Midterm 1

Math 252

Spring 2021

You have 50 minutes to complete this exam and scan and upload it to Canvas. **Show all your work. You may use a scientific calculator, but not a graphing one.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) Let f be a positive function on an interval $[a, b]$. The upper Riemann sum of f on $[a, b]$ with 5 subintervals chooses x_i^* to be

A) the left endpoint on each subinterval.

B) the right endpoint on each subinterval.

C) the x in the subinterval that has the largest value of $f(x)$.

D) the x in the subinterval that has the smallest value of $f(x)$.

b) (8 points) What is $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\, dx$?

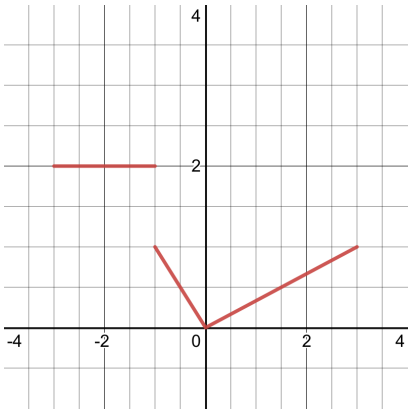
A) $\tan^{-1}(x) + C$.

B) $\sin^{-1}(x) + C$.

C) $\cos^{-1}(x) + C$.

D) $\tan^{-1}(x) + C$.

c) (8 points)



Let $f(t)$ be defined by the previous graph. Then $\int_{-2}^{1} f(t)\, dt$ is

A) positive.

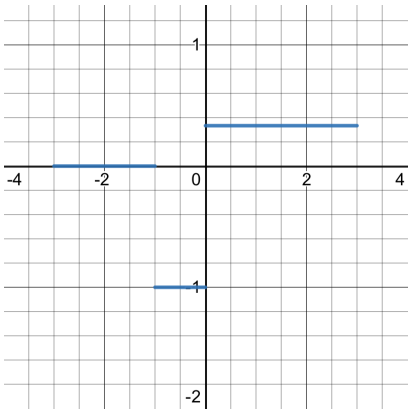
B) negative.

C) zero.

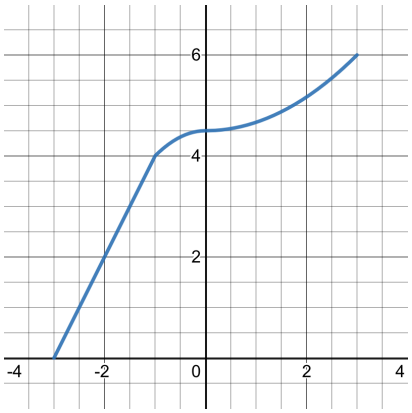
D) undefined.

d) (8 points) With f defined from the same graph as before, let $g(x) = \int_{-3}^{x} f(t)\,dt$. Which of the following could possibly be a graph of g?

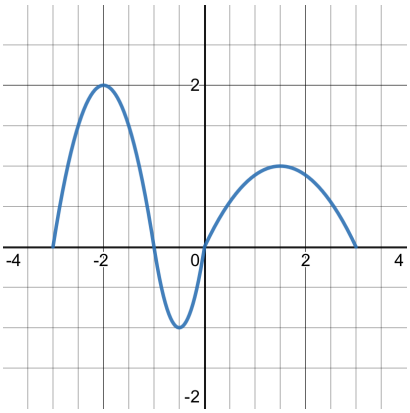
A)



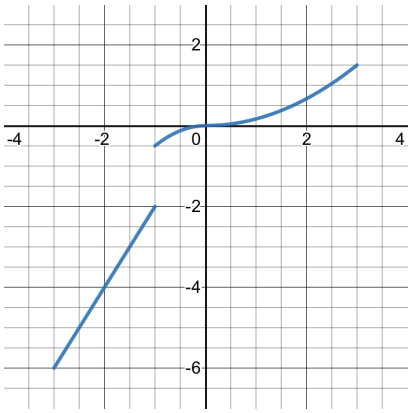
B)



C)



D)



2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Evaluate $1 + 2 + 3 + \cdots + 99 + 100$.

b) (8 points) Evaluate $\int (x^2 + \ln(x))\ dx$.

c) (8 points) Evaluate $\int_2^6 \frac{1}{r}\ dr$.

d) (8 points) Evaluate $\int \sin(t)\cos(t)\sin(\cos(2t))\ dt$. (Hint: $\sin(2t) = 2\sin(t)\cos(t)$)

3. (32 points) Let $v(t) = 2t + 2t^2$ be the velocity of a particle at time t.

a) (8 points) Find $a(t)$, the acceleration of the particle at time t.

b) (8 points) Find $s(t)$, the position of the particle at time t, given that $s(3) = 2$.

c) (8 points) Sketch graphs of $s(t)$, $v(t)$, and $a(t)$ on $[0, 5]$.

d) (8 points) What is the average position of the particle on $[0, 5]$?

e) (4 points extra credit) Let $e(x)$ be the average position of the particle on $[0, x]$. Find $e(x)$ and sketch a graph.