Final Exam Worksheet

Math 251

1. True or false: if $x = a$ is a critical point of $f(x)$ and $f''(a) > 0$, then a is a local minimum. If it's true, explain why, and if it's false, give an example of a function f and a critical point a that are a counterexample.

2. Evaluate $\lim_{x \to 2} (x-1)^{\ln(x-2)}$.

3. Let $g(x) = \dfrac{\sin(x^2)}{\sin^2(x)}$. What is $g'(2)$?

4. Find the equation of the tangent line to $y^2 + \ln(xy) = x$ at $(1, 1)$.

5. Define a function f by

$$f(x) = \begin{cases} x^3 \ln(x^2), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

on $[-2, 1]$.

a) Show that f is continuous.

b) Find and classify the critical points of f.

c) Find the inflection points of f.

d) Find the global maximum and minimum of f.

6. Let $f(x) = x^3 + x^2 + x + 1$. Find $f'(x)$ **using the limit definition of the derivative**.