CFA Program Level 1”

CFA ® Program Level I

FORMULA SHEET (2025)

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CFA Level 1 - Formula Sheet (2025)

Setting Up the Texas BA II Plus Financial Calculator

Video: https://youtu.be/OMS8d8QOFmc

Using Texas BA II Plus Financial Calculator

Video: https://youtu.be/LWmTTiZz8BU

Video (Requires Login to Facebook): https://fb.watch/nci5V7Dwtj/

VOLUME 1: QUANTITATIVE METHODS

Learning Module 1: Rates and Returns

Determinants of Interest Rates

Interest rate, r = Real risk-free rate + Inflation premium + Default risk premium

- Liquidity premium + Maturity premium (1+ Nominal risk-free rate) = (1+ Real risk-free rate) × (1+ Inflation premium)

Nominal risk-free rate = Real risk-free rate + Inflation premium

Maturity premium = Interest rate on longer-maturity, liquid Treasury debt

- Interest rate on short-term Treasury debt

Holding Period Return

$$R = \frac{P_1 - P_0 + I_1}{P_0}$$

where:

- P_0 = Price at the beginning of the period
- P_1 = Price at the end of the period
- I_1 = Income

If given holding period returns $R_1, R_2, \ldots, R_T$ over the holding period:

$$R = (1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_T) - 1$$

Arithmetic Return

$$\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it} = \frac{1}{T}(R_{i1} + R_{i2} + \cdots + R_{iT})$$

Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^{T}(1 + R_t) - 1} = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times \ldots \times (1 + R_{iT})} - 1$$

Harmonic Mean

$$\bar{X}_{Hi} = \frac{n}{\sum_{i=1}^{n}(1/X_i)} \quad \text{for } X_i > 0$$

Relationship between Arithmetic Mean, Geometric Mean, and Harmonic Mean

$$(\text{ Geometric mean })^2 = \text{ Arithmetic mean } \times \text{ Harmonic mean}$$

Money-Weighted Return (MWR)

$$\sum_{t=0}^{T} \frac{CF_t}{(1 + MWR)^t} = 0$$

Time-Weighted Return (TWR)

Given the holding period returns for each sub-period, $R_1, R_2, \ldots, R_T$

If T > 1 year, then

$$\text{Annualized TWR } = [(1+R_1) \times (1+R_2) \times \ldots \times (1+R_T)]^{1/T} - 1$$

If $T = 1$ year, then

$$\text{Annualized } TWR = (1+R_1) \times (1+R_2) \times \ldots \times (1+R_T) - 1$$

If $T < 1$ year, then

$$\text{TWR for holding period } = (1+R_1) \times (1+R_2) \times \ldots \times (1+R_T) - 1$$

Non-Annual Compounding

$$PV = FV_N \left(1 + \frac{R_S}{m}\right)^{-mN}$$

where:

- m = Number of compounding periods per year
- R_s = Quoted annual interest rate
- N = Number of years

Annualizing Returns

$R_{\text{annual }} = \left(1 + R_{\text{weekly }}\right)^{52} - 1$ $R_{\text{annual }} = \left(1 + R_{\text{monthly }}\right)^{12} - 1$ $R_{\text{annual }} = \left(1 + R_{\text{daily }}\right)^{252} - 1$ assuming 252 trading days per year $R_{\text{weekly }} = \left(1 + R_{\text{daily }}\right)^{5} - 1$ assuming 5 trading days per week

Continuously Compounded Returns

$$P_t = P_0 e^{r_{0,T}}$$

$$r_{0,T} = \ln\left(\frac{P_t}{P_0}\right)$$

$$r_{0,T} = r_{0,1} + r_{1,2} + \cdots + r_{T-2,T-1} + r_{T-1,T}$$

Real Returns

(1+ real return) = (1+ real risk-free rate) × (1+ risk premium)

Pre-Tax and After-Tax Nominal Return

After-tax nominal return = Pre-tax nominal return × (1 - Tax rate) After-tax real return $= \frac{[1+ \text{Pre-Tax nominal return } \times (1- \text{ Tax rate })]}{1+ \text{Inflation premium}} - 1$

Leveraged Return

Return on a leveraged portfolio

$$R_L = R_P + \frac{V_B}{V_E}\left(R_P - r_D\right)$$

where:

- R_P = Return on the investment portfolio (unleveraged)
- r_D = Cost of debt
- V_B = Debt/borrowed funds
- V_E = Equity of the portfolio

Learning Module 2: Time Value of Money in Finance

$$FV_t = PV(1+r)^t \quad PV = \frac{FV_t}{(1+r)^t}$$

where:

- FV_t = Future value at time t
- PV = Present value
- r = Discount rate per period
- t = Number of compounding periods

As compounding frequency becomes very large (i.e., continuous compounding)

$$FV_t = PVe^{rt} \quad PV = FV_t e^{-rt}$$

Present Value of Zero-Coupon Bond

$$PV(\text{ Discount Bond }) = \frac{FV}{(1+r)^t}$$

where:

- FV = Principal (or Face Value)
- r = Market discount rate per period
- t = Maturity of bond

$$r = \left(\frac{FV_t}{PV}\right)^{1/T} - 1$$

Present Value of Coupon Bond

$$PV(\text{ Coupon Bond }) = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}$$

where:

- PV = Bond's price
- PMT = Periodic coupon payment
- FV = Face value
- N = Number of periods
- r = Market discount rate per period

Present Value of a Perpetual Bond (Perpetuity)

$$PV(\text{ Perpetual Bond }) = \frac{PMT}{r}$$

Annuity Instruments (e.g., Mortgage)

$$A = \frac{rPV}{1-(1+r)^{-t}}$$

where:

- A = Periodic cash flow
- r = Market interest rate per period
- PV = Present value or principal amount of loan/bond
- t = Number of payment periods

Price of a Preferred Share

$$PV_t = \frac{D_t}{r}$$

where:

- D_t = Fixed periodic dividend
- r = Expected rate of return

Price of a Common Share

Constant Dividend Growth Rate into Perpetuity

$$PV_t = \frac{D_t(1+g)}{r-g} = \frac{D_{t+1}}{r-g} \quad r > g$$

where:

- D_t = Common dividend at time t
- g = Constant growth rate
- r = Expected rate of return

$$r = \frac{D_{t+1}}{PV_t} + g$$

$$\frac{PV_t}{E_t} = \frac{\frac{D_t}{E_t} \times (1+g)}{r-g}$$

$$\frac{PV_t}{E_{t+1}} = \frac{\frac{D_{t+1}}{E_{t+1}}}{r-g}$$

where:

- E_t = Earnings per share for period t
- $\frac{PV_t}{E_t}$ = Trailing price-to-earnings ratio
- $\frac{PV_t}{E_{t+1}}$ = Forward price-to-earnings ratio

Two-stage Dividend Discount Model

$$PV_t = \sum_{i=1}^{n} \frac{D_t \left(1+g_s\right)^i}{(1+r)^i} + \frac{E\left(S_{t+n}\right)}{(1+r)^n}$$

where:

- g_s = Higher short-term dividend growth rate
- g_L = Lower long-term dividend growth rate
- n = Initial growth phase
- $E\left(S_{t+n}\right)$ = Stock value in n periods (Terminal value)

$$= \frac{D_{t+n+1}}{r-g_L}$$

Forward Rate

$$F_{1,1} = \frac{\left(1+r_2\right)^2}{\left(1+r_1\right)} - 1$$

where:

- $F_{1,1}$ = One-year forward rate one year from now
- r_1 = Discount rate on one-year risk-free discount bond
- r_2 = Discount rate on two-year risk-free discount bond

Learning Module 3: Statistical Measures of Asset Returns

Measures of Central Tendency

$$\text{Sample Mean,} \quad \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

where:

- X_i = Observation $i (i = 1, 2, 3, \ldots, n)$

Median

$$\text{Position of median } = \frac{\text{Number of observations } + 1}{2}$$

Quantiles

$$\text{Interquartile range } = Q_3 - Q_1$$

where:

- Q_1 = First quartile
- Q_3 = Third quartile

Box and Whisker Plot

$$\begin{aligned}\text{Upper fence } &= Q_3 + 1.5 \times IQR \\ \text{Lower fence } &= Q_1 - 1.5 \times IQR\end{aligned}$$

Measures of Dispersion

$$\text{Range } = \text{ Maximum value } - \text{ Minimum value}$$

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} \left|X_i - \bar{X}\right|}{n}$$

Sample Variance

$$s^2 = \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2}{n-1}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2}{n-1}}$$

Sample Target Semideviation

$$s_{\text{Target}} = \sqrt{\frac{\sum_{X_i \leq B}^{n} \left(X_i - B\right)^2}{n-1}}$$

where:

- B = target
- n = total number of sample observations

Coefficient of Variation

$$CV = \frac{s}{\bar{X}}$$

Sample Skewness

$$\text{Skewness} \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^3}{s^3}$$

Sample Excess Kurtosis

$$K_E \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^4}{s^4} - 3$$

Sample Covariance

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^{n} \left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)$$

Sample Correlation Coefficient

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

Learning Module 4: Probability Trees and Conditional Expectations

Expected Value of a Discrete Random Variable

$$E(X) = \sum_{i=1}^{n} P(X_i)\, X_i$$

Variance of a Random Variable

$$\begin{aligned}\sigma^2(X) &= E[X - E(X)]^2 \\ &= \sum_{i=1}^{n} P(X_i)\,[X - E(X)]^2\end{aligned}$$

Conditional Expected Value of a Random Variable

$$E(X \mid S) = P(X_1 \mid S)\, X_1 + P(X_2 \mid S)\, X_2 + \cdots + P(X_n \mid S)\, X_n$$

Conditional Variance of a Random Variable

$$\begin{gathered}\sigma^2(X \mid S) = P(X_1 \mid S)\,[X_1 - E(X_1 \mid S)]^2 + P(X_2 \mid S)\,[X_2 - E(X_2 \mid S)]^2 + \cdots \\ + P(X_n \mid S)\,[X_n - E(X_n \mid S)]^2\end{gathered}$$

Total Probability Rule for Expected Value

$$E(X) = E(X \mid S_1)\, P(S_1) + E(X \mid S_2)\, P(S_2) + \cdots + E(X \mid S_n)\, P(S_n)$$

where:

- $S_1, S_2, \ldots, S_n$ are mutually exclusive and exhaustive events.

Bayes' Formula

$$P(A \mid B) = \frac{P(B \mid A)}{P(B)} \times P(A)$$

$$P(\text{ Event } \mid \text{ Information }) = \frac{P(\text{ Information } \mid \text{ Event })}{P(\text{ Information })} \times P(\text{ Event })$$

Learning Module 5: Portfolio Mathematics

For n assets in a portfolio

Expected return on portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \cdots + w_n E(R_n)$$

Variance on portfolio

$$\sigma^2(R_P) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \operatorname{Cov}(R_i, R_j)$$

Requires n variances and $\frac{n(n-1)}{2}$ distinct covariances to estimate portfolio variance.

Covariance

$$\begin{aligned} \operatorname{Cov}(R_i, R_j) &= E\left[(R_i - E(R_i))(R_j - E(R_j))\right] \\ &= \frac{1}{n-1} \sum_{t=1}^{n} (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) \end{aligned}$$

For a two-asset ($n = 2$) portfolio:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \operatorname{Cov}(R_1, R_2)$$

where:

- $\operatorname{Cov}(R_1, R_2) = \rho(R_1, R_2) \times \sigma(R_1) \times \sigma(R_2)$

Video: https://youtu.be/IUwuIZ9ONCO

For a three-asset ($n = 3$) portfolio:

$$\begin{aligned} \sigma^2(R_P) = w_1^2 \sigma_1^2 + & w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 \operatorname{Cov}(R_1, R_2) \\ & + 2 w_1 w_3 \operatorname{Cov}(R_1, R_3) + 2 w_2 w_3 \operatorname{Cov}(R_2, R_3) \end{aligned}$$

Covariance Given a Joint Probability Function

$$\operatorname{Cov}(R_A, R_B) = \sum_{i=1} \sum_{j=1} P\left(R_{A,i}, R_{B,j}\right) \times \left[R_{A,i} - E(R_A)\right] \times \left[R_{B,j} - E(R_B)\right]$$

If X and Y are uncorrelated, then $E(XY) = E(X)E(Y)$

If X and Y are independent, then $P(X, Y) = P(X)P(Y)$

Safety-First Optimal Portfolio

Safety-First Ratio

$$\text{SFRatio } = \frac{E(R_P) - R_L}{\sigma_P}$$

$$\text{Shortfall risk } = \Pr\left[E(R_P) < R_L\right] = \text{Normal}(-\text{ SFRatio })$$

where:

- R_L = Investor's threshold level
- $E(R_P)$ = Expected portfolio return
- σ_P = Portfolio standard deviation

Video: https://youtu.be/S3x5JrGIOUA

Learning Module 6: Simulation Methods

Lognormal Distribution

Mean of a lognormal random variable

$$\mu_L = \exp\left(\mu + 0.50\sigma^2\right)$$

Variance of a lognormal random variable

$$\sigma_L^2 = \exp\left(2\mu + \sigma^2\right) \times \left[\exp\left(\sigma^2\right) - 1\right]$$

where:

- μ = Mean of the normal random variable
- σ^2 = Variance of the normal random variable

Continuously Compounded Rates of Return

$$P_T = P_0 \exp\left(r_{0,T}\right)$$

where:

- P_0 = Current asset price
- P_T = Asset price at time T
- $r_{0,T}$ = Continuously compounded return from 0 to T

If returns are independently and identically distributed (i.i.d.), then

$$r_{0,T} = r_{0,1} + r_{1,2} + \cdots + r_{T-2,T-1} + r_{T-1,T}$$

If the one-period continuously compounded returns are i.i.d. random variables with mean μ and σ^2, then

$$\begin{aligned} E\left(r_{0,T}\right) &= \mu T \\ \sigma^2\left(r_{0,T}\right) &= \sigma^2 T \\ \sigma\left(r_{0,T}\right) &= \sigma\sqrt{T} \end{aligned}$$

Learning Module 7: Estimation and Inference

$$\text{Sharpe ratio } = \frac{R_P - R_F}{\sigma_P}$$

where:

- R_P = Portfolio return
- R_F = Risk-free rate
- σ_P = Portfolio standard deviation of return

$$\begin{aligned} &\text{Variance of the sampling distribution } = \frac{\sigma^2}{n} \\ &\text{of the sample means} \\ &\text{Standard error of } = \frac{\sigma}{\sqrt{n}} \end{aligned}$$

where:

- σ = Population standard deviation
- n = Sample size

Note: If σ is not known, use s, the sample standard deviation.

Bootstrap Resampling

$$s_{\bar{X}} = \sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}_b - \bar{\theta}\right)^2}$$

where:

- $s_{\bar{X}}$ = Estimate of the standard error of the sample mean
- B = Number of resamples drawn from the original sample
- $\hat{\theta}_b$ = Mean of a resample
- $\bar{\theta}$ = Mean across all the resample means

Learning Module 8: Hypothesis Testing

$$\begin{aligned} \text{Confidence level } &= 1-\alpha \\ \text{Power of the test } &= 1-\beta \end{aligned}$$

where:

- α = Significance level (Probability of Type I error)
- β = Probability of Type II error

Test of a Single Mean

Test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Degrees of freedom $= n-1$ $(1-\alpha)\%$ Confidence Interval $= \bar{X} +$ Critical value $\times \left(\frac{s}{\sqrt{n}}\right)$

Test of the Difference in Means

Test statistic

$$t = \frac{\left(\bar{X}_{d1} - \bar{X}_{d2}\right) - \left(\mu_{d1} - \mu_{d2}\right)}{\sqrt{\frac{s_p^2}{n_{d1}} + \frac{s_p^2}{n_{d2}}}}$$

Degrees of freedom $= n_{d1} + n_{d2} - 2$

$$s_p^2 = \frac{\left(n_{d1} - 1\right) s_{d1}^2 + \left(n_{d2} - 1\right) s_{d2}^2}{n_{d1} + n_{d2} - 2}$$

Test of the Mean of Differences

Test statistic

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

Degrees of freedom $= n - 1$

Test of a Single Variance

Test statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Degrees of freedom $= n - 1$

Test of the Difference in Variances

Test statistic

$$F = \frac{s_{\text{Before}}^2}{s_{\text{After}}^2}$$

Degrees of freedom $= n_1 - 1, n_2 - 1$

Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom $= n - 2$

Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^{m} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

Degrees of freedom $= (r-1)(c-1)$

where:

- m = Number of cells in the table
- O_{ij} = Number of observations in each cell of row i and column j
- E_{ij} = Expected number of observations in each cell of row i and column j

Learning Module 9: Parametric and Non-Parametric Tests of Independence

Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom $= n - 2$

Pearson Correlation (or Bivariate Correlation)

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

Spearman Rank Correlation Coefficient

$$r_S = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n\left(n^2-1\right)}$$

where:

- $d =$ Difference in ranks

Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^{m} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

Degrees of freedom $= (r-1)(c-1)$

where:

- $m =$ Number of cells in the table
- $O_{ij} =$ Number of observations in each cell of row i and column j
- $E_{ij} =$ Expected number of observations in each cell of row i and column j

$$= \frac{(\text{ Total row } i) \times (\text{ Total column } j)}{\text{Overall total}}$$

Standardized Residual (or Pearson Residual)

$$\text{Standardized Residual } = \frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}}$$

Learning Module 10: Simple Linear Regression

$$Y_i = b_0 + b_1 X_1 + \cdots + b_n X_n + \varepsilon_i, \quad i = 1, 2, \ldots, n$$

where:

- $Y =$ Dependent variable
- $X =$ Independent variable
- $b_0 =$ Intercept
- $b_i =$ Slope coefficient, $i = 1, 2, \ldots, n$
- $\varepsilon_i =$ Error term
- $b_0, b_1, \ldots, b_n =$ Regression coefficients

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i + e_i$$

where:

- $\hat{Y}_i =$ Estimated value on the regression line for the i th observation
- $\hat{b}_0 =$ Intercept
- $\hat{b}_1 =$ Slope
- $e_i =$ Residual for the i th observation $\hat{b}_1 = \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X} = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{n}(X_i - \bar{X})^2}$ $\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$ Sum of Squares Total, $SST = \sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2 = SSR + SSE$ Sum of Squares Regression, $SSR = \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$ Sum of Squares Error, $SSE = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} e_i^2$ Coefficient of Determination

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation coefficient

$$r = \frac{\text{Covariance of } X \text{ and } Y}{(\text{ Standard deviation of } X)(\text{ Standard deviation of } Y)}$$

Note: (Correlation coefficient) ^{2} = Coefficient of determination

Sample standard deviation of X

$$S_X = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}{n-1}}$$

Sample standard deviation of Y

$$S_Y = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}{n-1}}$$

Homoskedasticity

$$E\left(\varepsilon_i^2\right) = \sigma_\varepsilon^2, \quad i = 1, 2, \ldots, n$$

ANOVA F-Test

Mean square regression (MSR)

$$MSR = \frac{SSR}{k}$$

Mean square error (MSE)

$$MSE = \frac{SSE}{n-k-1}$$

F-distributed test statistic

$$F = \frac{MSR}{MSE}$$

where:

- n = Number of observations
- k = Number of independent variables

Standard error of estimate

$$s_e = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-k-1}}$$

Hypothesis Test of the Slope Coefficient

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

Degrees of freedom, $df = n - k - 1$

where:

- B_1 = Hypothesized population slope
- $s_{\hat{b}_1}$ = Standard error of the slope coefficient

$$= \frac{s_e}{\sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}}$$

Hypothesis Test of the Intercept

$$t_{\text{intercept }} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}}$$

Standard error of the intercept, $s_{\hat{b}_0}$

$$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}}$$

Prediction Intervals

$$\hat{Y}_f \pm t_{\alpha/2} \times s_f$$

where: $\hat{Y}_f = \hat{b}_0 + \hat{b}_1 X_f$

Variance of the prediction error of Y, given X

$$s_f^2 = s_e^2\left[1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}\right]$$

Standard error of the forecast

$$s_f = s_e\sqrt{1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{(n-1)s_X^2}}$$

The Log-Lin Model

$$\ln Y_i = b_0 + b_1 X_i$$

The Lin-Log Model

$$Y_i = b_0 + b_1 \ln X_i$$

The Log-Log Model

$$\ln Y_i = b_0 + b_1 \ln X_i$$

Learning Module 11: Introduction to Big Data Techniques

No formula.

VOLUME 2: ECONOMICS

Learning Module 1: The Firm and Market Structures

Total profit = Total revenue - Total cost

Economic profit = Total revenue - Total economic costs

Accounting profit = Total revenue - Total accounting costs

Total revenue = Price $\times$ Quantity = $P \times Q$

Average revenue = $\frac{\text{Total revenue}}{\text{Quantity}}$

Marginal cost = $\frac{\Delta TC}{\Delta Q}$

Average variable cost = $\frac{\text{Total variable cost}}{\text{Quantity}}$

Average fixed cost = $\frac{\text{Total fixed cost}}{\text{Quantity}}$

Total cost = Total fixed cost + Total variable cost

Average total cost = Average fixed cost + Average variable cost

Concentration Ratio

$$\text{Concentration ratio } = \sum_{i=1}^{n} (\text{ Market share })_i$$

Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{n} (\text{ Market share })_i^2$$

Learning Module 2: Understanding Business Cycles

No formula

Learning Module 3: Fiscal Policy

$$\text{Budget surplus/(deficit) } = G - T + B$$

where:

- G = Government spending
- T = Taxes
- B = Payments of transfer benefits

Disposable Income

$$YD = Y - NT = (1 - t)Y$$

where:

- t = Net tax rate
- NT = Net taxes = Taxes - Transfers
- tY = Total tax revenue

The Fiscal Multiplier

$$\text{Fiscal multiplier } = \frac{1}{1 - c(1 - t)}$$

where:

- c = Marginal propensity to consume
- t = Tax rate

Learning Module 4: Monetary Policy

Neutral rate = Trend growth + Inflation target

Learning Module 5: Introduction to Geopolitics

No formula

Learning Module 6: International Trade

No formula

Learning Module 7: Capital Flows and the FX Market

Real exchange rate $_{d/f} = S_{d/f} \times \frac{P_f}{P_d}$ % Change in real exchange rate $= \left(1 + \%\Delta S_{d/f}\right) \times \frac{(1+\%\Delta P_f)}{(1+\%\Delta P_d)} - 1$

$$\approx \%\Delta S_{d/f} + \%\Delta P_f - \%\Delta P_d$$

Percentage change in base currency f (vs currency d)

$$\frac{E\left(S_{d/f}\right) - S_{d/f}}{S_{d/f}}$$

where:

- $S_{d/f}$ = Spot exchange rate
- P_f = General price level of goods indexed in currency f
- P_d = General price level of goods indexed in currency d

Learning Module 8: Exchange Rate Calculations

Cross-Rate

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{D}$$

Forward Rate

$$F_{A/B} = S_{A/B} \times \left[\frac{1 + r_A \times T}{1 + r_B \times T}\right]$$

$$\begin{aligned}\text{Forward points } &= F_{A/B} - S_{A/B} \\ &= S_{A/B}\left(\frac{r_A - r_B}{1 + r_B}\right)T\end{aligned}$$

where:

- $S_{A/B}$ = Spot exchange rate
- $F_{A/B}$ = Forward exchange rate
- T = Time to maturity

Learning Module 1: Organizational Forms, Corporate Issuer Features, and Ownership

No formula

Learning Module 2: Investors and Other Stakeholders

No formula

Learning Module 3: Working Capital and Liquidity

$$\begin{array}{c}\text{Cash conversion} \\ \text{cycle}\end{array} = \begin{array}{c}\text{Days of inventory} \\ \text{on hand}\end{array} + \begin{array}{c}\text{Days sales} \\ \text{outstanding}\end{array} - \begin{array}{c}\text{Days payables} \\ \text{outstanding}\end{array}$$

$$\text{EAR of Supplier } \begin{array}{l}\text{Days in Year} \\ \text{Financing}\end{array} = \left(1 + \frac{\text{Discount \%}}{100\% - \text{ Discount \%}}\right)^{\frac{\text{Dayment Period-Discount Period}}{\text{Total working capital}}} = \text{Current assets} - \text{Current Liab}$$

$$\begin{aligned}\text{Net working} \\ \text{capital } &= \text{ Current assets (excluding cash and marketable securities)} \\ &- \text{Current Liabilities (excluding short-term and current debt)}\end{aligned}$$

Cash flow from operations = Cash received from customers

- Interest and dividends received on financial investments
- Cash paid to employees and suppliers
- Taxes paid to governments
- Interest paid to lenders

Free cash flow = Cash flow from operations - Investments in long-term assets

$$\text{Current ratio } = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio } = \frac{\text{Cash } + \text{ Short-term marketable instruments } + \text{ Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio } = \frac{\text{Cash } + \text{ Short-term marketable instruments}}{\text{Current liabilities}}$$

Learning Module 4: Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits

No formula

Learning Module 5: Capital Investments and Capital Allocation

Net Present Value

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

where:

- CF_t = After-tax cash flow at time t
- r = Required rate of return
- CF_0 = Initial outlay

Internal Rate of Return

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

Video: https://youtu.be/bzck7QLhICw

Return on Invested Capital

$$\begin{aligned}\text{ROIC} &= \frac{\text{After-tax operating profit}}{\text{Average invested capital}} \\ &= \frac{\text{Operating profit } t_t \times (1- \text{ Tax rate })}{\text{Average total long-term liabilities and equity }_{t-1,t}}\end{aligned}$$

$$\text{ROIC} = \frac{\text{After-tax operating profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Average invested capital}}$$

Real Options in Capital Budgeting

$$\begin{matrix}\text{Project NPV} \\ (\text{ with option })\end{matrix} = \begin{matrix}\text{Project NPV} \\ (\text{ without option })\end{matrix} - \text{ Option cost } + \text{ Option value}$$

Learning Module 6: Capital Structure

Weighted Average Cost of Capital

$$WACC = w_d r_d (1-t) + w_e r_e$$

where:

- w_d = Target weight of debt in capital structure = $\frac{D}{D+E}$
- w_e = Target weight of common stock in capital structure = $\frac{E}{D+E}$
- r_d = Before-tax marginal cost of debt
- t = Marginal tax rate
- $r_d(1-t)$ = After-tax marginal cost of debt
- r_e = Marginal cost of common stock

Operating Leverage

$$\text{Operating leverage } = \frac{\text{Fixed costs}}{\text{Total costs}}$$

Interest Coverage

$$\text{Interest coverage } = \frac{\text{Profit before interest and taxes}}{\text{Interest expense}}$$

Modigliani-Miller Capital Structure Propositions

$$V_L = V_U + tD$$
$$r_e = r_0 + (r_0 - r_d)(1-t)\frac{D}{E}$$
$$E = \frac{(CF_e - r_d D)(1-t)}{r_e}$$
$$V_L = \frac{CF_e(1-t)}{r_{\text{WACC}}}$$

where:

- V_L = Value of levered firm
- V_U = Value of unlevered firm
- t = Marginal tax rate
- r_e = Cost of equity
- r_d = Cost of debt
- r_0 = Cost of capital (for a 100% equity-financed company)
- D = Market value of debt
- E = Market value of equity
- CF_e = After-tax cash flows to shareholders
- $r_d D$ = Interest expense on debt

Static Trade-off Theory of Capital Structure

$$V_L = V_U + tD - PV(\text{ Costs of Financial Distress })$$

Learning Module 7: Business Models

No formula

VOLUME 4: FINANCIAL STATEMENT ANALYSIS

Learning Module 1: Introduction to Financial Statement Analysis

No formula

Learning Module 2: Analyzing Income Statements

Gross profit = Revenue - Cost of Goods Sold

Operating income = Gross margin - Selling, General, and Administrative Expense

Taxable income = Operating income - Interest expense

Net income = Taxable income - Taxes

Ending shareholders' equity = Beginning shareholders' equity + Net income

- Other comprehensive income
- Dividends
- Net capital contributions from shareholders

Ending retained earnings = Beginning retained earnings + Net income - Dividends

Return on Equity

$$ROE = \frac{\text{Net income}}{\text{Average shareholders' equity}}$$

Net Profit Margin

$$\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

Basic EPS

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

Diluted EPS (for convertible preferred stock)

$$\text{Diluted EPS} = \frac{\text{Net income}}{\begin{array}{c}\text{Weighted average number}\\ \text{of shares outstanding}\end{array} + \begin{array}{c}\text{New common shares that would}\\ \text{have been issued at conversion}\end{array}}$$

Diluted EPS (for convertible debt)

$$\text{Diluted EPS} = \frac{\begin{array}{c}\text{Net income} - \text{Preferred dividends} + \\ \text{After tax interest expense} \\ \text{on convertible debt}\end{array}}{\begin{array}{c}\text{Weighted average number} \\ \text{of shares outstanding}\end{array} + \begin{array}{c}\text{New common shares that would} \\ \text{have been issued at conversion}\end{array}}$$

Diluted EPS (for options)

$$\text{Diluted EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\begin{array}{c}\text{Weighted average number} \\ \text{of shares outstanding}\end{array} \quad \begin{array}{c}\text{Additional common} \\ \text{shares issued upon} \\ \text{conversion}\end{array}}$$

Treasury stock method

$$\begin{array}{c}\text{Additional common} \\ \text{shares issued upon} \\ \text{conversion}\end{array} = \left(\begin{array}{c}\text{New shares} \\ \text{issued at} \\ \text{option exercise}\end{array} \quad \begin{array}{c}\text{Shares repurchased} \\ - \\ \text{from option exercised}\end{array} \quad \text{with cash received} \right) \times \begin{array}{c}\text{Proportion of} \\ \text{during which o} \\ \text{were outstan}\end{array}$$

Video (Basic & Diluted EPS): https://youtu.be/2C-mwVqO2SQ

Learning Module 3: Analyzing Balance Sheets

Working capital = Current assets - Current liabilities

Liquidity Ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick (acid test) ratio} = \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}}$$

Solvency Ratios

$$\text{Long-term debt-to-equity} = \frac{\text{Long-term debt}}{\text{Total equity}}$$
$$\text{Debt-to-equity} = \frac{\text{Total debt}}{\text{Total equity}}$$
$$\text{Total debt} = \frac{\text{Total debt}}{\text{Total assets}}$$
$$\text{Financial leverage} = \frac{\text{Total assets}}{\text{Total equity}}$$

Learning Module 4: Analyzing Statements of Cash Flows I

Cash flow Cash flow Cash flow Ending cash = Beginning cash + from operating activities + from investing activities + from financing activities		
Ending accounts = Beginning accounts receivable receivable	+ Revenue -	*from customers*
Ending inventory = inventory	Cost of goods sold	
Ending accounts = Beginning accounts payable	+Purchases -	Cash paid to suppliers
Ending wages payable		Cash paid to employees
+ payable	Interest Cash paid expense −for interest	
______ taxpaple +	______	

Cash flow Cash flow Cash flow Cash flow Ending cash = Beginning cash + from operating activities + from investing activities + from financing activities	
Ending $PP\&E$ = Beginning $PP\&E$+ ______	Equipment sold *equipment sold*
Ending accumulated Beginning accumulated Depreciation depreciation depreciation ______ expense expense	

$$\begin{array}{c}\text{Gain on sale}\\ \text{of equipment}\end{array} = \begin{array}{c}\text{Cash received from}\\ \text{sale of equipment}\end{array} - \begin{array}{c}\text{Book value of}\\ \text{equipment sold}\end{array}$$

Note: $$\begin{array}{c}\text{Ending retained}\\ \text{earnings}\end{array} = \begin{array}{c}\text{Beginning retained}\\ \text{earnings}\end{array} + \begin{array}{c}\text{Net}\\ \text{income}\end{array} - \text{Dividends}$$

Learning Module 5: Analyzing Statements of Cash Flows II

Free Cash Flow To Firm (FCFF)

$$\begin{aligned} FCFF &= NI + NCC + \text{Int}(1 - \text{Tax rate}) - FC\text{Inv} - WC\text{Inv} \\ &= CFO + \text{Int}(1 - \text{Tax rate}) - FC\text{Inv} \end{aligned}$$

where:

- NI = Net income
- NCC = Non-cash charges (e.g., depreciation and amortization)
- Int = Interest expense
- FCInv = Capital expenditures
- WCInv = Working capital expenditures
- CFO = Cash flow from operating activities = NI + NCC - WCInv

Free Cash Flow to Equity (FCFE)

$FCFE = CFO - FCInv + NetBorrowing$

where:

- Net Borrowing = Debt issued - Debt repaid

Performance Ratios

Cash flow to revenue = $\frac{CFO}{\text{Revenue}}$

Cash return on assets = $\frac{CFO}{\text{Average total assets}}$

Cash return on equity = $\frac{\text{CFO}}{\text{Average shareholders equity}}$

Cash to income = $\frac{CFO}{\text{Operating income}}$

Cash flow per share = $\frac{\text{CFO} - \text{Preferred dividends}}{\text{Number of common shares outstanding}}$

Coverage Ratios

Debt coverage ratio = $\frac{CFO}{\text{Total debt}}$

Interest coverage ratio = $\frac{\text{CFO + Interest paid + Taxes paid}}{\text{Interest paid}}$ Reinvestment ratio = $\frac{\text{CFO}}{\text{Cash paid for long term assets}}$
Debt payment ratio = $\frac{CFO}{\text{Cash paid for long term debt repayment}}$ Dividend payment ratio = $\frac{\text{CFO}}{\text{Dividends paid}}$
Investing and financing ratio = $\frac{\text{CFO}}{\text{Cash flow for investing and financing activities}}$

Learning Module 6: Analysis of Inventories

IFRS

Inventories = Lower of Cost and Net Realizable Value (NRV) NRV = Estimated selling price less estimated costs of completion and costs necessary to complete the sale

US GAAP

Inventories = Lower of Cost and NRV For last-in, first-out (LIFO) method or retail inventory methods Inventories = Lower of Cost and Market Value

Market value = Current replacement cost (subject to lower and upper limits) Lower limit $= NRV -$ Normal profit margin Upper limit $= NRV$ Video: https://youtu.be/V8C31msIBzs Inventory turnover ratio $= \frac{\text{Cost of sales}}{\text{Average inventory}}$ Days of inventory on hand $= \frac{\text{Number of days in period}}{\text{Inventory turnover ratio}}$

Ending inventory (FIFO) = Ending inventory (LIFO)+ LIFO reserve

COGS $(FIFO)$ = COGS (LIFO) - Change in LIFO reserve

Learning Module 7: Analysis of Long-Term Assets

Net book value = Historical cost - Accumulated depreciation

Gain on sale of asset = Sale proceeds - Net book value $\begin{array}{c}\text{Estimated total}\\\text{useful life}\end{array} = \begin{array}{c}\text{Estimated age}\\\text{of equipment}\end{array} + \begin{array}{c}\text{Estimated}\\\text{remaining life}\end{array}$ $\begin{array}{c}\text{Estimated total}\\\text{useful life}\end{array} = \frac{\text{Gross PP\&E}}{\text{Annual depreciation expense}}$ $\begin{array}{c}\text{Estimated age}\\\text{of equipment}\end{array} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}}$ $\begin{array}{c}\text{Estimated}\\\text{remaining life}\end{array} = \frac{\text{Net PP\&E}}{\text{Annual depreciation expense}}$

Straight-line Depreciation

$$\text{Annual depreciation expense } = \frac{\text{Historical cost } - \text{ Salvage value}}{\text{Estimated useful life}}$$

Fixed Asset Turnover

$$\text{Fixed asset turnover } = \frac{\text{Revenue}}{\text{Average net PP\&E}}$$

Impairment of Long-Lived Assets

IFRS

Impairment = Carrying amount - Recoverable amount

where:

- Recoverable amount = max(Fair value less costs to sell, Value in use)

US GAAP

If asset's carrying amount > undiscounted expected future cash flows: Impairment = Carrying amount - Fair value

Learning Module 8: Topics in Long-Term Liabilities and Equity

Lessee Accounting - Finance Lease (IFRS)

$$\begin{aligned}
\text{Interest expense on lease} &= \text{Implied interest rate} \times \text{Beginning lease liability} \\
\text{Principal repayment} &= \text{Lease payment} - \text{Interest expense} \\
\text{Ending lease liability} &= \text{Beginning lease liability} + \text{Interest expense} - \text{Lease payment}
\end{aligned}$$

If ROU asset is amortized on a straight-line basis:

$$\text{Amortization expense} = \frac{\text{Initial ROU asset value - Salvage value}}{\text{Lease term}}$$

$$\underset{\text{asset}}{\text{Ending ROU}} = \underset{\text{asset}}{\text{Beginning ROU}} - \underset{\text{expense}}{\text{Amortization}}$$

Lessee Accounting - Operating Lease (US GAAP)

```
$\underset{\text { expense }}{\text { Amortization }}=$ Lease payment - Interest expense
```

```
$\underset{\text { asset }}{\text { Ending } R O U}=\underset{\text { asset }}{\text { Begin
\underset{\text { expense }}{\text { Amortization }}$
```

```
$\underset{\text { liability }}{\text { Ending lease }}=\underset{\text { liability }}{\text
\underset{\text { expense }}{\text { Amortization }}$
```

Stock Options

$$\text{Compensation expense} = \frac{\text{Fair value of options granted}}{\text{Vesting period}}$$

Learning Module 9: Analysis of Income Taxes

Deferred Tax Asset/Liability

For Assets:

$$\begin{array}{c}\text{Deferred tax}\\ \text{liability /(asset)}\end{array} = \text{Tax rate } \times \left(\begin{array}{c}\text{Carrying amount}\\ \text{of asset}\end{array} - \begin{array}{c}\text{Tax base}\\ \text{of asset}\end{array} \right)$$

For Liabilities:

$$\begin{array}{c}\text{Deferred tax}\\ \text{liability /(asset)}\end{array} = \text{Tax rate } \times \left(\begin{array}{c}\text{Tax base}\\ \text{of liability}\end{array} \quad \begin{array}{c}\text{Carrying amount}\\ \text{of liability}\end{array} \right)$$

$$\text{Income tax expense} = \text{Income tax payable} + \begin{array}{c}\text{Changes in deferred tax}\\ \text{assets and liabilities}\end{array}$$

Effective tax rate $= \frac{\text{Income tax expense}}{\text{Pre-tax income}}$

Cash tax rate $= \frac{\text{Cash tax}}{\text{Pre-tax income}}$

Learning Module 10: Financial Reporting Quality

Adjusted EBITDA

$$\underset{\text{EBITDA}}{\text{Adjusted}} = \underset{\text{EBIT}}{\text{Adjusted}} + \underset{\substack{\text{Software}\\ \text{and R\&D}\\ \text{amortization}}}{\text{Post-IPO}} + \underset{\substack{\text{Share-based}\\ \text{amortization}}}{\text{Depreciation}} + \underset{\text{shartion}}{\text{amortion}}$$

Straight-line method of depreciation

$$\text{Depreciation expense } = \frac{\text{Cost } - \text{ Salvage value}}{\text{Useful life}}$$

Double-Declining Balance method

$$\text{Depreciaton expense } = \frac{2}{\text{Useful life}} \times (\text{ Cost } - \text{ Accumulated depreciation })$$

Video: https://youtu.be/6RskYAxdAFk

Units-of-Production method

$$\text{Depreciation expense} = \frac{\text{Units produced}}{\text{Total units over useful life}} \times (\text{ Cost } - \text{ Salvage value })$$

Learning Module 11: Financial Analysis Techniques

Activity Ratios

$$\text{Inventory turnover} = \frac{\text{Cost of sales}}{\text{Average inventory}}$$

$$\text{Days of inventory on hand} = \frac{\text{Number of days in the period}}{\text{Inventory turnover}}$$

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Average receivables}}$$

$$\text{Days of sales outstanding} = \frac{\text{Number of days in the period}}{\text{Receivables turnover}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average payables}}$$

$$\text{Number of days of payables} = \frac{\text{Number of days in the period}}{\text{Payables turnover}}$$

$$\text{Working capital turnover} = \frac{\text{Revenue}}{\text{Average working capital}}$$

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average net fixed assets}}$$

$$\text{Total asset turnover} = \frac{\text{Revenue}}{\text{Average total assets}}$$

Liquidity Ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short term marketable investments}}{\text{Current liabilities}}$$

$$\text{Defensive interval} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Daily cash expenditures}}$$

$$\begin{array}{c}\text{Cash conversion}\\\text{cycle}\end{array} = \begin{array}{c}\text{Days of inventory}\\\text{on hand}\end{array} + \begin{array}{c}\text{Days of sales}\\\text{outstanding}\end{array} - \begin{array}{c}\text{Number of days}\\\text{of payables}\end{array}$$

Solvency Ratios

$$\underset{\text{("Total debt ratio")}}{\text{Debt-to-assets ratio}} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Debt-to-capital ratio} = \frac{\text{Total debt}}{\text{Total debt} + \text{Total equity}}$$

$$\text{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Financial leverage ratio} = \frac{\text{Average total assets}}{\text{Average total equity}}$$

$$\text{Debt-to-EBITDA ratio} = \frac{\text{Total or net debt}}{\text{EBITDA}}$$

Coverage Ratios

Interest coverage ratio $= \frac{\text{EBIT}}{\text{Interest payments}}$ Fixed charge coverage ratio $= \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$

Profitability Ratios

Gross profit margin $= \frac{\text{Gross profit}}{\text{Revenue}}$ Operating profit margin $= \frac{\text{Operating income}}{\text{Revenue}}$ Pretax margin $= \frac{EBT}{\text{Revenue}}$ Net profit margin $= \frac{\text{Net income}}{\text{Revenue}}$

Operating ROA $= \frac{\text{Operating income}}{\text{Average total assets}}$ ROA $= \frac{\text{Net income}}{\text{Average total assets}}$ Return on invested capital $= \frac{EBIT\times(1-Eff\text{ective tax rate })}{\text{Average total debt and equity}}$ ROE $= \frac{\text{Net income}}{\text{Average total equity}}$ Return on common equity $= \frac{\text{Net income} - \text{Preferred dividends}}{\text{Average common equity}}$

DuPont Analysis

ROE = ROA $\times$ Financial Leverage

ROE = Net profit margin $\times$ Total asset turnover $\times$ Financial leverage ROE $= \underset{\text{burden}}{\text{Tax}} \times \underset{\text{burden}}{\text{Interest}} \times \underset{\text{margin}}{\text{EBIT}} \times \underset{\text{turnover}}{\text{Total asset}} \times \underset{\text{leverage}}{\text{Financial}}$

where:

- Tax burden $= \frac{\text{Net income}}{\text{EBT}}$
- Interest burden $= \frac{EBT}{EBIT}$

Business Risk

$\text{Coefficient of variation of operating income} = \frac{\text{Standard deviation of operating income}}{\text{Average operating income}}$ $\text{Coefficient of variation of net income} = \frac{\text{Standard deviation of net income}}{\text{Average net income}}$ $\text{Coefficient of variation of revenue} = \frac{\text{Standard deviation of revenue}}{\text{Average revenue}}$

Financial Sector Ratios

$\text{Monetary reserve requirement (Cash reserve ratio)} = \frac{\text{Reserves held at central bank}}{\text{Specified deposit liabilities}}$ $\text{Net interest margin} = \frac{\text{Net interest income}}{\text{Total interest earning assets}}$ $\text{Liquid asset requirement} = \frac{\text{Approved readily marketable securities}}{\text{Specified deposit liabilities}}$ Net interest margin $= \frac{\text{Net interest income}}{\text{Total interest earning assets}}$

Learning Module 12: Introduction to Financial Statement Modeling

Nothing new.

VOLUME 5: EQUITY INVESTMENTS

Learning Module 1: Market Organization and Structure

Maximum leverage ratio $= \frac{1}{\text{Minimum margin requirement}}$ Total return on leveraged stock investment:

$$\text{Total Return} = \frac{\text{Sales proceeds} + \text{Dividends} - \text{Loan} - \frac{\text{Margin}}{\text{interest}} - \text{Sales commission}}{\text{Initial equity} + \text{Purchase commission}} - 1$$

Initial equity $= \text{Minimum margin requirement} \times$ Total purchase price Video (Return on Leveraged Stock Position): https://youtu.be/tZd4Xtvjill Margin Call Price $= \frac{P_0(1- \text{Initial Margin})}{(1- \text{Maintenance Margin})}$

Learning Module 2: Security Market Indexes

$$\text{Price Return Index,} \quad V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where:

- n_i = the number of units of constituent security i held in the index portfolio
- N = the number of constituent securities in the index
- P_i = the unit price of constituent security i
- D = value of the divisor

$$\text{Price return of an index,} \quad PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

$$\text{Total Return Index,} \quad TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:

- V_{PRI1} = value of the price return index at the end of the period
- V_{PRI0} = value of the price return index at the beginning of the period
- Inc_I = total income (dividends and/or interest) from all securities in the index held over the period

Weighting Methods

$$\text{Price weighting,} \quad w_i^P = \frac{P_i}{\sum_{j=1}^{N} P_j}$$

Video (Recalculating the divisor of a price weighted index): https://youtu.be/eYiZNK-ETrg

$$\text{Equal weighting,} \quad w_i^E = \frac{1}{N}$$

$$\text{Market-capitalization weighting,} \quad w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

$$\text{Float-adjusted market capitalization weighting,} \quad w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where:

- f_i = fraction of shares outstanding in the market float

- Q_i = number of shares outstanding of security i
- P_i = share price of security i
- N = number of securities in the index

$$\text{Fundamental weighting,} \quad w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where F_{i} denotes a fundamental size measure of company i

Learning Module 3: Market Efficiency

No formula

Learning Module 4: Overview of Equity Securities

Return on Equity (using average total book value of equity)

$$ROE_t = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Return on Equity (using beginning book value of equity)

$$ROE_t = \frac{NI_t}{BVE_{t-1}}$$

where BVE = book value (Assets - Liabilities)

Learning Module 5: Company Analysis: Past and Present

Market share = $\frac{\text{Revenue}}{\text{Market size}}$

Sales potential = 100%− Market share %

Net sales = Average selling price × Quantity sold

Take rate = $\frac{\text{Revenue earned from transactions}}{\text{Total transaction volume}} \times 100\%$

Operating income = $Q \times (P - VC) - FC$

where:

- Q = Units sold in a period
- P = Price per unit

- VC = Variable operating cost per unit
- FC = Fixed operating costs
- $P - VC$ = Contribution margin per unit

Degree of operating leverage $(DOL) = \frac{\%\Delta \text{ Operating income}}{\%\Delta \text{ Sales}}$ Degree of financial leverage $(DFL) = \frac{\%\Delta \text{ Net income}}{\%\Delta \text{ Operating income}}$

$$\text{WACC} = \begin{array}{c}\text{Weight}\\ \text{of debt}\end{array} \times \begin{array}{c}\text{Gross cost}\\ \text{of debt}\end{array} \times (1- \text{ tax rate }) + \underset{\text{of equity}}{\text{Weight}} \times \begin{array}{c}\text{Cost of}\\ \text{equity}\end{array}$$

Learning Module 6: Industry and Company Analysis

Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{\infty} s_i^2$$

where:

- s_i = Market share of participant i (stated as a whole number)

Learning Module 7: Company Analysis: Forecasting % Variable cost $\approx \frac{\%\Delta(\text{ Cost of revenue + Operating expense })}{\%\Delta \text{ Revenue}}$

% Fixed cost $\approx 1 -$ % Variable cost $\begin{array}{c}\text{Number of units sold}\\ \text{post-cannibalization}\end{array} = \begin{array}{c}\text{Number of units sold}\\ \text{pre-cannibalization}\end{array} - \begin{array}{c}\text{Expected}\\ \text{cannibalization}\end{array}$ $\underset{\text{cannibalization}}{\text{Expected}} = \underset{\text{pre-cannibalization}}{\text{Number of units sold}} \times \underset{\text{factor}}{\text{Cannibalization}}$

Learning Module 8: Equity Valuation: Concepts and Basic Tools

Dividend Discount Model (DDM)

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

where:

- V_0 = Intrinsic value of a share at $t = 0$
- D_t = expected dividend in year t
- r = required rate of return on stock
- P_n = expected price per share at $t = n$ (terminal value)

Free-cash-flow-to-equity (FCFE) Valuation Model

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$$

where:

- $FCFE = CFO - FCInv + NetBorrowing$
- FCInv = Fixed capital investment
- Net Borrowing = Borrowings minus repayments Value of preferred stock (non-callable, non-convertible, perpetual)

$$V_0 = \frac{D_0}{r}$$

Value of preferred stock (non-callable, non-convertible, maturity at time n)

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{\text{Par value}}{(1+r)^n}$$

Gordon Growth Model

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

where:

- D_0 = Most recent annual dividend
- D_1 = Expected dividend in the next period
- g = Constant growth rate
- r = Required return on equity

Sustainable growth rate

$$g = b \times ROE$$

where:

- b = earnings retention rate (= 1 - Dividend payout ratio)
- ROE = Return on equity

Video: https://youtu.be/MnfRRRhuGpA

Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^{n} \frac{D_0\left(1+g_s\right)^t}{(1+r)^t} + \frac{V_n}{(1+r)^t}$$

where:

- g_L = Long-term stable growth rate
- g_s = Short-term growth rate
- $V_n = \frac{D_{n+1}}{r-g_L} = \frac{D_0(1+g_s)^t(1+g_L)}{r-g_L}$

Justified forward P/E

$$\frac{P_0}{E_1} = \frac{\text{Dividend payout ratio}}{r-g}$$

Enterprise Value

$$EV = \begin{array}{c}\text{Market value}\\ \text{of equity}\end{array} + \begin{array}{c}\text{Market value}\\ \text{of preferred stock}\end{array} + \begin{array}{c}\text{Market value}\\ \text{of debt}\end{array} - \begin{array}{c}\text{Cash and}\\ \text{short term}\\ \text{investments}\end{array}$$

Asset-based Valuation

$$\begin{array}{c}\text{Adjusted}\\ \text{book value}\end{array} = \begin{array}{c}\text{Market value}\\ \text{of assets}\end{array} - \begin{array}{c}\text{Market value}\\ \text{of liabilities}\end{array}$$

VOLUME 6: FIXED INCOME

Learning Module 1: Fixed-Income Instrument Features Current yield = $\frac{\text{Annual coupon}}{\text{Bond price}}$ Bond price = $\frac{\text{Coupon}}{(1+r)^1} + \frac{\text{Coupon}}{(1+r)^2} + \cdots + \frac{\text{Coupon + Face value}}{(1+r)^n}$

where:

- Coupon per period = Coupon rate per period × Face value
- r = Yield to maturity per period
- n = Number of payments

Floating-rate Note (FRN) coupon rate = MRR + Spread

Learning Module 2: Fixed-Income Cash Flows and Types

Fully Amortizing Loan with Level Payment

$$A = \frac{r \times \text{ Principal}}{1 - (1 + r)^{-N}}$$

where:

- $A =$ Periodic payment
- $r =$ Market interest rate per period
- $N =$ Number of payment periods

If the periodic payment is monthly:

$$\begin{aligned}
\text{Monthly interest payment } &= \text{ Interest rate per month} \times \text{Beginning principal of loan} \\
\text{Monthly principal payment } &= \text{ Total monthly payment } - \text{ Monthly interest payment} \\
\text{Ending principal of loan } &= \text{ Beginning principal of loan } - \text{ Monthly principal payment}
\end{aligned}$$

Capital-Index Bond (e.g., TIPS)

Inflation-adjusted principal = Principal amount × (1+ Inflation adjustment)

Coupon per period = Coupon rate per period × Inflation-adjusted principal

Deferred Coupon Bond

Video: https://youtu.be/erRbAUOGIyM

Convertible Bonds

$$\begin{array}{c}\text{Conversion} \\ \text{ratio}\end{array} = \frac{\text{Convertible bond par value}}{\text{Conversion price}}$$

$$\begin{array}{c}\text{Conversion} \\ \text{value}\end{array} = \begin{array}{c}\text{Conversion} \\ \text{ratio}\end{array} \times \begin{array}{c}\text{Current share} \\ \text{price}\end{array}$$

Zero-Coupon Bond

Original issue discount = Bond par value - Issuance price

Learning Module 3: Fixed-Income Issuance and Trading

No formula

Learning Module 4: Fixed-Income Markets for Corporate Issuers

Repurchase Agreements

$$\text{Repurchase price} = \text{Price of bond} \times \left[1 + \text{Repo rate} \times \frac{\text{Repo term (in days)}}{\text{Number of days in a year}}\right. \text{Initial margin} = \frac{\text{Security pric}}{\text{Purchase pric}}$$

$$\text{Haircut} = \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Security price}_0}$$

$$\text{Variation margin} = \left(\text{Initial margin} \times \text{Purchase price}_t\right) - \text{Security price}_t$$

Learning Module 5: Fixed-Income Markets for Government Issuers

No formula.

Learning Module 6: Fixed-Income Bond Valuation: Prices and Yields

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \cdots + \frac{PMT_N + FV_N}{(1+r)^N}$$

where:

- PMT_t = Coupon that occurs in t periods
- r = Market discount rate per period
- N = Number of periods to maturity
- FV = Face value of bond

Full Price, Flat Price, and Accrued Interest (Video: https://youtu.be/I7G075JAu5w)

$$\begin{aligned} \text{PV}^{\text{Full}} &= \text{PV}^{\text{Flat}} + \text{Accrued Interest} \\ &= PV_{BOP} \times (1+r)^{t/T} \end{aligned}$$

where:

- Accrued Interest $= \frac{t}{T} \times PMT$
- t = number of days from the last coupon payment to the settlement date
- T = number of days in the coupon period

- t/T = fraction of the coupon period that has gone by since the last payment
- PV_{BOP} = price on the previous coupon date (before the settlement date)

Matrix Pricing

$$\text{Interpolated yield } = \text{ Yield }_S + \left(\frac{\text{Tenor }_{\text{Target}} - \text{ Tenor }_S}{\text{Tenor }_L - \text{ Tenor }_S}\right) \times (\text{ Yield }_L - \text{ Yield }_S)$$

where:

- Yield $_S$ = Yield of shorter-term bond
- Yield $_L$ = Yield of longer-term bond
- Tenor $_S$ = Tenor of shorter-term bond
- Tenor $_L$ = Tenor of longer-term bond
- Tenor $_{\text{Target}}$ = Tenor of the subject bond
- Tenor $_S$ < Tenor $_{\text{Target}}$ < Tenor $_L$ Required yield spread = Bond YTM - Government Bond YTM (Similar maturity)

Learning Module 7: Yield and Yield Spread Measures for Fixed Rate Bonds

Periodicity Conversion

$$\left(1+\frac{APR_m}{m}\right)^m = \left(1+\frac{APR_n}{n}\right)^n$$

where:

- APR_m = Annual percentage rate for m periods per year
- APR_n = Annual percentage rate for n periods per year Current yield $_t = \frac{\text{Annual coupon }_t}{\text{Bond price }_t}$ Government equivalent yield, Yield $_{\text{ACT / ACT}} = \frac{365}{360} \times$ Yield $_{30/360}$ Simple yield $= \frac{\text{Coupon } + \left(\frac{FV-PV}{N}\right)}{\text{Flat price}}$

Callable Bonds

$$PV = \frac{PMT}{(1+YTC)^1} + \frac{PMT}{(1+YTC)^2} + \cdots + \frac{PMT + \text{ Call price}}{(1+YTC)^N}$$

where:

- PV = Price of the callable bond
- PMT = Coupon payment per period
- YTC = Yield to call per period

- N = Number of periods to when the bond can be called at the call price

Option-adjusted price = Flat price of bond + Value of embedded call option

Value of call option = Price of option-free bond - Price of callable bond

G-spread = Bond YTM - Interpolated sovereign bond YTM

I-spread = Bond YTM - Swap rate

Z-Spread

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \cdots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

where:

- Z = Z-spread
- z_N = Spot rate for N periods

OAS = Z-spread - Option value (in basis points per year)

Learning Module 8: Yield and Yield Spread Measures for Floating-Rate Instruments

Value of Floating Rate Note (FRN)

$$PV = \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1+\frac{MRR+DM}{m}\right)^1} + \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1+\frac{MRR+DM}{m}\right)^2} + \cdots + \frac{\left(\frac{MRR+QM}{m}\right) \times FV + FV}{\left(1+\frac{MRR+DM}{m}\right)^n}$$

where:

- QM = Quoted Margin
- DM = Discount Margin
- MRR = Market reference rate
- m = Periodicity (i.e., number of payment periods per year)
- FV = Face Value of FRN
- N = Number of evenly spaced periods to maturity

Video: https://youtu.be/zqYOtVLkYR8

Yield Measures for Money Market Instruments

Discount Rate Basis

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$
$$DR = \frac{Year}{Days} \times \left(\frac{FV - PV}{FV}\right)$$

where:

- PV = present value of money market instrument
- FV = future value paid at maturity
- Days = number of days between settlement and maturity
- Year = number of days in the year
- DR = discount rate (stated as annual percentage rate)

Add-on Rate Basis

$$PV = \frac{FV}{\left(1 + \frac{\text{Days}}{\text{Year}} \times AOR\right)}$$
$$AOR = \frac{Year}{\text{Days}} \times \left(\frac{FV - PV}{PV}\right)$$
$$\text{Bond equivalent yield } = \frac{365}{Days} \times \left(\frac{FV - PV}{PV}\right)$$

Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves

Calculation of Bond Price Using Spot Rates

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT + FV}{(1+Z_N)^N}$$

where:

- PV = Price of bond
- PMT = Bond coupon payment
- Z_N = Spot rate (or zero-coupon yield or zero rate) for period N
- FV = Face value of bond

Given a Par Rate, $FV = PV$ and $PMT =$ Par Rate (%) $\times FV$

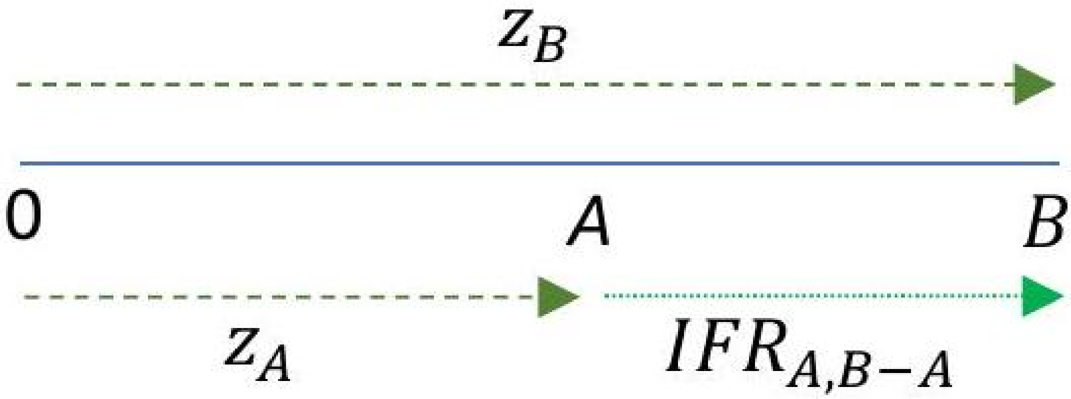
$$100 = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT + 100}{(1+Z_N)^N}$$

Forward Rates, IFR

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where:

- $IFR_{A,B-A}$ = Forward rate for ($B - A$) periods that starts in period A



Learning Module 10: Interest Rate Risk and Return

Duration Gap

Duration gap = Macaulay duration - Investment horizon

Macaulay Duration

$$\text{Macaulay duration} = (1 - \tfrac{t}{T}) \left[\frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{\text{Full}}} \right] + (2 - \tfrac{t}{T}) \left[\frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{\text{Full}}} \right] + \cdots$$

$$+ \left(N - \frac{t}{T}\right) \left[\frac{\frac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{Full}} \right]$$

$$\text{Macaulay duration} = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \frac{t}{T}$$

where:

- r = Yield per period
- c = Coupon rate per period
- N = Number of evenly spaced periods to maturity as of the beginning of the current period
- t = Number of days from the last coupon payment to the settlement date
- T = Number of days in the coupon period

Video: https://youtu.be/USgjcdCk7Fs

Learning Module 11: Yield-Based Bond Duration Measures and Properties

Modified Duration

$$\text{Modified Duration } = \frac{\text{Macaulay Duration}}{1+r}$$

Approximate Modified Duration

$$\begin{aligned} \text{AnnModDur } &\approx \frac{(PV_{-})-(PV_{+})}{2\times(\Delta \text{ Yield })\times(PV_0)} \\ \%\Delta PV^{\text{Full}} &\approx -\text{ AnnModDur } \times \Delta \text{ Yield} \end{aligned}$$

Money Duration

Money duration = AnnModDur $\times$ PV^{full}

$$\Delta PV^{\text{Full}} \approx -\text{ MoneyDur } \times \Delta \text{ Yield}$$

Duration of Zero-Coupon Bond

$$\begin{aligned} \text{MacDur } &= \text{ Time to maturity} \\ \text{ModDur } &= \frac{\text{Time to maturity}}{1+r} \end{aligned}$$

Duration of Perpetual Bond

$$\text{MacDur } = \frac{1+r}{r}$$
$$\text{ModDur } = \frac{1}{r}$$

Duration of Floating-Rate Notes

$$\text{MacDur } = \frac{T-t}{T} = \begin{array}{c}\text{Fraction of period remaining until} \\ \text{the next reset date}\end{array}$$

Learning Module 12: Yield-Based Bond Convexity and Portfolio Properties

Convexity

$$\text{Convexity } = \sum_{t=1}^{N} \frac{t(t+1) \times \frac{PV_t}{PV^{Full}}}{(1+YTM)^2}$$

Approximate Annualized Convexity

$$\text{ApproxConv } \approx \frac{(PV_{-}) + (PV_{+}) - 2\,(PV_0)}{(\Delta \text{ Yield })^2 \times (PV_0)}$$
$$\%\Delta PV^{\text{Full}} \approx - \text{ AnnModDur } \times \Delta \text{ Yield } + \frac{1}{2} \times \text{ AnnConvexity } \times (\Delta \text{ Yield })^2$$

Money Convexity

$$\text{MoneyCon } = \text{ AnnConvexity } \times PV^{\text{Full}}$$
$$\Delta PV^{\text{Full}} \approx -(\text{ MoneyDur } \times \Delta \text{ Yield }) + \left[\frac{1}{2} \times \text{ MoneyCon } \times (\Delta \text{ Yield })^2\right]$$

Portfolio Duration and Convexity

$$\text{Portfolio Modified Duration } = \sum_{i=1}^{N} w_i \times \text{ ModDur }_i$$
$$\text{Portfolio Convexity } = \sum_{i=1}^{N} w_i \times \text{ Convexity }_i$$

where:

- w_i = Weight of bond i, measured in market value

Learning Module 13: Curve-Based and Empirical Fixed-Income Risk Measures

Effective Duration

$$\text{EffDur } = \frac{(PV_{-}) - (PV_{+})}{2 \times (\Delta \text{ Curve }) \times PV_0}$$

Effective Convexity

$$\text{EffCon } = \frac{(PV_{-}) + (PV_{+}) - 2 \times PV_0}{(\Delta \text{ Curve })^2 \times PV_0}$$
$$\%\Delta PV^{\text{Full }} \approx - \text{ EffDur } \times \Delta \text{ Curve } + \frac{1}{2} \times \text{ EffCon } \times (\Delta \text{ Curve })^2$$

Key Rate Duration

$$\text{KeyRateDur}_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k}$$
$$\%\Delta PV = -\text{KeyRateDur}_k \times \Delta r_k$$
$$\sum_{k=1}^{n} \text{KeyRateDur}_k = \text{ EffDur}$$

where:

- $r_k = k$ th key rate

Learning Module 14: Credit Risk

Expected Loss

$$EL = LGD \times POD$$
$$LGD = EE \times (1 - RR)$$

where:

- POD = Probability of default
- LGD = Loss given default
- EE = Expected exposure
- RR = Recovery rate
- $1 - RR$ = Loss severity

Credit spread $\approx$ POD $\times$ LGD

Decomposing Bond Yields

Yield spread = Bond YTM - Government bond YTM (Similar maturity)

Liquidity spread = Bond YTM (Bid)$-$ Bond YTM (Offer)

Credit spread = Yield spread - Liquidity spread

Price Impact Given a Change in Yield Spread

$$\%\Delta PV^{\text{Full}} \approx -\text{ AnnModDur } \times \Delta \text{ Spread } + \frac{1}{2} \times \text{ AnnConvexity } \times (\Delta \text{ Spread })^2$$

where:

- AnnModDur $\approx \frac{(PV_{-})-(PV_{+})}{2\times(\Delta \text{ Yield })\times(PV_0)}$
- AnnConvexity $\approx \frac{(PV_{-})+(PV_{+})-2(PV_0)}{(\Delta \text{ Yield })^2\times(PV_0)}$

Learning Module 15: Credit Analysis for Government Issuers

No formula.

Learning Module 16: Credit Analysis for Corporate Issuers

$$\begin{aligned}
\text{EBIT margin } &= \frac{\text{Operating income}}{\text{Revenue}} \\
\text{EBIT to interest expense } &= \frac{\text{Operating income}}{\text{Interest expense}} \\
\text{Debt to EBITDA } &= \frac{\text{Debt}}{\text{EBITDA}} \\
\text{RCF to net debt } &= \frac{\text{Retained cash flow}}{\text{Debt - Cash and marketable securities}} \\
\text{FFO to debt } &= \frac{FFO}{\text{Debt}}
\end{aligned}$$

where:

- FFO = Net income from continuing operations + Depreciation & amortization
- Deferred income taxes + Other non-cash items

Learning Module 17: Fixed-Income Securitization

No formula.

Learning Module 18: Asset-Backed Security (ABS) Instrument and Market Features

No formula.

Learning Module 19: Mortgage-Backed Security (MBS) Instrument and Market Features

Loan-to-value (LTV) ratio

$$LTV = \frac{\text{Loan amount}}{\text{House price}}$$

Debt-to-income (DTI) ratio

$$DTI = \frac{\text{Monthly debt payment}}{\text{Monthly pre-tax gross income}}$$

Mortgage Pass-Through Securities

$$WAC = \sum_{i=1}^{N} c_i \left(\frac{CB_i}{CB}\right)$$
$$WAM = \sum_{i=1}^{N} MM_i \left(\frac{CB_i}{CB}\right)$$

where:

- WAC = Weighted average coupon
- WAM = Weighted average maturity
- c_i = Coupon rate on mortgage i
- MM_i = Number of months to maturity for mortgage i
- N = Number of mortgages in MBS
- CB_i = Current balance on mortgage i
- CB = Total current balance of mortgages in MBS

Commercial Mortgage-Backed Securities (CMBS)

Debt Service Coverage Ratio (DSCR)

$$\text{DSCR } = \frac{\text{Net operating income}}{\text{Debt service}}$$

Net Operating Income (NOI)

NOI = (Rental income - Cash operating expenses)− Replacement reserves

VOLUME 7: DERIVATIVES

Learning Module 1: Derivative Instrument and Derivatives Market Features

No formula.

Learning Module 2: Forward Commitments and Contingent Claim Features and Instruments

Forward Contract

$$\text{Buyer (Long) payoff } = S_T - F_0(T)$$
$$\text{Seller (Short) payoff } = -\left[S_T - F_0(T)\right]$$

where:

- S_T = Spot price on contract's maturity
- $F_0(T)$ = Forward price with maturity of T

Futures Contract

For one futures contract:

> Long Futures daily mark-to-market $= f_t(T) - f_{t-1}(T)$ Short Futures daily mark-to-market $= -\left[f_t(T) - f_{t-1}(T)\right]$

where:

- $f_t(T)$ = Closing price of futures contract on day t
- $f_{t-1}(T)$ = Closing price of futures contract on day $t-1$ T = Maturity of futures contract

If margin balance < maintenance margin:

> Variation Margin = Initial margin - Margin balance

Options Contract

LONG Call option

Payoff or Value at expiration, $c_T = \max(0, S_T - X)$

Profit at expiration, $\Pi = \max(0, S_T - X) - c_0$

where:

- c_0 = Call premium
- X = Exercise/Strike price S_T = Spot price at expiration

SHORT Call option

Payoff or Value at expiration, $c_T = -\max(0, S_T - X)$

Profit at expiration, $\Pi = -[\max(0, S_T - X) - c_0]$

LONG Put option

Payoff or Value at expiration, $p_T = \max(0, X - S_T)$

Profit at expiration, $\Pi = \max(0, X - S_T) - p_0$

SHORT Put option

Payoff or Value at expiration, $p_T = -\max(0, X - S_T)$

Profit at expiration, $\Pi = -[\max(0, X - S_T) - p_0]$

Credit Default Swap (CDS)

CDS MTM Change = $\triangle$ CDS Spread $\times$ CDS Notional $\times$ EffDur $_{\text{CDS}}$

In a credit event, payment from CDS seller to CDS buyer $\approx LGD(\%) \times$ Notional

Learning Module 3: Derivative Benefits, Risks, and Issuer and Investor Uses

No formula.

Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

If there are no underlying costs or benefits:

$$\text{Forward price, } F_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$\text{Forward price, } F_0(T) = [S_0 - PV_0(\text{ Income }) + PV_0(\text{ Cost })](1+r)^T$$

where:

- S_0 = Current spot price
- r = Risk-free rate
- T = Tenor of forward contract

Under continuous compounding, $F_0(T) = S_0 e^{rT}$ Under continuous compounding, with income (i) and cost (c) expressed in %:

$$F_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$F_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

where:

- $F_{0,f/d}$ = Forward exchange rate
- $S_{0,f/d}$ = Spot exchange rate
- r_f = Continuously compounded risk-free rate (for price/quote currency)
- r_d = Continuously compounded risk-free rate (for base currency)
- T = Maturity of forward contract

Learning Module 5: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

Value of LONG Forward Prior to Expiration

$$V_0(T) = 0$$
$$V_t(T) = S_t - \frac{F_0(T)}{(1+r)^{T-t}} = S_t - F_0(T) \times (1+r)^{-(T-t)}$$
$$V_T(T) = S_0 - F_0(T)$$

If the asset incurs cost or generates income from time t through maturity,

$$V_t(T) = [S_t - PV_t(\text{ Income }) + PV_t(\text{ Cost })] - F_0(T) \times (1+r)^{-(T-t)}$$

For foreign exchange forward contract,

$$V_t(T) = S_{t,f/d} - F_{0,f/d}(T) \times e^{-(r_f - r_d)(T-t)}$$

Value of SHORT Forward Prior to Expiration

$$V_0(T) = 0$$
$$V_t(T) = -\left[S_t - \frac{F_0(T)}{(1+r)^{T-t}}\right]$$
$$V_T(T) = -\left[S_0 - F_0(T)\right]$$

Interest Rate Forward Contracts (Forward Rate Agreements (FRA))

$$(1+z_A)^A \times \left(1+IFR_{A,B-A}\right)^{B-A} = (1+z_B)^B$$

where:

- z_A = Spot rate for A periods
- z_B = Spot rate for B periods
- $IFR_{A,B-A}$ = Implied forward rate for $(B-A)$ periods, starting in A periods

Payoff for a Long FRA $= \left(MRR_{B-A} - IFR_{A,B-A}\right) \times$ Notional principal $\times$ Period

Payoff for a Short FRA $= -\left(MRR_{B-A} - IFR_{A,B-A}\right) \times$ Notional principal $\times$ Period

Learning Module 6: Pricing and Valuation of Futures Contracts

If there are no underlying costs or benefits:

$$\text{Futures price, } f_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$f_0(T) = \left[S_0 - PV_0(\text{ Income }) + PV_0(\text{ Cost })\right](1+r)^T$$

Under continuous compounding, $f_0(T) = S_0 e^{rT}$

Under continuous compounding, with income (i) and cost (c) expressed in %:

$$f_0(T) = S_0 e^{(r+c-i)T}$$

Foreign Exchange Forward Contract

$$f_{0,f/d}(T) = S_{0,f/d}(T)e^{(r_f-r_d)T}$$

Interest Rate Futures Contract

$$f_{A,B-A} = 100 - \left(100 \times MRR_{A,B-A}\right)$$

where:

- $f_{A,B-A}$ = Futures price for a market reference rate for ($B-A$) periods that begins in A periods

Futures contract basis point value, BPV = Notional principal $\times 0.01\% \times$ Period

Learning Module 7: Pricing and Valuation of Interest Rates and Other Swaps

For a fixed-rate payer in an interest rate swap:

$$\text{Periodic settlement value } = (MRR - s_N) \times \text{ Swap Notional} \times \text{Period}$$

For a fixed-rate receiver in an interest rate swap:

$$\text{Periodic settlement value } = (s_N - MRR) \times \text{ Swap Notional} \times \text{Period}$$

where:

- s_N = Fixed swap rate
- MRR = Market reference rate

Calculating Par Swap Rate

$$\sum_{i=1}^{N} \frac{IFR}{(1+z_i)^i} = \sum_{i=1}^{N} \frac{s_N}{(1+z_i)^i}$$

where:

- IFR = Implied forward rates
- s_N = Fixed swap rate
- N = Tenor of swap contract

Valuation of Interest Rate Swap

Value of a pay-fixed interest rate swap on a settlement date after inception $= \begin{array}{c}\text{Current settlement}\\ \text{value}\end{array} + \Sigma(\text{ Floating payments }) - \Sigma(\text{ Fixed payments })$

Value of a receive-fixed interest rate swap on a settlement date after inception $= \begin{array}{c}\text{Current settlement}\\ \text{value}\end{array} + \Sigma(\text{ Fixed payments }) - \Sigma(\text{ Floating payments })$

Learning Module 8: Pricing and Valuation of Options

Option value = Exercise value + Time value

At time t (prior to option expiration):

Call option exercise value = Max $\left[0, S_t - X(1+r)^{-(T-t)}\right]$

Call option time value = $c_t -$ Max $\left[0, S_t - X(1+r)^{-(T-t)}\right]$

Put option exercise value = Max $\left[0, X(1+r)^{-(T-t)} - S_t\right]$

Put option time value = $p_t -$ Max $\left[0, X(1+r)^{-(T-t)} - S_t\right]$

$$
\begin{aligned}
\text{Lower bound of call option value } &= \text{Max}\left[0, S_t - X(1+r)^{-(T-t)}\right] \\
\text{Upper bound of call option value } &= S_t \\
\text{Lower bound of put option value } &= \text{Max}\left[0, X(1+r)^{-(T-t)} - S_t\right] \\
\text{Upper bound of put option value } &= X
\end{aligned}
$$

where:

- S_t = Spot price at time t
- X = Exercise price (or strike price)
- T = Maturity of option
- r = Risk-free rate

Learning Module 9: Option Replication Using Put-Call Parity

Put-Call Parity

$$S_0 + p_0 = c_0 + X(1+r)^{-T}$$

Put-Call Forward Parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

Value of the Firm

$$V_0 = c_0 + PV(\text{ Debt }) - p_0$$

Value of debt $= PV(\text{ Debt }) - p_0$ Value of equity $= c_0$

Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Risk-neutral probability of a price increase in underlying

$$\pi = \frac{1+r-R^d}{R^u-R^d}$$

where:

- $R^u = Up$ factor $= \frac{S_1^u}{S_0} > 1$
- $R^d =$ Down factor $= \frac{S_1^d}{S_0} < 1$
- $S_0 =$ Current asset price
- $S_1^u =$ One-period asset price when price moves up
- $S_1^d =$ One-period asset price when price moves down

Video: https://youtu.be/ymUIKgz-rAw

Hedge ratio

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

where:

- $c_1^u = \max(0, S_1^u - X)$
- $c_1^d = \max(0, S_1^d - X)$

Riskless portfolio with a Call: h of the underlying, S, and short call position, c $V_0 = hS_0 - c_0$ $V_1^u = hS_1^u - c_1^u$ $V_1^d = hS_1^d - c_1^d$

Riskless portfolio with a Put: h of the underlying, S, and long put position, p

$V_0 = hS_0 + p_0$ $V_1^u = hS_1^u + p_1^u$ $V_1^d = hS_1^d + p_1^d$

Value of a one-period call option

$$c_0 = \frac{\pi c_1^u + (1-\pi)c_1^d}{1+r}$$

Value of a one-period put option

$$p_0 = \frac{\pi p_1^u + (1-\pi)p_1^d}{1+r}$$

where:

- $p_1^u = \max\left(0, X - S_1^u\right)$
- $p_1^d = \max\left(0, X - S_1^d\right)$ Video: https://youtu.be/bXEC-78y AU

Learning Module 1: Alternative Investment Features, Methods, and Structures

GP Compensation Structure

Ignoring management fee; no catch-up clause

$$r_{GP} = \max\left[0, p\left(r - r_h\right)\right]$$

Ignoring management fee; with catch-up clause

$$r_{GP} = \max\left[0, r_{cu} + p\left(r - r_h - r_{cu}\right)\right]$$

where:

- $r_{GP} = \text{GP}'$ s rate of return
- $p =$ Performance fee as a percentage of total return
- $r =$ Single-period rate of return
- $r_h =$ Hard hurdle rate
- $r_{cu} =$ Catch-up clause

Learning Module 2: Alternative Investment Performance and Returns

Multiple on Invested Capital

$$MOIC = \frac{\text{Realized value of investment } + \text{ Unrealized value of investment}}{\text{Total amount of invested capital}}$$

Leveraged Portfolio Return

$$r_L = r + \frac{V_b}{V_c}(r - r_b)$$

where:

- r = Periodic rate of return on invested funds
- r_b = Periodic cost of borrowing
- V_b = Amount of borrowed funds
- V_c = Amount of cash (investor's own capital)

Investor's Return Net of Fees

$$r_i = \frac{P_1 - P_0 - R_{GP}}{P_0}$$
$$R_{GP} = (P_1 \times r_m) + \max\left[0, (P_1 - P_0) \times p\right]$$

where:

- P_0 = Beginning-of-period asset value
- P_1 = End-of-period asset value
- p = GP performance fee
- R_{GP} = GP$'$ s return in current terms
- r_m = GP's management fees as a percentage of assets under management

Calculating Hedge Fund Fees and Returns

Management Fee Based on Beginning Market Value

$$\underset{\text{Fee}}{\text{Management}} = \underset{\text{Fee}}{\%} = \text{Beginning Market}^{\text{Management}}$$

Management Fee Based on Ending Market Value

$$\underset{\text{Fee}}{\text{Management}} = \underset{\text{Fee}}{\%\ \text{Management}} \times \overset{\text{Ending Market}}{\text{Value}}$$

Incentive Fee Calculated Independent of Management Fee

$$\begin{array}{c}\text{Incentive}\\\text{Fee}\end{array} = \begin{array}{c}\text{\%Incentive}\\\text{Fee}\end{array} \times \text{ Gain}$$

Incentive Fee Calculated Net of Management Fee

$$\begin{array}{c}\text{Incentive}\\\text{Fee}\end{array} = \begin{array}{c}\text{\% Incentive}\\\text{Fee}\end{array} \times (\text{ Gain } - \text{ Management Fee })$$

Incentive Fee with Hard Hurdle (Independent of Management Fee)

$$\begin{array}{c}\text{Incentive}\\\text{Fee}\end{array} = \begin{array}{c}\text{\% Incentive}\\\text{Fee}\end{array} \times (\text{ Gain } - \text{ Hurdle })$$

Incentive Fee with Hard Hurdle (Net of Management Fee)

$$\begin{array}{c}\text{Incentive}\\\text{Fee}\end{array} = \begin{array}{c}\text{\% Incentive}\\\text{Fee}\end{array} \times (\text{ Gain } - \text{ Management Fee } - \text{ Hurdle })$$

$$\text{Hurdle} = \text{Hurdle Rate} \times \text{Beginning market value}$$

Note: 1) No incentive is paid if hedge fund incurs loss for the year. 2) Gain may be subject to high watermark.

[1] ## Learning Module 3: Investments in Private Capital: Equity and Debt

No formula.

Learning Module 4: Real Estate and Infrastructure

Loan-to-Value (LTV) Ratio

$$LTV = \frac{\text{Mortgage liability}}{\text{Portfolio value}}$$

Required reduction in mortgage liability = Mortgage liability - Required mortgage liability

[1] Video: https://youtu.be/ODKmCgsAAdc

Learning Module 5: Natural Resources

No formula.

Learning Module 6: Hedge Funds

No formula.

Learning Module 7: Introduction to Digital Assets

No formula.

VOLUME 9: PORTFOLIO MANAGEMENT

Learning Module 1: Portfolio Risk and Return: Part I

Expected Return on Asset

$$1 + E(R) = (1 + r_{rF}) \times [1 + E(\pi)] \times [1 + E(RP)]$$

where:

- r_{rF} = Real risk-free rate
- $E(\pi)$ = Expected inflation
- $E(RP)$ = Expected risk premium for the asset

Utility on Investment

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:

- U = Utility of investment
- $E(R)$ = Expected return of investment
- A = Risk aversion coefficient
- σ^2 = Variance of investment (Note: Substitute σ in decimals)

Capital Allocation Line (CAL)

For a portfolio of risky assets (Weight: w_i) and risk-free asset:

$$E\left(R_p\right) = R_f + \left[\frac{E\left(R_i\right) - R_f}{\sigma_i}\right]\sigma_p$$

where:

- R_f = Rate of return on risk-free asset
- $E\left(R_i\right)$ = Expected return of risky asset
- $E\left(R_p\right)$ = Expected return of portfolio
- σ_i = Standard deviation of risky asset's returns
- σ_p = Standard deviation of portfolio's returns = $w_i \times \sigma_i$
- $\frac{E(R_i)-R_f}{\sigma_i}$ = Market price of risk

Two-asset portfolio

Portfolio expected return, $E\left(R_p\right) = w_1 R_1 + w_2 R_2$

Portfolio variance, $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\,\mathrm{Cov}\left(R_1, R_2\right)$

Portfolio standard deviation, $\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\,\mathrm{Cov}\left(R_1, R_2\right)}$

Note: 1) $\mathrm{Cov}\left(R_1, R_2\right) = \rho_{12}\sigma_1\sigma_2$ 2) n securities requires n variances and $\frac{n(n-1)}{2}$ covariances

Video: https://youtu.be/IUwulZ9ONCO

Foreign Asset

Return of a foreign asset in domestic currency

$$R_D = \left(1 + R_{lc}\right) \times \left(1 + R_{FX}\right) - 1$$

Standard deviation of return of a foreign asset in domestic currency

$$\sigma_D = \sqrt{\sigma_{lc}^2 + \sigma_{FX}^2 + 2 \times \rho \times \sigma_{lc} \times \sigma_{FX}}$$

where:

- R_{lc} = Return of foreign asset (in local currency)
- R_{FX} = Change in exchange rate (FX rate quoted as domestic currency/foreign currency)

- σ_{lc} = Standard deviation of foreign asset's returns
- σ_{FX} = Standard deviation of the exchange rate (DC/FC)
- ρ = Correlation coefficient between returns on foreign asset and exchange rate

Portfolio of Many Risky Assets

$$\sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\overline{\text{Cov}} = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\rho\bar{\sigma}^2$$

where:

- N = Number of assets in portfolio
- $\bar{\sigma}^2$ = Average variance
- $\overline{\text{Cov}}$ = Average covariance

Learning Module 2: Portfolio Risk and Return: Part II

Capital Market Line (CML)

$E\left(R_p\right) = w_f R_f + \left(1 - w_f\right) E\left(R_m\right) = R_f + \left[\frac{E(R_m) - R_f}{\sigma_m}\right]\sigma_p$ $\sigma_p = \left(1 - w_f\right)\sigma_m$

Return-Generating Models

$$E\left(R_i\right) - R_f = \beta_{i1}\left[E\left(R_m\right) - R_f\right] + \sum_{j=2}^{k}\beta_{ij}E\left(F_j\right)$$

where:

- $E\left(R_i\right) - R_f$ = Expected excess return on asset i
- k = Number of factors β_{ij} = Factor weights (also called factor loadings) $E\left(R_m\right)$ = Expected return on market

The Single-Index Model

$$E\left(R_i\right) - R_f = \left(\frac{\sigma_i}{\sigma_m}\right)\left[E\left(R_m\right) - R_f\right]$$

where:

- $\frac{\sigma_i}{\sigma_m}$ = Factor loading (or factor weight)

Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i \left[E(R_m) - R_f\right]$$

The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Beta of security i

$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$ Portfolio beta, $\beta_p = \sum_{i=1}^{n} w_i \beta_i$ Total variance = Systematic variance + Nonsystematic variance

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

Total risk, $\sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_e^2}$

Arbitrage Pricing Theory (APT) Model

$$E(R_P) = R_F + \lambda_1 \beta_{P,1} + \cdots + \lambda_K \beta_{P,K}$$

where:

- $E(R_P)$ = Expected return on portfolio
- R_F = Risk-free rate
- λ_j = Risk premium for factor j
- $\beta_{P,1}$ = Sensitivity of the portfolio to factor j
- K = Number of risk factors

Fama-French Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t$$

Carhart Model

$$E\left(R_{it}\right)=\alpha_i+\beta_{i,MKT}MKT_t+\beta_{i,SMB}SMB_t+\beta_{i,HML}HML_t+\beta_{i,UMD}UMD_t$$

where:

- $E\left(R_i\right)$ = Return on an asset in excess of the one-month T-bill return
- MKT = Excess return on the market portfolio
- SMB = Difference in returns between small-capitalization stocks and large-capitalization stocks (Size)
- HML = Difference in returns between high-book-to-market stocks and low-book-to-market stocks (Value versus growth)
- UMD = Difference in returns of the prior year's winners versus losers (Momentum)

Portfolio Performance Appraisal Measures

Sharpe ratio $=\frac{R_p-R_f}{\sigma_p}$

Treynor ratio $=\frac{R_p-R_f}{\beta_p}$ $M^2=\left(R_p-R_f\right)\frac{\sigma_m}{\sigma_p}+R_f$ = Sharpe ratio $\times\sigma_m+R_f$ M^2 alpha $=M^2-R_m$

Jensen's Alpha, $\quad \alpha_p=R_p-\left[R_f+\beta_p\left(R_m-R_f\right)\right]$

Security Characteristic Line (SCL)

$$R_i-R_f=\alpha_i+\beta_i\left(R_m-R_f\right)$$

Information ratio $=\frac{\alpha_i}{\sigma_{ei}}$

Learning Module 3: Portfolio Management: An Overview

No formula.

Learning Module 4: Basics of Portfolio Planning and Construction

No formula.

Learning Module 5: The Behavioral Biases of Individuals

No formula.

Learning Module 6: Introduction to Risk Management

No formula.