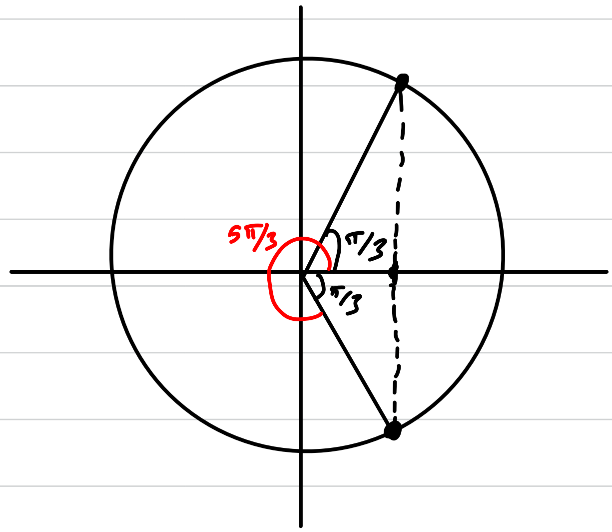
Quiz 6

$$\arccos(1/2) = \pi/3$$

① $$2\cos(3x-1) = 1$$

$$\cos(3x-1) = 1/2$$



$$3x - 1 = \pi/3 + 2\pi n$$

or

$$3x - 1 = 5\pi/3 + 2\pi n$$

$$\boxed{\begin{aligned} x &= \pi/9 + 1/3 + \frac{2\pi n}{3} \\ &\text{or} \\ x &= 5\pi/9 + 1/3 + \frac{2\pi n}{3} \end{aligned}}$$

② $n = 0$: $x = \boxed{\pi/9 + 1/3} = .682$ ✓ or $x = \boxed{5\pi/9 + 1/3} = 2.079$ ✓

$n = 1$: $x = \boxed{\pi/9 + 1/3 + \frac{2\pi}{3}} = 2.777$ ✓ or $x = 5\pi/9 + \frac{1}{3} + \frac{2\pi}{3} = 4.173$ ✗

$n = -1$: $x = \boxed{\pi/9 + 1/3 - \frac{2\pi}{3}} = -1.412$ ✓ or $x = \boxed{5\pi/9 + \frac{1}{3} - \frac{2\pi}{3}} = -.016$ ✓

$n = -2$: $x = \pi/9 + 1/3 - \frac{4\pi}{3} = -3.506$ ✗ or $x = \boxed{5\pi/9 + 1/3 - 4\pi/3} = -2.11$ ✓

③ f sinusoidal

amplitude 2
midline -1
period 2
contains $(1,1)$

$$f(\theta) = A\sin\left(B(\theta - h)\right) + k$$

A = amplitude $\qquad A = 2$
k = midline $\qquad k = -1$
$2\pi/B$ = period $\qquad 2\pi/B = 2$, so $B = \pi$

$f(1) = 1$ because f contains $(1,1)$

$\arcsin(1) = \pi/2$

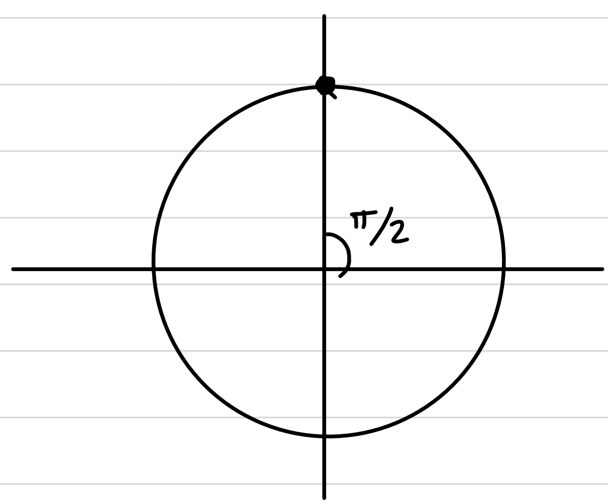
$$1 = 2\sin\left(\pi(1-h)\right) - 1$$

$$1 = \sin\left(\pi(1-h)\right)$$

$$\pi(1-h) = \pi/2 + 2\pi n$$

$$1-h = 1/2 + 2n$$

$$1 = 1/2 + 2n + h$$



$h = 1/2 - 2n$ ← pick any n since we only need one possible f, not all possible f

$n=0$: $h = 1/2$

$$f(\theta) = 2\sin(\pi(\theta - 1/2)) - 1.$$

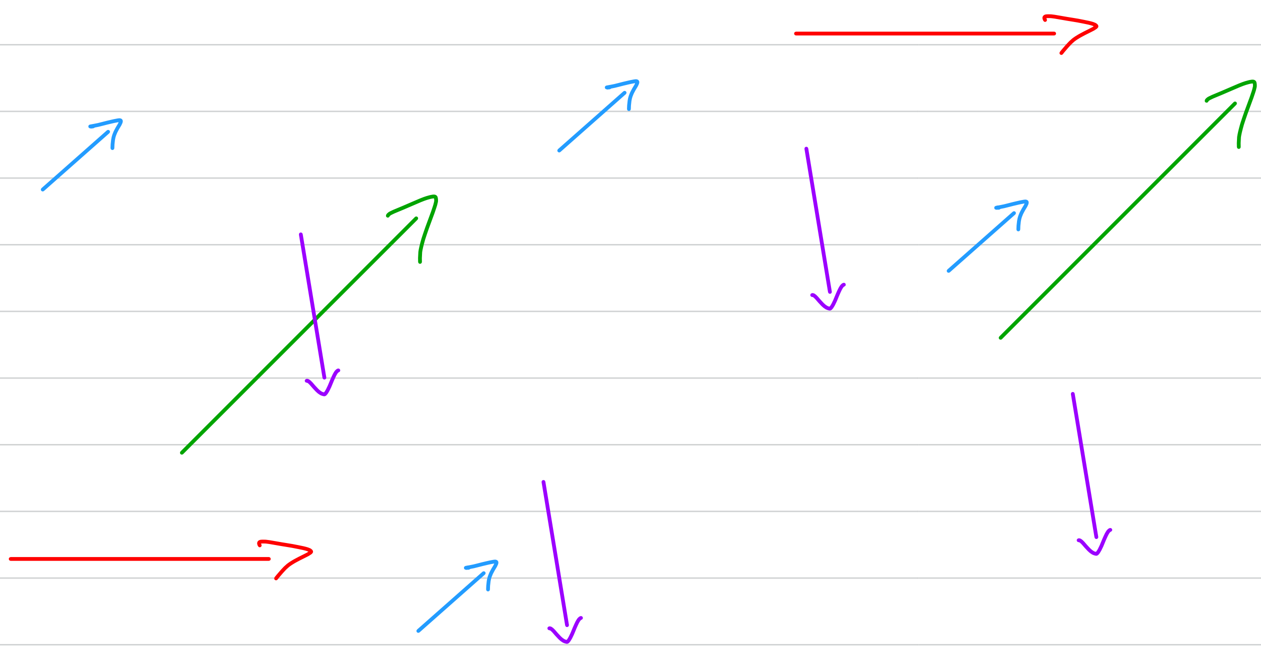
Midterm: Friday, covers through 3.5

Chapter IV: Vectors

Ex: you take a flight somewhere, starting at the Los Angeles airport. Then the information of that flight depends only on what direction you fly and how far (in particular, it doesn't depend on where you start)

Def: A vector is a quantity that consists of a direction and a magnitude. We draw them as arrows.

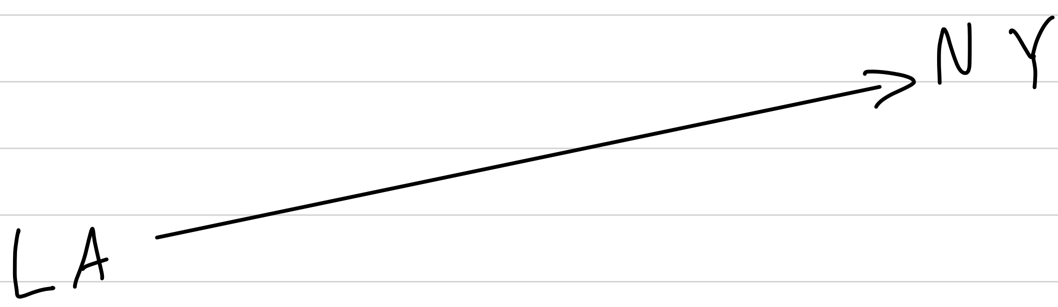
Ex: Some 2-dimensional vectors:



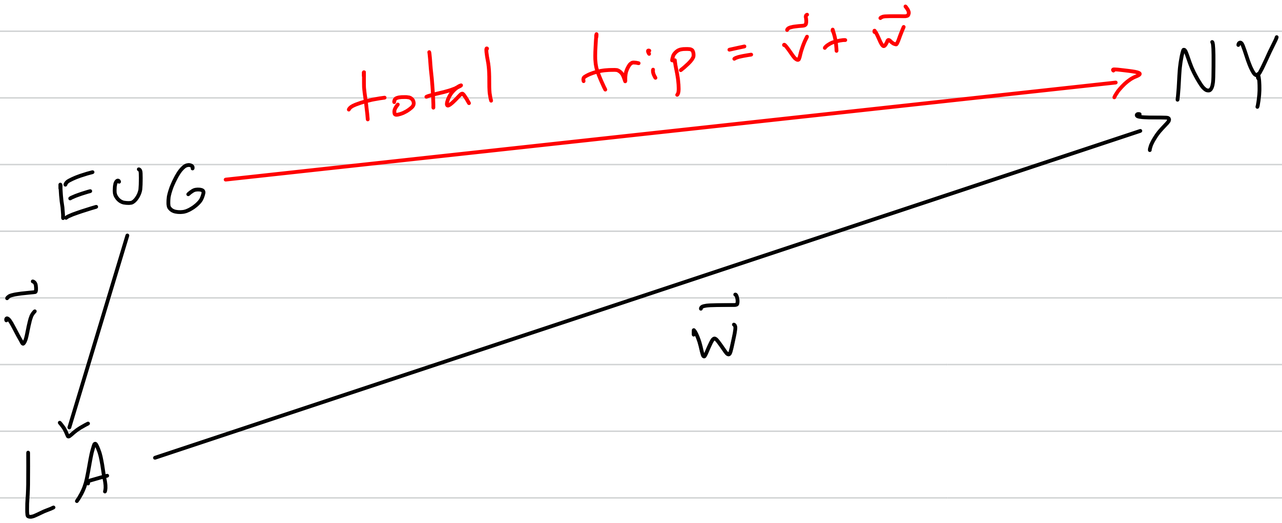
All the vectors of the same color are equal.

Comment: When we use a variable to represent a vector, we write an arrow over it — for example, $\vec{v}$, $\vec{w}$, and $\vec{x}$

Ex: For the flight from LA to NY, we have

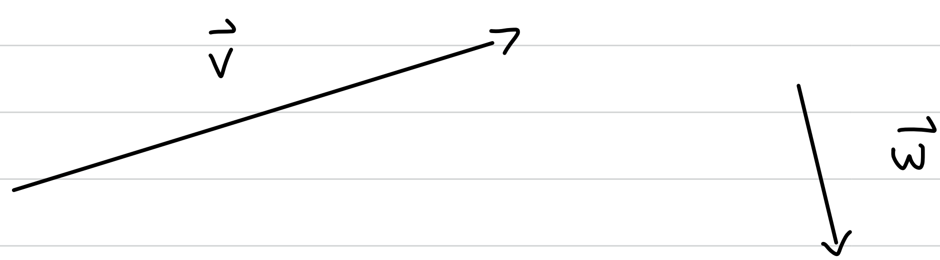


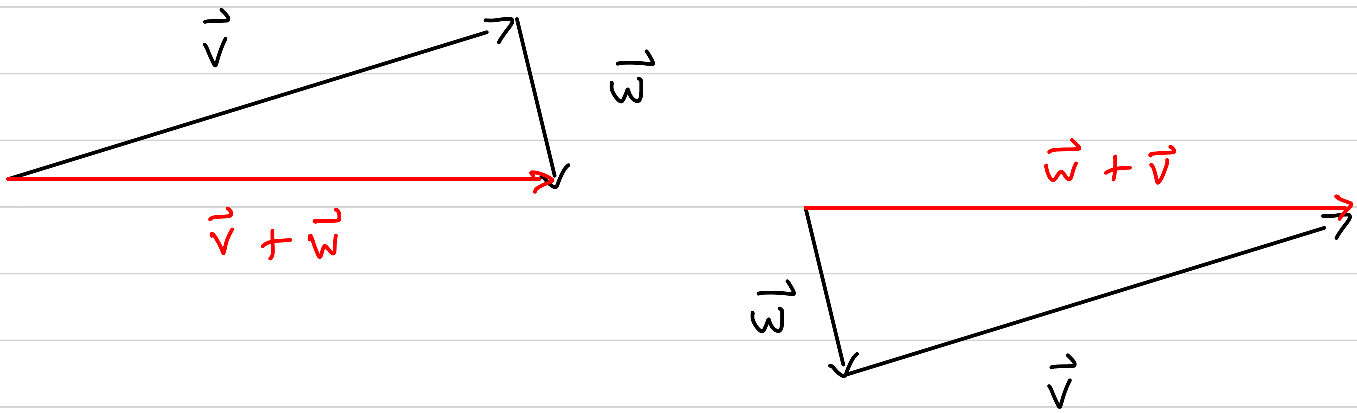
Now suppose you first flew from Eugene to LA. What is the vector corresponding to the entire trip?



<u>Def</u>: Let $\vec{v}$ and $\vec{w}$ be vectors. The sum of $\vec{v}$ and $\vec{w}$ is the vector starting at the start of $\vec{v}$ and ending at the end of $\vec{w}$, when the start of $\vec{w}$ is placed at the end of $\vec{v}$.

<u>Ex</u>:



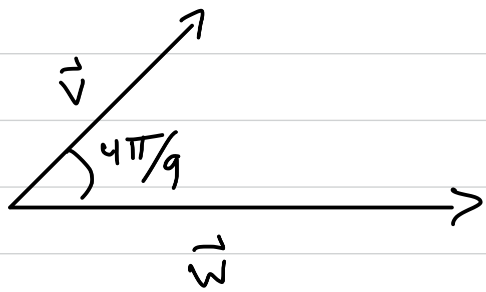


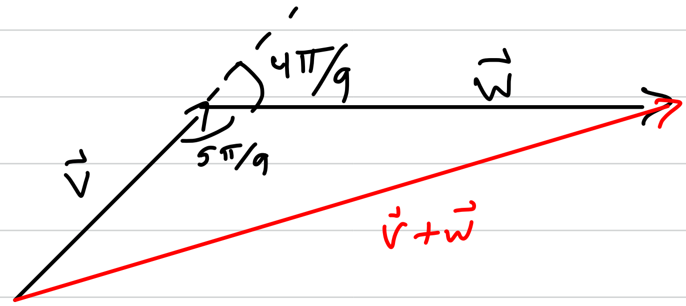
Prop: For vectors $\vec{v}$ and $\vec{w}$, $\vec{v}+\vec{w}=\vec{w}+\vec{v}$.

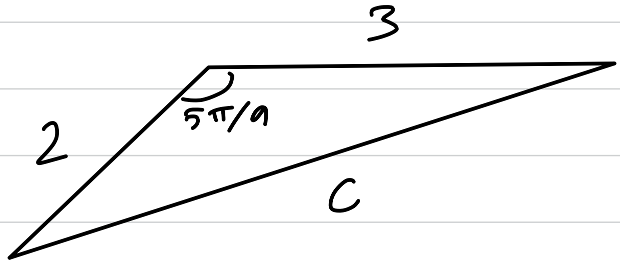
Def: Let $\vec{v}$ be a vector. The magnitude of $\vec{v}$ is the length of $\vec{v}$, written $\|\vec{v}\|$.

Comment: Think of magnitude as something similar to absolute value — you can treat a number as a vector from 0, and in that sense, the absolute value is the length. For example, $|-3|=3$ means -3 is 3 from 0.

Ex: Find $\|\vec{v}+\vec{w}\|$, given $\|\vec{v}\| = 2$ and $\|\vec{w}\| = 3$.







By the Law of Cosines,

$$c^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cos\left(5\pi/9\right)$$

$$c^2 = 4 + 9 + 2.0838 = 15.0838$$

$$c = 3.884.$$

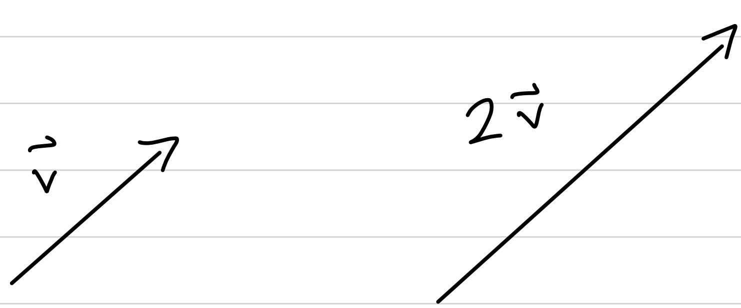
$$\|\vec{v}+\vec{w}\| = 3.884$$

Def: The <u>zero vector</u> is the unique vector with magnitude zero. It has no direction. We write $\vec{0}$.

Def: A <u>scalar</u> is a number (not a vector).

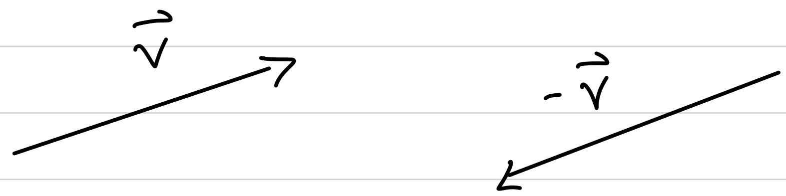
Def: Let $\vec{v}$ be a vector and c a scalar with $c>0$. The vector $c\vec{v}$ is the vector with the same direction as $\vec{v}$ and magnitude $c\,\|\vec{v}\|$.

Ex



Def: Let $\vec{v}$ be a vector. Then $-\vec{v}$ is the vector with the same magnitude as $\vec{v}$, but going in the opposite direction.

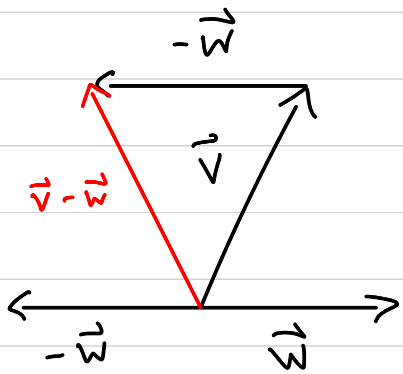
Ex:



Def: Let $\vec{v}$ be a vector. Then $0\vec{v} = \vec{0}$.

Prop: For any vector $\vec{v}$ and scalar c, $\|c\vec{v}\| = |c| \cdot \|\vec{v}\|$.

Ex: Find $\vec{v} - \vec{w}$. $\quad -\vec{w} = (-1)\vec{w}$, so $\vec{v} - \vec{w} = \vec{v} + (-1)\vec{w}$

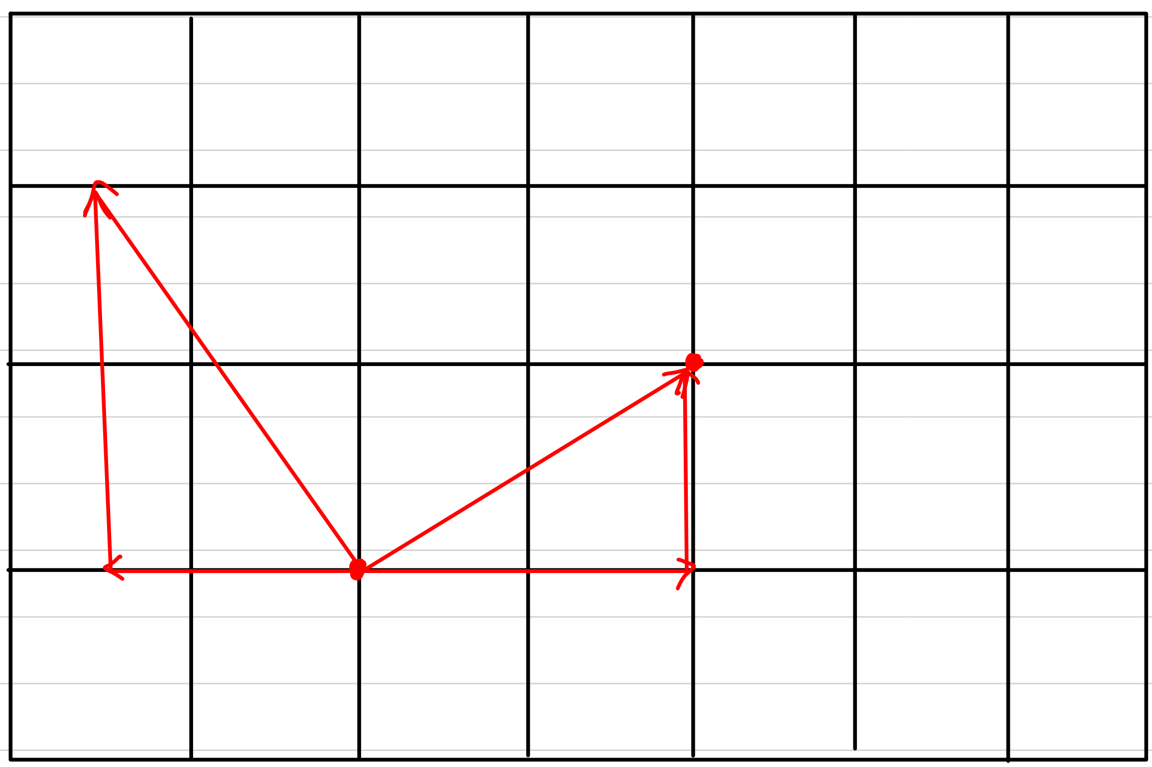


Prop:

1. $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.
2. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$.
3. $\vec{v} + \vec{0} = \vec{v}$.
4. $\vec{v} - \vec{v} = \vec{0}$.
5. $0\vec{v} = \vec{0}$.
6. $1\vec{v} = \vec{v}$.
7. $(cd)\vec{v} = c(d\vec{v})$.
8. $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$.
9. $(c+d)\vec{v} = c\vec{v} + d\vec{v}$.
10. If $\|\vec{v}\| = 0$, then $\vec{v} = \vec{0}$.
11. $\|c\vec{v}\| = |c| \cdot \|\vec{v}\|$.

Vectors as Algebraic Objects

<u>Ex</u>: In a city with street blocks, every trip can be described as some number of blocks north / south and some number east/west.



<u>Comment</u>: We'll be able to write any vector <u>uniquely</u> as a sum of east/west and north/south vectors.

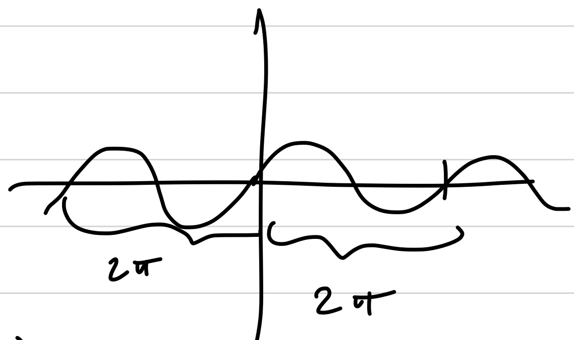
$$\cos(\theta+2\theta) = \cos(\theta)\cos(2\theta) - \sin(\theta)\sin(2\theta)$$

$$= \cos(\theta)\left(\cos(\theta)^2 - \sin(\theta)^2\right) - \sin(\theta)\left(2\sin(\theta)\cos(\theta)\right)$$

Since $\cos(\theta)$ is even, $\cos(-\theta) = \cos(\theta)$

$$\cos\left(\frac{4\pi}{5}\right) = \cos\left(-\frac{4\pi}{5}\right)$$

$$\cos(\theta+2\pi) = \cos(\theta)$$



$$\cos\left(-\frac{4\pi}{5}+2\pi\right) = \cos\left(-\frac{4\pi}{5}\right)$$

$$\cos\left(\frac{6\pi}{5}\right) = \cos\left(\frac{4\pi}{5}\right)$$

$\alpha = \frac{2\pi}{5}$ $\quad 2\alpha = \frac{4\pi}{5}$ $\quad 3\alpha = \frac{6\pi}{5}$

$$\cos(2\alpha) = \cos(3\alpha)$$

$$\cos(\alpha)^2 - \sin(\alpha)^2 = \cos(\alpha)^3 - 3\cos(\alpha)\sin(\alpha)^2$$

$$\cos(\theta)\left(\cos(\theta)^2 - \sin(\theta)^2\right) - \sin(\theta)\left(2\sin(\theta)\cos(\theta)\right)$$

$$\cos(\theta)^3 - \cos(\theta)\sin(\theta)^2 - 2\sin(\theta)^2\cos(\theta)$$

$$\cos(\theta)^3 - 3\cos(\theta)\sin(\theta)^2$$

$$\sin(\alpha)^2 = 1 - \cos(\alpha)^2$$

$$V(t) = A\sin(B(t-h)) + \cancel{k}^{\,0}$$

$M = A$

$m = -A$

amplitude: $\frac{M-m}{2} = \frac{A-(-A)}{2} = A$

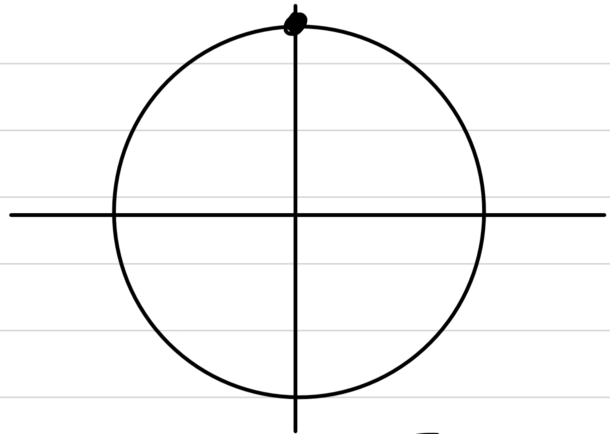
midline: $\frac{M+m}{2} = \frac{A-A}{2} = 0$

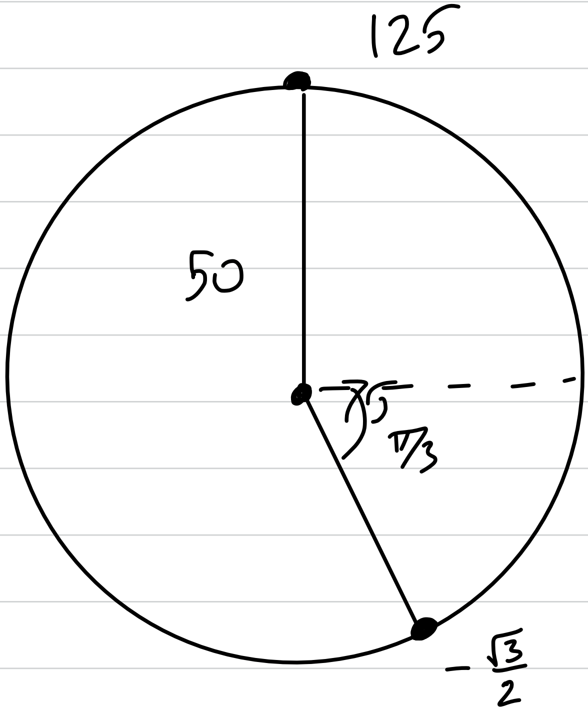
$$\frac{2\pi}{B} = \frac{1}{60}$$

$$V(0) = 0$$

$\sin\left(\frac{\pi}{6}(\ \ -h)\right) = 1 \qquad \arcsin(1) = \frac{\pi}{2}$

$\frac{\pi}{6}(-h) = \frac{\pi}{2} + 2\pi n$





$y = 50 \overbrace{\sin(B(t-h))}^{-\frac{\sqrt{3}}{2}} + 75$

$B = \frac{2\pi}{3}$

$y = 50\left(-\frac{\sqrt{3}}{2}\right) + 75 \quad = \quad -25\sqrt{3} + 75$

$-25\sqrt{3} + 75 = 50 \sin\left(\frac{2\pi}{3}(0-h)\right) + 75$

$$\cos(\underset{\alpha}{\theta} + \underset{\beta}{2\theta}) = \cos(\theta)\cos(2\theta) - \sin(\theta)\sin(2\theta)$$

$$= \cos(\theta)\left((\cos(\theta))^2 - (\sin(\theta))^2\right) - \sin(\theta)\left(2\sin(\theta)\cos(\theta)\right)$$

$$= \cos(\theta)^3 - \sin(\theta)^2\cos(\theta) - 2\sin(\theta)^2\cos(\theta)$$

$$= \cos(\theta)^3 - 3\sin(\theta)^2\cos(\theta)$$

b) $\cos(-\theta) = \cos(\theta)$ even

$\cos(\theta + 2\pi) = \cos(\theta)$ period 2π

$$\cos\left(\frac{4\pi}{5}\right) = \cos\left(-\frac{4\pi}{5}\right) = \cos\left(-\frac{4\pi}{5} + 2\pi\right)$$

$$= \cos\left(\frac{6\pi}{5}\right)$$

c) $\alpha = \frac{2\pi}{5}$ $\quad 2\alpha = \frac{4\pi}{5}$ $\quad 3\alpha = \frac{6\pi}{5}$

$$\cos(2\alpha) = \cos(3\alpha)$$

$$\cos(\alpha)^2 - \sin(\alpha)^2 = \cos(\alpha)^3 - 3\sin(\alpha)^2\cos(\alpha)$$

substitute $\sin(\alpha)^2 = 1 - \cos(\alpha)^2$

$$f(x) = A\sin(B(x-h)) + k$$

$$A = 23$$
$$k = -51$$
$$B = 2\pi/11$$

$$f(x) = 23\sin\left(2\pi/11\,(x-h)\right) - 51$$

$$f(0) = -51$$

$$-51 = 23\sin\left(2\pi/11\,(-h)\right) - 51$$

$f(x) = A\sin(B(x-h)) + k$

$A = 6$
$k = 7$
$B = 2\pi/4 = \pi/2$

$f(x) = 6\sin(\pi/2\,(x-h)) + 7$

$f(\frac{4}{3}) = 10$ f increasing when $x = 0$

$10 = 6\sin(\pi/2\,(\frac{4}{3} - h)) + 7$

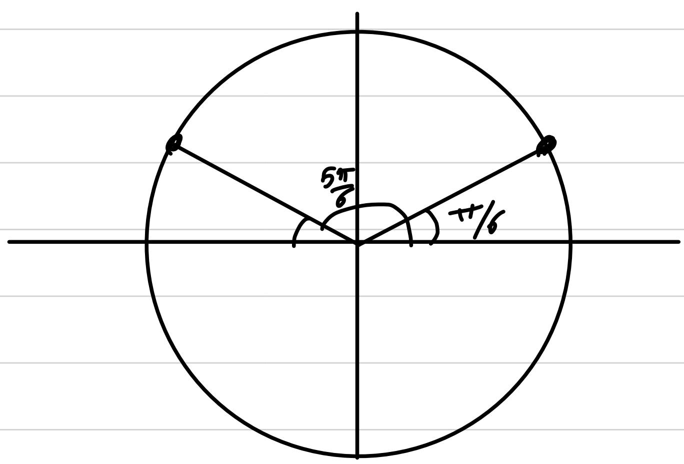
$\frac{1}{2} = \sin(\pi/2\,(4/3 - h))$

$\arcsin(1/2) = \pi/6$

$\pi/2\,(\frac{4}{3} - h) = \frac{\pi}{6} + 2\pi n$

or

$\pi/2\,(\frac{4}{3} - h) = \frac{5\pi}{6} + 2\pi n$



take n=0

$\frac{4}{3} - h = \frac{1}{3} + 4n$ $\quad h = 1 - 4n \quad \Rightarrow \quad h = 1$

or or or

$\frac{4}{3} - h = \frac{5}{3} + 4n$ $\quad h = -\frac{1}{3} - 4n \quad h = -\frac{1}{3}$

$$v(t) = A \sin\left(120\pi\,(t - h)\right)$$