Name: ______________________ Homework 8 | Math 253 | Cruz Godar

Due Wednesday of Week 10 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–5, find a Taylor series for $f(x)$ centered at $x = a$ and determine its interval of convergence.

1. $f(x) = \cos(x)$, $a = 0$.

2. $f(x) = \sin(2x)$, $a = \dfrac{\pi}{2}$.

3. $f(x) = \ln(-x)$, $a = -1$.

4. $f(x) = \dfrac{1}{x}$, $a = 1$.

5. $f(x) = \sqrt[3]{x+1}$, $a = 0$.

In problems 6–9, compute the value of the series and justify your answer.

6. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$.

7. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$.

8. $\displaystyle\sum_{n=0}^{\infty} (n+1)\left(\frac{1}{2}\right)^n$.

9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!}$.

In problems 10–13, approximate the value to within 0.01 using Taylor series.

10. $\cos(2)$.

11. $\frac{1}{e^2}$.

12. $\ln\left(\frac{1}{2}\right)$.

13. $\sqrt[4]{\frac{1}{2}}$.

14. Using the fact that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, evaluate $\int_0^1 \frac{\ln(1+x)}{x}$.

15. Find a series solution to $\frac{1}{x} + y'' = \frac{y}{x}$.

16. The function $\sin(x)$ has zeroes at exactly $x = \pi n$ for integers n, and this allows us to write $\sin(x)$ as

$$\sin(x) = x\left(1 - \frac{x}{\pi}\right)\left(1 + \frac{x}{\pi}\right)\left(1 - \frac{x}{2\pi}\right)\left(1 + \frac{x}{2\pi}\right)\cdots.$$

Note that this isn't true for functions in general! There can often be a lot more going on with a function than we can determine by its zeros.

a) Divide by x to find an expression for $\frac{\sin(x)}{x}$.

b) Use the difference of squares formula $(a-b)(a+b) = a^2 - b^2$ to group each two consecutive factors in the product.

c) Expand $\frac{\sin(x)}{x}$ as a Maclaurin series. What is the coefficient of x^2?

d) In the grouped product, what is the coefficient of x^2? You'll likely need to express it as a sum.

e) Set the two sides equal to one another. What have you shown?