Digital Communications 4: Tutorial on Information Theory

You may use a spreadsheet, python or equivalent to aid your calculations.

1.

   (a) Assuming that letters occur with equal probability, calculate the information (in bits) for the message "the quick brown fox jumps over the lazy dog" (ignore spaces). You may find the following identity useful if you are using a calculator:

$$\log_2 x = (\log_{10} x)/(\log_{10} 2)$$

   (b) The table shows the probability distribution of letters in the English alphabet. How do you think these values are arrived at?

k	x_k	$P(x_k)$	k	x_k	$P(x_k)$	k	x_k	$P(x_k)$
1	a	8.167%	10	j	0.153%	19	s	6.327%
2	b	1.492%	11	k	0.772%	20	t	9.056%
3	c	2.782%	12	l	4.025%	21	u	2.758%
4	d	4.253%	13	m	2.406%	22	v	0.978%
5	e	12.702%	14	n	6.749%	23	w	2.360%
6	f	2.228%	15	o	7.507%	24	x	0.150%
7	g	2.015%	16	p	1.929%	25	y	1.974%
8	h	6.094%	17	q	0.095%	26	z	0.074%
9	i	6.966%	18	r	5.987%			

   (c) Using this probability distribution, calculate the information in the given message.

   (d) Again using this probability distribution, calculate the entropy of the English alphabet.

   (e) Is this a true Discrete Memoryless Source (DMS)?

2. Two six-sided dice are thrown and the numbers shown are added together.

   (a) Is this a true Discrete Memoryless Source (DMS)?

   (b) Calculate the probabilities for each sum total.

   (c) Hence, determine the information (in bits) for each sum total.

   (d) Hence, calculate the entropy for the symbol set consisting of the sum totals.

   (e) Devise a Huffman code for this alphabet.

3. Repeat the previous question, but now take the absolute value of the difference between the two thrown dice.

4. The LZW algorithm was used in the lecture notes to compress the first line of the following tongue-twister. Continue with applying the LZW source coding to the rest of the verse. Remember the space is a character in the initial dictionary.

   ```
   she sells sea shells on the sea shore
   the shells she sells are sea shells im sure
   and if she sells sea shells on the sea shore
   then im sure she sells seashore shells
   ```

   What compression ratio did you achieve? Check your answer by attempting to decode your result.