PRML-Assignment 2

Rohith Ingilela, EE19BTECH11005

April 8, 2023

1 Problem Statement

In Figure 1, ABCD is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If AB = 16~cm, AE = 8~cm and CF = 10~cm, find AD. Construct the parallelogram.

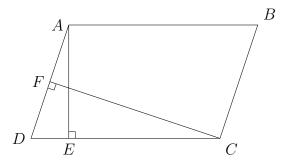


Figure 1: Parallelogram ABCD

2 Solution

Given,

LHS	RHS
A-B	C-D
$(\boldsymbol{A}-\boldsymbol{E})^T(\boldsymbol{C}-\boldsymbol{D})$	0
$(\boldsymbol{C} - \boldsymbol{F})^T (\boldsymbol{A} - \boldsymbol{D})$	0
$\ A-B\ $	16 cm
$\ oldsymbol{A} - oldsymbol{E}\ $	8 cm
$\ C-F\ $	10 cm

To find: ||AD||

We know that,

$$Ar(ABCD) = \|\mathbf{A} - \mathbf{D}\| \times \|\mathbf{C} - \mathbf{F}\| = \|\mathbf{A} - \mathbf{E}\| \times \|\mathbf{C} - \mathbf{D}\|$$

 $\|\mathbf{A} - \mathbf{D}\| \times 10 = 8 \times 16 = 128$
 $\|\mathbf{A} - \mathbf{D}\| = 12.8 \text{ cm}$
 $\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A}\| = 12.8 \text{ cm}$

To find: \boldsymbol{A}

Let $\theta = \angle ADE$

$$\sin \theta = \frac{\|\mathbf{A} - \mathbf{E}\|}{\|\mathbf{A} - \mathbf{D}\|}$$

$$\theta = \sin^{-1} \left(\frac{\|\mathbf{A} - \mathbf{E}\|}{\|\mathbf{A} - \mathbf{D}\|} \right)$$

$$(1)$$

$$\boldsymbol{A} = \|\boldsymbol{A} - \boldsymbol{D}\| \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$
 (2)

Substituting ||AE|| and ||AD|| in (1) and (2)

$$\mathbf{A} \approx \begin{pmatrix} 10\\8 \end{pmatrix} \tag{3}$$

To find: \boldsymbol{F}

Equation of line passing through AD:

Direction vector,
$$\mathbf{m} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

Normal vector,

$$\implies n = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$$

Equation of line passing through D with normal vector n is

$$\boldsymbol{n}^T(\boldsymbol{x} - \boldsymbol{D}) = 0$$

$$\boldsymbol{n}^T \boldsymbol{x} = 0 \tag{4}$$

Since ${m F}$ is foot of perpendicular from ${m C}$ to line ${m A}{m D}$

$$\begin{pmatrix} m & n \end{pmatrix}^T \boldsymbol{F} = \begin{pmatrix} m^T C \\ 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 10 & 8 \\ 8 & -10 \end{pmatrix}^{-1} \begin{pmatrix} 160 \\ 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{5}$$

3 Code

https://github.com/1ROH1TH/PRML/blob/main/9.9.2.1/codes/9.9.2.1.py

4 Plot

The above code plots Figure 2. .

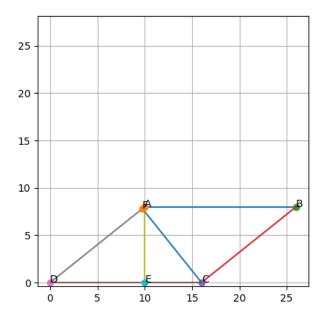


Figure 2: Parallelogram ABCD