Ex: The function $S(T) = \begin{cases} 0, & -273 \le T \le 0 \\ 1, & 0 < T < 100 \\ 2, & 100 \le T \end{cases}$

gives the state of water at temperature $T\,^\circ C$ and standard pressure, where 0 is solid, 1 is liquid, and 2 is gas.

Given that $T\,^\circ C = \frac{9}{5}T + 32\,^\circ F$, find a formula for S when T is measured in $^\circ F$.

Since we're trying to rescale the input, we a horizontal transformation. The naive approach is to take $S(\frac{9}{5}T+32)$, but this doesn't work! It's always helpful to draw a digram:

$$°C \xrightarrow{S} \text{state}$$

$\Rightarrow$ S will always take inputs in °C, so we need to turn our °F input into °C, not vice versa.

What we were trying with $S(\frac{9}{5}T + 32)$ looks like:

$$°C \rightarrow °F \xrightarrow{S} \text{state}$$

S doesn't take in °F as input!

Instead, we need

$$°F \rightarrow °C \xrightarrow{S} \text{state}$$

To do that, we need to know °F → °C, which we can find from

$T\ °C = \frac{9}{5}T + 32\ °F$

$T - 32\ °C = \frac{9}{5}T\ °F$

$\frac{5}{9}(T - 32)\ °C = T\ °F$

In total, we have our new function is $y = S\left(\frac{5}{9}(T-32)\right)$.

Moral of this: always make sure that the units you're plugging into a function are the units it accepts.

Combinations of Transformations

Def: Let f be a function. A <u>tranformation</u> of f is a function

$$g(x) = \pm a \cdot f\left(\pm b\,(x-h)\right) + k,$$

for $a > 0$, $b > 0$, and h and k any numbers.

a : vertical stretch (+ maybe vertical reflection)

b : horizontal stretch (+ maybe horizontal reflection)

k : vertical shift

h : horizontal shift

Theorem: To graph a transformation of a function f:

1. Find x and start there.
2. Horizontally stretch by a factor of $1/b$.
3. Horizontally reflect, if needed.
4. Horizontally shift h units to the left.

these look backward!

⑤ vertically stretch by a factor of a.
⑥ Vertically reflect, if needed.
⑦ vertically shift k units up.

Ex: Graph the function $y = -2\sqrt[3]{2x-5} + 4$ and label at least 3 points at every step.

Correct form: $-2\sqrt[3]{2(x-\frac{5}{2})} + 4$

① Parent function: $y = \sqrt[3]{x}$

② Horizontal stretch by a factor of $1/2$

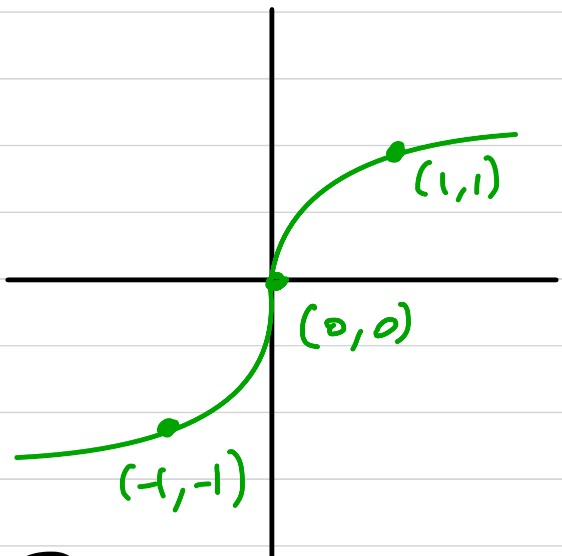
③ Horizontal shift $5/2$ to the right

④ vertical stretch by a factor of 2

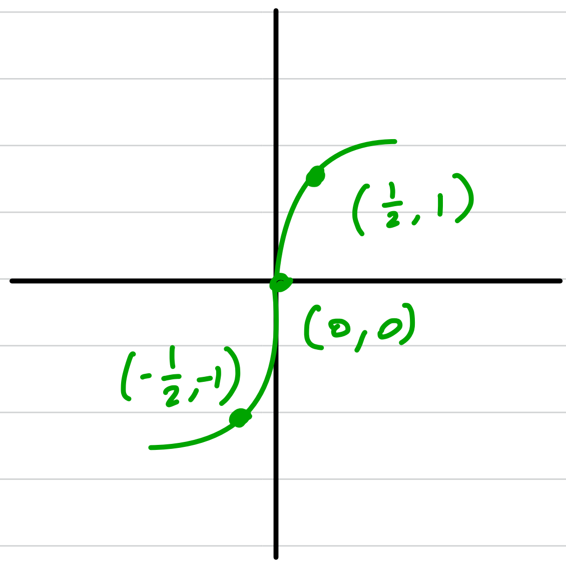
⑤ vertical reflection

⑥ vertical shift 4 up

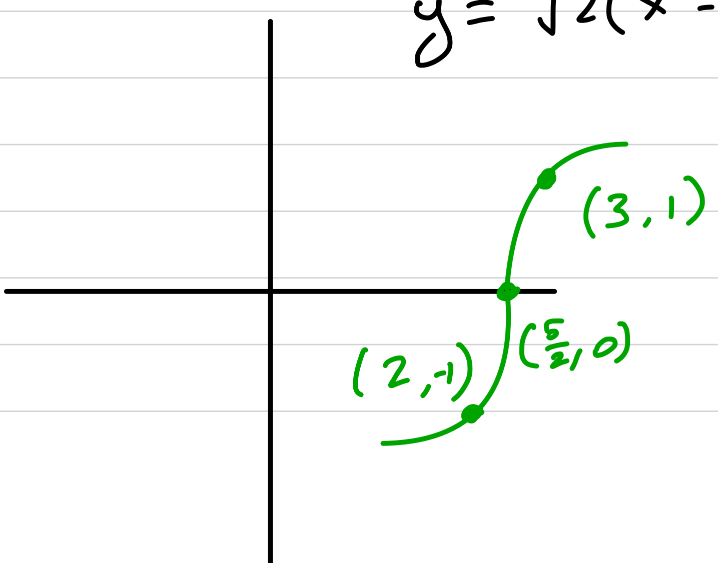
① $y = \sqrt[3]{x}$



② $y = \sqrt[3]{2x}$



③ $y = \sqrt[3]{2(x - 5/2)}$



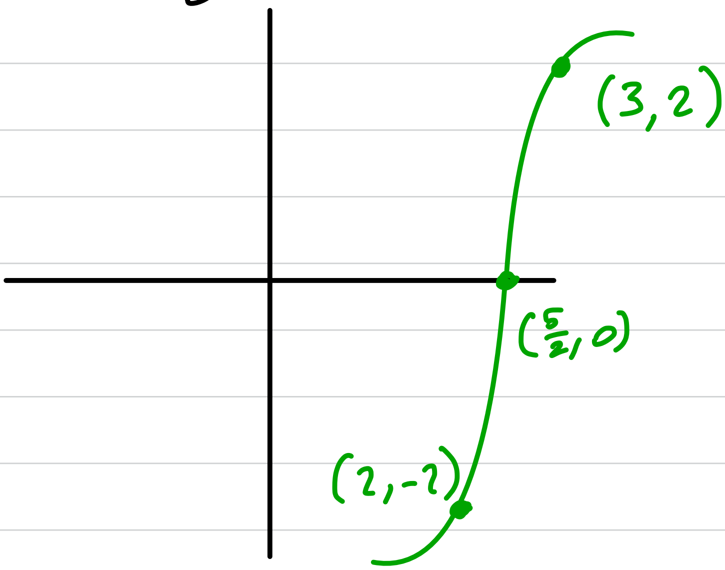
$$-\frac{1}{2} + \frac{5}{2} = \frac{-1+5}{2}$$

$$= 4/2$$

$$= 2$$

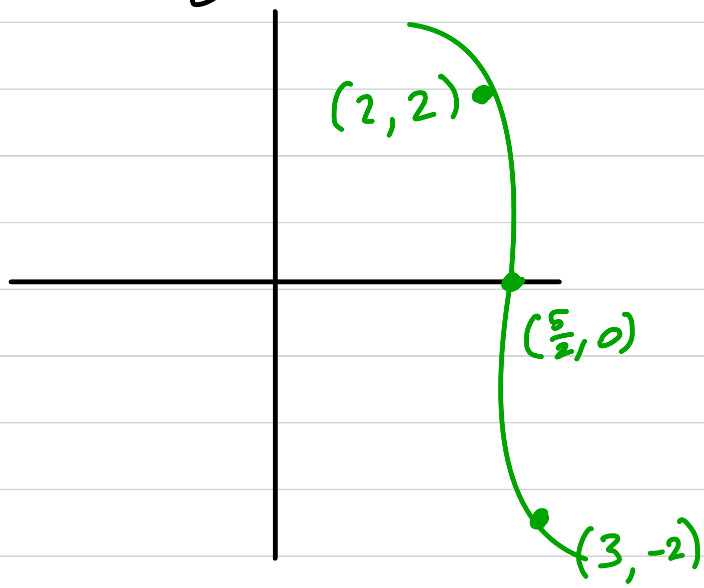
4

$$y = 2\sqrt[3]{2\left(x - \tfrac{5}{2}\right)}$$



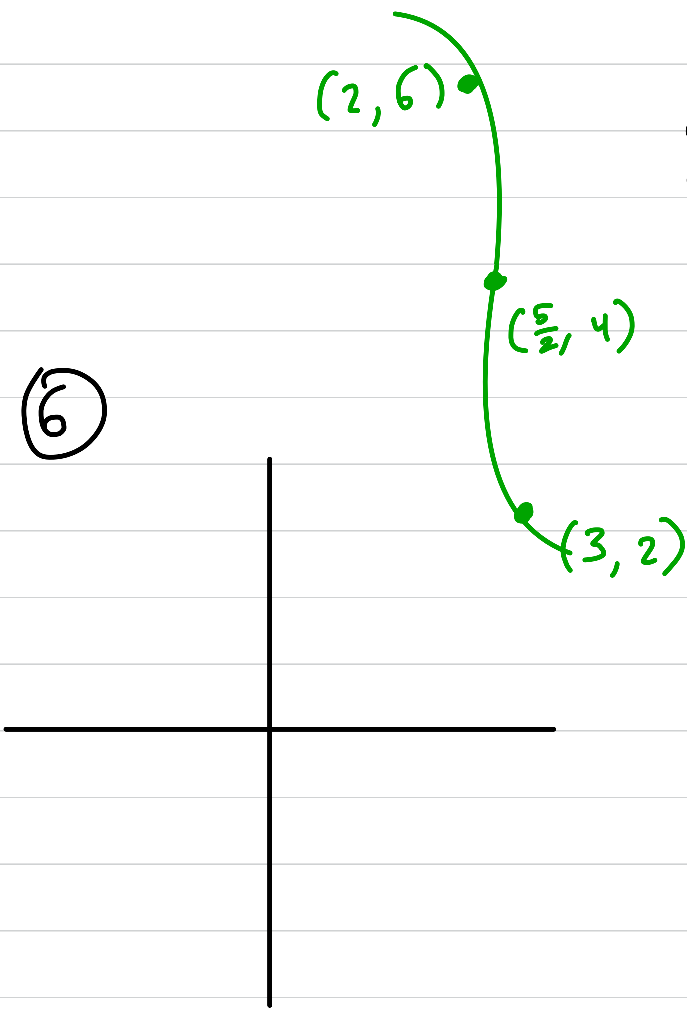
5

$$y = -2\sqrt[3]{2\left(x - \tfrac{5}{2}\right)}$$



6

$$y = -2\sqrt[3]{2\left(x - \tfrac{5}{2}\right)} + 4$$



<u>Ex</u>: You burn about 200 calories per mile when running. The function $C(d) = 200d$ gives the approximate number of calories burned when running d miles. Given that 1 mile is 1.61 km and 1 calorie = 4184 Joules, write a function $B(d)$ that gives the number of Joules burned when running d km.

$$\underbrace{\text{km} \longrightarrow \text{miles} \xrightarrow{C} \text{calories} \longrightarrow \text{Joules}}_{B}$$

d km $= \dfrac{d}{1.61}$ miles, so

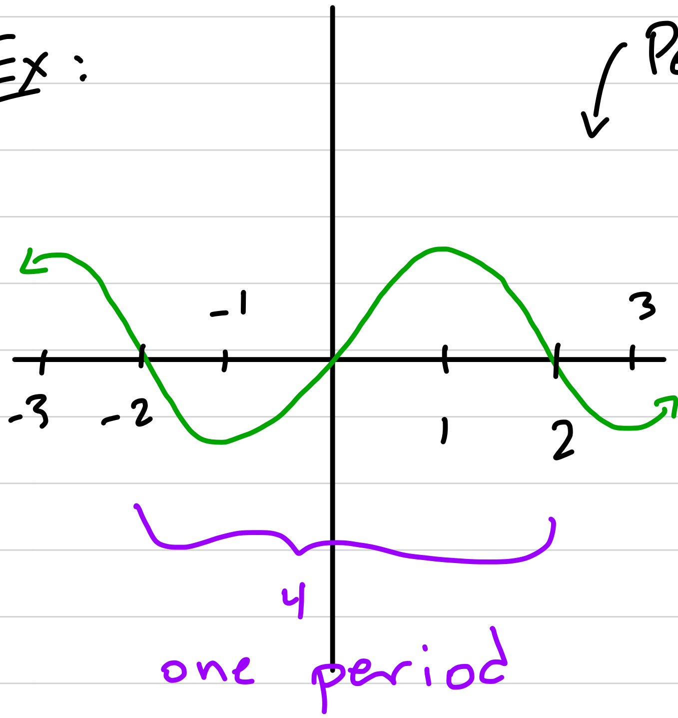
$$B(d) = 4184\,C\left(\frac{d}{1.61}\right) = 4184\left(200\left(\frac{d}{1.61}\right)\right).$$

Periodic Functions

Def: A function f is <u>periodic</u> if there is a number n so that $f(x+n) = f(x)$ for all x in the domain of f. The <u>period</u> of f is the smallest n that does this. Periodic functions are ones whose graph repeats.

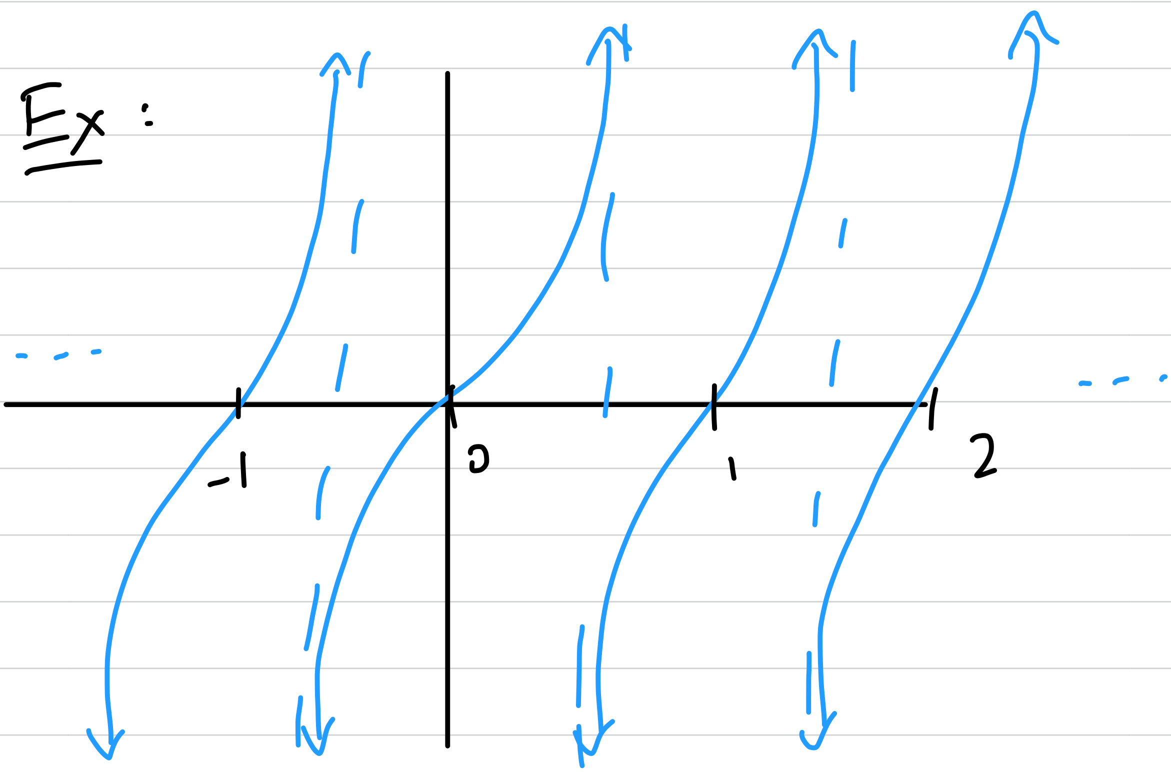
Ex:

Periodic with period 4.

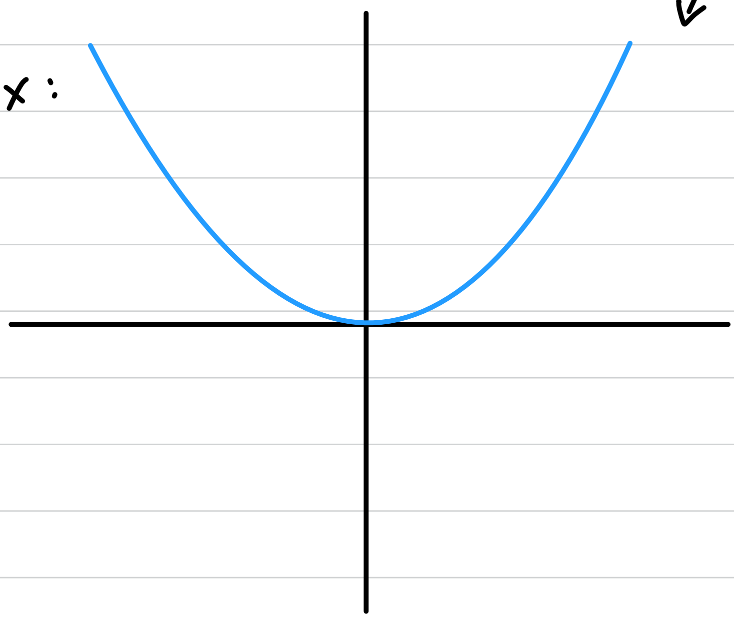


Periodic with period 1.

Ex:



Ex:



not periodic

Ex: A function f is periodic with period 5. For x with $-2 \le x < 3$, $f(x) = -x^2 - 2x + 3$. Find $f(1)$, $f(-6)$, $f(3)$, all of the x-values that make $f(x) = 0$, and sketch a graph of f.

$f(1) = -(1)^2 - 2(1) + 3 = -1 - 2 + 3 = 0.$

$f(-6) = f(-6+5) = f(-1) = -(-1)^2 - 2(-1) + 3$

$= -1 + 2 + 3 = 4.$

$$f(3) = f(3-5) = f(-2) = -(-2)^2 - 2(-2) + 3$$
$$= -4 + 4 + 3 = 3.$$

Solve $f(x) = 0$. If we know all the places $f(x) = 0$ in a single period, then we can add or subtract the period any number of times to get the rest of the zeros.

$$-x^2 - 2x + 3 = 0$$
$$(-1)(-x^2) + (-1)(-2x) + (-1)(3) = (-1)(0)$$

So solve $-x^2 - 2x + 3 = 0$

$$x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$

$x + 3 = 0$ $\quad$ $x - 1 = 0$

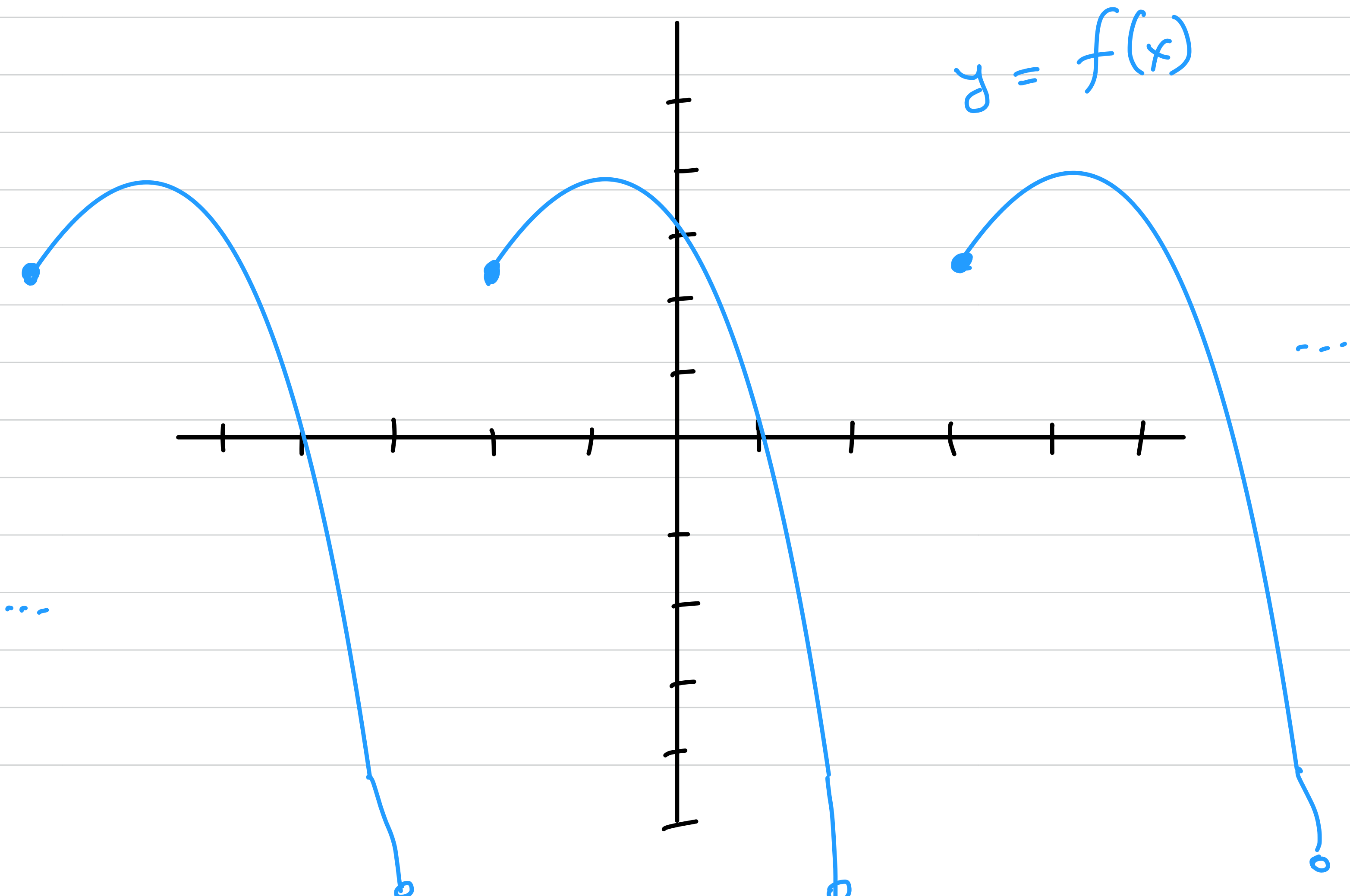
$x = -3$ $\quad$ $x = 1$

We only want x-values with $-2 \leq x < 3$.

$\Rightarrow x = 1$

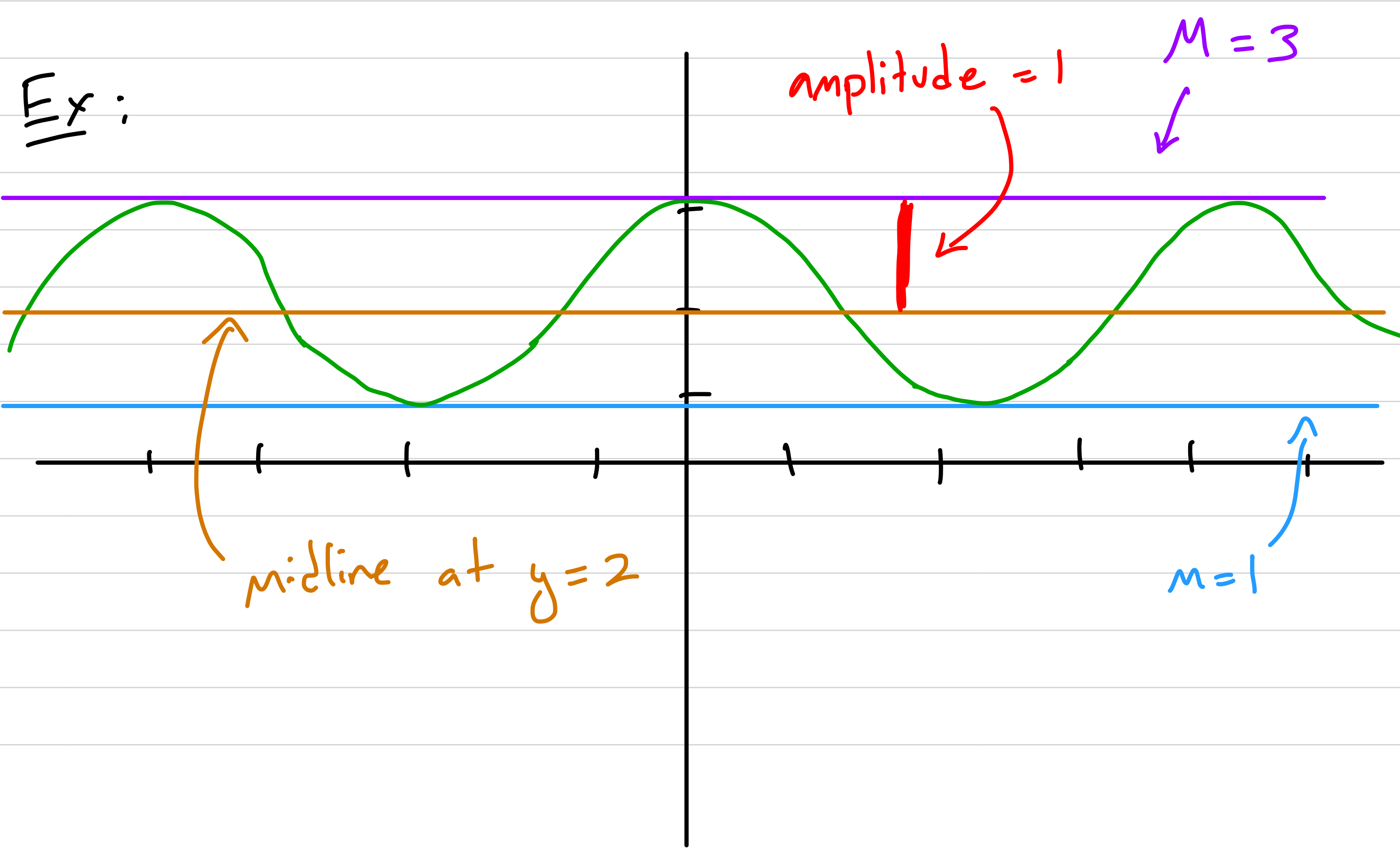
To write down every zero, we can just write "$x = 1 + 5k$ for any integer k".

To graph f, first graph it on $[-2, 3)$ and then we copy the graph and paste it every 5 units.



Def: Let f be a periodic function. If f has a maximum y-value M and a minimum y-value m, we define the <u>midline</u> of f to be the y-value $\frac{M+m}{2}$. The <u>amplitude</u> of f is $\frac{M-m}{2}$, which is the farthest the function ever gets from its midline.

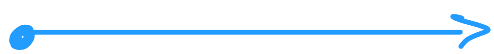
<u>Ex</u>:



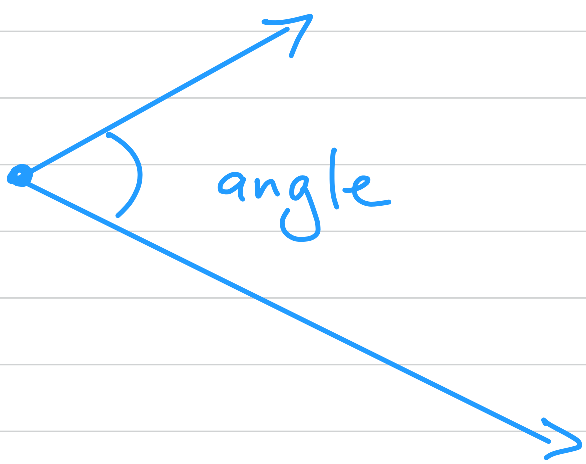
Chapter 2

Geometry Review

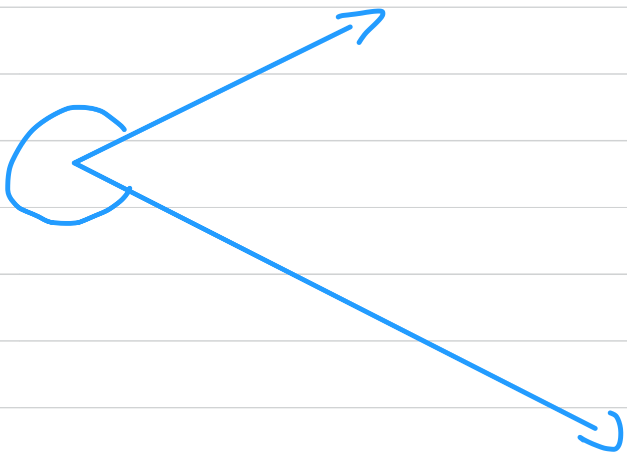
Comment: Recall some basic geometry definitions:

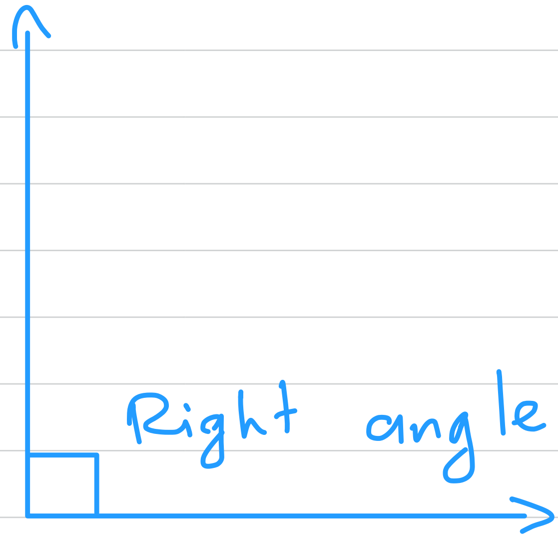


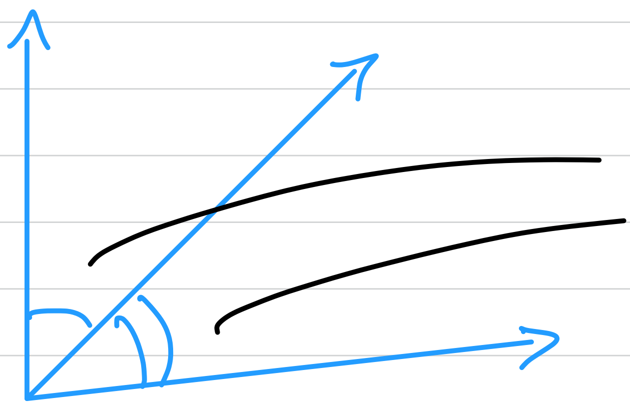
Ray



(remember, always label the arc you're refering to!)

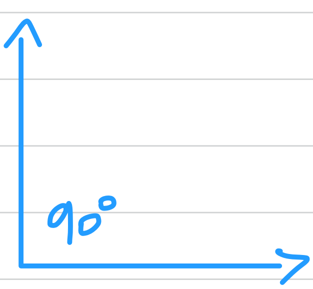


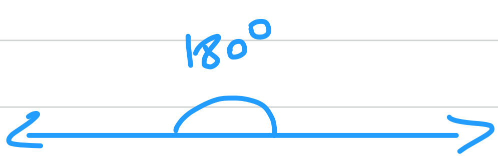


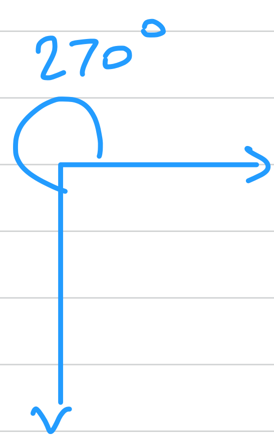


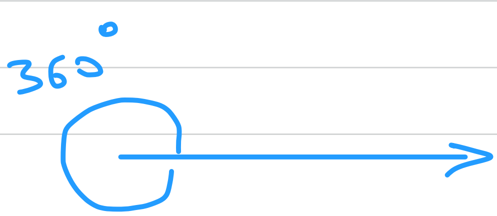
adjacent angles

A right angle is 90°







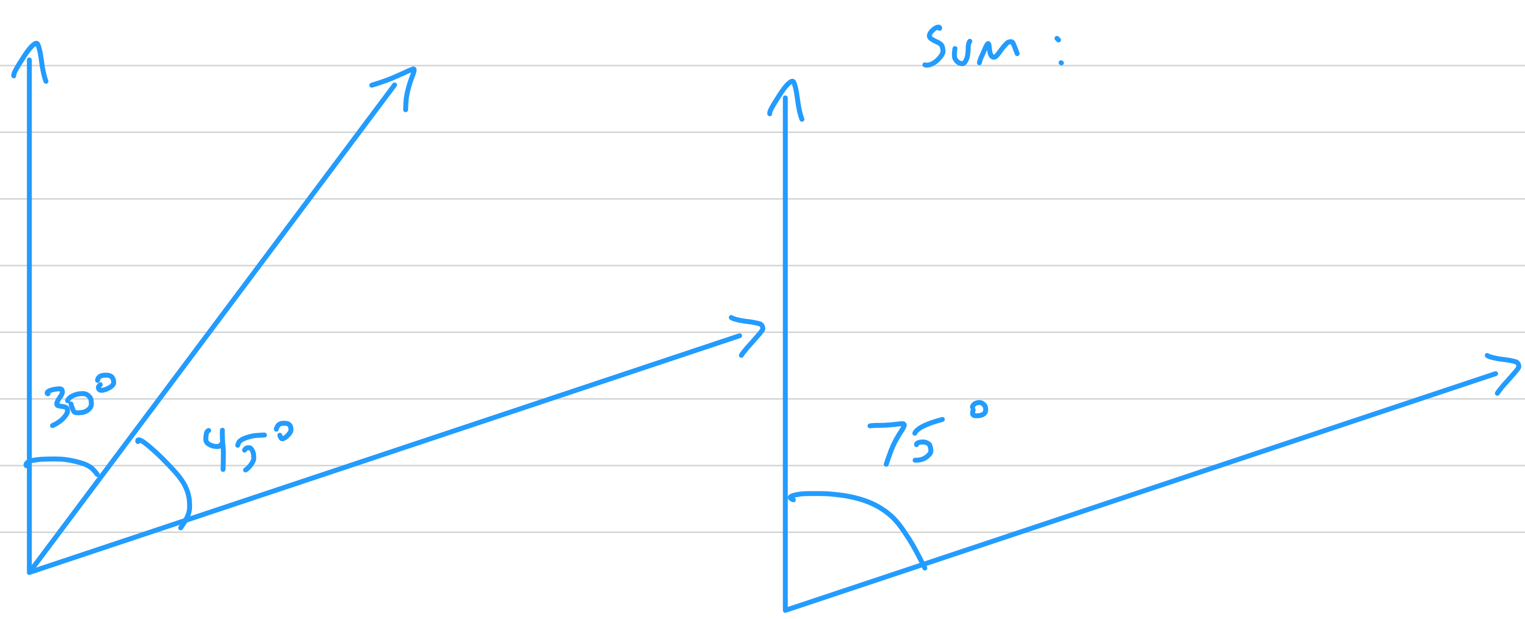


Acute angles are ones $< 90°$

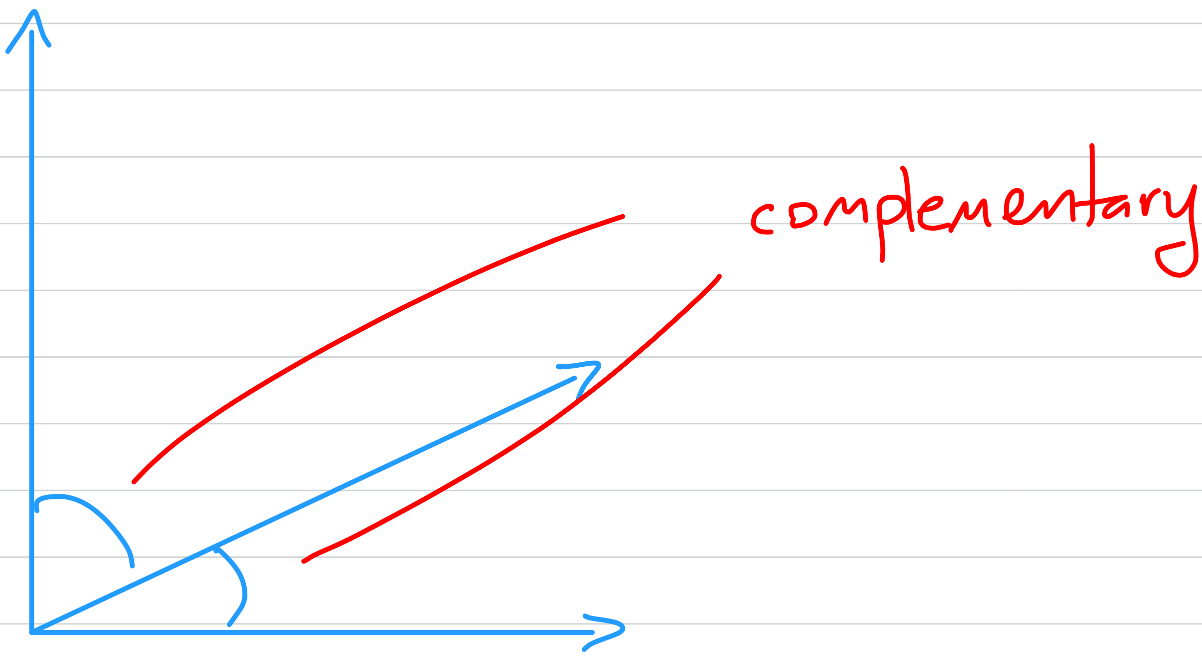
Obtuse angles are ones between $90°$ and $180°$

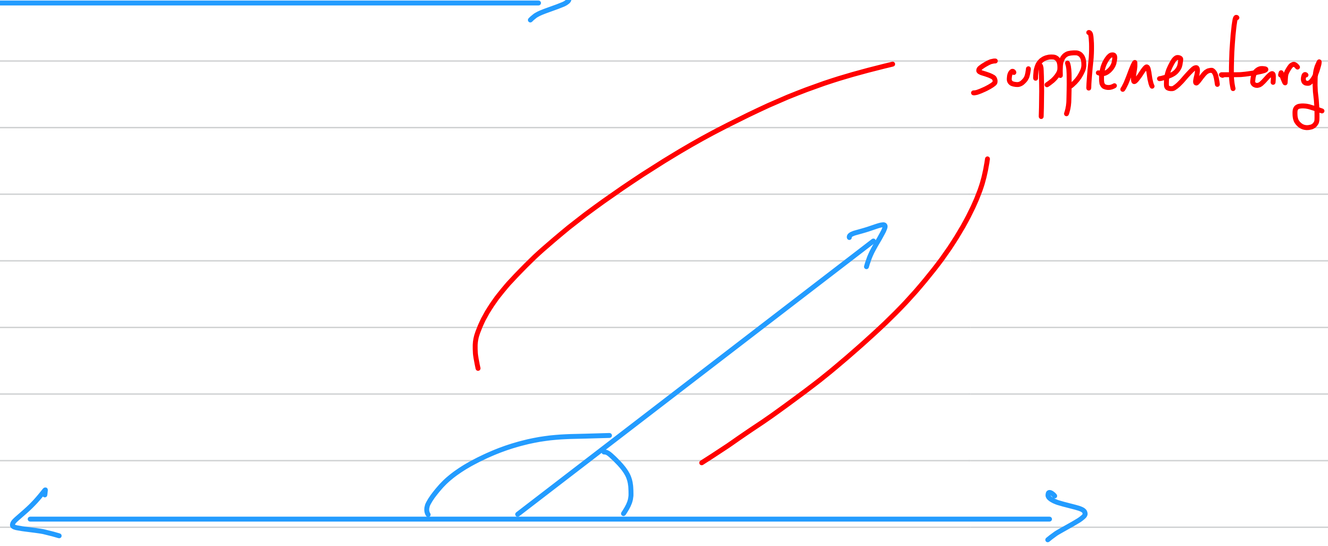
Reflex angles are ones between $180°$ and $360°$

We can add adjacent angles:

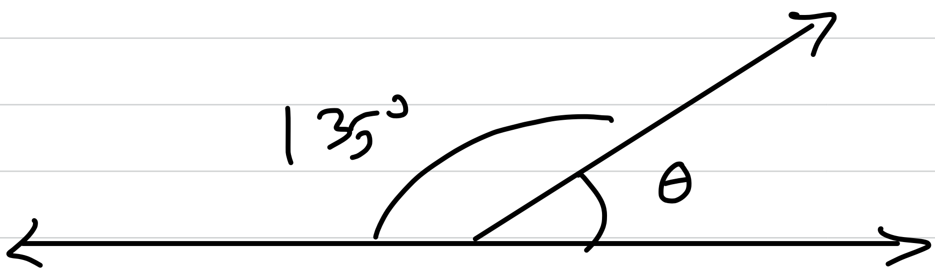


Two angles are <u>complementary</u> if they sum to 90° and <u>supplementary</u> if they sum to 180°.





Ex: find θ so that the two angles below are supplementary:

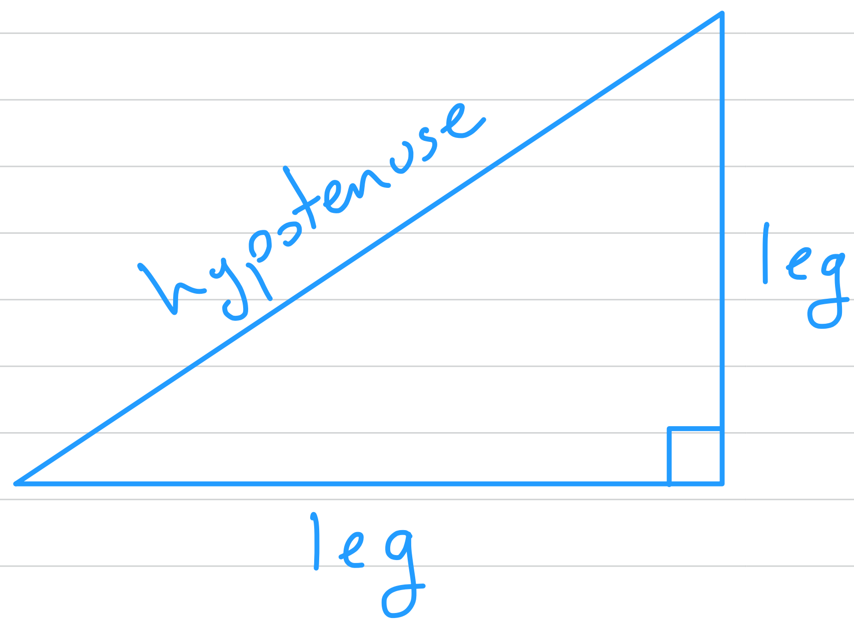


$135 + \theta = 180$, so $\theta = 45$.

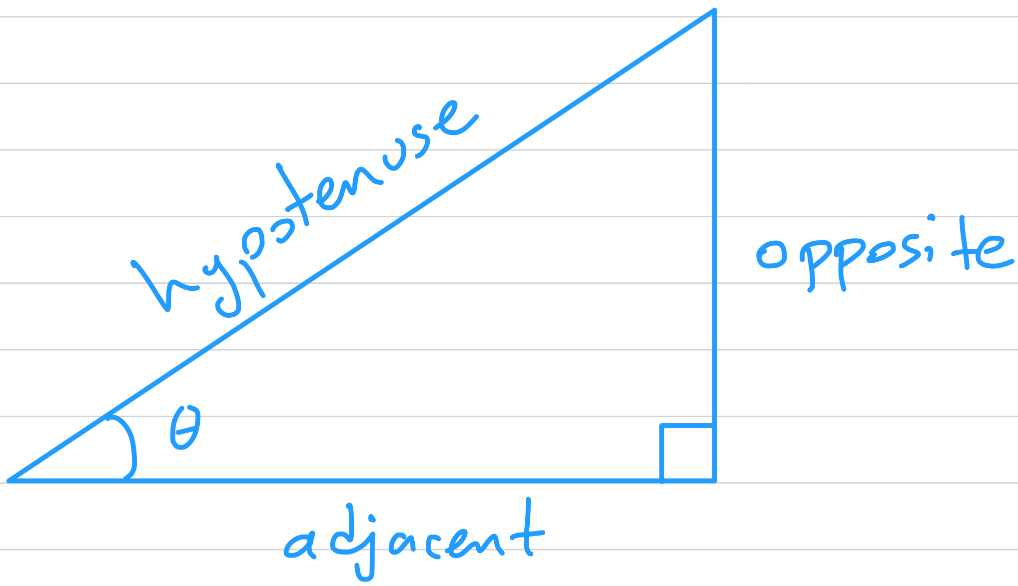
Angles are typically written with Greek letters. Some common ones are:

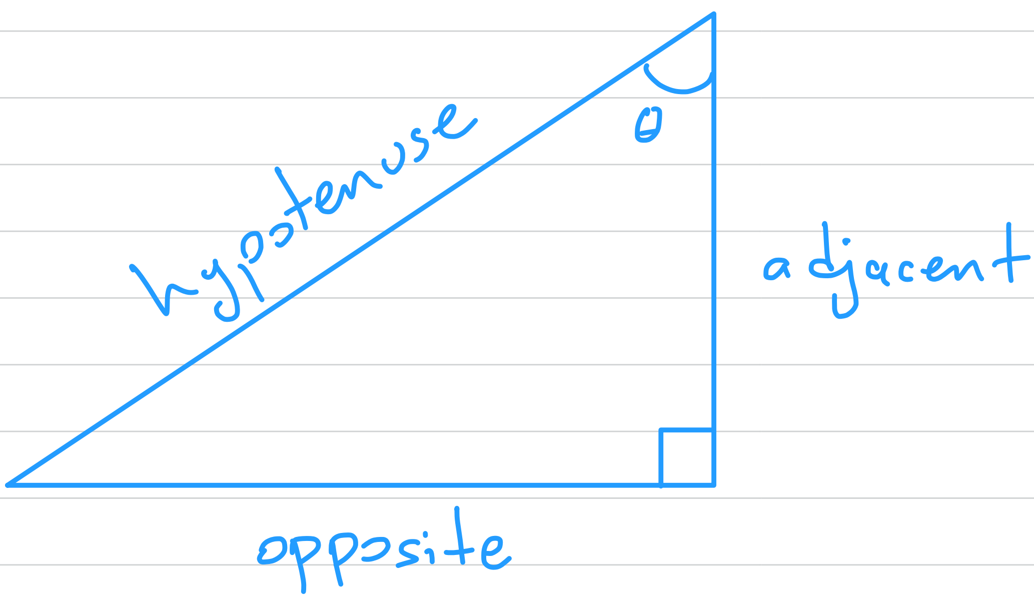
θ	— theta	α	— alpha
φ	— phi	β	— beta
ψ	— psi	γ	— gamma

A right triangle is a triangle with one right angle.

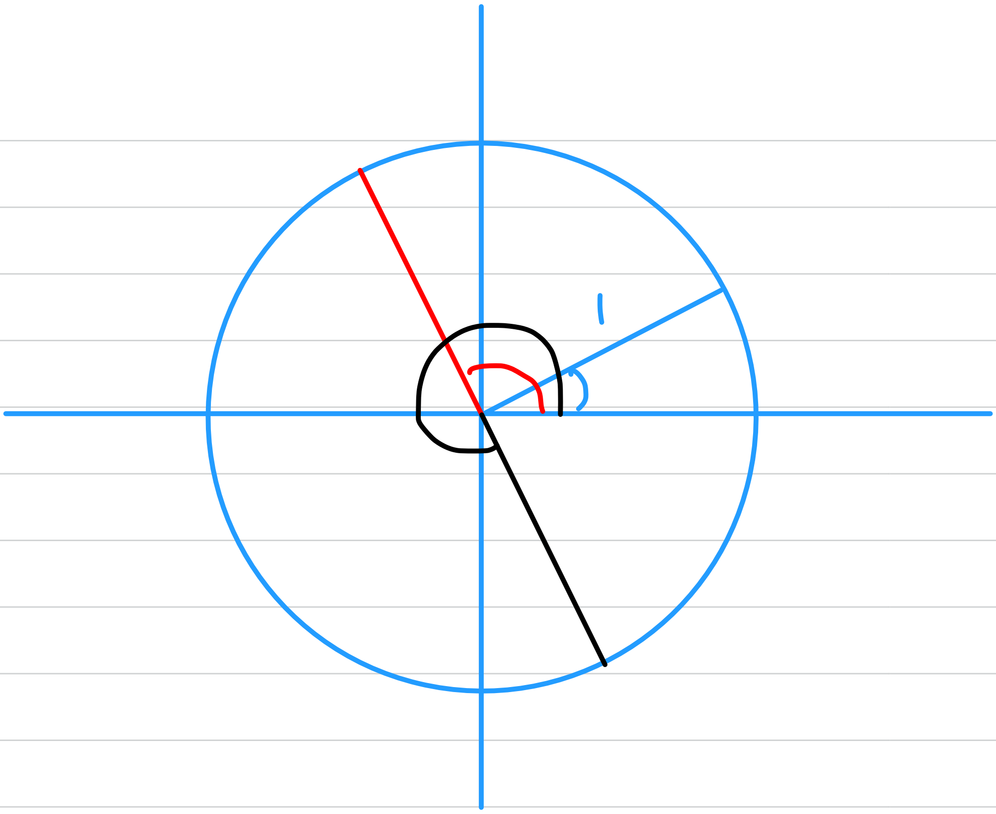


If we choose an angle θ in a right triangle (where θ is not the right angle), the leg touching θ is called the <u>adjacent</u> side of the triangle and the other leg is called the <u>opposite</u> side.





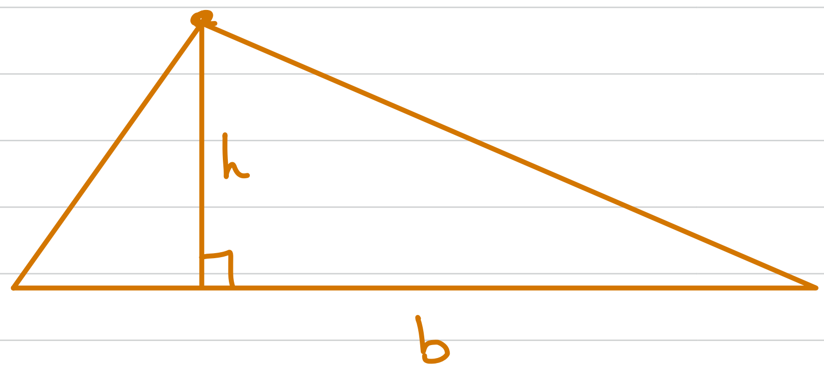
The unit circle is the circle of radius 1 centered at the origin.



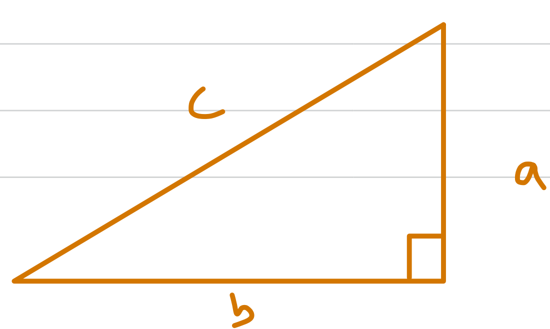
An angle on the unit circle is an angle formed by the positive x-axis and a radius of the circle.

<u>Prop</u>: the angles of any triangle sum to $180°$. In particular, the non-right angles of a right triangle sum to $90°$.

Prop: Let b be one side of a triangle and h the shortest distance from b to the vertex opposite b. Then the area of the triangle is $\frac{1}{2}bh$.



The Pythagorean Theorem: In a right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.



Because of this, any point (x,y) on the unit circle has $x^2 + y^2 = 1$.