Chapter IV: Vectors

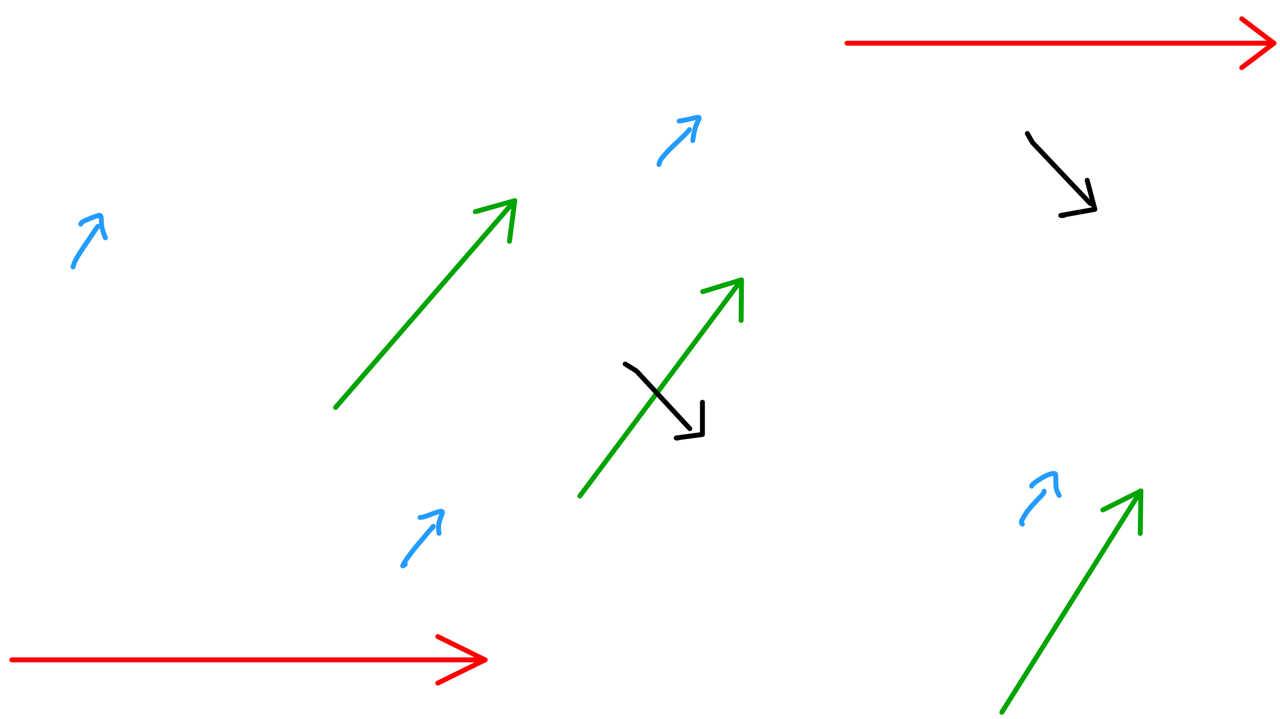
If you take a flight from here to New York, that flight's information depends only on the direction you fly and the distance. Specifically, you don't need the information of where the flight starts — you're already at the airport.

Def: A vector is a quantity that consists of a direction and a magnitude.

We typically draw vectors as arrows.

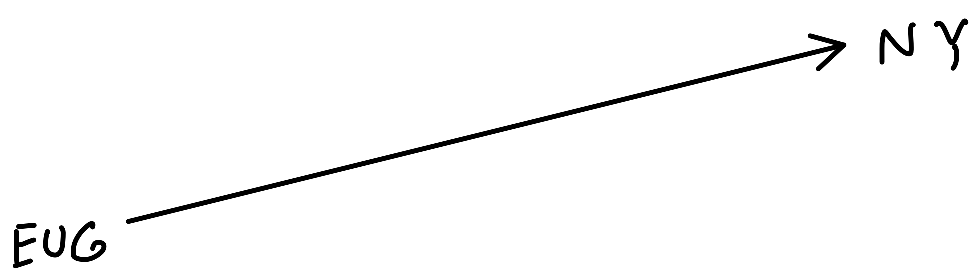
Comment: Vectors don't care where they start or end — only their direction and length.

Ex: Some 2-dimensional vectors.

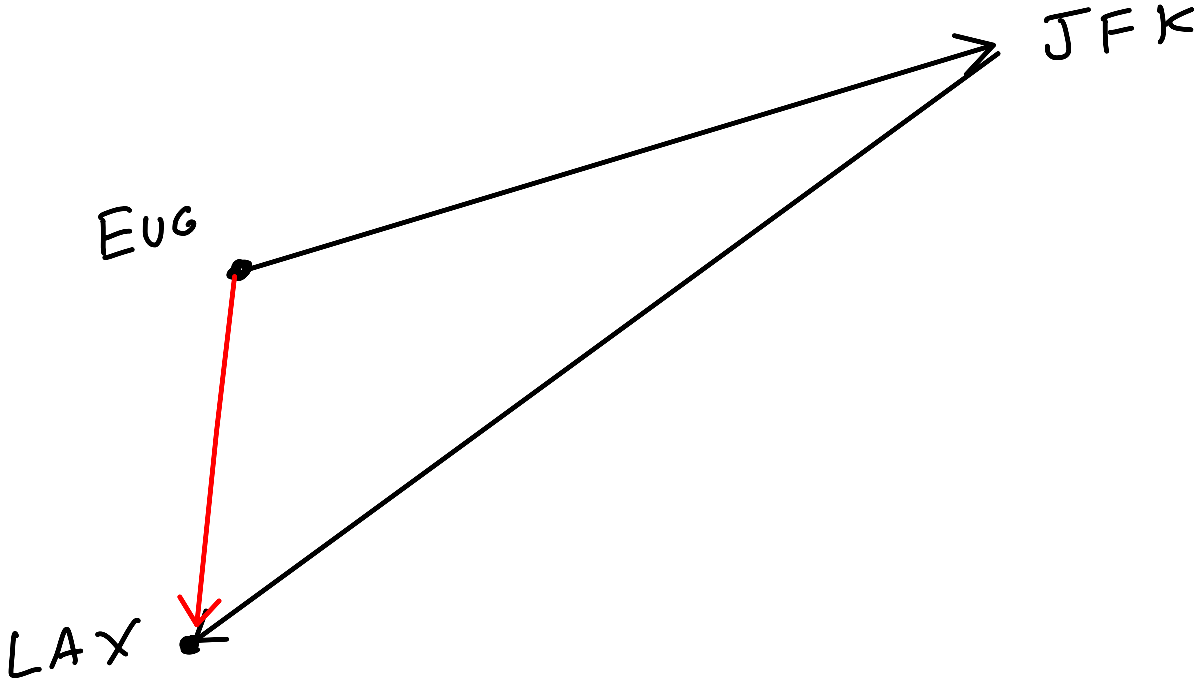


Comment: We typically write variables that correspond to vectors with an arrow symbol — for example, $\vec{v}$, $\vec{w}$, or $\vec{u}$.

Ex: For the flight from here to NY, we have



Now suppose you fly from NY to LA.



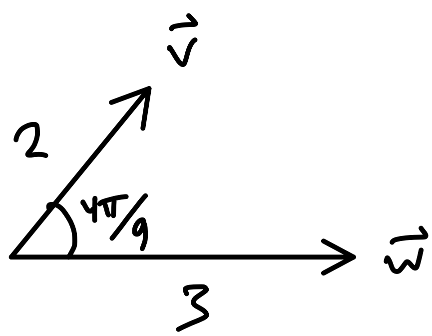
In total, you flew from EUG to LAX.

<u>Def</u>: Let $\vec{v}$ and $\vec{w}$ be vectors. The sum of $\vec{v}$ and $\vec{w}$ is the vector formed by placing the start of $\vec{w}$ at the end of $\vec{v}$ and taking the vector from the start of $\vec{v}$ to the end of $\vec{w}$.

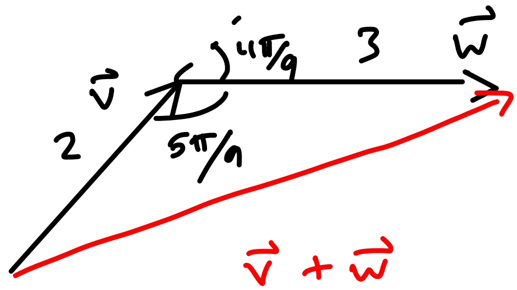
<u>Comment</u>: Let $\vec{v}$ be a vector. We write $\|\vec{v}\|$ to represent the magnitude of $\vec{v}$. In general,

$\|\vec{v}+\vec{w}\| \neq \|\vec{v}\| + \|\vec{w}\|$.

<u>Ex</u>: Find $\|\vec{v}+\vec{w}\|$.



First, shift $\vec{w}$ so that its start is at $\vec{v}$'s end.



By the Law of Cosines,

$$\|\vec{v}+\vec{w}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2 - 2 \cdot \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos 5\pi/9 .$$

$$\vdots$$

$$\|\vec{v} + \vec{w}\| = 3.88$$

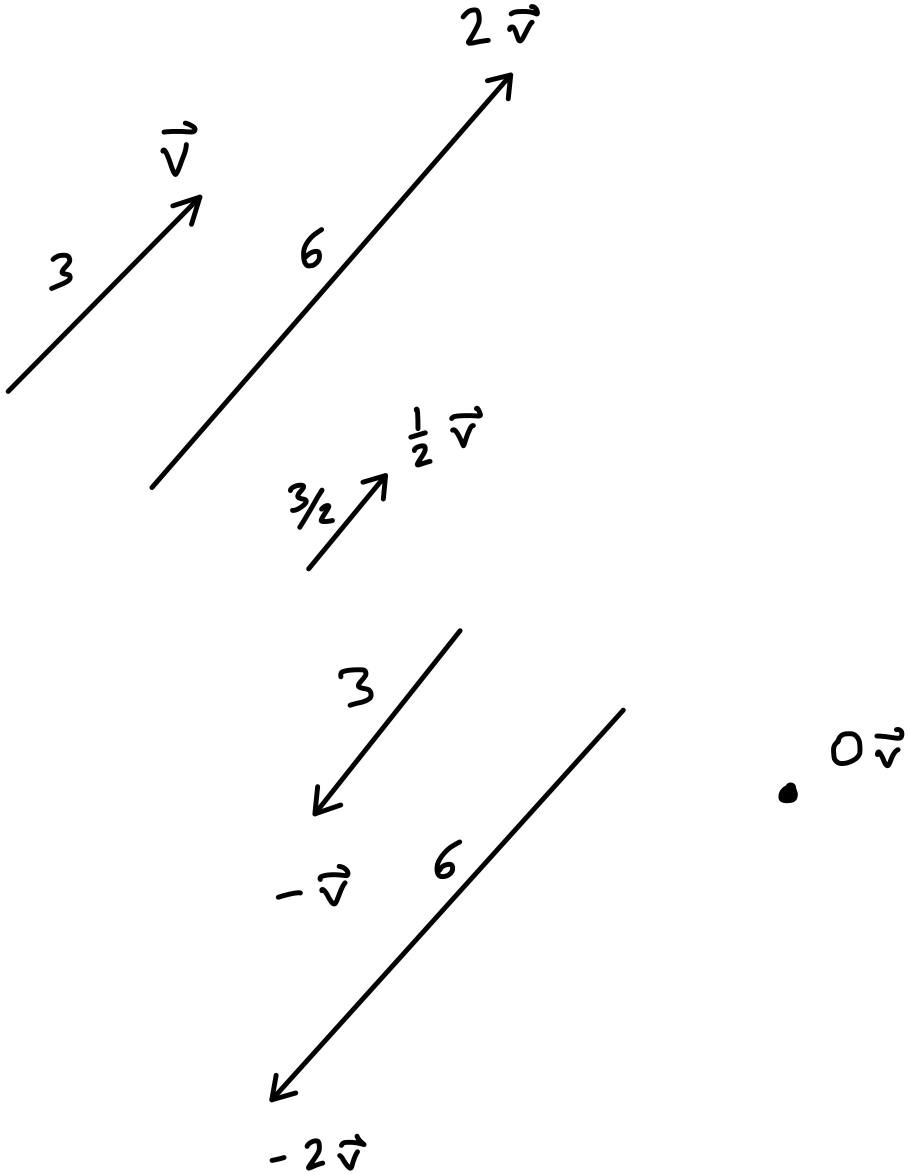
Def: The <u>zero vector</u> is the vector with magnitude 0, written $\vec{0}$. It does not have a direction.

Def: A <u>scalar</u> is a number that isn't a vector. We use this word to distinguish between things that are vectors and things that aren't.

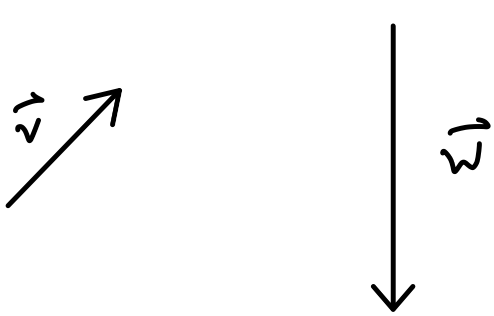
Def: Let $\vec{v}$ be a vector and c be a scalar. The vector $c\vec{v}$ is:

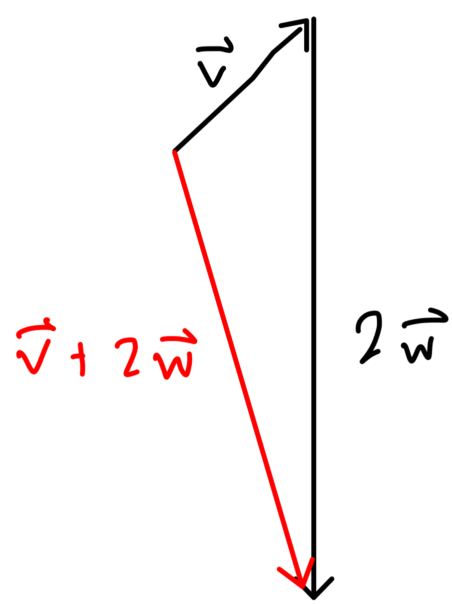
1. If $c > 0$, $c\vec{v}$ is the vector in the same direction but with length $c\|\vec{v}\|$.
2. If $c < 0$, $c\vec{v}$ is the vector in the opposite direction to $\vec{v}$ with length $|c| \cdot \|\vec{v}\|$.
3. If $c = 0$, $c\vec{v}$ is the zero vector.

Ex:



Ex: Find $\vec{v} + 2\vec{w}$.

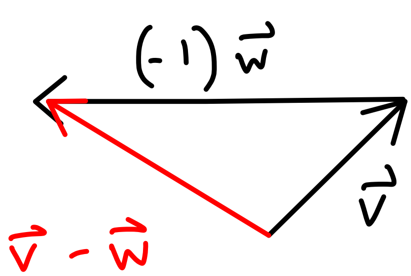




Comment: To subtract vectors, multiply one by (-1) and add them.

Ex: Find $\vec{v} - \vec{w}$.

First find $(-1)\vec{w}$. Then $\vec{v} - \vec{w} = \vec{v} + (-1)\vec{w}$



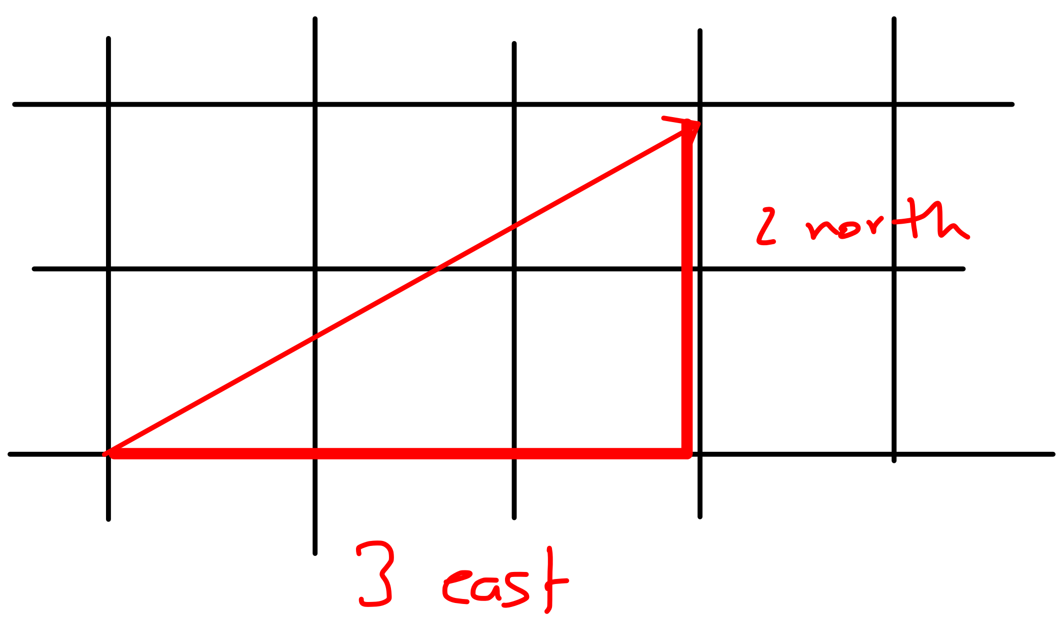
<u>Theorem</u> (Properties of vectors): Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be vectors and c and d scalars.

1. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.
2. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$.
3. $\vec{v} + \vec{0} = \vec{v}$.
4. $\vec{v} - \vec{v} = \vec{0}$.
5. $0 \cdot \vec{v} = \vec{0}$.
6. $1 \cdot \vec{v} = \vec{v}$.
7. $(cd)\vec{v} = c(d\vec{v})$.
8. $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$.
9. $(c+d)\vec{v} = c\vec{v} + d\vec{v}$.
10. If $\|\vec{v}\| = 0$, then $\vec{v} = \vec{0}$.
11. $\|c\vec{v}\| = |c| \cdot \|\vec{v}\|$.

Vectors as Algebraic Objects

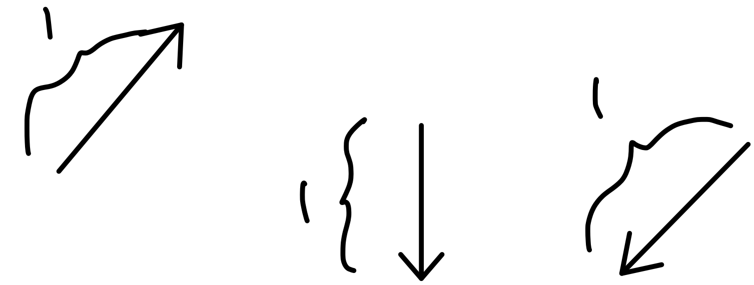
Comment: In a city grid, a vector that describes a trip from one part of the city to another could be written as a bunch of blocks.

Ex

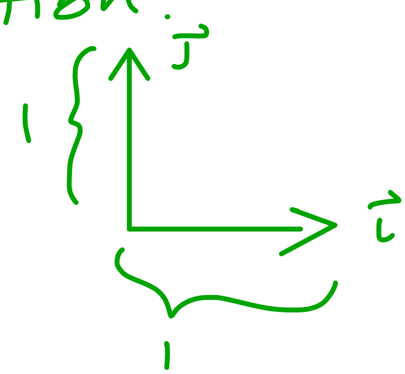


Def: A <u>unit vector</u> is a vector $\vec{v}$ with $\|\vec{v}\| = 1$.

Ex:



Def: The first two <u>standard unit vectors</u> are $\vec{\imath}$ and $\vec{\jmath}$. $\vec{\imath}$ is the unit vector in the positive-x direction and $\vec{\jmath}$ is the unit vector in the positive-y direction.



<u>Def</u>: The unit vector decomposition of a vector $\vec{v}$ is a sum of $\vec{i}$ and $\vec{j}$ that equals $\vec{v}$.

<u>Ex</u>: Find the unit vector decomposition of each vector

$\vec{y}$

$\vec{u} = \vec{i} + \vec{j}$

$\vec{v} = 2\vec{i} - \vec{j}$

$\vec{w} = -6\vec{i} - 2\vec{j}$

$\vec{x} = -4\vec{j}$