Formulário

Sejam $f, g\colon I \longrightarrow \mathbb{R}$ funções deriváveis no intervalo real I.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$(f(x) \times g(x))' = f'(x)g(x) + g'(x)f(x) \qquad \left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}$$

$$(f[g(x)])' = g'(x)f'[g(x)] \qquad [f^{-1}(y)]' = \frac{1}{f'[f^{-1}(y)]}$$

Sejam $f: I \longrightarrow \mathbb{R}$ uma função derivável no intervalo real I e $\mathcal{C}$ uma constante real arbitrária e $\{k, n\} \in \mathbb{R}$.

1.$Pk = kx + \mathcal{C}$

2.$P\ f'(x)f^n(x) = \dfrac{f^{n+1}(x)}{n+1} + \mathcal{C} \quad (n \neq -1)$

3.$P\ \dfrac{f'(x)}{f(x)} = \ln|f(x)| + \mathcal{C}$

4.$P\ f'(x)e^{f(x)} = e^{f(x)} + \mathcal{C}$

5.$P\ a^{f(x)}\, f'(x) = \dfrac{a^{f(x)}}{\ln a} + \mathcal{C} \quad (a \in \mathbb{R}^+ \backslash 1)$

6.$P\ f'(x)\cos(f(x)) = \operatorname{sen}(f(x)) + \mathcal{C}$

7.$P\ f'(x)\operatorname{sen}(f(x)) = -\cos(f(x)) + \mathcal{C}$

8.$P\ f'(x)\sec^2(f(x)) = \operatorname{tg}(f(x)) + \mathcal{C}$

9.$P\ f'(x)\operatorname{cosec}^2(f(x)) = -\operatorname{cotg}(f(x)) + \mathcal{C}$

10.$P\ f'(x)\sec(f(x)) = \ln|\sec(f(x)) + \operatorname{tg}(f(x))| + \mathcal{C}$

11.$P\ f'(x)\operatorname{cosec}(f(x)) = \ln|\operatorname{cosec}(f(x)) - \operatorname{cotg}(f(x))| + \mathcal{C}$

12.$P\ \dfrac{f'(x)}{\sqrt{1 - f^2(x)}} = \operatorname{arcsen}(f(x)) + \mathcal{C}$

13.$P\ \dfrac{-f'(x)}{\sqrt{1 - f^2(x)}} = \arccos(f(x)) + \mathcal{C}$

14.$P\ \dfrac{f'(x)}{1 + f^2(x)} = \operatorname{arctg}(f(x)) + \mathcal{C}$

15.$P\ \dfrac{-f'(x)}{1 + f^2(x)} = \operatorname{arccotg}(f(x)) + \mathcal{C}$

16.$P\ \dfrac{f'(x)}{|f(x)|\sqrt{f^2(x) - 1}} = \operatorname{arcsec}(f(x)) + \mathcal{C}$

17.$P\ f'(x)\operatorname{ch}(f(x)) = \operatorname{sh}(f(x)) + \mathcal{C}$

18.$P\ f'(x)\operatorname{sh}(f(x)) = \operatorname{ch}(f(x)) + \mathcal{C}$

19.$P\ \dfrac{f'(x)}{\operatorname{ch}^2(f(x))} = \operatorname{th}(f(x)) + \mathcal{C}$

20.$P\ \dfrac{f'(x)}{\operatorname{sh}^2(f(x))} = -\operatorname{coth}(f(x)) + \mathcal{C}$

21.$P\ \dfrac{f'(x)}{\sqrt{f^2(x) + 1}} = \operatorname{argsh}(f(x)) + \mathcal{C}$

22.$P\ \dfrac{f'(x)}{\sqrt{f^2(x) - 1}} = \operatorname{argch}(f(x)) + \mathcal{C}$

23.$P\ \dfrac{f'(x)}{1 - f^2(x)} = \operatorname{argth}(f(x)) + \mathcal{C}$

24.$P\ \dfrac{f'(x)}{1 - f^2(x)} = \operatorname{argcoth}(f(x)) + \mathcal{C}$

$\operatorname{sen}\frac{\pi}{6} = \cos\frac{\pi}{3} = \frac{1}{2}$	$\operatorname{sen}\frac{\pi}{3} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\operatorname{sen}\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \operatorname{sen}\alpha\operatorname{sen}\beta$	$\cos^2\alpha = \dfrac{1 + \cos 2\alpha}{2}$
$\sec\alpha = \frac{1}{\cos\alpha}$	$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \operatorname{sen}\beta\cos\alpha$	$\operatorname{sen}^2\alpha = \dfrac{1 - \cos 2\alpha}{2}$
$\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha}$	$\sec^2\alpha = 1 + \operatorname{tg}^2\alpha$	$\operatorname{cosec}^2\alpha = 1 + \operatorname{cotg}^2\alpha$
$\operatorname{ch} u = \dfrac{e^u + e^{-u}}{2}$	$\operatorname{sh} u = \dfrac{e^u - e^{-u}}{2}$	$\operatorname{ch}^2 u - \operatorname{sh}^2 u = 1$
$\operatorname{th} u = \dfrac{\operatorname{sh} u}{\operatorname{ch} u}$	$\operatorname{ch}(u \pm v) = \operatorname{ch} u\operatorname{ch} v \pm \operatorname{sh} u\operatorname{sh} v$	$\operatorname{ch}^2 u = \dfrac{\operatorname{ch}(2u) + 1}{2}$
$\operatorname{coth} u = \dfrac{1}{\operatorname{th} u}$	$\operatorname{sh}(u \pm v) = \operatorname{sh} u\operatorname{ch} v \pm \operatorname{sh} v\operatorname{ch} u$	$\operatorname{sh}^2 u = \dfrac{\operatorname{ch}(2u) - 1}{2}$

Primitivação por Substituição

Na lista de substituições que se segue, a, b e c são constantes reais arbitrárias. A notação $R(\cdots)$ indica uma função racional dos monómios que se encontram dentro dos parêntesis. Na coluna da esquerda, figuram diferentes tipos de funções primitiváveis Na coluna da direita sugere-se, em cada caso, uma substituição adequada à função indicada na coluna da esquerda.

Tipo de Função	Substituição
1. $\dfrac{1}{(x^2+a^2)^k}$, $k \in \mathbb{N}$, $k > 1$	$x = a \operatorname{tg} t$
2. $R(a^{rx},\ a^{sx},\ \ldots)$	$a^{mx} = t$ com $m = \text{m.d.c.}(r, s, \ldots)$
3. $R(\log_a x)$	$t = \log_a x$
4. $R\left(x, \left(\dfrac{ax+b}{cx+d}\right)^{p/q}, \left(\dfrac{ax+b}{cx+d}\right)^{r/s}, \ldots\right)$	$\dfrac{ax+b}{cx+d} = t^m$ com $m = \text{m.m.c.}(q, s, \ldots)$
5. $R\left(x, (ax+b)^{p/q},\ (ax+b)^{r/s}, \ldots\right)$	$(ax+b) = t^m$ com $m = \text{m.m.c.}(q, s, \ldots)$
6. $R\left(x, x^{p/q},\ x^{r/s}, \ldots\right)$	$x = t^m$ com $m = \text{m.m.c.}(q, s, \ldots)$
7. $R\left(x, \sqrt{a^2 - b^2x^2}\right)$	$x = \dfrac{a}{b} \operatorname{sen} t$ ou $x = \dfrac{a}{b} \cos t$ ou $x = \dfrac{a}{b} \operatorname{th} t$
8. $R\left(x, \sqrt{a^2 + b^2x^2}\right)$	$x = \dfrac{a}{b} \operatorname{tg} t$ ou $x = \dfrac{a}{b} \operatorname{sh} t$
9. $R\left(x, \sqrt{b^2x^2 - a^2}\right)$	$x = \dfrac{a}{b} \sec t$ ou $x = \dfrac{a}{b} \operatorname{ch} t$
10. $R(\operatorname{sen} x, \cos x)$ com	
(a) R ímpar em $\operatorname{sen} x$ isto é $R(-\operatorname{sen} x, \cos x) = -R(\operatorname{sen} x, \cos x)$	$\cos x = t$
(b) R ímpar em $\cos x$ isto é $R(\operatorname{sen} x, -\cos x) = -R(\operatorname{sen} x, \cos x)$	$\operatorname{sen} x = t$
(c) R par em $(\operatorname{sen} x, \cos x)$ isto é $R(-\operatorname{sen} x, -\cos x) = R(\operatorname{sen} x, \cos x)$	$\operatorname{tg} x = t$, sendo então (supondo $x \in]0, \pi/2[$) $\operatorname{sen} x = \dfrac{t}{\sqrt{1+t^2}}$, $\cos x = \dfrac{1}{\sqrt{1+t^2}}$
(d) nos restantes casos (e até nos anteriores)	$\operatorname{tg} \dfrac{x}{2} = t$, sendo $\operatorname{sen} x = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$
11. $R(\operatorname{sh} x, \operatorname{ch} x)$ com	
(a) R ímpar em $\operatorname{sh} x$	$\operatorname{ch} x = t$
(b) R ímpar em $\operatorname{ch} x$	$\operatorname{sh} x = t$
(c) R par em $(\operatorname{sh} x, \operatorname{ch} x)$	$\operatorname{th} x = t$, sendo $\operatorname{sh} x = \dfrac{t}{\sqrt{1-t^2}}$, $\operatorname{ch} x = \dfrac{1}{\sqrt{1-t^2}}$
(d) nos restantes casos (e até nos anteriores)	$\operatorname{th} \dfrac{x}{2} = t$, sendo $\operatorname{sh} x = \dfrac{2t}{1-t^2}$, $\operatorname{ch} x = \dfrac{1+t^2}{1-t^2}$