Quiz 4 Solutions:

① $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$E_1 = \{7\}$

$E_2 = \{2, 4, 6, 8\}$

$E_3 = \{3, 4, 5, 6\}$

② $p(E_1) = \frac{1}{8}$ $\qquad o(E_1) = 1:7$

$p(E_2) = \frac{4}{8} = \frac{1}{2}$ $\qquad o(E_2) = 4:4$

$p(E_3) = \frac{4}{8} = \frac{1}{2}$ $\qquad o(E_3) = 4:4$

Let E be the event of rolling a 3 on the 8-sided die and F the event of rolling a 3 on the 4-sided one. Then we want

$$p(E \cup F) = p(E) + p(F) - p(E \cap F)$$

$n(E) = 4 \qquad p(E) = 4/32 = 1/8$

$n(F) = 8 \qquad p(F) = 8/32 = 1/4$

$n(E \cap F) = 1 \qquad p(E \cap F) = 1/32$

$n(S) = 32$

$$p(E \cup F) = 1/8 + 1/4 - 1/32 = 11/32.$$

3.7: Independence

Def: Events A and B are independent if $p(A|B) = p(A)$. What this means is that B taking place has no effect on the chance that A will take place.

Ex: if you toss two coins, the result of the second toss doesn't depend on the result of the first, so they're independent. In symbols, if A is the event of getting heads on the first toss and B is the event of getting heads on the second, then

$p(A) = 1/2$

$p(B) = 1/2$

$p(B \mid A) = 1/2$

Since $p(B \mid A) = p(B)$, A and B are independent.

Ex: If A is the event of drawing a heart off the top of a 52-card and B is the event of the card underneath it also being a heart, then

$p(A) = 13/52 = 1/4$

$p(A \mid B) = 12/51$

$\Longrightarrow$ A and B are dependent.

<u>Comment</u>: Independent vs Mutually exclusive

Independent means $p(A|B) = p(A)$.

Mutually exclusive means $p(A \cap B) = 0$.

<u>Ex</u>: Are the events A and B independent or mutually exclusive or neither, where A is having freckles and B is having red hair.

Since it's possible to have both freckles and red hair at the same time, A and B are mutually exclusive.

But having one makes you more likely to have the other, so A and B are dependent.

Theorem (Product rule for independent events)

If A and B are independent, then

$p(A \cap B) = p(A) \cdot p(B)$.

Ex: If A is the event of rolling a 3 on an 8-sided die and B is the event of rolling a 3 on a 4-sided die, then

$p(A \cap B) = \frac{1}{8} \cdot \frac{1}{4} = 1/32$ because

$p(A) = 1/8$

$p(B) = 1/4$

A and B are independent.

The Final

- 12 – 1:50 on Friday
- 1.5x midterm length (expect ~12 Qs)
- No outside resources (including a calculator)

- I'll post a list of topics

- Tue, Wed, Thu are open for questions, so come with questions ready

- office hours Wed + Fri as usual

If the first card is the 2 of spades, what is the probability that the second card is a spade?

Intuition: there are 12 spades and 51 cards left, so it should be $12/51$.

A: ~~getting~~ a spade, B: getting the 2 of spades

this is hard because $p(A)$ is hard.

If the first card is the 2 of spades, what is the probability that the second card is the ace of spades?

$$p(2 \cap \text{ace}) = \frac{1}{{}_{52}P_2} = \frac{1}{52 \cdot 51}$$

not asking for this.

$p(\text{ace} \mid 2) = 1/51$

12 burritos

5 spicy

7 not spicy

choose 3 at random

$$n(S) = {}_{12}C_3 = \frac{12!}{3!\,9!} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2}$$

$$= 2 \cdot 11 \cdot 10 = 220$$

(23) How many ways to choose 3 where all 3 are spicy?

$$_5C_3 = \frac{5!}{3!\,2!} = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{2}} = 10$$

$\Rightarrow$ prob is $10/220 = 1/22$.

(24) How many ways to choose 3 where none is spicy?

$$_7C_3 = \frac{7!}{3!\,4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 7 \cdot 5 = 35$$

$35/220$

(25) Exactly one is spicy.

______ ______ ______

How many ways are there to choose 1 spicy burrito and 2 nonspicy ones?

$${}_5C_1 \cdot {}_7C_2 = \frac{5!}{1!\,4!} \; \frac{7!}{2!\;5!}$$

$$= 5 \cdot \frac{7 \cdot 6}{2} = 5 \cdot 7 \cdot 3$$

$$= 105$$

$105/220$

(26) Exactly two are spicy.

$${}_5C_2 \cdot {}_7C_1 = \frac{5!}{2!\,3!} \cdot \frac{7!}{1!\,6!}$$

$$= \frac{5 \cdot 4}{2} \cdot 7$$

$$= 5 \cdot 2 \cdot 7 = 70$$

$70/220$

(27) At most one is spicy.
(so either none or one)

since those are mutually exclusive,

we get $35/220 + 105/220$

$$= 140/220$$

28 At least one is spicy.

(this is the same as not none being spicy)

so it's $1 - 35/220 = 185/220$.

29 At least two are spicy.

2 or 3

$\Rightarrow$ not (0 or 1)

$\Rightarrow 1 - 140/220 = 80/220$

30 At most two are spicy

0, 1, or 2

$\Rightarrow$ not 3

$\Rightarrow 1 - 10/220 = 210/220$

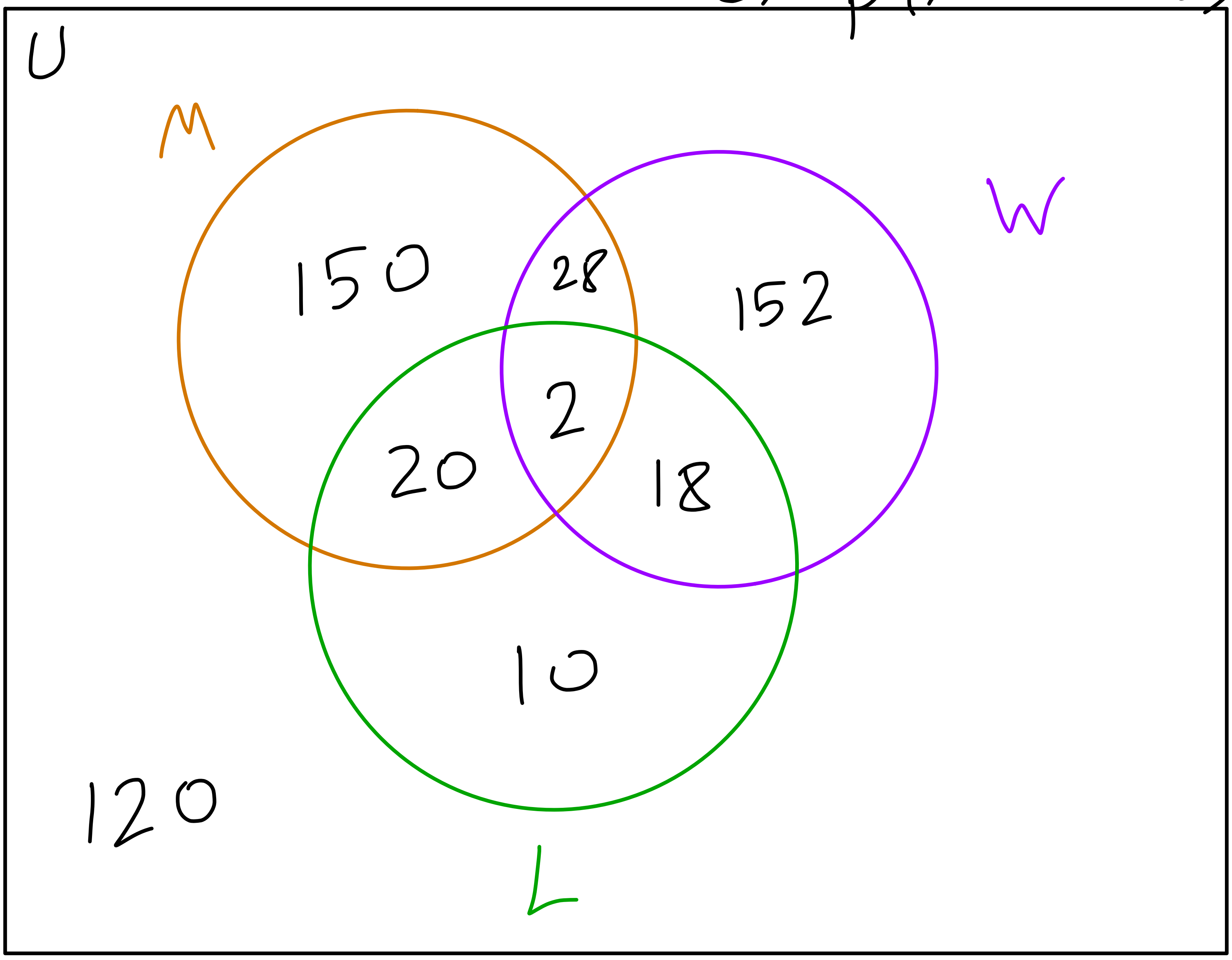
HW 3, 1

a) $p(M \cap W) = 30/500$

b) $p(L') = 450/500$

c) $p(M' \cap W' \cap L') = 120/500$

d) $p(M \cap W \cap L) = 2/500$



$n(U) = 500$

$n(M) = 200$

$n(W) = 200$

$n(L) = 50$

$n(M \cap W) = 30$

$n(W \cap L) = 20$

$n(M \cap W \cap L) = 2$

$n(M \cap W' \cap L') = 150$

a) $n(M \cap W \cap L') = 28$

b) $n(M \cap W' \cap L) = 20$

c) $n(M' \cap W' \cap L') = 120$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

A : the event of being dealt a 4-ace hand

B : the event of being dealt an ace off the top of the deck

$p(A \mid B)$ = the probability of being dealt a 4-ace hand when you're already been dealt one ace.

$$\Rightarrow \frac{\text{\# ways to get dealt 3 more aces}}{\text{\# ways to get dealt 4 more cards}} = {}^{48}\!/_{249900}$$

${}^{2}\!/_{10000}$

48

${}_{51}C_4 = 249900$

$$\frac{48}{{}_{50}C_3} = \frac{48}{19600} = 2/1000$$

3.4 #1

$$\frac{\text{\# ways to pick 30 birthdays where two are the same}}{\text{\# ways to pick 30 birthdays}}$$

If this is E, $n(E') = {}_{365}P_{30}$

365^{30}

$$\Rightarrow p(E') = \frac{{}_{365}P_{30}}{365^{30}}$$

So $p(E) = 1 - p(E') = .7$

If you have 20 objects and you want to order them, the number of ways is $20!$

If you have 20 objects and you want to choose 5 and order them, the number of ways is ${}_{20}P_5$

If you have 20 objects and you want to pick 5 and not order them, there are ${}_{20}C_5$ ways.

If you have 5 slots and you want to put one of the 20 objects in each slot, but you can reuse the objects and order matters, there are 20^5 ways to put them there.