Inverse Function Example

Math 111

The **suspended sediment concentration** is the concentration of sediment in a river. A certain river is flooded, and t days after the flood starts, the suspended sediment concentration is $C = S(t) = \dfrac{500t + 5000}{.25 + t} \, \dfrac{mg}{L}$.

a) Find $S(0)$ and write a sentence interpreting it.

$S(0) = \frac{5000}{.25} = 20000$. Therefore, immediately after the flood begins, the concentration os sediment is $20000\frac{mg}{L}$.

b) Find the behavior of S as $t \longrightarrow \infty$ and as $t \longrightarrow -\infty$, and for each, either write a sentence interpreting it or explain why it's not meaningful.

As $t \longrightarrow \infty$, $S(t) \longrightarrow \frac{500t}{t} = 500$. The same is true as $t \longrightarrow -\infty$. The first statement means that over time, the concentration of sediment in the river will settle down to $500\frac{mg}{L}$. The second statement is meaningless, since we have negative time.

c) Find the mathematical and practical domains of S.

The mathematical domain is $(-\infty, -.25) \cup (-.25, \infty)$, since we just need the denominator to be nonzero in a rational function. The practical domain is $[0, \infty)$, since we need positive time.

d) Using the method from Quiz 5, find the behavior of S as $t \longrightarrow -.25$ with $t > -.25$ and as $t \longrightarrow -.25$ with $t < -.25$.

Plugging in numbers close to $-.25$ but slightly bigger, we have (for example) $S(-.249) = 4875500$ and $S(-.2499) = 48750500$. It's pretty clear that as $t \longrightarrow -.25$ with $t > -.25$, $S(t) \longrightarrow \infty$. Doing the same process with numbers slightly less than $-.25$ shows us that as $t \longrightarrow -.25$ with $t < -.25$, $S(t) \longrightarrow -\infty$.

e) Find $S^{-1}(C)$ and write a sentence interpreting what it does.

We have $C = \dfrac{500t + 5000}{.25 + t}$, so $.25C + Ct = 500t + 5000$. Thus $Ct - 500t = 5000 - .25C$, and so $t = S^{-1}(C) = \dfrac{5000 - .25C}{C - 500}$. S^{-1} takes the output of S to whatever input produced that output. In this example, that means that $S(C)$ gives the number of days after which the concentration has fallen to C.

f) Find the domain of S^{-1}.

S^{-1} is also a rational function, so we just need to figure out when the denominator is nonzero. This gives us $(-\infty, 500) \cup (500, \infty)$.

g) Using your answer to part f), what must the image of S be?

The image of S is the domain of S^{-1}, so it's also $(-\infty, 500) \cup (500, \infty)$.

h) Find the behavior of S^{-1} as $C \longrightarrow \infty$ and interpret it.

As $C \longrightarrow \infty$, $S^{-1}(C) \longrightarrow \dfrac{-.25C}{C} = -.25$. This is apparently saying that if we want the time until we hit larger and larger concentrations, we should wait about $-.25$ days after the flood. This is clearly nonsense, but it should also not be too surprising: after the flood, the concentration starts at $20000\frac{mg}{L}$ and only decreases, so asking for the time until larger and larger concentrations happen (eventually over 20000) won't produce a meaningful response.