

Practice Final

Math 252

Exercise 1:

- a) What is the order of a differential equation?
- b) Given a continuous function f , what is an antiderivative of f ?
- c) What does it mean for $\int_0^\infty f(x) dx$ to converge?

Exercise 2: Let R be the region bounded by the x -axis and the curve $y = \cos(x)$ on $[\frac{\pi}{2}, \frac{3\pi}{2}]$. Find the centroid of R .

Exercise 3: A thin rope of length 1 meter has linear mass density of $\rho(x) = x^2 e^{5x}$ milligrams per meter, x meters from the left endpoint of the rope. What is the mass of the rope? Include units.

Exercise 4: Is $y = e^{3x}$ a solution to the differential equation $\frac{y'}{y} = x + y - e^{3x}$?

Exercise 5: A mug of tea is initially at $210^\circ F$ and is placed in a room at $72^\circ F$. The temperature in the mug is initially declining at a rate of $4^\circ F$ per minute. Find the temperature of the mug after 15 minutes.

Hint: think carefully about how to turn the initial cooling rate into an equation.

Exercise 6: Find the general solution to the differential equation $y'(t) = \frac{\ln|t|}{ty^4}$.

Exercise 7: Find the arc length of the curve $f(x) = \sqrt{1-x^2}$ on $\left[0, \frac{\sqrt{3}}{2}\right]$.

Exercise 8: Find the average value of $\frac{1}{\sqrt{1-x^2}}$ on $\left[0, \frac{1}{2}\right]$.

Exercise 9: Compute $\int_0^\infty xe^{-x^2} dx$. Show all your work and use good notation.

Exercise 10: (Adapted from a bonus problem) In a branch of math called Number Theory, a famous theorem states that for a fixed large number x , the probability of a random positive integer less than or equal to x being a prime number is approximately $\frac{1}{\ln(x)}$. Therefore, to count the number of primes less than or equal to x , a good approximation is $\int_2^x \frac{1}{\ln(t)} dt$. Unfortunately, there isn't an exact solution for this integral, so we just call it $\text{Li}(x)$.

a) Show that

$$\text{Li}(x) = \frac{x}{\ln(x)} - \frac{2}{\ln(2)} + \int_2^x \frac{1}{\ln^2(t)} dt.$$

b) The term $\int_2^x \frac{1}{\ln^2(t)} dt$ is called the error term because it is small compared to $\text{Li}(x)$. Specifically,

$$\lim_{x \rightarrow \infty} \frac{\int_2^x \frac{1}{\ln^2(t)} dt}{\text{Li}(x)} = 0.$$

Show this (hint: use L'Hôpital).