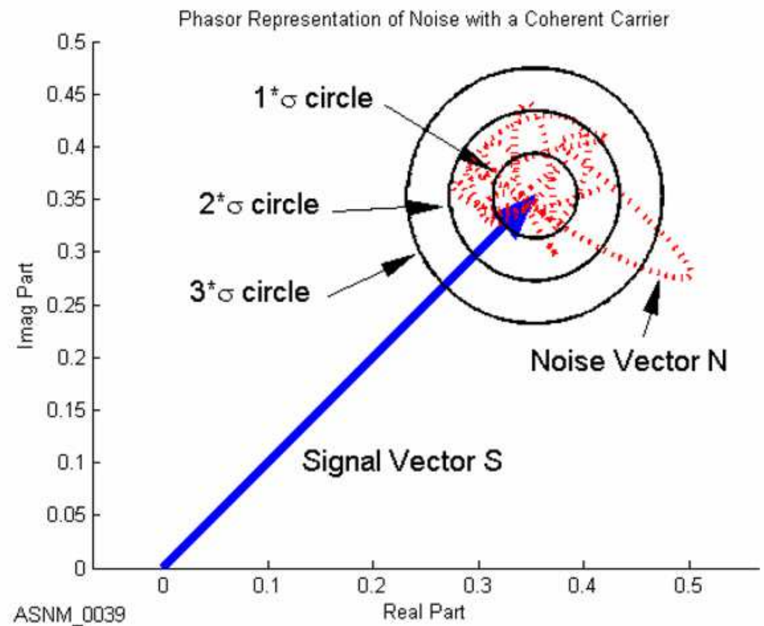
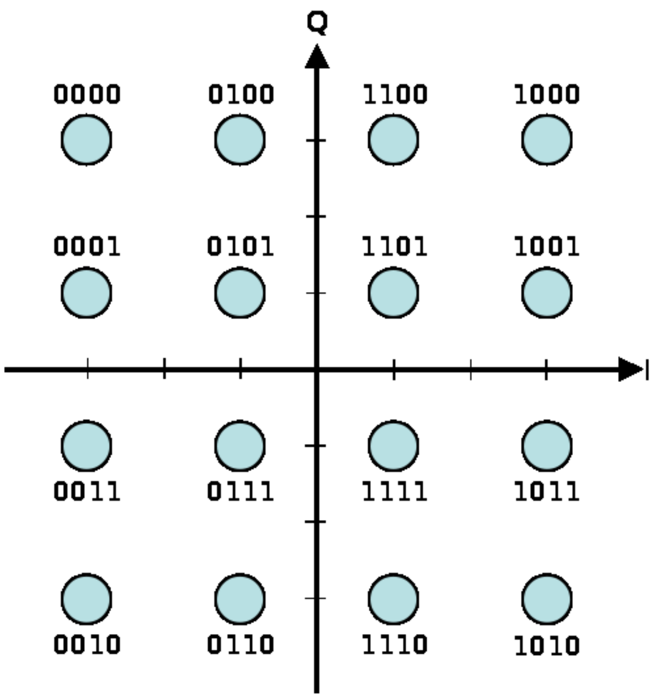
Additive White Gaussian Noise

Additive white Gaussian noise (AWGN) is a basic noise model used in Information theory to mimic the effect of many random processes that occur in nature.

- Additive because it is added to any intrinsic noise.
- White refers to a uniform power across the frequency band for the information system.
- Gaussian is a normal distribution in the time domain with an average of zero.



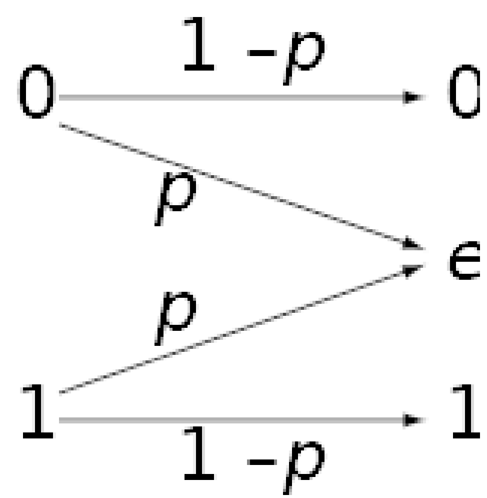
Quadrature Amplitude Modulation



Gray coded square 16-QAM

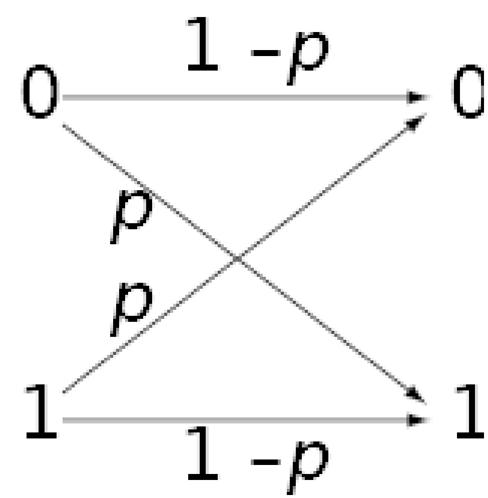
- most common square 16-QAM, 64-QAM and 256-QAM
- UK digital terrestrial television: 64-QAM (256-QAM for Freeview HD)
- extensively used in digital telecommunication systems

Binary Erasure Channel



- binary input, ternary output
- The erasure e with probability p represents complete loss of information about an input bit
- capacity $1-p$ bits/channel

Binary Symmetric Channel



- binary input, binary output
- flips the input bit with probability p
- capacity $1-H(p)$ bits/channel

Channel Capacity

Shannon showed that the capacity of an additive white Gaussian noise channel

$$C = B\log_2\left(1 + \frac{S}{N}\right)$$

- C channel capacity (bps)
- B bandwidth (Hz)
- $\frac{S}{N}$ signal-to-noise ratio, where S is the signal power and $N = N_0 B$
- where N_0 is the noise power spectrum (expressed as watts per hertz of bandwidth).

We shall not prove this rigorously but the following argument justifies the assertion.

- Consider a signal which is band limited to B Hz. Using the sampling theorem, i.e. the sampling rate must be at least twice the bandwidth, we have $R_{\text{samp}} = 2B$.

Channel Capacity

- In the presence of noise, the quantisation spacing in the presence of noise with mean power, N must be $\sigma_n = \sqrt{N}$ (square root to convert power to voltage).
- Hence the maximum number of discernible levels is

$$M = \frac{\sqrt{S+N}}{\sqrt{N}} = \sqrt{1 + \frac{S}{N}}$$

- Therefore the maximum entropy H is

$$H_{\text{max}} = \sum_{i=0}^{M-1} \frac{1}{M} \log_2 M = \log_2 M = \frac{1}{2} \log_2 \left(1 + \frac{S}{N}\right)$$

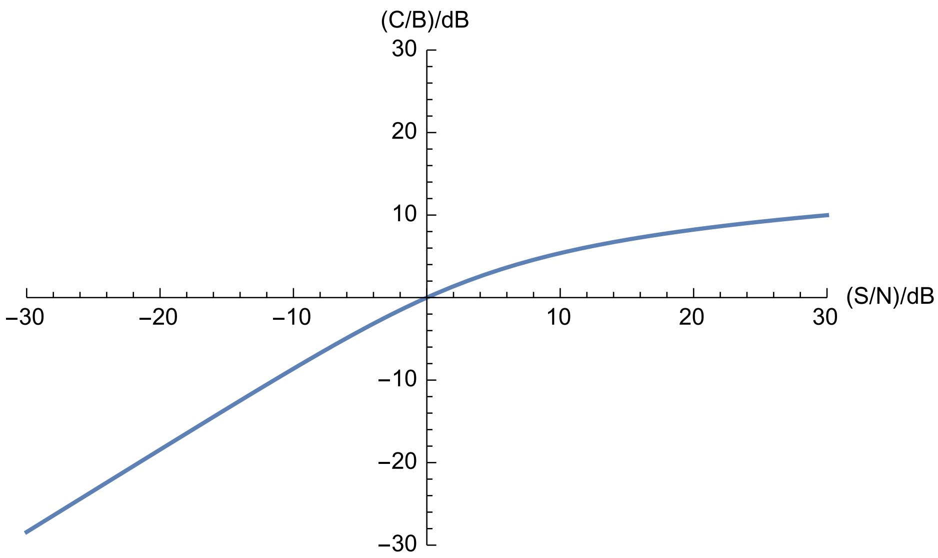
$$C_{\text{max}} = R_{\text{samp}} H_{\text{max}} = B \log_2 \left(1 + \frac{S}{N}\right)$$

Example

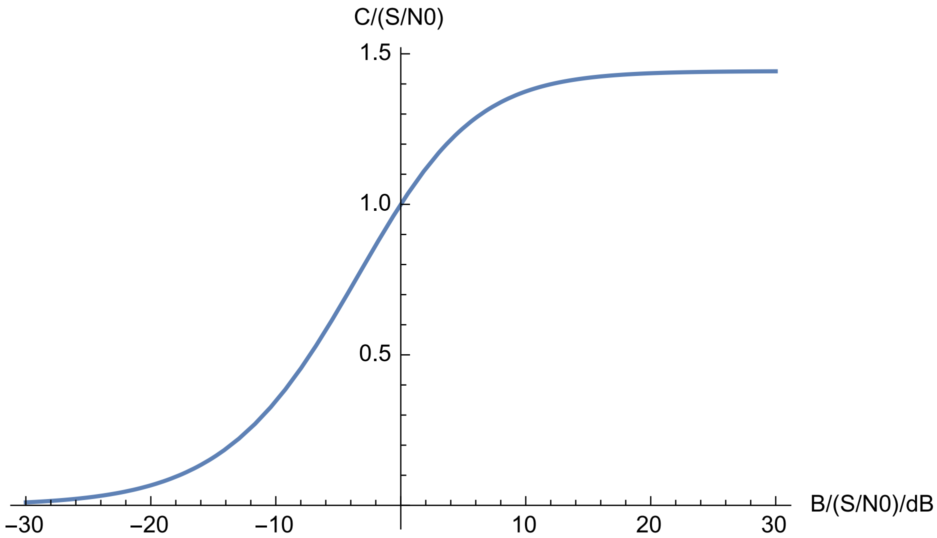
An SD terrestial digital TV requires a bit rate of $24\,\mathrm{Mbit\,s^{-1}}$. If the power to noise ratio is $30\,\mathrm{dB}$, what bandwidth is required?

Bounds on Channel Capacity

The bit rate R_b for any practical communications system must have $R_b < C$



Dependence on signal power S



Dependence on bandwidth B

Limit for large bandwidth

$$\lim_{B \to \infty} C = \frac{S}{N_0} \log_2 \mathrm{e} = 1.44 \frac{S}{N_0}$$

Types of Bit Errors

In, for example a Binary Symmetric Channel, if a bit 1 at the source is interpreted as a 0 or bit 0 interpreted as a 1 due to noise or distortion, this is called bit error.

single bit error e.g. 11010100 $\rightarrow$ 11**1**10100

multiple bit errors two or more single bit errors, not necessarily consecutive e.g. 11010100 $\rightarrow$ 1**0**010101

burst error a sequence of consecutive bit errors; the burst may contain zero error items in addition to the error bits.

e.g. 11010100 $\rightarrow$ 1101**101**0 or 11010100 $\rightarrow$ 1101**1111**

In a communication system, the receiver side bit errors can arise due to transmission channel noise, interference, distortion, bit synchronization problems, attenuation, wireless multipath fading, etc.

Bit Error Ratio (BER)

- The bit error ratio (BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unitless performance measure, often expressed as a percentage.
- For example, the bit string 0110001011 (sent) $\rightarrow$ 0**0**10**1**010**0**1 (received). The number of bit errors (bold) is, in this case, 3. The BER is 3 incorrect bits divided by 10 transferred bits, resulting in a BER of 0.3 or 30%.
- The BER may be evaluated using stochastic (Monte Carlo) computer simulations. If a simple transmission channel model and data source model is assumed, the BER may also be calculated analytically.

Bit Error Ratio (BER)

- DMS data source model
- Binary symmetric channel with non-bursty bit errors
- Additive white Gaussian Noise

For BPSK or QPSK, the BER as a function of energy per bit to noise power spectral density ratio is

$$\text{BER} = \frac{1}{2}\text{erfc}\sqrt{\frac{E_b}{N_0}}$$

where energy per bit is signal power divided by bit rate, $E_b = S/R_b$