Name: _____ Homework 2 | Math 342 | Cruz Godar

Due Wednesday of Week 3 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 1

1. Suppose every 5 years, an average of 5% of people from California move to Oregon, and 10% move to Washington; 15% from Oregon move to California and 10% to Washington; and 5% from Washington move to Oregon and 10% to California. California currently has a population of 40 million people, Oregon has a population of 4 million, and Washington has a population of 8 million. With this model, assuming no one arrives from or leaves to anywhere else, what will the population settle down to over time?

Section 2

In problems 2–7, find the eigenvalues and eigenvectors of A and the algebraic and geometric multiplicity of the eigenvalues. Then diagonalize A if possible, using block diagonalization with at most 2×2 blocks if necessary.

2. $A = \begin{bmatrix} -8 & 10 \\ -5 & 7 \end{bmatrix}$.

3. $A = \begin{bmatrix} -1 & -1 \\ 10 & 5 \end{bmatrix}$.

4. $A = \begin{bmatrix} -4 & -6 & 12 \\ 9 & 11 & -18 \\ 3 & 3 & -4 \end{bmatrix}$.

5. $A = \begin{bmatrix} -10 & 4 & -4 \\ -13 & 4 & -10 \\ 1 & 0 & 2 \end{bmatrix}$

6. $A = \begin{bmatrix} 11 & -6 & -4 & -8 \\ -4 & 1 & -12 & -4 \\ -12 & 6 & 3 & 8 \\ 20 & -10 & 0 & -11 \end{bmatrix}$.

7. $A = \begin{bmatrix} 10 & 6 & -7 & 12 \\ -4 & -3 & 4 & -12 \\ 7 & 6 & -4 & 12 \\ -1 & 0 & 1 & 3 \end{bmatrix}$.

8. Let $A = BDB^{-1}$ be a diagonalized $n \times n$ matrix, so that the columns of B are eigenvectors of A. Use this factorization to describe the eigenvectors of A^T (remember that transposing a product reverses the order of the factors).

9. Give an example of an invertible 3×3 matrix that is not diagonalizable, and an example of a diagonalizable 3×3 matrix that is not invertible.