Name: _____ Homework 7 | Math 253 | Cruz Godar

Due Wednesday of Week 8 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–3, find a power series for the function using power series multiplication and determine its interval of convergence.

1. $\left(\dfrac{1}{1-x}\right)\left(\dfrac{1}{1+x}\right)$.

2. $\left(\dfrac{1}{1-x}\right)\left(\dfrac{1}{(1+x)^2}\right)$.

3. $\left(\dfrac{1}{(1-x)^2}\right)\left(\dfrac{1}{(1+x)^2}\right)$.

In problems 4–5, find a power series for the given function using calculus and determine its interval of convergence.

4. $\ln\left(1+x^2\right)$.

5. $\dfrac{1}{(1+2x)^4}$.

6. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} c_n x^n$ on $(-R, R)$, so that the series does not converge at $x = R$. Is it possible for the series for $f'(x)$ to converge at $x = R$? Give an example of a series where this is true or justify why no such series can exist.

7. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} c_n x^n$ on $(-R, R)$, so that the series does not converge at $x = R$. Is it possible for the series for $\displaystyle\int f(x)\,dx$ to converge at $x = R$? Give an example of a series where this is true or justify why no such series can exist.

8. Let $f(x) = \sin(x)$.

a) What are $f(0)$ and $f'(0)$?

b) Suppose $f(x) = \displaystyle\sum_{n=0}^{\infty} c_n x^n$. What are c_0 and c_1?

c) Since $f''(x) = -\sin(x) = -f(x)$, the coefficient sequence (c_n) satisfies a recurrence. Write that recurrence down.

d) Find an explicit formula for c_n. (Hint: handle even and odd n separately).

e) What is the interval of convergence for $\displaystyle\sum_{n=0}^{\infty} c_n x^n$?