PRML-Assignment 2

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1   Problem Statement

In Figure 1, $ABCD$ is a parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16\,cm$, $AE = 8\,cm$ and $CF = 10\,cm$, find AD. Construct the paralellogram.
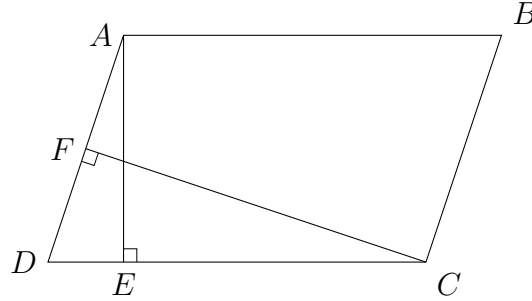

Figure 1: Parallelogram ABCD

2   Solution

Given,

$$AE \perp DC \implies (\boldsymbol{A} - \boldsymbol{E})^T (\boldsymbol{C} - \boldsymbol{D}) = 0 \tag{1}$$

$$CF \perp AD \implies (\boldsymbol{C} - \boldsymbol{F})^T (\boldsymbol{A} - \boldsymbol{D}) = 0 \tag{2}$$

$$\|AB\| = \|\boldsymbol{A} - \boldsymbol{B}\| = 16cm \tag{3}$$

$$\|AE\| = \|\boldsymbol{A} - \boldsymbol{E}\| = 8cm \tag{4}$$

$$\|CF\| = \|\boldsymbol{C} - \boldsymbol{F}\| = 10cm \tag{5}$$

To find: $\|AD\|$

We know that,

$$Ar(ABCD) = \|AD\| \times \|CF\| = \|AE\| \times \|CD\|$$

$$\|AD\| \times 10 = 8 \times 16 = 128$$

$$\|AD\| = 12.8 \; cm$$

$$\|AD\| = \|\boldsymbol{A} - \boldsymbol{D}\| = \|\boldsymbol{A}\| = 12.8 \; cm$$

To find: $\boldsymbol{A}$

Let $\theta = \angle ADE$

$$sin\theta = \frac{\|AE\|}{\|AD\|}$$

$$\theta = sin^{-}1\left(\frac{\|AE\|}{\|AD\|}\right) \tag{6}$$

$$\boldsymbol{A} = \|AD\| \begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix} \tag{7}$$

Substituting $\|AE\|$ and $\|AD\|$ in (6) and (7)

$$\boldsymbol{A} \approx \begin{pmatrix} 10 \\ 8 \end{pmatrix} \tag{8}$$

To find: $\boldsymbol{F}$

Equation of line passing through AD:

$$\text{Direction vector, } \boldsymbol{m} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

Normal vector,

$$\implies \boldsymbol{n} = \begin{pmatrix} 8 \\ -10 \end{pmatrix}$$

Equation of line passing through D with normal vector n is

$$\boldsymbol{n}^T(\boldsymbol{x} - \boldsymbol{D}) = 0$$

$$\boldsymbol{n}^T \boldsymbol{x} = 0 \tag{9}$$

Since $\boldsymbol{F}$ is foot of perpendicular from $\boldsymbol{C}$ to line $\boldsymbol{AD}$

$$\begin{pmatrix} m & n \end{pmatrix}^T \boldsymbol{F} = \begin{pmatrix} m^T C \\ 0 \end{pmatrix}$$

$$\boldsymbol{F} = \begin{pmatrix} 10 & 8 \\ 8 & -10 \end{pmatrix}^{-1} \begin{pmatrix} 160 \\ 0 \end{pmatrix}$$

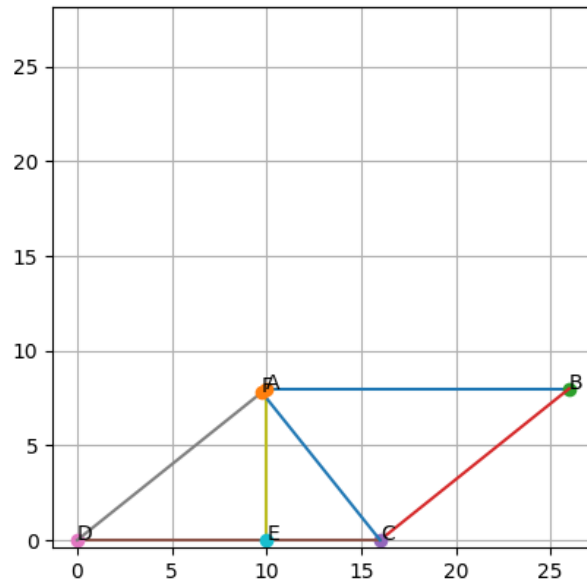$$\boldsymbol{F} = \begin{pmatrix} 9.75 \\ 7.8 \end{pmatrix} \tag{10}$$

3   Code

4   Plot

The above code plots Figure 2. .

Figure 2: Parallelogram ABCD