PRML-Assignment 2

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1 Problem Statement

In Figure 1, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\,cm$, $AE = 8\,cm$ and $CF = 10\,cm$, find AD. Construct the paralellogram.

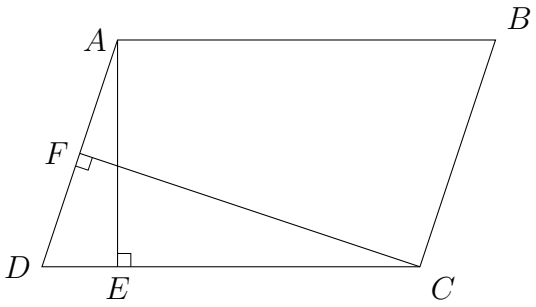

Figure 1: Parallelogram ABCD

2 Solution

Given,

LHS	**RHS**
$\boldsymbol{A-B}$	$\boldsymbol{C-D}$
$(\boldsymbol{A-E})^T(\boldsymbol{C-D})$	0
$(\boldsymbol{C-F})^T(\boldsymbol{A-D})$	0
$\|\boldsymbol{A-B}\|$	16 cm
$\|\boldsymbol{A-E}\|$	8 cm
$\|\boldsymbol{C-F}\|$	10 cm

To find: $\|AD\|$

We know that,

$$Ar(ABCD) = \|\boldsymbol{A}-\boldsymbol{D}\| \times \|\boldsymbol{C}-\boldsymbol{F}\| = \|\boldsymbol{A}-\boldsymbol{E}\| \times \|\boldsymbol{C}-\boldsymbol{D}\|$$

$$\|\boldsymbol{A}-\boldsymbol{D}\| \times 10 = 8 \times 16 = 128$$

$$\|\boldsymbol{A}-\boldsymbol{D}\| = 12.8\ cm$$

$$\|\boldsymbol{A}-\boldsymbol{D}\| = \|\boldsymbol{A}\| = 12.8\ cm$$

To find: $\boldsymbol{A}$

Let $\theta = \angle ADE$

$$\sin\theta = \frac{\|\boldsymbol{A}-\boldsymbol{E}\|}{\|\boldsymbol{A}-\boldsymbol{D}\|}$$

$$\theta = \sin^{-}1\left(\frac{\|\boldsymbol{A}-\boldsymbol{E}\|}{\|\boldsymbol{A}-\boldsymbol{D}\|}\right) \tag{1}$$

$$\boldsymbol{A} = \|\boldsymbol{A}-\boldsymbol{D}\|\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix} \tag{2}$$

Substituting $\|AE\|$ and $\|AD\|$ in (1) and (2)

$$\boldsymbol{A} \approx \begin{pmatrix}10\\ 8\end{pmatrix} \tag{3}$$

To find: $\boldsymbol{F}$

Equation of line passing through AD:

$$\text{Direction vector, } \boldsymbol{m} = \begin{pmatrix}10\\ 8\end{pmatrix}$$

Normal vector,

$$\implies \boldsymbol{n} = \begin{pmatrix}-8\\ 10\end{pmatrix}$$

Equation of line passing through D with normal vector n is

$$\boldsymbol{n}^T(\boldsymbol{x}-\boldsymbol{D}) = 0$$

$$\boldsymbol{n}^T\boldsymbol{x} = 0 \tag{4}$$

Since $\boldsymbol{F}$ is foot of perpendicular from $\boldsymbol{C}$ to line $\boldsymbol{AD}$

$$\begin{pmatrix} m & n \end{pmatrix}^T \boldsymbol{F} = \begin{pmatrix} m^T C \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 10 & 8 \\ 8 & -10 \end{pmatrix}^{-1} \begin{pmatrix} 160 \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{5}$$

3 Code

https://github.com/1ROH1TH/PRML/blob/main/9.9.2.1/codes/9.9.2.1.py

4 Plot

The above code plots Figure 2. .

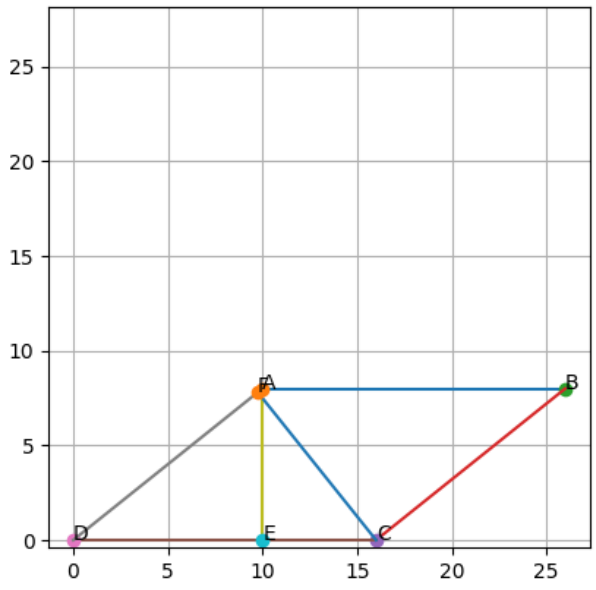

Figure 2: Parallelogram ABCD