[redacted] Ficha 7

Integral Impróprio de 2ª espécie — I. Imp. 1ª esp.

F10.1

1a) $\int_0^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \int_0^{1} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx + \int_1^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} =$

a função integranda é ilimitada $x \to 0^+$

1

$= -2\lim_{r\to 0^+} \int_r^1 -\frac{1}{2}\frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx + -2\lim_{r\to +\infty} \int_1^r -\frac{1}{2}\frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

$= -2\lim_{r\to 0^+} \left[e^{-\sqrt{x}}\right]_r^1 - 2\lim_{r\to +\infty} \left[e^{-\sqrt{x}}\right]_1^r$

$= -2\lim_{r\to 0^+} \left[e^{-\sqrt{1}} - e^{-\sqrt{r}}\right] - 2\lim_{r\to +\infty} \left[e^{-\sqrt{r}} - e^{-\sqrt{1}}\right]$

$= -2\lim_{r\to 0^+} \left[\frac{1}{e} - \frac{1}{e^{\sqrt{r}}}\right] - 2\lim_{r\to +\infty} \left[\frac{1}{e^{\sqrt{r}}} - \frac{1}{e}\right]$

$= -2\times\left[\frac{1}{e} - \frac{1}{1}\right] - 2\times\left[\frac{1}{\infty} - \frac{1}{e}\right]$

$= -\frac{2}{e} + 2 - 2\times\left[0 - \frac{1}{e}\right] = -\cancel{\frac{2}{e}} + 2 + \cancel{\frac{2}{e}} = 2$

O integral é convergente!

b) $\int_{-\infty}^{+\infty} \frac{e^{x}}{1+e^{2x}}\,dx = \int_{-\infty}^{0} \frac{e^{x}}{1+e^{2x}}\,dx + \int_0^{+\infty} \frac{e^{x}}{1+e^{2x}}$

$= \lim_{r\to -\infty} \int_r^0 \frac{e^{x}}{1+e^{2x}}\,dx + \lim_{r\to +\infty} \int_0^r \frac{e^{x}}{1+e^{2x}}\,dx$

$= \lim_{r\to -\infty} \left[\operatorname{arctg} e^{x}\right]_r^0 + \lim_{r\to +\infty} \left[\operatorname{arctg} e^{x}\right]_0^r$

$= \lim_{r\to -\infty} \left[\operatorname{arctg} 1 - \operatorname{arctg} e^{r}\right] + \lim_{r\to +\infty} \left[\operatorname{arctg} e^{r} - \operatorname{arctg} 1\right]$

$= \cancel{\operatorname{arctg} 1} - \operatorname{arctg}(e^{-\infty} = 0) + \operatorname{arctg} e^{\infty} - \cancel{\operatorname{arctg} 1}$

$=$ [redacted] $+ \frac{\pi}{2} = \frac{\pi}{2}$

O integral é convergente!

(2)

c) $\int_0^1 \ln x \, dx \underset{\uparrow}{=} \lim_{r\to 0^+} \int_r^1 1 \cdot \ln x \, dx =$

função integranda é ilimitada
f.d.c $x \to 0^+$

$= \lim_{r\to 0^+} \left[ \left[x \cdot \ln x\right]_r^1 - \int_r^1 x \cdot \frac{1}{x} \, dx \right]$

$= \lim_{r\to 0^+} \left[ 1 \cdot \overset{0}{\ln 1} - r \cdot \ln r - [x]_r^1 \right]$

$= \lim_{r\to 0^+} \left[ \underbrace{-r \ln r}_{0\times\infty} - 1 + r \right] \underset{\substack{\uparrow \\ CA}}{=} -0 - 1 + 0 = -1$

O Integral é convergente!

C.A: $\lim_{r\to 0^+} r \cdot \ln r \overset{0\times\infty}{=} \lim_{r\to 0^+} \frac{\ln r}{\frac{1}{r}} \overset{\frac{\infty}{\infty}}{=} \lim_{r\to 0^+} \frac{\frac{1}{r}}{-\frac{1}{r^2}}$ Aplicar Regra de ~~Hopital~~ L'Hopital

$= \lim_{r\to 0^+} -r = 0$

d) $\int_0^{\frac{\pi}{2}} \frac{\operatorname{tag} x}{1+\sin^2 x} \, dx$

. Mud. var: $t = \operatorname{tg} x \iff$ . $x = \operatorname{arctg} t$

. $dx = \frac{1}{1+t^2}$

. Da fórmula: . $\operatorname{sen}^2 x + \cos^2 x = 1$ $\quad : \cos^2 x$

$\operatorname{tg}^2 x + 1 = \frac{1}{\cos^2 x} \iff t^2 + 1 = \frac{1}{\cos^2 x} \iff \cos^2 x = \frac{1}{t^2+1}$

. Assim $\sin^2 x = 1 - \cos^2 x \Rightarrow \sin^2 x = 1 - \frac{1}{t^2+1} = \frac{t^2}{t^2+1}$

. Se $x = 0 \to t = \operatorname{tg} 0 = \frac{\operatorname{sen} 0}{\cos 0} = \frac{0}{1} = 0$

Se $x = \frac{\pi}{2} \to t = \operatorname{tg} \frac{\pi}{2} = +\infty$

Assim,

$$\int_0^{\frac{\pi}{2}} \frac{tg\,x}{1+\sin^2 x}\,dx = \int_0^{+\infty} \frac{t}{1+\frac{t^2}{1+t^2}} \cdot \frac{1}{1+t^2}\cdot dt$$

$$= \int_0^{+\infty} \frac{t}{\frac{1+t^2+t^2}{\cancel{1+t^2}}} \cdot \frac{1}{\cancel{1+t^2}}\,dt = \int_0^{+\infty} \frac{t}{1+2t^2}\,dt$$

$$= \frac{1}{4} \lim_{r\to+\infty} \int_0^r \frac{4t}{1+2t^2}\,dt = \frac{1}{4} \lim_{r\to+\infty} \left[\ln(1+2t^2)\right]_0^r$$

$$= \frac{1}{4} \lim_{r\to+\infty} \left[\ln(1+2r^2) - \ln 1\right] = +\infty$$

O Integral é divergente

e) $$\int_{-1}^{1} \frac{1}{2x-1}\,dx = \int_{-1}^{\frac{1}{2}} \frac{1}{2x-1}\,dx + \int_{\frac{1}{2}}^{1} \frac{1}{2x-1}\,dx$$

a função integranda é ilimitada p/ $x = \frac{1}{2}$

$$= \frac{1}{2} \lim_{r\to\frac{1}{2}^-} \int_{-1}^{r} \frac{2\times 1}{2x-1}\,dx + \frac{1}{2} \lim_{r\to\frac{1}{2}^+} \int_{r}^{1} \frac{1\times 2}{2x-1}\,dx$$

$$= \underbrace{\frac{1}{2} \lim_{r\to\frac{1}{2}^-} \left[\ln|2x-1|\right]_{-1}^{r}}_{①} + \underbrace{\frac{1}{2} \lim_{r\to\frac{1}{2}^+} \left[\ln|2x-1|\right]_{r}^{1}}_{②}$$

CA ①: $$\lim_{r\to\frac{1}{2}^-} \left[\ln|2r-1| - \ln|-2-1|\right] = \ln|1^- - 1| - \ln 3$$

$$= \ln 0^+ - \ln 3 = -\infty$$ ∴ divergente

Como um dos integrais é divergente, então

$$\int_{-1}^{1} \frac{1}{2x-1}\,dx$$ é divergente.

2.

2a) $\text{Área} = \int_1^{+\infty} \left(\frac{4}{2x+1} - \frac{2}{x+2}\right) dx =$

$= \lim_{r\to+\infty} \left[\int_1^r \left(\frac{4}{2x+1}\right) dx - \int_1^r \frac{2}{x+2} dx\right]$

$= \lim_{r\to+\infty} \left[2\left[\ln|2x+1|\right]_1^r - 2\left[\ln(x+2)\right]_1^r\right]$

$= \lim_{r\to+\infty} \left[2\left(\ln|2r+1| - \ln|3|\right) - 2\left(\ln|r+2| - \ln 3\right)\right]$

$= \lim_{r\to+\infty} \left[2\ln(2r+1) - 2\cancel{\ln 3} - 2\ln(r+2) + 2\cancel{\ln 3}\right] =$

$\overset{-\infty+\infty}{=} \lim_{r\to+\infty} \left[2\ln\left(\overset{\infty/\infty}{\frac{2r+1}{r+2}}\right)\right] = 2\ln 2$

CA:

CA:

$\lim_{r\to+\infty} \frac{2r+1}{r+2} \overset{\infty/\infty}{=} \lim_{r\to+\infty} \frac{2}{1} = 2$ regra L'Hôpital

1f) $\int_0^{+\infty} \frac{1}{\sqrt{e^x}} dx = \int_1^{+\infty} \frac{1}{t^{\frac{1}{2}}} \cdot \frac{1}{t} dt = \rightarrow$

Mud. var.

Mud. var.: $e^x = t \Rightarrow x = \ln t$

$dx = \frac{1}{t} \cdot dt$

- $x = 0 \Rightarrow t = 1$
- $x = +\infty \Rightarrow t = +\infty$

outro processo de resolução

$\int_0^{+\infty} (e^x)^{-\frac{1}{2}} dx$

$= -2\int_0^{+\infty} -\frac{1}{2} e^{-\frac{x}{2}} dx$

$= -2 \left[e^{-\frac{x}{2}}\right]_0^{+\infty}$

$= -2\left[e^{-\infty} - e^0\right]$

$= 2$

(+ simples!)

F10.3

5

$$= \lim_{r\to+\infty} \int_1^r t^{-\frac{3}{2}} dt = \lim_{r\to+\infty} \left[\frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right]_1^r$$

$$= -2 \lim_{r\to+\infty} \left[\frac{1}{\sqrt{t}}\right]_1^r = -2 \lim_{r\to+\infty} \left[\frac{1}{\sqrt{r}} - 1\right]$$

$$= -2\left[\frac{1}{\infty} - 1\right] = -2[0-1] = 2$$

O integral é convergente!

1g) $\int_0^{+\infty} \frac{t}{t^2+1} dt = \frac{1}{2} \lim_{r\to+\infty} \int_0^r \frac{2t}{t^2+1} dt$

$$= \frac{1}{2} \lim_{r\to+\infty} \left[\ln(t^2+1)\right]_0^r = \frac{1}{2} \lim_{r\to+\infty} \left[\ln(r^2+1) - \underbrace{\ln 1}_{0}\right]$$

$= +\infty$ $\therefore$ divergente

1h) $\int_0^{+\infty} \frac{1}{\sqrt{x^2+1}} dx = \lim_{r\to+\infty} \int_0^r \frac{1}{\sqrt{x^2+1}} dx =$

$$= \lim_{r\to+\infty} \left[\operatorname{argsh} x\right]_0^r = \lim_{r\to+\infty} \left[\operatorname{argsh} r - \underbrace{\operatorname{argsh} 0}_{0}\right]$$

$= +\infty$ $\therefore$ divergente

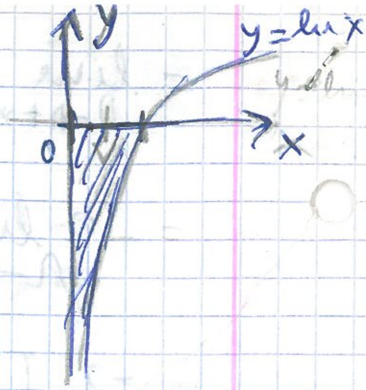
1i) $\int_1^2 \frac{1}{\sqrt[3]{x-1}} dx = \lim_{r\to 1^+} \int_r^2 (x-1)^{-\frac{1}{3}} dx$

função integranda é ilimitada $x=1$

$$= \lim_{r\to 1^+} \left[\frac{(x-1)^{\frac{2}{3}}}{\frac{2}{3}}\right]_r^2 = \frac{3}{2} \times \lim_{r\to 1^+} \left[\sqrt[3]{(x-1)^2}\right]_r^2$$

$$= \frac{3}{2} \lim_{r\to 1^+} \left[\underbrace{\sqrt[3]{1}}_{1} - \underbrace{\sqrt[3]{(1^+-1)^2}}_{0}\right] \quad \frac{3}{2} \times 1 = \frac{3}{2}$$

2b) $\text{Área} = \int_0^{1} (0 - \ln x)\, dx$ ⑥




$= - \lim_{r \to 0^+} \int_r^1 \ln x \, dx$

função integrada ilimitada p/ $x=0$

R.P.P. $= - \lim_{r\to 0^+} \left[ [x \ln x]_r^1 - \int_r^1 x \cdot \frac{1}{x} dx \right]$

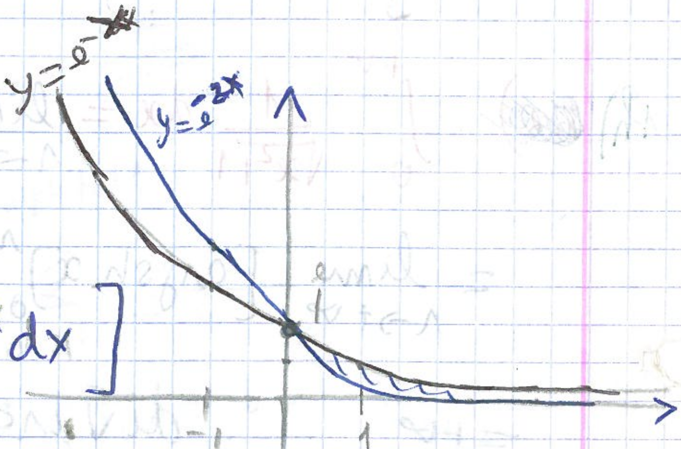
$= - \lim_{r \to 0^+} \left[ 1 \overset{=0}{\ln 1} - r \ln r - [x]_r^1 \right]$

$= - \lim_{r\to 0^+} \left[ - r \ln r - (1 - r) \right]$

$= \lim_{r \to 0^+} \underbrace{r \times \ln r}_{0 \times \infty \text{ C.A.}} + \lim_{r \to 0^+} (1 - r) \overset{\text{C.A.}}{=} 0 + 1 = 1$

C.A. $\lim_{r \to 0^+} \frac{\ln r}{\frac{1}{r}} \overset{\frac{\infty}{\infty}}{=} \overset{\text{Regra L'Hôpital}}{=} \lim_{r \to 0^+} \frac{\frac{1}{r}}{-\frac{1}{r^2}} = \lim_{r \to 0^+} -r = 0$

2c) $\text{Área} = \int_0 e^{-x} - e^{-2x}$




$\text{Área} = \int_0^{+\infty} (e^{-x} - e^{-2x}) dx$

$= \lim_{r \to +\infty} \left[ -\int_0^r -e^{-x} dx - \frac{1}{-2} \int_0^r (-2) e^{-2x} dx \right]$

$= \lim_{r \to +\infty} \left[ -[e^{-x}]_0^r + \frac{1}{2} [e^{-2x}]_0^r \right]$

$= \lim_{r \to +\infty} \left[ -[e^{-r} - 1] + \frac{1}{2} [e^{-2r} - 1] \right]$

$= \lim_{r \to +\infty} \left[ -\frac{1}{e^r} + 1 + \frac{1}{2} \times \frac{1}{e^{2r}} - \frac{1}{2} \right] = 0 + 1 + 0 - \frac{1}{2} = \frac{1}{2}$ //

$$3.\quad M = \underbrace{-c}_{+} \int_0^{+\infty} t\, e^{ct}\, dt \underset{\uparrow \text{R.P.P}}{=} -c \lim_{r\to+\infty} \frac{1}{c}\int_0^{r} \underset{g}{ct}\, \underset{f}{e^{ct}}\, dt$$

$$= -c \lim_{r\to+\infty} \left[\left[\frac{1}{c}e^{ct}\cdot t\right]_0^r - \int_0^r \frac{e^{ct}}{c}\, dt\right]$$

$$= -c \lim_{r\to+\infty} \left[\frac{r}{c}e^{cr} - \frac{1}{c}\frac{1}{c}\int_0^r c\, e^{ct}\, dt\right]$$

$$= -c \lim_{r\to+\infty} \left[\frac{r}{c}\, e^{cr} - \frac{1}{c^2}\left[e^{ct}\right]_0^r\right]$$

$$= \underbrace{-\cancel{c}}_{+} \lim_{r\to+\infty} \left[\frac{r}{\cancel{c}}e^{cr} - \frac{1}{c^2}\left(e^{cr}-1\right)\right] = \lim\left[e^{cr}\cdot\left(r-\frac{1}{c}\right)\right] + \frac{1}{c}$$

$$= +\infty$$

$$4.\quad \bar{v} = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \cdot \underbrace{\int_0^{+\infty} v^3 \cdot e^{-\overbrace{\frac{M \cdot v^2}{2RT}}^{=c}}}_{A}\, dv$$

Representado $\frac{M}{2RT} = c$, o integral dado em A vem:

$$\int_0^{+\infty} v^3 \cdot e^{-cv^2}\, dv \underset{\uparrow \text{R.P.P.}}{=} \lim_{r\to+\infty} \int_0^r \underset{g}{v^2} \cdot \underset{f}{v\, e^{-cv^2}}\, dv =$$

$$\left(F = \frac{-1}{2c}\int -2cv\, e^{-cv^2}\, dv = \frac{-1}{2c}\cdot e^{-cv^2}\ ;\ g'(v) = 2v\right)$$

$$= \lim_{r\to+\infty} \left(\left[\overbrace{-\frac{1}{2c}e^{-cv^2}}^{F} \cdot \underset{g}{v^2}\right]_0^r - \left(\frac{-1}{2c}\right)\frac{-1}{c}\int_0^r \overbrace{-cv \cdot e^{-cv^2}}^{F} \times \underset{g'}{1}\, dv\right)$$

$$= \lim_{r\to+\infty} \left[-\frac{1}{2c}e^{-cr^2}\cdot r^2 - 0 - \frac{1}{2c^2}\left[e^{-cv^2}\right]_0^r\right)$$

$$= \lim_{r\to+\infty} \left[-\frac{1}{2c}e^{-cr^2}\cdot r^2 - \frac{1}{2c^2}\left(e^{-cr^2}-1\right)\right] =$$

$$= \lim_{r\to+\infty} \left(\underbrace{-\frac{r^2}{2c\, e^{cr^2}}}_{\text{R L'Hôp}} - \underbrace{\frac{1}{2c^2 e^{cr^2}}}_{\to 0} + \underbrace{\frac{1}{2c^2}}_{\text{cont}}\right) =$$

$$= \lim_{r\to+\infty} \frac{-2r}{4c^2 r\, e^{cr^2}} - 0 + \frac{1}{2c^2} = 0 + \frac{1}{2c^2} = \frac{1}{2c^2}$$

Assim,

$$V = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \times \frac{1}{2c^2} = \frac{4}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{3/2} \times \frac{1}{2\left(\frac{M}{2RT}\right)^2}$$

mas $c = \frac{M}{2RT}$

$$= \frac{\overset{2}{\cancel{4}}}{\sqrt{\pi}} \times \left(\frac{M}{2RT}\right)^{3/2} \times \frac{1}{\cancel{2}} \times \frac{4R^2T^2}{M^2}$$

$$= \frac{2^3}{\sqrt{\pi}} \cdot \frac{M^{3/2}}{2^{3/2} \cdot R^{3/2} \cdot T^{3/2}} \times \frac{R^2 \times T^2}{M^2} = \frac{2^{3-\frac{3}{2}}}{\sqrt{\pi}} \cdot M^{\frac{3}{2}-2} \times R^{2-\frac{3}{2}} \times T^{2-\frac{3}{2}}$$

$$= \frac{2^{3/2}}{\sqrt{\pi}} \times M^{-\frac{1}{2}} \times R^{+1/2} \times T^{+1/2} = \sqrt{\frac{8RT}{\pi M}}$$