Name: ______________________ Homework 4 | Math 341 | Cruz Godar

Due Wednesday of Week 5 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 4

In problems 1–3, compute $\vec{v} \bullet \vec{w}$, $||\vec{v}||$, $||\vec{w}||$, the distance between $\vec{v}$ and $\vec{w}$, and the angle between them (in radians).

1. $\vec{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$.

2. $\vec{v} = \begin{bmatrix} -7 \\ 2 \\ 0 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

3. $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ -4 \end{bmatrix}$.

4. Let $X = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Find a basis for $X^\perp$.

5. Let $X = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \\ -1 \end{bmatrix} \right\}$. Find a basis for $X^\perp$.

6. Let X be a subspace of $\mathbb{R}^n$. Show that $\dim X + \dim X^\perp = n$ by constructing a linear map whose kernel is exactly $X^\perp$. You may find it useful to recall that a matrix's image has dimension equal to the number of

linearly independent rows.

In problems 7–9, show that the set of vectors is orthogonal and then normalize them all to produce an orthonormal basis. Then express the given vector $\vec{v}$ in that basis.

7. $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} -6 \\ 3 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$.

8. $\vec{v_1} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} -1 \\ 2 \\ -4 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

9. $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ 2 \\ -6 \\ 5 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} -2 \\ 7 \\ 12 \\ 12 \end{bmatrix}$, $\vec{v_4} = \begin{bmatrix} 5 \\ -2 \\ 1 \\ 1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$.

10. Let A and B be $n \times n$ unitary matrices. Is AB always unitary? If so, explain why, and if not, give a brief counterexample.

11. Let A be a matrix whose columns are orthogonal (but not necessarily orthonormal). Does A still preserve lengths? If so, explain why, and if not, give a brief counterexample.

12. Continuing with the idea from the previous problem, let A be a matrix whose columns are orthogonal, but not necessarily orthonormal. Is A still necessarily invertible? If so, explain why, and if not, give a brief counterexample.