PRML-Assignment 2

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1 Problem Statement

In Figure 1, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\,cm$, $AE = 8\,cm$ and $CF = 10\,cm$, find AD. Construct the paralellogram.

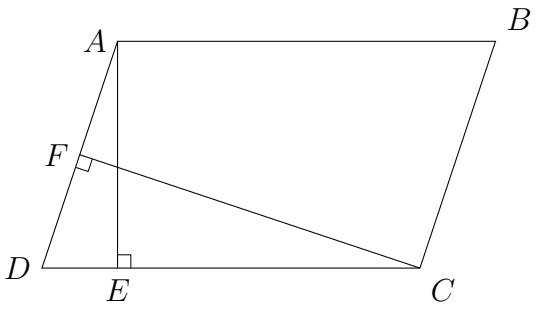

Figure 1: Parallelogram ABCD

2 Solution

Given,

$$AE \perp DC \implies (\boldsymbol{A}-\boldsymbol{E})^T(\boldsymbol{C}-\boldsymbol{D}) = 0 \tag{1}$$

$$CF \perp AD \implies (\boldsymbol{C}-\boldsymbol{F})^T(\boldsymbol{A}-\boldsymbol{D}) = 0 \tag{2}$$

$$\|AB\| = \|\boldsymbol{A}-\boldsymbol{B}\| = 16cm \tag{3}$$

$$\|AE\| = \|\boldsymbol{A}-\boldsymbol{E}\| = 8cm \tag{4}$$

$$\|CF\| = \|\boldsymbol{C}-\boldsymbol{F}\| = 10cm \tag{5}$$

To find: $\|AD\|$

We know that,

$$Ar(ABCD) = \|AD\| \times \|CF\| = \|AE\| \times \|CD\|$$

$$\|AD\| \times 10 = 8 \times 16 = 128$$

$$\|AD\| = 12.8\ cm \tag{6}$$

$$\|AD\| = \|\boldsymbol{A} - \boldsymbol{D}\| = 12.8\ cm$$

To find: $\boldsymbol{A}$

Let $\boldsymbol{A} = \begin{pmatrix} x \\ 8 \end{pmatrix}$

$$\|\boldsymbol{A}\| = 12.8$$

$$x^2 + 8^2 = 12.8^2$$

$$x \approx 10 \tag{7}$$

$$\boldsymbol{A} = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \tag{8}$$

To find: $\boldsymbol{F}$

Given,

$$\|CF\| = \|\boldsymbol{F} - \boldsymbol{C}\| = 10$$

Squaring on both sides,

$$\boldsymbol{F}^T\boldsymbol{F} - 2\boldsymbol{C}^T\boldsymbol{F} + \boldsymbol{C}^T\boldsymbol{C} = 100$$

$$2\boldsymbol{C}^T\boldsymbol{F} - \boldsymbol{F}^T\boldsymbol{F} = 156 \quad (\because \boldsymbol{C}^T\boldsymbol{C} = 256) \tag{9}$$

From Figure 1, $DF \perp CF$

$$(\boldsymbol{F} - \boldsymbol{D})^T(\boldsymbol{C} - \boldsymbol{F}) = 0$$

$$\boldsymbol{C}^T\boldsymbol{F} - \boldsymbol{F}^T\boldsymbol{F} = 0 \tag{10}$$

From (9) and (10),

$$\boldsymbol{C}^T\boldsymbol{F} = 156 \tag{11}$$

Equation of line passing through AD:

$$\text{Direction vector, } \boldsymbol{m} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

Normal vector,

$$\implies \boldsymbol{n} = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$$

Equation of line passing through D with normal vector n is

$$\boldsymbol{n}^T(\boldsymbol{x} - \boldsymbol{D}) = 0$$

Since $\boldsymbol{F}$ passes through AD,

$$\boldsymbol{n}^T\boldsymbol{F} = 0 \tag{12}$$

From (11) and (12),

$$\begin{pmatrix} \boldsymbol{C}^T \\ \boldsymbol{n}^T \end{pmatrix} \boldsymbol{F} = \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

Substituting values of $\boldsymbol{C}$ and $\boldsymbol{n}$

$$\begin{pmatrix} 16 & 0 \\ -8 & 10 \end{pmatrix} \boldsymbol{F} = \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 16 & 0 \\ -8 & 10 \end{pmatrix}^{-1} \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \tfrac{1}{160} \begin{pmatrix} 10 & 0 \\ 8 & 16 \end{pmatrix} \begin{pmatrix} 156 \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{13}$$

3 Code

https://github.com/1ROH1TH/PRML/blob/main/9.9.2.1/codes/9.9.2.1.py

4 Plot

The above code plots Figure 2. .

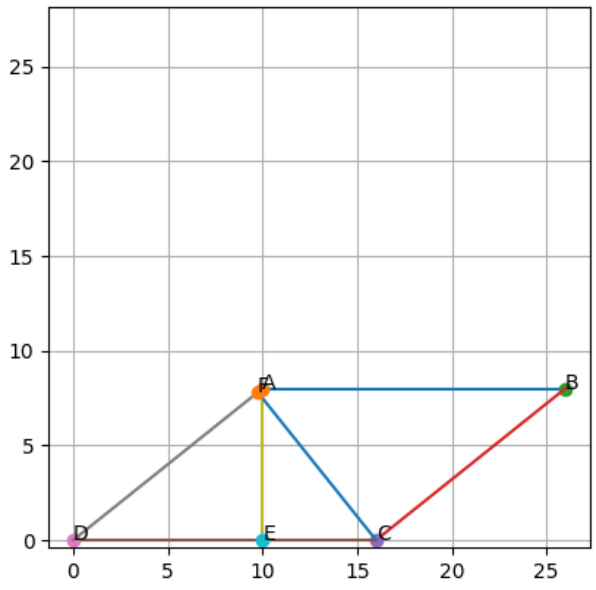

Figure 2: Parallelogram ABCD