Digital Communications 4: Tutorial 3

1. Consider the (7,4) cyclic code defined by the generator polynomial $x^3 + x^2 + 1$, equivalent to Hamming (7,4). Construct a lookup table to identify the bit error from the non-zero syndrome. Hint: since 0000000 is a valid codeword you can calculate the syndrome for the seven error codewords 1000000, 0100000, . . .

2. A cyclic $n = 6$, $k = 4$ code has the generator polynomial $x^2 + x + 1$.

   (a) Construct a table of all possible 4-bit input data and the corresponding codewords for this cyclic code

   (b) What is the minimum Hamming distance for this code?

   (c) Hence, identify how many bit errors this code is able to detect

   (d) Can this code be used for forward error correction?

3. Find a generator polynomial and the corresponding code words for a (7,3) cyclic code. Hint: consider the factorisation of $x^7 + 1$ in your lecture notes. What is the minimum Hamming distance for this code?

4. Using the generator polynomial $x^4 + x + 1$, find the generator matrix and the parity check matrix (in systematic form) of a (15,11) cyclic code.

5. A convolutional encoder is shown in Fig. 1, where the two outputs shown are bit interleaved (S P S P. . .) into a single output stream.

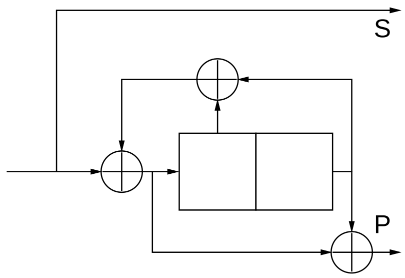

Figure 1: A convolutional encoder

   (a) What is the rate of this convolutional code?

   (b) Is this code recursive or non-recursive?

   (c) Is this code systematic or non-systematic?

   (d) Draw a state transition diagram for this convolutional encoder.

   (e) Using a Trellis diagram, or otherwise, find the convolutional code corresponding to the digital input 1001. You can assume the shift register is initially in the default all-zero state.

   (f) A received code, which may contain an error, is 11110001. Using the Viterbi algorithm and a Trellis diagram, determine the most likely binary data sequence that generated this code.

   (g) What is the rate 2/3 punctured code corresponding to part (e) using the puncturing matrix $\left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$