Final Exam Review

Math 112

Exercise 1:

a) Let $f(x) = x^2 + 6x + 9$. f is a transformation of the function $y = x^2$ — identify the transformation(s) and graph each step.

b) Apply a horizontal stretch by a factor of $\frac{1}{5}$ to f and graph the result.

Exercise 2:

a) Find $\sin(120°)$, $\cos(120°)$, and $\tan(120°)$ by drawing a picture and **using a reference angle**.

b) Find $\arcsin(-1)$, $\arccos(-1)$, and $\arctan(-1)$ in degrees. Draw a picture for each.

c) You lean a ladder up against a wall. The ladder makes an angle of 30° with the top of the wall, and the base of the ladder is 5 feet from the base of the wall. How long is the ladder, and how far does it reach up the wall?

Exercise 3:

a) What is the arc length of a 120° arc in a circle of radius 3?

b) Consider a triangle with one side of length 10, another of length 8, and an angle of $\frac{\pi}{3}$ between them. Find the third side of the triangle.

c) Find a sinusoidal function $f(x)$ with amplitude 4, midline 2, and period 1, such that the graph passes through the point $\left(\frac{1}{3}, 4\right)$ and is increasing there.

Exercise 4:

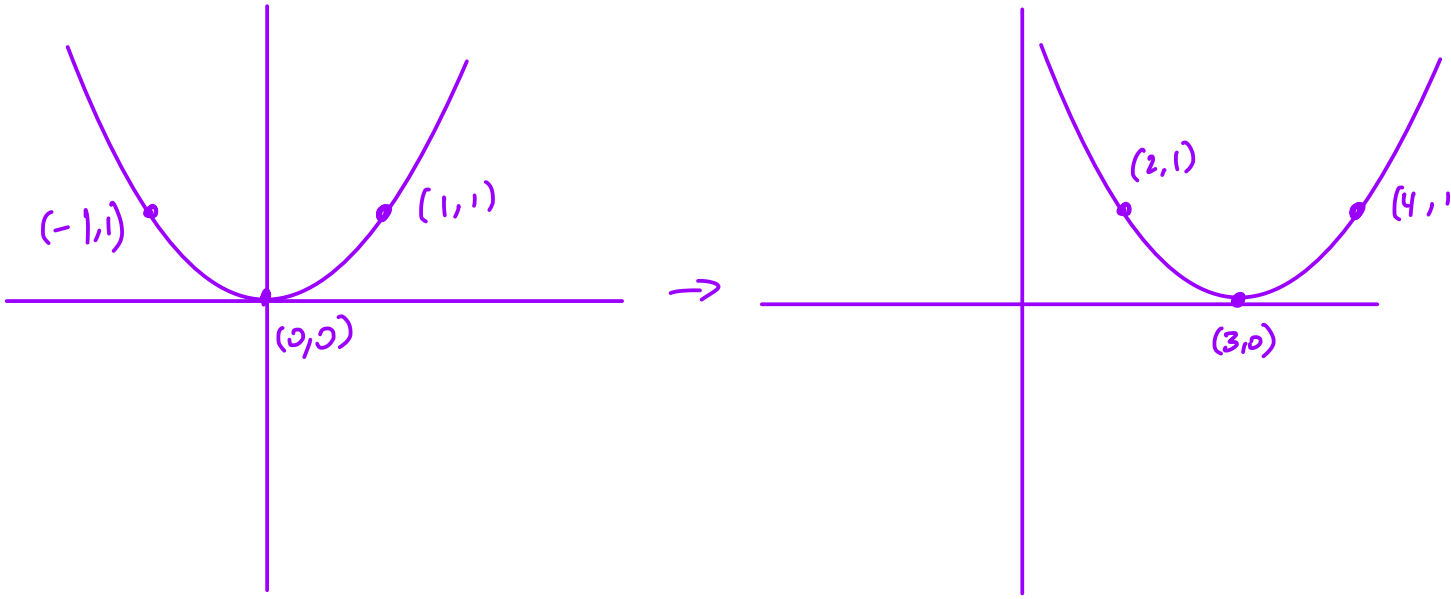
a) Find the unit vector decomposition of a vector with magnitude 7 and an angle of $\frac{5\pi}{4}$ from the horizontal.

b) Find the magnitude and direction of the vector $\vec{v} = -\sqrt{3}\vec{i} + 2\vec{j}$.

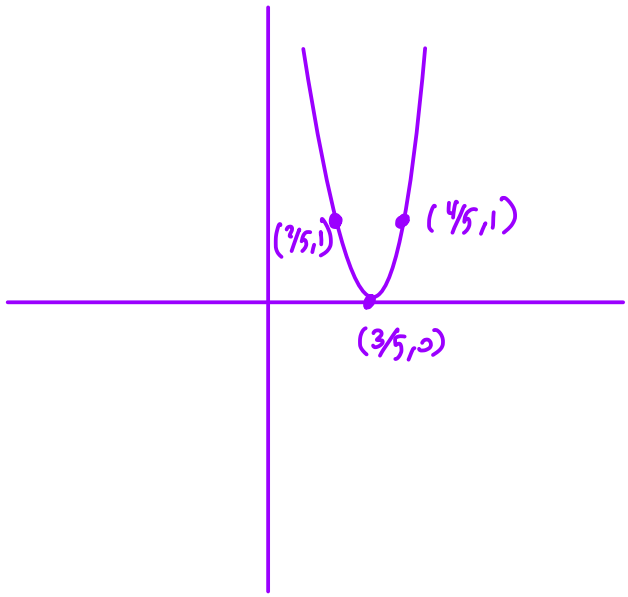
c) Find the dot product of the vectors in parts a) and b).

d) Find the angle between the vectors in parts a) and b).

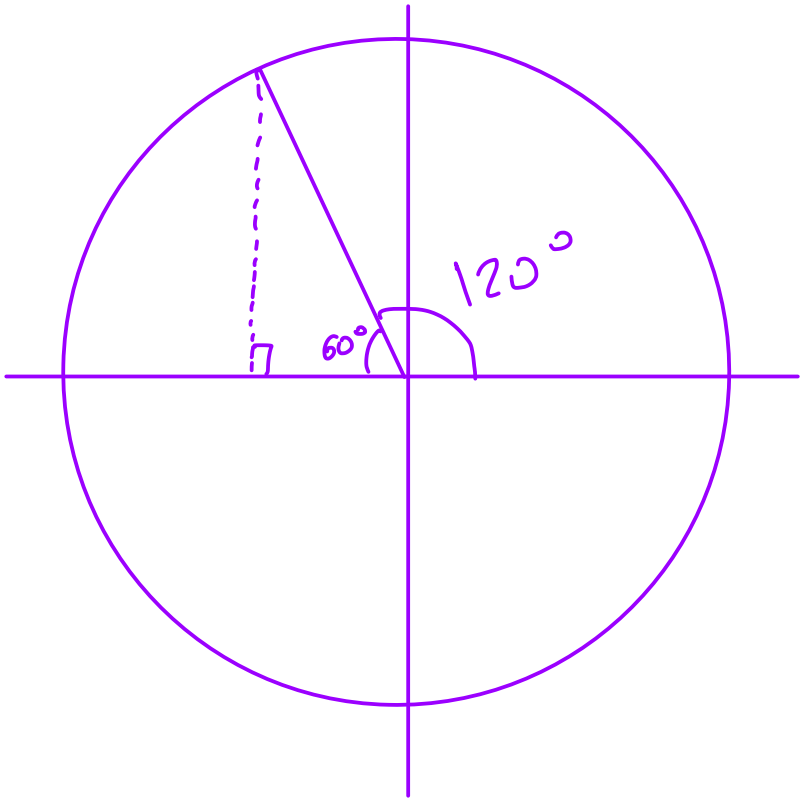
1) a) $f(x) = (x+3)^2$ $\Rightarrow$ horizontal shift 3 left



b) $y = (5x + 3)^2$



(2) a)

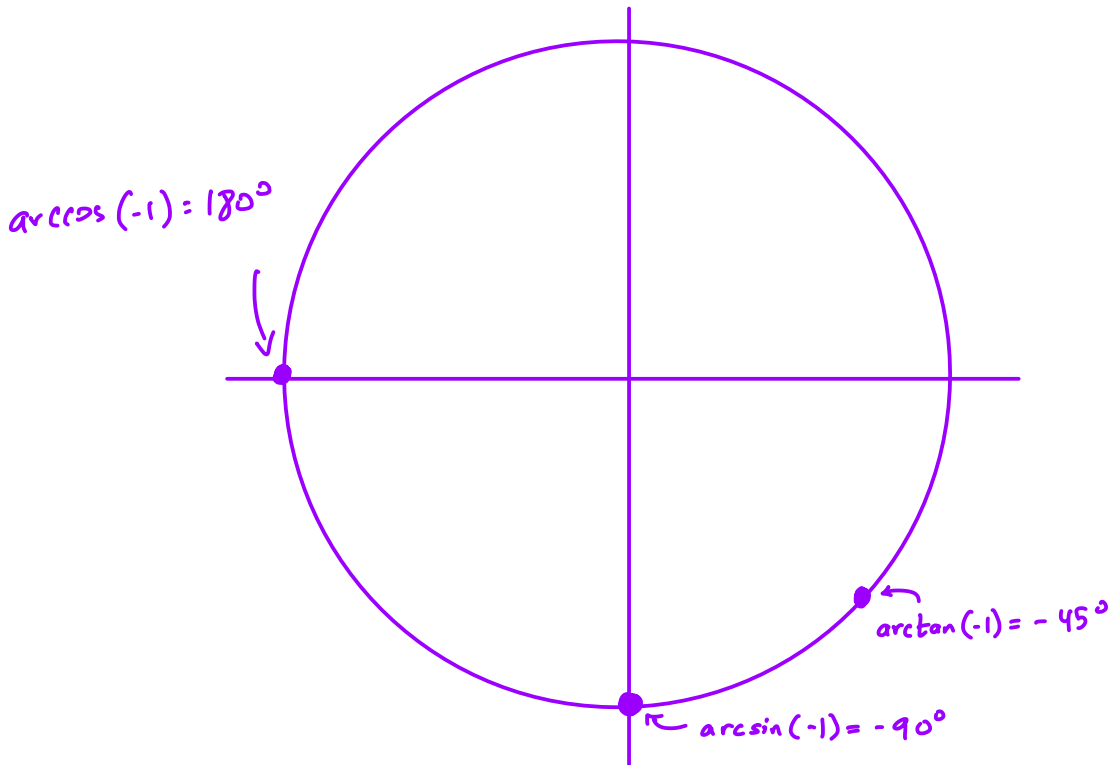


$\sin(120°) = \sin(60°) = \sqrt{3}/2$

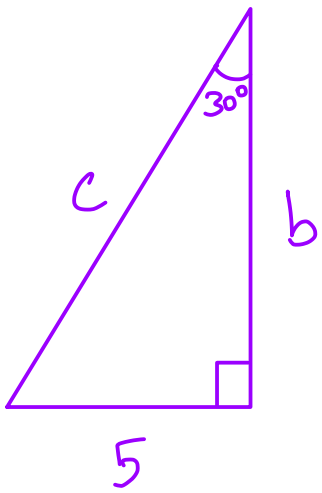
$\cos(120°) = -\cos(60°) = -1/2$

$\tan(120°) = -\tan(60°) = -\sqrt{3}$

b)



c)



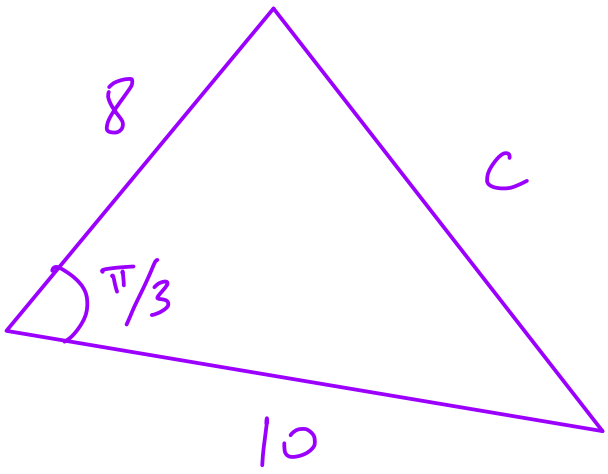
$\sin(30^\circ) = \frac{5}{c} \Rightarrow c = 10$

$\tan(30^\circ) = \frac{5}{b} \Rightarrow b = 5\sqrt{3}$

(3) a) $120^\circ = 2\pi/3$

$r\theta = 3\left(2\pi/3\right) = 2\pi$

b)



$c^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cos\left(\frac{\pi}{3}\right)$

$c^2 = 64 + 100 - 80 = 84$

$c = \sqrt{84}$

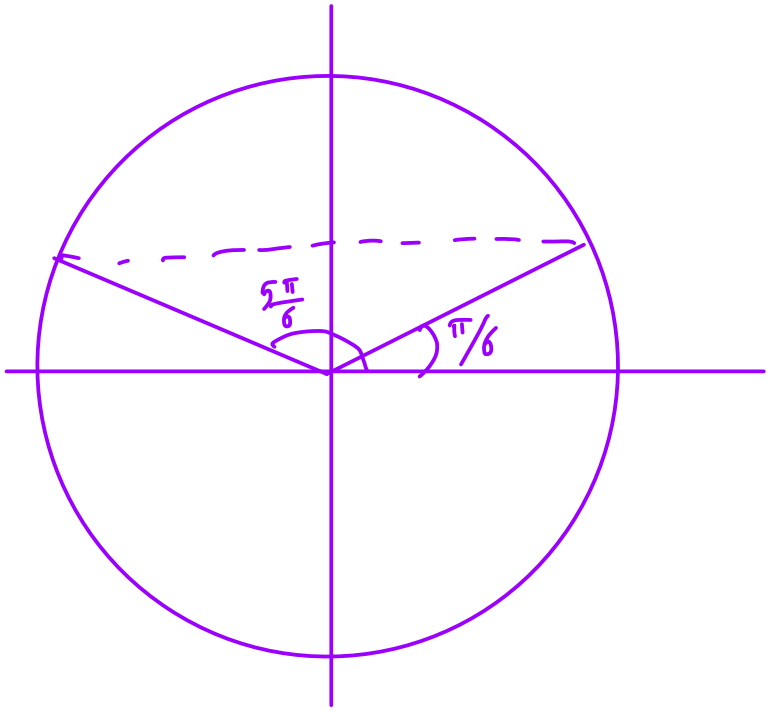
c) $A = 4 \quad k = 2 \quad \frac{2\pi}{B} = 1 \Rightarrow B = 2\pi$

$f(x) = 4\sin\left(2\pi(x-h)\right) + 2$

$$4 = 4\sin\left(2\pi\left(\frac{1}{3}-h\right)\right)+2$$

$$\sin\left(2\pi\left(\frac{1}{3}-h\right)\right) = \frac{1}{2}$$

$$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$



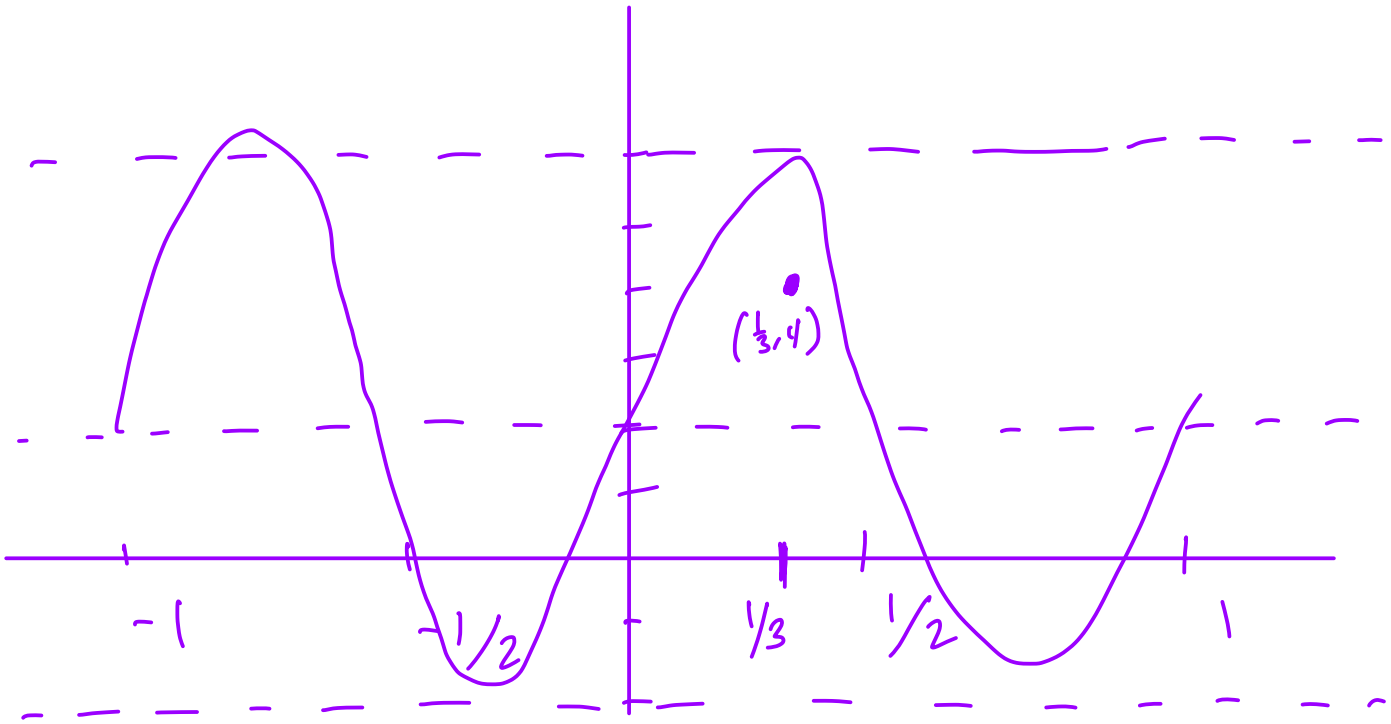
$$2\pi\left(\frac{1}{3}-h\right) = \frac{\pi}{6}+2\pi n \quad \text{or} \quad 2\pi\left(\frac{1}{3}-h\right) = \frac{5\pi}{6}+2\pi n$$

Can take $n = 0$

$$\frac{1}{3}-h = \frac{1}{12} \quad \text{or} \quad \frac{1}{3}-h = \frac{5}{12}$$

$h = \frac{1}{4}$ or $h = -\frac{1}{12}$

Graph:

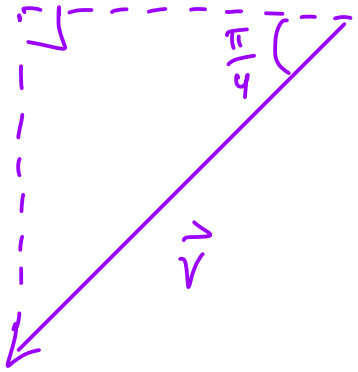


Need to move right, so $h = \frac{1}{4}$

$$f(x) = 4 \sin\left(2\pi\left(x - \tfrac{1}{4}\right)\right) + 2$$

(4) (Haven't covered all this yet)

a)

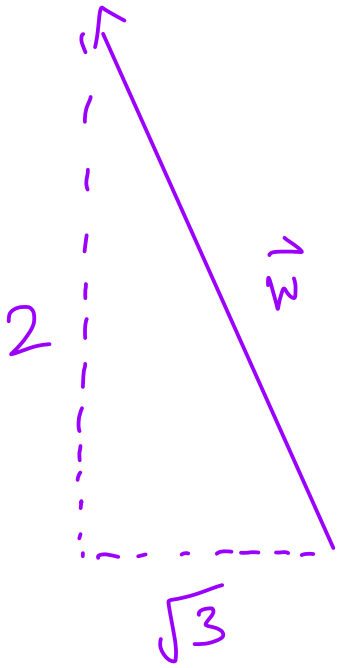


$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\Rightarrow \quad \vec{v} = -7\frac{\sqrt{2}}{2}\vec{i} - 7\frac{\sqrt{2}}{2}\vec{j}$

b)



$\|\vec{w}\| = \sqrt{2^2 + 3} = \sqrt{7}$

angle: $\arctan\left(\frac{2}{\sqrt{3}}\right) = .857$

so from the horizontal, it's $\pi - .857$

$= -2.285$

c) $\vec{v} \cdot \vec{w} = \left(-\frac{7\sqrt{2}}{2}\right)(-\sqrt{3}) + \left(-\frac{7\sqrt{2}}{2}\right)(2)$

$= \frac{7}{2}\sqrt{6} - 7\sqrt{2}$

≈ -1.326

d) $-1.326 = \|\vec{v}\| \, \|\vec{w}\| \cos(\theta)$

$-1.326 = 7\sqrt{7} \cos(\theta)$

$\theta = \arccos(-.0716)$

$= 1.642$