Homework 5

Math 105

Due July 14th at 11:59 PM

Textbook Exercises

3.4: 1, 3, 5, 7, 23, 24, 25, 26, 27, 28

3.5: 1, 3, 5, 7, 9, 11, 17

3.6: 1, 5, 11, 13, 15, 16, 17, 19, 21

Exercise 1: In class on Thursday, we found that the probability of being dealt a five-card hand with four aces from a 52-card deck is $\frac{48}{2598960} \approx .0000185$. Now find the probability of being dealt a hand with four aces in each of the following cases.

- The first card dealt is an ace.
- The first two cards dealt are aces.
- The first three cards dealt are aces.
- The first four cards dealt are aces.
- The first card dealt is not an ace.
- The first two cards dealt are not aces.

Exercise 2: One of (if not the) most important results in statistics is *Bayes' theorem*, which allows us to calculate $p(A \mid B)$ for events A and B, as long as we know $p(A)$, $p(B)$, and $p(B \mid A)$. The statement of the theorem isn't too complicated:

$$p(A \mid B) = \frac{p(B \mid A)p(A)}{p(B)}$$

and yet it can often lead to very surprising results. We'll explore one in this exercise.

Suppose 2% of the population uses a specific drug, and that the test for that drug is 99% accurate — that is, 1% of tests on nonusers will come back positive, and 1% of tests on users will come back negative.

a) Let A be the event that a randomly chosen person uses the drug and B the event that a test on that person comes back positive. Find and interpret $p(A)$ and $p(B \mid A)$.

b) Find and interpret $p(B)$.

c) Finally, use Bayes' theorem to find $p(A \mid B)$. Interpret it.

d) And now the most important part: find $p(A' \mid B)$, which is just $1 - p(A \mid B)$, and interpret it.

This is not a manufactured example. Some quick research shows that the incidence of heroin usage in the US is about 2%, and the standard urine test reports results with roughly 99% accuracy. Especially in the context of the country's prison system, it's a sobering thought.