Noise & Distortion in Communication Systems

Intrinsic noise Thermal noise, shot noise

Extrinsic noise Atmospheric noise, industrial noise, solar and cosmic noise

Crosstalk signal transmitted in one circuit or channel of a transmission systems creates undesired interference onto a signal in another channel.

Multipath Propagation creates delayed replicas of signal, which may cause amplitude and/or phase fluctuation

Bandwidth limited pulse has components outwith bit timing bin

Group velocity dispersion in dispersive media, pulse is distorted (normally spread) due to frequency dependent velocity

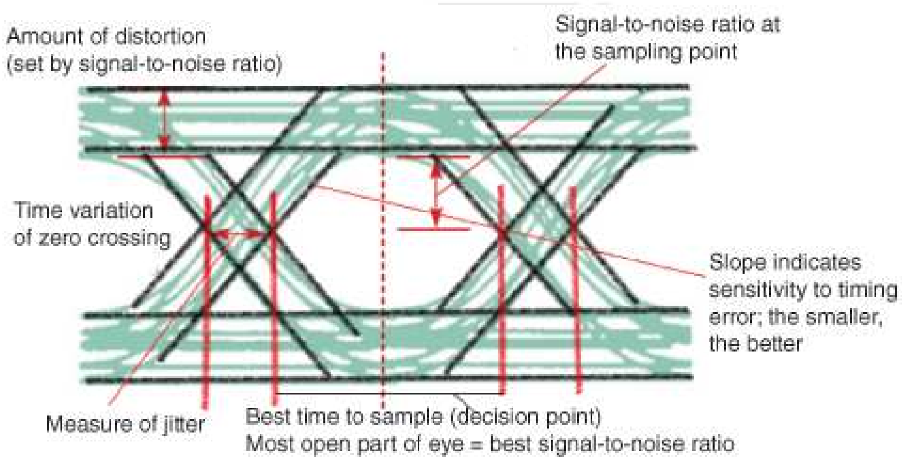
Jitter synchronisation offset of a signal in relation to a reference clock signal, or timing jitter with clock recovery

InterSymbol Interference a symbol interferes with prior or subsequent symbols, due to e.g. multipath propagation, timing jitter, limited bandwidth

Eye Diagram

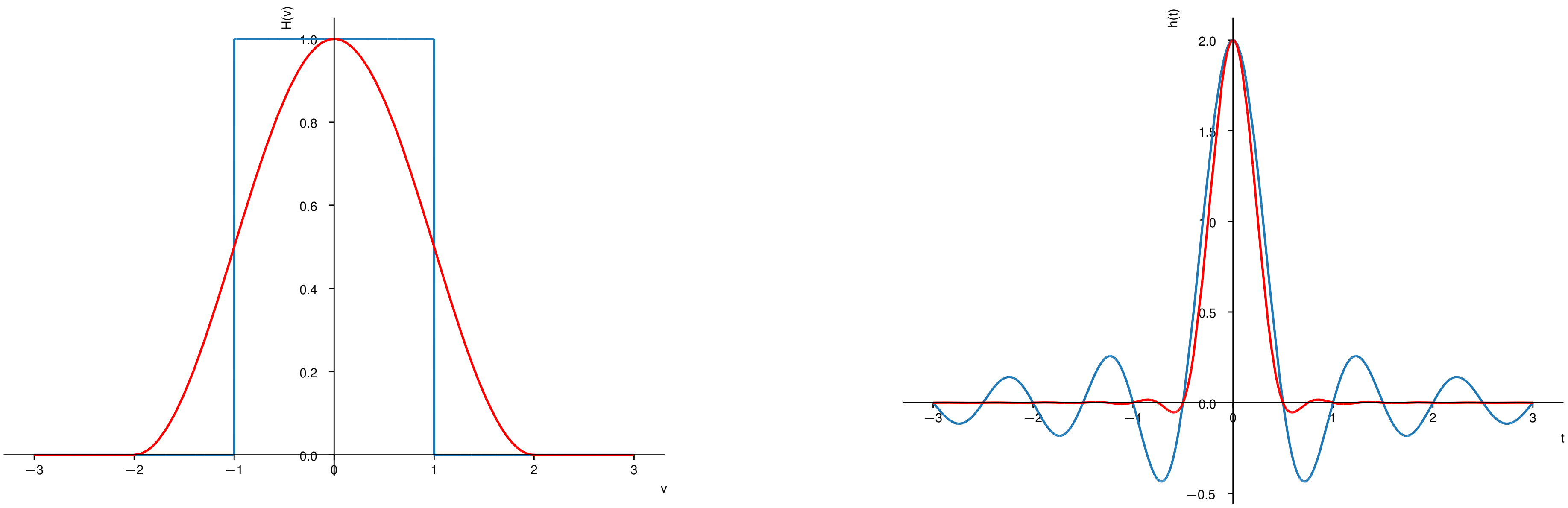
In telecommunication, an eye pattern, also known as an eye diagram, is an oscilloscope display in which a digital signal from a receiver is repetitively sampled and applied to the vertical input, while the system clock is used to trigger the horizontal sweep.

Eye opening (height, peak to peak) Additive noise in the signal
Eye overshoot/undershoot Peak distortion due to interruptions in the signal path
Eye width Timing synchronization & jitter effects
Eye closure Intersymbol interference, additive noise



Pulse shape in time

- sharp bit transitions require ∞ bandwidth
  - give rise to interference in other channels
  - impossible to realise with physical components/channels
- usual to require a limited bandwidth $B_{\text{DSB}} = 2B_{\text{DC}}$
- examples below are rect filter and raised cosine filter



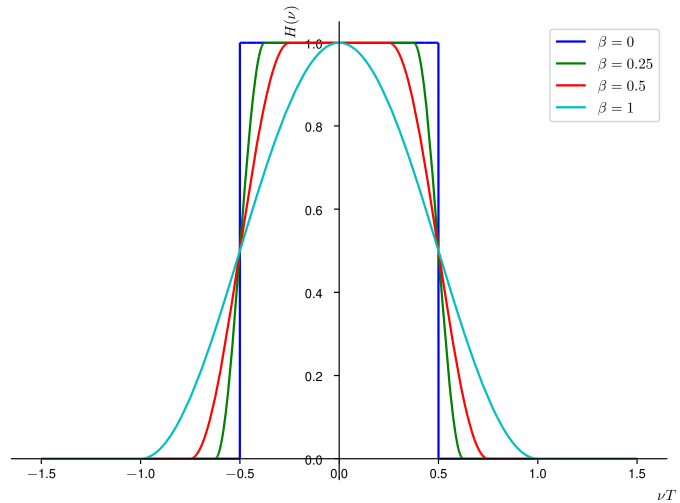
Rectangular bandwidth limit

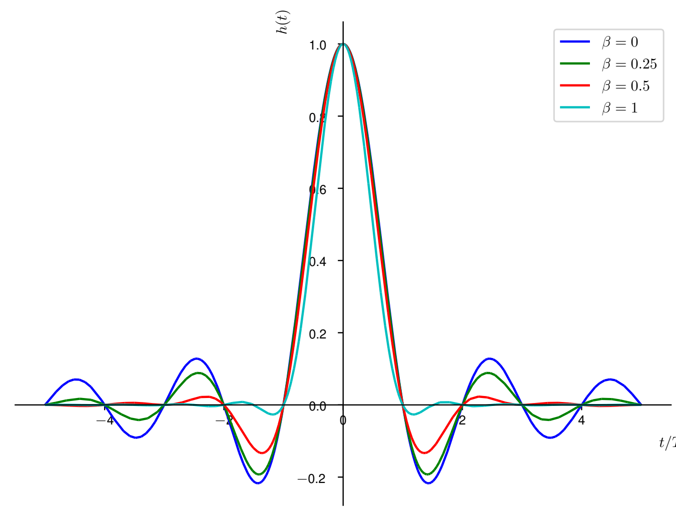
$$\mathcal{F}^{-1}\mathrm{rect}\frac{\nu}{2B_{\mathsf{DC}}} = 2B_{\mathsf{DC}}\mathrm{sinc}(2\pi B_{\mathsf{DC}}t)$$

- Note that the sinc function has zeros at non-zero integer number of π
- Arrange for the mid-bit sample points to coincide with zeros would avoid ISI
- This corresponds to the **Nyquist rate** $\frac{1}{T_{\mathsf{sym}}} = 2B_{\mathsf{DC}} = B_{\mathsf{DSB}}$ for the symbol-rate

- However, slow decay of sinc can still give rise to ISI with multipath interference or inaccurate timing.

Raised-Cosine Filter with Roll-Off





$$H(\nu) = \begin{cases} 1 & |\nu| \leq \frac{1-\beta}{2T_{\text{sym}}} \\ \frac{1}{2}\left[1 + \cos \frac{\pi T_{\text{sym}}}{\beta}\left(|\nu| - \frac{1-\beta}{2T_{\text{sym}}}\right)\right] & \frac{1-\beta}{2T_{\text{sym}}} < |\nu| < \frac{1+\beta}{2T_{\text{sym}}} \\ 0 & \text{otherwise} \end{cases}$$

Bandwidth is increased by factor: $B_{\text{DSB}} = \frac{1}{T_{\text{sym}}}(\beta + 1)$

Raised Cosine bandwidth limit

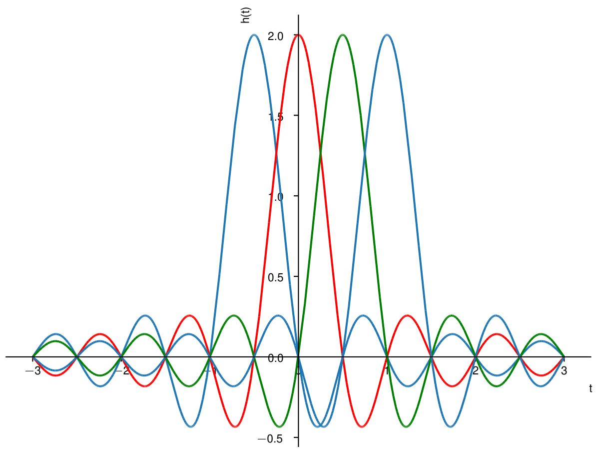
$$h(t) = \frac{1}{T_{\text{sym}}} \text{sinc}\left(\frac{\pi t}{T_{\text{sym}}}\right) \frac{\cos\left(\frac{\pi \beta t}{T_{\text{sym}}}\right)}{1 - \left(\frac{2\beta t}{T_{\text{sym}}}\right)^2} \qquad 0 \leq \beta \leq 1$$

- Again the time domain function has zeros at non-zero integer number of π
- Arrange for the mid-bit sample points to coincide with zeros would avoid ISI
- This corresponds to a modified rate $\frac{1}{T_{\text{sym}}} = \frac{B_{\text{DSB}}}{1+\beta}$ for the symbol-rate
- Faster decay relatively reduces ISI with multipath interference or inaccurate timing for raised cosine filter compared to rect.

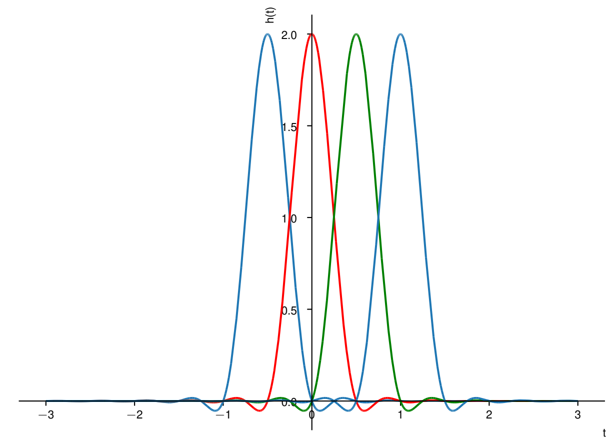
Bandwidth limited digital modulation

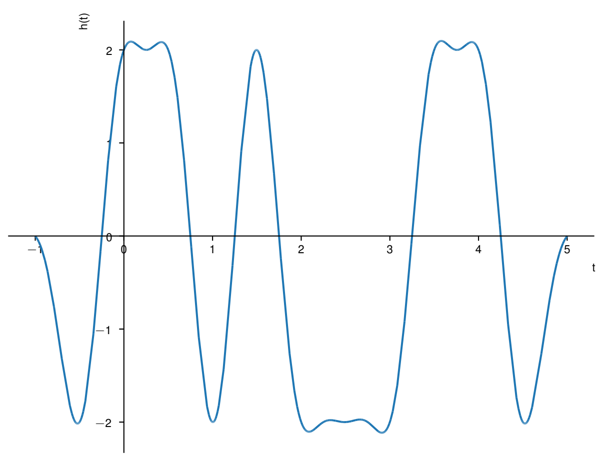
Bit maxima align with zeros of other bits for rate $\frac{1}{T_{\text{sym}}} = \frac{B_{\text{DSB}}}{1+\beta}$

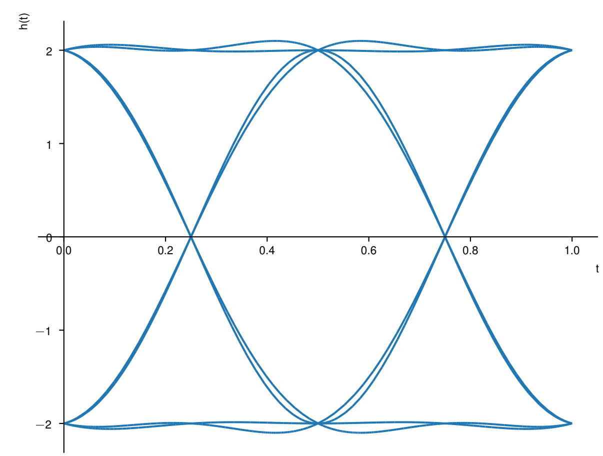
$\beta = 0$



$\beta = 1$

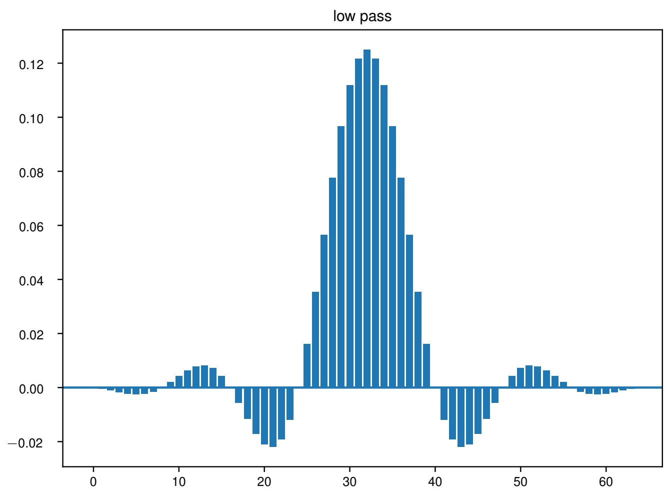


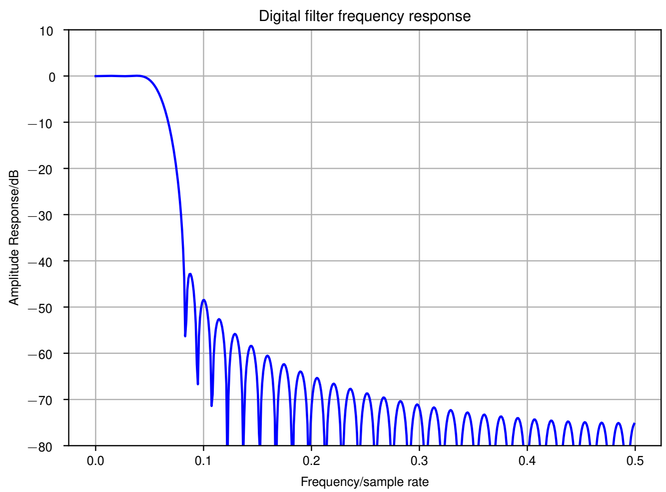




Raised-Cosine Digital Filter with Roll-Off

β=0.33





For UMTS (3G) mobile communications we use a roll-off β=0.22

Matched Filter

One trick we use in wireless communications is called matched filtering. To understand matched filtering you must first understand these points:

- The pulses we discussed above only have to be aligned perfectly at the receiver prior to sampling.
- We want a filter in our transmitter to reduce the amount of spectrum our signal uses.
- But the receiver also needs a filter to eliminate as much noise/interference next to the signal as possible.
- As a result, we have a filter at the transmitter (Tx) and another at the receiver (Rx), then sampling occurs after both filters.

What we do in modern communications is split the pulse shaping filter equally between the Tx and Rx. They don't have to be identical filters, but, theoretically, the optimal linear filter for maximizing the SNR in the presence of AWGN is to use the same filter at both the Tx and Rx. This is called the **matched filter** concept.

Root Raised-Cosine Filter

Filtering in the frequency domain,

$$G(\nu) = H(\nu)F(\nu) = \sqrt{H(\nu)}\sqrt{H(\nu)}F(\nu)$$

- Root raised-cosine (RRC) filter is implemented in both our Tx and Rx.
- Combined they form a normal raised-cosine filter.
- Because splitting a filter in half involves a frequency-domain square root, the impulse response gets a bit complicated.
- Luckily it's a widely used filter and there are plenty of implementations, e.g. komm package for python.

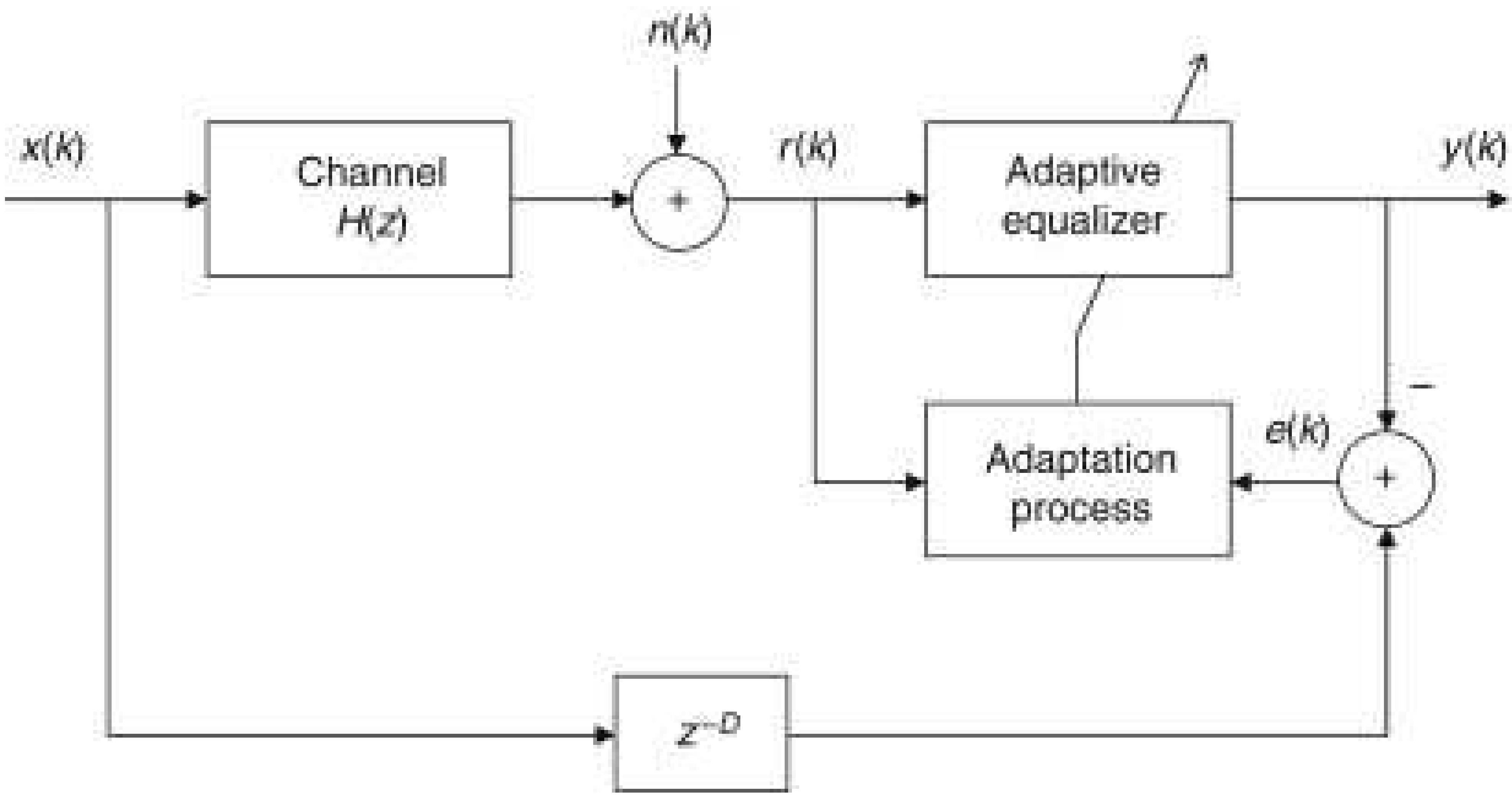
Equalisation

- In telecommunication, equalisation is the reversal of distortion incurred by a signal transmitted through a channel
- Equalisers are used to render the frequency response flat from end-to-end
- Equalising filters must cancel out any group delay and phase delay between different frequency components
- In digital communications, the equaliser's purpose is to reduce intersymbol interference to allow recovery of the transmit symbols
- Linear equalizer: processes the incoming signal with a linear filter

Zero forcing equalizer approximates the inverse of the channel with a linear filter.

MMSE equalizer designs the filter to minimize $E(|e|^2)$, where e is the error signal, which is the filter output minus the transmitted signal

Adaptive Equaliser



Adaptive Equaliser

- An adaptive equalizer automatically adapts to time-varying properties of the communication channel
- Frequently used with coherent modulations such as phase shift keying, mitigating the effects of multipath propagation and Doppler spreading
- Basic idea: transmit known training symbols and optimise digital filter weights to minimise “error” due to interference

Least mean squares algorithms are a class of adaptive filter used to mimic a desired filter by finding the filter coefficients that relate to producing the least mean square of the error signal

Stochastic gradient descent is an iterative method for optimizing an objective function with suitable smoothness properties

Recursive least squares algorithm that recursively finds the coefficients that minimize a weighted linear least squares cost function relating to the input signals. For RLS, the input signals are considered deterministic, while for the LMS and similar algorithm they are considered stochastic.

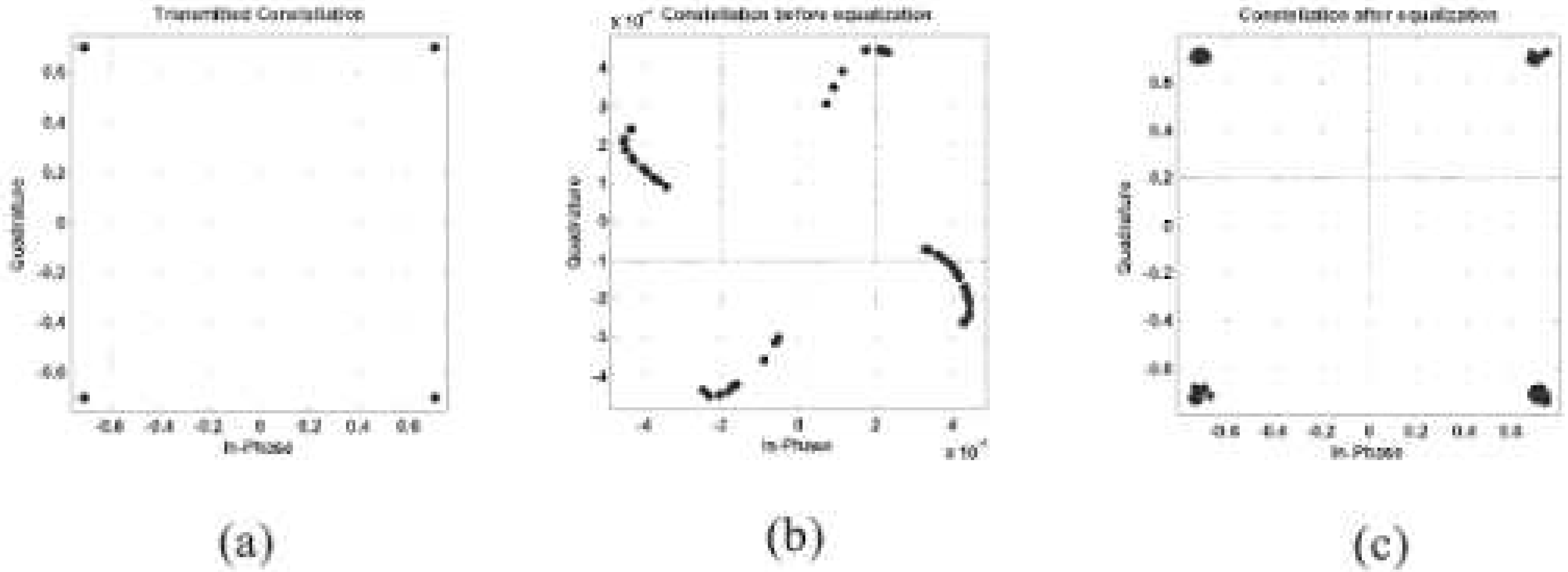


Figure 13 Constellations during channel equalization process, (a) transmitted constellation, (b) constellation before equalization, (c) constellation after equalization