Midterm 1 Practice

Math 111

Exercise 1: The percentage of registered voters that vote in an election can be modeled as a function of the money spent on marketing. One model is

$$V(m) = \begin{cases} 0, & m < 0 \\ \dfrac{100m}{m+80}, & 0 \leq m \leq 720 \\ 90, & m > 720 \end{cases},$$

where m million dollars are spent on marketing and $V(m)$ percent of registered voters actually vote.

a) Find $V(50)$, $V(-5)$, and $V(1000)$. Then **interpret** the values you found or explain why they are not meaningful.

$V(50) = \dfrac{100 \cdot 50}{50+80} = \dfrac{5000}{130} = 38.46$, so about 38% of registered voters will vote when 50 million dollars are spent on marketing. $V(-5) = 0$. This isn't meaningful, since we can't spend a negative amount of money. Finally, $V(1000) = 90$, so spending 1 billion dollars will only result in 90% of the population voting.

b) How much money would need to be spent on marketing to ensure that 80% of the population that can vote does?

We want the value or values of m so that $V(m) = 80$. Clearly this can only happen in the second piece of the function, since the first and third are constant and never equal to 80. Now if $\dfrac{100m}{m+80} = 80$, $100m = 80m + 6400$, so $20m = 6400$. Thus $m = 320$, so 320 million dollars need to be spent to make 80% of registered voters vote.

c) Is it ever possible to make 100% of registered voters vote? Explain.

Now we want m so that $V(m) = 100$. As in part b), this can't happen in the first or third pieces. In the second piece, we would have $\dfrac{100m}{m+80} = 100$, so $100m = 100m + 8000$. But then $0 = 8000$, which is nonsense! Therefore, it's not possible to make everyone vote, no matter how much you spend.

d) What is the mathematical domain of M?

On the first piece, there is no division or even roots, so the only restriction is $m < 0$. Thus the domain is $(-\infty, 0)$. On the second piece, there is division — therefore, we want $m \neq -80$. We also have $0 \leq m \leq 720$, so in total, we have $[0, 720]$. The third piece is similar to the first, and its domain is $(720, \infty)$. In total, the domain of V is $(-\infty, 0) \cup [0, 720] \cup (720, \infty)$ (which is just $(-\infty, \infty)$).

e) What is the practical domain of M?

It doesn't make sense to spend a negative amount of money, so we want $m \geq 0$. There isn't a limit to how much you could possibly spend, though, so in total, we have $[0, \infty)$.

Exercise 2: The velocity of a baseball thrown straight into the air has a constant average rate of change. It is thrown with an initial velocity of 20 feet per second upward, and every second, it slows down by 32 feet per second.

a) Find a formula for the function $v(t)$ that gives the ball's velocity t seconds after it was thrown.

Since v has a constant average rate of change, it must be a linear function. Because of that, we can write $v(t) = mt + b$, where b is the velocity of the ball at $t = 0$ and m is the amount the velocity changes each second. Therefore, $v(t) = -32t + 20$.

b) The ball hits the ground after spending 2 seconds in the air. How fast is it going when it hits the ground?

This is the velocity when $t = 2$, which is $v(2) = -32(2) + 20 = -64 + 20 = -44$. Therefore, the ball is moving at 44 feet per second (downward, since the velocity is negative) when it hits the ground.

c) Using the information from part b), what is the practical domain of $v(t)$?

We certainly don't want negative time, so $t \geq 0$. Also, once the ball hits the ground, its velocity shouldn't keep decreasing like the function says it does. Therefore, $v(t)$ should only make sense for $t \leq 2$. In total, our practical domain is $[0, 2]$.

d) Find the horizontal and vertical axis intercept(s) of $v(t)$, **and interpret your answers**.

The vertical axis intercept is 20, which means that at time 0, the ball is moving at 20 feet per second. The horizontal intercept is when $v(t) = 0$, so $-32t + 20 = 0$. This occurs when $t = \frac{20}{32} = .625$ which means that the velocity of the ball has a velocity of 0 — that is, the ball hits its peak height .625 seconds after it's thrown.

Exercise 3: The function $R(p) = -2(p-5)^2 + 50$ gives the revenue (total income) a company earns from selling their items at \$$p$ each.

a) What is the mathematical domain of R? What is the practical domain?

Since there is no division or even roots, the mathematical domain is $(-\infty, \infty)$. For the practical domain, it doesn't make sense for the company to price items at a negative dollar amount, so we should require $p > 0$. Therefore, the practical domain is $(0, \infty)$.

b) Find **and interpret** the horizontal and vertical axis intercept(s).

The vertical axis intercept is $-2(0-5)^2 + 50 = 0$. This means that if the company gives all their product away for free, they won't make any money. The horizontal axis intercepts are the values of p for which $-2(p-5)^2 + 50 = 0$, so $(p-5)^2 = 25$. Thus $p - 5 = \pm\sqrt{5}$ (don't forget the $\pm$!), and so $p = 5 \pm 5 = 0$ or 10. This means that if the company *wants* to make no profit at all, then it should price its items either at \$0 or \$10 each.

c) If the company wants to maximize their revenue, how much should they charge for each item?

This is the p-coordinate of the vertex, which is $(5, 50)$ since the function is given in vertex form. Therefore, they should price their items at \$5 each to maximize profit.