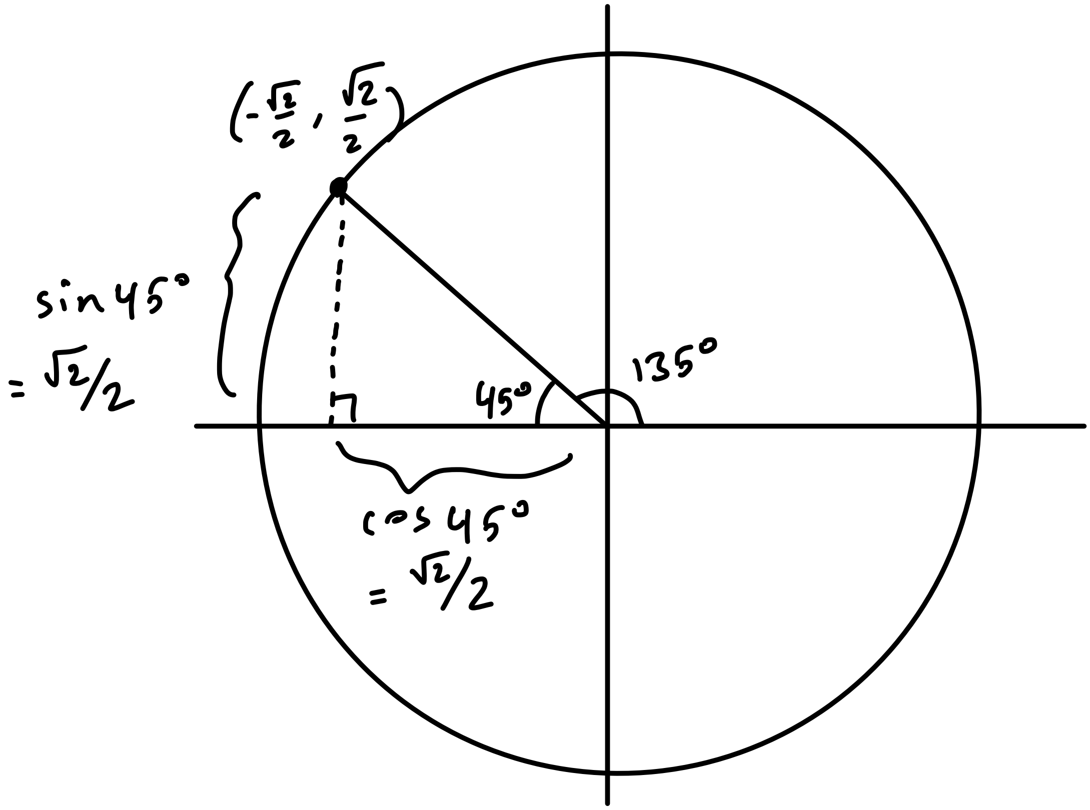
Ex: Find $\sin 135°$ and $\cos 135°$.



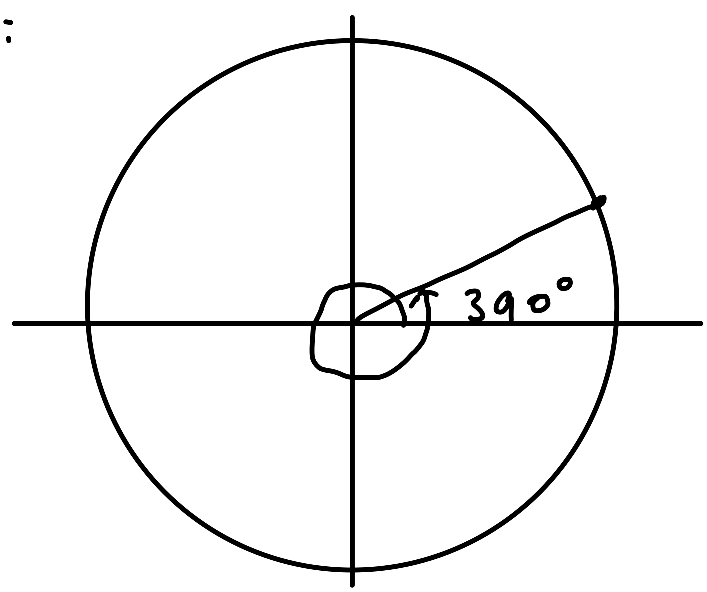
So $\sin 135° = \frac{\sqrt{2}}{2}$ and $\cos 135° = -\frac{\sqrt{2}}{2}$.

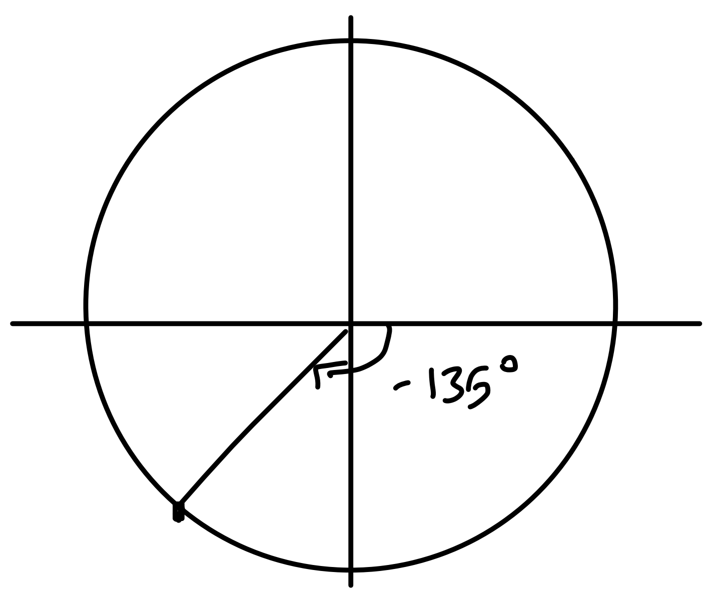
Comment: The table for special angles of cos is "reversed" from the one for sin θ.

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

Def: An angle larger than 360° corresponds to wrapping around the unit circle more than once, still c.c.w. from the positive x-axis. Negative angles correspond to moving clockwise from the positive x-axis.

Ex:

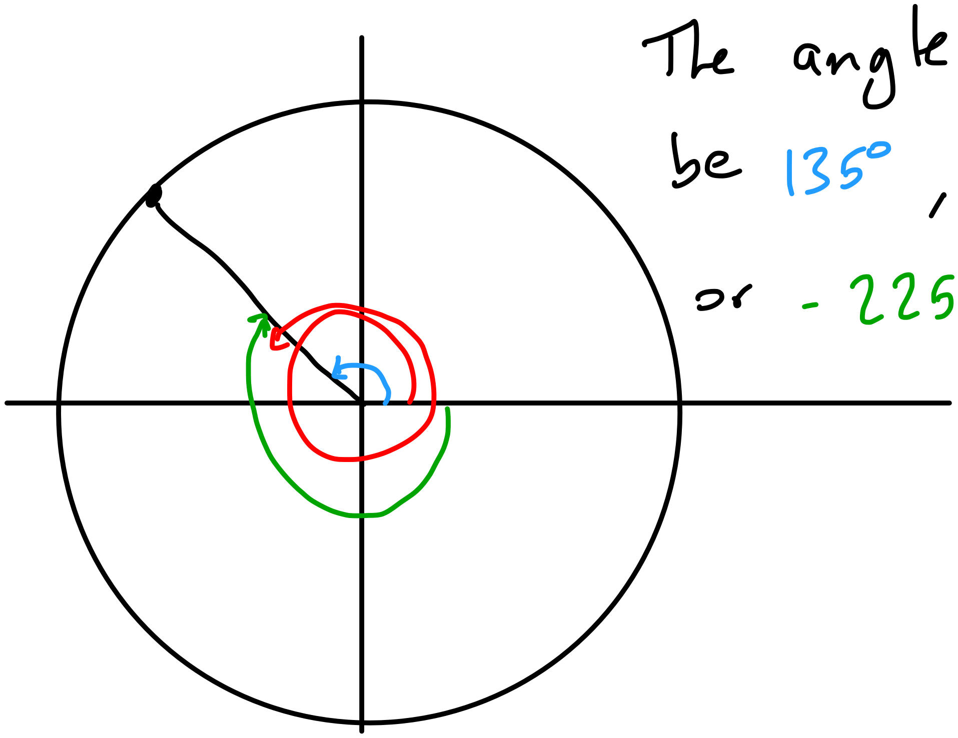




Comment: Any point on the unit circle can have many angles corresponding to it.

Specifically, adding or subtracting 360° from a point's angle won't change the point.

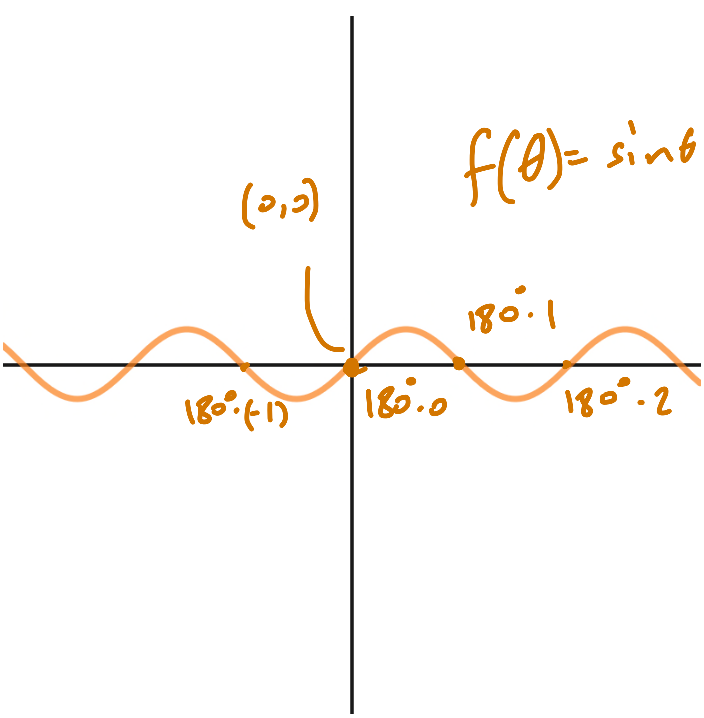
Ex:

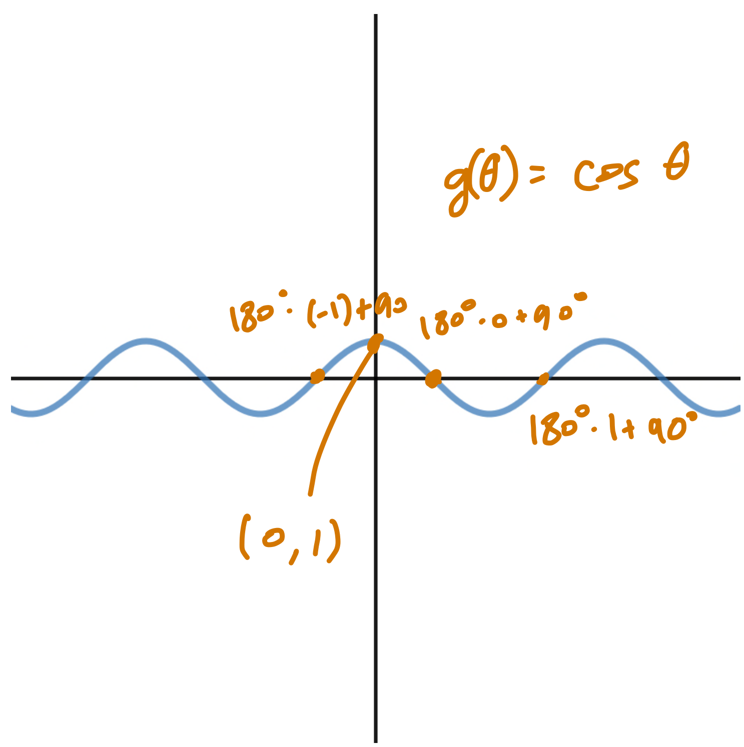


The angle could be 135°, 495°, or -225°.

The Graphs of sin and cos

Theorem: ① The graphs of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ are:





② The domain of $\sin\theta$ and $\cos\theta$ is $(-\infty, \infty)$ — any real number θ is fine to plug into both sin and cos.

③ The image of $\sin\theta$ and $\cos\theta$ is $[-1, 1]$.

④ The roots of $\sin\theta$ are $\theta = 180^\circ n$ for any integer n (remember that integers can be negative!)

The roots of $\cos\theta$ are $\theta = 180^\circ n + 90^\circ$ for any integer n.

⑤ The y-intercept of $\sin\theta$ is 0 and the y-intercept of $\cos\theta$ is 1.

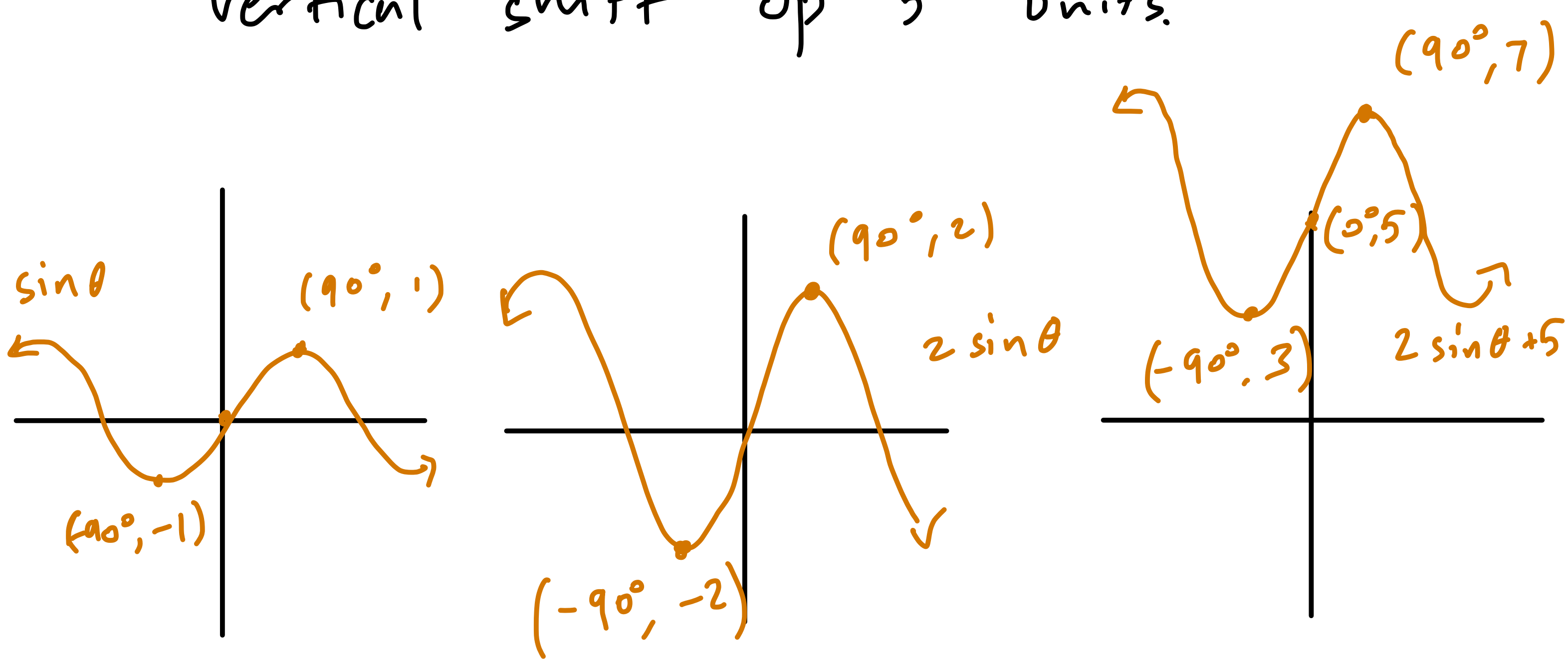
⑥ $\sin\theta$ is an odd function, and $\cos\theta$ is an even one.

⑦ Both $\sin\theta$ and $\cos\theta$ are periodic with period $360°$.

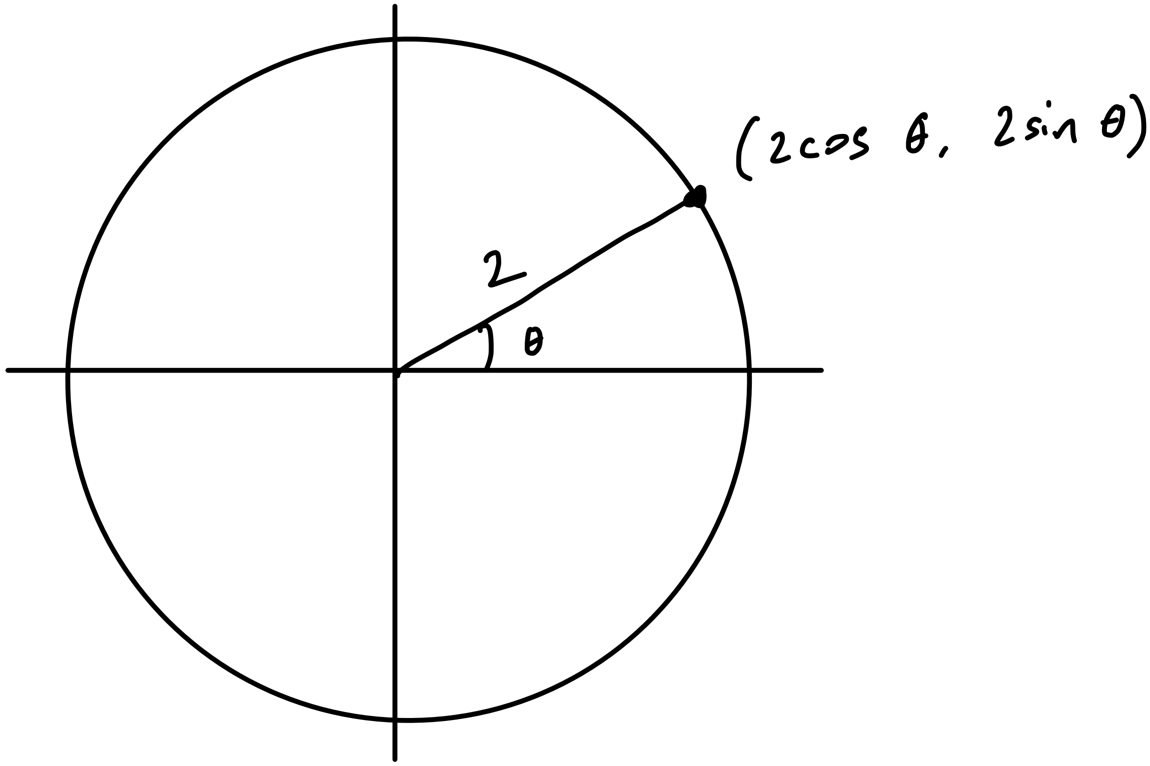
⑧ The midline of $\sin\theta$ and $\cos\theta$ is 0, and the amplitude is 1.

Ex: Graph $f(\theta) = 2\sin\theta + 5$.

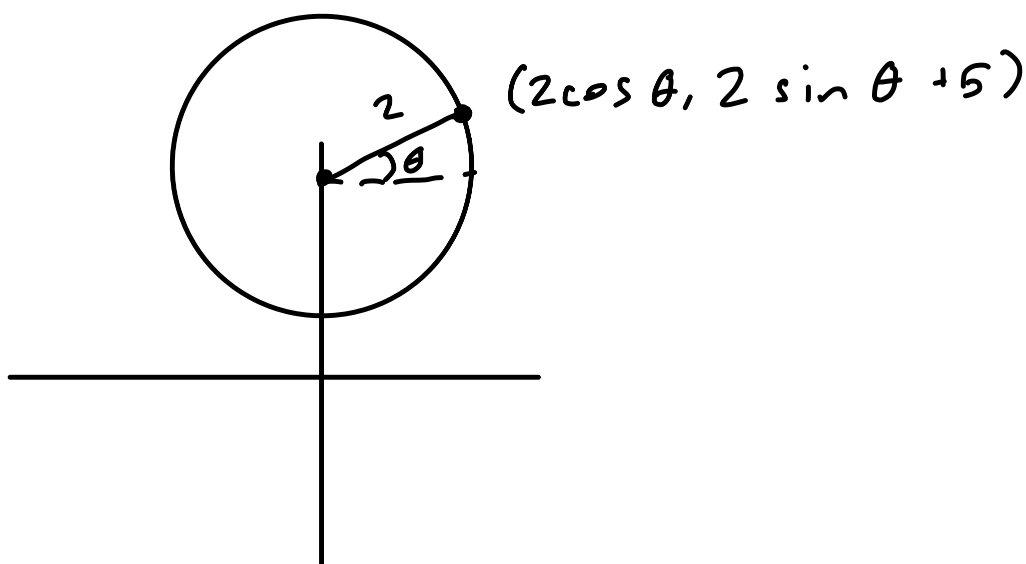
This is a vertical stretch of $\sin\theta$ by a factor of 2, followed by a vertical shift up 5 units.



Comment: If we were still thinking about $\sin\theta$ as the y-coordinate on a circle, then $2\sin\theta$ would be giving the y-coordinate on a circle of radius 2.



Comment: $2\sin\theta + 5$ is now the y-coordinate of a circle of radius 2 that's been shifted up 5 units.



Theorem: Let $f(\theta) = r\cos\theta + h$

and $g(\theta) = r\sin\theta + k$.

Then $(f(\theta), g(\theta))$ gives

the coordinates of a point

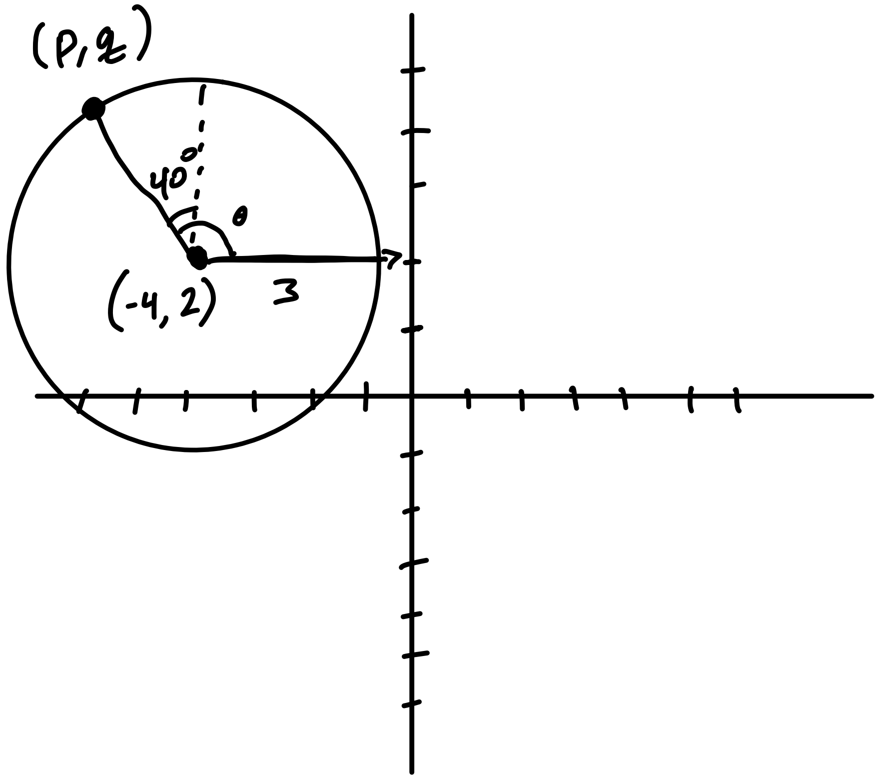
on the circle of radius

r centered at (h, k).

Note: the r values must

be the same.

Ex: Find p and q:

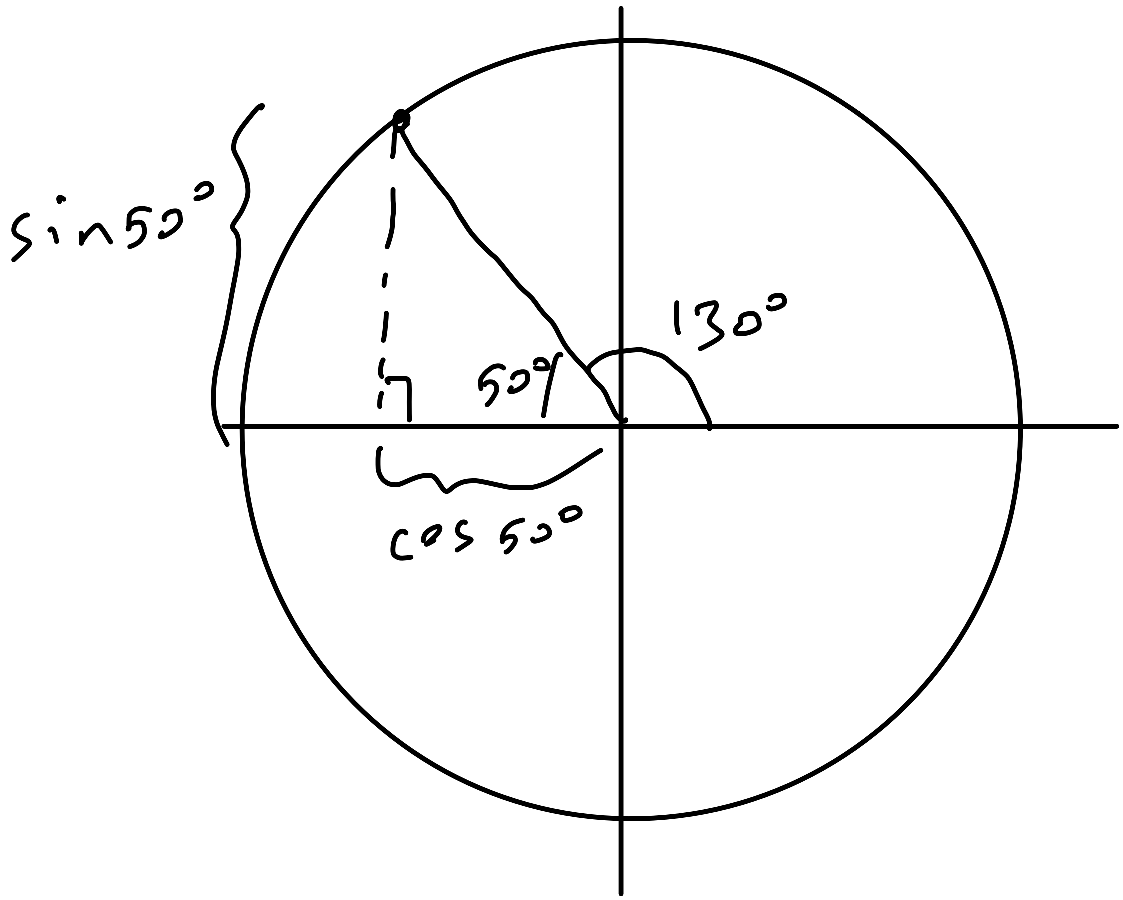


Applying the previous theorem, a point with angle θ on this circle has coordinates

$$(3\cos\theta - 4,\ 3\sin\theta + 2).$$

Here, $\theta = 90^\circ + 40^\circ = 130^\circ$. So we need $\cos 130^\circ$ and $\sin 130^\circ$.

Now we'll go back to the unit circle to find these:



50° is <u>not</u> a special angle, so we have to use a calculator: $\cos 50° = .643$ and $\sin 50° = .766$.

So $\sin 130° = .766$ and $\cos 130°$ $= -.643$. Finally,

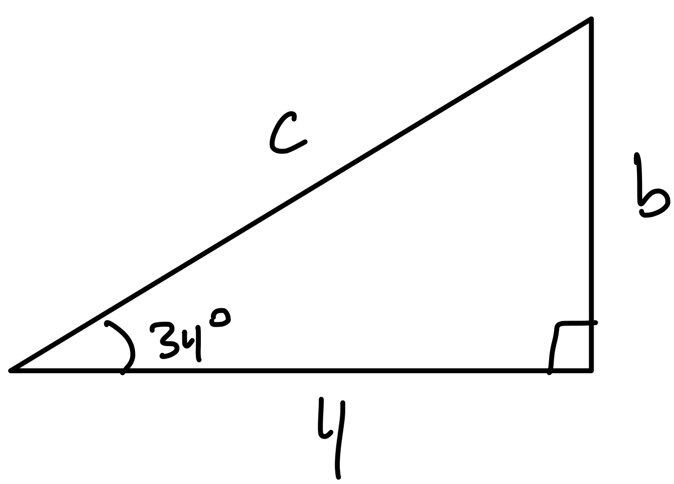
$$3\cos 130° - 4 = 3(-.643) - 4$$

$= -5.93$ and $3\sin 130° + 2$ $= 3(.766) + 2 = 4.3$. So the coordinates of a point on a circle of radius 3 centered at $(-4, 2)$, where the point has angle $130°$, are $(-5.93, 4.3)$.

The Tangent Function

Comment: In total, there are 6 trig functions. We've seen two: sin and cos. The good news is, we really only care about one of the other 4.

Ex: Find b:



We know that $\cos 34° = \frac{4}{c}$ and $\sin 34° = \frac{b}{c}$, but neither one of these lets us solve for b. But notice: $\dfrac{\sin 34^\circ}{\cos 34^\circ} = \dfrac{b/c}{4/c} = \dfrac{b}{c} \cdot \dfrac{c}{4} = \dfrac{b}{4}$.

And we know $\dfrac{\sin 34^\circ}{\cos 34^\circ}$. Thus $b = 4\left(\dfrac{\sin 34^\circ}{\cos 34^\circ}\right)$ $= 2.7$. We'd like to automate this reasoning.

Def: The <u>tangent</u> function is $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

Ex:

$$\tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = \frac{0}{1} = 0.$$

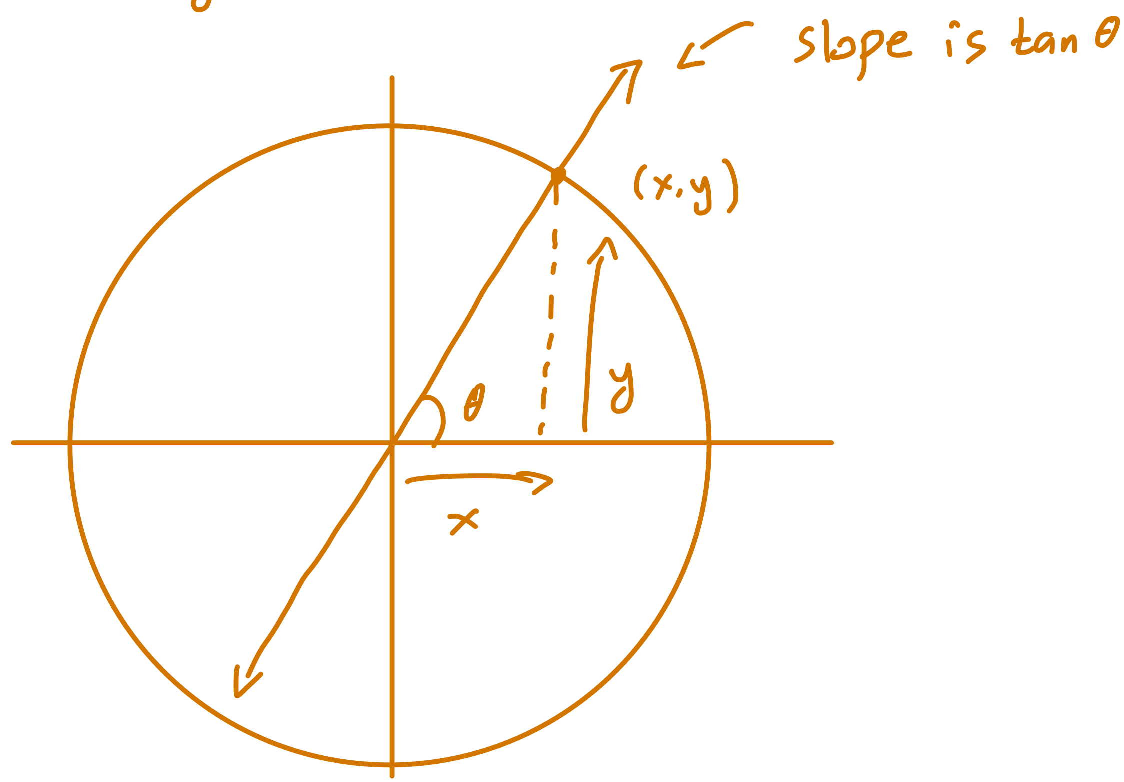
$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1.$$

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}.$$

$$\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0} \Rightarrow \text{undefined!}$$

<u>Theorem:</u> Let (x,y) be a point on the unit circle with angle θ. $\tan\theta$ is the slope of the line passing through $(0,0)$ and (x,y).



Proof: $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} = \frac{y-0}{x-0} = \frac{\text{rise}}{\text{run}}$.