$$X = \begin{bmatrix} x_0 & x_1 & 0 & 0 \\ 0 & 0 & x_0 & x_1 \end{bmatrix}$$

$$\hat{X} = \begin{bmatrix} x_0 & x_1 & 0 & 0 \\ 0 & 0 & x_0 & x_1 \\ x_1 & -x_0 & 0 & 0 \\ 0 & 0 & x_1 & -x_0 \end{bmatrix}.$$

$\hat{X}$ is orthogonal when the signal x is constrained to have unit energy. Some of the properties of $\hat{X}$ are discussed in Bershad.[5] Then (2) becomes

$$y] = \{U_{nm}\}\hat{X}\, a] \tag{6}$$

and

$$\langle yy_t\rangle = \{U_{nm}\}\hat{X}\langle aa_t\rangle\hat{X}^{-1}\{U_{nm}\}^{-1} \tag{7}$$

since the $\hat{X}$ are deterministic and $\hat{X}_t = \hat{X}^{-1}$. For any channel $\langle a\, a_t\rangle$ is fixed *a priori* and can always be written as

$$\langle aa_t\rangle = Q\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{mn} \end{bmatrix}Q^{-1} \quad 0 \leq \lambda_i \leq \lambda_j, \quad i < j \tag{8}$$

since $\langle a\, a_t\rangle$ is a positive definite symmetric matrix. Hence (7) becomes

$$\langle yy_t\rangle = \{U_{nm}\}\hat{X}Q\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{mn} \end{bmatrix}Q^{-1}\hat{X}^{-1}\{U_{nm}\}^{-1}, \tag{9}$$

but the eigenvalues of R

$$R = \hat{X}Q\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_{mn} \end{bmatrix}Q^{-1}\hat{X}^{-1} \tag{10}$$

are $\lambda_1, \lambda_2, \cdots, \lambda_{mn}$ since (10) constitutes a similarity transformation. Finally, by use of a theorem[6] on truncated matrices, the eigenvalues of (10), λ_i, and of (9), λ_i', are related as follows:

$$\lambda_{m-j} \leq \lambda'_{m-j} \leq \lambda_{mn-j} \quad \text{for} \quad j = 0, 1, \cdots, m. \tag{11}$$

That is to say, the m eigenvalues of the truncated matrix are less than the corresponding m largest of R but greater than the corresponding m smallest. The usefulness of (11) depends upon the difference between λ_0 and λ_{mn} of $\langle a\, a_t\rangle$. Suppose the eigenvalues of $\langle a\, a_t\rangle$ are close together, then those of ϕ_{yy} must also be close together since $\lambda_{mn} - \lambda_0 = \epsilon$, $\epsilon > 0$ implies $\lambda'_m - \lambda'_0 \leq \epsilon$. However, since the eigenvalues of $\langle a\, a_t\rangle$ are independent of the signal x, this must also be the maximum difference (for fixed signal energy) between the largest and smallest eigenvalues of ϕ_{yy}. Hence if the channel is "good" in the sense that $\langle a\, a_t\rangle$ has almost equal eigenvalues, it makes little difference what signal of a given energy is transmitted. This is an interesting generalization of the results for the purely additive Gaussian channel. If, on the other hand, the eigenvalues of $\langle a\, a_t\rangle$ are widely separated, then signal shaping may allow control of the eigenvalues of ϕ_{yy} within the limits defined by (11).

To add a comment with respect to the assumed optimum distribution of eigenvalues for ϕ_{yy}, suppose that ϕ_{yy} has the optimum set of eigenvalues. Then, it is easily shown that ϕ_{yy} is a scalar multiple of the identity matrix.

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N. J. Bershad
Data Science Lab.
Air Force Cambridge
Research Labs.
Bedford, Mass.

[5] N. J. Bershad, "On the Optimum Design of Multipath Signals," AF Cambridge Research Labs., Bedford, Mass., Tech. Rept. (to be published).
[6] R. Courant and D. Hilbert, "Methods of Mathematical Physics," Interscience Publishers, New York, N. Y., vol. I, p. 33; 1953.

Table of Generators for Bose-Chaudhuri Codes

This communication presents a table of the polynomials which are the generators of all possible nontrivial Bose-Chaudhuri codes of length up to 255 bits. This group of codes was first presented by Bose and Chaudhuri,[1] and is a group of cyclic codes for which the packing density is predetermined. The shortest of these codes are good approximations to close-packed codes, and have the advantage of using a relatively small amount of equipment to encode and decode.

The polynomials are represented in octal form, with the coefficients represented by either one or zero as determined by the binary number represented by the octal number listed. For example, the third entry in the table lists the generator as 721, which in binary form is 111010001, and therefore the generator for a Bose-Chaudhuri code of length 15 bits, of which 7 are information symbols, is $x^8 + x^7 + x^6 + x^4 + 1$. Also listed in the table are n the number of bits in a code word; k the number of information bits in the code word; and t the maximum number of random errors which can be corrected using the code listed. Additional properties may be of interest if the code is used only for error detection, in which case any $2t$ errors will be detected, and any burst of errors of length $n - k$ or less will also be detected.

This table was derived in the manner described by W. W. Peterson,[2] and it was checked by insuring that the fundamental polynomial was a factor of all succeeding polynomials.

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[1] R. C. Bose and D. K. Ray-Chaudhuri, "On a class of error correcting binary group codes," *Information and Control*, vol. 3, pp. 68–79; September, 1960.
[2] W. W. Peterson, "Error Correcting Codes," M.I.T. Technology Press, Cambridge, Mass., and John Wiley and Sons, Inc., New York, N. Y.; 1961.

n	k	t	$g(x)$
7	4	1	13
15	11	1	23
	7	2	721
	5	3	2467
31	26	1	45
	21	2	3551
	16	3	107657
	11	5	5423325
	6	7	313365047
63	57	1	103
	51	2	12471
	45	3	1701317
	39	4	166623567
	36	5	1033500423
	30	6	157464165547
	24	7	17323260404441
	18	10	1363026512351725
	16	11	6331141367235453
	10	13	472622305527250155
	7	15	5231045543503271737
127	120	1	211
	113	2	41567
	106	3	11554743
	99	4	3447023271
	92	5	624730022327
	85	6	130704476322273
	78	7	26230002166130115
	71	9	6255010713253127753
	64	10	1206534025570773100045
	57	11	335265252505705053517721
	50	13	54446512523314012421501421
	43	14	17721772213651227521220574343
	36	15	3146074666522075044764574721735
	29	21	403114461367670603667530141176155
	22	23	123376070404722522435445626637647043
	15	27	22057042445604554770523013762217604353
	8	31	7047264052751030651476224271567733130217
255	247	1	435
	239	2	267543
	231	3	156720665
	223	4	75626641375
	215	5	23157564726421
	207	6	16176560567636227
	199	7	7633031270420722341
	191	8	2663470176115333714567
	187	9	52755313540001322236351
	179	10	22624710717340432416300455
255	171	11	15416214212342356077061630637
	163	12	7500415510075602551574724514601
	155	13	3757513005407665015722506464677633
	147	14	1642130173537165525304165305441011711
	139	15	461401732060175561570722730247453567445
	131	18	2157133314715101512612502774421420241 65471
	123	19	1206140522420660037172103265161412262 72506267
	115	21	6052666557210024726363640460027635255 6313472737
	107	22	2220577232206625631241730023534742017 6574750154441
	99	23	1065666725347317422274141620157433225 2411076432303431
	91	25	6750265030327444172723631724732511075 550762720724344561
	87	26	1101367634147432364352316343071720462 06722545273311721317
	79	27	6670003563765750002027034420736617462 1015326711766541342355
	71	29	2402471052064432151555417211233116320 5444250362557643221706035
	63	30	1075447505516354432531521735770700366 6111726455267613656702543301
	55	31	7315425203501100133015275306032054325 414326755010557044426035473617
	47	42	2533542017062646563033041377406233175 123334145446045005066024552543173
	45	43	1520205605523416113110134637642370156 3670024470762373033202157025051541
	37	45	5136330255067007414177447245437530420 735706174323432347644354737403044003
	29	47	3025715536673071465527064012361377115 34224232420117411406025475741040356 5037
	21	55	1256215257060332656001773153607612103 22734140565307454252115312161446651 3473725
	13	59	4641732005052564544426573714250066004 33067744547656140317467721357026134 460500547
	9	63	1572602521747246320103104325535513461 41623672120440745451127661155477055 61677516057

John P. Stenbit
Aerospace Corp.
Los Angeles, Calif.