

Name: \_\_\_\_\_

## Homework 7 | Math 253 | Cruz Godar

*Due Wednesday of Week 8 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–3, find a power series for the function using power series multiplication and determine its interval of convergence.

1.  $\left(\frac{1}{1-x}\right)\left(\frac{1}{1+x}\right).$

2.  $\left(\frac{1}{1-x}\right)\left(\frac{1}{(1+x)^2}\right).$

3.  $\left(\frac{1}{(1-x)^2}\right)\left(\frac{1}{(1+x)^2}\right).$

In problems 4–5, find a power series for the given function using calculus and determine its interval of convergence.

4.  $\ln(1+x^2).$

5.  $\frac{1}{(1+2x)^4}.$

6. Let  $f(x) = \sum_{n=0}^{\infty} c_n x^n$  on  $(-R, R)$ , so that the series does not converge at  $x = R$ . Is it possible for the series for  $f'(x)$  to converge at  $x = R$ ? Give an example of a series where this is true or justify why no such series can exist.

7. Let  $f(x) = \sum_{n=0}^{\infty} c_n x^n$  on  $(-R, R)$ , so that the series does not converge at  $x = R$ . Is it possible for the series for  $\int f(x) dx$  to converge at  $x = R$ ? Give an example of a series where this is true or justify why no such series can exist.

8. Let  $f(x) = \sin(x)$ .

a) What are  $f(0)$  and  $f'(0)$ ?

b) Suppose  $f(x) = \sum_{n=0}^{\infty} c_n x^n$ . What are  $c_0$  and  $c_1$ ?

c) Since  $f''(x) = -\sin(x) = -f(x)$ , the coefficient sequence  $(c_n)$  satisfies a recurrence. Write that recurrence down.

d) Find an explicit formula for  $c_n$ . (Hint: handle even and odd  $n$  separately).

e) What is the interval of convergence for  $\sum_{n=0}^{\infty} c_n x^n$ ?