Demodulation of DSB-SC

- Recovery of the original signal (demodulation) could be accomplished by another frequency shift, i.e. modulating with $\cos\omega_c t$ again.

$$\phi(t)\cos\omega_c t = f(t)\cos^2\omega_c t = \frac{1}{2}f(t)[1+\cos 2\omega_c t]$$

- Taking the Fourier transform,

$$\mathcal{F}\{\phi(t)\cos\omega_c t\} = \frac{1}{2}F(\omega) + \frac{1}{4}F(\omega+2\omega_c) + \frac{1}{4}F(\omega-2\omega_c)$$

- We can reject the signals around $\pm 2\nu_c$ with a low-pass filter and obtain output $e_0(t) = \frac{1}{2}f(t)$.
- However, we'll see with demodulation of DSB-SC that phase and frequency must exactly match.

- Introduce an error in the phase ($\Delta\theta$) into the locally generated carrier frequency.

$$\begin{aligned}\phi(t)\cos(\omega_c t + \Delta\theta) &= f(t)\cos\omega_c t\cos(\omega_c t + \Delta\theta) \\ &= \frac{1}{2}f(t)\cos\Delta\theta + \frac{1}{2}f(t)\cos(2\omega_c t + \Delta\theta)\end{aligned}$$

- After a low-pass filter get output

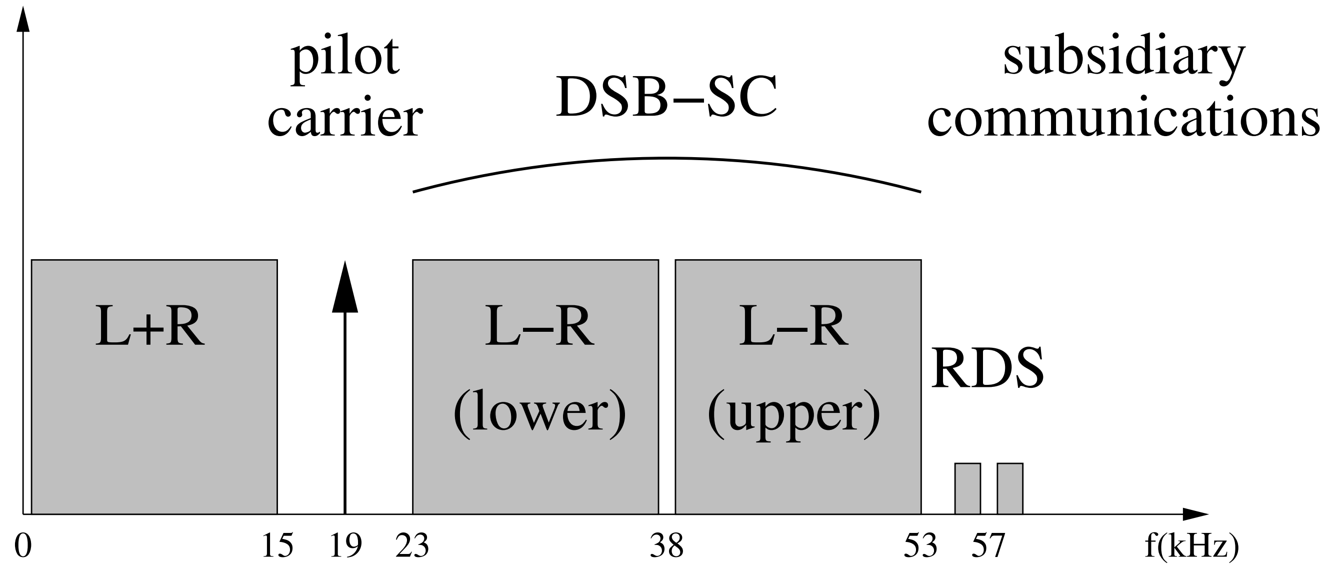
$$e_0(t) = \frac{1}{2}f(t)\cos\Delta\theta$$

- A phase error causes a variable output (drops to zero if $\Delta\theta = \pm 90°$).
- A frequency mis-match $\Delta\theta = \Delta\omega t$ results in a low-frequency oscillation.
- Therefore with DSB-SC it is necessary to use a synchronised oscillator — termed *synchronous* detection or *coherent* detection.

Include Carrier in Transmission

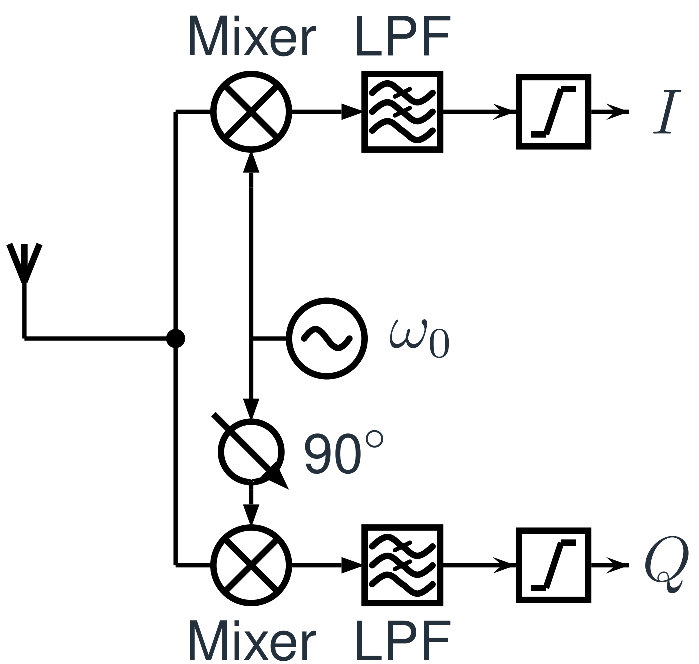
DSB-LC Double sideband large carrier allows asynchronous demodulation, e.g. envelope detector, but sidebands are limited to <33% power, therefore inefficient

pilot carrier Include a pilot carrier. For example, in stereo FM there is a 19 kHz signal included (limited to 10% of peak frequency deviation) which allows generation of 38 kHz carrier for DSB-SC demodulation of $(L - R)$ component, and 57 kHz for Radio Data System (RDS) demodulation.



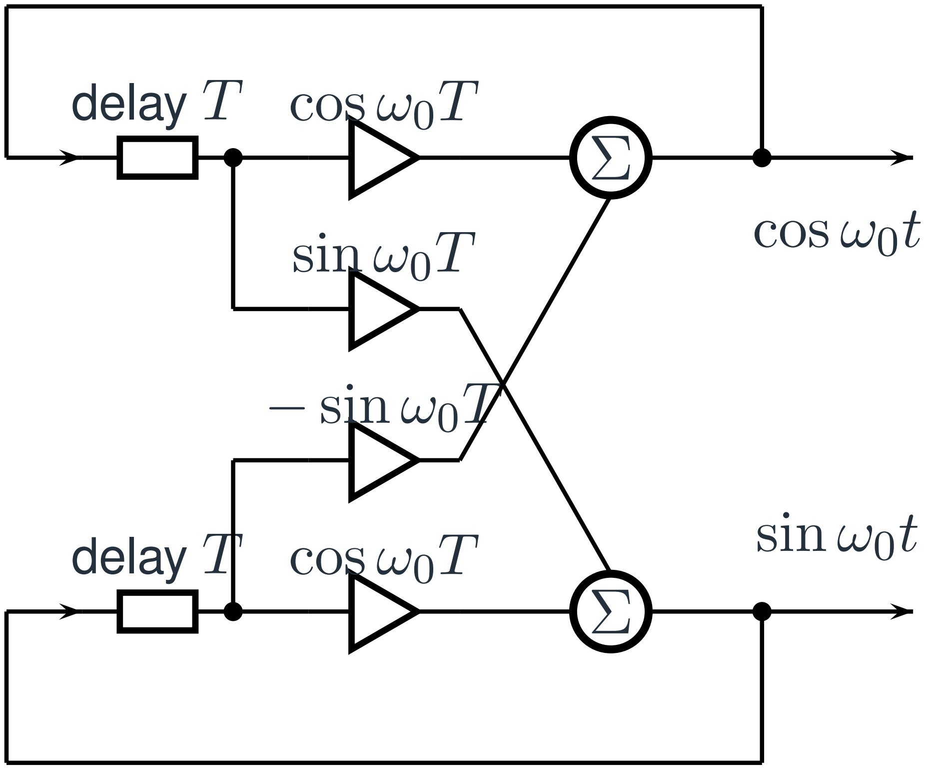
Carrier Recovery

- How do we regenerate a phase-locked carrier at the receiver where there is no explicit carrier in the data stream?
- Need a controllable oscillator at the receiver, such as a voltage controlled oscillator (VCO), e.g. $\omega_0 = \omega_i + \alpha v$.
- Need a means to phase lock the oscillator to the input waveform.

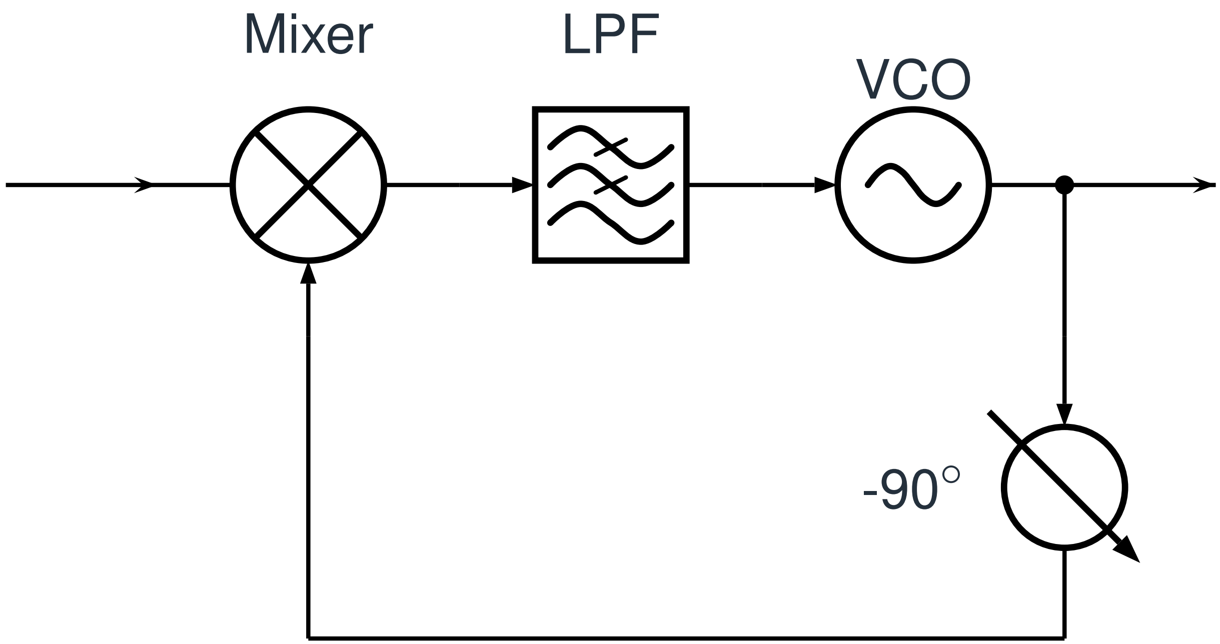


Digital Oscillator: Cordic Algorithm

$$\cos\omega_0(t+T) = \cos\omega_0 T\cos\omega_0 t - \sin\omega_0 T\sin\omega_0 t$$
$$\sin\omega_0(t+T) = \cos\omega_0 T\sin\omega_0 t + \sin\omega_0 T\cos\omega_0 t$$



Phase-Locked Loop



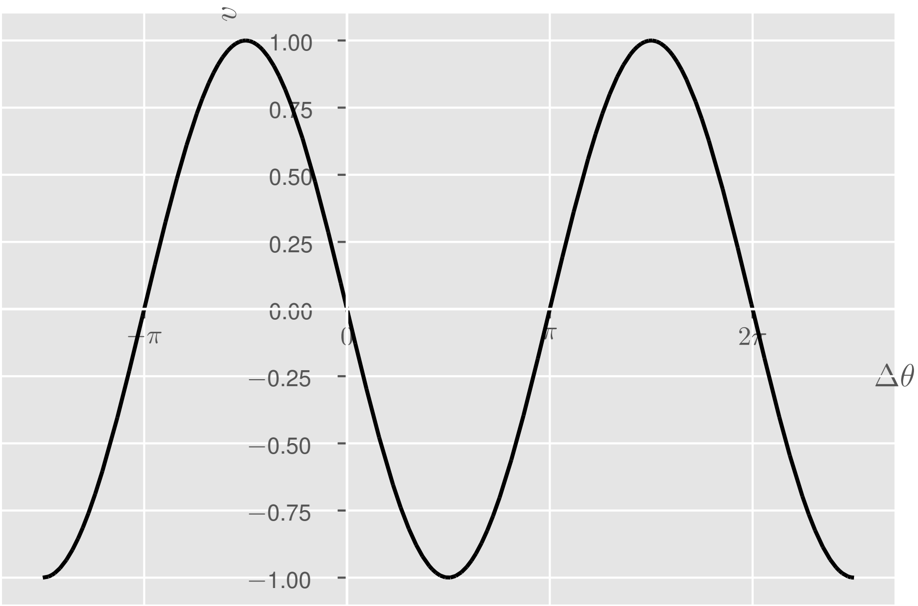
input: $\cos(\omega_{\text{ref}} t + \theta_{\text{ref}})$

mixed with: $-\sin\left(\omega_0 t + \theta_0\right)$

after LPF: $-\frac{1}{2}\sin\left(\Delta\omega t + \Delta\theta\right)$

stationary solution: $\Delta\omega = 0$ and hence $\sin\Delta\theta = \frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$.

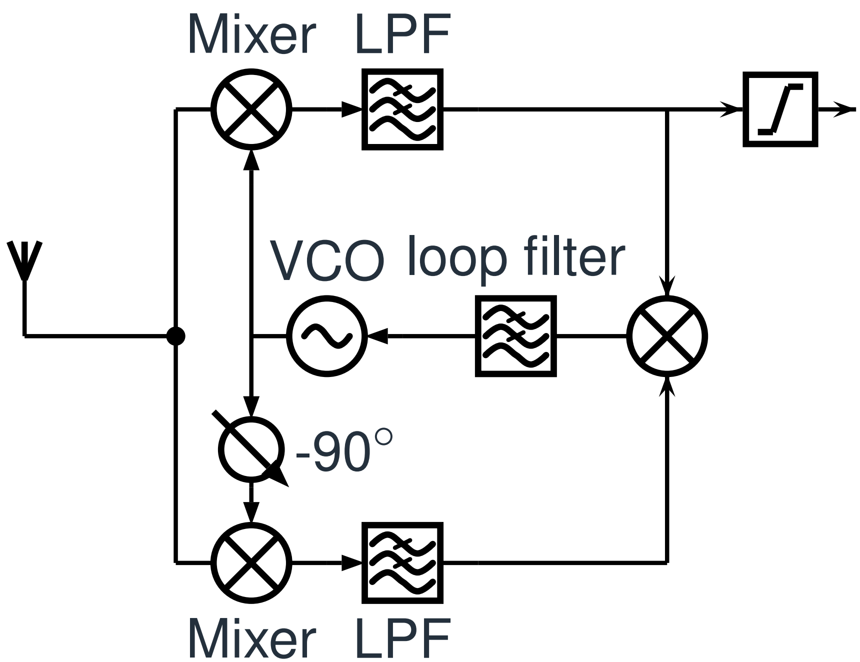
Phase-Locked Loop



- phase is integral of frequency $\theta_0(t) = \int_0^t \omega_0(\tau)\,\mathrm{d}\tau + \theta_0(t=0)$
- $\Delta\theta = \sin^{-1}\frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$ is a **stable** stationary point (negative feedback)
- $\Delta\theta = \pi - \sin^{-1}\frac{2(\omega_i - \omega_{\text{ref}})}{\alpha}$ is an **unstable** stationary point (positive feedback)
- PLL locks in frequency and phase to a sinusoidal reference

Costas Loop

- but phase-modulated input would continually be trying to lock to instantaneous phase (suppressed carrier)
- we could initialise with an unmodulated carrier first, and then switch off lock (analogue PAL for colour info)
- **Costas Loops** allow the symbol phase info to be eliminated.
- In classical Costas loop use both in-phase and quadrature VCO outputs and combine LPF outputs to drive VCO.



input: $s(t)\cos(\omega_{\text{ref}}t + \theta_{\text{ref}})$

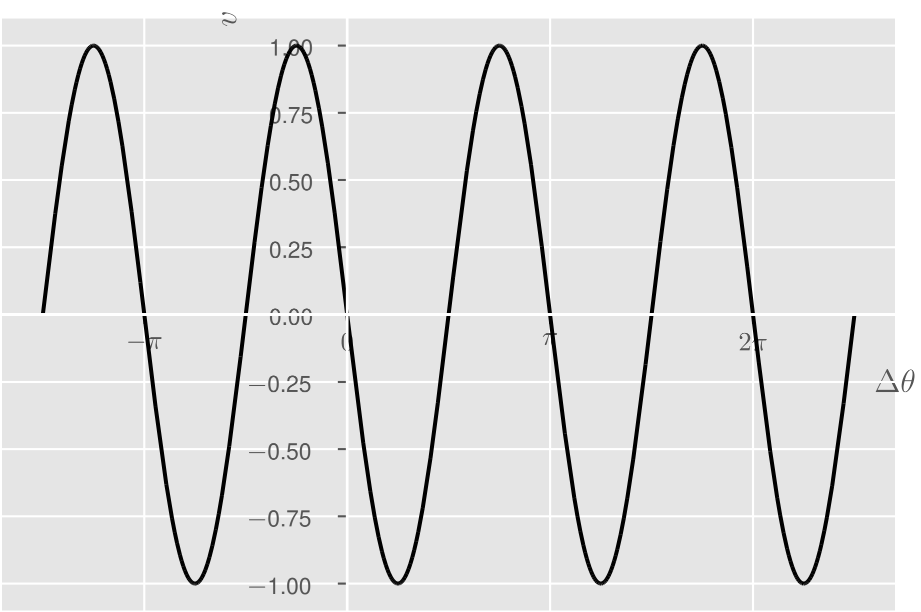
after LPF1: $\frac{1}{2}s(t)\cos\left(\Delta\omega t + \Delta\theta\right)$

after LPF2: $-\frac{1}{2}s(t)\sin\left(\Delta\omega t + \Delta\theta\right)$

VCO input: $-\frac{1}{8}s^2(t)\sin 2\left(\Delta\omega t + \Delta\theta\right)$ with $s(t) = \pm 1$

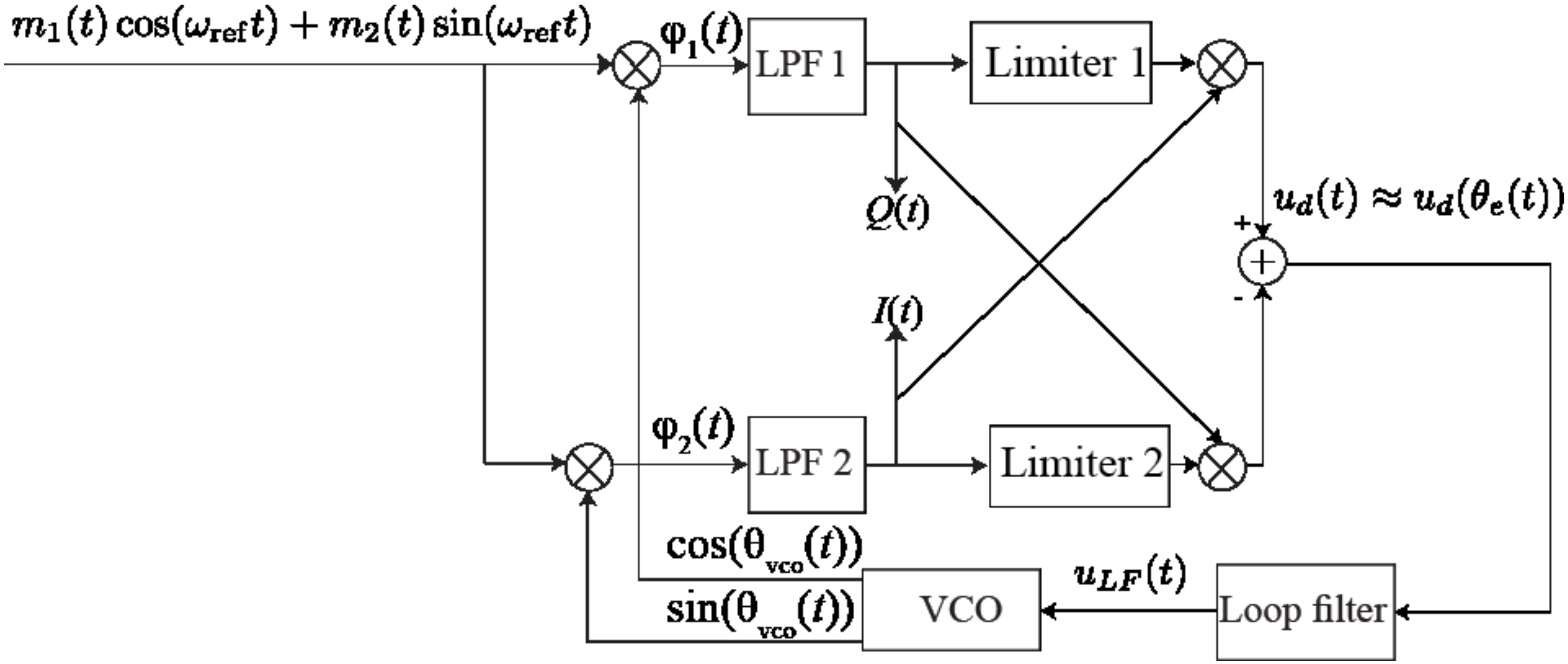
stationary solution: $\Delta\omega = 0$ and hence $\sin 2\Delta\theta = \frac{8(\omega_i - \omega_{\text{ref}})}{\alpha}$.

Costas Loop



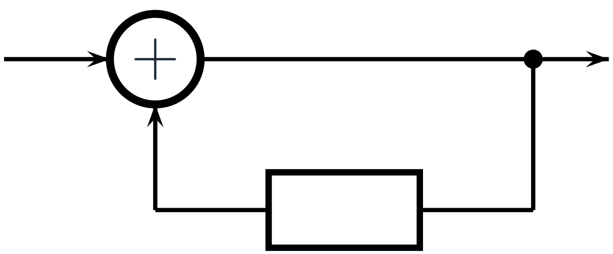
- phase is integral of frequency $\theta_0(t) = \int_0^t \omega_0(\tau)\,\mathrm{d}\tau + \theta_0(t=0)$
- $\Delta\theta = \frac{1}{2}\sin^{-1}\frac{8(\omega_i - \omega_{\mathrm{ref}})}{\alpha}$ is a **stable** stationary point (negative feedback)
- $\Delta\theta = \pi + \frac{1}{2}\sin^{-1}\frac{8(\omega_i - \omega_{\mathrm{ref}})}{\alpha}$ is also a **stable** stationary point
- Requires a check to see if BPSK binary data is inverted or not.

QPSK Costas Loop



Differential Encoding

Differential encoding prevents inversion of the signal (and symbols) from affecting the data. Note: adds an additional symbol requiring initialisation.

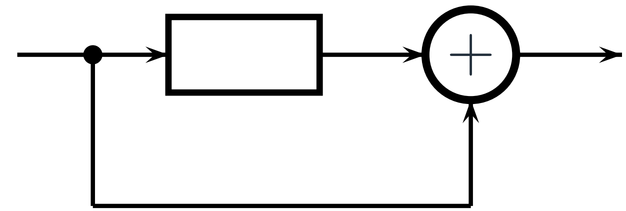


Register

Differential Encoder

$$y_i = y_{i-1} \oplus x_i$$

Register



Differential Decoder

$$x_i = y_i \oplus y_{i-1}$$

$\oplus$ represents component-wise modulo 2 addition (XOR) for binary symbols.

There are also line codes designed to be polarity insensitive, such as Differential Manchester encoding.