

# CFA Program Level 1”

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## CFA® Program Level I

### FORMULA SHEET (2025)

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## **CFA Level 1 - Formula Sheet (2025)**

### **Setting Up the Texas BA II Plus Financial Calculator**

Video: <https://youtu.be/OMS8d8QOFmc>

### **Using Texas BA II Plus Financial Calculator**

Video: <https://youtu.be/LWmTTiZz8BU>

Video (Requires Login to Facebook): <https://fb.watch/nci5V7Dwtj/>

## **VOLUME 1: QUANTITATIVE METHODS**

### **Learning Module 1: Rates and Returns**

#### **Determinants of Interest Rates**

Interest rate,  $r = \text{Real risk-free rate} + \text{Inflation premium} + \text{Default risk premium}$

- Liquidity premium + Maturity premium  $(1 + \text{Nominal risk-free rate}) = (1 + \text{Real risk-free rate}) \times (1 + \text{Inflation premium})$

Nominal risk-free rate = Real risk-free rate + Inflation premium

Maturity premium = Interest rate on longer-maturity, liquid Treasury debt

- Interest rate on short-term Treasury debt

## Holding Period Return

$$R = \frac{P_1 - P_0 + I_1}{P_0}$$

where:

- $P_0$  = Price at the beginning of the period
- $P_1$  = Price at the end of the period
- $I_1$  = Income

If given holding period returns  $R_1, R_2, \dots, R_T$  over the holding period:

$$R = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

## Arithmetic Return

$$\bar{R}_i = \frac{1}{T} \sum_{t=1}^T R_{it} = \frac{1}{T} (R_{i1} + R_{i2} + \dots + R_{iT})$$

## Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^T (1 + R_t)} - 1 = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times \dots \times (1 + R_{iT})} - 1$$

## Harmonic Mean

$$\bar{X}_{Hi} = \frac{n}{\sum_{i=1}^n (1/X_i)} \quad \text{for } X_i > 0$$

## Relationship between Arithmetic Mean, Geometric Mean, and Harmonic Mean

$$(\text{Geometric mean})^2 = \text{Arithmetic mean} \times \text{Harmonic mean}$$

## Money-Weighted Return (MWR)

$$\sum_{t=0}^T \frac{CF_t}{(1 + MWR)^t} = 0$$

## Time-Weighted Return (TWR)

Given the holding period returns for each sub-period,  $R_1, R_2, \dots, R_T$

If  $T > 1$  year, then

$$\text{Annualized TWR} = [(1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T)]^{1/T} - 1$$

If  $T = 1$  year, then

$$\text{Annualized TWR} = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

If  $T < 1$  year, then

$$\text{TWR for holding period} = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

## Non-Annual Compounding

$$PV = FV_N \left(1 + \frac{R_s}{m}\right)^{-mN}$$

where:

- $m$  = Number of compounding periods per year
- $R_s$  = Quoted annual interest rate
- $N$  = Number of years

## Annualizing Returns

$$R_{\text{annual}} = (1 + R_{\text{weekly}})^{52} - 1 \quad R_{\text{annual}} = (1 + R_{\text{monthly}})^{12} - 1 \quad R_{\text{annual}} = (1 + R_{\text{daily}})^{252} - 1$$

assuming 252 trading days per year  $R_{\text{weekly}} = (1 + R_{\text{daily}})^5 - 1$  assuming 5 trading days per week

## Continuously Compounded Returns

$$P_t = P_0 e^{r_{0,T}}$$
$$r_{0,T} = \ln \left( \frac{P_t}{P_0} \right)$$
$$r_{0,T} = r_{0,1} + r_{1,2} + \dots + r_{T-2,T-1} + r_{T-1,T}$$

## Real Returns

$$(1 + \text{real return}) = (1 + \text{real risk-free rate}) \times (1 + \text{risk premium})$$

## Pre-Tax and After-Tax Nominal Return

$$\begin{aligned} \text{After-tax nominal return} &= \text{Pre-tax nominal return} \times (1 - \text{Tax rate}) \\ &= \frac{[1 + \text{Pre-Tax nominal return} \times (1 - \text{Tax rate})]}{1 + \text{Inflation premium}} - 1 \end{aligned}$$

## Leveraged Return

Return on a leveraged portfolio

$$R_L = R_P + \frac{V_B}{V_E} (R_P - r_D)$$

where:

- $R_P$  = Return on the investment portfolio (unleveraged)
- $r_D$  = Cost of debt
- $V_B$  = Debt/borrowed funds
- $V_E$  = Equity of the portfolio

## Learning Module 2: Time Value of Money in Finance

$$FV_t = PV(1 + r)^t \quad PV = \frac{FV_t}{(1 + r)^t}$$

where:

- $FV_t$  = Future value at time  $t$
- $PV$  = Present value
- $r$  = Discount rate per period
- $t$  = Number of compounding periods

As compounding frequency becomes very large (i.e., continuous compounding)

$$FV_t = PV e^{rt} \quad PV = FV_t e^{-rt}$$

## **Present Value of Zero-Coupon Bond**

$$PV(\text{ Discount Bond }) = \frac{FV}{(1+r)^t}$$

where:

- $FV$  = Principal (or Face Value)
- $r$  = Market discount rate per period
- $t$  = Maturity of bond

$$r = \left( \frac{FV_t}{PV} \right)^{1/T} - 1$$

## **Present Value of Coupon Bond**

$$PV(\text{ Coupon Bond }) = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}$$

where:

- $PV$  = Bond's price
- $PMT$  = Periodic coupon payment
- $FV$  = Face value
- $N$  = Number of periods
- $r$  = Market discount rate per period

## **Present Value of a Perpetual Bond (Perpetuity)**

$$PV(\text{ Perpetual Bond }) = \frac{PMT}{r}$$

## **Annuity Instruments (e.g., Mortgage)**

$$A = \frac{rPV}{1 - (1+r)^{-t}}$$

where:

- $A$  = Periodic cash flow
- $r$  = Market interest rate per period
- $PV$  = Present value or principal amount of loan/bond
- $t$  = Number of payment periods

## Price of a Preferred Share

$$PV_t = \frac{D_t}{r}$$

where:

- $D_t$  = Fixed periodic dividend
- $r$  = Expected rate of return

## Price of a Common Share

### Constant Dividend Growth Rate into Perpetuity

$$PV_t = \frac{D_t(1+g)}{r-g} = \frac{D_{t+1}}{r-g} \quad r > g$$

where:

- $D_t$  = Common dividend at time  $t$
- $g$  = Constant growth rate
- $r$  = Expected rate of return

$$\begin{aligned} r &= \frac{D_{t+1}}{PV_t} + g \\ \frac{PV_t}{E_t} &= \frac{\frac{D_t}{E_t} \times (1+g)}{r-g} \\ \frac{PV_t}{E_{t+1}} &= \frac{\frac{D_{t+1}}{E_{t+1}}}{r-g} \end{aligned}$$

where:

- $E_t$  = Earnings per share for period  $t$
- $\frac{PV_t}{E_t}$  = Trailing price-to-earnings ratio
- $\frac{PV_t}{E_{t+1}}$  = Forward price-to-earnings ratio

## Two-stage Dividend Discount Model

$$PV_t = \sum_{i=1}^n \frac{D_t (1 + g_s)^i}{(1 + r)^i} + \frac{E(S_{t+n})}{(1 + r)^n}$$

where:

- $g_s$  = Higher short-term dividend growth rate
- $g_L$  = Lower long-term dividend growth rate
- $n$  = Initial growth phase
- $E(S_{t+n})$  = Stock value in  $n$  periods (Terminal value)

$$= \frac{D_{t+n+1}}{r - g_L}$$

## Forward Rate

$$F_{1,1} = \frac{(1 + r_2)^2}{(1 + r_1)} - 1$$

where:

- $F_{1,1}$  = One-year forward rate one year from now
- $r_1$  = Discount rate on one-year risk-free discount bond
- $r_2$  = Discount rate on two-year risk-free discount bond

## Learning Module 3: Statistical Measures of Asset Returns

### Measures of Central Tendency

Sample Mean,  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$

where:

- $X_i$  = Observation  $i$  ( $i = 1, 2, 3, \dots, n$ )

### Median

$$\text{Position of median} = \frac{\text{Number of observations} + 1}{2}$$

## Quantiles

$$\text{Interquartile range} = Q_3 - Q_1$$

where:

- $Q_1$  = First quartile
- $Q_3$  = Third quartile

## Box and Whisker Plot

$$\text{Upper fence} = Q_3 + 1.5 \times IQR$$

$$\text{Lower fence} = Q_1 - 1.5 \times IQR$$

## Measures of Dispersion

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

## Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^n |X_i - \bar{X}|}{n}$$

## Sample Variance

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$$

## Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

## Sample Target Semideviation

$$s_{\text{Target}} = \sqrt{\frac{\sum_{X_i \leq B}^n (X_i - B)^2}{n - 1}}$$

where:

- $B$  = target
- $n$  = total number of sample observations

## Coefficient of Variation

$$CV = \frac{s}{\bar{X}}$$

## Sample Skewness

$$\text{Skewness} \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^n (X_i - \bar{X})^3}{s^3}$$

## Sample Excess Kurtosis

$$K_E \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^n (X_i - \bar{X})^4}{s^4} - 3$$

## Sample Covariance

$$s_{XY} = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})$$

## Sample Correlation Coefficient

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

## **Learning Module 4: Probability Trees and Conditional Expectations**

### **Expected Value of a Discrete Random Variable**

$$E(X) = \sum_{i=1}^n P(X_i) X_i$$

### **Variance of a Random Variable**

$$\begin{aligned}\sigma^2(X) &= E[X - E(X)]^2 \\ &= \sum_{i=1}^n P(X_i) [X - E(X)]^2\end{aligned}$$

### **Conditional Expected Value of a Random Variable**

$$E(X | S) = P(X_1 | S) X_1 + P(X_2 | S) X_2 + \dots + P(X_n | S) X_n$$

### **Conditional Variance of a Random Variable**

$$\begin{aligned}\sigma^2(X | S) &= P(X_1 | S) [X_1 - E(X_1 | S)]^2 + P(X_2 | S) [X_2 - E(X_2 | S)]^2 + \dots \\ &\quad + P(X_n | S) [X_n - E(X_n | S)]^2\end{aligned}$$

### **Total Probability Rule for Expected Value**

$$E(X) = E(X | S_1) P(S_1) + E(X | S_2) P(S_2) + \dots + E(X | S_n) P(S_n)$$

where:

- $S_1, S_2, \dots, S_n$  are mutually exclusive and exhaustive events.

### **Bayes' Formula**

$$\begin{aligned}P(A | B) &= \frac{P(B | A)}{P(B)} \times P(A) \\ P(\text{ Event } | \text{ Information }) &= \frac{P(\text{ Information } | \text{ Event })}{P(\text{ Information })} \times P(\text{ Event })\end{aligned}$$

## Learning Module 5: Portfolio Mathematics

For  $n$  assets in a portfolio

### Expected return on portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

### Variance on portfolio

$$\sigma^2(R_P) = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}(R_i, R_j)$$

Requires  $n$  variances and  $\frac{n(n-1)}{2}$  distinct covariances to estimate portfolio variance.

### Covariance

$$\begin{aligned} \text{Cov}(R_i, R_j) &= E[(R_i - E(R_i))(R_j - E(R_j))] \\ &= \frac{1}{n-1} \sum_{t=1}^n (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) \end{aligned}$$

For a two-asset ( $n = 2$ ) portfolio:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)$$

where:

- $\text{Cov}(R_1, R_2) = \rho(R_1, R_2) \times \sigma(R_1) \times \sigma(R_2)$

**Video:** <https://youtu.be/IUwulZ9ONCO>

For a three-asset ( $n = 3$ ) portfolio:

$$\begin{aligned} \sigma^2(R_P) &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2w_1 w_2 \text{Cov}(R_1, R_2) \\ &\quad + 2w_1 w_3 \text{Cov}(R_1, R_3) + 2w_2 w_3 \text{Cov}(R_2, R_3) \end{aligned}$$

## Covariance Given a Joint Probability Function

$$\text{Cov}(R_A, R_B) = \sum_{i=1} \sum_{j=1} P(R_{A,i}, R_{B,j}) \times [R_{A,i} - E(R_A)] \times [R_{B,j} - E(R_B)]$$

If  $X$  and  $Y$  are uncorrelated, then  $E(XY) = E(X)E(Y)$

If  $X$  and  $Y$  are independent, then  $P(X, Y) = P(X)P(Y)$

## Safety-First Optimal Portfolio

### Safety-First Ratio

$$\text{SFRatio} = \frac{E(R_P) - R_L}{\sigma_P}$$

$$\text{Shortfall risk} = \Pr[E(R_P) < R_L] = \text{Normal}(-\text{SFRatio})$$

where:

- $R_L$  = Investor's threshold level
- $E(R_P)$  = Expected portfolio return
- $\sigma_P$  = Portfolio standard deviation

**Video:** <https://youtu.be/S3x5JrGIOUA>

## Learning Module 6: Simulation Methods

### Lognormal Distribution

#### Mean of a lognormal random variable

$$\mu_L = \exp(\mu + 0.50\sigma^2)$$

#### Variance of a lognormal random variable

$$\sigma_L^2 = \exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$$

where:

- $\mu$  = Mean of the normal random variable
- $\sigma^2$  = Variance of the normal random variable

## Continuously Compounded Rates of Return

$$P_T = P_0 \exp(r_{0,T})$$

where:

- $P_0$  = Current asset price
- $P_T$  = Asset price at time  $T$
- $r_{0,T}$  = Continuously compounded return from 0 to  $T$

If returns are independently and identically distributed (i.i.d.), then

$$r_{0,T} = r_{0,1} + r_{1,2} + \dots + r_{T-2,T-1} + r_{T-1,T}$$

If the one-period continuously compounded returns are i.i.d. random variables with mean  $\mu$  and  $\sigma^2$ , then

$$\begin{aligned} E(r_{0,T}) &= \mu T \\ \sigma^2(r_{0,T}) &= \sigma^2 T \\ \sigma(r_{0,T}) &= \sigma\sqrt{T} \end{aligned}$$

## Learning Module 7: Estimation and Inference

$$\text{Sharpe ratio} = \frac{R_P - R_F}{\sigma_P}$$

where:

- $R_P$  = Portfolio return
- $R_F$  = Risk-free rate
- $\sigma_P$  = Portfolio standard deviation of return

$$\begin{aligned} \text{Variance of the sampling distribution} &= \frac{\sigma^2}{n} \\ \text{of the sample means} \\ \text{Standard error of} &= \frac{\sigma}{\sqrt{n}} \end{aligned}$$

where:

- $\sigma$  = Population standard deviation
- $n$  = Sample size

Note: If  $\sigma$  is not known, use  $s$ , the sample standard deviation.

### Bootstrap Resampling

$$s_{\bar{X}} = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}_b - \bar{\theta})^2}$$

where:

- $s_{\bar{X}}$  = Estimate of the standard error of the sample mean
- $B$  = Number of resamples drawn from the original sample
- $\hat{\theta}_b$  = Mean of a resample
- $\bar{\theta}$  = Mean across all the resample means

## Learning Module 8: Hypothesis Testing

Confidence level =  $1 - \alpha$

Power of the test =  $1 - \beta$

where:

- $\alpha$  = Significance level (Probability of Type I error)
- $\beta$  = Probability of Type II error

### Test of a Single Mean

Test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Degrees of freedom =  $n - 1$   $(1 - \alpha)\%$  Confidence Interval =  $\bar{X} + \text{Critical value} \times \left(\frac{s}{\sqrt{n}}\right)$

## **Test of the Difference in Means**

Test statistic

$$t = \frac{(\bar{X}_{d1} - \bar{X}_{d2}) - (\mu_{d1} - \mu_{d2})}{\sqrt{\frac{s_p^2}{n_{d1}} + \frac{s_p^2}{n_{d2}}}}$$

Degrees of freedom =  $n_{d1} + n_{d2} - 2$

$$s_p^2 = \frac{(n_{d1} - 1)s_{d1}^2 + (n_{d2} - 1)s_{d2}^2}{n_{d1} + n_{d2} - 2}$$

## **Test of the Mean of Differences**

Test statistic

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

Degrees of freedom =  $n - 1$

## **Test of a Single Variance**

Test statistic

$$\chi^2 = \frac{(n - 1)s^2}{\sigma_0^2}$$

Degrees of freedom =  $n - 1$

## **Test of the Difference in Variances**

Test statistic

$$F = \frac{s_{\text{Before}}^2}{s_{\text{After}}^2}$$

Degrees of freedom =  $n_1 - 1, n_2 - 1$

## **Test of a Correlation**

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom =  $n - 2$

## **Test of Independence (Categorical Data)**

Test statistic

$$\chi^2 = \sum_{i=1}^m \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Degrees of freedom =  $(r - 1)(c - 1)$

where:

- $m$  = Number of cells in the table
- $O_{ij}$  = Number of observations in each cell of row  $i$  and column  $j$
- $E_{ij}$  = Expected number of observations in each cell of row  $i$  and column  $j$

## **Learning Module 9: Parametric and Non-Parametric Tests of Independence**

### **Test of a Correlation**

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom =  $n - 2$

### **Pearson Correlation (or Bivariate Correlation)**

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

## Spearman Rank Correlation Coefficient

$$r_S = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}$$

where:

- $d$  = Difference in ranks

## Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^m \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Degrees of freedom =  $(r - 1)(c - 1)$

where:

- $m$  = Number of cells in the table
- $O_{ij}$  = Number of observations in each cell of row  $i$  and column  $j$
- $E_{ij}$  = Expected number of observations in each cell of row  $i$  and column  $j$

$$= \frac{(\text{Total row } i) \times (\text{Total column } j)}{\text{Overall total}}$$

## Standardized Residual (or Pearson Residual)

$$\text{Standardized Residual} = \frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}}$$

## Learning Module 10: Simple Linear Regression

$$Y_i = b_0 + b_1 X_1 + \cdots + b_n X_n + \varepsilon_i, \quad i = 1, 2, \dots, n$$

where:

- $Y$  = Dependent variable
- $X$  = Independent variable
- $b_0$  = Intercept
- $b_i$  = Slope coefficient,  $i = 1, 2, \dots, n$
- $\varepsilon_i$  = Error term
- $b_0, b_1, \dots, b_n$  = Regression coefficients

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i + e_i$$

where:

- $\hat{Y}_i$  = Estimated value on the regression line for the  $i$  th observation
- $\hat{b}_0$  = Intercept
- $\hat{b}_1$  = Slope
- $e_i$  = Residual for the  $i$  th observation  $\hat{b}_1 = \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X} = \frac{\sum_{i=1}^n (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^n (X_i - \bar{X})^2}$   $\hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$  Sum of Squares Total,  $SST = \sum_{i=1}^n (Y_i - \bar{Y})^2 = SSR + SSE$  Sum of Squares Regression,  $SSR = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$  Sum of Squares Error,  $SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n e_i^2$  Coefficient of Determination

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation coefficient

$$r = \frac{\text{Covariance of } X \text{ and } Y}{(\text{Standard deviation of } X)(\text{Standard deviation of } Y)}$$

Note:  $(\text{Correlation coefficient})^2 = \text{Coefficient of determination}$

Sample standard deviation of X

$$S_X = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

Sample standard deviation of Y

$$S_Y = \sqrt{\frac{\sum_{i=1}^n (Y_i - \bar{Y})^2}{n-1}}$$

Homoskedasticity

$$E(\varepsilon_i^2) = \sigma_\varepsilon^2, \quad i = 1, 2, \dots, n$$

## ANOVA F-Test

Mean square regression (MSR)

$$MSR = \frac{SSR}{k}$$

Mean square error (MSE)

$$MSE = \frac{SSE}{n-k-1}$$

F-distributed test statistic

$$F = \frac{MSR}{MSE}$$

where:

- $n$  = Number of observations
- $k$  = Number of independent variables

Standard error of estimate

$$s_e = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-k-1}}$$

Hypothesis Test of the Slope Coefficient

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

Degrees of freedom,  $df = n - k - 1$

where:

- $B_1$  = Hypothesized population slope
- $s_{\hat{b}_1}$  = Standard error of the slope coefficient

$$= \frac{s_e}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Hypothesis Test of the Intercept

$$t_{\text{intercept}} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}}$$

Standard error of the intercept,  $s_{\hat{b}_0}$

$$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Prediction Intervals

$$\hat{Y}_f \pm t_{\alpha/2} \times s_f$$

where:  $\hat{Y}_f = \hat{b}_0 + \hat{b}_1 X_f$

Variance of the prediction error of Y, given X

$$s_f^2 = s_e^2 \left[ 1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{(n-1)s_X^2} \right]$$

Standard error of the forecast

$$s_f = s_e \sqrt{1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{(n-1)s_X^2}}$$

## The Log-Lin Model

$$\ln Y_i = b_0 + b_1 X_i$$

## The Lin-Log Model

$$Y_i = b_0 + b_1 \ln X_i$$

## The Log-Log Model

$$\ln Y_i = b_0 + b_1 \ln X_i$$

## Learning Module 11: Introduction to Big Data Techniques

No formula.

## VOLUME 2: ECONOMICS

### Learning Module 1: The Firm and Market Structures

Total profit = Total revenue - Total cost

Economic profit = Total revenue - Total economic costs

Accounting profit = Total revenue - Total accounting costs

Total revenue = Price  $\times$  Quantity =  $P \times Q$

Average revenue =  $\frac{\text{Total revenue}}{\text{Quantity}}$

Marginal cost =  $\frac{\Delta TC}{\Delta Q}$

Average variable cost =  $\frac{\text{Total variable cost}}{\text{Quantity}}$

Average fixed cost =  $\frac{\text{Total fixed cost}}{\text{Quantity}}$

Total cost = Total fixed cost + Total variable cost

Average total cost = Average fixed cost + Average variable cost

### Concentration Ratio

$$\text{Concentration ratio} = \sum_{i=1}^n (\text{Market share})_i$$

## **Herfindahl-Hirschman Index (HHI)**

$$HHI = \sum_{i=1}^n (\text{Market share})_i^2$$

## **Learning Module 2: Understanding Business Cycles**

### **No formula**

## **Learning Module 3: Fiscal Policy**

$$\text{Budget surplus/(deficit)} = G - T + B$$

where:

- $G$  = Government spending
- $T$  = Taxes
- $B$  = Payments of transfer benefits

## **Disposable Income**

$$YD = Y - NT = (1 - t)Y$$

where:

- $t$  = Net tax rate
- $NT$  = Net taxes = Taxes - Transfers
- $tY$  = Total tax revenue

## **The Fiscal Multiplier**

$$\text{Fiscal multiplier} = \frac{1}{1 - c(1 - t)}$$

where:

- $c$  = Marginal propensity to consume
- $t$  = Tax rate

## **Learning Module 4: Monetary Policy**

Neutral rate = Trend growth + Inflation target

## **Learning Module 5: Introduction to Geopolitics**

No formula

## **Learning Module 6: International Trade**

No formula

## **Learning Module 7: Capital Flows and the FX Market**

Real exchange rate  $S_{d/f} = P_f / P_d$  % Change in real exchange rate =  $(1 + \% \Delta S_{d/f}) \times \frac{(1 + \% \Delta P_f)}{(1 + \% \Delta P_d)} - 1$

$$\approx \% \Delta S_{d/f} + \% \Delta P_f - \% \Delta P_d$$

Percentage change in base currency  $f$  (vs currency  $d$ )

$$\frac{E(S_{d/f}) - S_{d/f}}{S_{d/f}}$$

where:

- $S_{d/f}$  = Spot exchange rate
- $P_f$  = General price level of goods indexed in currency  $f$
- $P_d$  = General price level of goods indexed in currency  $d$

## **Learning Module 8: Exchange Rate Calculations**

### **Cross-Rate**

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{D}$$

## Forward Rate

$$F_{A/B} = S_{A/B} \times \left[ \frac{1 + r_A \times T}{1 + r_B \times T} \right]$$

$$\begin{aligned}\text{Forward points} &= F_{A/B} - S_{A/B} \\ &= S_{A/B} \left( \frac{r_A - r_B}{1 + r_B} \right) T\end{aligned}$$

where:

- $S_{A/B}$  = Spot exchange rate
- $F_{A/B}$  = Forward exchange rate
- $T$  = Time to maturity

Learning Module 1: Organizational Forms, Corporate Issuer Features, and Ownership

No formula

## Learning Module 2: Investors and Other Stakeholders

### No formula

Learning Module 3: Working Capital and Liquidity

$$\begin{aligned}\text{Cash conversion cycle} &= \frac{\text{Days of inventory on hand}}{\text{Days sales outstanding}} - \frac{\text{Days payables outstanding}}{\text{Days sales outstanding}} \\ \text{EAR of Supplier Financing} &= \left( 1 + \frac{\text{Discount \%}}{100\% - \text{Discount \%}} \right)^{\frac{\text{Days in Year}}{\text{Days Period-Discount Period}}} = \frac{\text{Current assets} - \text{Current Liabilities}}{\text{Total working capital}} \\ \text{Net working capital} &= \text{Current assets (excluding cash and marketable securities)} \\ &\quad - \text{Current Liabilities (excluding short-term and current debt)}\end{aligned}$$

Cash flow from operations = Cash received from customers

- Interest and dividends received on financial investments
- Cash paid to employees and suppliers
- Taxes paid to governments
- Interest paid to lenders

Free cash flow = Cash flow from operations - Investments in long-term assets

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Short-term marketable instruments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short-term marketable instruments}}{\text{Current liabilities}}$$

#### **Learning Module 4: Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits**

##### **No formula**

#### **Learning Module 5: Capital Investments and Capital Allocation**

##### **Net Present Value**

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^T \frac{CF_t}{(1+r)^t}$$

where:

- $CF_t$  = After-tax cash flow at time  $t$
- $r$  = Required rate of return
- $CF_0$  = Initial outlay

##### **Internal Rate of Return**

$$\sum_{t=0}^T \frac{CF_t}{(1+IRR)^t} = 0$$

**Video:** <https://youtu.be/bzck7QLhICw>

## Return on Invested Capital

$$\begin{aligned} \text{ROIC} &= \frac{\text{After-tax operating profit}}{\text{Average invested capital}} \\ &= \frac{\text{Operating profit } t_t \times (1 - \text{Tax rate })}{\text{Average total long-term liabilities and equity }_{t-1,t}} \end{aligned}$$

$$\text{ROIC} = \frac{\text{After-tax operating profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Average invested capital}}$$

## Real Options in Capital Budgeting

$$\frac{\text{Project NPV}}{(\text{with option})} = \frac{\text{Project NPV}}{(\text{without option})} - \text{Option cost} + \text{Option value}$$

## Learning Module 6: Capital Structure

### Weighted Average Cost of Capital

$$WACC = w_d r_d (1 - t) + w_e r_e$$

where:

- $w_d$  = Target weight of debt in capital structure =  $\frac{D}{D+E}$
- $w_e$  = Target weight of common stock in capital structure =  $\frac{E}{D+E}$
- $r_d$  = Before-tax marginal cost of debt
- $t$  = Marginal tax rate
- $r_d(1 - t)$  = After-tax marginal cost of debt
- $r_e$  = Marginal cost of common stock

## Operating Leverage

$$\text{Operating leverage} = \frac{\text{Fixed costs}}{\text{Total costs}}$$

## Interest Coverage

$$\text{Interest coverage} = \frac{\text{Profit before interest and taxes}}{\text{Interest expense}}$$

## **Modigliani-Miller Capital Structure Propositions**

$$\begin{aligned}V_L &= V_U + tD \\r_e &= r_0 + (r_0 - r_d)(1-t)\frac{D}{E} \\E &= \frac{(CF_e - r_d D)(1-t)}{r_e} \\V_L &= \frac{CF_e(1-t)}{r_{WACC}}\end{aligned}$$

where:

- $V_L$  = Value of levered firm
- $V_U$  = Value of unlevered firm
- $t$  = Marginal tax rate
- $r_e$  = Cost of equity
- $r_d$  = Cost of debt
- $r_0$  = Cost of capital (for a 100% equity-financed company)
- $D$  = Market value of debt
- $E$  = Market value of equity
- $CF_e$  = After-tax cash flows to shareholders
- $r_d D$  = Interest expense on debt

## **Static Trade-off Theory of Capital Structure**

$$V_L = V_U + tD - PV(\text{Costs of Financial Distress})$$

## **Learning Module 7: Business Models**

No formula

## **VOLUME 4: FINANCIAL STATEMENT ANALYSIS**

### **Learning Module 1: Introduction to Financial Statement Analysis**

No formula

## **Learning Module 2: Analyzing Income Statements**

Gross profit = Revenue - Cost of Goods Sold

Operating income = Gross margin - Selling, General, and Administrative Expense

Taxable income = Operating income - Interest expense

Net income = Taxable income - Taxes

Ending shareholders' equity = Beginning shareholders' equity + Net income

- Other comprehensive income
- Dividends
- Net capital contributions from shareholders

Ending retained earnings = Beginning retained earnings + Net income - Dividends

### **Return on Equity**

$$ROE = \frac{\text{Net income}}{\text{Average shareholders' equity}}$$

### **Net Profit Margin**

$$\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

### **Basic EPS**

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

### **Diluted EPS (for convertible preferred stock)**

$$\text{Diluted EPS} = \frac{\text{Net income}}{\frac{\text{Weighted average number of shares outstanding}}{\text{New common shares that would have been issued at conversion}} + \text{New common shares that would have been issued at conversion}}$$

### Diluted EPS (for convertible debt)

$$\text{Diluted EPS} = \frac{\text{Net income} - \text{Preferred dividends} + \text{After tax interest expense}}{\text{Weighted average number of shares outstanding} + \text{New common shares that would have been issued at conversion}}$$

### Diluted EPS (for options)

$$\text{Diluted EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding} + \text{Additional common shares issued upon conversion}}$$

### Treasury stock method

$$\text{Additional common shares issued upon conversion} = \left( \begin{array}{ccc} \text{New shares issued at option exercise} & & \text{Shares repurchased from option exercised} \\ & - & \\ & & \end{array} \right) \times \text{Proportion of shares with cash received during which options were outstanding}$$

Video (Basic & Diluted EPS): <https://youtu.be/2C-mwVqO2SQ>

### Learning Module 3: Analyzing Balance Sheets

Working capital = Current assets - Current liabilities

### Liquidity Ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick (acid test) ratio} = \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}}$$

## Solvency Ratios

$$\text{Long-term debt-to-equity} = \frac{\text{Long-term debt}}{\text{Total equity}}$$

$$\text{Debt-to-equity} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Total debt} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Financial leverage} = \frac{\text{Total assets}}{\text{Total equity}}$$

## Learning Module 4: Analyzing Statements of Cash Flows I

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|                  |   |
|------------------|---|
| Cash flow        | Cash flow                                     |
| flow             | Cash flow                                     |
| Ending           | =   |
| cash             |   |
| Beginning        | +<br>cash                                     |
| from operating   | +<br>activities                               |
| from investing   | +<br>activities                               |
| from financing   | +<br>activities                               |
| Ending accounts  | + Revenue - <i>from customers</i>             |
| = Beginning      |   |
| accounts         |   |
| receivable       |   |
| receivable       |   |
| Ending inventory | Cost of goods<br>sold                         |
| inventory =      |   |
| inventory        |   |
| Ending accounts  | +Purchases -      Cash paid to<br>= Beginning |
| accounts payable | suppliers                                     |
| Ending wages     |   |
| payable          |   |
| + payable        | Cash paid to<br>employees                     |
|                  |   |
|                  | Interest Cash<br>paid expense                 |
|                  | -for interest                                 |
| _____ taxpable   | _____   |
| +                |   |

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|                         |                       |
|-------------------------|-----------------------|
| Cash flow               | Cash flow             |
| flow                    | Cash flow             |
| Ending =                |                       |
| cash                    |                       |
| Beginning +             |                       |
| cash                    |                       |
| from operating +        |                       |
| activities              |                       |
| from investing +        |                       |
| activities              |                       |
| from financing          |                       |
| activities              |                       |
| Ending                  | Equipment sold        |
| <i>PP&amp;E</i> =       |                       |
| Beginning               |                       |
| <i>PP&amp;E</i> + _____ |                       |
| Ending                  | <i>equipment sold</i> |
| accumulated             |                       |
| Beginning               |                       |
| accumulated             |                       |
| Depreciation            |                       |
| depreciation            |                       |
| depreciation            |                       |
| _____ expense           |                       |
| expense                 |                       |

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$$\text{Gain on sale} = \frac{\text{Cash received from sale of equipment}}{\text{Book value of equipment}} - \frac{\text{Equipment sold}}{\text{Book value of equipment}}$$

Note:  $\frac{\text{Ending retained earnings}}{\text{Beginning retained earnings}} = \frac{\text{Beginning retained earnings}}{\text{Net income}} + \frac{\text{Dividends}}{\text{Net income}}$

## Learning Module 5: Analyzing Statements of Cash Flows II

### Free Cash Flow To Firm (FCFF)

$$\begin{aligned} FCFF &= NI + NCC + \text{Int}(1 - \text{Tax rate}) - FCInv - WCInv \\ &= CFO + \text{Int}(1 - \text{Tax rate}) - FCInv \end{aligned}$$

where:

- NI = Net income
- NCC = Non-cash charges (e.g., depreciation and amortization)
- Int = Interest expense
- FCInv = Capital expenditures
- WCInv = Working capital expenditures
- CFO = Cash flow from operating activities = NI + NCC - WCInv

## Free Cash Flow to Equity (FCFE)

$$FCFE = CFO - FCInv + NetBorrowing$$

where:

- Net Borrowing = Debt issued - Debt repaid

## Performance Ratios

$$\text{Cash flow to revenue} = \frac{CFO}{\text{Revenue}}$$

$$\text{Cash return on assets} = \frac{CFO}{\text{Average total assets}}$$

$$\text{Cash return on equity} = \frac{CFO}{\text{Average shareholders equity}}$$

$$\text{Cash to income} = \frac{CFO}{\text{Operating income}}$$

$$\text{Cash flow per share} = \frac{\text{CFO} - \text{Preferred dividends}}{\text{Number of common shares outstanding}}$$

## Coverage Ratios

$$\text{Debt coverage ratio} = \frac{CFO}{\text{Total debt}}$$

$$\text{Interest coverage ratio} = \frac{\text{CFO} + \text{Interest paid} + \text{Taxes paid}}{\text{Interest paid}} \quad \text{Reinvestment ratio} = \frac{\text{CFO}}{\text{Cash paid for long term assets}}$$

$$\text{Debt payment ratio} = \frac{\text{CFO}}{\text{Cash paid for long term debt repayment}} \quad \text{Dividend payment ratio} = \frac{\text{CFO}}{\text{Dividends paid}}$$

$$\text{Investing and financing ratio} = \frac{\text{CFO}}{\text{Cash flow for investing and financing activities}}$$

## Learning Module 6: Analysis of Inventories

### IFRS

Inventories = Lower of Cost and Net Realizable Value (NRV)  $NRV$  = Estimated selling price less estimated costs of completion and costs necessary to complete the sale

## US GAAP

Inventories = Lower of Cost and NRV For last-in, first-out (LIFO) method or retail inventory methods Inventories = Lower of Cost and Market Value

Market value = Current replacement cost (subject to lower and upper limits) Lower limit

=  $NRV - \text{Normal profit margin}$  Upper limit =  $NRV$  Video: <https://youtu.be/V8C31msIBzs>

Inventory turnover ratio =  $\frac{\text{Cost of sales}}{\text{Average inventory}}$  Days of inventory on hand =  $\frac{\text{Number of days in period}}{\text{Inventory turnover ratio}}$

Ending inventory ( FIFO ) = Ending inventory ( LIFO ) + LIFO reserve

COGS (FIFO) = COGS ( LIFO ) - Change in LIFO reserve

## Learning Module 7: Analysis of Long-Term Assets

Net book value = Historical cost - Accumulated depreciation

$$\begin{aligned} \text{Gain on sale of asset} &= \text{Sale proceeds} - \text{Net book value} & \frac{\text{Estimated total useful life}}{\text{Estimated age of equipment}} &= \frac{\text{Estimated age of equipment}}{\text{of equipment}} + \\ \frac{\text{Estimated remaining life}}{\text{Estimated useful life}} &= \frac{\text{Gross PP&E}}{\text{Annual depreciation expense}} & \frac{\text{Estimated age of equipment}}{\text{of equipment}} &= \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}} \\ \text{Estimated remaining life} &= \frac{\text{Net PP&E}}{\text{Annual depreciation expense}} \end{aligned}$$

## Straight-line Depreciation

$$\text{Annual depreciation expense} = \frac{\text{Historical cost} - \text{Salvage value}}{\text{Estimated useful life}}$$

## Fixed Asset Turnover

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average net PP&E}}$$

## Impairment of Long-Lived Assets

### IFRS

Impairment = Carrying amount - Recoverable amount

where:

- Recoverable amount =  $\max(\text{Fair value less costs to sell}, \text{Value in use})$

## **US GAAP**

If asset's carrying amount > undiscounted expected future cash flows: Impairment = Carrying amount - Fair value

## **Learning Module 8: Topics in Long-Term Liabilities and Equity**

### **Lessee Accounting - Finance Lease (IFRS)**

Interest expense = Implied interest rate  $\times$  Beginning lease liability  
on lease

Principal repayment = Lease payment - Interest expense

Ending lease liability = Beginning lease liability + Interest expense - Lease payment

If ROU asset is amortized on a straight-line basis:

$$\text{Amortization expense} = \frac{\text{Initial ROU asset value} - \text{Salvage value}}{\text{Lease term}}$$

Ending ROU asset = Beginning ROU asset - Amortization expense

### **Lessee Accounting - Operating Lease (US GAAP)**

$\$ \text{Lease payment} - \text{Interest expense}$   
 $\$ \text{Beginning ROU asset} - \text{Ending ROU asset}$   
 $\$ \text{Beginning lease liability} - \text{Ending lease liability}$

### **Stock Options**

$$\text{Compensation expense} = \frac{\text{Fair value of options granted}}{\text{Vesting period}}$$

## **Learning Module 9: Analysis of Income Taxes**

### **Deferred Tax Asset/Liability**

For Assets:

$$\text{Deferred tax liability } /(\text{ asset }) = \text{Tax rate} \times \left( \frac{\text{Carrying amount of asset}}{\text{Tax base of asset}} - 1 \right)$$

### **For Liabilities:**

$$\text{Deferred tax liability } /(\text{ asset }) = \text{Tax rate} \times \left( \frac{1 - \text{Carrying amount of liability}}{\text{Tax base of liability}} \right)$$

$$\text{Income tax expense} = \text{Income tax payable} + \frac{\text{Changes in deferred tax assets and liabilities}}{\text{Pre-tax income}}$$

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pre-tax income}}$$

$$\text{Cash tax rate} = \frac{\text{Cash tax}}{\text{Pre-tax income}}$$

## **Learning Module 10: Financial Reporting Quality**

### **Adjusted EBITDA**

$$\text{Adjusted EBITDA} = \text{Adjusted EBIT} + \text{Post-IPO Software and R&D amortization} + \text{Depreciation Share-based amortization}$$

Straight-line method of depreciation

$$\text{Depreciation expense} = \frac{\text{Cost} - \text{Salvage value}}{\text{Useful life}}$$

### **Double-Declining Balance method**

$$\text{Depreciation expense} = \frac{2}{\text{Useful life}} \times (\text{Cost} - \text{Accumulated depreciation})$$

**Video:** <https://youtu.be/6RskYAxAFk>

### **Units-of-Production method**

$$\text{Depreciation expense} = \frac{\text{Units produced}}{\text{Total units over useful life}} \times (\text{Cost} - \text{Salvage value})$$

## **Learning Module 11: Financial Analysis Techniques**

### **Activity Ratios**

$$\text{Inventory turnover} = \frac{\text{Cost of sales}}{\text{Average inventory}}$$

$$\text{Days of inventory on hand} = \frac{\text{Number of days in the period}}{\text{Inventory turnover}}$$

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Average receivables}}$$

$$\text{Days of sales outstanding} = \frac{\text{Number of days in the period}}{\text{Receivables turnover}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average payables}}$$

$$\text{Number of days of payables} = \frac{\text{Number of days in the period}}{\text{Payables turnover}}$$

$$\text{Working capital turnover} = \frac{\text{Revenue}}{\text{Average working capital}}$$

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average net fixed assets}}$$

$$\text{Total asset turnover} = \frac{\text{Revenue}}{\text{Average total assets}}$$

### Liquidity Ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short term marketable investments}}{\text{Current liabilities}}$$

$$\text{Defensive interval} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Daily cash expenditures}}$$

$$\text{Cash conversion cycle} = \frac{\text{Days of inventory on hand}}{\text{Days of sales outstanding}} + \frac{\text{Number of days of payables}}{\text{Days of sales outstanding}}$$

## Solvency Ratios

$$\text{Debt-to-assets ratio} = \frac{\text{Total debt}}{\text{Total assets}}$$

( "Total debt ratio" )

$$\text{Debt-to-capital ratio} = \frac{\text{Total debt}}{\text{Total debt} + \text{Total equity}}$$

$$\text{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Financial leverage ratio} = \frac{\text{Average total assets}}{\text{Average total equity}}$$

$$\text{Debt-to-EBITDA ratio} = \frac{\text{Total or net debt}}{\text{EBITDA}}$$

## Coverage Ratios

$$\text{Interest coverage ratio} = \frac{\text{EBIT}}{\text{Interest payments}} \quad \text{Fixed charge coverage ratio} = \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$$

## Profitability Ratios

$$\text{Gross profit margin} = \frac{\text{Gross profit}}{\text{Revenue}} \quad \text{Operating profit margin} = \frac{\text{Operating income}}{\text{Revenue}} \quad \text{Pretax margin}$$

$$= \frac{\text{EBT}}{\text{Revenue}} \quad \text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

$$\text{Operating ROA} = \frac{\text{Operating income}}{\text{Average total assets}} \quad \text{ROA} = \frac{\text{Net income}}{\text{Average total assets}} \quad \text{Return on invested capital}$$

$$= \frac{\text{EBIT} \times (1 - \text{Effective tax rate})}{\text{Average total debt and equity}} \quad \text{ROE} = \frac{\text{Net income}}{\text{Average total equity}} \quad \text{Return on common equity}$$

$$= \frac{\text{Net income} - \text{Preferred dividends}}{\text{Average common equity}}$$

## DuPont Analysis

$$\text{ROE} = \text{ROA} \times \text{Financial Leverage}$$

$$\text{ROE} = \text{Net profit margin} \times \text{Total asset turnover} \times \text{Financial leverage} \quad \text{ROE} = \frac{\text{Tax burden}}{\text{Interest burden}} \times \frac{\text{EBIT margin}}{\text{EBIT}} \times \frac{\text{Total asset turnover}}{\text{turnover}} \times \frac{\text{Financial leverage}}{\text{leverage}}$$

where:

- Tax burden =  $\frac{\text{Net income}}{\text{EBT}}$
- Interest burden =  $\frac{\text{EBT}}{\text{EBIT}}$

## Business Risk

$$\begin{aligned} \text{Coefficient of variation of operating income} &= \frac{\text{Standard deviation of operating income}}{\text{Average operating income}} & \text{Coefficient of variation of net income} &= \\ \frac{\text{Standard deviation of net income}}{\text{Average net income}} & & \text{Coefficient of variation of revenue} &= \frac{\text{Standard deviation of revenue}}{\text{Average revenue}} \end{aligned}$$

## Financial Sector Ratios

$$\begin{aligned} \text{Monetary reserve requirement ( Cash reserve ratio )} &= \frac{\text{Reserves held at central bank}}{\text{Specified deposit liabilities}} & \text{Net interest margin} &= \\ \frac{\text{Net interest income}}{\text{Total interest earning assets}} & & \frac{\text{Approved readily marketable securities}}{\text{Specified deposit liabilities}} &= \text{Net} \\ \text{interest margin} &= \frac{\text{Net interest income}}{\text{Total interest earning assets}} \end{aligned}$$

## Learning Module 12: Introduction to Financial Statement Modeling

Nothing new.

## VOLUME 5: EQUITY INVESTMENTS

### Learning Module 1: Market Organization and Structure

Maximum leverage ratio =  $\frac{1}{\text{Minimum margin requirement}}$  Total return on leveraged stock investment:

$$\text{Total Return} = \frac{\frac{\text{Sales proceeds}}{\text{Initial equity}} + \frac{\text{Dividends}}{\text{Purchase commission}} - \frac{\text{Loan}}{\text{Margin interest}} - \frac{\text{Sales commission}}{\text{Interest}} - 1}{1}$$

Initial equity =  $\frac{\text{Minimum margin requirement}}{\text{Total purchase price Video (Return on Leveraged Stock Position)}} \times \text{Position}$   
 $\text{Margin Call Price} = \frac{P_0(1 - \text{Initial Margin})}{(1 - \text{Maintenance Margin})}$

## Learning Module 2: Security Market Indexes

$$\text{Price Return Index, } V_{PRI} = \frac{\sum_{i=1}^N n_i P_i}{D}$$

where:

- $n_i$  = the number of units of constituent security  $i$  held in the index portfolio
- $N$  = the number of constituent securities in the index
- $P_i$  = the unit price of constituent security  $i$
- $D$  = value of the divisor

$$\text{Price return of an index, } PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

$$\text{Total Return Index, } TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:

- $V_{PRI1}$  = value of the price return index at the end of the period
- $V_{PRI0}$  = value of the price return index at the beginning of the period
- $Inc_I$  = total income (dividends and/or interest) from all securities in the index held over the period

## Weighting Methods

$$\text{Price weighting, } w_i^P = \frac{P_i}{\sum_{j=1}^N P_j}$$

Video (Recalculating the divisor of a price weighted index): <https://youtu.be/eYiZNK-ETrg>

$$\text{Equal weighting, } w_i^E = \frac{1}{N}$$

$$\text{Market-capitalization weighting, } w_i^M = \frac{Q_i P_i}{\sum_{j=1}^N Q_j P_j}$$

$$\text{Float-adjusted market capitalization weighting, } w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^N f_j Q_j P_j}$$

where:

- $f_i$  = fraction of shares outstanding in the market float

- $Q_i$  = number of shares outstanding of security  $i$
- $P_i$  = share price of security  $i$
- $N$  = number of securities in the index

$$\text{Fundamental weighting, } w_i^F = \frac{F_i}{\sum_{j=1}^N F_j}$$

where  $F_i$  denotes a fundamental size measure of company  $i$

### **Learning Module 3: Market Efficiency**

#### **No formula**

### **Learning Module 4: Overview of Equity Securities**

Return on Equity (using average total book value of equity)

$$ROE_t = \frac{NI_t}{(BVE_t + BVE_{t-1}) / 2}$$

Return on Equity (using beginning book value of equity)

$$ROE_t = \frac{NI_t}{BVE_{t-1}}$$

where BVE = book value (Assets - Liabilities)

### **Learning Module 5: Company Analysis: Past and Present**

Market share =  $\frac{\text{Revenue}}{\text{Market size}}$

Sales potential = 100% – Market share %

Net sales = Average selling price × Quantity sold

Take rate =  $\frac{\text{Revenue earned from transactions}}{\text{Total transaction volume}} \times 100\%$

Operating income =  $Q \times (P - VC) - FC$

where:

- $Q$  = Units sold in a period
- $P$  = Price per unit

- $VC$  = Variable operating cost per unit
- $FC$  = Fixed operating costs
- $P - VC$  = Contribution margin per unit

Degree of operating leverage ( $DOL$ ) =  $\frac{\% \Delta \text{ Operating income}}{\% \Delta \text{ Sales}}$  Degree of financial leverage ( $DFL$ ) =  $\frac{\% \Delta \text{ Net income}}{\% \Delta \text{ Operating income}}$

$$\text{WACC} = \frac{\text{Weight of debt}}{\text{Gross cost of debt}} \times (1 - \text{tax rate}) + \frac{\text{Weight of equity}}{\text{Cost of equity}}$$

## Learning Module 6: Industry and Company Analysis

### Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{\infty} s_i^2$$

where:

- $s_i$  = Market share of participant  $i$  (stated as a whole number)

Learning Module 7: Company Analysis: Forecasting % Variable cost  $\approx \frac{\% \Delta (\text{Cost of revenue} + \text{Operating expense})}{\% \Delta \text{ Revenue}}$   
 $\% \text{ Fixed cost} \approx 1 - \% \text{ Variable cost}$        $\frac{\text{Number of units sold}}{\text{post-cannibalization}} = \frac{\text{Number of units sold}}{\text{pre-cannibalization}}$        $-$   
 $\frac{\text{Expected cannibalization}}{\text{cannibalization}} = \frac{\text{Expected cannibalization}}{\text{pre-cannibalization}} = \frac{\text{Number of units sold}}{\text{pre-cannibalization}} \times \text{Cannibalization factor}$

## Learning Module 8: Equity Valuation: Concepts and Basic Tools

### Dividend Discount Model (DDM)

$$V_0 = \sum_{t=1}^n \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

where:

- $V_0$  = Intrinsic value of a share at  $t = 0$
- $D_t$  = expected dividend in year  $t$
- $r$  = required rate of return on stock
- $P_n$  = expected price per share at  $t = n$  (terminal value)

## **Free-cash-flow-to-equity (FCFE) Valuation Model**

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$$

where:

- $FCFE = CFO - FCInv + NetBorrowing$
- $FCInv$  = Fixed capital investment
- Net Borrowing = Borrowings minus repayments Value of preferred stock (non-callable, non-convertible, perpetual)

$$V_0 = \frac{D_0}{r}$$

Value of preferred stock (non-callable, non-convertible, maturity at time  $n$ )

$$V_0 = \sum_{t=1}^n \frac{D_t}{(1+r)^t} + \frac{\text{Par value}}{(1+r)^n}$$

## **Gordon Growth Model**

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

where:

- $D_0$  = Most recent annual dividend
- $D_1$  = Expected dividend in the next period
- $g$  = Constant growth rate
- $r$  = Required return on equity

## **Sustainable growth rate**

$$g = b \times ROE$$

where:

- $b$  = earnings retention rate ( $= 1 - \text{Dividend payout ratio}$ )
- $ROE$  = Return on equity

**Video:** <https://youtu.be/MnfRRRhGpA>

## Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^n \frac{D_0 (1 + g_s)^t}{(1 + r)^t} + \frac{V_n}{(1 + r)^t}$$

where:

- $g_L$  = Long-term stable growth rate
- $g_s$  = Short-term growth rate
- $V_n = \frac{D_{n+1}}{r - g_L} = \frac{D_0 (1 + g_s)^t (1 + g_L)}{r - g_L}$

## Justified forward P/E

$$\frac{P_0}{E_1} = \frac{\text{Dividend payout ratio}}{r - g}$$

## Enterprise Value

$$EV = \frac{\text{Market value of equity}}{} + \frac{\text{Market value of preferred stock}}{} + \frac{\text{Market value of debt}}{} - \frac{\text{Cash and short term investments}}{}$$

## Asset-based Valuation

$$\frac{\text{Adjusted book value}}{} = \frac{\text{Market value of assets}}{} - \frac{\text{Market value of liabilities}}{}$$

## VOLUME 6: FIXED INCOME

Learning Module 1: Fixed-Income Instrument Features Current yield =  $\frac{\text{Annual coupon}}{\text{Bond price}}$  Bond price =  $\frac{\text{Coupon}}{(1+r)^1} + \frac{\text{Coupon}}{(1+r)^2} + \dots + \frac{\text{Coupon} + \text{Face value}}{(1+r)^n}$

where:

- Coupon per period = Coupon rate per period  $\times$  Face value
- $r$  = Yield to maturity per period
- $n$  = Number of payments

Floating-rate Note (FRN) coupon rate = MRR + Spread

## Learning Module 2: Fixed-Income Cash Flows and Types

### Fully Amortizing Loan with Level Payment

$$A = \frac{r \times \text{Principal}}{1 - (1 + r)^{-N}}$$

where:

- $A$  = Periodic payment
- $r$  = Market interest rate per period
- $N$  = Number of payment periods

If the periodic payment is monthly:

Monthly interest payment = Interest rate per month  $\times$  Beginning principal of loan

Monthly principal payment = Total monthly payment  $-$  Monthly interest payment

Ending principal of loan = Beginning principal of loan  $-$  Monthly principal payment

### Capital-Index Bond (e.g., TIPS)

Inflation-adjusted principal = Principal amount  $\times$  (1+ Inflation adjustment )

Coupon per period = Coupon rate per period  $\times$  Inflation-adjusted principal

### Deferred Coupon Bond

Video: <https://youtu.be/erRbAUOGIyM>

### Convertible Bonds

$$\begin{aligned} \text{Conversion ratio} &= \frac{\text{Convertible bond par value}}{\text{Conversion price}} = \frac{\text{Conversion value}}{\text{Current share price}} \\ &= \text{Conversion ratio} \times \text{Current share price} \end{aligned}$$

### Zero-Coupon Bond

Original issue discount = Bond par value - Issuance price

### **Learning Module 3: Fixed-Income Issuance and Trading**

No formula

Learning Module 4: Fixed-Income Markets for Corporate Issuers

### **Repurchase Agreements**

$$\text{Repurchase price} = \text{Price of bond} \times \left[ 1 + \frac{\text{Repo rate} \times \frac{\text{Repo term (in days)}}{\text{Number of days in a year}}}{\text{Initial margin}} \right] = \frac{\text{Security price}_t}{\text{Purchase price}_0}$$

$$\text{Haircut} = \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Security price}_0}$$

$$\text{Variation margin} = (\text{Initial margin} \times \text{Purchase price}_t) - \text{Security price}_t$$

### **Learning Module 5: Fixed-Income Markets for Government Issuers**

**No formula.**

### **Learning Module 6: Fixed-Income Bond Valuation: Prices and Yields**

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \cdots + \frac{PMT_N + FV_N}{(1+r)^N}$$

where:

- $PMT_t$  = Coupon that occurs in  $t$  periods
- $r$  = Market discount rate per period
- $N$  = Number of periods to maturity
- $FV$  = Face value of bond

Full Price, Flat Price, and Accrued Interest (Video: <https://youtu.be/I7G075JAu5w>)

$$\begin{aligned} PV^{\text{Full}} &= PV^{\text{Flat}} + \text{Accrued Interest} \\ &= PV_{BOP} \times (1+r)^{t/T} \end{aligned}$$

where:

- Accrued Interest =  $\frac{t}{T} \times PMT$
- $t$  = number of days from the last coupon payment to the settlement date
- $T$  = number of days in the coupon period

- $t/T$  = fraction of the coupon period that has gone by since the last payment
- $PV_{BOP}$  = price on the previous coupon date (before the settlement date)

## Matrix Pricing

$$\text{Interpolated yield} = \text{Yield}_S + \left( \frac{\text{Tenor}_{\text{Target}} - \text{Tenor}_S}{\text{Tenor}_L - \text{Tenor}_S} \right) \times (\text{Yield}_L - \text{Yield}_S)$$

where:

- $\text{Yield}_S$  = Yield of shorter-term bond
- $\text{Yield}_L$  = Yield of longer-term bond
- $\text{Tenor}_S$  = Tenor of shorter-term bond
- $\text{Tenor}_L$  = Tenor of longer-term bond
- $\text{Tenor}_{\text{Target}}$  = Tenor of the subject bond
- $\text{Tenor}_S < \text{Tenor}_{\text{Target}} < \text{Tenor}_L$  Required yield spread = Bond YTM - Government Bond YTM (Similar maturity)

## Learning Module 7: Yield and Yield Spread Measures for Fixed Rate Bonds

### Periodicity Conversion

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n$$

where:

- $APR_m$  = Annual percentage rate for  $m$  periods per year
- $APR_n$  = Annual percentage rate for  $n$  periods per year Current yield  $t = \frac{\text{Annual coupon}_t}{\text{Bond price}_t}$   
Government equivalent yield,  $\text{Yield}_{\text{ACT/ACT}} = \frac{365}{360} \times \text{Yield}_{30/360}$  Simple yield  
 $= \frac{\text{Coupon} + (\frac{FV - PV}{N})}{\text{Flat price}}$

### Callable Bonds

$$PV = \frac{PMT}{(1 + YTC)^1} + \frac{PMT}{(1 + YTC)^2} + \dots + \frac{PMT + \text{Call price}}{(1 + YTC)^N}$$

where:

- $PV$  = Price of the callable bond
- $PMT$  = Coupon payment per period
- $YTC$  = Yield to call per period

- $N$  = Number of periods to when the bond can be called at the call price

Option-adjusted price = Flat price of bond + Value of embedded call option

Value of call option = Price of option-free bond - Price of callable bond

G-spread = Bond YTM - Interpolated sovereign bond YTM

I-spread = Bond YTM - Swap rate

## Z-Spread

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \dots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

where:

- $Z$  = Z-spread
- $z_N$  = Spot rate for  $N$  periods

OAS = Z-spread - Option value (in basis points per year)

## Learning Module 8: Yield and Yield Spread Measures for Floating-Rate Instruments

### Value of Floating Rate Note (FRN)

$$PV = \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1 + \frac{MRR+DM}{m}\right)^1} + \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1 + \frac{MRR+DM}{m}\right)^2} + \dots + \frac{\left(\frac{MRR+QM}{m}\right) \times FV + FV}{\left(1 + \frac{MRR+DM}{m}\right)^n}$$

where:

- $QM$  = Quoted Margin
- $DM$  = Discount Margin
- $MRR$  = Market reference rate
- $m$  = Periodicity (i.e., number of payment periods per year)
- $FV$  = Face Value of FRN
- $N$  = Number of evenly spaced periods to maturity

**Video:** <https://youtu.be/zqYotVLkYR8>

## **Yield Measures for Money Market Instruments**

### **Discount Rate Basis**

$$PV = FV \times \left(1 - \frac{\text{Days}}{\text{Year}} \times DR\right)$$
$$DR = \frac{\text{Year}}{\text{Days}} \times \left(\frac{FV - PV}{FV}\right)$$

where:

- $PV$  = present value of money market instrument
- $FV$  = future value paid at maturity
- Days = number of days between settlement and maturity
- Year = number of days in the year
- $DR$  = discount rate (stated as annual percentage rate)

### **Add-on Rate Basis**

$$PV = \frac{FV}{\left(1 + \frac{\text{Days}}{\text{Year}} \times AOR\right)}$$
$$AOR = \frac{\text{Year}}{\text{Days}} \times \left(\frac{FV - PV}{PV}\right)$$
$$\text{Bond equivalent yield} = \frac{365}{\text{Days}} \times \left(\frac{FV - PV}{PV}\right)$$

## **Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves**

### **Calculation of Bond Price Using Spot Rates**

$$PV = \frac{PMT}{(1 + Z_1)^1} + \frac{PMT}{(1 + Z_2)^2} + \cdots + \frac{PMT + FV}{(1 + Z_N)^N}$$

where:

- $PV$  = Price of bond
- $PMT$  = Bond coupon payment
- $Z_N$  = Spot rate (or zero-coupon yield or zero rate) for period  $N$
- $FV$  = Face value of bond

Given a Par Rate,  $FV = PV$  and  $PMT = \text{Par Rate (\%)} \times FV$

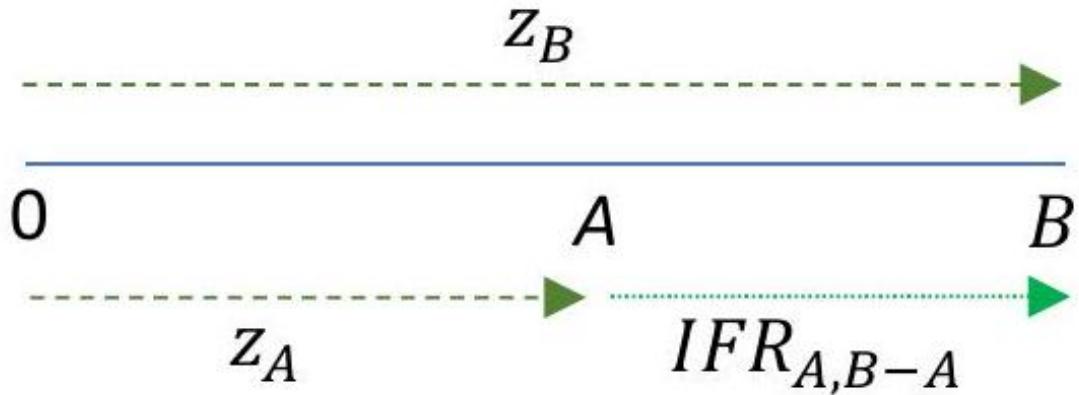
$$100 = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+100}{(1+Z_N)^N}$$

### Forward Rates, IFR

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where:

- $IFR_{A,B-A}$  = Forward rate for ( $B - A$ ) periods that starts in period  $A$



### Learning Module 10: Interest Rate Risk and Return

#### Duration Gap

Duration gap = Macaulay duration - Investment horizon

#### Macaulay Duration

$$\begin{aligned} \text{Macaulay duration} &= \left(1 - \frac{t}{T}\right) \left[ \frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{\text{Full}}} \right] + \left(2 - \frac{t}{T}\right) \left[ \frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{\text{Full}}} \right] + \cdots \\ &\quad + \left(N - \frac{t}{T}\right) \left[ \frac{\frac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{\text{Full}}} \right] \end{aligned}$$

$$\text{Macaulay duration} = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \frac{t}{T}$$

where:

- $r$  = Yield per period
- $c$  = Coupon rate per period
- $N$  = Number of evenly spaced periods to maturity as of the beginning of the current period
- $t$  = Number of days from the last coupon payment to the settlement date
- $T$  = Number of days in the coupon period

**Video:** <https://youtu.be/USgjcdCk7Fs>

## Learning Module 11: Yield-Based Bond Duration Measures and Properties

### Modified Duration

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + r}$$

### Approximate Modified Duration

$$\begin{aligned}\text{AnnModDur} &\approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{ Yield}) \times (PV_0)} \\ \% \Delta PV^{\text{Full}} &\approx -\text{AnnModDur} \times \Delta \text{ Yield}\end{aligned}$$

### Money Duration

$$\text{Money duration} = \text{AnnModDur} \times PV^{\text{full}}$$

$$\Delta PV^{\text{Full}} \approx -\text{MoneyDur} \times \Delta \text{ Yield}$$

### Duration of Zero-Coupon Bond

$$\begin{aligned}\text{MacDur} &= \text{Time to maturity} \\ \text{ModDur} &= \frac{\text{Time to maturity}}{1 + r}\end{aligned}$$

## Duration of Perpetual Bond

$$\begin{aligned}\text{MacDur} &= \frac{1+r}{r} \\ \text{ModDur} &= \frac{1}{r}\end{aligned}$$

## Duration of Floating-Rate Notes

$$\text{MacDur} = \frac{T-t}{T} = \begin{array}{l}\text{Fraction of period remaining until} \\ \text{the next reset date}\end{array}$$

## Learning Module 12: Yield-Based Bond Convexity and Portfolio Properties

### Convexity

$$\text{Convexity} = \sum_{t=1}^N \frac{t(t+1) \times \frac{PV_t}{PV^{\text{Full}}}}{(1+YTM)^2}$$

### Approximate Annualized Convexity

$$\begin{aligned}\text{ApproxConv} &\approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta \text{ Yield})^2 \times (PV_0)} \\ \% \Delta PV^{\text{Full}} &\approx -\text{AnnModDur} \times \Delta \text{ Yield} + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{ Yield})^2\end{aligned}$$

### Money Convexity

$$\begin{aligned}\text{MoneyCon} &= \text{AnnConvexity} \times PV^{\text{Full}} \\ \% \Delta PV^{\text{Full}} &\approx -(\text{MoneyDur} \times \Delta \text{ Yield}) + \left[ \frac{1}{2} \times \text{MoneyCon} \times (\Delta \text{ Yield})^2 \right]\end{aligned}$$

## Portfolio Duration and Convexity

$$\text{Portfolio Modified Duration} = \sum_{i=1}^N w_i \times \text{ModDur}_i$$

$$\text{Portfolio Convexity} = \sum_{i=1}^N w_i \times \text{Convexity}_i$$

where:

- $w_i$  = Weight of bond  $i$ , measured in market value

## Learning Module 13: Curve-Based and Empirical Fixed-Income Risk Measures

### Effective Duration

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{ Curve}) \times PV_0}$$

### Effective Convexity

$$\begin{aligned} \text{EffCon} &= \frac{(PV_-) + (PV_+) - 2 \times PV_0}{(\Delta \text{ Curve})^2 \times PV_0} \\ \% \Delta PV^{\text{Full}} &\approx -\text{EffDur} \times \Delta \text{ Curve} + \frac{1}{2} \times \text{EffCon} \times (\Delta \text{ Curve})^2 \end{aligned}$$

### Key Rate Duration

$$\text{KeyRateDur}_k = -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k}$$

$$\% \Delta PV = -\text{KeyRateDur}_k \times \Delta r_k$$

$$\sum_{k=1}^n \text{KeyRateDur}_k = \text{EffDur}$$

where:

- $r_k$  =  $k$  th key rate

## Learning Module 14: Credit Risk

### Expected Loss

$$EL = LGD \times POD$$
$$LGD = EE \times (1 - RR)$$

where:

- $POD$  = Probability of default
- $LGD$  = Loss given default
- $EE$  = Expected exposure
- $RR$  = Recovery rate
- $1 - RR$  = Loss severity

Credit spread  $\approx$   $POD \times LGD$

### Decomposing Bond Yields

Yield spread = Bond YTM - Government bond YTM (Similar maturity)

Liquidity spread = Bond YTM ( Bid ) - Bond YTM ( Offer )

Credit spread = Yield spread - Liquidity spread

### Price Impact Given a Change in Yield Spread

$$\% \Delta PV^{\text{Full}} \approx - \text{AnnModDur} \times \Delta \text{ Spread} + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{ Spread})^2$$

where:

- $\text{AnnModDur} \approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{ Yield}) \times (PV_0)}$
- $\text{AnnConvexity} \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta \text{ Yield})^2 \times (PV_0)}$

## **Learning Module 15: Credit Analysis for Government Issuers**

No formula.

Learning Module 16: Credit Analysis for Corporate Issuers

$$\begin{aligned} \text{EBIT margin} &= \frac{\text{Operating income}}{\text{Revenue}} \\ \text{EBIT to interest expense} &= \frac{\text{Operating income}}{\text{Interest expense}} \\ \text{Debt to EBITDA} &= \frac{\text{Debt}}{\text{EBITDA}} \\ \text{RCF to net debt} &= \frac{\text{Retained cash flow}}{\text{Debt - Cash and marketable securities}} \\ \text{FFO to debt} &= \frac{\text{FFO}}{\text{Debt}} \end{aligned}$$

where:

- FFO = Net income from continuing operations + Depreciation & amortization
- Deferred income taxes + Other non-cash items

## **Learning Module 17: Fixed-Income Securitization**

No formula.

## **Learning Module 18: Asset-Backed Security (ABS) Instrument and Market Features**

No formula.

Learning Module 19: Mortgage-Backed Security (MBS) Instrument and Market Features

### **Loan-to-value (LTV) ratio**

$$LTV = \frac{\text{Loan amount}}{\text{House price}}$$

### **Debt-to-income (DTI) ratio**

$$DTI = \frac{\text{Monthly debt payment}}{\text{Monthly pre-tax gross income}}$$

### **Mortgage Pass-Through Securities**

$$WAC = \sum_{i=1}^N c_i \left( \frac{CB_i}{CB} \right)$$
$$WAM = \sum_{i=1}^N MM_i \left( \frac{CB_i}{CB} \right)$$

where:

- WAC = Weighted average coupon
- WAM = Weighted average maturity
- $c_i$  = Coupon rate on mortgage  $i$
- $MM_i$  = Number of months to maturity for mortgage  $i$
- $N$  = Number of mortgages in MBS
- $CB_i$  = Current balance on mortgage  $i$
- $CB$  = Total current balance of mortgages in MBS

### **Commercial Mortgage-Backed Securities (CMBS)**

#### **Debt Service Coverage Ratio (DSCR)**

$$\text{DSCR} = \frac{\text{Net operating income}}{\text{Debt service}}$$

#### **Net Operating Income (NOI)**

$\text{NOI} = (\text{Rental income} - \text{Cash operating expenses}) - \text{Replacement reserves}$

## **VOLUME 7: DERIVATIVES**

### **Learning Module 1: Derivative Instrument and Derivatives Market Features**

No formula.

## **Learning Module 2: Forward Commitments and Contingent Claim Features and Instruments**

### **Forward Contract**

$$\begin{aligned}\text{Buyer (Long) payoff} &= S_T - F_0(T) \\ \text{Seller (Short) payoff} &= -[S_T - F_0(T)]\end{aligned}$$

where:

- $S_T$  = Spot price on contract's maturity
- $F_0(T)$  = Forward price with maturity of  $T$

### **Futures Contract**

For one futures contract:

$$\begin{aligned}\text{Long Futures daily mark-to-market} &= f_t(T) - f_{t-1}(T) \\ \text{Short Futures daily mark-to-market} &= -[f_t(T) - f_{t-1}(T)]\end{aligned}$$

where:

- $f_t(T)$  = Closing price of futures contract on day  $t$
- $f_{t-1}(T)$  = Closing price of futures contract on day  $t - 1$
- $T$  = Maturity of futures contract

If margin balance < maintenance margin:

$$\text{Variation Margin} = \text{Initial margin} - \text{Margin balance}$$

### **Options Contract**

#### **LONG Call option**

Payoff or Value at expiration,  $c_T = \max(0, S_T - X)$

Profit at expiration,  $\Pi = \max(0, S_T - X) - c_0$

where:

- $c_0$  = Call premium
- $X$  = Exercise/Strike price
- $S_T$  = Spot price at expiration

### **SHORT Call option**

Payoff or Value at expiration,  $c_T = -\max(0, S_T - X)$

Profit at expiration,  $\Pi = -[\max(0, S_T - X) - c_0]$

### **LONG Put option**

Payoff or Value at expiration,  $p_T = \max(0, X - S_T)$

Profit at expiration,  $\Pi = \max(0, X - S_T) - p_0$

### **SHORT Put option**

Payoff or Value at expiration,  $p_T = -\max(0, X - S_T)$

Profit at expiration,  $\Pi = -[\max(0, X - S_T) - p_0]$

### **Credit Default Swap (CDS)**

CDS MTM Change =  $\Delta$  CDS Spread  $\times$  CDS Notional  $\times$  EffDur<sub>CDS</sub>

In a credit event, payment from CDS seller to CDS buyer  $\approx LGD(\%) \times$  Notional

### **Learning Module 3: Derivative Benefits, Risks, and Issuer and Investor Uses**

No formula.

### **Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives**

If there are no underlying costs or benefits:

Forward price,  $F_0(T) = S_0(1 + r)^T$

If there are underlying costs or benefits in present value terms:

Forward price,  $F_0(T) = [S_0 - PV_0(\text{ Income }) + PV_0(\text{ Cost })] (1 + r)^T$

where:

- $S_0$  = Current spot price
- $r$  = Risk-free rate
- $T$  = Tenor of forward contract

Under continuous compounding,  $F_0(T) = S_0 e^{rT}$  Under continuous compounding, with income (i) and cost (c) expressed in %:

$$F_0(T) = S_0 e^{(r+c-i)T}$$

### Foreign Exchange Forward Contract

$$F_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

where:

- $F_{0,f/d}$  = Forward exchange rate
- $S_{0,f/d}$  = Spot exchange rate
- $r_f$  = Continuously compounded risk-free rate (for price/quote currency)
- $r_d$  = Continuously compounded risk-free rate (for base currency)
- T = Maturity of forward contract

### Learning Module 5: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

#### Value of LONG Forward Prior to Expiration

$$\begin{aligned} V_0(T) &= 0 \\ V_t(T) &= S_t - \frac{F_0(T)}{(1+r)^{T-t}} = S_t - F_0(T) \times (1+r)^{-(T-t)} \\ V_T(T) &= S_0 - F_0(T) \end{aligned}$$

If the asset incurs cost or generates income from time  $t$  through maturity,

$$V_t(T) = [S_t - PV_t(\text{ Income }) + PV_t(\text{ Cost })] - F_0(T) \times (1+r)^{-(T-t)}$$

For foreign exchange forward contract,

$$V_t(T) = S_{t,f/d} - F_{0,f/d}(T) \times e^{-(r_f - r_d)(T-t)}$$

## Value of SHORT Forward Prior to Expiration

$$\begin{aligned}V_0(T) &= 0 \\V_t(T) &= - \left[ S_t - \frac{F_0(T)}{(1+r)^{T-t}} \right] \\V_T(T) &= - [S_0 - F_0(T)]\end{aligned}$$

## Interest Rate Forward Contracts (Forward Rate Agreements (FRA))

$$(1+z_A)^A \times (1+IFR_{A,B-A})^{B-A} = (1+z_B)^B$$

where:

- $z_A$  = Spot rate for  $A$  periods
- $z_B$  = Spot rate for  $B$  periods
- $IFR_{A,B-A}$  = Implied forward rate for  $(B-A)$  periods, starting in  $A$  periods

Payoff for a Long FRA =  $(MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional principal} \times \text{Period}$

Payoff for a Short FRA =  $-(MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional principal} \times \text{Period}$

## Learning Module 6: Pricing and Valuation of Futures Contracts

If there are no underlying costs or benefits:

$$\text{Futures price, } f_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$f_0(T) = [S_0 - PV_0(\text{ Income }) + PV_0(\text{ Cost })](1+r)^T$$

Under continuous compounding,  $f_0(T) = S_0 e^{rT}$

Under continuous compounding, with income ( $i$ ) and cost ( $c$ ) expressed in %:

$$f_0(T) = S_0 e^{(r+c-i)T}$$

## Foreign Exchange Forward Contract

$$f_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

## Interest Rate Futures Contract

$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$$

where:

- $f_{A,B-A}$  = Futures price for a market reference rate for ( $B - A$ ) periods that begins in  $A$  periods

Futures contract basis point value,  $BPV$  = Notional principal  $\times 0.01\% \times$  Period

## Learning Module 7: Pricing and Valuation of Interest Rates and Other Swaps

For a fixed-rate payer in an interest rate swap:

$$\text{Periodic settlement value} = (MRR - s_N) \times \text{Swap Notional} \times \text{Period}$$

For a fixed-rate receiver in an interest rate swap:

$$\text{Periodic settlement value} = (s_N - MRR) \times \text{Swap Notional} \times \text{Period}$$

where:

- $s_N$  = Fixed swap rate
- $MRR$  = Market reference rate

## Calculating Par Swap Rate

$$\sum_{i=1}^N \frac{IFR}{(1+z_i)^i} = \sum_{i=1}^N \frac{s_N}{(1+z_i)^i}$$

where:

- $IFR$  = Implied forward rates
- $s_N$  = Fixed swap rate
- $N$  = Tenor of swap contract

## Valuation of Interest Rate Swap

Value of a pay-fixed interest rate swap on a settlement date after inception =  
Current settlement value +  $\Sigma(\text{Floating payments}) - \Sigma(\text{Fixed payments})$

Value of a receive-fixed interest rate swap on a settlement date after inception  
= Current settlement value +  $\Sigma(\text{Fixed payments}) - \Sigma(\text{Floating payments})$

## Learning Module 8: Pricing and Valuation of Options

Option value = Exercise value + Time value

At time  $t$  (prior to option expiration):

Call option exercise value =  $\max[0, S_t - X(1+r)^{-(T-t)}]$

Call option time value =  $c_t - \max[0, S_t - X(1+r)^{-(T-t)}]$

Put option exercise value =  $\max[0, X(1+r)^{-(T-t)} - S_t]$

Put option time value =  $p_t - \max[0, X(1+r)^{-(T-t)} - S_t]$

Lower bound of call option value =  $\max[0, S_t - X(1+r)^{-(T-t)}]$

Upper bound of call option value =  $S_t$

Lower bound of put option value =  $\max[0, X(1+r)^{-(T-t)} - S_t]$

Upper bound of put option value =  $X$

where:

- $S_t$  = Spot price at time  $t$
- $X$  = Exercise price (or strike price)
- $T$  = Maturity of option
- $r$  = Risk-free rate

## Learning Module 9: Option Replication Using Put-Call Parity

### Put-Call Parity

$$S_0 + p_0 = c_0 + X(1+r)^{-T}$$

## Put-Call Forward Parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

## Value of the Firm

$$V_0 = c_0 + PV(\text{Debt}) - p_0$$

Value of debt =  $PV(\text{Debt}) - p_0$  Value of equity =  $c_0$

## Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Risk-neutral probability of a price increase in underlying

$$\pi = \frac{1+r-R^d}{R^u-R^d}$$

where:

- $R^u = Up \text{ factor} = \frac{S_1^u}{S_0} > 1$
- $R^d = Down \text{ factor} = \frac{S_1^d}{S_0} < 1$
- $S_0 = Current \text{ asset price}$
- $S_1^u = One-period \text{ asset price when price moves up}$
- $S_1^d = One-period \text{ asset price when price moves down}$

**Video:** <https://youtu.be/ymUIKgz-rAw>

## Hedge ratio

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

where:

- $c_1^u = \max(0, S_1^u - X)$
- $c_1^d = \max(0, S_1^d - X)$

Riskless portfolio with a Call:  $h$  of the underlying,  $S$ , and short call position,  $c$   $V_0 = hS_0 - c_0$   
 $V_1^u = hS_1^u - c_1^u$   $V_1^d = hS_1^d - c_1^d$

### Riskless portfolio with a Put: $h$ of the underlying, $S$ , and long put position, $p$

$$V_0 = hS_0 + p_0 \quad V_1^u = hS_1^u + p_1^u \quad V_1^d = hS_1^d + p_1^d$$

Value of a one-period call option

$$c_0 = \frac{\pi c_1^u + (1 - \pi)c_1^d}{1 + r}$$

Value of a one-period put option

$$p_0 = \frac{\pi p_1^u + (1 - \pi)p_1^d}{1 + r}$$

where:

- $p_1^u = \max(0, X - S_1^u)$
- $p_1^d = \max(0, X - S_1^d)$  Video: <https://youtu.be/bXEC-78y AU>

Learning Module 1: Alternative Investment Features, Methods, and Structures

### GP Compensation Structure

Ignoring management fee; no catch-up clause

$$r_{GP} = \max[0, p(r - r_h)]$$

Ignoring management fee; with catch-up clause

$$r_{GP} = \max[0, r_{cu} + p(r - r_h - r_{cu})]$$

where:

- $r_{GP}$  = GP's rate of return
- $p$  = Performance fee as a percentage of total return
- $r$  = Single-period rate of return
- $r_h$  = Hard hurdle rate
- $r_{cu}$  = Catch-up clause

## **Learning Module 2: Alternative Investment Performance and Returns**

### **Multiple on Invested Capital**

$$MOIC = \frac{\text{Realized value of investment} + \text{Unrealized value of investment}}{\text{Total amount of invested capital}}$$

### **Leveraged Portfolio Return**

$$r_L = r + \frac{V_b}{V_c} (r - r_b)$$

where:

- $r$  = Periodic rate of return on invested funds
- $r_b$  = Periodic cost of borrowing
- $V_b$  = Amount of borrowed funds
- $V_c$  = Amount of cash (investor's own capital)

### **Investor's Return Net of Fees**

$$r_i = \frac{P_1 - P_0 - R_{GP}}{P_0}$$
$$R_{GP} = (P_1 \times r_m) + \max [0, (P_1 - P_0) \times p]$$

where:

- $P_0$  = Beginning-of-period asset value
- $P_1$  = End-of-period asset value
- $p$  = GP performance fee
- $R_{GP}$  = GP's return in current terms
- $r_m$  = GP's management fees as a percentage of assets under management

### **Calculating Hedge Fund Fees and Returns**

Management Fee Based on Beginning Market Value

$$\text{Management Fee} = \% = \frac{\text{Management Fee}}{\text{Beginning Market Value}}$$

Management Fee Based on Ending Market Value

$$\text{Management Fee} = \% \text{ Management Fee} \times \frac{\text{Ending Market Value}}{\text{Beginning Market Value}}$$

Incentive Fee Calculated Independent of Management Fee

$$\text{Incentive Fee} = \frac{\% \text{ Incentive Fee}}{\text{Gain}}$$

Incentive Fee Calculated Net of Management Fee

$$\text{Incentive Fee} = \frac{\% \text{ Incentive Fee}}{\text{Gain}} \times (\text{Gain} - \text{Management Fee})$$

Incentive Fee with Hard Hurdle (Independent of Management Fee)

$$\text{Incentive Fee} = \frac{\% \text{ Incentive Fee}}{\text{Gain}} \times (\text{Gain} - \text{Hurdle})$$

Incentive Fee with Hard Hurdle (Net of Management Fee)

$$\text{Incentive Fee} = \frac{\% \text{ Incentive Fee}}{\text{Gain}} \times (\text{Gain} - \text{Management Fee} - \text{Hurdle})$$

$$\text{Hurdle} = \text{Hurdle Rate} \times \text{Beginning market value}$$

Note: 1) No incentive is paid if hedge fund incurs loss for the year. 2) Gain may be subject to high watermark.

<sup>1</sup> ## Learning Module 3: Investments in Private Capital: Equity and Debt

No formula.

## Learning Module 4: Real Estate and Infrastructure

### Loan-to-Value (LTV) Ratio

$$LTV = \frac{\text{Mortgage liability}}{\text{Portfolio value}}$$

Required reduction in mortgage liability = Mortgage liability - Required mortgage liability

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<sup>1</sup>Video: <https://youtu.be/ODKmCgsAAdc>

## **Learning Module 5: Natural Resources**

No formula.

## **Learning Module 6: Hedge Funds**

No formula.

## **Learning Module 7: Introduction to Digital Assets**

No formula.

# **VOLUME 9: PORTFOLIO MANAGEMENT**

## **Learning Module 1: Portfolio Risk and Return: Part I**

### **Expected Return on Asset**

$$1 + E(R) = (1 + r_{rF}) \times [1 + E(\pi)] \times [1 + E(RP)]$$

where:

- $r_{rF}$  = Real risk-free rate
- $E(\pi)$  = Expected inflation
- $E(RP)$  = Expected risk premium for the asset

### **Utility on Investment**

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:

- $U$  = Utility of investment
- $E(R)$  = Expected return of investment
- $A$  = Risk aversion coefficient
- $\sigma^2$  = Variance of investment (Note: Substitute  $\sigma$  in decimals)

## Capital Allocation Line (CAL)

For a portfolio of risky assets (Weight:  $w_i$ ) and risk-free asset:

$$E(R_p) = R_f + \left[ \frac{E(R_i) - R_f}{\sigma_i} \right] \sigma_p$$

where:

- $R_f$  = Rate of return on risk-free asset
- $E(R_i)$  = Expected return of risky asset
- $E(R_p)$  = Expected return of portfolio
- $\sigma_i$  = Standard deviation of risky asset's returns
- $\sigma_p$  = Standard deviation of portfolio's returns =  $w_i \times \sigma_i$
- $\frac{E(R_i) - R_f}{\sigma_i}$  = Market price of risk

## Two-asset portfolio

Portfolio expected return,  $E(R_p) = w_1 R_1 + w_2 R_2$

Portfolio variance,  $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)$

Portfolio standard deviation,  $\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)}$

Note: 1)  $\text{Cov}(R_1, R_2) = \rho_{12} \sigma_1 \sigma_2$  2)  $n$  securities requires  $n$  variances and  $\frac{n(n-1)}{2}$  covariances

Video: <https://youtu.be/IUwulZ9ONCO>

## Foreign Asset

Return of a foreign asset in domestic currency

$$R_D = (1 + R_{lc}) \times (1 + R_{FX}) - 1$$

Standard deviation of return of a foreign asset in domestic currency

$$\sigma_D = \sqrt{\sigma_{lc}^2 + \sigma_{FX}^2 + 2 \times \rho \times \sigma_{lc} \times \sigma_{FX}}$$

where:

- $R_{lc}$  = Return of foreign asset (in local currency)
- $R_{FX}$  = Change in exchange rate (FX rate quoted as domestic currency/foreign currency)

- $\sigma_{lc}$  = Standard deviation of foreign asset's returns
- $\sigma_{FX}$  = Standard deviation of the exchange rate (DC/FC)
- $\rho$  = Correlation coefficient between returns on foreign asset and exchange rate

## Portfolio of Many Risky Assets

$$\sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N} \overline{\text{Cov}} = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N} \rho \bar{\sigma}^2$$

where:

- $N$  = Number of assets in portfolio
- $\bar{\sigma}^2$  = Average variance
- $\overline{\text{Cov}}$  = Average covariance

## Learning Module 2: Portfolio Risk and Return: Part II

### Capital Market Line (CML)

$$E(R_p) = w_f R_f + (1 - w_f) E(R_m) = R_f + \left[ \frac{E(R_m) - R_f}{\sigma_m} \right] \sigma_p \sigma_p = (1 - w_f) \sigma_m$$

### Return-Generating Models

$$E(R_i) - R_f = \beta_{i1} [E(R_m) - R_f] + \sum_{j=2}^k \beta_{ij} E(F_j)$$

where:

- $E(R_i) - R_f$  = Expected excess return on asset  $i$
- $k$  = Number of factors  $\beta_{ij}$  = Factor weights (also called factor loadings)  $E(R_m)$  = Expected return on market

### The Single-Index Model

$$E(R_i) - R_f = \left( \frac{\sigma_i}{\sigma_m} \right) [E(R_m) - R_f]$$

where:

- $\frac{\sigma_i}{\sigma_m}$  = Factor loading (or factor weight)

## Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

## The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

### Beta of security $i$

$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$  Portfolio beta,  $\beta_p = \sum_{i=1}^n w_i \beta_i$  Total variance = Systematic variance + Nonsystematic variance

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

$$\text{Total risk, } \sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_e^2}$$

## Arbitrage Pricing Theory (APT) Model

$$E(R_P) = R_F + \lambda_1 \beta_{P,1} + \dots + \lambda_K \beta_{P,K}$$

where:

- $E(R_P)$  = Expected return on portfolio
- $R_F$  = Risk-free rate
- $\lambda_j$  = Risk premium for factor  $j$
- $\beta_{P,j}$  = Sensitivity of the portfolio to factor  $j$
- $K$  = Number of risk factors

## Fama-French Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t$$

## Carhart Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,UMD}UMD_t$$

where:

- $E(R_i)$  = Return on an asset in excess of the one-month T-bill return
- $MKT$  = Excess return on the market portfolio
- $SMB$  = Difference in returns between small-capitalization stocks and large-capitalization stocks (Size)
- $HML$  = Difference in returns between high-book-to-market stocks and low-book-to-market stocks (Value versus growth)
- $UMD$  = Difference in returns of the prior year's winners versus losers (Momentum)

## Portfolio Performance Appraisal Measures

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\begin{aligned}\text{Treynor ratio} &= \frac{R_p - R_f}{\beta_p} M^2 = (R_p - R_f) \frac{\sigma_m}{\sigma_p} + R_f = \text{Sharpe ratio} \times \sigma_m + R_f M^2 \text{ alpha} \\ &= M^2 - R_m\end{aligned}$$

$$\text{Jensen's Alpha}, \quad \alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]$$

Security Characteristic Line (SCL)

$$R_i - R_f = \alpha_i + \beta_i (R_m - R_f)$$

$$\text{Information ratio} = \frac{\alpha_i}{\sigma_{e,i}}$$

## Learning Module 3: Portfolio Management: An Overview

No formula.

Learning Module 4: Basics of Portfolio Planning and Construction

No formula.

Learning Module 5: The Behavioral Biases of Individuals

No formula.

Learning Module 6: Introduction to Risk Management

No formula.