Chapter 1

- Parent functions and their graphs
  - Lines
  - x^2, x^3, x^p
  - $1/x$, $1/x^2$, $1/x^p$
  - $\sqrt{x}$, $\sqrt[3]{x}$, $x^{1/p}$
  - e^x, $\ln(x)$
  - trig functions

- Even and odd functions
  - Definition
  - Geometric interpretation (symmetry)

- Transformations
  - Vertical and horizontal stretches, reflections, and shifts
  - The order to apply them when there's

more than one

- Periodic functions
  - Definition
  - Period, amplitude, and midline
  - Graphing them given a small section

Chapter 2

- Basic geometry
  - Finding angles via complementary, supplementary, etc.
  - Reference angles
  - Area of a triangle
  - Pythagorean theorem
  - Opposite, adjacent, hypotenuse
  - The unit circle

- The three standard trig functions
  - Definition of sin, cos, tan
  - Special angles
  - $\sin(\theta)^2 + \cos(\theta)^2 = 1$
  - Reference angles with trig functions
  - Trig functions in right triangles
  - Graphs
  - Transformations of sin and cos interpreted as coordinates on a non-unit circle
  - The arc functions

Chapter 3

- Radians
  - Definition
  - Arc length

  $\}$ basically the same
  - Trig functions of angles in radians

- Non-right triangles
  - Law of Cosines
  - Law of Sines

- Trig equations
  - Finding one solution with an arc function
  - Finding all the others

- Sinusoidal functions
  - Finding amplitude, midline, and period
  - Finding horizontal shift via a trig equation

- When and how to use the double-angle, half-angle, and sum and difference formulas (but not exactly what they all are)

Chapter 4

- Vectors as quantities that measure a change in position

- Vector arithmetic

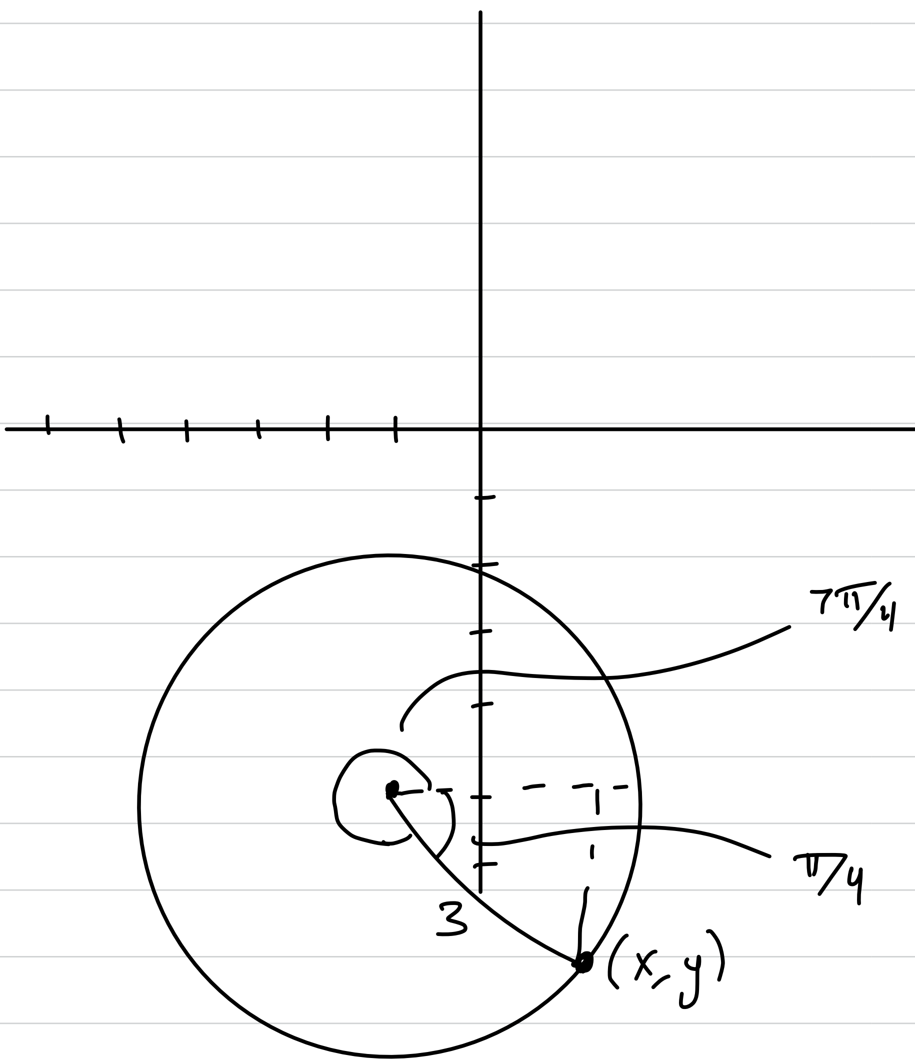
- Magnitude and direction

- Unit vector decompositions

- Changing between a unit vector decomposition and a magnitude-angle description

- The dot product
  - Unit vector definition
  - Magnitude-angle definition
  - Finding angle between vectors
  - Orthogonal vectors

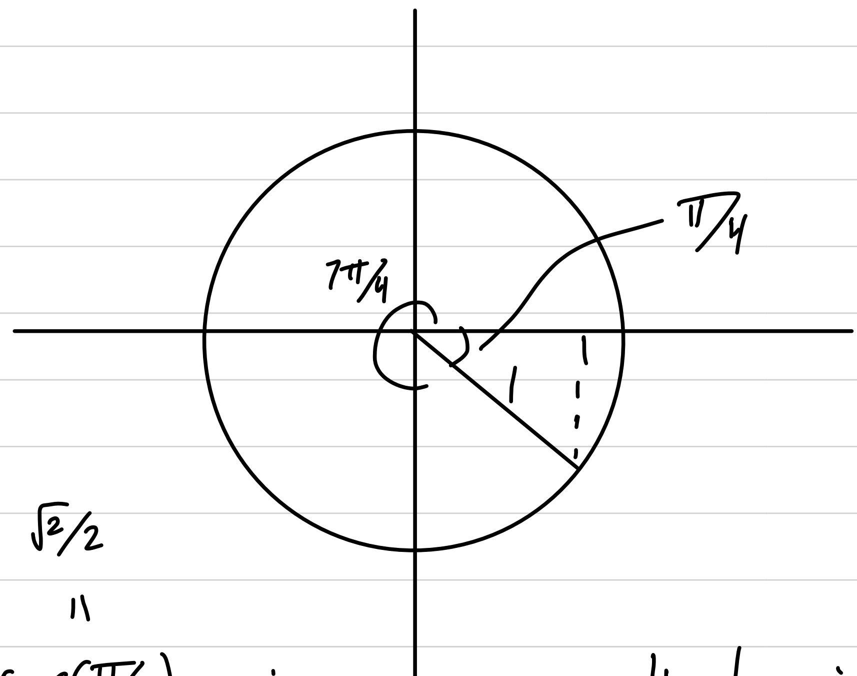
Find the coordinates of a point on a circle with radius 3 and center $(-1, -5)$, where the point has angle $7\pi/4$ from the horizontal.



$$x = 3\cos(\theta) + (-1) = 3\cos\left(\frac{7\pi}{4}\right) - 1 = 3\left(\frac{\sqrt{2}}{2}\right) - 1$$

$$y = 3\sin(\theta) + (-5) = 3\sin\left(\frac{7\pi}{4}\right) - 5 = 3\left(-\frac{\sqrt{2}}{2}\right) - 5$$

Now we need to find $\cos(7\pi/4)$ and $\sin(7\pi/4)$: so we use the unit circle



$$\cos(7\pi/4) = \underbrace{\cos(\pi/4)}_{= \sqrt{2}/2}$$ since x-coordinate is positive

$$\sin(7\pi/4) = \underbrace{-\sin(\pi/4)}_{= -\sqrt{2}/2}$$ since y-coordinate is negative

$$x = \frac{3\sqrt{2}}{2} - 1$$

$$y = -\frac{3\sqrt{2}}{2} - 5$$

Use the trig identity formulas when you're trying to find the exact value of a non-special angle

Ex: $\tan(15^\circ) = \tan\left(30^\circ/2\right) \overset{\text{half-angle}}{=} \dfrac{\sin(30^\circ)}{1+\cos(30^\circ)} = \dfrac{1/2}{1+\sqrt{3}/2}$

$\tan(15^\circ) = \tan(45^\circ - 30^\circ) \underset{\text{difference formula}}{=} \dfrac{\tan(45^\circ)-\tan(30^\circ)}{1+\tan(45^\circ)\tan(30^\circ)} = \dfrac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$

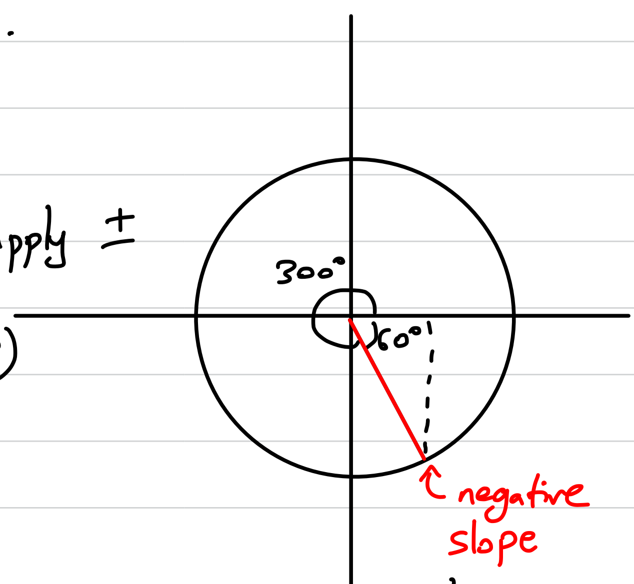
Quiz 4

Exact value of $\tan(300^\circ)$.

Either: know $\tan(60^\circ)$ and how to apply $\pm$

or

know $\sin(300^\circ)$ and $\cos(300^\circ)$



$\tan(60°) = \sqrt{3}$

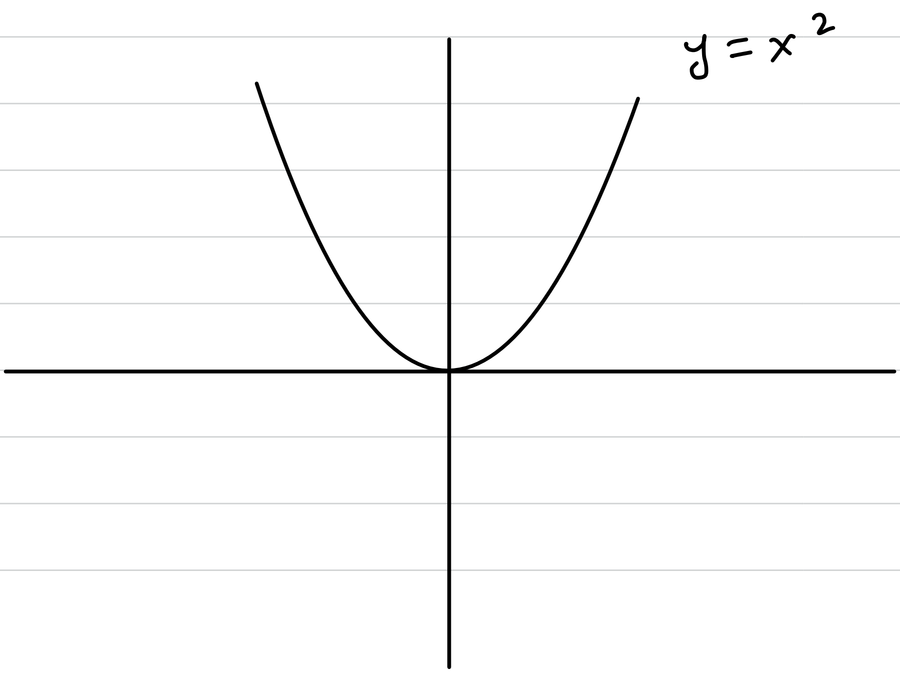
$\tan(300°) = -\sqrt{3}$

Or:

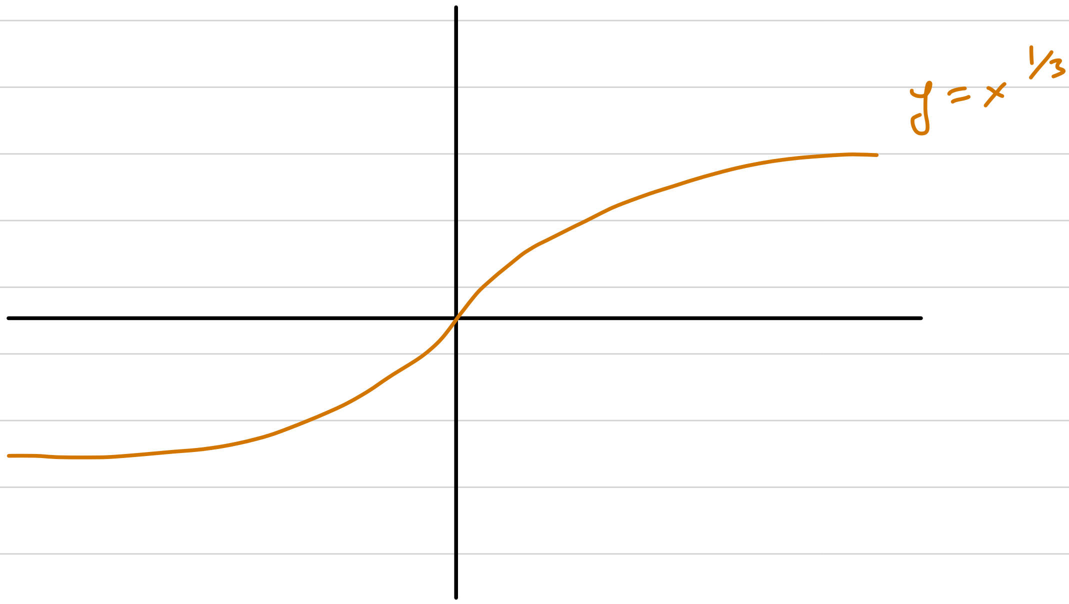
$$\tan(300°) = \frac{\overset{-\sin(60°)}{\overset{\shortparallel}{\sin(300°)}}}{\underset{\cos(60°)}{\underset{\shortparallel}{\cos(300°)}}} = \frac{-\sqrt{3}/2}{1/2} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

$y = x^{1/3}$ one-to-one ?

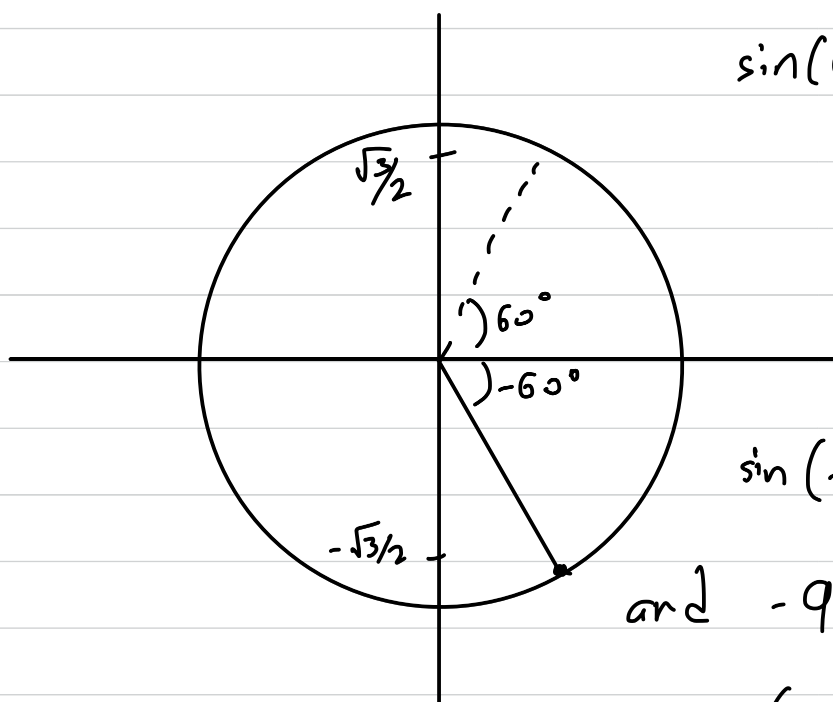
This means every y-value comes from only one x-value.



$y = x^{1/3}$ is one-to-one because it passes the HLT.



$\arcsin\left(-\frac{\sqrt{3}}{2}\right)$: this is the angle whose sin is $-\frac{\sqrt{3}}{2}$



$\sin(60^\circ) = \sqrt{3}/2$

$\sin(-60^\circ) = -\sqrt{3}/2$

and $-90^\circ \le -60^\circ \le 90^\circ$, so

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ$.

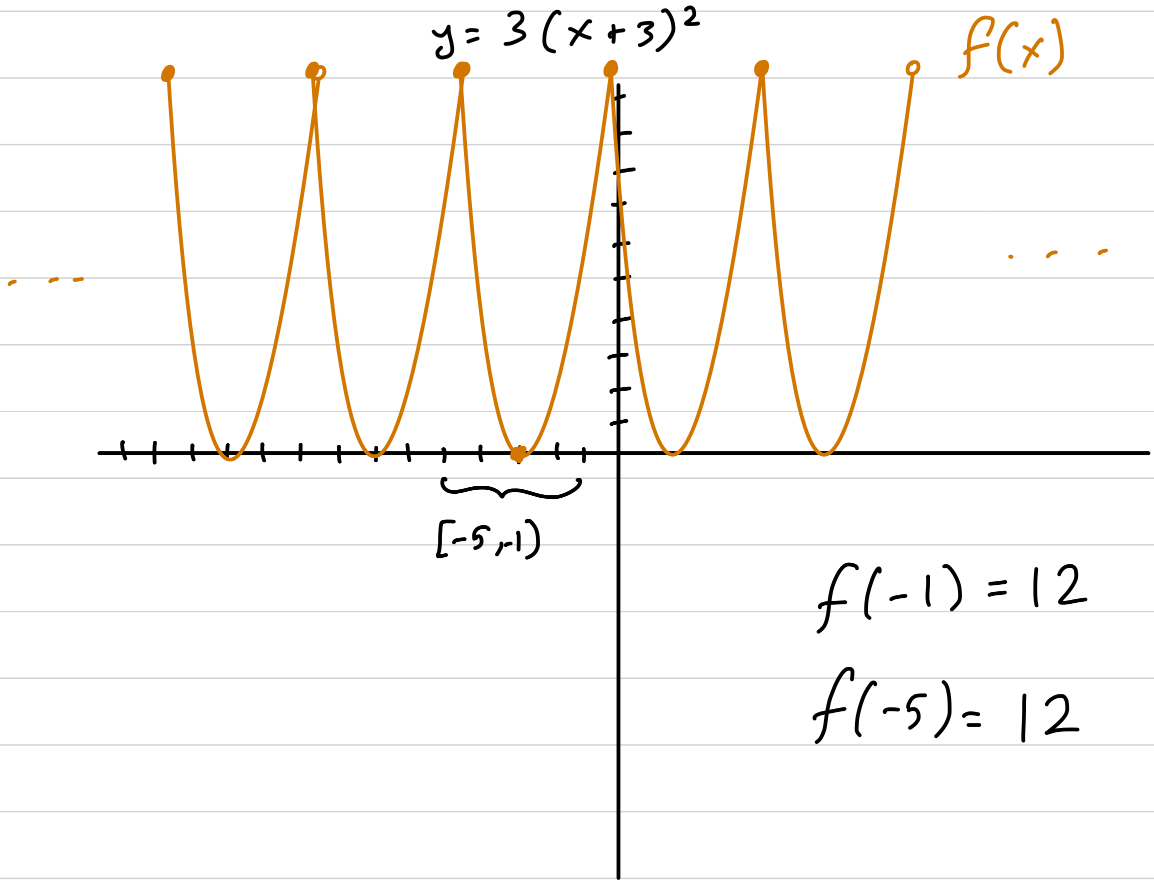
Let f be a periodic function with period 4 such that on the interval $[-5,-1)$, $f(x) = 3(x+3)^2$.

Graph f

Parent function: x^2

Horizontal shift 3 left

Vertical stretch by a factor of 3.



$f(-1) = 12$

$f(-5) = 12$

$\vec{u} = 3\vec{i} - 2\vec{j}$

$\vec{v} = (t-1)\vec{i} + (t+1)\vec{j}$

angle between $\vec{u}$ and $\vec{v}$ is $2\pi/3$ } dot product

$\|\vec{v}\| \geq 4$

$\vec{u} \bullet \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos(2\pi/3)$

$\|\vec{u}\| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\vec{u} \bullet \vec{v} = (\sqrt{13}) \|\vec{v}\| (-1/2)$

$\vec{u} \bullet \vec{v} = 3(t-1) - 2(t+1) = 3t - 3 - 2t - 2 = t - 5$

$t - 5 = (\sqrt{13}) \|\vec{v}\| (-1/2)$

$\|\vec{v}\| = \sqrt{(t-1)^2 + (t+1)^2} = \sqrt{t^2 - 2t + 1 + t^2 + 2t + 1}$

$= \sqrt{2t^2 + 2}$

$t - 5 = (\sqrt{13})(\sqrt{2t^2+2})(-1/2)$

$$\sqrt{2t^2+2} = \frac{-2t+10}{\sqrt{13}}$$

$$2t^2+2 = \left(\frac{-2t+10}{\sqrt{13}}\right)^2 = \frac{(-2t+10)^2}{13} = \frac{4t^2-40t+100}{13}$$

$$26t^2+26 = 4t^2-40t+100$$

$$22t^2+40t-74=0$$

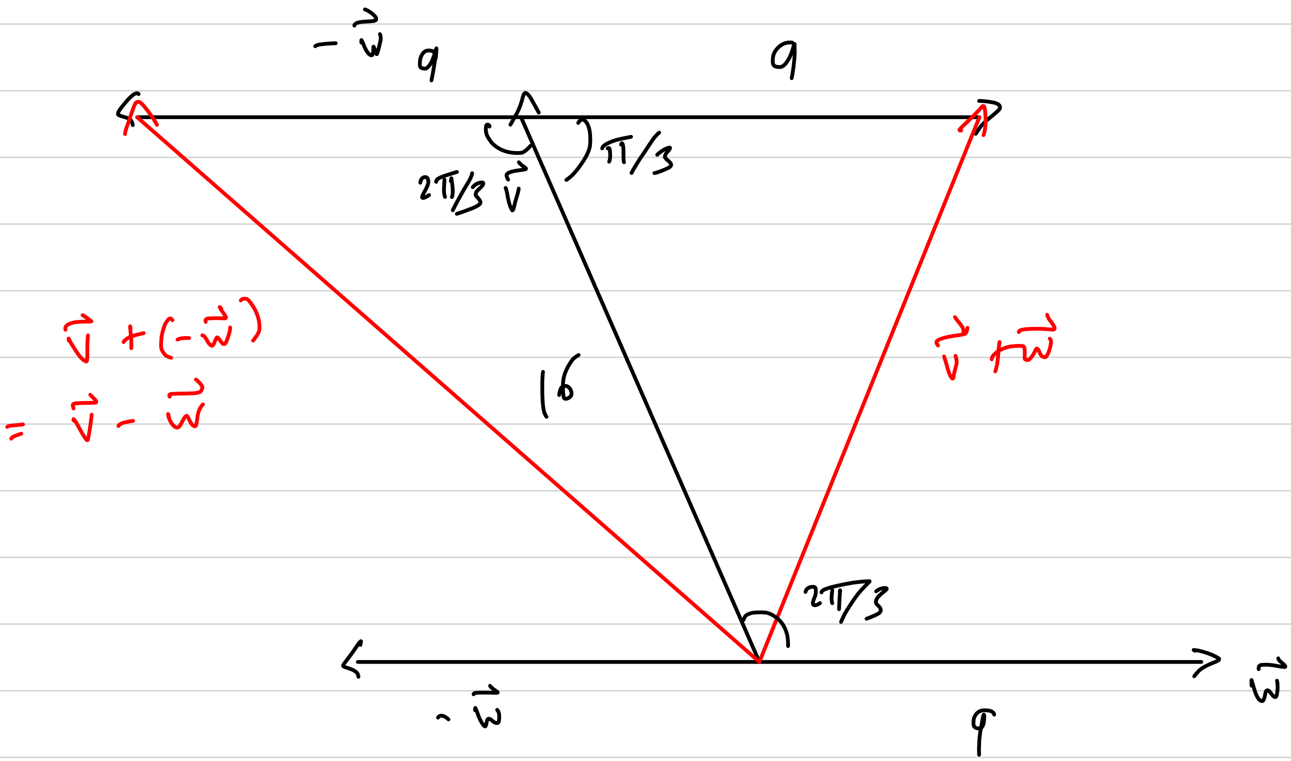
$$t = \frac{-40 \pm \sqrt{40^2-4(22)(-74)}}{2(22)} = -2.956 \text{ or } 1.138$$

$$t = \boxed{-2.956}: \quad \|\vec{v}\| = \sqrt{2t^2+2} = \sqrt{2(-2.956)^2+2} = 4.413 \checkmark$$

$$t = 1.138: \quad \cancel{\|\vec{v}\| = 2.142}$$

$\|\vec{v}\| = 16$

$\|\vec{w}\| = 9$



Let $c = \|\vec{v} - \vec{w}\|$

By LoC, $c^2 = 9^2 + 16^2 - 2 \cdot 9 \cdot 16 \cdot \cos(2\pi/3)$

$c^2 = 81 + 256 + 144$

$c = 21.93$

Find a sinusoidal function $f(x)$ such that:

① f has amplitude π, midline $-\pi$, period 3

② the graph of f passes through the point $(1,-2)$ and it is decreasing there.

$$f(x) = A\sin(B(x-h)) + k$$

$A = \pi$

$k = -\pi$

$2\pi/B = 3 \quad \Rightarrow \quad B = 2\pi/3$

$$f(1) = -2$$

$$-2 = \pi \sin\left(\frac{2\pi}{3}(1-h)\right) - \pi$$

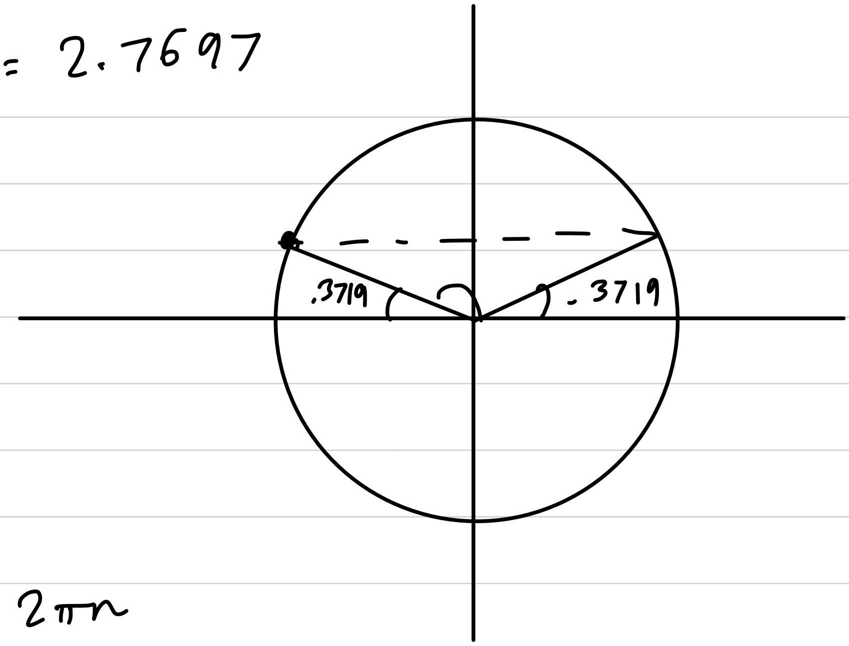
$$\frac{\pi - 2}{\pi} = \sin\left(\frac{2\pi}{3}(1-h)\right)$$

$\arcsin\left(\frac{\pi-2}{\pi}\right)$ (want to write $= \frac{2\pi}{3}(1-h)$, but that misses solutions)

$\parallel$

$.3719$

other angle $= \pi - .3719 = 2.7697$



$$\tfrac{2\pi}{3}(1-h) = .3719 + 2\pi n$$

or

$$\tfrac{2\pi}{3}(1-h) = 2.7697 + 2\pi n$$

$$(1-h) = (.3719 + 2\pi n)\left(\frac{3}{2\pi}\right) = .1776 + 3n$$

or

$$(1-h) = (2.7697 + 2\pi n)\left(\frac{3}{2\pi}\right) = 1.3224 + 3n$$

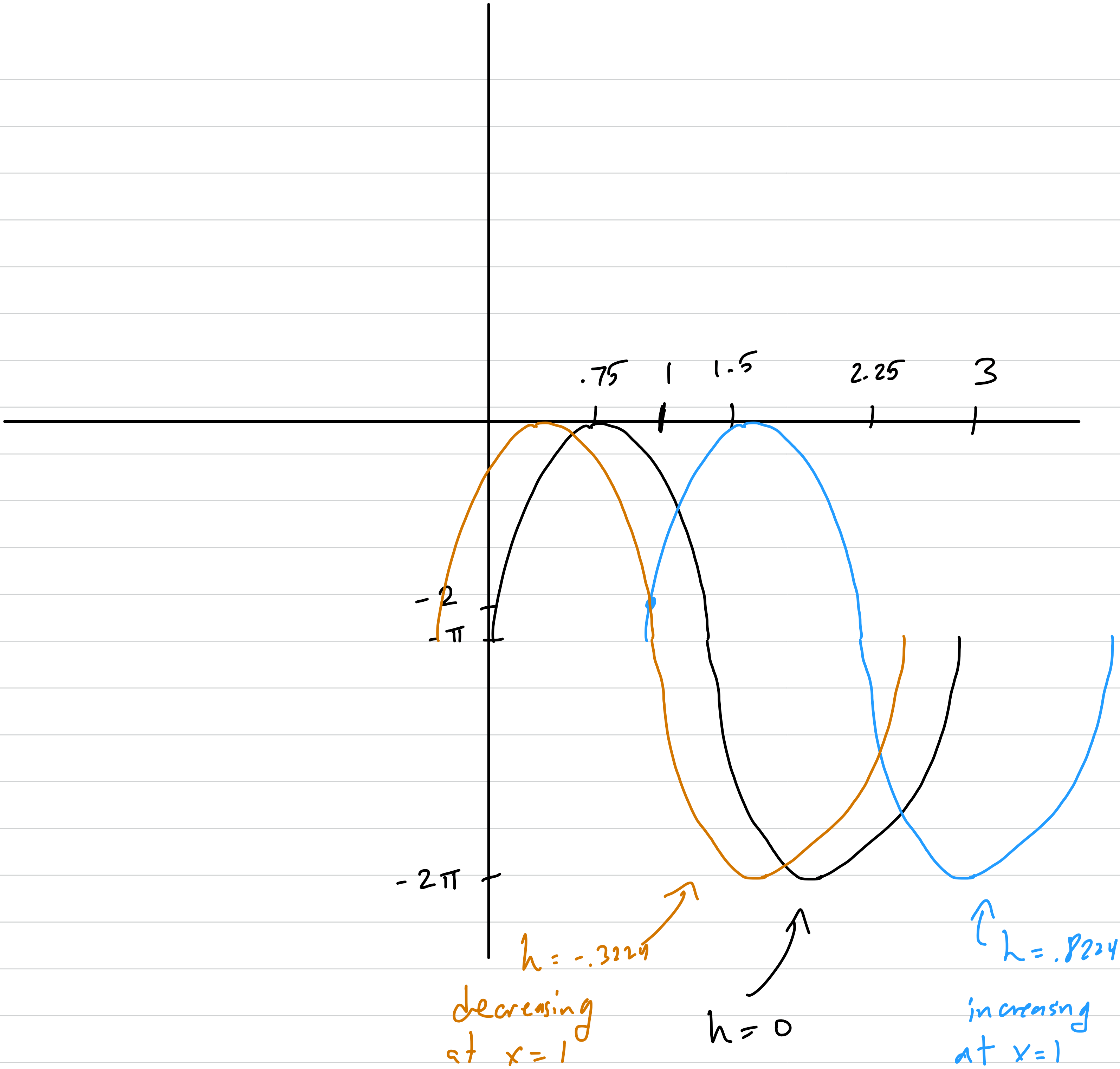
$$h = .8224 - 3n$$

or

$$h = -.3224 - 3n$$

Try $h = .8224$. $f(x) = \pi \sin\left(\frac{2\pi}{3}(x - .8224)\right) - \pi$

Is this decreasing at $(1,-2)$? We need to graph it.



h = -.3224 is what we want.

$$f(x) = \pi \sin\left(\frac{2\pi}{3}\left(x + .3224\right)\right) - \pi.$$

$\vec{v} = 2\vec{i} - 3.5\vec{j}$

$\vec{w}$ magnitude 3 and angle $2\pi/5$ clockwise from the horizontal

Find the angle between them

Need $\vec{v} \cdot \vec{w}$. Two formulas:

① $\vec{i}$ and $\vec{j}$

② magnitude and angle between ← trying to find this

$\vec{w} = 3\cos(-2\pi/5)\,\vec{i} + 3\sin(-2\pi/5)\,\vec{j}$

$.9271\,\vec{i} - 2.8532\,\vec{j}$

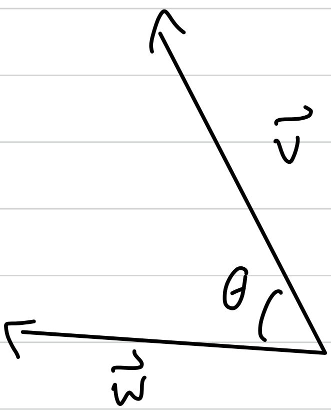
$\vec{v} \cdot \vec{w} = 2(.9271) + (-3.5)(-2.8532) = 11.8404$.

$11.8404 = \|\vec{v}\|\,\|\vec{w}\|\cos(\theta) = \left(\sqrt{2^2 + (-3.5)^2}\right)(3)\cos(\theta)$

$\cos(\theta) = .5765$

$\theta = \arccos(.5765) = .9551$

this is okay because arccos outputs angles between 0 and π, and two vectors have the angle between them always in that range.



$0 \leq \theta \leq \pi$

Reminder: fill out course eval!