Midterm 1

Math 243

Summer 2021

You have 1 hour and 50 minutes to complete this exam and scan and upload it to Canvas. **Show all your work. You may use a scientific calculator or a graphing one, and you may use your notes and textbook, but not the internet or other people.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

1. (32 points) Multiple choice. You don't need to show your work.

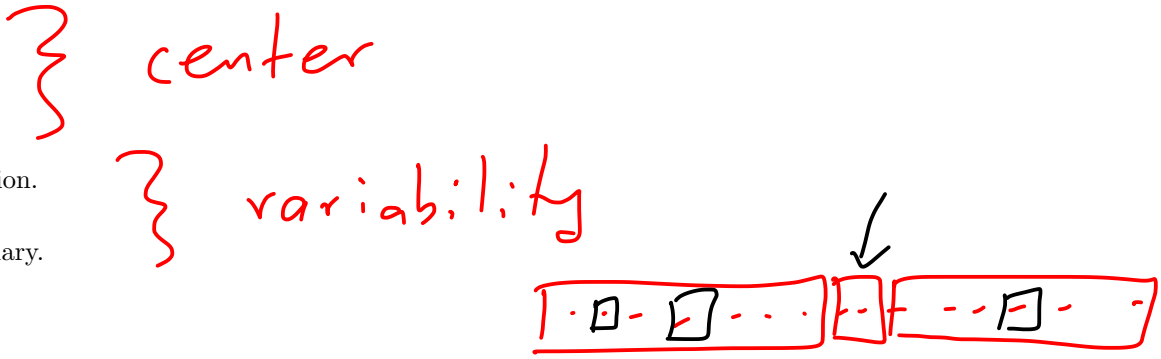
a) (8 points) Which of the following is a **non**-resistant measure of variability?

A) The mean.

B) The median.

C) The standard deviation.

D) The 5-number summary.



b) (8 points) In a data set, the first quartile Q_1 is defined such that

A) The mean of the data less than Q_1 is approximately one-fourth of the mean of the entire data set.

B) The median of the data less than Q_1 is approximately one-fourth of the median of the entire data set.

C) The number of data points less than Q_1 is approximately one-fourth of the total number of data points in the set.

D) None of the above.

c) (8 points) We flip a fair coin 2 times and record the total number of tails in a random variable X. What is the sample space?

A) $S = \{\text{heads}, \text{tails}\}$.

B) $S = \{0, 1, 2\}$.

C) $S = \{(\text{heads}, \text{heads}), (\text{heads}, \text{tails}), (\text{tails}, \text{heads}), (\text{tails}, \text{tails})\}$.

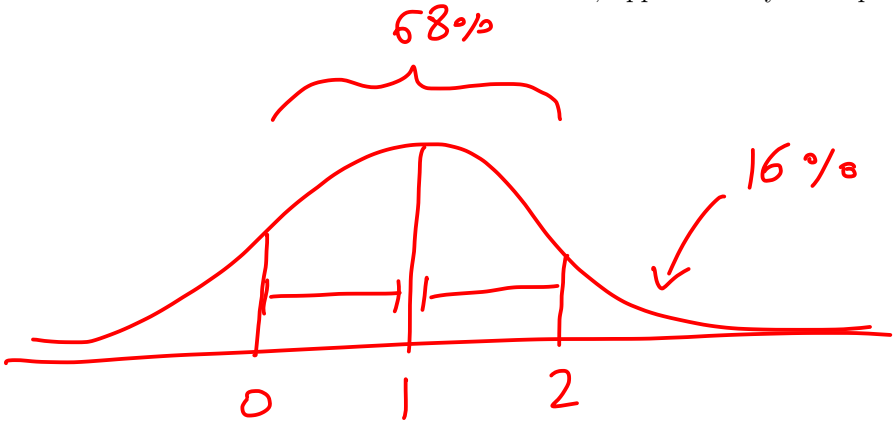
d) (8 points) In a normal distribution with mean 1 and standard deviation 1, approximately what percent of the data is larger than 0?

A) 84%.

B) 68%.

C) 32%.

D) 95%.

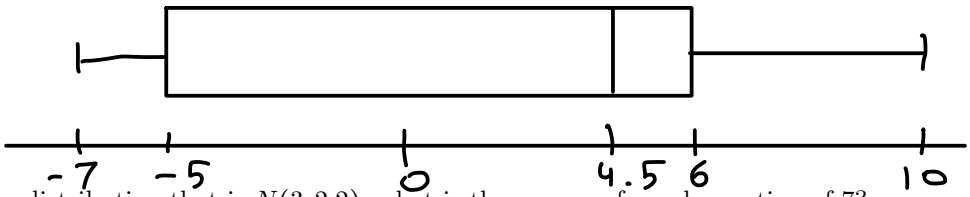


2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Draw a box-plot of the following data set. You don't need to mark outliers.

$10, -5, -1, 4, 5, 6, -7, 5$

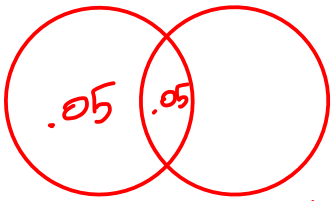
min = -1, $Q_1 = -5$, $Q_3 = 6$, max = 10, med = 4.5



b) (8 points) In a distribution that is $N(3, 2.2)$, what is the z-score of an observation of 7?

$$z = \frac{7-3}{2.2} = 1.82$$

c) (8 points) You reach into a drawer full of differently-colored clothing and pull something out at random. If the probability of pulling out a shirt is .4, the probability of pulling out something green is .1, and the probability of pulling out either a green object or a shirt is .45, what is the probability of pulling out a green shirt?



Total area = .45

Green shirt = .05

Alt: disjoint probs add

P(green shirt) + P(green nonshirt) + P(nongreen shirt) = P(green or shirt)

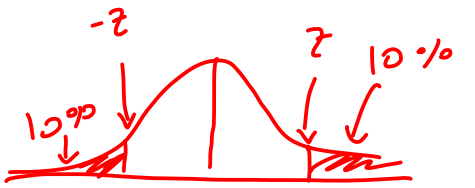
d) (8 points) With a population of 10000 people, how do we take a simple random sample of 100? You don't need to state the definition of an SRS verbatim, as long as your explanation is equivalent.

Assign ID #s and randomly generate 100

3. (32 points) The distribution of ACT scores of high school students in the US is approximately Normally distributed with mean 21 and standard deviation 5.

a) (8 points) If you'd want to score in the 90th percentile, what is the lowest score you can get?

want .9 of the population to be below



$z = -1.28 \Rightarrow .1$

$z = 1.28 \Rightarrow .9$

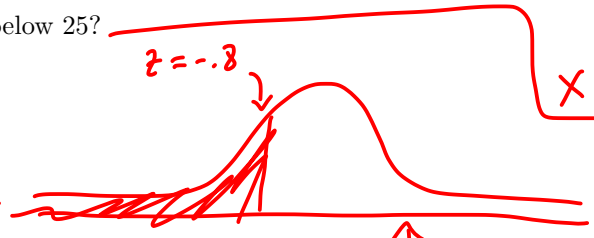
$z = \frac{x-21}{5}$

$1.28 = \frac{x-21}{5}$

$x = 27.4$.

b) (8 points) What proportion of scores are below 25?

$z = \frac{25-21}{5} = .8$



.2119

$1 - .2119 = .7881$

c) (8 points) In reality, the ACT has a maximum score of 36. If the distribution were actually normal, and 100000 students took the exam, how many would the theoretical distribution predict got over the maximum score?

$36 = 21 + 3(5)$

99.7% of scores are within 6 and 36,

so .3% are <6 or >36

.15% are above 36, so $\sim 100000(.0015) = 150$

d) (8 points) If you sample a single student from the distribution, what is the approximate probability that they scored between a 16 and 26?

got >36.

21 - 5

21 + 5

68%

4. (32 points) You roll a fair 6-sided die (with sides labeled 1 through 6) three times and record the number shown each time.

a) (8 points) Write three elements in the sample space.

(4, 3, 5), (3, 2, 6), (1, 1, 1) (e.g.)

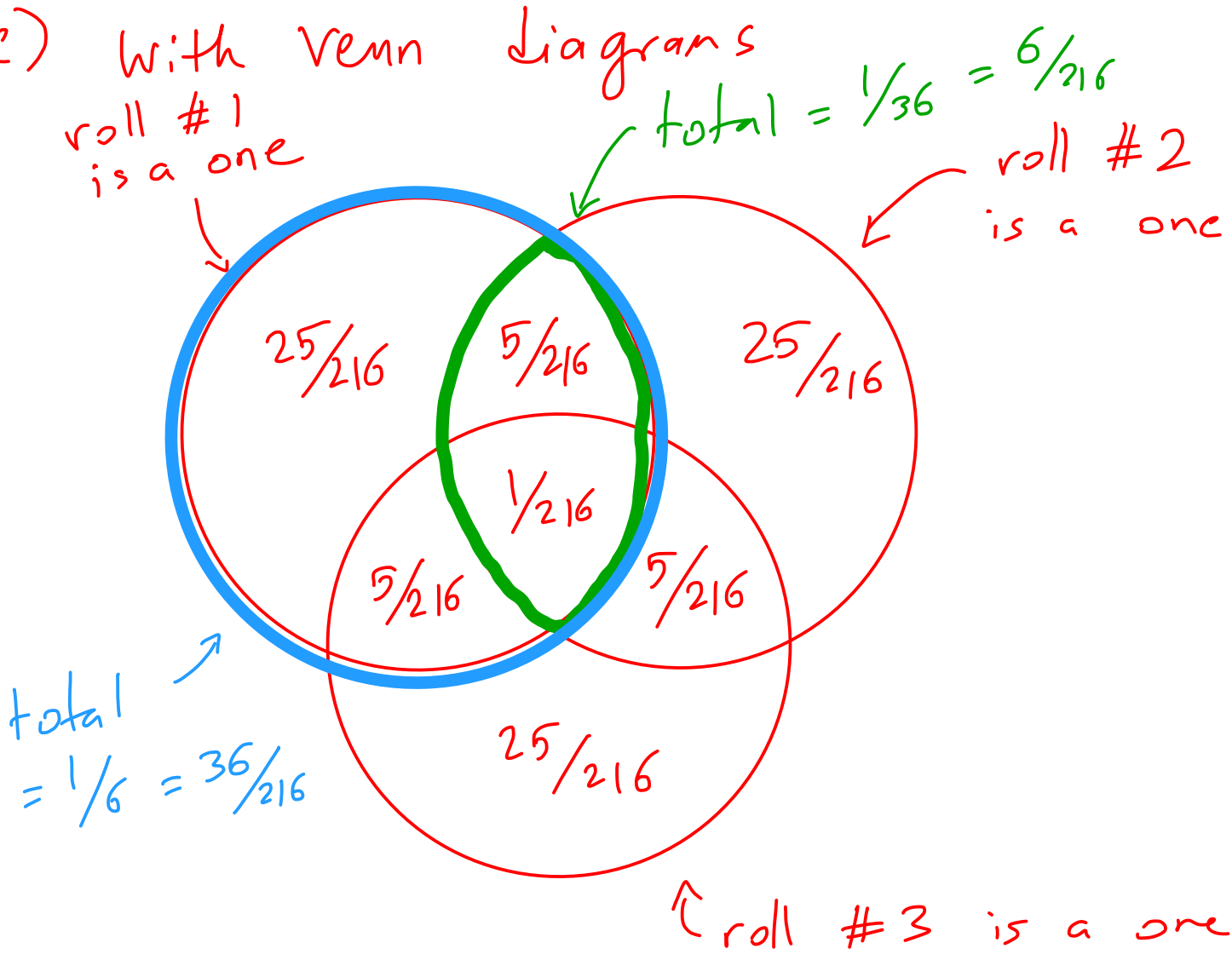
b) (8 points) Is this a discrete or continuous process?

Discrete, because there are a finite # of outcomes

c) (8 points) What is the probability of rolling exactly one 1?

d) (8 points) What is the probability of rolling at least one 1? (Hint: break this down into cases where you roll exactly one, two or three 1s)

c) With Venn diagrams



These three events are not disjoint, but they are independent, because rolling a one on the first roll doesn't affect the chance of rolling a one on the next rolls.

$$\Rightarrow P(3 \text{ ones}) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

$$P(\text{roll \#1 and roll \#2 are both ones}) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(\text{exactly 1 one}) = \frac{25}{216} + \frac{25}{216} + \frac{25}{216} = \frac{75}{216}$$

Without Venn diagrams:

How can we get exactly 1 one?

$(1, -, -)$ $(-, 1, -)$ ← 25

$(-, -, 1)$ ← 25

25 total

not ones (5 possibilities per $-$)

Total outcomes: $6 \cdot 6 \cdot 6 = 216$

$\Rightarrow {}^{75}/_{216}$

d) By Venn: $\frac{75}{216} + \frac{15}{216} + \frac{1}{216}$

(1 one, 2 ones, 3 ones)

$= {}^{91}/_{216}$

Or: to get exactly two ones:

$(1, 1, -), (1, -, 1), (-, 1, 1)$

5 5 5

exactly 3: (1, 1, 1) : 1

75 + 15 + 1 ways to get at least one 1

Or: $P(\geq 1 \text{ one}) = 1 - P(\text{no ones})$

$\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6}$

$= 1 - \frac{125}{216}$

$= 91/216$