Name: ______________________ Homework 6 | Math 342 | Cruz Godar

Due Wednesday of Week 7 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 5

In problems 1–4, find the least-squares solution $\vec{x}'$ to the matrix equation $A\vec{x} = \vec{b}$ and the distance between $A\vec{x}'$ and $\vec{b}$. Also determine if that least-squares solution is unique.

1. $\begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 0 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$

2. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}.$

3. $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 8 \\ 5 \\ 6 \end{bmatrix}.$

4. $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -3 \end{bmatrix}.$

5. Let A be an $m \times n$ unitary matrix (i.e. not necessarily square). What is the least-squares solution to $A\vec{x} = \vec{b}$ in terms of A and $\vec{b}$?

Section 6

In problems 6–8, compute $\langle \vec{v}, \vec{w} \rangle$, $||\vec{v}||$, $||\vec{w}||$, and the angle between $\vec{v}$ and $\vec{w}$ for the given inner product V.

6. $\vec{v} = 1 + 2x$ and $\vec{w} = 2x + x^2 - x^3$ for $V = \operatorname{span}\left\{1, x, x^2, x^3\right\}$ with the inner product

$$\langle p, q \rangle = \sum_{n=0}^{3} p(n)q(n).$$

7. $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ for $V = \mathbb{R}^3$ with the inner product

$$\langle \vec{v}, \vec{w} \rangle = 2v_1w_1 + v_2w_2 + 50v_3w_3.$$

8. $\vec{v}(x) = e^x$ and $\vec{w}(x) = x$ for $V = C[0,1]$ with the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,\mathrm{d}x.$$

9. Let $V = C[0,1]$. For each of the following possible inner product formulas, give an example showing that it does not define an inner product.

a) $\langle f, g \rangle = \displaystyle\int_0^1 |f(x) + g(x)|\,\mathrm{d}x.$

b) $\langle f, g \rangle = \displaystyle\int_0^1 (f(x)g(x))^2\,\mathrm{d}x.$

c) $\langle f, g \rangle = \displaystyle\int_0^1 f(g(x))\,\mathrm{d}x.$

d) $\langle f, g \rangle = \displaystyle\int_{1/4}^{3/4} f(x)g(x)\,\mathrm{d}x.$