

Name: \_\_\_\_\_

Homework 9 | Math 256 | Cruz Godar

*Due Friday of Week 10 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 11

In problems 1–4, solve the system of DEs.

1.

$$\begin{aligned}x_1' &= 2x_1 - x_2 \\x_2' &= 3x_1 - 2x_2.\end{aligned}$$

For this problem, also sketch a vector field.

2.

$$\begin{aligned}x_1' &= 19x_1 - 4x_2 + 8x_3 \\x_2' &= -8x_1 + 5x_2 - 10x_3 \\x_3' &= -x_1 - 2x_2 + 4x_3.\end{aligned}$$

3.

$$\begin{aligned}x' &= 2x + 2y - 2z \\y' &= -3x + 7y + 3z \\z' &= -5x + 5y + 5z.\end{aligned}$$

4.

$$a' = 5a + 6b + 4c - 4d$$

$$b' = 3a + 8b - 2c + 2d$$

$$c' = 3a - 3b + 9c + 2d$$

$$d' = 11d.$$

5. Two tanks are set up in a cyclical cascade. Tank 1 initially contains 100 gallons of water and 10 pounds of sugar, and tank 2 initially contains 50 gallons of water and no sugar. At time  $t = 0$ , two valves are opened — the well-mixed solution in tank 1 flows into tank 2 at a rate of 5 gallons per second, and the well-mixed solution in tank 2 flows *back* into tank 1 at 5 gallons per second. After one minute, what is the concentration of sugar in each tank?

6. Let  $f(\lambda) = \lambda^2 + a\lambda + b$  be a polynomial. The **companion matrix** to  $f$  is the  $2 \times 2$  matrix

$$\mathbf{C}(f) = \begin{bmatrix} 0 & -b \\ 1 & -a \end{bmatrix}.$$

Show that the characteristic polynomial of  $\mathbf{C}(f)$  is  $f$ .

7. Using companion matrices, create  $2 \times 2$  matrices with real entries and the following eigenvalues:

a)  $\lambda_1 = 1, \quad \lambda_2 = 1.$

b)  $\lambda_1 = 1, \quad \lambda_2 = -1.$

c)  $\lambda_1 = -1, \quad \lambda_2 = -1.$

d)  $\lambda_1 = 1 + 2i, \quad \lambda_2 = 1 - 2i.$

e)  $\lambda_1 = -1 + 2i, \quad \lambda_2 = -1 - 2i.$

f)  $\lambda_1 = i, \quad \lambda_2 = -i.$

8. For each of the six matrices in the previous problem, sketch a vector field for  $-2 \leq x \leq 2$  and  $-2 \leq y \leq 2$ . Check your answer with the vector fields applet — for example, to plot

$$x' = 2x - y$$

$$y' = x + 4y,$$

enter `(2x - y, x + 4y)` in the box and hit generate.

## Section 13

In problems 9–12, solve the system of DEs for an implicit function of  $x$  and  $y$ , and determine the critical points. Verify your answers with the vector fields applet, and use it to classify the critical points as stable, unstable, or neither.

9.

$$x' = xy$$

$$y' = 2x - 1$$

10.

$$x' = x - xy$$

$$y' = xy - y$$

11.

$$x' = (y - 1)(y + 1)$$

$$y' = (x - 1)(x + 1)$$

12.

$$x' = x$$

$$y' = y \log |x|$$