Homework 2

Math 112

Due April 17th at the start of class

Textbook Exercises

1.4: 1.4.1A, 1.4.2A, 1.4.11A

1.6: 1.6.1A, 1.6.2A, 1.6.3A, 1.6.4A, 1.6.5A, 1.6.10A, 1.6.11A, 1.6.12A

Exercise 1: A bowl of boiling water is placed in a room. After t hours, its new temperature, in $°F$, is $T(t) = 212e^{-t} + 65$.

a) Sketch a graph of T.

b) We want to measure the temperature in Celsius, instead. Apply a transformation to T that accomplishes this, and graph the resulting function. Recall that $d\ °F = \frac{5}{9}(d - 32)\ °C$.

c) We also want to measure the time in minutes instead of hours. Once again, apply a transformation (on top of the one from the previous part) that does this, and graph the result.

d) Finally, we decide to perform the same experiment in space, and are forced by NASA to record all of our temperatures in Kelvin. If you're unfamiliar, Kelvin uses the same scale as Celsius, but shifted so that $0\ K$ is absolute zero. Therefore, $d\ K = d - 273\ °C$. As with the previous two parts, apply a transformation that makes the input be in Kelvin and graph it.

   Note: never say "degrees Kelvin" or write $°K$! The word degrees simply isn't ever used with Kelvin. So although it sounds a little funny, boiling water is 100 degrees Celsius and −173 Kelvin.

Exercise 2: We've talked a bit about periodic functions and given examples, but those examples have been a little contrived — either we have a function that we define to be periodic or we just draw a periodic graph. In this exercise we'll find and examine a naturally-occurring periodic function.

a) Let x be a real number. The **floor** of x, written $\lfloor x \rfloor$, is the largest integer that is less than or equal to x. For example, $\lfloor 3.5 \rfloor = 3$. Find $\lfloor 1.2 \rfloor$, $\lfloor 4 \rfloor$, and $\lfloor -2.01 \rfloor$.

b) Let's consider the function $f(x) = x - \lfloor x \rfloor$. Find $f(3.5)$, $f(1.2)$, $f(4)$, and $f(-2.01)$, and make a guess at what the function f actually does.

c) Using your answer to the previous part, graph f. Take care to draw open and closed circles when the function jumps around to indicate where the endpoints are located.

d) If all went well, you should now have a periodic function. Find its period, midline, and amplitude, and make sure they match up with the graph.

Bonus: The function in exercise 2 doesn't actually have a maximum y-value, but finding its midline and amplitude wasn't a problem. Rather than taking M to be the maximum value the function achieves and m the minimum one, is there a slightly better choice of M and m we could have used that would give us the same definition of midline and amplitude but also work on functions like the one in exercise 2?