Ex: Let $h(t) = 2^t$. Want a function whose graph is the graph of h shifted to the left 5 units.

$\hookrightarrow$ horizontal shift, so we need to modify the input, t.

We know that $h(t-k)$ shifts the graph to the right k units, so we want $k=-5$. So, the final function is $y = h(t+5) = 2^{t+5}$

Ex: The function $S(T) = \begin{cases} 0, & -273 \le T \le 0 \\ 1, & 0 \le T \le 100 \\ 2, & 100 \le T \end{cases}$

gives the state that water takes at standard pressure and T °C.

Write a function that does the

same, but that takes in $^\circ F$.

This is the input, so we want a horizontal transformation.

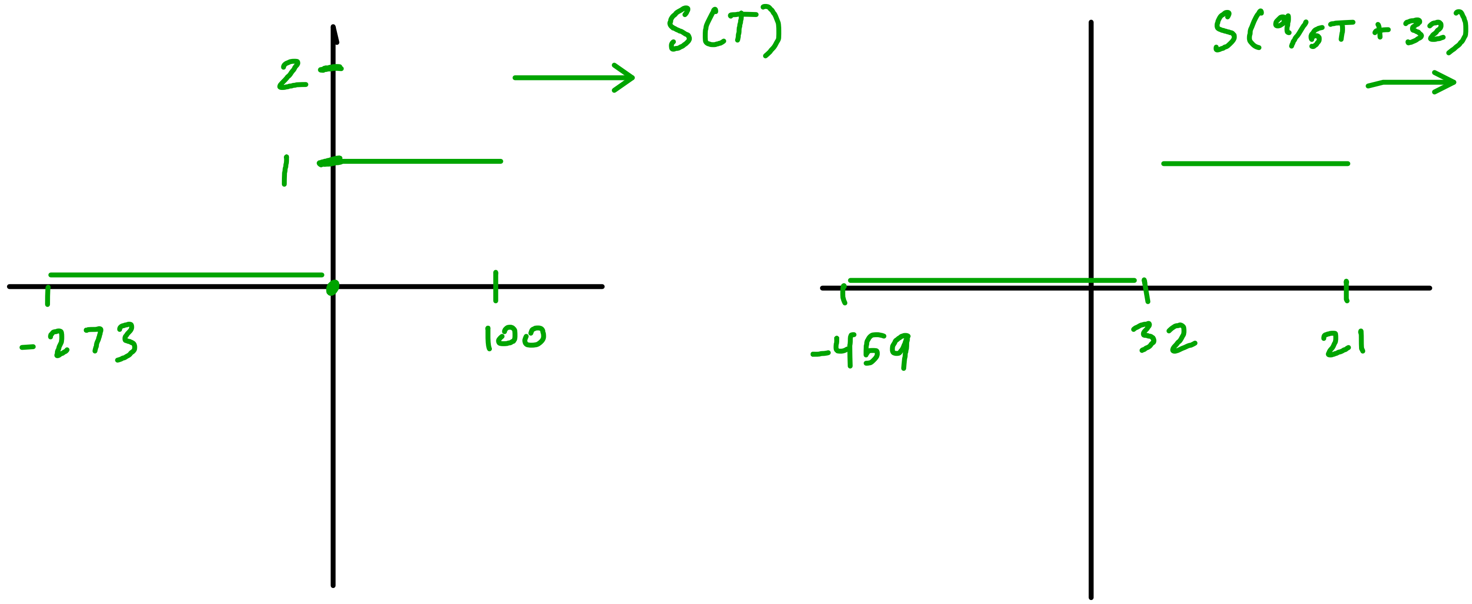
Recall: if T is in $^\circ C$, then

$\frac{9}{5}T + 32$ is T in $^\circ F$ (e.g. $0^\circ C$ is $\frac{9}{5}(0) + 32 = 32\,^\circ F$ and $100^\circ C$ is $\frac{9}{5}(100) + 32 = 180 + 32 = 212\,^\circ F$).

So the function we want is

$$y = S\left(\tfrac{9}{5}T + 32\right) = \begin{cases} 0, & -273 \le \frac{9}{5}T + 32 \le 0 \\ 1, & 0 \le \frac{9}{5}T + 32 \le 100 \\ 2, & 100 \le \frac{9}{5}T + 32 \end{cases}$$

$$= \begin{cases} 0, & -459 \le T \le 32 \\ 1, & 32 \le T \le 212 \\ 2, & 212 \le T \end{cases}$$



Comment: We can convert a T in °F to °C with the inverse function: $\frac{5}{9}(T-32)$

Combinations of Transformations

Def: Let f be a function. A function g is a transformation of f is $g(x) = a(f(b(x-h))) + k$ for some real numbers a, b, h, and k.

<u>Theorem</u>: To graph a transformation of a function f:

1. Find x and start there.
2. Preform the horizontal stretch and horizontal shift, in that order.
3. Preform the vertical stretch and vertical shift, in that order.

Ex: Graph the function $y = -2\sqrt[3]{3x-1} + 4$.

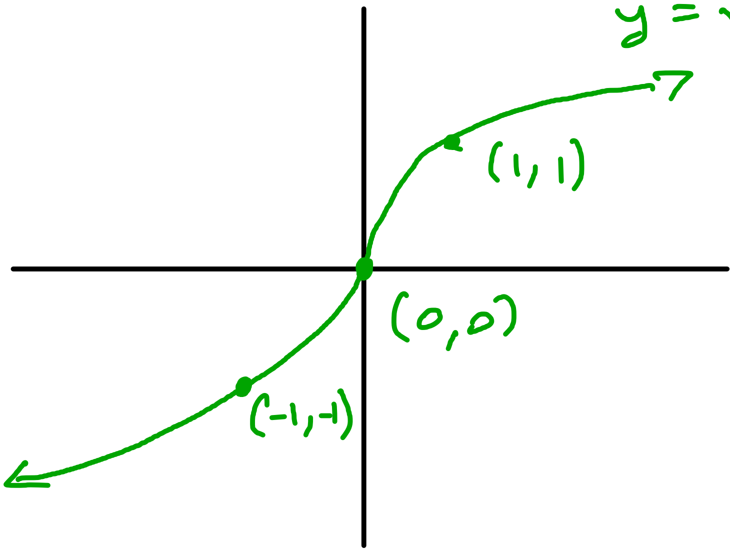
Parent function is $y = \sqrt[3]{x}$

We have both a horizontal stretch and shift, so we need to factor out the stretch. So $3x - 1 = 3(x - \frac{1}{3})$.

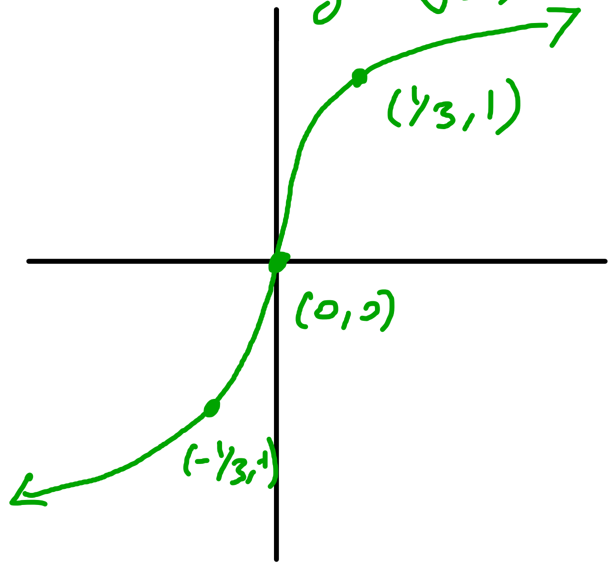
Now we will perform, in this order:

1. Horizontal stretch by a factor of $1/3$
2. Horizontal shift by $\frac{1}{3}$ units to the right
3. Vertical stretch by a factor of 2 and a vertical reflection
4. Vertical shift 4 units up.

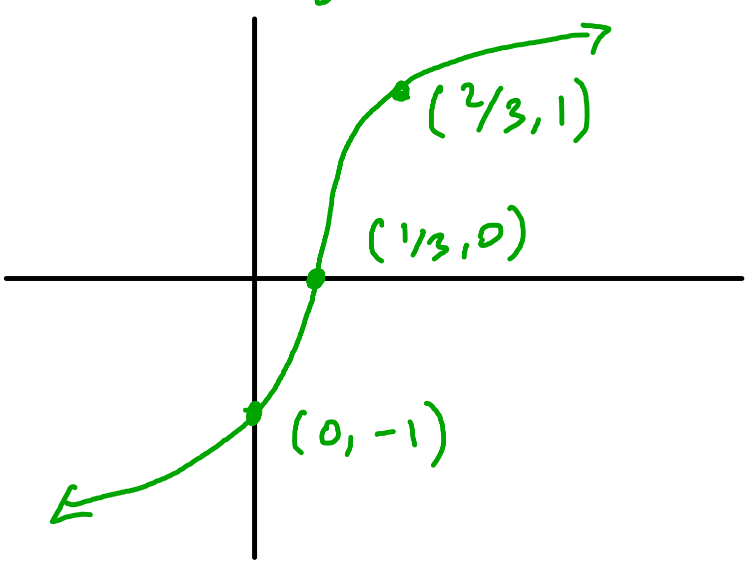
$y = \sqrt[3]{x}$

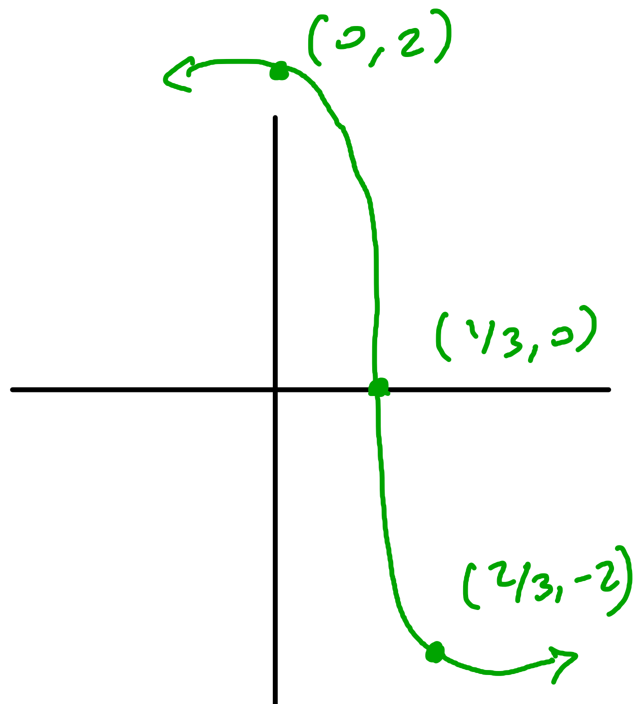


$y = \sqrt[3]{3x}$

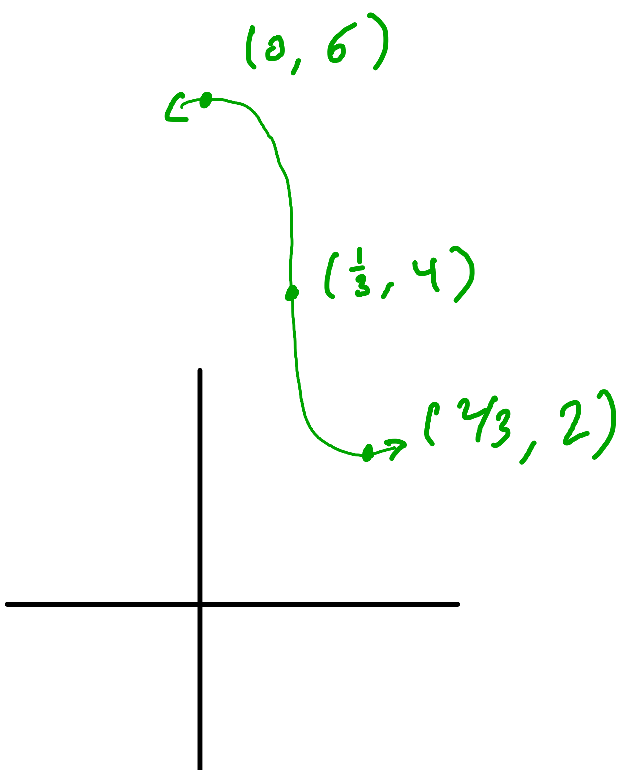


$y = \sqrt[3]{3(x - 1/3)}$





$y = -2\sqrt[3]{3(x - 1/3)}$



Ex: You burn roughly 200 calories per mile that you run. The function $C(d) = 200d$ gives the approximate number of calories burned by running d miles. Convert this to a function that takes in km and outputs Joules.

First, we know 1 mile = 1.61 km and 1 calorie = 4184 Joules. We want the function

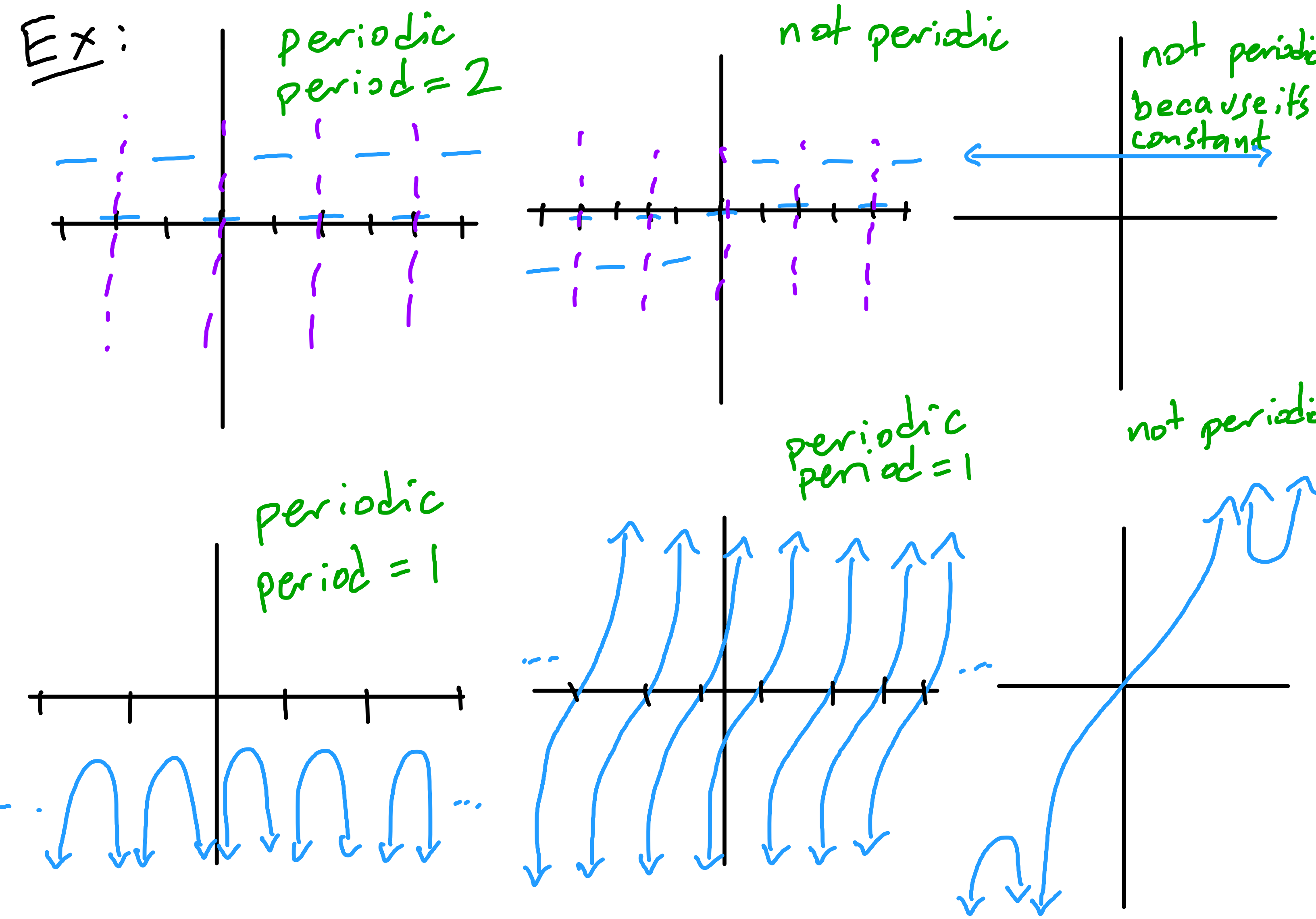
$$y = \underbrace{4184}_{\text{to J}}\left(200\left(\underbrace{\tfrac{1}{1.61}}_{\text{to miles}}\underbrace{d}_{\text{km}}\right)\right).$$

Periodic Functions

<u>Def</u>: A nonconstant function f is <u>periodic</u> if there is some number n such that for all x in the domain of f, $f(x) = f(x+n)$. The <u>period</u> of f is the smallest n that works.

<u>Comment</u>: Periodic functions "repeat" every n units. Of course, n is different for every periodic function.

Ex:



Ex: A function f is periodic with period 5. For x with $-2 \le x < 3$, $f(x) = -x^2 - 2x + 3$. Find $f(1)$, $f(-6)$, $f(3)$, all of f's roots, and sketch a graph.

Since $-2 \leq 1 < 3$, $f(1) = -(1)^2 - 2(1) + 3$

$= -1 - 2 + 3 = 0$.

-6 is not in the interval $[-2, 3)$.

However, $f(-6) = f(-6+5) = f(-1)$

$= -(-1)^2 - 2(-1) + 3 = -1 + 2 + 3 = 4$.

$f(3) = f(3-5) = f(-2) = -(-2)^2 - 2(-2) + 3$

$= -4 + 4 + 3 = 3$.

Recall that x is a <u>root</u> of a function f if $f(x) = 0$. What we can do is find the roots in $[-2, 3)$, then add or subtract 5.

So we solve $f(x)=0$, so

$$-x^2-2x+3=0.$$

$$x^2+2x-3=0$$

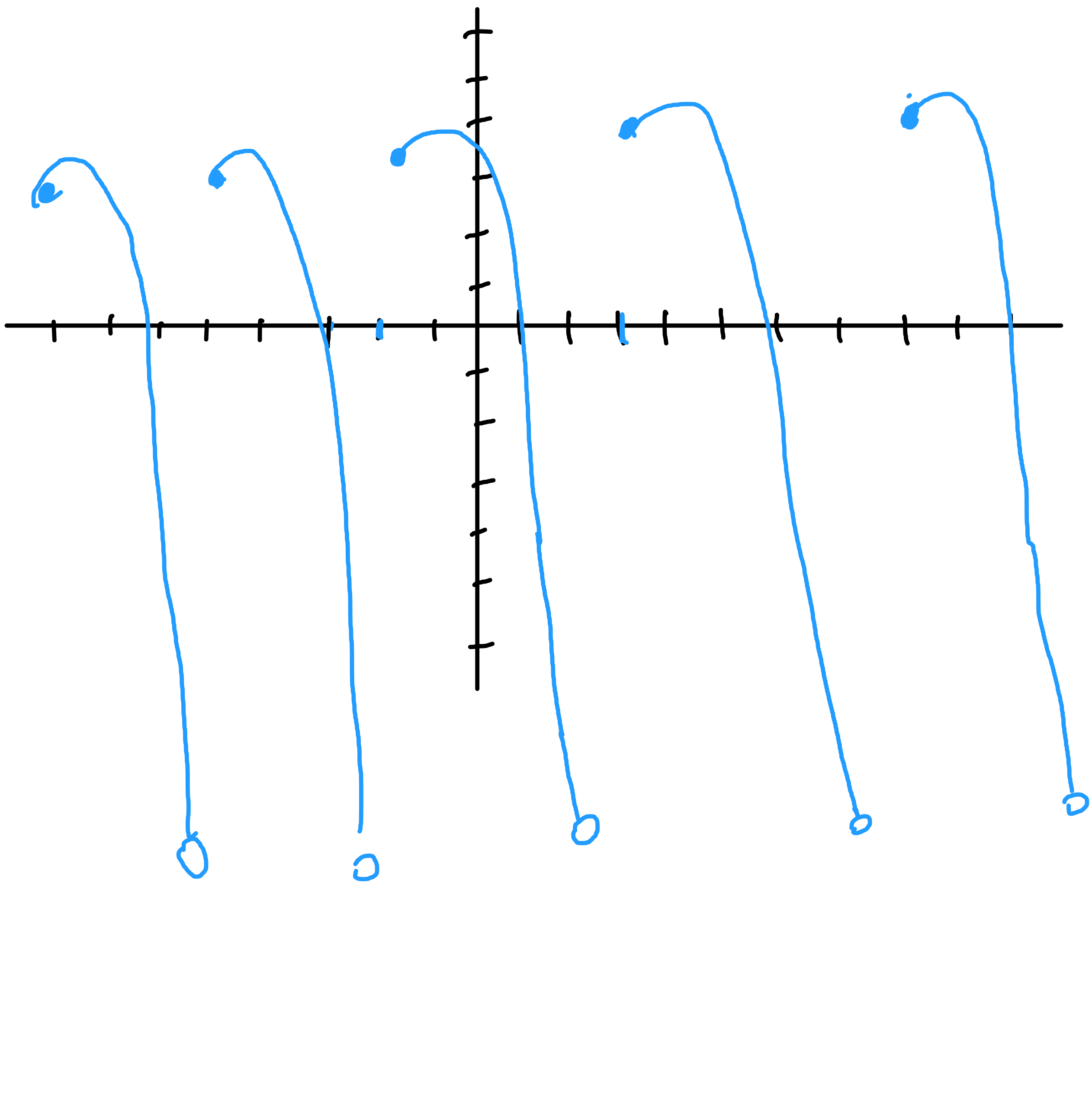
$$(x+3)(x-1)=0$$

$x=-3$ $\qquad\qquad$ $x=1$

not in $[-2, 3)$ $\qquad\qquad$ in $[-2,3)$

So the roots are $1+5n$ for any integer n. In a list, this would be $\ldots, -9, -4, 1, 6, 11, \ldots$

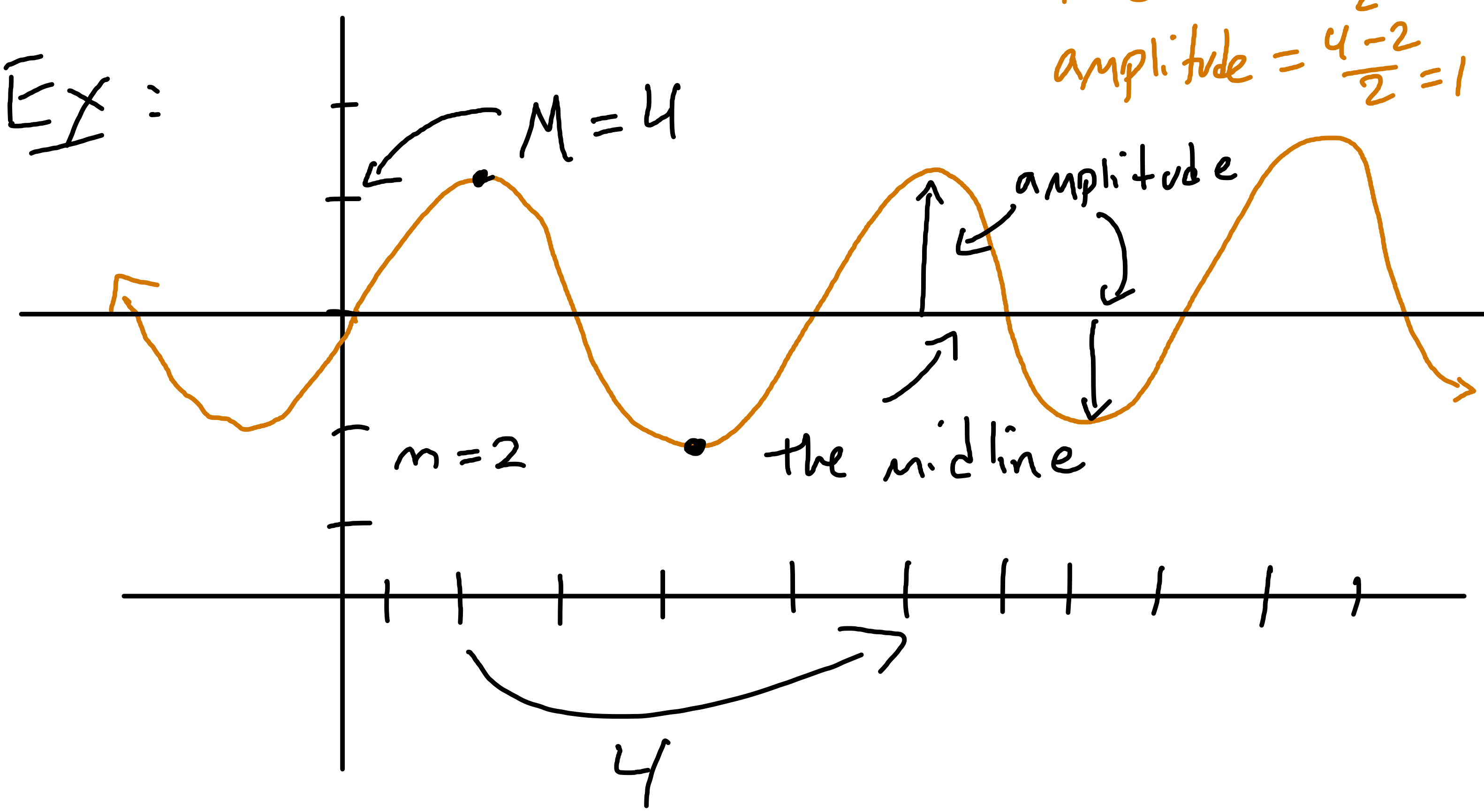
To graph f, first graph it on [-2, 3), then copy the graph and paste it every 5 units.



Def: Let f be a periodic function. If f has a maximum y-value, M, and a minimum y-value, m, we define the <u>midline</u> of f to be $\dfrac{M+m}{2}$ and the amplitude of f to be $\dfrac{M-m}{2}$.

periodic
period $= 4$
midline $= \dfrac{4+2}{2} = 3$
amplitude $= \dfrac{4-2}{2} = 1$

Ex:



Comment: The <u>midline</u> is the line through the average y-value of the function, and the amplitude is the farthest away that the function ever gets from its midline.