PRML-Assignment 2

Rohith Ingilela, EE19BTECH11005

April 8, 2023

1 Problem Statement

In Figure 1, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\,cm$, $AE = 8\,cm$ and $CF = 10\,cm$, find AD.

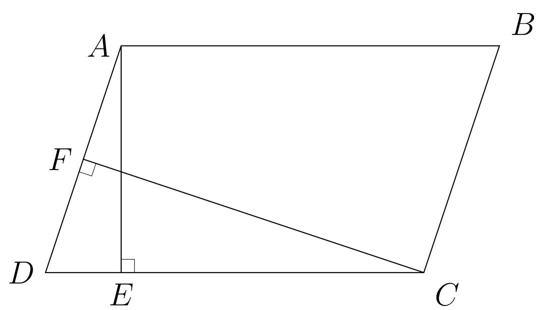

Figure 1: Parallelogram ABCD

2 Solution

Let $A = \begin{pmatrix} x \\ 8 \end{pmatrix}$, $B = \begin{pmatrix} x+16 \\ 8 \end{pmatrix}$, $C = \begin{pmatrix} 16 \\ 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $E = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$$Ar(ABCD) = \|AD\| \times \|CF\| = \|AE\| \times \|CD\|$$

$$\|AD\| \times 10 = 8 \times 16 = 128$$

$$\|AD\| = 12.8\ cm \tag{1}$$

$$\|AD\| = \|A - D\| = 12.8\ cm$$

$$\|A\| = 12.8$$

$$x^2 + 8^2 = 12.8^2$$

$$x \approx 10 \tag{2}$$

$$A = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \tag{3}$$

Given,

$$\|CF\| = \|F - C\| = 10$$

Squaring on both sides,

$$F^T F - 2C^T F + C^T C = 100$$

$$2C^T F - F^T F = 156 \qquad (\because C^T C = 256) \tag{4}$$

From Figure 1, $DF \perp CF$

$$(F - D)^T (C - F) = 0$$

$$C^T F - F^T F = 0 \tag{5}$$

From (4) and (5),

$$C^T F = 156$$

$$\begin{pmatrix} 16 & 0 \end{pmatrix} F = 156 \tag{6}$$

Equation of line passing through AD:

$$\text{Direction vector, } m = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

Normal vector,

$$\implies n = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$$

Equation of line passing through D with normal vector n is

$$n^T (x - D) = 0$$

$$\implies \begin{pmatrix} -8 & 10 \end{pmatrix} x = 0$$

Since F passes through AD,

$$\begin{pmatrix} -8 & 10 \end{pmatrix} F = 0 \tag{7}$$

From (6) and (7),

$$\begin{pmatrix} 16 & 0 \\ -8 & 10 \end{pmatrix} F = \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$F = \begin{pmatrix} 16 & 0 \\ -8 & 10 \end{pmatrix}^{-1} \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$F = \tfrac{1}{160} \begin{pmatrix} 10 & 0 \\ 8 & 16 \end{pmatrix} \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$F = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{8}$$

3 Code

https://github.com/1R0H1TH/PRML/blob/main/9.9.2.1/codes/9.9.2.1.py

4 Plot

The above code plots Figure 2. .

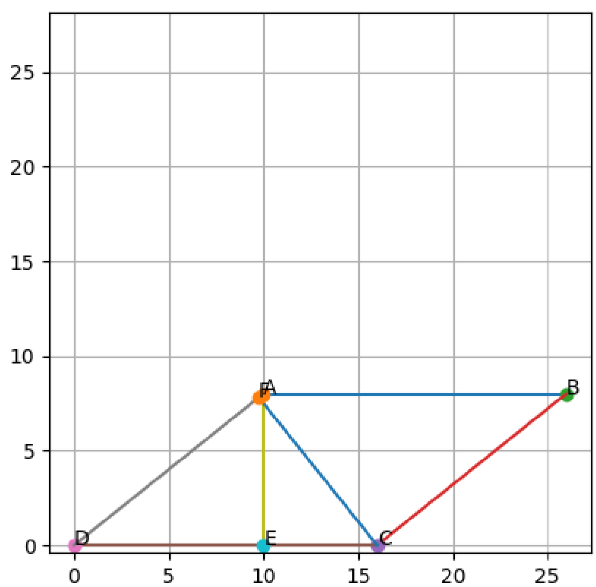

Figure 2: Parallelogram ABCD