1.4 Converse / Inverse / Contrapositive

Only if / if and only if

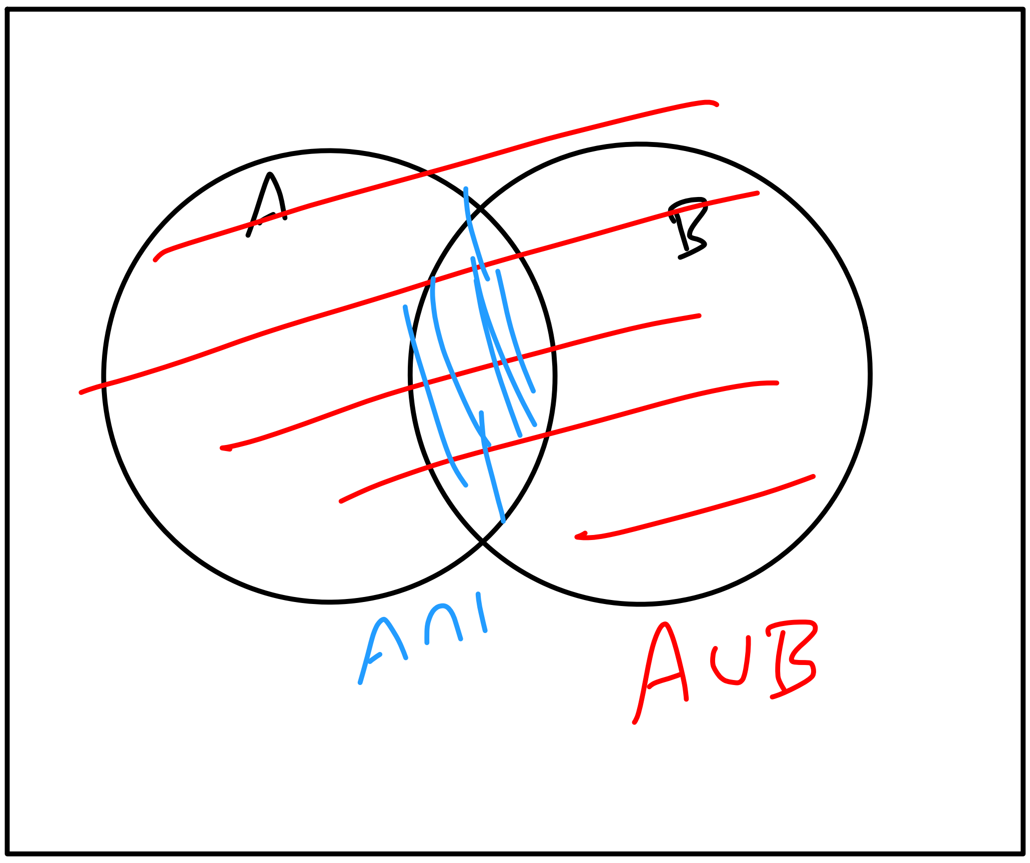
"p only if q" means "if p, then q"

$p \rightarrow q$

"p if and only if q" means "if p, then q, and if q, then p" $\quad p \leftrightarrow q$

P_1	$p \rightarrow q$	
P_2	$\sim q$	
C	$\sim p$	

$P_1 \wedge P_2 \rightarrow C$



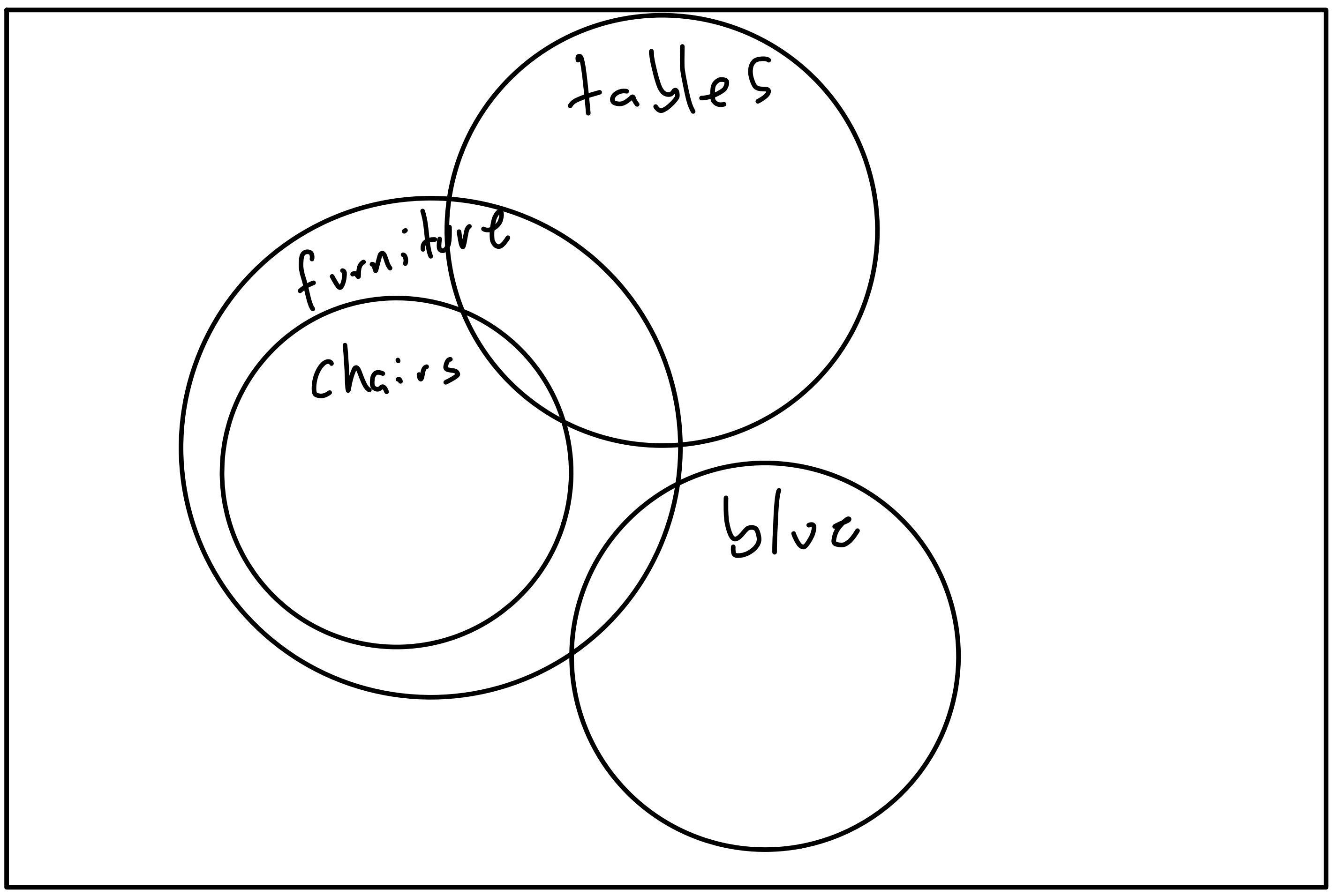
HW 1 solutions

① general facts
1. All chairs are furniture.
2. Some furniture is blue.
3. Nothing blue is a table.

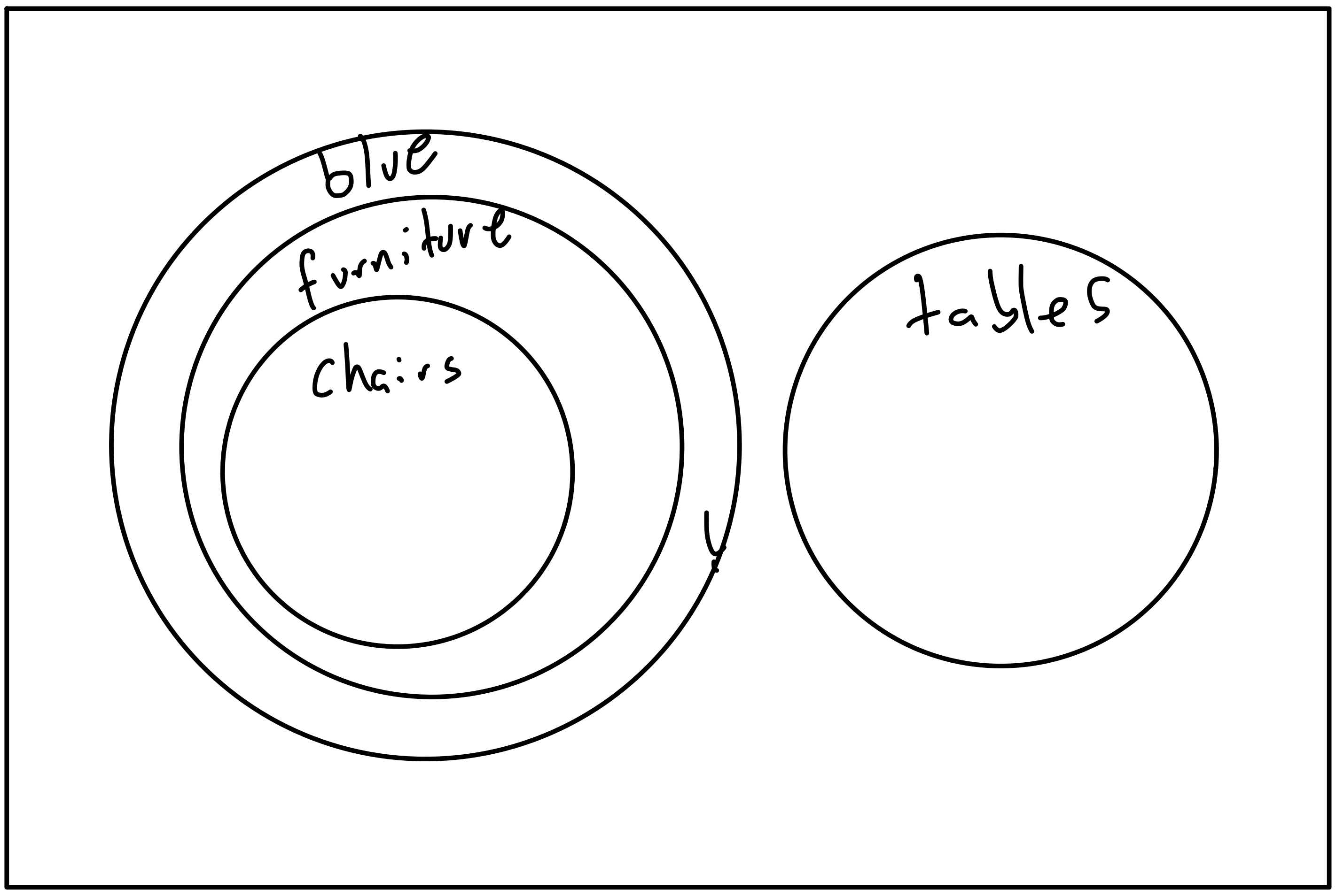
No chairs are tables.

a) Inductive or $\boxed{\text{deductive}}$

b) Let's try to prove that it's invalid. ✓



c)



(2) p : "all chairs are furniture"

q : "no furniture is blue"

r : "some blue objects are tables"

s : "no chairs are tables"

a)

$$\begin{array}{l} p \\ \sim q \\ \sim r \\ \hline s \end{array}$$

$p \wedge \sim q \wedge \sim r \longrightarrow s$

p	q	r	s	$p \wedge \sim q \wedge \sim r$	$p \wedge \sim q \wedge \sim r \to s$
T	T	T	T	F	T
T	T	T	F	F	T
T	T	F	T	F	T
T	T	F	F	F	T
T	F	T	T	F	T
T	F	T	F	F	T
T	F	F	T	T	T
T	F	F	F	T	$\boxed{F}$
F	T	T	T	F	T
F	T	T	F	F	T
F	T	F	T	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	T	F	F	T
F	F	F	T	F	T
F	F	F	F	F	T

c) $x \rightarrow y \equiv \sim x \vee y$

$$p \wedge \sim q \wedge \sim r \rightarrow s$$

$$\equiv \sim(p \wedge \sim q \wedge \sim r) \vee s$$

$$\equiv \sim p \vee \sim(\sim q) \vee \sim(\sim r) \vee s$$

$$\equiv \sim p \vee q \vee r \vee s$$

2.2: More Venn Diagrams

Ex: The results of a survey tell us:

213 people have tablets

294 have cell phones

337 have Blu-Ray players

109 have all three

64 have none

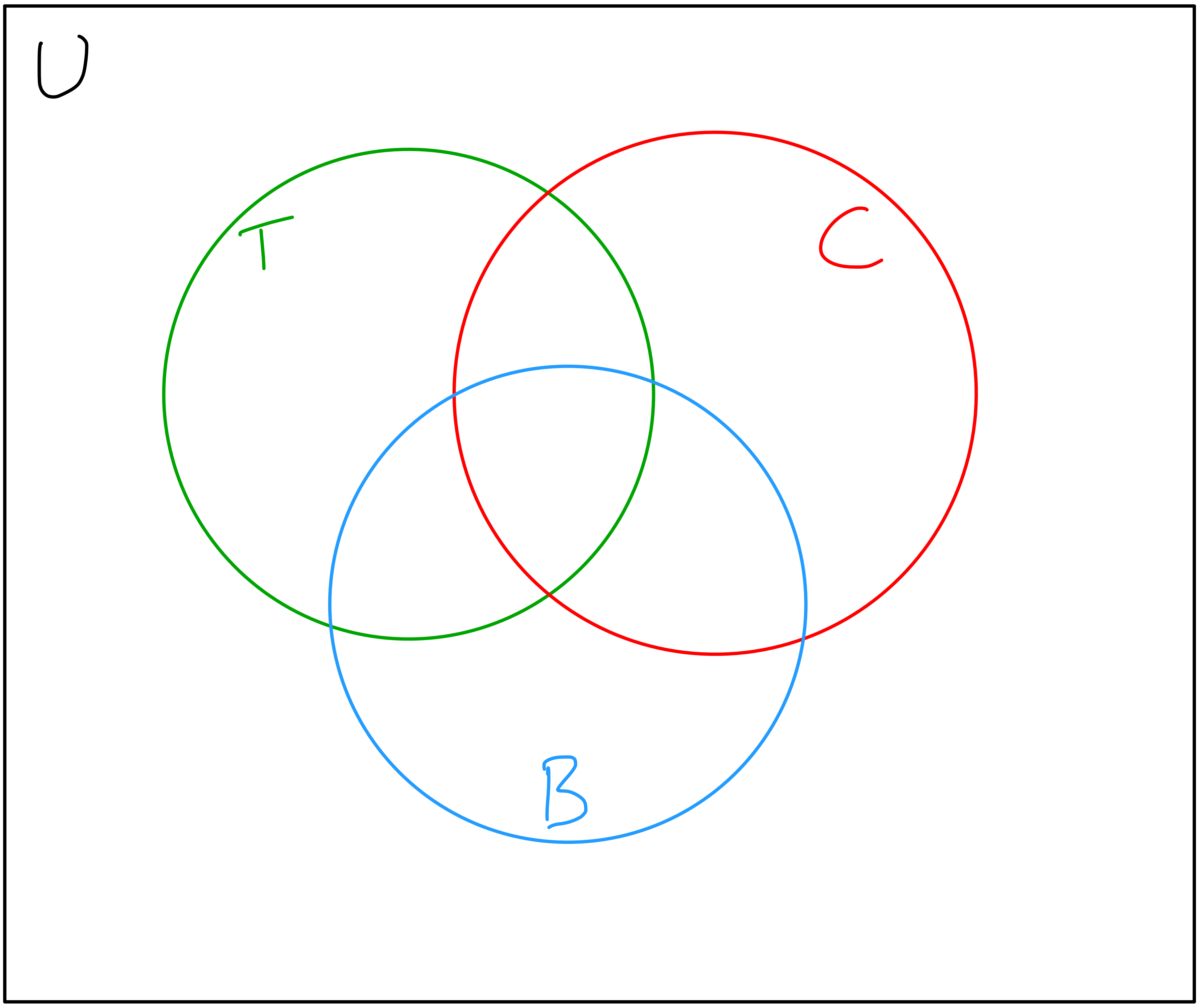
198 have cell phones and Blu-Ray players

382 have cell phones or tablets

61 have tablets and Blu-Ray players, but not cell phones

a) How many people surveyed own tablets but neither Blu-Ray players or cell phones?

b) How many own a Blu-Ray player but not a tablet or cell phone?



If U is the set of people surveyed, C is the set of people with cell phones, T is the set of people with tablets, and B is the set with Blu-Ray players, then:

$$n(T) = 213$$

$$n(C) = 294$$

$$n(B) = 337$$

$$n(C \cap T \cap B) = 109$$

$$n(C' \cap T' \cap B') = 64$$

$$n(C \cap B) = 198$$

$$n(C \cup T) = 382$$

$$n(T \cap B \cap C') = 61$$

Want: $n(T \cap B' \cap C')$

$n(B \cap T' \cap C')$

To solve these problems, fill in sections of the Venn diagram with cardinality when we know them.

Important: only write in numbers for sets that are not split into smaller sets. E.g. don't write the cardinality of B or $C \cap T$. Then use

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ to solve for the rest.