

# Demodulation of DSB-SC

- Recovery of the original signal (demodulation) could be accomplished by another frequency shift, i.e. modulating with  $\cos \omega_c t$  again.

$$\phi(t) \cos \omega_c t = f(t) \cos^2 \omega_c t = \frac{1}{2} f(t) [1 + \cos 2\omega_c t]$$

- Taking the Fourier transform,

$$\mathcal{F}\{\phi(t) \cos \omega_c t\} = \frac{1}{2} F(\omega) + \frac{1}{4} F(\omega + 2\omega_c) + \frac{1}{4} F(\omega - 2\omega_c)$$

- We can reject the signals around  $\pm 2\nu_c$  with a low-pass filter and obtain output  $e_0(t) = \frac{1}{2} f(t)$ .
- However, we'll see with demodulation of DSB-SC that phase and frequency must exactly match.



- Introduce an error in the phase ( $\Delta\theta$ ) into the locally generated carrier frequency.

$$\begin{aligned}\phi(t) \cos(\omega_c t + \Delta\theta) &= f(t) \cos \omega_c t \cos(\omega_c t + \Delta\theta) \\ &= \frac{1}{2} f(t) \cos \Delta\theta + \frac{1}{2} f(t) \cos(2\omega_c t + \Delta\theta)\end{aligned}$$

- After a low-pass filter get output

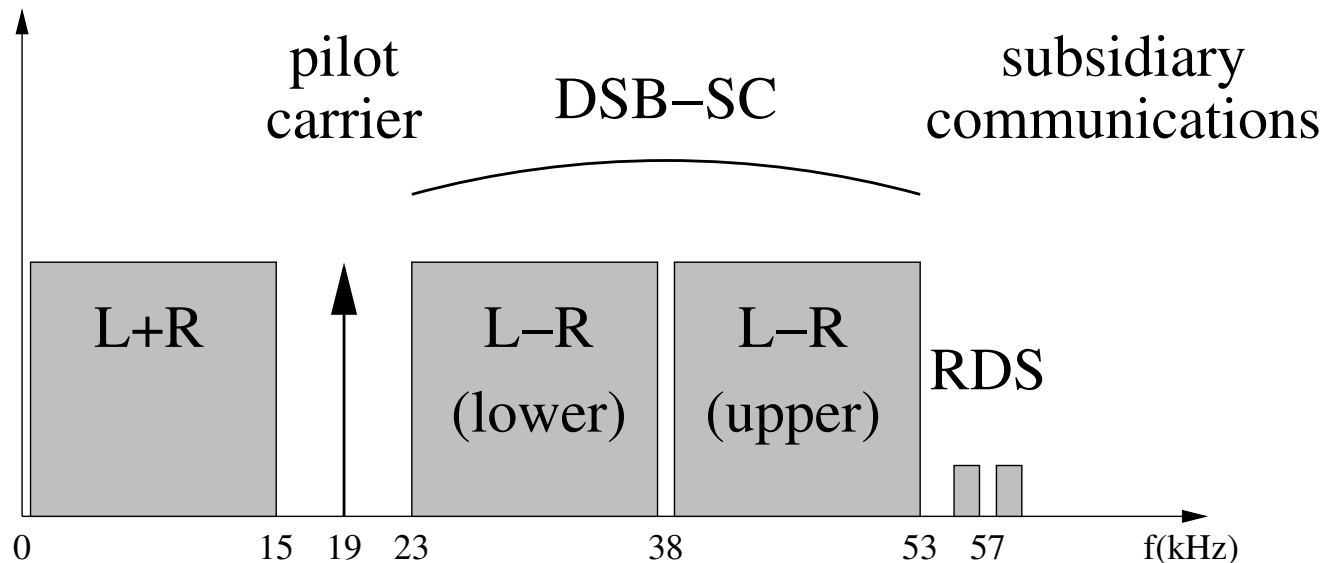
$$e_0(t) = \frac{1}{2} f(t) \cos \Delta\theta$$

- A phase error causes a variable output (drops to zero if  $\Delta\theta = \pm 90^\circ$ ).
- A frequency mis-match  $\Delta\theta = \Delta\omega t$  results in a low-frequency oscillation.
- Therefore with DSB-SC it is necessary to use a synchronised oscillator — termed *synchronous* detection or *coherent* detection.

# Include Carrier in Transmission

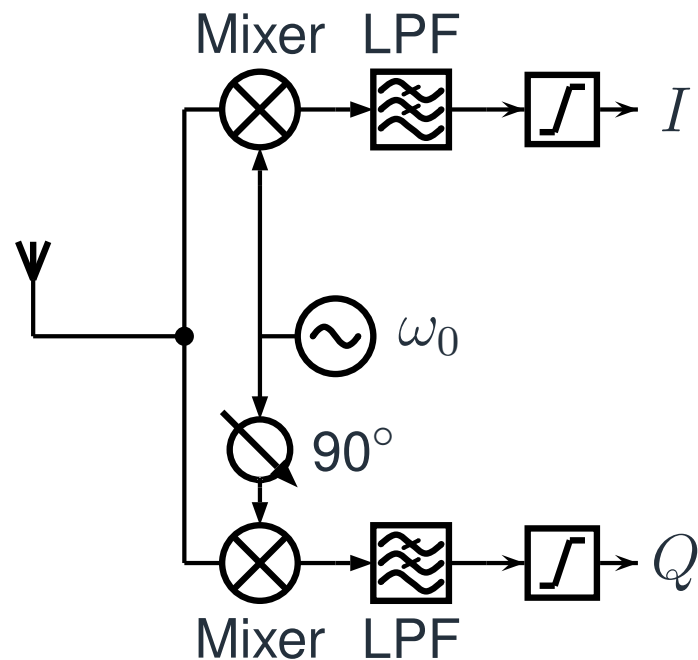
**DSB-LC** Double sideband large carrier allows asynchronous demodulation, e.g. envelope detector, but sidebands are limited to  $<33\%$  power, therefore inefficient

**pilot carrier** Include a pilot carrier. For example, in stereo FM there is a 19 kHz signal included (limited to 10% of peak frequency deviation) which allows generation of 38 kHz carrier for DSB-SC demodulation of  $(L - R)$  component, and 57 kHz for Radio Data System (RDS) demodulation.



# Carrier Recovery

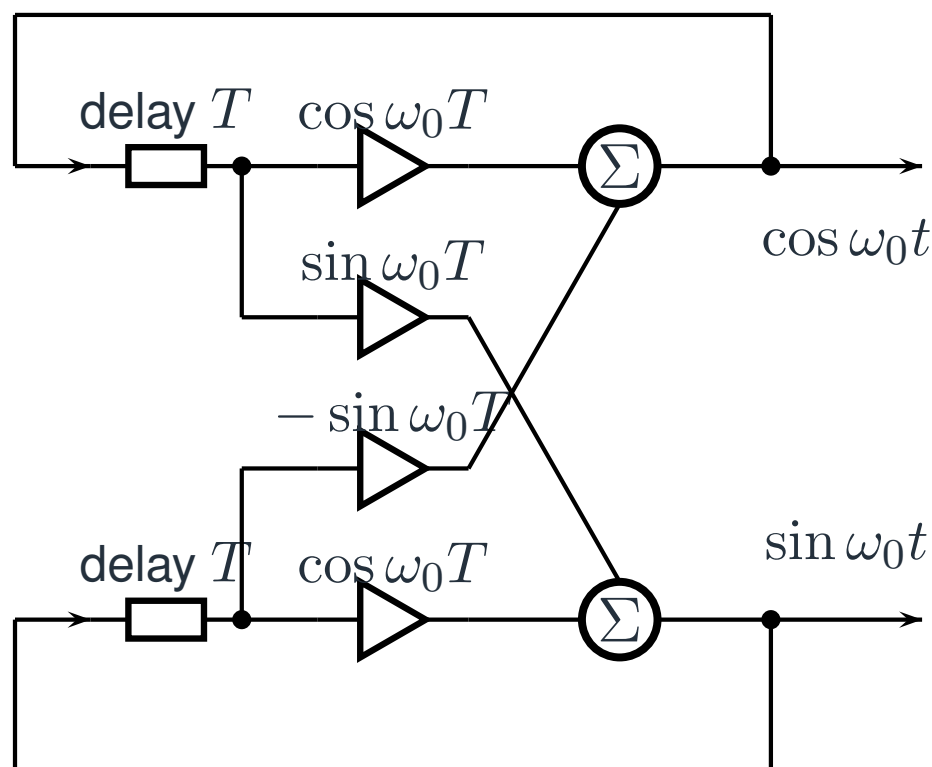
- How do we regenerate a phase-locked carrier at the receiver where there is no explicit carrier in the data stream?
- Need a controllable oscillator at the receiver, such as a voltage controlled oscillator (VCO), e.g.  $\omega_0 = \omega_i + \alpha v$ .
- Need a means to phase lock the oscillator to the input waveform.



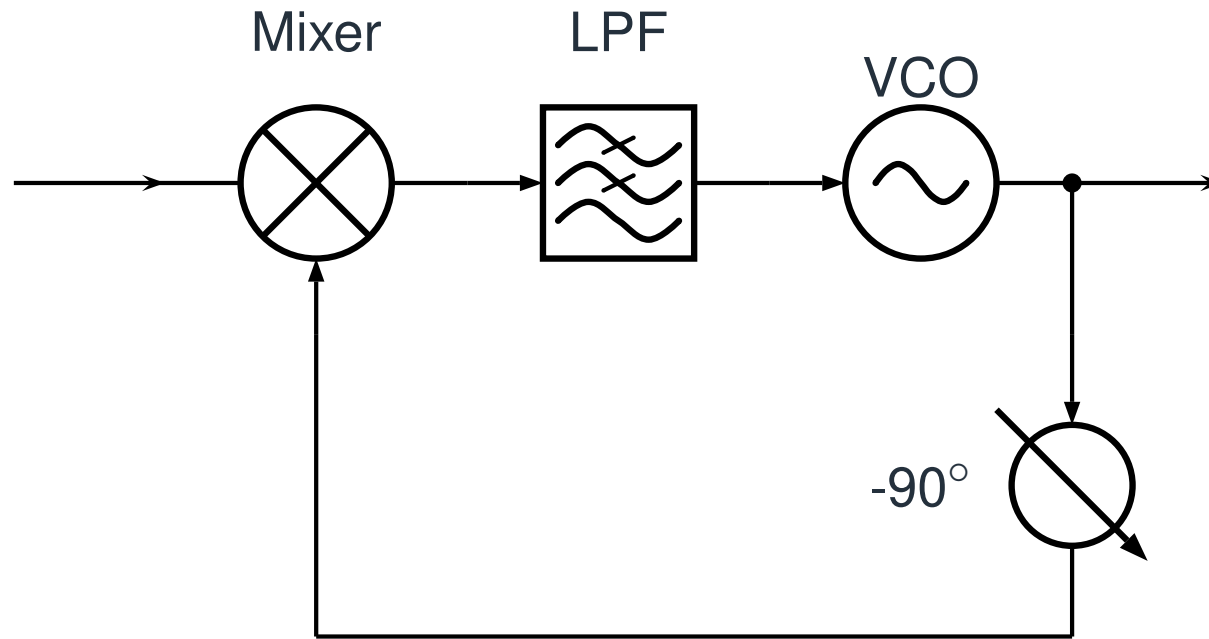
# Digital Oscillator: CORDIC Algorithm

$$\cos \omega_0(t + T) = \cos \omega_0 T \cos \omega_0 t - \sin \omega_0 T \sin \omega_0 t$$

$$\sin \omega_0(t + T) = \cos \omega_0 T \sin \omega_0 t + \sin \omega_0 T \cos \omega_0 t$$



# Phase-Locked Loop



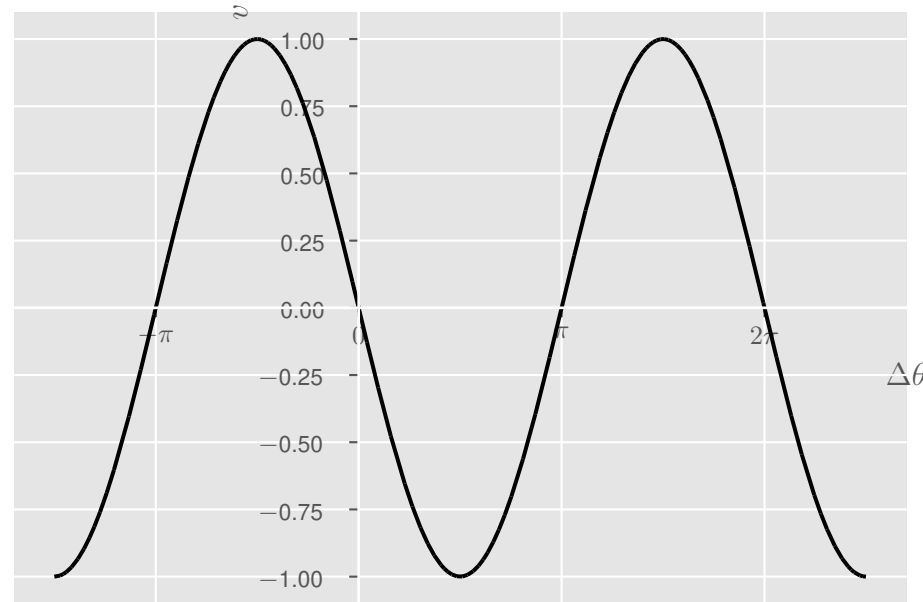
**input:**  $\cos(\omega_{\text{ref}}t + \theta_{\text{ref}})$

**mixed with:**  $-\sin(\omega_0t + \theta_0)$

**after LPF:**  $-\frac{1}{2} \sin(\Delta\omega t + \Delta\theta)$

**stationary solution:**  $\Delta\omega = 0$  and hence  $\sin \Delta\theta = \frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$ .

# Phase-Locked Loop

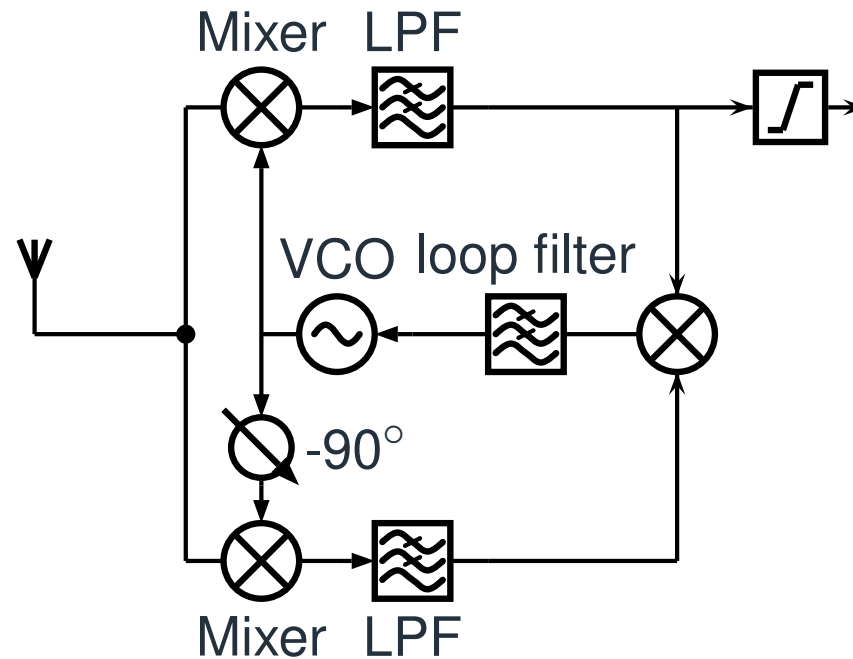


- phase is integral of frequency  $\theta_0(t) = \int_0^t \omega_0(\tau) d\tau + \theta_0(t = 0)$
- $\Delta\theta = \sin^{-1} \frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$  is a **stable** stationary point (negative feedback)
- $\Delta\theta = \pi - \sin^{-1} \frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$  is an **unstable** stationary point (positive feedback)
- PLL locks in frequency and phase to a sinusoidal reference

- but phase-modulated input would continually be trying to lock to instantaneous phase (suppressed carrier)
- we could initialise with an unmodulated carrier first, and then switch off lock (analogue PAL for colour info)
- **Costas Loops** allow the symbol phase info to be eliminated.
- In classical Costas loop use both in-phase and quadrature VCO outputs and combine LPF outputs to drive VCO.



# Costas Loop



**input:**  $s(t) \cos(\omega_{\text{ref}} t + \theta_{\text{ref}})$

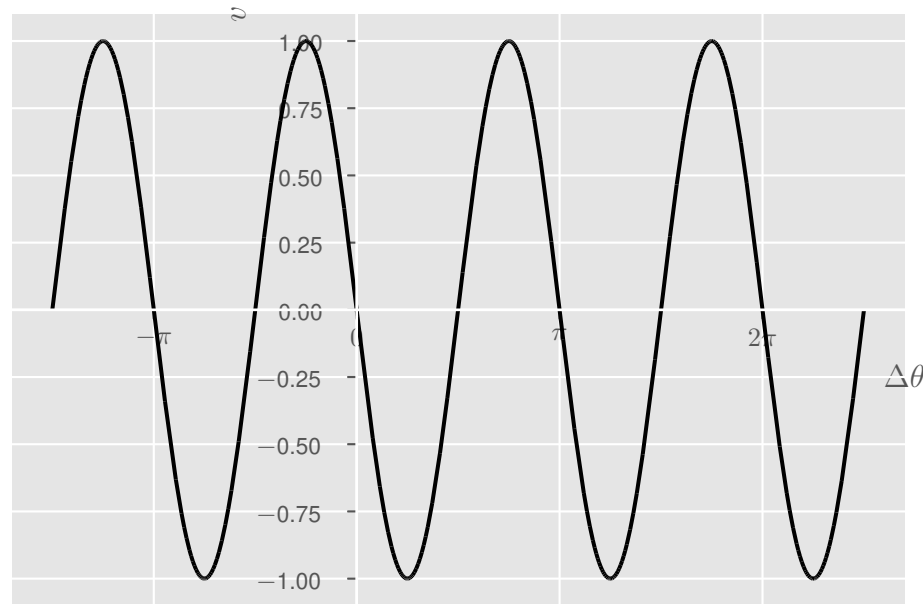
**after LPF1:**  $\frac{1}{2} s(t) \cos(\Delta\omega t + \Delta\theta)$

**after LPF2:**  $-\frac{1}{2} s(t) \sin(\Delta\omega t + \Delta\theta)$

**VCO input:**  $-\frac{1}{8} s^2(t) \sin 2(\Delta\omega t + \Delta\theta)$  with  $s(t) = \pm 1$

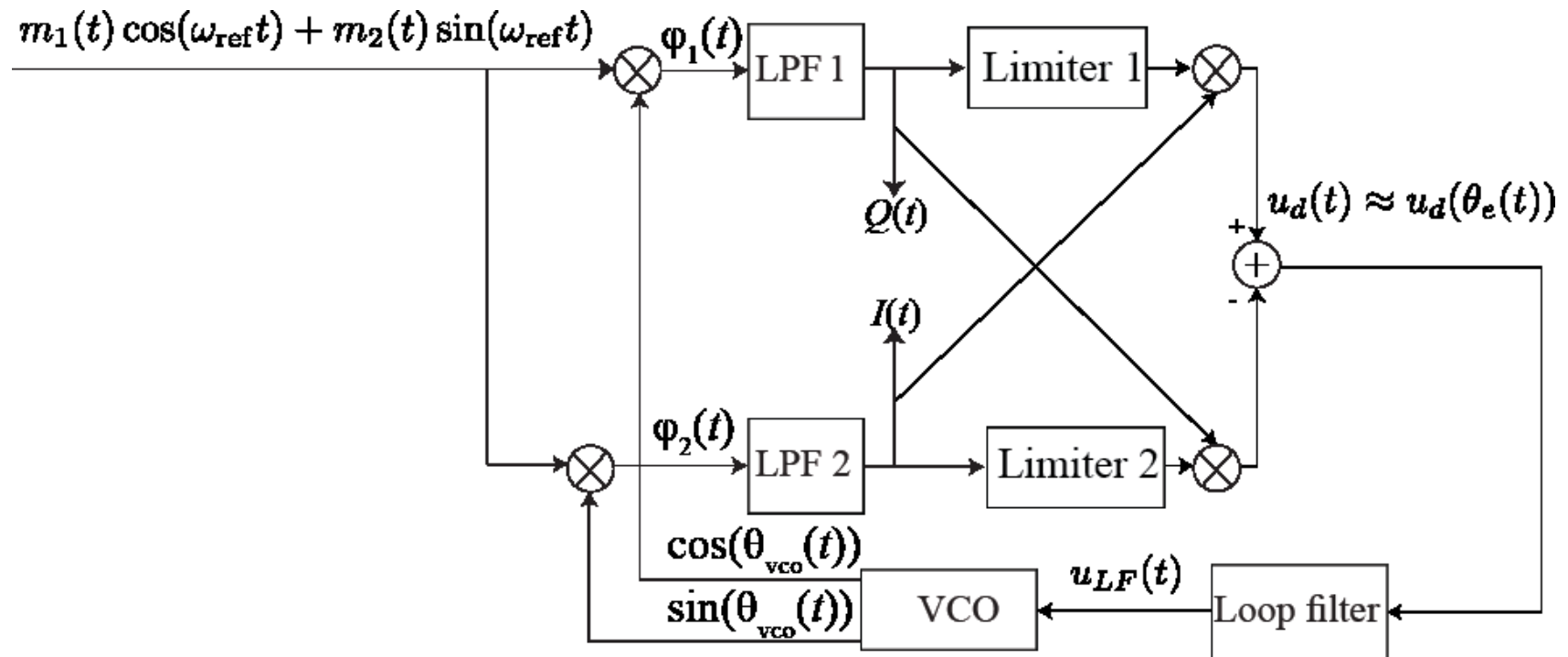
**stationary solution:**  $\Delta\omega = 0$  and hence  $\sin 2\Delta\theta = \frac{8(\omega_i - \omega_{\text{ref}})}{\alpha}$ .

# Costas Loop



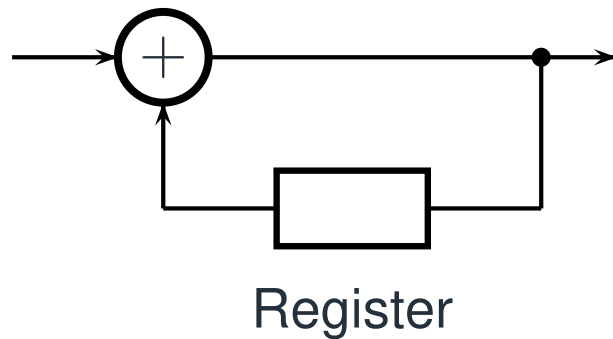
- phase is integral of frequency  $\theta_0(t) = \int_0^t \omega_0(\tau) d\tau + \theta_0(t = 0)$
- $\Delta\theta = \frac{1}{2} \sin^{-1} \frac{8(\omega_i - \omega_{\text{ref}})}{\alpha}$  is a **stable** stationary point (negative feedback)
- $\Delta\theta = \pi + \frac{1}{2} \sin^{-1} \frac{8(\omega_i - \omega_{\text{ref}})}{\alpha}$  is also a **stable** stationary point
- Requires a check to see if BPSK binary data is inverted or not.

# QPSK Costas Loop



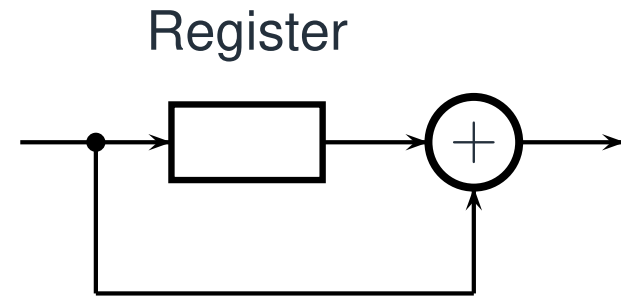
# Differential Encoding

Differential encoding prevents inversion of the signal (and symbols) from affecting the data. Note: adds an additional symbol requiring initialisation.



Differential Encoder

$$y_i = y_{i-1} \oplus x_i$$



Differential Decoder

$$x_i = y_i \oplus y_{i-1}$$

$\oplus$  represents component-wise modulo 2 addition (XOR) for binary symbols.

There are also line codes designed to be polarity insensitive, such as Differential Manchester encoding.