

# CFA Program Level 1”

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## CFA® Program Level I

FORMULA SHEET (2025)

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## **CFA Level 1 - Formula Sheet (2025)**

### **Setting Up the Texas BA II Plus Financial Calculator**

Video: <https://youtu.be/OMS8d8QOFmc>

### **Using Texas BA II Plus Financial Calculator**

Video: <https://youtu.be/LWmTTiZz8BU>

Video (Requires Login to Facebook): <https://fb.watch/nci5V7Dwtj/>

## **VOLUME 1: QUANTITATIVE METHODS**

### **Learning Module 1: Rates and Returns**

#### **Determinants of Interest Rates**

Interest rate,  $r$  = Real risk-free rate + Inflation premium + Default risk premium

- Liquidity premium + Maturity premium  $(1 + \text{Nominal risk-free rate}) = (1 + \text{Real risk-free rate}) \times (1 + \text{Inflation premium})$

Nominal risk-free rate = Real risk-free rate + Inflation premium

Maturity premium = Interest rate on longer-maturity, liquid Treasury debt

- Interest rate on short-term Treasury debt

## Holding Period Return

$$R = \frac{P_1 - P_0 + I_1}{P_0}$$

where:

- $P_0$  = Price at the beginning of the period
- $P_1$  = Price at the end of the period
- $I_1$  = Income

If given holding period returns  $R_1, R_2, \dots, R_T$  over the holding period:

$$R = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

## Arithmetic Return

$$\bar{R}_i = \frac{1}{T} \sum_{t=1}^T R_{it} = \frac{1}{T} (R_{i1} + R_{i2} + \dots + R_{iT})$$

## Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{\prod_{t=1}^T (1 + R_{it})} - 1 = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times \dots \times (1 + R_{iT})} - 1$$

## Harmonic Mean

$$\bar{X}_{Hi} = \frac{n}{\sum_{i=1}^n (1/X_i)} \quad \text{for } X_i > 0$$

## Relationship between Arithmetic Mean, Geometric Mean, and Harmonic Mean

$$(\text{Geometric mean})^2 = \text{Arithmetic mean} \times \text{Harmonic mean}$$

## Money-Weighted Return (MWR)

$$\sum_{t=0}^T \frac{CF_t}{(1 + MWR)^t} = 0$$

## Time-Weighted Return (TWR)

Given the holding period returns for each sub-period,  $R_1, R_2, \dots, R_T$

If  $T > 1$  year, then

$$\text{Annualized TWR} = [(1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T)]^{1/T} - 1$$

If  $T = 1$  year, then

$$\text{Annualized TWR} = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

If  $T < 1$  year, then

$$\text{TWR for holding period} = (1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T) - 1$$

## Non-Annual Compounding

$$PV = FV_N \left(1 + \frac{R_s}{m}\right)^{-mN}$$

where:

- $m$  = Number of compounding periods per year
- $R_s$  = Quoted annual interest rate
- $N$  = Number of years

## Annualizing Returns

$R_{\text{annual}} = (1 + R_{\text{weekly}})^{52} - 1$   $R_{\text{annual}} = (1 + R_{\text{monthly}})^{12} - 1$   $R_{\text{annual}} = (1 + R_{\text{daily}})^{252} - 1$   
assuming 252 trading days per year  $R_{\text{weekly}} = (1 + R_{\text{daily}})^5 - 1$  assuming 5 trading days per week

## Continuously Compounded Returns

$$P_t = P_0 e^{r_{0,T}}$$

$$r_{0,T} = \ln \left( \frac{P_t}{P_0} \right)$$

$$r_{0,T} = r_{0,1} + r_{1,2} + \dots + r_{T-2,T-1} + r_{T-1,T}$$

## Real Returns

$$(1 + \text{real return}) = (1 + \text{real risk-free rate}) \times (1 + \text{risk premium})$$

## Pre-Tax and After-Tax Nominal Return

$$\begin{aligned} \text{After-tax nominal return} &= \text{Pre-tax nominal return} \times (1 - \text{Tax rate}) \\ \text{After-tax real return} &= \frac{[1 + \text{Pre-Tax nominal return} \times (1 - \text{Tax rate})]}{1 + \text{Inflation premium}} - 1 \end{aligned}$$

## Leveraged Return

Return on a leveraged portfolio

$$R_L = R_P + \frac{V_B}{V_E} (R_P - r_D)$$

where:

- $R_P$  = Return on the investment portfolio (unleveraged)
- $r_D$  = Cost of debt
- $V_B$  = Debt/borrowed funds
- $V_E$  = Equity of the portfolio

## Learning Module 2: Time Value of Money in Finance

$$FV_t = PV(1 + r)^t \quad PV = \frac{FV_t}{(1 + r)^t}$$

where:

- $FV_t$  = Future value at time  $t$
- $PV$  = Present value
- $r$  = Discount rate per period
- $t$  = Number of compounding periods

As compounding frequency becomes very large (i.e., continuous compounding)

$$FV_t = PVe^{rt} \quad PV = FV_te^{-rt}$$

### Present Value of Zero-Coupon Bond

$$PV(\text{ Discount Bond } ) = \frac{FV}{(1 + r)^t}$$

where:

- $FV$  = Principal (or Face Value)
- $r$  = Market discount rate per period
- $t$  = Maturity of bond

$$r = \left( \frac{FV_t}{PV} \right)^{1/T} - 1$$

### Present Value of Coupon Bond

$$PV(\text{ Coupon Bond } ) = \frac{PMT}{(1 + r)^1} + \frac{PMT}{(1 + r)^2} + \cdots + \frac{PMT + FV}{(1 + r)^N}$$

where:

- $PV$  = Bond's price
- $PMT$  = Periodic coupon payment
- $FV$  = Face value
- $N$  = Number of periods
- $r$  = Market discount rate per period

### Present Value of a Perpetual Bond (Perpetuity)

$$PV(\text{ Perpetual Bond } ) = \frac{PMT}{r}$$

### Annuity Instruments (e.g., Mortgage)

$$A = \frac{rPV}{1 - (1 + r)^{-t}}$$

where:

- $A$  = Periodic cash flow
- $r$  = Market interest rate per period
- $PV$  = Present value or principal amount of loan/bond
- $t$  = Number of payment periods

## Price of a Preferred Share

$$PV_t = \frac{D_t}{r}$$

where:

- $D_t$  = Fixed periodic dividend
- $r$  = Expected rate of return

## Price of a Common Share

### Constant Dividend Growth Rate into Perpetuity

$$PV_t = \frac{D_t(1+g)}{r-g} = \frac{D_{t+1}}{r-g} \quad r > g$$

where:

- $D_t$  = Common dividend at time  $t$
- $g$  = Constant growth rate
- $r$  = Expected rate of return

$$\begin{aligned} r &= \frac{D_{t+1}}{PV_t} + g \\ \frac{PV_t}{E_t} &= \frac{\frac{D_t}{E_t} \times (1+g)}{r-g} \\ \frac{PV_t}{E_{t+1}} &= \frac{\frac{D_{t+1}}{E_{t+1}}}{r-g} \end{aligned}$$

where:

- $E_t$  = Earnings per share for period  $t$
- $\frac{PV_t}{E_t}$  = Trailing price-to-earnings ratio
- $\frac{PV_t}{E_{t+1}}$  = Forward price-to-earnings ratio

## Two-stage Dividend Discount Model

$$PV_t = \sum_{i=1}^n \frac{D_t (1 + g_s)^i}{(1 + r)^i} + \frac{E(S_{t+n})}{(1 + r)^n}$$

where:

- $g_s$  = Higher short-term dividend growth rate
- $g_L$  = Lower long-term dividend growth rate
- $n$  = Initial growth phase
- $E(S_{t+n})$  = Stock value in  $n$  periods (Terminal value)

$$= \frac{D_{t+n+1}}{r - g_L}$$

## Forward Rate

$$F_{1,1} = \frac{(1 + r_2)^2}{(1 + r_1)} - 1$$

where:

- $F_{1,1}$  = One-year forward rate one year from now
- $r_1$  = Discount rate on one-year risk-free discount bond
- $r_2$  = Discount rate on two-year risk-free discount bond

## Learning Module 3: Statistical Measures of Asset Returns

### Measures of Central Tendency

$$\text{Sample Mean, } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

where:

- $X_i$  = Observation  $i$  ( $i = 1, 2, 3, \dots, n$ )

### Median

$$\text{Position of median} = \frac{\text{Number of observations} + 1}{2}$$

## Quantiles

$$\text{Interquartile range} = Q_3 - Q_1$$

where:

- $Q_1$  = First quartile
- $Q_3$  = Third quartile

## Box and Whisker Plot

$$\text{Upper fence} = Q_3 + 1.5 \times IQR$$

$$\text{Lower fence} = Q_1 - 1.5 \times IQR$$

## Measures of Dispersion

$$\text{Range} = \text{Maximum value} - \text{Minimum value}$$

## Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^n |X_i - \bar{X}|}{n}$$

## Sample Variance

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$$

## Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

### Sample Target Semideviation

$$s_{\text{Target}} = \sqrt{\frac{\sum_{X_i \leq B}^n (X_i - B)^2}{n - 1}}$$

where:

- $B$  = target
- $n$  = total number of sample observations

### Coefficient of Variation

$$CV = \frac{s}{\bar{X}}$$

### Sample Skewness

$$\text{Skewness} \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^n (X_i - \bar{X})^3}{s^3}$$

### Sample Excess Kurtosis

$$K_E \approx \left(\frac{1}{n}\right) \frac{\sum_{i=1}^n (X_i - \bar{X})^4}{s^4} - 3$$

### Sample Covariance

$$s_{XY} = \frac{1}{n - 1} \sum_{i=1}^n (X_i - \bar{X}) (Y_i - \bar{Y})$$

### Sample Correlation Coefficient

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

## Learning Module 4: Probability Trees and Conditional Expectations

### Expected Value of a Discrete Random Variable

$$E(X) = \sum_{i=1}^n P(X_i) X_i$$

### Variance of a Random Variable

$$\begin{aligned}\sigma^2(X) &= E[X - E(X)]^2 \\ &= \sum_{i=1}^n P(X_i) [X - E(X)]^2\end{aligned}$$

### Conditional Expected Value of a Random Variable

$$E(X | S) = P(X_1 | S) X_1 + P(X_2 | S) X_2 + \dots + P(X_n | S) X_n$$

### Conditional Variance of a Random Variable

$$\begin{aligned}\sigma^2(X | S) &= P(X_1 | S) [X_1 - E(X_1 | S)]^2 + P(X_2 | S) [X_2 - E(X_2 | S)]^2 + \dots \\ &\quad + P(X_n | S) [X_n - E(X_n | S)]^2\end{aligned}$$

### Total Probability Rule for Expected Value

$$E(X) = E(X | S_1) P(S_1) + E(X | S_2) P(S_2) + \dots + E(X | S_n) P(S_n)$$

where:

- $S_1, S_2, \dots, S_n$  are mutually exclusive and exhaustive events.

### Bayes' Formula

$$\begin{aligned}P(A | B) &= \frac{P(B | A)}{P(B)} \times P(A) \\ P(\text{Event} | \text{Information}) &= \frac{P(\text{Information} | \text{Event})}{P(\text{Information})} \times P(\text{Event})\end{aligned}$$

## Learning Module 5: Portfolio Mathematics

For  $n$  assets in a portfolio

### Expected return on portfolio

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

### Variance on portfolio

$$\sigma^2(R_P) = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \text{Cov}(R_i, R_j)$$

Requires  $n$  variances and  $\frac{n(n-1)}{2}$  distinct covariances to estimate portfolio variance.

### Covariance

$$\begin{aligned} \text{Cov}(R_i, R_j) &= E[(R_i - E(R_i))(R_j - E(R_j))] \\ &= \frac{1}{n-1} \sum_{t=1}^n (R_{i,t} - \bar{R}_i)(R_{j,t} - \bar{R}_j) \end{aligned}$$

For a two-asset ( $n = 2$ ) portfolio:

$$\sigma^2(R_P) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)$$

where:

- $\text{Cov}(R_1, R_2) = \rho(R_1, R_2) \times \sigma(R_1) \times \sigma(R_2)$

**Video:** <https://youtu.be/IUwulZ9ONCO>

For a three-asset ( $n = 3$ ) portfolio:

$$\begin{aligned} \sigma^2(R_P) &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2w_1 w_2 \text{Cov}(R_1, R_2) \\ &\quad + 2w_1 w_3 \text{Cov}(R_1, R_3) + 2w_2 w_3 \text{Cov}(R_2, R_3) \end{aligned}$$

## Covariance Given a Joint Probability Function

$$\text{Cov}(R_A, R_B) = \sum_{i=1} \sum_{j=1} P(R_{A,i}, R_{B,j}) \times [R_{A,i} - E(R_A)] \times [R_{B,j} - E(R_B)]$$

If  $X$  and  $Y$  are uncorrelated, then  $E(XY) = E(X)E(Y)$

If  $X$  and  $Y$  are independent, then  $P(X, Y) = P(X)P(Y)$

## Safety-First Optimal Portfolio

### Safety-First Ratio

$$\text{SFRatio} = \frac{E(R_P) - R_L}{\sigma_P}$$

$$\text{Shortfall risk} = \Pr[E(R_P) < R_L] = \text{Normal}(-\text{SFRatio})$$

where:

- $R_L$  = Investor's threshold level
- $E(R_P)$  = Expected portfolio return
- $\sigma_P$  = Portfolio standard deviation

**Video:** <https://youtu.be/S3x5JrGIOUA>

## Learning Module 6: Simulation Methods

### Lognormal Distribution

#### Mean of a lognormal random variable

$$\mu_L = \exp(\mu + 0.50\sigma^2)$$

#### Variance of a lognormal random variable

$$\sigma_L^2 = \exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$$

where:

- $\mu$  = Mean of the normal random variable
- $\sigma^2$  = Variance of the normal random variable

## Continuously Compounded Rates of Return

$$P_T = P_0 \exp(r_{0,T})$$

where:

- $P_0$  = Current asset price
- $P_T$  = Asset price at time  $T$
- $r_{0,T}$  = Continuously compounded return from 0 to  $T$

If returns are independently and identically distributed (i.i.d.), then

$$r_{0,T} = r_{0,1} + r_{1,2} + \cdots + r_{T-2,T-1} + r_{T-1,T}$$

If the one-period continuously compounded returns are i.i.d. random variables with mean  $\mu$  and  $\sigma^2$ , then

$$\begin{aligned} E(r_{0,T}) &= \mu T \\ \sigma^2(r_{0,T}) &= \sigma^2 T \\ \sigma(r_{0,T}) &= \sigma\sqrt{T} \end{aligned}$$

## Learning Module 7: Estimation and Inference

$$\text{Sharpe ratio} = \frac{R_P - R_F}{\sigma_P}$$

where:

- $R_P$  = Portfolio return
- $R_F$  = Risk-free rate
- $\sigma_P$  = Portfolio standard deviation of return

$$\begin{aligned} \text{Variance of the sampling distribution} &= \frac{\sigma^2}{n} \\ \text{of the sample means} \\ \text{Standard error of} &= \frac{\sigma}{\sqrt{n}} \end{aligned}$$

where:

- $\sigma$  = Population standard deviation
- $n$  = Sample size

Note: If  $\sigma$  is not known, use  $s$ , the sample standard deviation.

### Bootstrap Resampling

$$s_{\bar{X}} = \sqrt{\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}_b - \bar{\theta})^2}$$

where:

- $s_{\bar{X}}$  = Estimate of the standard error of the sample mean
- $B$  = Number of resamples drawn from the original sample
- $\hat{\theta}_b$  = Mean of a resample
- $\bar{\theta}$  = Mean across all the resample means

### Learning Module 8: Hypothesis Testing

$$\text{Confidence level} = 1 - \alpha$$

$$\text{Power of the test} = 1 - \beta$$

where:

- $\alpha$  = Significance level (Probability of Type I error)
- $\beta$  = Probability of Type II error

### Test of a Single Mean

Test statistic

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

$$\text{Degrees of freedom} = n - 1 \quad (1 - \alpha)\% \text{ Confidence Interval} = \bar{X} \pm \text{Critical value} \times \left( \frac{s}{\sqrt{n}} \right)$$

## Test of the Difference in Means

Test statistic

$$t = \frac{(\bar{X}_{d1} - \bar{X}_{d2}) - (\mu_{d1} - \mu_{d2})}{\sqrt{\frac{s_p^2}{n_{d1}} + \frac{s_p^2}{n_{d2}}}}$$

Degrees of freedom =  $n_{d1} + n_{d2} - 2$

$$s_p^2 = \frac{(n_{d1} - 1) s_{d1}^2 + (n_{d2} - 1) s_{d2}^2}{n_{d1} + n_{d2} - 2}$$

## Test of the Mean of Differences

Test statistic

$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$$

Degrees of freedom =  $n - 1$

## Test of a Single Variance

Test statistic

$$\chi^2 = \frac{(n - 1) s^2}{\sigma_0^2}$$

Degrees of freedom =  $n - 1$

## Test of the Difference in Variances

Test statistic

$$F = \frac{s_{\text{Before}}^2}{s_{\text{After}}^2}$$

Degrees of freedom =  $n_1 - 1, n_2 - 1$

## Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom =  $n - 2$

## Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^m \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Degrees of freedom =  $(r - 1)(c - 1)$

where:

- $m$  = Number of cells in the table
- $O_{ij}$  = Number of observations in each cell of row  $i$  and column  $j$
- $E_{ij}$  = Expected number of observations in each cell of row  $i$  and column  $j$

## Learning Module 9: Parametric and Non-Parametric Tests of Independence

### Test of a Correlation

Test statistic

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degrees of freedom =  $n - 2$

### Pearson Correlation (or Bivariate Correlation)

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

## Spearman Rank Correlation Coefficient

$$r_S = 1 - \frac{6 \sum_{i=1}^n d_i^2}{n(n^2 - 1)}$$

where:

- $d$  = Difference in ranks

## Test of Independence (Categorical Data)

Test statistic

$$\chi^2 = \sum_{i=1}^m \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Degrees of freedom =  $(r - 1)(c - 1)$

where:

- $m$  = Number of cells in the table
- $O_{ij}$  = Number of observations in each cell of row  $i$  and column  $j$
- $E_{ij}$  = Expected number of observations in each cell of row  $i$  and column  $j$

$$= \frac{(\text{Total row } i) \times (\text{Total column } j)}{\text{Overall total}}$$

## Standardized Residual (or Pearson Residual)

$$\text{Standardized Residual} = \frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}}}$$

## Learning Module 10: Simple Linear Regression

$$Y_i = b_0 + b_1X_1 + \dots + b_nX_n + \varepsilon_i, \quad i = 1, 2, \dots, n$$

where:

- $Y$  = Dependent variable
- $X$  = Independent variable
- $b_0$  = Intercept
- $b_i$  = Slope coefficient,  $i = 1, 2, \dots, n$
- $\varepsilon_i$  = Error term
- $b_0, b_1, \dots, b_n$  = Regression coefficients

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1X_i + e_i$$

where:

- $\hat{Y}_i$  = Estimated value on the regression line for the  $i$  th observation
- $\hat{b}_0$  = Intercept
- $\hat{b}_1$  = Slope
- $e_i$  = Residual for the  $i$  th observation  $\hat{b}_1 = \frac{\text{Covariance of } X \text{ and } Y}{\text{Variance of } X} = \frac{\sum_{i=1}^n (Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^n (X_i - \bar{X})^2}$   $\hat{b}_0 = \bar{Y} - \hat{b}_1\bar{X}$  Sum of Squares Total,  $SST = \sum_{i=1}^n (Y_i - \bar{Y})^2 = SSR + SSE$  Sum of Squares Regression,  $SSR = \sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2$  Sum of Squares Error,  $SSE = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^n e_i^2$  Coefficient of Determination

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

Correlation coefficient

$$r = \frac{\text{Covariance of } X \text{ and } Y}{(\text{Standard deviation of } X)(\text{Standard deviation of } Y)}$$

Note: (Correlation coefficient)<sup>2</sup> = Coefficient of determination

Sample standard deviation of X

$$S_X = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}}$$

Sample standard deviation of Y

$$S_Y = \sqrt{\frac{\sum_{i=1}^n (Y_i - \bar{Y})^2}{n-1}}$$

Homoskedasticity

$$E(\varepsilon_i^2) = \sigma_\varepsilon^2, \quad i = 1, 2, \dots, n$$

## ANOVA F-Test

Mean square regression (MSR)

$$MSR = \frac{SSR}{k}$$

Mean square error (MSE)

$$MSE = \frac{SSE}{n-k-1}$$

F-distributed test statistic

$$F = \frac{MSR}{MSE}$$

where:

- $n$  = Number of observations
- $k$  = Number of independent variables

Standard error of estimate

$$s_e = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-k-1}}$$

Hypothesis Test of the Slope Coefficient

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

Degrees of freedom,  $df = n - k - 1$

where:

- $B_1$  = Hypothesized population slope
- $s_{\hat{b}_1}$  = Standard error of the slope coefficient

$$= \frac{s_e}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Hypothesis Test of the Intercept

$$t_{\text{intercept}} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}}$$

Standard error of the intercept,  $s_{\hat{b}_0}$

$$s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^n (X_i - \bar{X})^2}}$$

Prediction Intervals

$$\hat{Y}_f \pm t_{\alpha/2} \times s_f$$

where:  $\hat{Y}_f = \hat{b}_0 + \hat{b}_1 X_f$

Variance of the prediction error of Y, given X

$$s_f^2 = s_e^2 \left[ 1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{(n-1)s_X^2} \right]$$

Standard error of the forecast

$$s_f = s_e \sqrt{1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{(n-1)s_X^2}}$$

## The Log-Lin Model

$$\ln Y_i = b_0 + b_1 X_i$$

## The Lin-Log Model

$$Y_i = b_0 + b_1 \ln X_i$$

## The Log-Log Model

$$\ln Y_i = b_0 + b_1 \ln X_i$$

## Learning Module 11: Introduction to Big Data Techniques

No formula.

## VOLUME 2: ECONOMICS

### Learning Module 1: The Firm and Market Structures

Total profit = Total revenue - Total cost

Economic profit = Total revenue - Total economic costs

Accounting profit = Total revenue - Total accounting costs

Total revenue = Price  $\times$  Quantity =  $P \times Q$

Average revenue =  $\frac{\text{Total revenue}}{\text{Quantity}}$

Marginal cost =  $\frac{\Delta TC}{\Delta Q}$

Average variable cost =  $\frac{\text{Total variable cost}}{\text{Quantity}}$

Average fixed cost =  $\frac{\text{Total fixed cost}}{\text{Quantity}}$

Total cost = Total fixed cost + Total variable cost

Average total cost = Average fixed cost + Average variable cost

### Concentration Ratio

$$\text{Concentration ratio} = \sum_{i=1}^n (\text{Market share})_i$$

## Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^n (\text{Market share})_i^2$$

## Learning Module 2: Understanding Business Cycles

No formula

## Learning Module 3: Fiscal Policy

$$\text{Budget surplus/(deficit)} = G - T + B$$

where:

- $G$  = Government spending
- $T$  = Taxes
- $B$  = Payments of transfer benefits

## Disposable Income

$$YD = Y - NT = (1 - t)Y$$

where:

- $t$  = Net tax rate
- $NT$  = Net taxes = Taxes - Transfers
- $tY$  = Total tax revenue

## The Fiscal Multiplier

$$\text{Fiscal multiplier} = \frac{1}{1 - c(1 - t)}$$

where:

- $c$  = Marginal propensity to consume
- $t$  = Tax rate

## Learning Module 4: Monetary Policy

Neutral rate = Trend growth + Inflation target

## Learning Module 5: Introduction to Geopolitics

No formula

## Learning Module 6: International Trade

No formula

## Learning Module 7: Capital Flows and the FX Market

Real exchange rate  $_{d/f} = S_{d/f} \times \frac{P_f}{P_d}$  % Change in real exchange rate =  $\left(1 + \% \Delta S_{d/f}\right) \times \frac{(1 + \% \Delta P_f)}{(1 + \% \Delta P_d)} - 1$

$$\approx \% \Delta S_{d/f} + \% \Delta P_f - \% \Delta P_d$$

Percentage change in base currency  $f$  (vs currency  $d$ )

$$\frac{E(S_{d/f}) - S_{d/f}}{S_{d/f}}$$

where:

- $S_{d/f}$  = Spot exchange rate
- $P_f$  = General price level of goods indexed in currency  $f$
- $P_d$  = General price level of goods indexed in currency  $d$

## Learning Module 8: Exchange Rate Calculations

### Cross-Rate

$$\frac{A}{B} = \frac{A}{C} \times \frac{C}{D}$$

## Forward Rate

$$F_{A/B} = S_{A/B} \times \left[ \frac{1 + r_A \times T}{1 + r_B \times T} \right]$$

$$\begin{aligned} \text{Forward points} &= F_{A/B} - S_{A/B} \\ &= S_{A/B} \left( \frac{r_A - r_B}{1 + r_B} \right) T \end{aligned}$$

where:

- $S_{A/B}$  = Spot exchange rate
- $F_{A/B}$  = Forward exchange rate
- $T$  = Time to maturity

Learning Module 1: Organizational Forms, Corporate Issuer Features, and Ownership

No formula

## Learning Module 2: Investors and Other Stakeholders

No formula

Learning Module 3: Working Capital and Liquidity

$$\begin{aligned} \text{Cash conversion cycle} &= \text{Days of inventory on hand} + \text{Days sales outstanding} - \text{Days payables outstanding} \\ \text{EAR of Supplier} \frac{\text{Days in Year}}{\text{Financing}} &= \left( 1 + \frac{\text{Discount \%}}{100\% - \text{Discount \%}} \right)^{\frac{\text{Dayment Period} - \text{Discount Period}}{\text{Total working capital}}} - 1 = \frac{\text{Current assets} - \text{Current Liabilities}}{\text{Net working capital}} \\ \text{capital} &= \text{Current assets (excluding cash and marketable securities)} \\ &\quad - \text{Current Liabilities (excluding short-term and current debt)} \end{aligned}$$

Cash flow from operations = Cash received from customers

- Interest and dividends received on financial investments
- Cash paid to employees and suppliers
- Taxes paid to governments
- Interest paid to lenders

Free cash flow = Cash flow from operations - Investments in long-term assets

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Short-term marketable instruments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short-term marketable instruments}}{\text{Current liabilities}}$$

#### **Learning Module 4: Corporate Governance: Conflicts, Mechanisms, Risks, and Benefits**

**No formula**

#### **Learning Module 5: Capital Investments and Capital Allocation**

##### **Net Present Value**

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_T}{(1+r)^T} = \sum_{t=0}^T \frac{CF_t}{(1+r)^t}$$

where:

- $CF_t$  = After-tax cash flow at time  $t$
- $r$  = Required rate of return
- $CF_0$  = Initial outlay

##### **Internal Rate of Return**

$$\sum_{t=0}^T \frac{CF_t}{(1+IRR)^t} = 0$$

**Video:** <https://youtu.be/bzck7QLhICw>

### **Return on Invested Capital**

$$\begin{aligned}\text{ROIC} &= \frac{\text{After-tax operating profit}}{\text{Average invested capital}} \\ &= \frac{\text{Operating profit } t_t \times (1 - \text{Tax rate})}{\text{Average total long-term liabilities and equity}_{t-1,t}} \\ \text{ROIC} &= \frac{\text{After-tax operating profit}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Average invested capital}}\end{aligned}$$

### **Real Options in Capital Budgeting**

$$\begin{aligned}\text{Project NPV} \\ (\text{with option}) &= \text{Project NPV} \\ (\text{without option}) &- \text{Option cost} + \text{Option value}\end{aligned}$$

## **Learning Module 6: Capital Structure**

### **Weighted Average Cost of Capital**

$$WACC = w_d r_d (1 - t) + w_e r_e$$

where:

- $w_d$  = Target weight of debt in capital structure =  $\frac{D}{D+E}$
- $w_e$  = Target weight of common stock in capital structure =  $\frac{E}{D+E}$
- $r_d$  = Before-tax marginal cost of debt
- $t$  = Marginal tax rate
- $r_d(1 - t)$  = After-tax marginal cost of debt
- $r_e$  = Marginal cost of common stock

### **Operating Leverage**

$$\text{Operating leverage} = \frac{\text{Fixed costs}}{\text{Total costs}}$$

### **Interest Coverage**

$$\text{Interest coverage} = \frac{\text{Profit before interest and taxes}}{\text{Interest expense}}$$

## Modigliani-Miller Capital Structure Propositions

$$\begin{aligned}V_L &= V_U + tD \\r_e &= r_0 + (r_0 - r_d)(1 - t)\frac{D}{E} \\E &= \frac{(CF_e - r_d D)(1 - t)}{r_e} \\V_L &= \frac{CF_e(1 - t)}{r_{WACC}}\end{aligned}$$

where:

- $V_L$  = Value of levered firm
- $V_U$  = Value of unlevered firm
- $t$  = Marginal tax rate
- $r_e$  = Cost of equity
- $r_d$  = Cost of debt
- $r_0$  = Cost of capital (for a 100% equity-financed company)
- $D$  = Market value of debt
- $E$  = Market value of equity
- $CF_e$  = After-tax cash flows to shareholders
- $r_d D$  = Interest expense on debt

## Static Trade-off Theory of Capital Structure

$$V_L = V_U + tD - PV(\text{Costs of Financial Distress})$$

## Learning Module 7: Business Models

No formula

## VOLUME 4: FINANCIAL STATEMENT ANALYSIS

## Learning Module 1: Introduction to Financial Statement Analysis

No formula

## Learning Module 2: Analyzing Income Statements

Gross profit = Revenue - Cost of Goods Sold

Operating income = Gross margin - Selling, General, and Administrative Expense

Taxable income = Operating income - Interest expense

Net income = Taxable income - Taxes

Ending shareholders' equity = Beginning shareholders' equity + Net income

- Other comprehensive income
- Dividends
- Net capital contributions from shareholders

Ending retained earnings = Beginning retained earnings + Net income - Dividends

### Return on Equity

$$ROE = \frac{\text{Net income}}{\text{Average shareholders' equity}}$$

### Net Profit Margin

$$\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

### Basic EPS

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

### Diluted EPS (for convertible preferred stock)

$$\text{Diluted EPS} = \frac{\text{Net income}}{\text{Weighted average number of shares outstanding} + \text{New common shares that would have been issued at conversion}}$$

### Diluted EPS (for convertible debt)

$$\text{Diluted EPS} = \frac{\text{Net income} - \text{Preferred dividends} + \text{After tax interest expense on convertible debt}}{\text{Weighted average number of shares outstanding} + \text{New common shares that would have been issued at conversion}}$$

### Diluted EPS (for options)

$$\text{Diluted EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding} + \text{Additional common shares issued upon conversion}}$$

### Treasury stock method

$$\text{Additional common shares issued upon conversion} = \left( \begin{array}{cc} \text{New shares issued at} & \text{Shares repurchased} \\ \text{option exercise} & \text{from option exercised} \end{array} - \begin{array}{c} \text{with cash received} \end{array} \right) \times \begin{array}{c} \text{Proportion of} \\ \text{during which c} \\ \text{were outstan} \end{array}$$

Video (Basic & Diluted EPS): <https://youtu.be/2C-mwVqO2SQ>

### Learning Module 3: Analyzing Balance Sheets

Working capital = Current assets - Current liabilities

### Liquidity Ratios

$$\begin{aligned} \text{Current ratio} &= \frac{\text{Current assets}}{\text{Current liabilities}} \\ \text{Quick ( acid test ) ratio} &= \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current liabilities}} \\ \text{Cash ratio} &= \frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}} \end{aligned}$$

## Solvency Ratios

$$\text{Long-term debt-to-equity} = \frac{\text{Long-term debt}}{\text{Total equity}}$$

$$\text{Debt-to-equity} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Total debt} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Financial leverage} = \frac{\text{Total assets}}{\text{Total equity}}$$

## Learning Module 4: Analyzing Statements of Cash Flows I

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Cash flow Cash  
 flow Cash flow  
 Ending =  
<sub>cash</sub>  
 Beginning +  
<sub>cash</sub>  
 from operating +  
<sub>activities</sub>  
 from investing +  
<sub>activities</sub>  
 from financing  
<sub>activities</sub>

---

Ending accounts + Revenue - *from customers*  
 = Beginning  
 accounts  
 receivable  
 receivable

Ending Cost of goods  
 inventory = sold  
 inventory

Ending accounts + Purchases - Cash paid to  
 = Beginning suppliers  
 accounts payable

Ending wages Cash paid to  
 payable employees  
 + payable

Interest Cash  
 paid expense  
 - for interest

\_\_\_\_\_ taxpaple \_\_\_\_\_  
 +

---

Cash flow Cash  
 flow Cash flow  
 Ending =  
     cash  
 Beginning +  
     cash  
 from operating +  
     activities  
 from investing +  
     activities  
 from financing  
     activities

---

Ending Equipment sold

*PP&E* =  
 Beginning  
*PP&E* + \_\_\_\_\_

Ending *equipment sold*  
 accumulated  
 Beginning  
 accumulated  
 Depreciation  
 depreciation  
 depreciation  
 \_\_\_\_\_ expense  
 expense

---

$$\text{Gain on sale of equipment} = \text{Cash received from sale of equipment} - \text{Book value of equipment sold}$$

Note: 
$$\text{Ending retained earnings} = \text{Beginning retained earnings} + \text{Net income} - \text{Dividends}$$

## Learning Module 5: Analyzing Statements of Cash Flows II

### Free Cash Flow To Firm (FCFF)

$$\begin{aligned} FCFF &= NI + NCC + \text{Int}(1 - \text{Tax rate}) - FC\text{Inv} - WC\text{Inv} \\ &= CFO + \text{Int}(1 - \text{Tax rate}) - FC\text{Inv} \end{aligned}$$

where:

- NI = Net income
- $NCC$  = Non-cash charges (e.g., depreciation and amortization)
- Int = Interest expense
- $FCInv$  = Capital expenditures
- $WCInv$  = Working capital expenditures
- $CFO$  = Cash flow from operating activities =  $NI + NCC - WCInv$

## Free Cash Flow to Equity (FCFE)

$$FCFE = CFO - FCInv + NetBorrowing$$

where:

- Net Borrowing = Debt issued - Debt repaid

## Performance Ratios

$$\text{Cash flow to revenue} = \frac{CFO}{\text{Revenue}}$$

$$\text{Cash return on assets} = \frac{CFO}{\text{Average total assets}}$$

$$\text{Cash return on equity} = \frac{CFO}{\text{Average shareholders equity}}$$

$$\text{Cash to income} = \frac{CFO}{\text{Operating income}}$$

$$\text{Cash flow per share} = \frac{CFO - \text{Preferred dividends}}{\text{Number of common shares outstanding}}$$

## Coverage Ratios

$$\text{Debt coverage ratio} = \frac{CFO}{\text{Total debt}}$$

$$\text{Interest coverage ratio} = \frac{CFO + \text{Interest paid} + \text{Taxes paid}}{\text{Interest paid}} \quad \text{Reinvestment ratio} = \frac{CFO}{\text{Cash paid for long term assets}}$$

$$\text{Debt payment ratio} = \frac{CFO}{\text{Cash paid for long term debt repayment}} \quad \text{Dividend payment ratio} = \frac{CFO}{\text{Dividends paid}}$$

$$\text{Investing and financing ratio} = \frac{CFO}{\text{Cash flow for investing and financing activities}}$$

## Learning Module 6: Analysis of Inventories

### IFRS

Inventories = Lower of Cost and Net Realizable Value (NRV)  $NRV$  = Estimated selling price less estimated costs of completion and costs necessary to complete the sale

## US GAAP

Inventories = Lower of Cost and NRV For last-in, first-out (LIFO) method or retail inventory methods Inventories = Lower of Cost and Market Value

Market value = Current replacement cost (subject to lower and upper limits) Lower limit =  $NRV - \text{Normal profit margin}$  Upper limit =  $NRV$  Video: <https://youtu.be/V8C31msIBzs>

Inventory turnover ratio =  $\frac{\text{Cost of sales}}{\text{Average inventory}}$  Days of inventory on hand =  $\frac{\text{Number of days in period}}{\text{Inventory turnover ratio}}$

Ending inventory ( FIFO ) = Ending inventory ( LIFO ) + LIFO reserve

COGS ( FIFO ) = COGS ( LIFO ) - Change in LIFO reserve

## Learning Module 7: Analysis of Long-Term Assets

Net book value = Historical cost - Accumulated depreciation

Gain on sale of asset = Sale proceeds - Net book value

$$\frac{\text{Estimated remaining life}}{\text{Estimated useful life}} = \frac{\text{Estimated total useful life}}{\frac{\text{Gross PP\&E}}{\text{Annual depreciation expense}}}$$
$$\frac{\text{Estimated remaining life}}{\text{Estimated useful life}} = \frac{\text{Estimated age of equipment}}{\frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}}}$$
$$\frac{\text{Estimated remaining life}}{\text{Estimated useful life}} = \frac{\text{Net PP\&E}}{\text{Annual depreciation expense}}$$

## Straight-line Depreciation

$$\text{Annual depreciation expense} = \frac{\text{Historical cost} - \text{Salvage value}}{\text{Estimated useful life}}$$

## Fixed Asset Turnover

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average net PP\&E}}$$

## Impairment of Long-Lived Assets

### IFRS

Impairment = Carrying amount - Recoverable amount

where:

- Recoverable amount = max(Fair value less costs to sell, Value in use)

## US GAAP

If asset's carrying amount > undiscounted expected future cash flows: Impairment = Carrying amount - Fair value

## Learning Module 8: Topics in Long-Term Liabilities and Equity

### Lessee Accounting - Finance Lease (IFRS)

Interest expense = Implied interest rate  $\times$  Beginning lease liability  
on lease

Principal repayment = Lease payment - Interest expense

Ending lease liability = Beginning lease liability + Interest expense - Lease payment

If ROU asset is amortized on a straight-line basis:

$$\text{Amortization expense} = \frac{\text{Initial ROU asset value} - \text{Salvage value}}{\text{Lease term}}$$

Ending ROU asset = Beginning ROU asset - Amortization expense

### Lessee Accounting - Operating Lease (US GAAP)

$\text{Amortization expense} = \text{Lease payment} - \text{Interest expense}$

$\text{Ending ROU asset} = \text{Beginning ROU asset} - \text{Amortization expense}$

$\text{Ending lease liability} = \text{Beginning lease liability} - \text{Amortization expense}$

## Stock Options

$$\text{Compensation expense} = \frac{\text{Fair value of options granted}}{\text{Vesting period}}$$

## Learning Module 9: Analysis of Income Taxes

### Deferred Tax Asset/Liability

For Assets:

$$\frac{\text{Deferred tax liability}}{\text{asset}} = \text{Tax rate} \times \left( \frac{\text{Carrying amount of asset}}{\text{asset}} - \frac{\text{Tax base of asset}}{\text{asset}} \right)$$

**For Liabilities:**

$$\frac{\text{Deferred tax liability}}{\text{asset}} = \text{Tax rate} \times \left( \frac{\text{Tax base of liability}}{\text{liability}} - \frac{\text{Carrying amount of liability}}{\text{liability}} \right)$$

$$\text{Income tax expense} = \text{Income tax payable} + \frac{\text{Changes in deferred tax assets and liabilities}}{\text{assets and liabilities}}$$

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pre-tax income}}$$

$$\text{Cash tax rate} = \frac{\text{Cash tax}}{\text{Pre-tax income}}$$

## Learning Module 10: Financial Reporting Quality

### Adjusted EBITDA

$$\text{Adjusted EBITDA} = \text{Adjusted EBIT} + \text{Post-IPO Software and R\&D amortization} + \text{Depreciation} + \text{amortization share-based amortization}$$

Straight-line method of depreciation

$$\text{Depreciation expense} = \frac{\text{Cost} - \text{Salvage value}}{\text{Useful life}}$$

### Double-Declining Balance method

$$\text{Depreciation expense} = \frac{2}{\text{Useful life}} \times (\text{Cost} - \text{Accumulated depreciation})$$

**Video:** <https://youtu.be/6RskYAxdAFk>

### **Units-of-Production method**

$$\text{Depreciation expense} = \frac{\text{Units produced}}{\text{Total units over useful life}} \times (\text{Cost} - \text{Salvage value})$$

## **Learning Module 11: Financial Analysis Techniques**

### **Activity Ratios**

$$\text{Inventory turnover} = \frac{\text{Cost of sales}}{\text{Average inventory}}$$

$$\text{Days of inventory on hand} = \frac{\text{Number of days in the period}}{\text{Inventory turnover}}$$

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Average receivables}}$$

$$\text{Days of sales outstanding} = \frac{\text{Number of days in the period}}{\text{Receivables turnover}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average payables}}$$

$$\text{Number of days of payables} = \frac{\text{Number of days in the period}}{\text{Payables turnover}}$$

$$\text{Working capital turnover} = \frac{\text{Revenue}}{\text{Average working capital}}$$

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average net fixed assets}}$$

$$\text{Total asset turnover} = \frac{\text{Revenue}}{\text{Average total assets}}$$

### **Liquidity Ratios**

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Quick ratio} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Current liabilities}}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short term marketable investments}}{\text{Current liabilities}}$$

$$\text{Defensive interval} = \frac{\text{Cash} + \text{Short term marketable investments} + \text{Receivables}}{\text{Daily cash expenditures}}$$

$$\text{Cash conversion cycle} = \frac{\text{Days of inventory on hand}}{\text{on hand}} + \frac{\text{Days of sales outstanding}}{\text{outstanding}} - \frac{\text{Number of days of payables}}{\text{of payables}}$$

## Solvency Ratios

$$\text{Debt-to-assets ratio} = \frac{\text{Total debt}}{\text{Total assets}} \\ (\text{"Total debt ratio"})$$

$$\text{Debt-to-capital ratio} = \frac{\text{Total debt}}{\text{Total debt} + \text{Total equity}}$$

$$\text{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Total equity}}$$

$$\text{Financial leverage ratio} = \frac{\text{Average total assets}}{\text{Average total equity}}$$

$$\text{Debt-to-EBITDA ratio} = \frac{\text{Total or net debt}}{\text{EBITDA}}$$

## Coverage Ratios

$$\text{Interest coverage ratio} = \frac{\text{EBIT}}{\text{Interest payments}} \quad \text{Fixed charge coverage ratio} = \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$$

## Profitability Ratios

$$\text{Gross profit margin} = \frac{\text{Gross profit}}{\text{Revenue}} \quad \text{Operating profit margin} = \frac{\text{Operating income}}{\text{Revenue}} \quad \text{Pretax margin} = \frac{\text{EBT}}{\text{Revenue}} \quad \text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

$$\text{Operating ROA} = \frac{\text{Operating income}}{\text{Average total assets}} \quad \text{ROA} = \frac{\text{Net income}}{\text{Average total assets}} \quad \text{Return on invested capital} = \frac{\text{EBIT} \times (1 - \text{Effective tax rate})}{\text{Average total debt and equity}} \quad \text{ROE} = \frac{\text{Net income}}{\text{Average total equity}} \quad \text{Return on common equity} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Average common equity}}$$

## DuPont Analysis

$$\text{ROE} = \text{ROA} \times \text{Financial Leverage}$$

$$\text{ROE} = \text{Net profit margin} \times \text{Total asset turnover} \times \text{Financial leverage} \quad \text{ROE} = \frac{\text{Tax}}{\text{burden}} \times \frac{\text{Interest}}{\text{burden}} \times \frac{\text{EBIT}}{\text{margin}} \times \frac{\text{Total asset}}{\text{turnover}} \times \frac{\text{Financial}}{\text{leverage}}$$

where:

- Tax burden =  $\frac{\text{Net income}}{\text{EBT}}$
- Interest burden =  $\frac{\text{EBT}}{\text{EBIT}}$

## Business Risk

$$\begin{aligned} \text{Coefficient of variation of operating income} &= \frac{\text{Standard deviation of operating income}}{\text{Average operating income}} & \text{Coefficient of variation of net income} &= \frac{\text{Standard deviation of net income}}{\text{Average net income}} \\ \text{Coefficient of variation of revenue} &= \frac{\text{Standard deviation of revenue}}{\text{Average revenue}} \end{aligned}$$

## Financial Sector Ratios

$$\begin{aligned} \text{Monetary reserve requirement (Cash reserve ratio)} &= \frac{\text{Reserves held at central bank}}{\text{Specified deposit liabilities}} & \text{Net interest margin} &= \frac{\text{Net interest income}}{\text{Total interest earning assets}} \\ \text{Liquid asset requirement} &= \frac{\text{Approved readily marketable securities}}{\text{Specified deposit liabilities}} & \text{Net interest margin} &= \frac{\text{Net interest income}}{\text{Total interest earning assets}} \end{aligned}$$

## Learning Module 12: Introduction to Financial Statement Modeling

Nothing new.

## VOLUME 5: EQUITY INVESTMENTS

### Learning Module 1: Market Organization and Structure

Maximum leverage ratio =  $\frac{1}{\text{Minimum margin requirement}}$  Total return on leveraged stock investment:

$$\text{Total Return} = \frac{\text{Sales proceeds} + \text{Dividends} - \text{Loan} - \frac{\text{Margin}}{\text{interest}} - \text{Sales commission}}{\text{Initial equity} + \text{Purchase commission}} - 1$$

Initial equity =  $\frac{\text{Minimum margin requirement}}{\text{requirement}} \times \text{Total purchase price}$  Video (Return on Leveraged Stock

Position): <https://youtu.be/tZd4Xtvjill> Margin Call Price =  $\frac{P_0(1 - \text{Initial Margin})}{(1 - \text{Maintenance Margin})}$

## Learning Module 2: Security Market Indexes

$$\text{Price Return Index, } V_{PRI} = \frac{\sum_{i=1}^N n_i P_i}{D}$$

where:

- $n_i$  = the number of units of constituent security  $i$  held in the index portfolio
- $N$  = the number of constituent securities in the index
- $P_i$  = the unit price of constituent security  $i$
- $D$  = value of the divisor

$$\text{Price return of an index, } PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

$$\text{Total Return Index, } TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:

- $V_{PRI1}$  = value of the price return index at the end of the period
- $V_{PRI0}$  = value of the price return index at the beginning of the period
- $Inc_I$  = total income (dividends and/or interest) from all securities in the index held over the period

## Weighting Methods

$$\text{Price weighting, } w_i^P = \frac{P_i}{\sum_{j=1}^N P_j}$$

Video (Recalculating the divisor of a price weighted index): <https://youtu.be/eYiZNK-ETrg>

$$\text{Equal weighting, } w_i^E = \frac{1}{N}$$

$$\text{Market-capitalization weighting, } w_i^M = \frac{Q_i P_i}{\sum_{j=1}^N Q_j P_j}$$

$$\text{Float-adjusted market capitalization weighting, } w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^N f_j Q_j P_j}$$

where:

- $f_i$  = fraction of shares outstanding in the market float

- $Q_i$  = number of shares outstanding of security  $i$
- $P_i$  = share price of security  $i$
- $N$  = number of securities in the index

$$\text{Fundamental weighting, } w_i^F = \frac{F_i}{\sum_{j=1}^N F_j}$$

where  $F_i$  denotes a fundamental size measure of company  $i$

### Learning Module 3: Market Efficiency

No formula

### Learning Module 4: Overview of Equity Securities

Return on Equity (using average total book value of equity)

$$ROE_t = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Return on Equity (using beginning book value of equity)

$$ROE_t = \frac{NI_t}{BVE_{t-1}}$$

where BVE = book value (Assets - Liabilities)

### Learning Module 5: Company Analysis: Past and Present

$$\text{Market share} = \frac{\text{Revenue}}{\text{Market size}}$$

$$\text{Sales potential} = 100\% - \text{Market share } \%$$

$$\text{Net sales} = \text{Average selling price} \times \text{Quantity sold}$$

$$\text{Take rate} = \frac{\text{Revenue earned from transactions}}{\text{Total transaction volume}} \times 100\%$$

$$\text{Operating income} = Q \times (P - VC) - FC$$

where:

- $Q$  = Units sold in a period
- $P$  = Price per unit

- $VC$  = Variable operating cost per unit
- $FC$  = Fixed operating costs
- $P - VC$  = Contribution margin per unit

$$\text{Degree of operating leverage (DOL)} = \frac{\frac{\% \Delta \text{ Operating income}}{\% \Delta \text{ Sales}}}{\frac{\% \Delta \text{ Net income}}{\% \Delta \text{ Operating income}}} \quad \text{Degree of financial leverage (DFL)} =$$

$$\text{WACC} = \frac{\text{Weight of debt}}{\text{Gross cost of debt}} \times (1 - \text{tax rate}) + \frac{\text{Weight of equity}}{\text{Cost of equity}}$$

## Learning Module 6: Industry and Company Analysis

### Herfindahl-Hirschman Index (HHI)

$$HHI = \sum_{i=1}^{\infty} s_i^2$$

where:

- $s_i$  = Market share of participant  $i$  (stated as a whole number)

$$\text{Learning Module 7: Company Analysis: Forecasting \% Variable cost} \approx \frac{\% \Delta (\text{Cost of revenue} + \text{Operating expense})}{\% \Delta \text{ Revenue}}$$

$$\% \text{ Fixed cost} \approx 1 - \% \text{ Variable cost} \quad \frac{\text{Number of units sold post-cannibalization}}{\text{Number of units sold pre-cannibalization}} = \frac{\text{Number of units sold pre-cannibalization}}{\text{Number of units sold pre-cannibalization}} -$$

$$\frac{\text{Expected cannibalization}}{\text{Expected cannibalization}} = \frac{\text{Number of units sold pre-cannibalization}}{\text{Number of units sold pre-cannibalization}} \times \frac{\text{Cannibalization factor}}{\text{Cannibalization factor}}$$

## Learning Module 8: Equity Valuation: Concepts and Basic Tools

### Dividend Discount Model (DDM)

$$V_0 = \sum_{t=1}^n \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

where:

- $V_0$  = Intrinsic value of a share at  $t = 0$
- $D_t$  = expected dividend in year  $t$
- $r$  = required rate of return on stock
- $P_n$  = expected price per share at  $t = n$  (terminal value)

## Free-cash-flow-to-equity (FCFE) Valuation Model

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$$

where:

- $FCFE = CFO - FCInv + NetBorrowing$
- $FCInv$  = Fixed capital investment
- $Net\ Borrowing$  = Borrowings minus repayments Value of preferred stock (non-callable, non-convertible, perpetual)

$$V_0 = \frac{D_0}{r}$$

Value of preferred stock (non-callable, non-convertible, maturity at time  $n$  )

$$V_0 = \sum_{t=1}^n \frac{D_t}{(1+r)^t} + \frac{\text{Par value}}{(1+r)^n}$$

## Gordon Growth Model

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$$

where:

- $D_0$  = Most recent annual dividend
- $D_1$  = Expected dividend in the next period
- $g$  = Constant growth rate
- $r$  = Required return on equity

## Sustainable growth rate

$$g = b \times ROE$$

where:

- $b$  = earnings retention rate (= 1 - Dividend payout ratio)
- $ROE$  = Return on equity

**Video:** <https://youtu.be/MnfRRRhuGpA>

## Two-Stage Dividend Discount Model

$$V_0 = \sum_{t=1}^n \frac{D_0 (1 + g_s)^t}{(1 + r)^t} + \frac{V_n}{(1 + r)^t}$$

where:

- $g_L$  = Long-term stable growth rate
- $g_s$  = Short-term growth rate
- $V_n = \frac{D_{n+1}}{r - g_L} = \frac{D_0(1+g_s)^t(1+g_L)}{r - g_L}$

## Justified forward P/E

$$\frac{P_0}{E_1} = \frac{\text{Dividend payout ratio}}{r - g}$$

## Enterprise Value

$$EV = \begin{array}{c} \text{Market value} \\ \text{of equity} \end{array} + \begin{array}{c} \text{Market value} \\ \text{of preferred stock} \end{array} + \begin{array}{c} \text{Market value} \\ \text{of debt} \end{array} - \begin{array}{c} \text{Cash and} \\ \text{short term} \\ \text{investments} \end{array}$$

## Asset-based Valuation

$$\begin{array}{c} \text{Adjusted} \\ \text{book value} \end{array} = \begin{array}{c} \text{Market value} \\ \text{of assets} \end{array} - \begin{array}{c} \text{Market value} \\ \text{of liabilities} \end{array}$$

## VOLUME 6: FIXED INCOME

Learning Module 1: Fixed-Income Instrument Features Current yield =  $\frac{\text{Annual coupon}}{\text{Bond price}}$  Bond price =  $\frac{\text{Coupon}}{(1+r)^1} + \frac{\text{Coupon}}{(1+r)^2} + \dots + \frac{\text{Coupon} + \text{Face value}}{(1+r)^n}$

where:

- Coupon per period = Coupon rate per period  $\times$  Face value
- $r$  = Yield to maturity per period
- $n$  = Number of payments

Floating-rate Note (FRN) coupon rate = MRR + Spread

## Learning Module 2: Fixed-Income Cash Flows and Types

### Fully Amortizing Loan with Level Payment

$$A = \frac{r \times \text{Principal}}{1 - (1 + r)^{-N}}$$

where:

- $A$  = Periodic payment
- $r$  = Market interest rate per period
- $N$  = Number of payment periods

If the periodic payment is monthly:

Monthly interest payment = Interest rate per month  $\times$  Beginning principal of loan

Monthly principal payment = Total monthly payment  $-$  Monthly interest payment

Ending principal of loan = Beginning principal of loan  $-$  Monthly principal payment

### Capital-Index Bond (e.g., TIPS)

Inflation-adjusted principal = Principal amount  $\times$  (1 + Inflation adjustment )

Coupon per period = Coupon rate per period  $\times$  Inflation-adjusted principal

### Deferred Coupon Bond

Video: <https://youtu.be/erRbAUOGIyM>

### Convertible Bonds

$$\begin{array}{c} \text{Conversion} \\ \text{ratio} \end{array} = \frac{\text{Convertible bond par value}}{\begin{array}{c} \text{Conversion price} \\ \text{Conversion} \\ \text{value} \end{array}} = \begin{array}{c} \text{Conversion} \\ \text{ratio} \end{array} \times \begin{array}{c} \text{Current share} \\ \text{price} \end{array}$$

### Zero-Coupon Bond

Original issue discount = Bond par value - Issuance price

## Learning Module 3: Fixed-Income Issuance and Trading

No formula

Learning Module 4: Fixed-Income Markets for Corporate Issuers

## Repurchase Agreements

$$\begin{aligned}\text{Repurchase price} &= \text{Price of bond} \times \left[ 1 + \text{Repo rate} \times \frac{\text{Repo term (in days)}}{\text{Number of days in a year}} \right] \\ \text{Initial margin} &= \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Purchase price}_0} \\ \text{Haircut} &= \frac{\text{Security price}_0 - \text{Purchase price}_0}{\text{Security price}_0}\end{aligned}$$

$$\text{Variation margin} = (\text{Initial margin} \times \text{Purchase price}_t) - \text{Security price}_t$$

## Learning Module 5: Fixed-Income Markets for Government Issuers

No formula.

## Learning Module 6: Fixed-Income Bond Valuation: Prices and Yields

$$PV = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} + \dots + \frac{PMT_N + FV_N}{(1+r)^N}$$

where:

- $PMT_t$  = Coupon that occurs in  $t$  periods
- $r$  = Market discount rate per period
- $N$  = Number of periods to maturity
- $FV$  = Face value of bond

Full Price, Flat Price, and Accrued Interest (Video: <https://youtu.be/I7G075JAu5w>)

$$\begin{aligned}PV^{\text{Full}} &= PV^{\text{Flat}} + \text{Accrued Interest} \\ &= PV_{BOP} \times (1+r)^{t/T}\end{aligned}$$

where:

- $\text{Accrued Interest} = \frac{t}{T} \times PMT$
- $t$  = number of days from the last coupon payment to the settlement date
- $T$  = number of days in the coupon period

- $t/T$  = fraction of the coupon period that has gone by since the last payment
- $PV_{BOP}$  = price on the previous coupon date (before the settlement date)

## Matrix Pricing

$$\text{Interpolated yield} = \text{Yield}_S + \left( \frac{\text{Tenor}_{\text{Target}} - \text{Tenor}_S}{\text{Tenor}_L - \text{Tenor}_S} \right) \times (\text{Yield}_L - \text{Yield}_S)$$

where:

- $\text{Yield}_S$  = Yield of shorter-term bond
- $\text{Yield}_L$  = Yield of longer-term bond
- $\text{Tenor}_S$  = Tenor of shorter-term bond
- $\text{Tenor}_L$  = Tenor of longer-term bond
- $\text{Tenor}_{\text{Target}}$  = Tenor of the subject bond
- $\text{Tenor}_S < \text{Tenor}_{\text{Target}} < \text{Tenor}_L$  Required yield spread = Bond YTM - Government Bond YTM (Similar maturity)

## Learning Module 7: Yield and Yield Spread Measures for Fixed Rate Bonds

### Periodicity Conversion

$$\left( 1 + \frac{APR_m}{m} \right)^m = \left( 1 + \frac{APR_n}{n} \right)^n$$

where:

- $APR_m$  = Annual percentage rate for  $m$  periods per year
  - $APR_n$  = Annual percentage rate for  $n$  periods per year
- Current yield  $_t = \frac{\text{Annual coupon}_t}{\text{Bond price}_t}$
- Government equivalent yield,  $\text{Yield}_{\text{ACT} / \text{ACT}} = \frac{365}{360} \times \text{Yield}_{30/360}$  Simple yield
- $= \frac{\text{Coupon} + (\frac{FV - PV}{N})}{\text{Flat price}}$

### Callable Bonds

$$PV = \frac{PMT}{(1 + YTC)^1} + \frac{PMT}{(1 + YTC)^2} + \dots + \frac{PMT + \text{Call price}}{(1 + YTC)^N}$$

where:

- $PV$  = Price of the callable bond
- $PMT$  = Coupon payment per period
- $YTC$  = Yield to call per period

- $N$  = Number of periods to when the bond can be called at the call price

Option-adjusted price = Flat price of bond + Value of embedded call option

Value of call option = Price of option-free bond - Price of callable bond

G-spread = Bond YTM - Interpolated sovereign bond YTM

I-spread = Bond YTM - Swap rate

## Z-Spread

$$PV = \frac{PMT}{(1 + z_1 + Z)^1} + \frac{PMT}{(1 + z_2 + Z)^2} + \dots + \frac{PMT + FV}{(1 + z_N + Z)^N}$$

where:

- $Z$  = Z-spread
- $z_N$  = Spot rate for  $N$  periods

OAS = Z-spread - Option value (in basis points per year)

## Learning Module 8: Yield and Yield Spread Measures for Floating-Rate Instruments

### Value of Floating Rate Note (FRN)

$$PV = \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1 + \frac{MRR+DM}{m}\right)^1} + \frac{\left(\frac{MRR+QM}{m}\right) \times FV}{\left(1 + \frac{MRR+DM}{m}\right)^2} + \dots + \frac{\left(\frac{MRR+QM}{m}\right) \times FV + FV}{\left(1 + \frac{MRR+DM}{m}\right)^n}$$

where:

- $QM$  = Quoted Margin
- $DM$  = Discount Margin
- $MRR$  = Market reference rate
- $m$  = Periodicity (i.e., number of payment periods per year)
- $FV$  = Face Value of FRN
- $N$  = Number of evenly spaced periods to maturity

**Video:** <https://youtu.be/zqY0tVLkYR8>

## **Yield Measures for Money Market Instruments**

### **Discount Rate Basis**

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$
$$DR = \frac{Year}{Days} \times \left(\frac{FV - PV}{FV}\right)$$

where:

- $PV$  = present value of money market instrument
- $FV$  = future value paid at maturity
- Days = number of days between settlement and maturity
- Year = number of days in the year
- $DR$  = discount rate (stated as annual percentage rate)

### **Add-on Rate Basis**

$$PV = \frac{FV}{\left(1 + \frac{Days}{Year} \times AOR\right)}$$
$$AOR = \frac{Year}{Days} \times \left(\frac{FV - PV}{PV}\right)$$
$$\text{Bond equivalent yield} = \frac{365}{Days} \times \left(\frac{FV - PV}{PV}\right)$$

## **Learning Module 9: The Term Structure of Interest Rates: Spot, Par, and Forward Curves**

### **Calculation of Bond Price Using Spot Rates**

$$PV = \frac{PMT}{(1 + Z_1)^1} + \frac{PMT}{(1 + Z_2)^2} + \dots + \frac{PMT + FV}{(1 + Z_N)^N}$$

where:

- $PV$  = Price of bond
- PMT = Bond coupon payment
- $Z_N$  = Spot rate (or zero-coupon yield or zero rate) for period  $N$
- $FV$  = Face value of bond

Given a Par Rate,  $FV = PV$  and  $PMT = \text{Par Rate (\%)} \times FV$

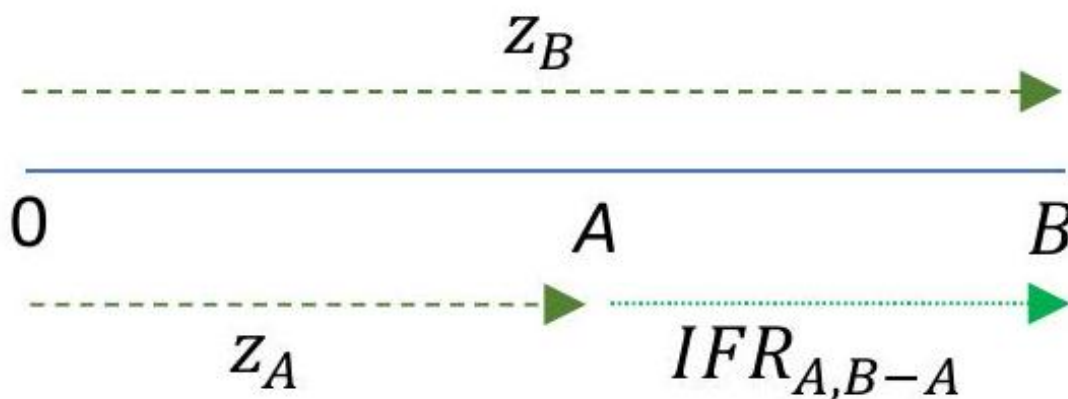
$$100 = \frac{PMT}{(1 + Z_1)^1} + \frac{PMT}{(1 + Z_2)^2} + \dots + \frac{PMT + 100}{(1 + Z_N)^N}$$

## Forward Rates, IFR

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$$

where:

- $IFR_{A,B-A}$  = Forward rate for  $(B - A)$  periods that starts in period  $A$



## Learning Module 10: Interest Rate Risk and Return

### Duration Gap

Duration gap = Macaulay duration - Investment horizon

### Macaulay Duration

$$\begin{aligned} \text{Macaulay duration} = & \left(1 - \frac{t}{T}\right) \left[ \frac{\frac{PMT}{(1+r)^{1-t/T}}}{PV^{Full}} \right] + \left(2 - \frac{t}{T}\right) \left[ \frac{\frac{PMT}{(1+r)^{2-t/T}}}{PV^{Full}} \right] + \dots \\ & + \left(N - \frac{t}{T}\right) \left[ \frac{\frac{PMT+FV}{(1+r)^{N-t/T}}}{PV^{Full}} \right] \end{aligned}$$

$$\text{Macaulay duration} = \left\{ \frac{1+r}{r} - \frac{1+r+[N \times (c-r)]}{c \times [(1+r)^N - 1] + r} \right\} - \frac{t}{T}$$

where:

- $r$  = Yield per period
- $c$  = Coupon rate per period
- $N$  = Number of evenly spaced periods to maturity as of the beginning of the current period
- $t$  = Number of days from the last coupon payment to the settlement date
- $T$  = Number of days in the coupon period

**Video:** <https://youtu.be/USgjcdCk7Fs>

## **Learning Module 11: Yield-Based Bond Duration Measures and Properties**

### **Modified Duration**

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1 + r}$$

### **Approximate Modified Duration**

$$\begin{aligned}\text{AnnModDur} &\approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{Yield}) \times (PV_0)} \\ \% \Delta PV^{\text{Full}} &\approx - \text{AnnModDur} \times \Delta \text{Yield}\end{aligned}$$

### **Money Duration**

$$\text{Money duration} = \text{AnnModDur} \times PV^{\text{full}}$$

$$\Delta PV^{\text{Full}} \approx - \text{MoneyDur} \times \Delta \text{Yield}$$

### **Duration of Zero-Coupon Bond**

$$\begin{aligned}\text{MacDur} &= \text{Time to maturity} \\ \text{ModDur} &= \frac{\text{Time to maturity}}{1 + r}\end{aligned}$$

## Duration of Perpetual Bond

$$\text{MacDur} = \frac{1+r}{r}$$
$$\text{ModDur} = \frac{1}{r}$$

## Duration of Floating-Rate Notes

$$\text{MacDur} = \frac{T-t}{T} = \begin{array}{l} \text{Fraction of period remaining until} \\ \text{the next reset date} \end{array}$$

## Learning Module 12: Yield-Based Bond Convexity and Portfolio Properties

### Convexity

$$\text{Convexity} = \sum_{t=1}^N \frac{t(t+1) \times \frac{PV_t}{PV^{\text{Full}}}}{(1+YTM)^2}$$

### Approximate Annualized Convexity

$$\text{ApproxConv} \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta \text{Yield})^2 \times (PV_0)}$$
$$\% \Delta PV^{\text{Full}} \approx -\text{AnnModDur} \times \Delta \text{Yield} + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Yield})^2$$

### Money Convexity

$$\text{MoneyCon} = \text{AnnConvexity} \times PV^{\text{Full}}$$
$$\Delta PV^{\text{Full}} \approx -(\text{MoneyDur} \times \Delta \text{Yield}) + \left[ \frac{1}{2} \times \text{MoneyCon} \times (\Delta \text{Yield})^2 \right]$$

## Portfolio Duration and Convexity

$$\text{Portfolio Modified Duration} = \sum_{i=1}^N w_i \times \text{ModDur}_i$$

$$\text{Portfolio Convexity} = \sum_{i=1}^N w_i \times \text{Convexity}_i$$

where:

- $w_i$  = Weight of bond  $i$ , measured in market value

## Learning Module 13: Curve-Based and Empirical Fixed-Income Risk Measures

### Effective Duration

$$\text{EffDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{ Curve}) \times PV_0}$$

### Effective Convexity

$$\begin{aligned} \text{EffCon} &= \frac{(PV_-) + (PV_+) - 2 \times PV_0}{(\Delta \text{ Curve})^2 \times PV_0} \\ \% \Delta PV^{\text{Full}} &\approx - \text{EffDur} \times \Delta \text{ Curve} + \frac{1}{2} \times \text{EffCon} \times (\Delta \text{ Curve})^2 \end{aligned}$$

### Key Rate Duration

$$\begin{aligned} \text{KeyRateDur}_k &= -\frac{1}{PV} \times \frac{\Delta PV}{\Delta r_k} \\ \% \Delta PV &= - \text{KeyRateDur}_k \times \Delta r_k \\ \sum_{k=1}^n \text{KeyRateDur}_k &= \text{EffDur} \end{aligned}$$

where:

- $r_k$  =  $k$  th key rate

## Learning Module 14: Credit Risk

### Expected Loss

$$EL = LGD \times POD$$
$$LGD = EE \times (1 - RR)$$

where:

- $POD$  = Probability of default
- $LGD$  = Loss given default
- $EE$  = Expected exposure
- $RR$  = Recovery rate
- $1 - RR$  = Loss severity

Credit spread  $\approx$   $POD \times LGD$

### Decomposing Bond Yields

Yield spread = Bond YTM - Government bond YTM (Similar maturity)

Liquidity spread = Bond YTM ( Bid ) – Bond YTM ( Offer )

Credit spread = Yield spread - Liquidity spread

### Price Impact Given a Change in Yield Spread

$$\% \Delta PV^{\text{Full}} \approx - \text{AnnModDur} \times \Delta \text{Spread} + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta \text{Spread})^2$$

where:

- $\text{AnnModDur} \approx \frac{(PV_-) - (PV_+)}{2 \times (\Delta \text{Yield}) \times (PV_0)}$
- $\text{AnnConvexity} \approx \frac{(PV_-) + (PV_+) - 2(PV_0)}{(\Delta \text{Yield})^2 \times (PV_0)}$

## Learning Module 15: Credit Analysis for Government Issuers

No formula.

Learning Module 16: Credit Analysis for Corporate Issuers

$$\begin{aligned}\text{EBIT margin} &= \frac{\text{Operating income}}{\text{Revenue}} \\ \text{EBIT to interest expense} &= \frac{\text{Operating income}}{\text{Interest expense}} \\ \text{Debt to EBITDA} &= \frac{\text{Debt}}{\text{EBITDA}} \\ \text{RCF to net debt} &= \frac{\text{Retained cash flow}}{\text{Debt - Cash and marketable securities}} \\ \text{FFO to debt} &= \frac{\text{FFO}}{\text{Debt}}\end{aligned}$$

where:

- FFO = Net income from continuing operations + Depreciation & amortization
- Deferred income taxes + Other non-cash items

## Learning Module 17: Fixed-Income Securitization

No formula.

## Learning Module 18: Asset-Backed Security (ABS) Instrument and Market Features

No formula.

Learning Module 19: Mortgage-Backed Security (MBS) Instrument and Market Features

## Loan-to-value (LTV) ratio

$$LTV = \frac{\text{Loan amount}}{\text{House price}}$$

### Debt-to-income (DTI) ratio

$$DTI = \frac{\text{Monthly debt payment}}{\text{Monthly pre-tax gross income}}$$

### Mortgage Pass-Through Securities

$$WAC = \sum_{i=1}^N c_i \left( \frac{CB_i}{CB} \right)$$
$$WAM = \sum_{i=1}^N MM_i \left( \frac{CB_i}{CB} \right)$$

where:

- $WAC$  = Weighted average coupon
- $WAM$  = Weighted average maturity
- $c_i$  = Coupon rate on mortgage  $i$
- $MM_i$  = Number of months to maturity for mortgage  $i$
- $N$  = Number of mortgages in MBS
- $CB_i$  = Current balance on mortgage  $i$
- $CB$  = Total current balance of mortgages in MBS

### Commercial Mortgage-Backed Securities (CMBS)

#### Debt Service Coverage Ratio (DSCR)

$$DSCR = \frac{\text{Net operating income}}{\text{Debt service}}$$

#### Net Operating Income (NOI)

$NOI = (\text{Rental income} - \text{Cash operating expenses}) - \text{Replacement reserves}$

## VOLUME 7: DERIVATIVES

### Learning Module 1: Derivative Instrument and Derivatives Market Features

No formula.

## Learning Module 2: Forward Commitments and Contingent Claim Features and Instruments

### Forward Contract

$$\begin{aligned}\text{Buyer (Long) payoff} &= S_T - F_0(T) \\ \text{Seller (Short) payoff} &= -[S_T - F_0(T)]\end{aligned}$$

where:

- $S_T$  = Spot price on contract's maturity
- $F_0(T)$  = Forward price with maturity of  $T$

### Futures Contract

For one futures contract:

$$\begin{aligned}\text{Long Futures daily mark-to-market} &= f_t(T) - f_{t-1}(T) \\ \text{Short Futures daily mark-to-market} &= -[f_t(T) - f_{t-1}(T)]\end{aligned}$$

where:

- $f_t(T)$  = Closing price of futures contract on day  $t$
- $f_{t-1}(T)$  = Closing price of futures contract on day  $t - 1$   $T$  = Maturity of futures contract

If margin balance < maintenance margin:

$$\text{Variation Margin} = \text{Initial margin} - \text{Margin balance}$$

### Options Contract

#### LONG Call option

Payoff or Value at expiration,  $c_T = \max(0, S_T - X)$

Profit at expiration,  $\Pi = \max(0, S_T - X) - c_0$

where:

- $c_0$  = Call premium
- $X$  = Exercise/Strike price  $S_T$  = Spot price at expiration

### **SHORT Call option**

Payoff or Value at expiration,  $c_T = -\max(0, S_T - X)$

Profit at expiration,  $\Pi = -[\max(0, S_T - X) - c_0]$

### **LONG Put option**

Payoff or Value at expiration,  $p_T = \max(0, X - S_T)$

Profit at expiration,  $\Pi = \max(0, X - S_T) - p_0$

### **SHORT Put option**

Payoff or Value at expiration,  $p_T = -\max(0, X - S_T)$

Profit at expiration,  $\Pi = -[\max(0, X - S_T) - p_0]$

### **Credit Default Swap (CDS)**

CDS MTM Change =  $\Delta$  CDS Spread  $\times$  CDS Notional  $\times$  EffDur<sub>CDS</sub>

In a credit event, payment from CDS seller to CDS buyer  $\approx LGD(\%) \times$  Notional

## **Learning Module 3: Derivative Benefits, Risks, and Issuer and Investor Uses**

No formula.

## **Learning Module 4: Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives**

If there are no underlying costs or benefits:

$$\text{Forward price, } F_0(T) = S_0(1 + r)^T$$

If there are underlying costs or benefits in present value terms:

$$\text{Forward price, } F_0(T) = [S_0 - PV_0(\text{Income}) + PV_0(\text{Cost})](1 + r)^T$$

where:

- $S_0$  = Current spot price
- $r$  = Risk-free rate
- $T$  = Tenor of forward contract

Under continuous compounding,  $F_0(T) = S_0 e^{rT}$  Under continuous compounding, with income (i) and cost (c) expressed in %:

$$F_0(T) = S_0 e^{(r+c-i)T}$$

## Foreign Exchange Forward Contract

$$F_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

where:

- $F_{0,f/d}$  = Forward exchange rate
- $S_{0,f/d}$  = Spot exchange rate
- $r_f$  = Continuously compounded risk-free rate (for price/quote currency)
- $r_d$  = Continuously compounded risk-free rate (for base currency)
- $T$  = Maturity of forward contract

## Learning Module 5: Pricing and Valuation of Forward Contracts and for an Underlying with Varying Maturities

### Value of LONG Forward Prior to Expiration

$$\begin{aligned} V_0(T) &= 0 \\ V_t(T) &= S_t - \frac{F_0(T)}{(1+r)^{T-t}} = S_t - F_0(T) \times (1+r)^{-(T-t)} \\ V_T(T) &= S_0 - F_0(T) \end{aligned}$$

If the asset incurs cost or generates income from time  $t$  through maturity,

$$V_t(T) = [S_t - PV_t(\text{Income}) + PV_t(\text{Cost})] - F_0(T) \times (1+r)^{-(T-t)}$$

For foreign exchange forward contract,

$$V_t(T) = S_{t,f/d} - F_{0,f/d}(T) \times e^{-(r_f - r_d)(T-t)}$$

## Value of SHORT Forward Prior to Expiration

$$\begin{aligned}V_0(T) &= 0 \\V_t(T) &= - \left[ S_t - \frac{F_0(T)}{(1+r)^{T-t}} \right] \\V_T(T) &= - [S_0 - F_0(T)]\end{aligned}$$

## Interest Rate Forward Contracts (Forward Rate Agreements (FRA))

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)^B$$

where:

- $z_A$  = Spot rate for  $A$  periods
- $z_B$  = Spot rate for  $B$  periods
- $IFR_{A,B-A}$  = Implied forward rate for  $(B - A)$  periods, starting in  $A$  periods

Payoff for a Long FRA =  $(MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional principal} \times \text{Period}$

Payoff for a Short FRA =  $-(MRR_{B-A} - IFR_{A,B-A}) \times \text{Notional principal} \times \text{Period}$

## Learning Module 6: Pricing and Valuation of Futures Contracts

If there are no underlying costs or benefits:

$$\text{Futures price, } f_0(T) = S_0(1+r)^T$$

If there are underlying costs or benefits in present value terms:

$$f_0(T) = [S_0 - PV_0(\text{Income}) + PV_0(\text{Cost})] (1+r)^T$$

Under continuous compounding,  $f_0(T) = S_0 e^{rT}$

Under continuous compounding, with income (  $i$  ) and cost (  $c$  ) expressed in %:

$$f_0(T) = S_0 e^{(r+c-i)T}$$

## Foreign Exchange Forward Contract

$$f_{0,f/d}(T) = S_{0,f/d}(T) e^{(r_f - r_d)T}$$

## Interest Rate Futures Contract

$$f_{A,B-A} = 100 - (100 \times MRR_{A,B-A})$$

where:

- $f_{A,B-A}$  = Futures price for a market reference rate for (  $B - A$  ) periods that begins in  $A$  periods

Futures contract basis point value,  $BPV = \text{Notional principal} \times 0.01\% \times \text{Period}$

## Learning Module 7: Pricing and Valuation of Interest Rates and Other Swaps

For a fixed-rate payer in an interest rate swap:

$$\text{Periodic settlement value} = (MRR - s_N) \times \text{Swap Notional} \times \text{Period}$$

For a fixed-rate receiver in an interest rate swap:

$$\text{Periodic settlement value} = (s_N - MRR) \times \text{Swap Notional} \times \text{Period}$$

where:

- $s_N$  = Fixed swap rate
- $MRR$  = Market reference rate

## Calculating Par Swap Rate

$$\sum_{i=1}^N \frac{IFR}{(1 + z_i)^i} = \sum_{i=1}^N \frac{s_N}{(1 + z_i)^i}$$

where:

- $IFR$  = Implied forward rates
- $s_N$  = Fixed swap rate
- $N$  = Tenor of swap contract

## Valuation of Interest Rate Swap

Value of a pay-fixed interest rate swap on a settlement date after inception =  
Current settlement value +  $\Sigma(\text{Floating payments}) - \Sigma(\text{Fixed payments})$

Value of a receive-fixed interest rate swap on a settlement date after inception  
= Current settlement value +  $\Sigma(\text{Fixed payments}) - \Sigma(\text{Floating payments})$

## Learning Module 8: Pricing and Valuation of Options

Option value = Exercise value + Time value

At time  $t$  (prior to option expiration):

Call option exercise value =  $\text{Max}[0, S_t - X(1+r)^{-(T-t)}]$

Call option time value =  $c_t - \text{Max}[0, S_t - X(1+r)^{-(T-t)}]$

Put option exercise value =  $\text{Max}[0, X(1+r)^{-(T-t)} - S_t]$

Put option time value =  $p_t - \text{Max}[0, X(1+r)^{-(T-t)} - S_t]$

Lower bound of call option value =  $\text{Max}[0, S_t - X(1+r)^{-(T-t)}]$

Upper bound of call option value =  $S_t$

Lower bound of put option value =  $\text{Max}[0, X(1+r)^{-(T-t)} - S_t]$

Upper bound of put option value =  $X$

where:

- $S_t$  = Spot price at time  $t$
- $X$  = Exercise price (or strike price)
- $T$  = Maturity of option
- $r$  = Risk-free rate

## Learning Module 9: Option Replication Using Put-Call Parity

### Put-Call Parity

$$S_0 + p_0 = c_0 + X(1+r)^{-T}$$

## Put-Call Forward Parity

$$F_0(T)(1+r)^{-T} + p_0 = c_0 + X(1+r)^{-T}$$

## Value of the Firm

$$V_0 = c_0 + PV(\text{Debt}) - p_0$$

Value of debt =  $PV(\text{Debt}) - p_0$  Value of equity =  $c_0$

## Learning Module 10: Valuing a Derivative Using a One-Period Binomial Model

Risk-neutral probability of a price increase in underlying

$$\pi = \frac{1+r-R^d}{R^u-R^d}$$

where:

- $R^u = \text{Up factor} = \frac{S_1^u}{S_0} > 1$
- $R^d = \text{Down factor} = \frac{S_1^d}{S_0} < 1$
- $S_0 = \text{Current asset price}$
- $S_1^u = \text{One-period asset price when price moves up}$
- $S_1^d = \text{One-period asset price when price moves down}$

**Video:** <https://youtu.be/ymUIKgZ-rAw>

## Hedge ratio

$$h^* = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

where:

- $c_1^u = \max(0, S_1^u - X)$
- $c_1^d = \max(0, S_1^d - X)$

Riskless portfolio with a Call:  $h$  of the underlying,  $S$ , and short call position,  $c$   $V_0 = hS_0 - c_0$   
 $V_1^u = hS_1^u - c_1^u$   $V_1^d = hS_1^d - c_1^d$

**Riskless portfolio with a Put:**  $h$  of the underlying,  $S$ , and long put position,  $p$

$$V_0 = hS_0 + p_0 \quad V_1^u = hS_1^u + p_1^u \quad V_1^d = hS_1^d + p_1^d$$

Value of a one-period call option

$$c_0 = \frac{\pi c_1^u + (1 - \pi)c_1^d}{1 + r}$$

Value of a one-period put option

$$p_0 = \frac{\pi p_1^u + (1 - \pi)p_1^d}{1 + r}$$

where:

- $p_1^u = \max(0, X - S_1^u)$
- $p_1^d = \max(0, X - S_1^d)$  Video: <https://youtu.be/bXEC-78yAU>

Learning Module 1: Alternative Investment Features, Methods, and Structures

## GP Compensation Structure

Ignoring management fee; no catch-up clause

$$r_{GP} = \max[0, p(r - r_h)]$$

Ignoring management fee; with catch-up clause

$$r_{GP} = \max[0, r_{cu} + p(r - r_h - r_{cu})]$$

where:

- $r_{GP}$  = GP' s rate of return
- $p$  = Performance fee as a percentage of total return
- $r$  = Single-period rate of return
- $r_h$  = Hard hurdle rate
- $r_{cu}$  = Catch-up clause

## Learning Module 2: Alternative Investment Performance and Returns

### Multiple on Invested Capital

$$MOIC = \frac{\text{Realized value of investment} + \text{Unrealized value of investment}}{\text{Total amount of invested capital}}$$

### Leveraged Portfolio Return

$$r_L = r + \frac{V_b}{V_c} (r - r_b)$$

where:

- $r$  = Periodic rate of return on invested funds
- $r_b$  = Periodic cost of borrowing
- $V_b$  = Amount of borrowed funds
- $V_c$  = Amount of cash (investor's own capital)

### Investor's Return Net of Fees

$$r_i = \frac{P_1 - P_0 - R_{GP}}{P_0}$$

$$R_{GP} = (P_1 \times r_m) + \max[0, (P_1 - P_0) \times p]$$

where:

- $P_0$  = Beginning-of-period asset value
- $P_1$  = End-of-period asset value
- $p$  = GP performance fee
- $R_{GP}$  = GP's return in current terms
- $r_m$  = GP's management fees as a percentage of assets under management

### Calculating Hedge Fund Fees and Returns

Management Fee Based on Beginning Market Value

$$\text{Management Fee} = \frac{\text{Management}}{\text{Beginning Market}} \%$$

Management Fee Based on Ending Market Value

$$\text{Management Fee} = \% \text{ Management Fee} \times \text{Ending Market Value}$$

Incentive Fee Calculated Independent of Management Fee

$$\text{Incentive Fee} = \% \text{ Incentive Fee} \times \text{Gain}$$

Incentive Fee Calculated Net of Management Fee

$$\text{Incentive Fee} = \% \text{ Incentive Fee} \times ( \text{Gain} - \text{Management Fee} )$$

Incentive Fee with Hard Hurdle (Independent of Management Fee)

$$\text{Incentive Fee} = \% \text{ Incentive Fee} \times ( \text{Gain} - \text{Hurdle} )$$

Incentive Fee with Hard Hurdle (Net of Management Fee)

$$\text{Incentive Fee} = \% \text{ Incentive Fee} \times ( \text{Gain} - \text{Management Fee} - \text{Hurdle} )$$

$$\text{Hurdle} = \text{Hurdle Rate} \times \text{Beginning market value}$$

Note: 1) No incentive is paid if hedge fund incurs loss for the year. 2) Gain may be subject to high watermark.

<sup>1</sup> ## Learning Module 3: Investments in Private Capital: Equity and Debt

No formula.

## Learning Module 4: Real Estate and Infrastructure

### Loan-to-Value (LTV) Ratio

$$LTV = \frac{\text{Mortgage liability}}{\text{Portfolio value}}$$

Required reduction in mortgage liability = Mortgage liability - Required mortgage liability

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<sup>1</sup>Video: <https://youtu.be/ODKmcGsAAdc>

## **Learning Module 5: Natural Resources**

No formula.

## **Learning Module 6: Hedge Funds**

No formula.

## **Learning Module 7: Introduction to Digital Assets**

No formula.

## **VOLUME 9: PORTFOLIO MANAGEMENT**

### **Learning Module 1: Portfolio Risk and Return: Part I**

#### **Expected Return on Asset**

$$1 + E(R) = (1 + r_{rF}) \times [1 + E(\pi)] \times [1 + E(RP)]$$

where:

- $r_{rF}$  = Real risk-free rate
- $E(\pi)$  = Expected inflation
- $E(RP)$  = Expected risk premium for the asset

#### **Utility on Investment**

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:

- $U$  = Utility of investment
- $E(R)$  = Expected return of investment
- $A$  = Risk aversion coefficient
- $\sigma^2$  = Variance of investment (Note: Substitute  $\sigma$  in decimals)

## Capital Allocation Line (CAL)

For a portfolio of risky assets (Weight:  $w_i$ ) and risk-free asset:

$$E(R_p) = R_f + \left[ \frac{E(R_i) - R_f}{\sigma_i} \right] \sigma_p$$

where:

- $R_f$  = Rate of return on risk-free asset
- $E(R_i)$  = Expected return of risky asset
- $E(R_p)$  = Expected return of portfolio
- $\sigma_i$  = Standard deviation of risky asset's returns
- $\sigma_p$  = Standard deviation of portfolio's returns =  $w_i \times \sigma_i$
- $\frac{E(R_i) - R_f}{\sigma_i}$  = Market price of risk

## Two-asset portfolio

Portfolio expected return,  $E(R_p) = w_1 R_1 + w_2 R_2$

Portfolio variance,  $\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)$

Portfolio standard deviation,  $\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}(R_1, R_2)}$

Note: 1)  $\text{Cov}(R_1, R_2) = \rho_{12} \sigma_1 \sigma_2$  2)  $n$  securities requires  $n$  variances and  $\frac{n(n-1)}{2}$  covariances

Video: <https://youtu.be/IUwulZ9ONCO>

## Foreign Asset

Return of a foreign asset in domestic currency

$$R_D = (1 + R_{lc}) \times (1 + R_{FX}) - 1$$

Standard deviation of return of a foreign asset in domestic currency

$$\sigma_D = \sqrt{\sigma_{lc}^2 + \sigma_{FX}^2 + 2 \times \rho \times \sigma_{lc} \times \sigma_{FX}}$$

where:

- $R_{lc}$  = Return of foreign asset (in local currency)
- $R_{FX}$  = Change in exchange rate (FX rate quoted as domestic currency/foreign currency)

- $\sigma_{lc}$  = Standard deviation of foreign asset's returns
- $\sigma_{FX}$  = Standard deviation of the exchange rate (DC/FC)
- $\rho$  = Correlation coefficient between returns on foreign asset and exchange rate

## Portfolio of Many Risky Assets

$$\sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N} \overline{\text{Cov}} = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N} \rho \bar{\sigma}^2$$

where:

- $N$  = Number of assets in portfolio
- $\bar{\sigma}^2$  = Average variance
- $\overline{\text{Cov}}$  = Average covariance

## Learning Module 2: Portfolio Risk and Return: Part II

### Capital Market Line (CML)

$$E(R_p) = w_f R_f + (1 - w_f) E(R_m) = R_f + \left[ \frac{E(R_m) - R_f}{\sigma_m} \right] \sigma_p \sigma_p = (1 - w_f) \sigma_m$$

### Return-Generating Models

$$E(R_i) - R_f = \beta_{i1} [E(R_m) - R_f] + \sum_{j=2}^k \beta_{ij} E(F_j)$$

where:

- $E(R_i) - R_f$  = Expected excess return on asset  $i$
- $k$  = Number of factors  $\beta_{ij}$  = Factor weights (also called factor loadings)  $E(R_m) =$  Expected return on market

### The Single-Index Model

$$E(R_i) - R_f = \left( \frac{\sigma_i}{\sigma_m} \right) [E(R_m) - R_f]$$

where:

- $\frac{\sigma_i}{\sigma_m}$  = Factor loading (or factor weight)

## Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

## The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

## Beta of security $i$

$\beta_i = \frac{\text{Cov}(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m} \sigma_i}{\sigma_m}$  Portfolio beta,  $\beta_p = \sum_{i=1}^n w_i \beta_i$  Total variance = Systematic variance + Nonsystematic variance

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_e^2$$

Total risk,  $\sigma_i = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_e^2}$

## Arbitrage Pricing Theory (APT) Model

$$E(R_P) = R_F + \lambda_1 \beta_{P,1} + \cdots + \lambda_K \beta_{P,K}$$

where:

- $E(R_P)$  = Expected return on portfolio
- $R_F$  = Risk-free rate
- $\lambda_j$  = Risk premium for factor  $j$
- $\beta_{P,1}$  = Sensitivity of the portfolio to factor  $j$
- $K$  = Number of risk factors

## Fama-French Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t$$

## Carhart Model

$$E(R_{it}) = \alpha_i + \beta_{i,MKT}MKT_t + \beta_{i,SMB}SMB_t + \beta_{i,HML}HML_t + \beta_{i,UMD}UMD_t$$

where:

- $E(R_i)$  = Return on an asset in excess of the one-month T-bill return
- $MKT$  = Excess return on the market portfolio
- $SMB$  = Difference in returns between small-capitalization stocks and large-capitalization stocks (Size)
- $HML$  = Difference in returns between high-book-to-market stocks and low-book-to-market stocks (Value versus growth)
- $UMD$  = Difference in returns of the prior year's winners versus losers (Momentum)

## Portfolio Performance Appraisal Measures

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p} \quad M^2 = (R_p - R_f) \frac{\sigma_m}{\sigma_p} + R_f = \text{Sharpe ratio} \times \sigma_m + R_f \quad M^2 \text{ alpha} = M^2 - R_m$$

$$\text{Jensen's Alpha, } \alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]$$

Security Characteristic Line (SCL)

$$R_i - R_f = \alpha_i + \beta_i (R_m - R_f)$$

$$\text{Information ratio} = \frac{\alpha_i}{\sigma_{ei}}$$

## Learning Module 3: Portfolio Management: An Overview

No formula.

Learning Module 4: Basics of Portfolio Planning and Construction

No formula.

Learning Module 5: The Behavioral Biases of Individuals

No formula.

Learning Module 6: Introduction to Risk Management

No formula.