Name: _____

Midterm 2

Math 111

Winter 2020

You have 50 minutes to complete this exam. Other than a writing tool, you're allowed to use a scientific calculator only (one listed on the syllabus). When you're finished, first check your work if there is time remaining, then turn the exam in. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Let $g(x) = \dfrac{1+3x}{1+2x+x^2}$.

a) (8 points) Is g a rational function? Why or why not?

b) (8 points) Evaluate $g(0)$ and $g(1)$.

c) (8 points) What is the mathematical domain of g?

d) (8 points) What is the behavior of g as $x \longrightarrow \infty$? What about as $x \longrightarrow -\infty$?

2. (32 points) You own a bakery and sell scones. You raise the price very slightly over time to account for inflation. Specifically, d days after you open the bakery, you sell each scone for $P(d) = 3.75 + .0001d + .00001d^2$ dollars.

a) (8 points) Is P a polynomial, a rational function, both, or neither? Explain.

b) (8 points) Find and interpret the behavior of P as $d \longrightarrow \infty$.

c) (8 points) As you sell the scones for more and more, your customers buy fewer of them. $N(d) = 1000 - .001d$ gives the number of scones you sell per day, d days after you open the bakery. Write and simplify a function $R(d)$ that gives your daily revenue (total money earned, not including your own expenses) d days after you open the bakery.

d) (8 points) Write a sentence interpreting what the inverse function R^{-1} does. Specify what it takes as input and what it produces as output. You do **not** need to find a formula for R^{-1}.

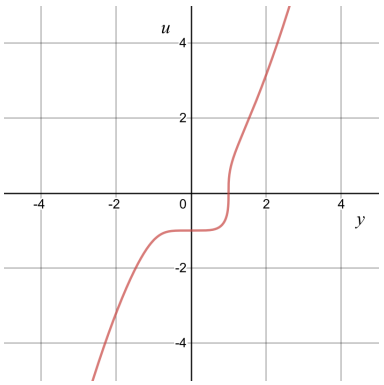
3. (32 points) Let $u = H(y) = \sqrt[3]{y^5 - 1}$ (that's a cube root, not a square root).

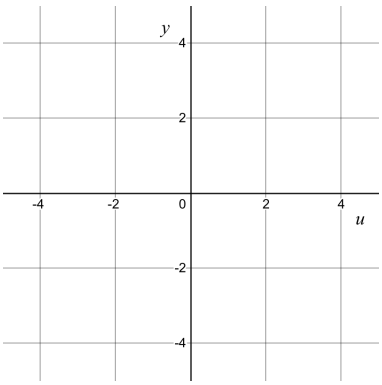
a) (8 points) H is a one-to-one function. Given that $H(2) = 6.232$, find $H^{-1}(6.232)$ without finding a formula for H^{-1}.

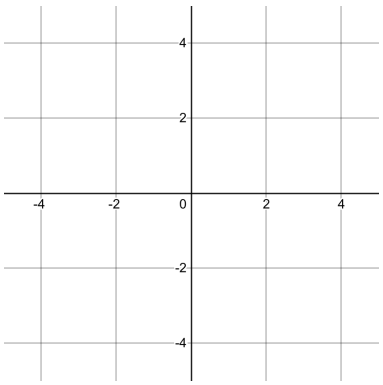
b) (8 points) Now find a formula for $H^{-1}(u)$.

c) (8 points) Find the image of H^{-1}.

d) (8 points) The left graph below is a graph of $H(y)$. Sketch a graph of $H^{-1}(u)$ on the middle graph. On the right graph, sketch a graph of $(H^{-1} \circ H)(y)$.







4. (32 points) You decide to start running regularly. t weeks after you start, you can run $D(t) = \sqrt{t} + 1$ miles.

a) (8 points) What is the average rate of change of D on the interval $[0, 9]$?

b) (8 points) When you run m miles, you burn roughly $C(m) = 200m$ calories. Write and simplify a function that gives the number of calories burned as a function of the number of weeks since you started running.

c) (8 points) What is the domain of the function you found in part b)?

d) (8 points) After going for a run on which you burned c calories, you need to rest for $B(c)$ minutes. Write a function that gives the amount of rest time you need per run, t weeks after you start running. Leave D and C as letters if use you use them (i.e. don't use their formulas).