Binary Convolutional Codes

- A binary convolutional code is denoted by a three-tuple (n, k, L).
- n output bits are generated whenever k input bits are received. The rate of such a code is given

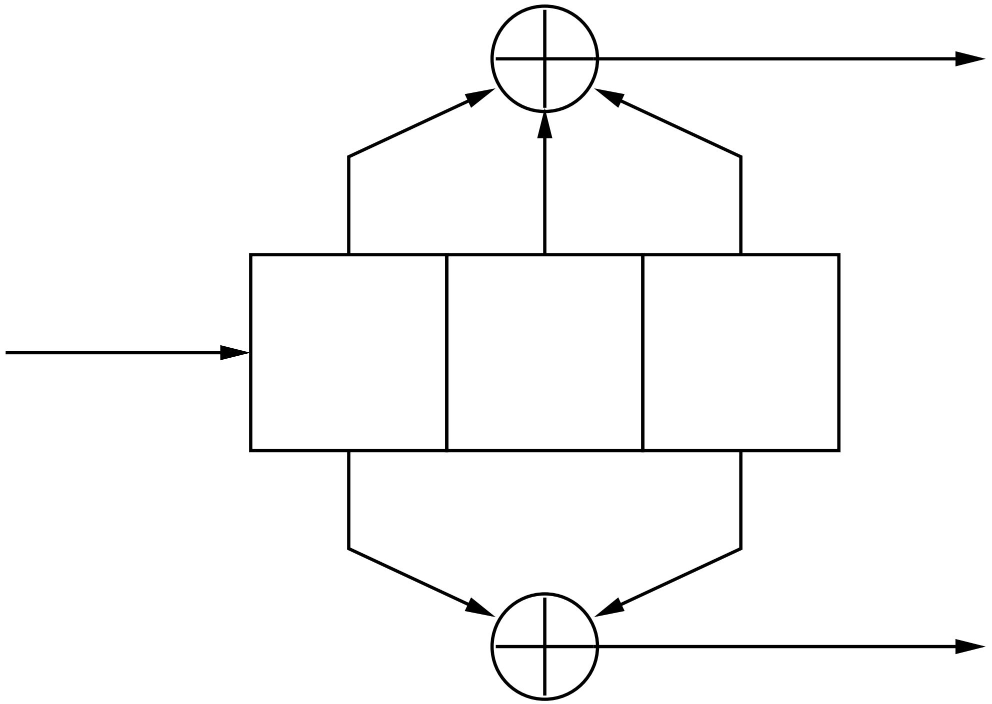
$$R_c = \frac{k}{n}$$

- L is called the depth or the constraint length of the convolutional code.
- The current n outputs are linear combinations of the present k input bits and the previous $(L-1) \times k$ input bits.
- Using the "convolutional" terminology, a classic convolutional code might be considered a Finite impulse response (FIR) filter, while a recursive convolutional code might be considered an Infinite impulse response (IIR) filter.

Binary Convolutional Codes

- A binary convolutional encoder is conveniently structured as a mechanism of shift registers and modulo-2 adders (XOR), where the output bits are modulo-2 additions of selective shift register contents and present input bits.
- Encoders are termed systematic if the input data is also used in the output symbols. Codes with output symbols that do not include the input data are called non-systematic.
- Recursive codes are typically systematic and, conversely, non-recursive codes are typically non-systematic. It isn't a strict requirement, but a common practice.
- Recursive systematic convolutional (RSC) codes have become more popular due to their use in Turbo Codes.
- Several algorithms exist for decoding convolutional codes. For relatively small values of k, the Viterbi algorithm is universally used as it provides maximum likelihood performance and is highly parallelizable.

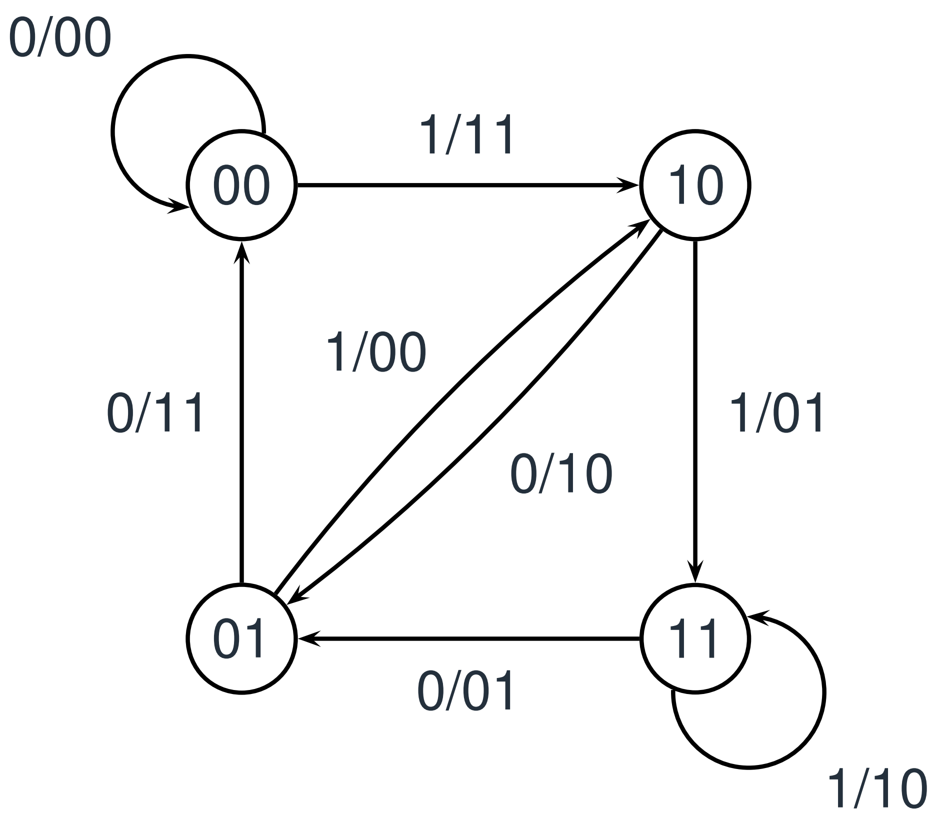
Example Convolutional Codes



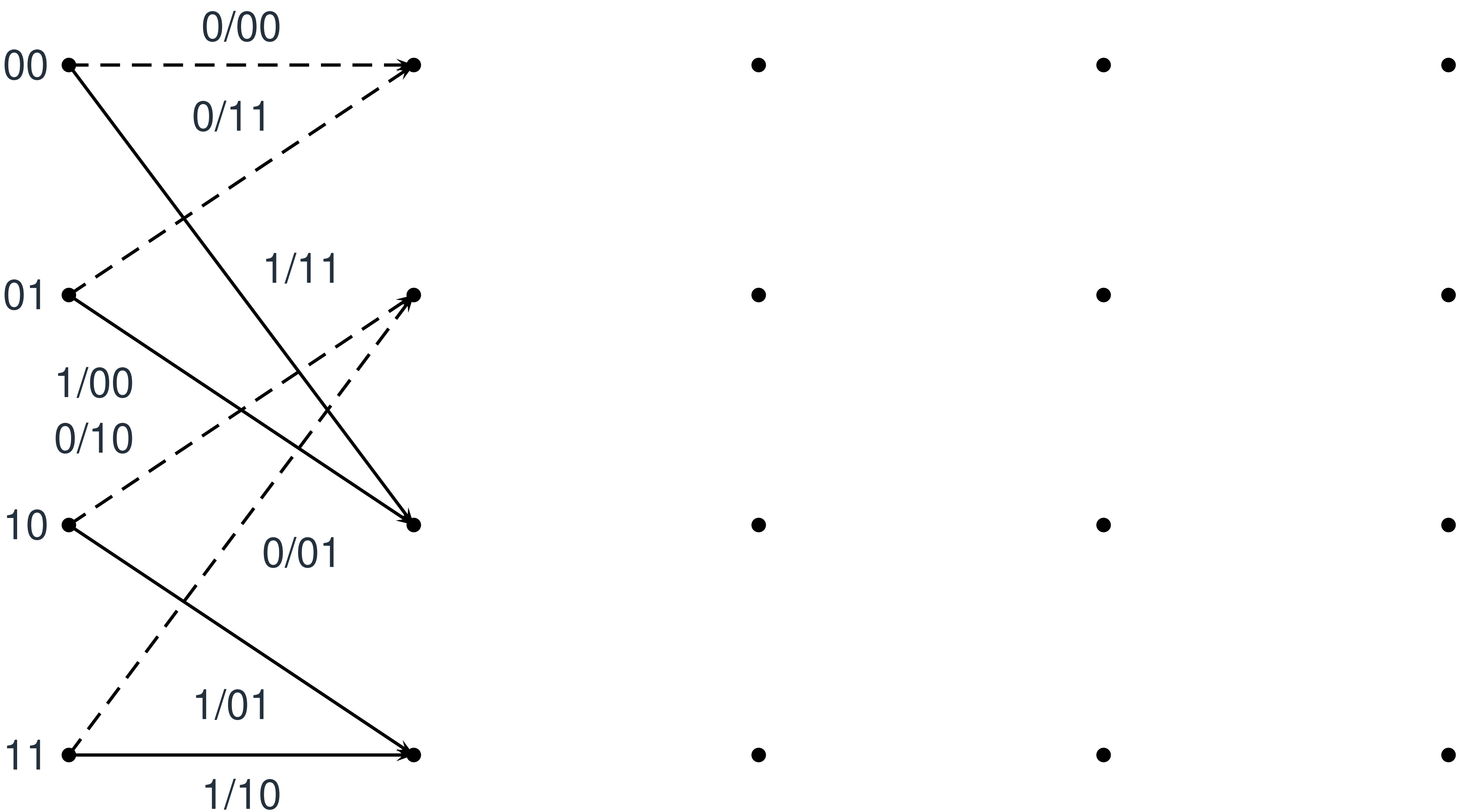
A rate $\frac{1}{2}$ convolutional encoder.

Example encoding: 11010 $\rightarrow$ 1101010010

State Diagram

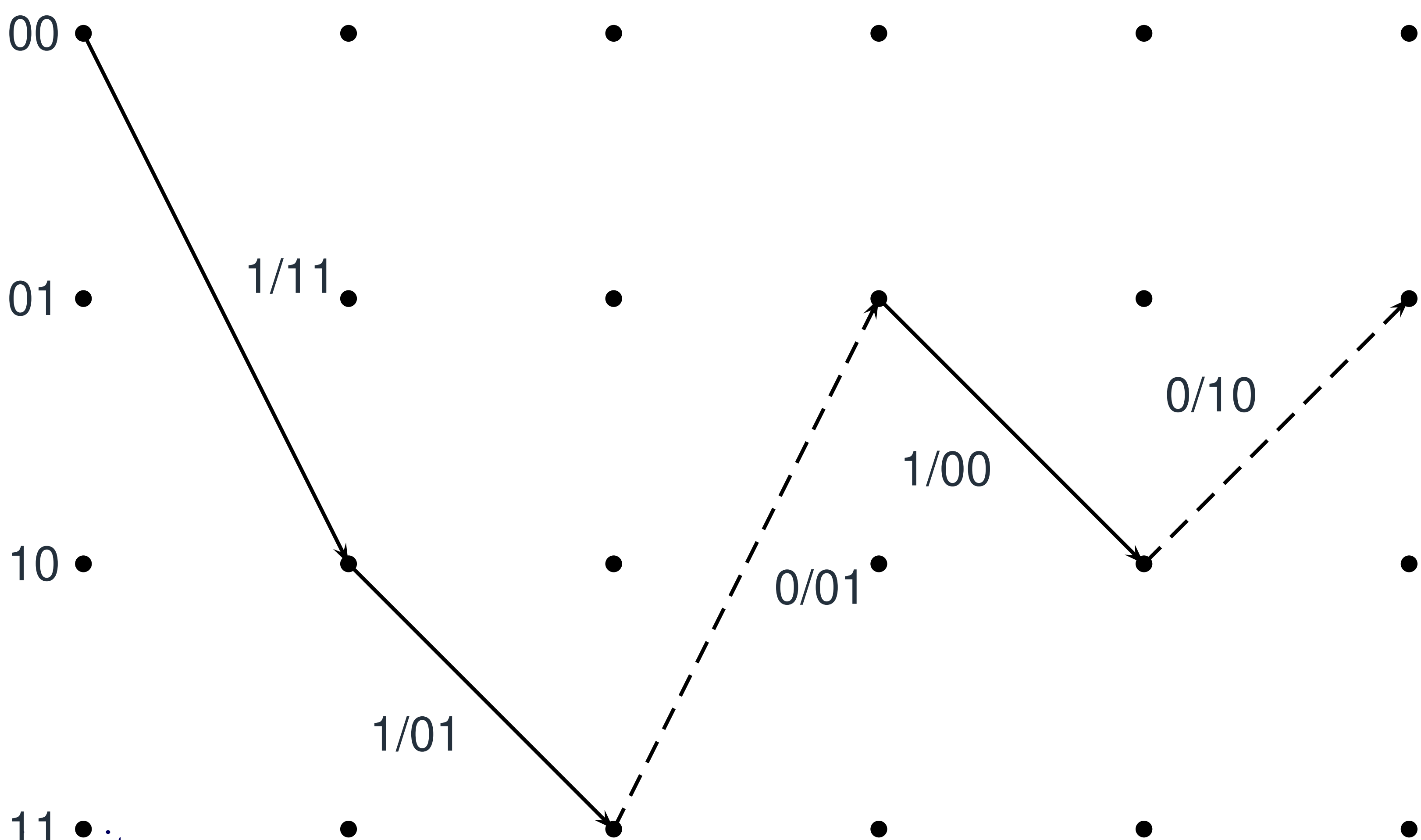


Trellis diagram

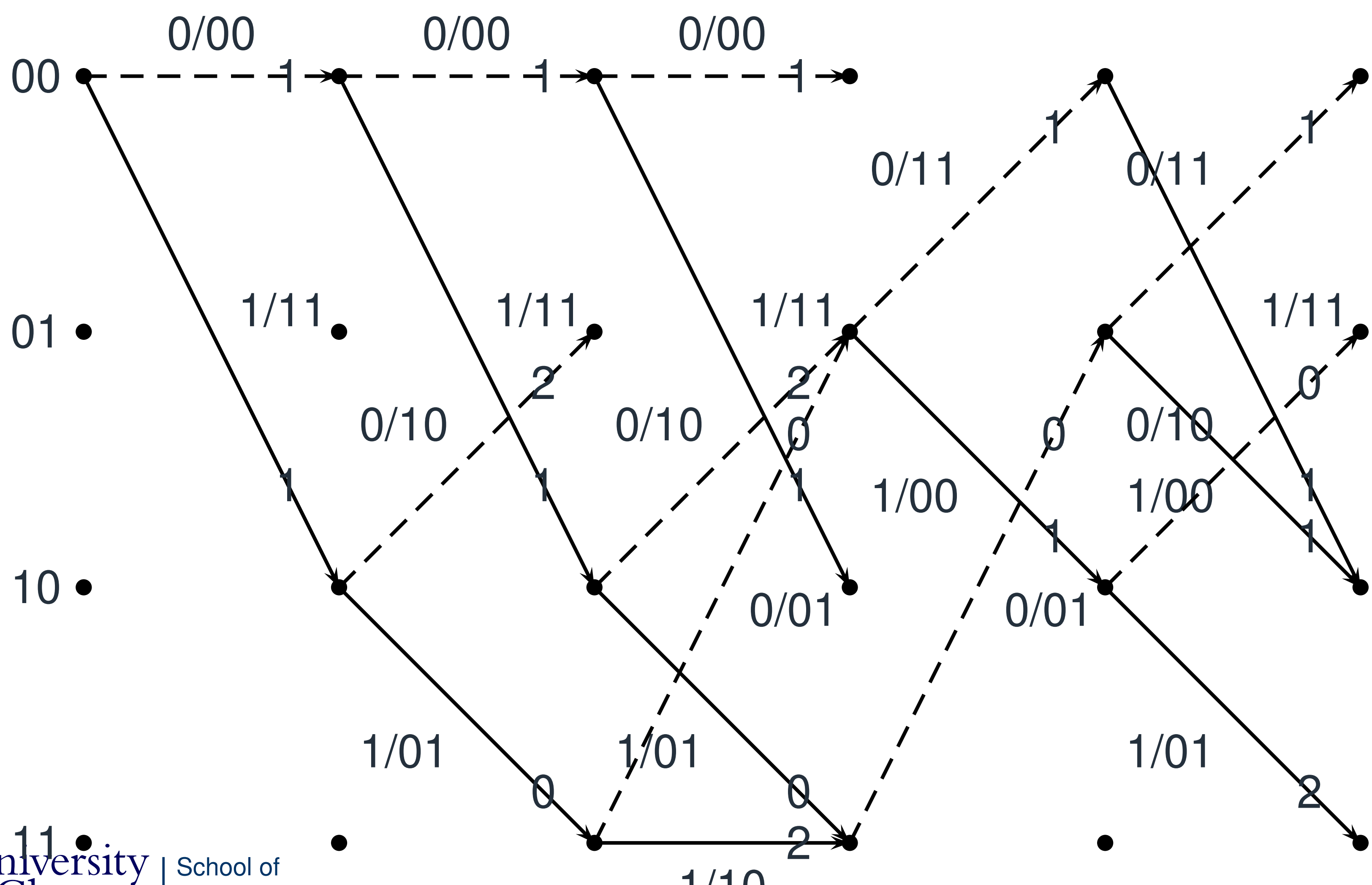


Trellis diagram

Encode 11010 $\rightarrow$ 1101010010



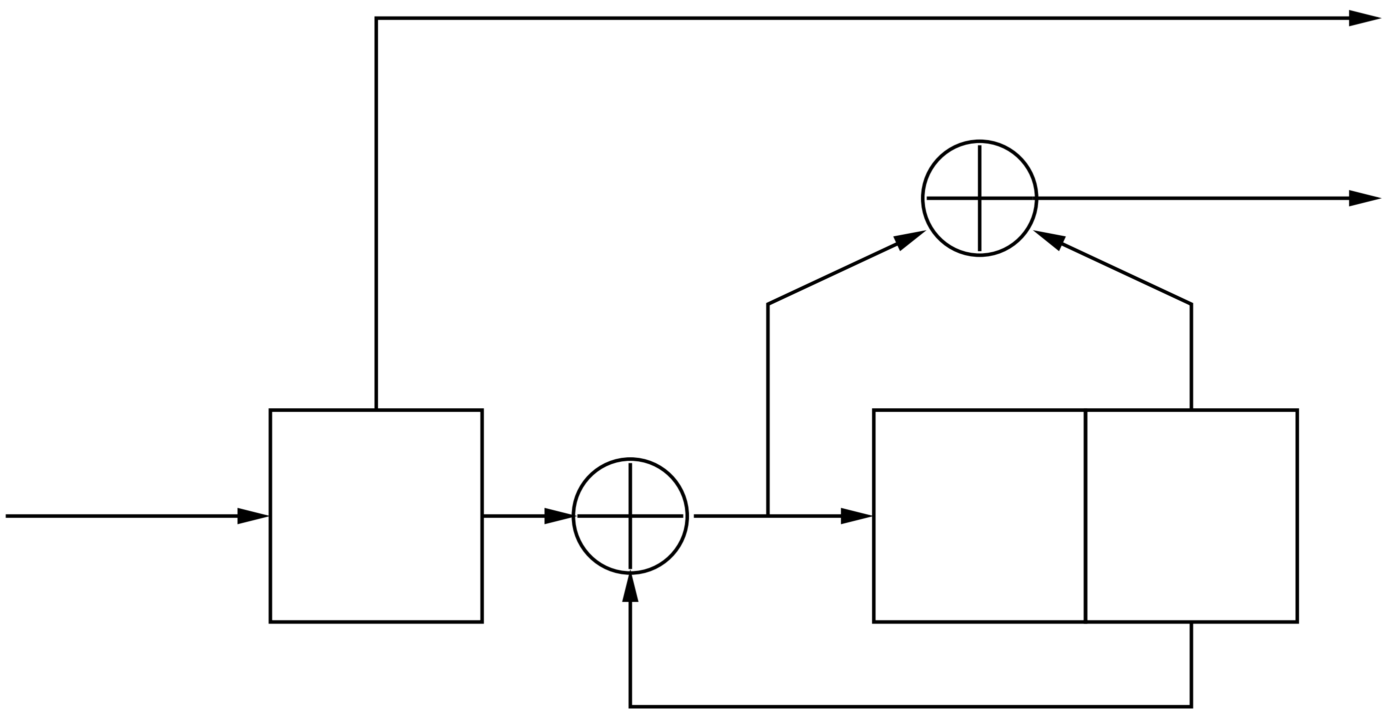
Decode with 2 bit errors 1**0**010101**1**10 $\rightarrow$ 11010



Soft- and Hard-Decision Decoding

- a *hard*-decision decoder operates on data that take on a fixed set of possible values (e.g. the binary values 0 or 1)
- the inputs to a *soft*-decision decoder may take on a whole range of values in-between
- this extra information indicates the reliability of each input data point, and is used to form better estimates of the original data
- a soft-decision decoder will typically perform better in the presence of corrupted data than its hard-decision counterpart.
- soft-decision decoders are often used in Viterbi decoders and turbo code decoders

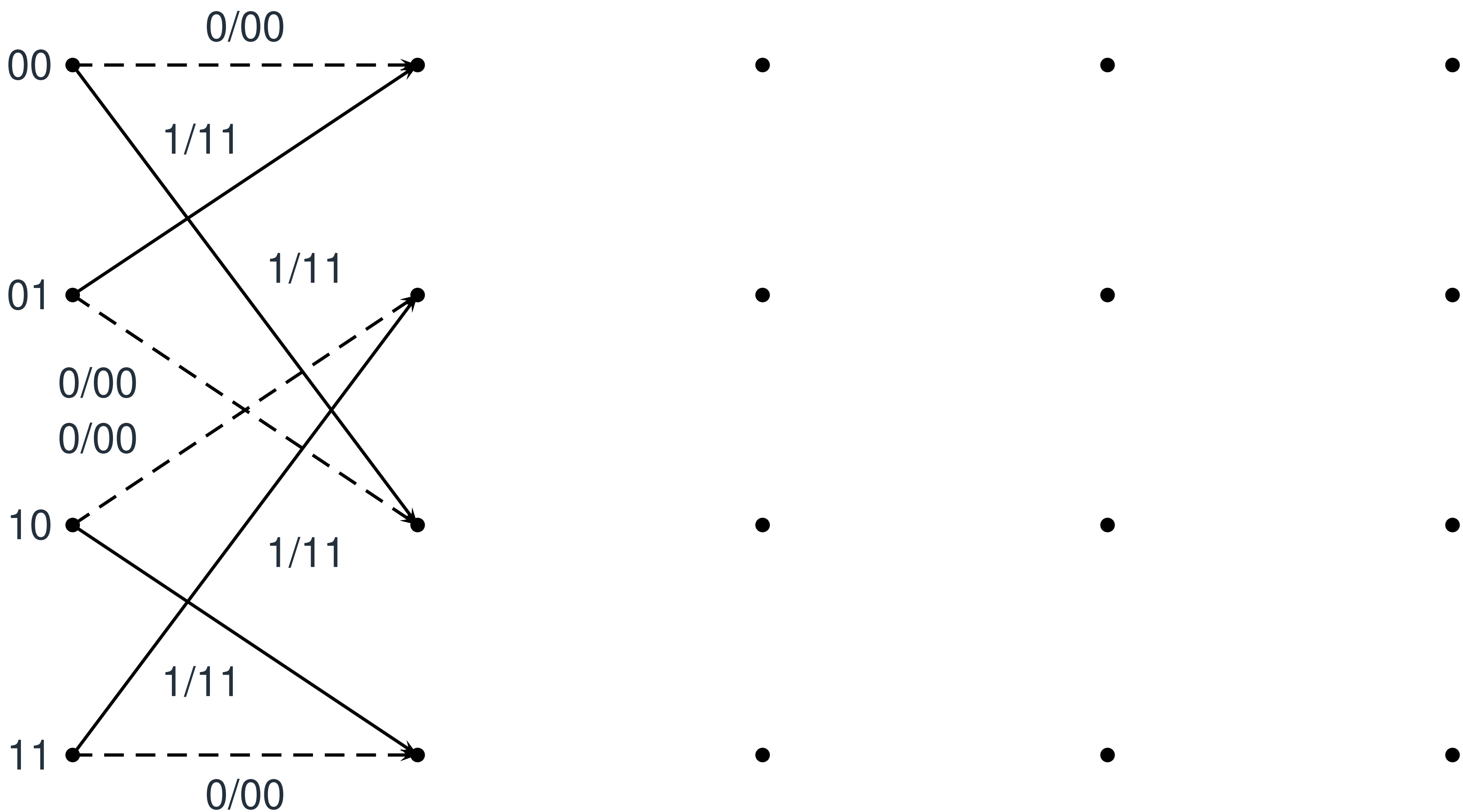
Recursive Convolutional Codes



A rate $\frac{1}{2}$ four-state recursive systematic convolutional (RSC) code.

Useful for serial concatenated convolutional codes (SCCCs) and multidimensional turbo codes.

Trellis diagram for RSC



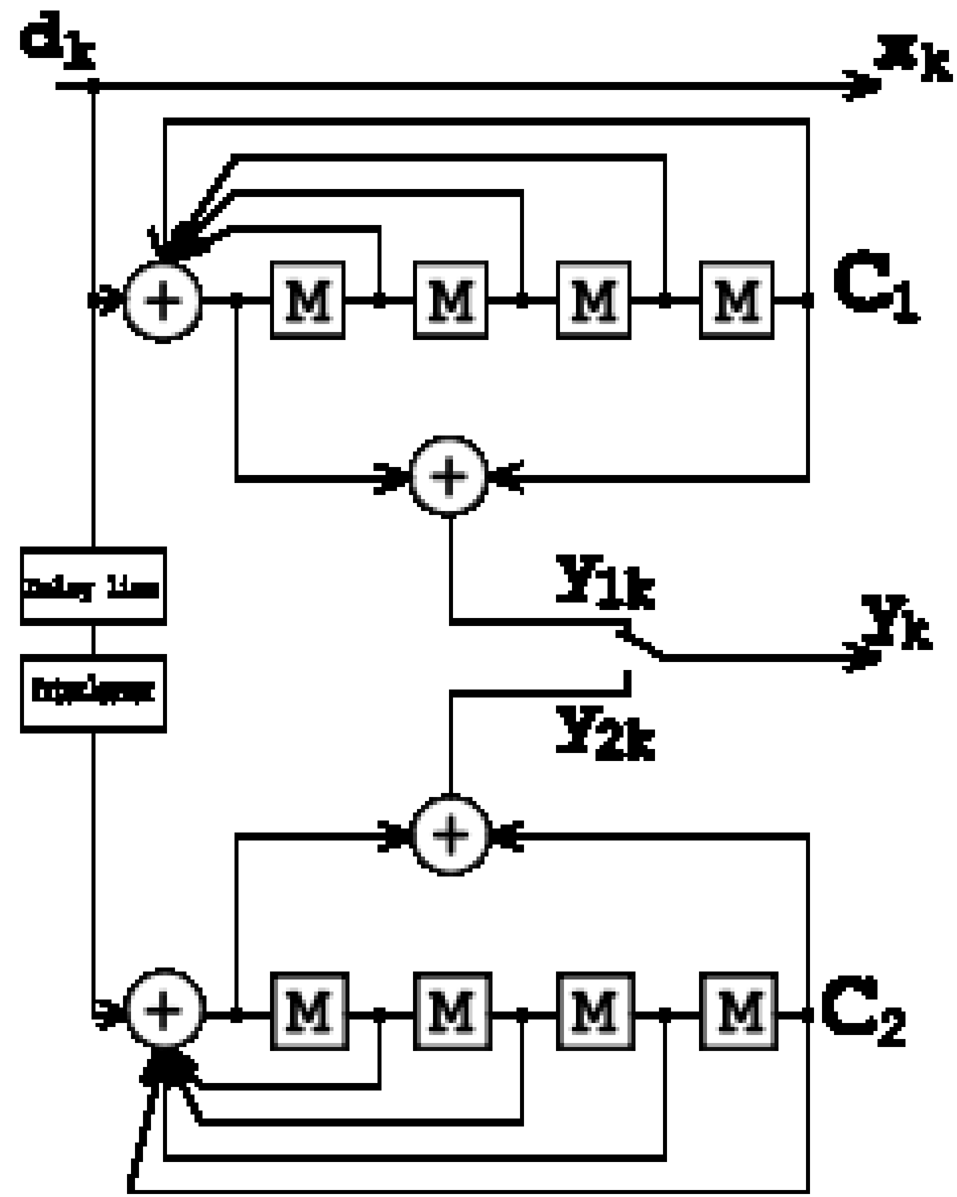
Punctured convolutional codes

- Puncturing is a technique used to make a m/n rate code from a basic low-rate (e.g., 1/n) code.
- It is reached by deletion of some bits in the encoder output: bits are deleted according to a puncturing matrix.

code rate	puncturing matrix
1/2	$\begin{matrix} 1 \\ 1 \end{matrix}$
2/3	$\begin{matrix} 1 & 0 \\ 1 & 1 \end{matrix}$
3/4	$\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$
5/6	$\begin{matrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{matrix}$
6/7	$\begin{matrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{matrix}$

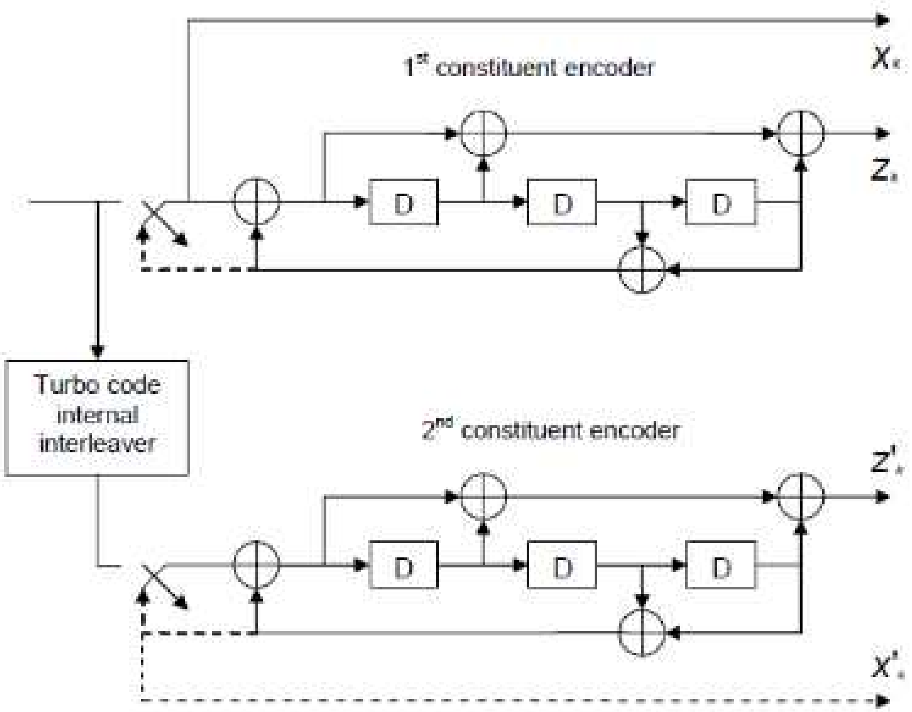
Turbo Codes

- Turbo codes are a class of FEC codes developed around 1990–91
- First practical codes to closely approach the channel capacity
- Turbo codes are used in 3G/4G mobile communications (e.g., in UMTS and LTE) and in (deep space) satellite communications
- The name *turbo code* arose from the feedback loop used during normal turbo code decoding



Turbo Encoder for LTE

The scheme of the Turbo encoder for LTE mobile networks is a Parallel Concatenated Convolutional Code (PCCC) with two 8-state constituent encoders and one Turbo code internal interleaver.



State of the art

Textbook convolutional codes The de facto standard error-correcting code for satellite communications is a convolutional code with constraint length 7.

Concatenated convolutional codes The above convolutional code can be concatenated with a Reed-Solomon code with eight-bit symbols. This code was used in deep space communications such as the Voyager spacecraft.

The code for Galileo A code using the same format but using a longer constraint length of 15 for its convolutional code and a larger Reed-Solomon code was developed by the Jet Propulsion Laboratory (Swanson, 1988). In 1992, this was the best code known of rate 1/4.

Turbo codes In 1993, Berrou, Glavieux and Thitimajshima reported work on turbo codes. The encoder of a turbo code is based on the encoders of two convolutional codes.

Gallager's low-density parity-check block codes The best block codes known for Gaussian channels were invented by Gallager in 1962 and rediscovered in 1995 and shown to have outstanding theoretical and practical properties.

Low-density parity-check code

- In 2003, an irregular repeat accumulate (IRA) style LDPC code beat six turbo codes to become the error correcting code in the new DVB-S2 standard for the satellite transmission of digital television.
- In 2008, LDPC beat convolutional turbo codes as the forward error correction (FEC) system for the ITU-T G.hn standard, a specification for home networking with data rates up to 1 Gbit/s and operation over three types of legacy wires
- LDPC codes are also used for 10GBase-T Ethernet, which sends data at 10 gigabits per second over twisted-pair cables. As of 2009, LDPC codes are also part of the Wi-Fi 802.11 standard as an optional part of 802.11n and 802.11ac, in the High Throughput (HT) PHY specification
- LDPC codes are used in the 5G mobile network standard for the data channels.