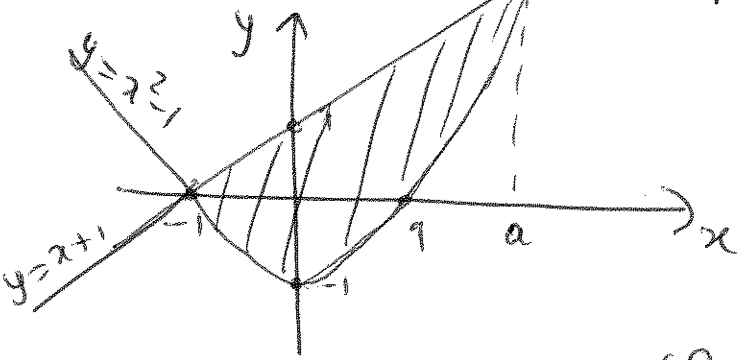
$y=-(x-4)(x-1)$

$x=+\frac{5}{2} \Longrightarrow y=-\left(-\frac{3}{2}\right)\left(+\frac{3}{2}\right)=\frac{9}{4}$

$$\int_1^3 (-x^2+5x-4)-(x^2-4x+3)\,dx =$$

$$=\int_1^3 (-2x^2+9x-7)\,dx.$$

d) $\{(x,y)\in\mathbb{R}^2 \in\mathbb{R}^2 : x^2-1\leq y\leq x+1\}$



$$\int_{-1}^{a} (x+1)-(x^2-1)\,dx =$$

$$=\int_{-1}^{a} (x+1-x^2+1)\,dx =$$

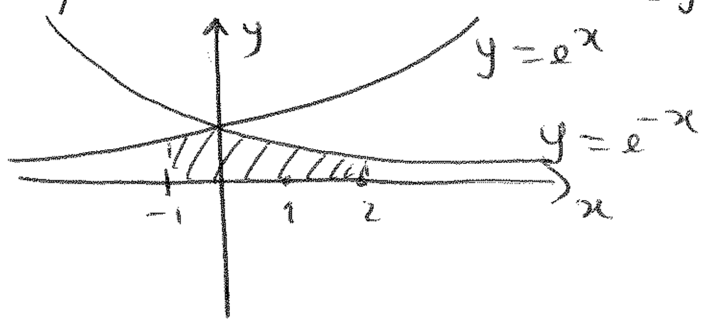
$$=\int_{-1}^{a} (-x^2+x+2)\,dx$$

onde a:

$\begin{cases} y=x+1 \\ y=x^2-1 \end{cases}$ $\begin{cases} x^2-1=x+1 \end{cases}$ $\begin{cases} x^2-x-2=0 \Leftrightarrow x=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm 3}{2} \\ x=2 \vee x=-1 \end{cases}$

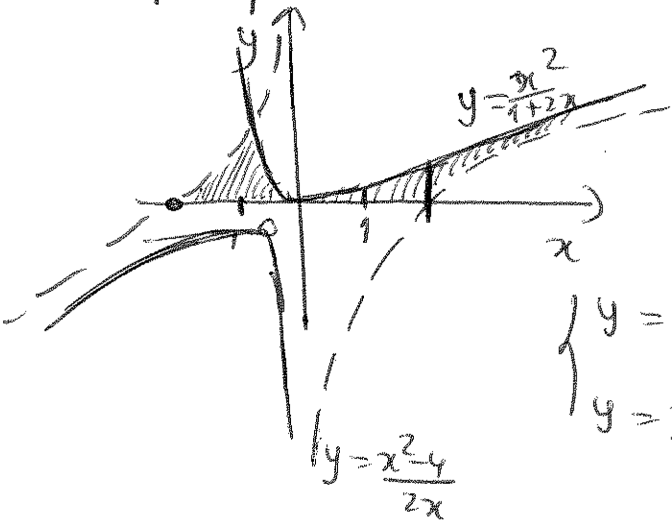
$$\int_{-1}^{2} (-x^2+x+2)\,dx.$$

e) $\{(x,y)\in\mathbb{R}^2 : -1\leq x\leq 2 \ e \ 0\leq y\leq e^x$ $e \ 0\leq y\leq e^{-x}\}$



$$\int_{-1}^{0} e^x\,dx+\int_0^2 e^{-x}\,dx.$$

2. f) $\{(x,y)\in\mathbb{R}^2 : y \geq 0 \wedge y \geq x^2-2xy \leq 4\}$



$$y = x^2-2xy \Leftrightarrow y = \frac{x^2}{1+2x}$$

$$x^2-2xy=4 \Leftrightarrow y = \frac{x^2-4}{2x}$$

$$\begin{cases} y = \frac{x^2}{1+2x} \\ y = \frac{x^2-4}{2x} \end{cases}$$

$$\frac{x^2}{1+2x} = \frac{x^2-4}{2x} \Leftrightarrow \frac{x^2-8x-4}{2x(1+2x)} = 0$$

$$\Leftrightarrow x = \frac{8\pm\sqrt{80}}{2} \Leftrightarrow x = \frac{8\pm 4\sqrt5}{2}$$

$$x = 4\pm 2\sqrt5$$

$$\int_{-2}^{4-2\sqrt5} \frac{x^2-4}{2x}\,dx + \int_{4-2\sqrt5}^{2} \frac{x^2}{1+2x}\,dx + \int_{2}^{4+2\sqrt5}\left(\frac{x^2}{1+2x} - \frac{x^2-4}{2x}\right)dx \,.$$