

## Solving a problem relating to camera rotation

Given the view plane, we have a unit vector representing the y axis and another representing the x axis, both parallel to the plane.

We have a point on the plane representing the middle of the screen, or (0, 0).

I want to find what combination of the two vectors offset from the middle point is equal to some arbitrary point on the view plane.

Essentially turning a 3D coordinate on a plane into a 2D coordinate.

$$\text{Plane } Ax + By + Cz - D = 0$$

$$\text{With normal } N = (A, B, C)$$

$$\text{Arbitrary point on plane } P = (x, y, z)$$

Two vectors parallel to plane

$$V_1 = (a_1, b_1, c_1)$$

$$V_2 = (a_2, b_2, c_2)$$

Two scalars  $m$  &  $n$

$$\text{Starting point on plane } P_0 = (x_0, y_0, z_0)$$

Given the following, find  $m$  &  $n$

$$P = P_0 + m V_1 + n V_2 \quad \text{we know: } P, P_0, V_1 \text{ \& } V_2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + m \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} + n \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$$

Any component of  $V_1$  or  $V_2$  may be 0, but at least one is not

$$x = x_0 + m a_1 + n a_2$$

$$y = y_0 + m b_1 + n b_2$$

$$z = z_0 + m c_1 + n c_2$$

$$x = x_0 + m a_1 + n a_2 \quad (1)$$

$$y = y_0 + m b_1 + n b_2 \quad (2)$$

$$z = z_0 + m c_1 + n c_2 \quad (3)$$

Solving for  $m$

From (1)

$$x = x_0 + m a_1 + n a_2$$

$$m a_1 = x - x_0 + n a_2$$

$$m = (x - x_0 + n a_2) / a_1 \quad (4)$$

Sub (4) into (2)

$$y = y_0 + \frac{x - x_0 + n a_2}{a_1} b_1 + n b_2$$

Solve for  $n$

$$n = \left[ y - y_0 - \frac{x - x_0 + n a_2}{a_1} b_1 \right] \div b_2$$

In this case  $a_1$  &  $b_2 \neq 0$

From here we can see six cases

$$\text{If } a_1 \neq 0$$

$$m = (x - x_0 + n a_2) / a_1$$

$$\& \text{ If } b_2 \neq 0$$

$$n = \left[ y - y_0 - \frac{x - x_0 + n a_2}{a_1} b_1 \right] \div b_2$$

$$\& \text{ If } c_2 \neq 0$$

$$n = \left[ z - z_0 - \frac{x - x_0 + n a_2}{a_1} c_1 \right] \div c_2$$

$$\text{If } b_1 \neq 0$$

$$m = (y - y_0 + n b_2) / b_1$$

$$\& \text{ If } a_2 \neq 0$$

$$n = \left[ x - x_0 - \frac{y - y_0 + n b_2}{b_1} a_1 \right] \div a_2$$

$$\& \text{ If } c_2 \neq 0$$

$$n = \left[ z - z_0 - \frac{y - y_0 + n b_2}{b_1} c_1 \right] \div c_2$$

$$\text{If } c_1 \neq 0$$

$$m = (z - z_0 + n c_2) / c_1$$

$$\& \text{ If } a_2 \neq 0$$

$$n = \left[ x - x_0 - \frac{z - z_0 + n c_2}{c_1} a_1 \right] \div a_2$$

$$\& \text{ If } b_2 \neq 0$$

$$n = \left[ y - y_0 - \frac{z - z_0 + n c_2}{c_1} b_1 \right] \div b_2$$