

Name: _____

Homework 8 | Math 342 | Cruz Godar

Due Wednesday of Week 10 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–5, do the following:

- a) Find the eigenvalues of A .
- b) Find the corresponding eigenvectors and generalized eigenvectors of A .
- c) Determine if A is diagonalizable. If it is, write $A = BDB^{-1}$ for a diagonal matrix D , and if not, write $A = BJB^{-1}$ for a matrix J in Jordan normal form (you don't need to invert B).
- d) If A is diagonalizable, determine if there is an orthonormal basis of eigenvectors; if so, find it.
- e) If the eigenvalues of A are distinct, find the general solution to the system of differential equations $\vec{x}' = A\vec{x}$.

1. $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$.

2. $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}$.

3. $A = \begin{bmatrix} -3 & -1 & -3 \\ -8 & -3 & -8 \\ 4 & 1 & 3 \end{bmatrix}$.

4. $A = \begin{bmatrix} -9 & -10 & -10 \\ 4 & 4 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

$$5. A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -3 & 1 \end{bmatrix}.$$

In problems 6–8, do the following:

- a) Find a singular value decomposition $A = U\Sigma V^T$.
- b) Find a least-squares solution to $A\vec{x} = \vec{b}$.
- c) Determine if the least-squares solution is unique.

$$6. A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

$$7. A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

$$8. A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

In problems 9–12, do the following:

- a) Find an orthonormal basis for the given inner product space X , and then extend it to an orthonormal basis for V .
- b) Find the orthogonal decomposition of the vector \vec{v} as $\vec{v} = \vec{x} + \vec{x}'$ for $\vec{x} \in X$ and $\vec{x}' \in X^\perp$.

$$9. V = \mathbb{R}^3 \text{ with } \langle \vec{v}, \vec{w} \rangle = \vec{v} \bullet \vec{w}, X = \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \right\}, \text{ and } \vec{v} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}.$$

10. $V = \text{span}\{1, x, x^2\}$ with $\langle p, q \rangle = \sum_{n=0}^2 p(n)q(n)$, X is the subspace of polynomials p with $p'(0) = 0$, and $\vec{v} = 1 + 2x + x^2$.
11. (optional) $V = \text{span}\{1, \sin(x), \cos(x)\}$ with $\langle f, g \rangle = \int_0^1 f(x)g(x) \, dx$, X is the subspace of functions f with $f'(0) = 0$, and $\vec{v} = \sin(x) + 2\cos(x) - 1$.
12. (optional; more recommended than the previous problem) $V = \text{span}\{1, \cos(x)\}$ with $\langle f, g \rangle = \int_0^{\pi/2} f(x)g(x) \, dx$, $X = \text{span}\{1\}$, and $\vec{v} = \cos(x) - 2$.