

Name: \_\_\_\_\_

Homework 2 | Math 256 | Cruz Godar

*Due Wednesday of Week 3 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 2

In problems 1–10, solve the DE. Whenever possible, fully evaluate all integrals.

1.  $y' + 2y = e^{-2t}$ ,  $y(0) = -1$ .

2.  $y' + \frac{y}{t} = \sin(t)$ ,  $y(\pi) = 0$ .

3.  $xy' = \sqrt{y^2x^2 - y^2}$ . Hint: during the integration, you'll want to set  $x = \sec(\theta)$  and use trig sub.

4.  $y' = y + t^2e^t$ .

5.  $y' = \frac{t}{1+y}$ ,  $y(0) = -2$ .

6.  $y' + ty^2 + ty + 2y^2 + 2y = 0$ .

7.  $ty' + (t+1)y = t+2$ ,  $y(1) = 2$ .

8.  $\cos^2(t)y' = 1 + y$ .

9.  $y' = t + \frac{t}{y}$ .

10.  $y' = e^{x+y}$ ,  $y(0) = 0$ .

11. You invest \$1000 into an account which compounds interest **continuously** — in other words, your balance is constantly increasing at a rate proportional to its current value. If the continuous interest rate is 0.2% per year, how much money will be in the account after 10 years?

12. The population  $P(t)$  of a colony of bacteria after  $t$  hours (in thousands) follows the equation  $P' + P = 2^{-t} + 1$ . After enough time, what will the population settle down to?

13. The function  $y$  satisfies  $y' = y \log(x)$  with  $y(1) = \frac{1}{e}$ .

a) Express  $y''$  in terms of  $x, y$ , and  $y'$ .

b) Solve the original DE, find  $y''$  in terms of  $x$  only, and verify that your answer to part a) was correct.

14. Solve the DE  $y' + xy = x$  in two ways: first with integrating factors and then with separation of variables. Make sure your answers are the same!

15. True or false: if a DE is separable, then its direction field is symmetric about the origin — i.e. the slope at  $(x, y)$  is the same as the slope at  $(-x, -y)$ . If it's true, briefly explain why, and if it's false, give a counterexample.

16. A tank is filled with 1 L of water. We begin pouring in a salt solution with a concentration of  $2 \frac{\text{g}}{\text{mL}}$  at a rate of  $10 \frac{\text{mL}}{\text{s}}$ , and also begin draining the tank at a rate of  $10 \frac{\text{mL}}{\text{s}}$ . What will the concentration of salt be after an hour?

17. A tank is filled with 100 L of uncolored paint. We begin pouring in a solution of red paint, which contains  $0.5 \frac{\text{kg}}{\text{L}}$  of dye, at a rate of  $1 \frac{\text{L}}{\text{s}}$ . We also begin draining the well-mixed paint from the bottom of the tank at a rate of  $1 \frac{\text{L}}{\text{s}}$ . How long will it take for the paint in the tank to have half the concentration of dye that the incoming red paint does?