

Name: _____

Homework 9 | Math 256 | Cruz Godar

Due Friday of Week 10 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 11

In problems 1–4, solve the system of DEs.

1.

$$\begin{aligned}x_1' &= 2x_1 - x_2 \\x_2' &= 3x_1 - 2x_2.\end{aligned}$$

For this problem, also sketch a vector field.

2.

$$\begin{aligned}x_1' &= 19x_1 - 4x_2 + 8x_3 \\x_2' &= -8x_1 + 5x_2 - 10x_3 \\x_3' &= -x_1 - 2x_2 + 4x_3.\end{aligned}$$

3.

$$\begin{aligned}x' &= 2x + 2y - 2z \\y' &= -3x + 7y + 3z \\z' &= -5x + 5y + 5z.\end{aligned}$$

4.

$$a' = 5a + 6b + 4c - 4d$$

$$b' = 3a + 8b - 2c + 2d$$

$$c' = 3a - 3b + 9c + 2d$$

$$d' = 11d.$$

5. Two tanks are set up in a cyclical cascade. Tank 1 initially contains 100 gallons of water and 10 pounds of sugar, and tank 2 initially contains 50 gallons of water and no sugar. At time $t = 0$, two valves are opened — the well-mixed solution in tank 1 flows into tank 2 at a rate of 5 gallons per second, and the well-mixed solution in tank 2 flows *back* into tank 1 at 5 gallons per second. After one minute, what is the concentration of sugar in each tank?

6. Let $f(\lambda) = \lambda^2 + a\lambda + b$ be a polynomial. The **companion matrix** to f is the 2×2 matrix

$$\mathbf{C}(f) = \begin{bmatrix} 0 & -b \\ 1 & -a \end{bmatrix}.$$

Show that the characteristic polynomial of $\mathbf{C}(f)$ is f .

7. Using companion matrices, create 2×2 matrices with real entries and the following eigenvalues:

a) $\lambda_1 = 1, \quad \lambda_2 = 1.$

b) $\lambda_1 = 1, \quad \lambda_2 = -1.$

c) $\lambda_1 = -1, \quad \lambda_2 = -1.$

d) $\lambda_1 = 1 + 2i, \quad \lambda_2 = 1 - 2i.$

e) $\lambda_1 = -1 + 2i, \quad \lambda_2 = -1 - 2i.$

f) $\lambda_1 = i, \quad \lambda_2 = -i.$

8. For each of the six matrices in the previous problem, sketch a vector field for $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Check your answer with the vector fields applet — for example, to plot

$$x' = 2x - y$$

$$y' = x + 4y,$$

enter `(2x - y, x + 4y)` in the box and hit generate.

Section 13

In problems 9–12, solve the system of DEs for an implicit function of x and y , and determine the critical points. Verify your answers with the vector fields applet, and use it to classify the critical points as stable, unstable, or neither.

9.

$$x' = xy$$

$$y' = 2x - 1$$

10.

$$x' = x - xy$$

$$y' = xy - y$$

11.

$$x' = (y - 1)(y + 1)$$

$$y' = (x - 1)(x + 1)$$

12.

$$x' = x$$

$$y' = y \log |x|$$