Digital Communications 4: Tutorial on Block Codes

1. A microwave digital signal is received from a satellite by a ground station. The noise spectral density at the receiver is $3 \times 10^{-20}\,\mathrm{W\,Hz^{-1}}$, the bandwidth is 32 MHz and the required bit rate is $64\,\mathrm{kbit\,s^{-1}}$. What is the minimum received energy per bit in order to achieve error free transmission?

2. Write down a generator matrix and a corresponding parity check matrix for the Hamming (7,4) code in systematic form, where the parity bits p_1, p_2 and p_3 obey:

$$p_1 \oplus d_1 \oplus d_2 \oplus d_4 = 0$$
$$p_2 \oplus d_1 \oplus d_3 \oplus d_4 = 0$$
$$p_3 \oplus d_2 \oplus d_3 \oplus d_4 = 0$$

   (a) Obtain the 16 valid code words.

   (b) Identify a mapping between single bit errors and the syndrome vector by considering the syndrome for the 7 codes $\mathbf{e}_i$ consisting of a 1 in the ith position and 0 elsewhere.

   (c) Verify your result with a few examples by taking a valid codeword and introducing a single bit error.

   (d) If the single bit error ratio is p, and you can assume $p \ll 1$, then what is the probability of an error in a transmission of the original 4 bit code word?

   (e) What is the probability of bit errors resulting in a valid but incorrect received 7 bit code word?

   (f) What is the probability that there is an error in the decoded received 7 bit code word?

3. Consider an (8,4) block code by adding an additional overall parity bit p_4 to a set of Hamming (7,4) valid code words, e.g. taken from the previous question and in addition:

$$p_1 \oplus p_2 \oplus p_3 \oplus p_4 \oplus d_1 \oplus d_2 \oplus d_3 \oplus d_4 = 0$$

   (a) What is the minimum Hamming distance for this code?

   (b) Therefore identify the number of errors which can be detected for this code.

   (c) Therefore identify the number of errors which can be corrected for this code.

   (d) Write down the generator and the parity check matrices for this code.

   (e) Verify the application of this code with a few examples by taking a valid 8 bit codeword and introducing single bit errors and two bit errors.