Chapter I: Logic

Problem Solving

Ex: Solve $x^2 + 2x = -1$ for x.

$x^2 + 2x + 1 = 0$ ← specific situation

general fact ↓

Use the quadratic formula:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$= \frac{-2 \pm 0}{2} = -1.$$

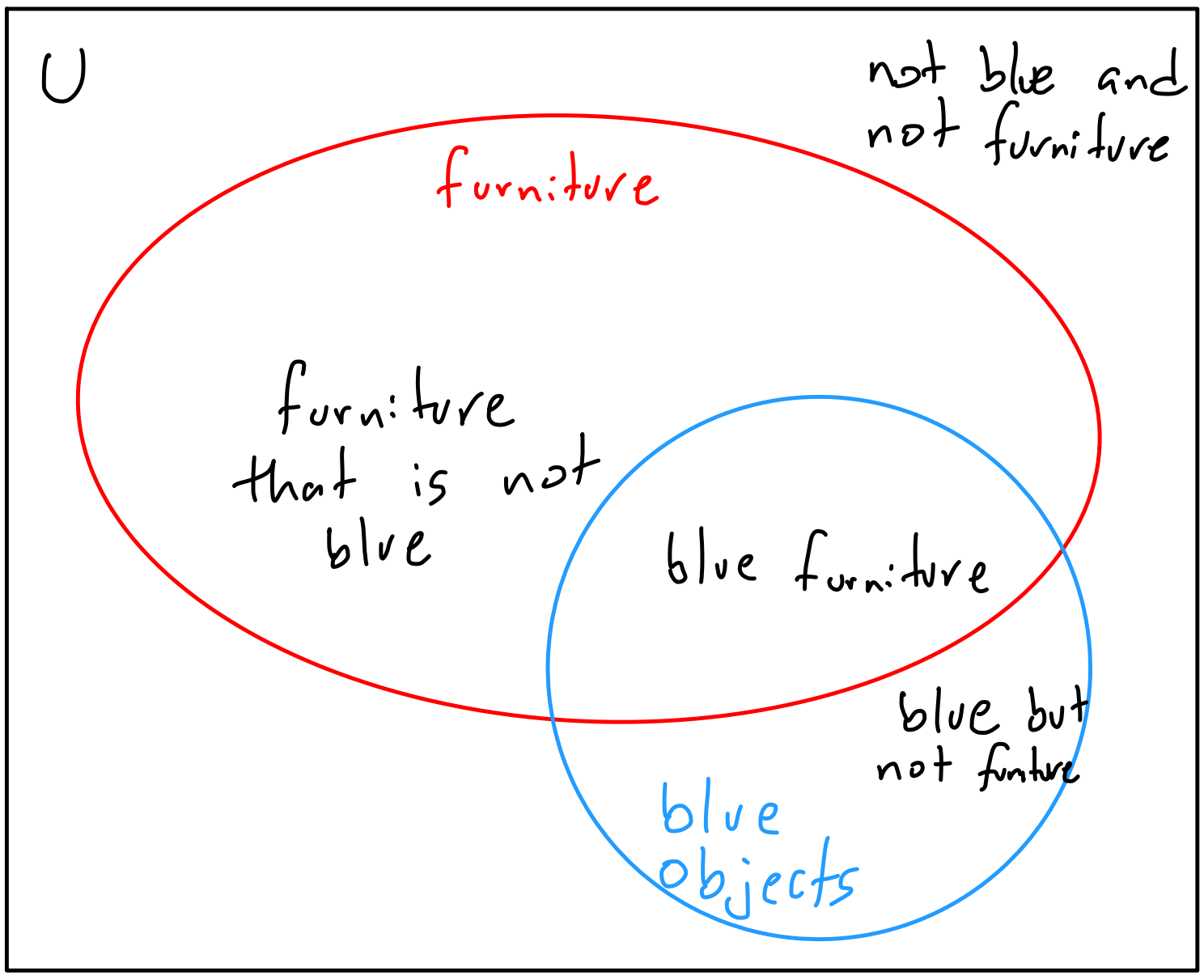
Comment: how did know how to solve this?

Well, we know that we can use the quadratic formula whenever we have an equation of the form $ax^2+bx+c=0$. Here, $ax^2+bx+c=0$ is a general kind of problem, and $x^2+2x+1=0$ is a specific instance.

Def: Deductive reasoning is a method to solve problems by applying general knowledge to a specific situation.

Def: A Venn diagram is a set of overlapping figures that are contained within a universe U, typically drawn as a rectangle.

Ex:



Def: An argument is valid if the conclusion follows logically from the statements before it. It doesn't matter whether those statements or the conclusion are true.

Ex: 1. All humans are mammals.

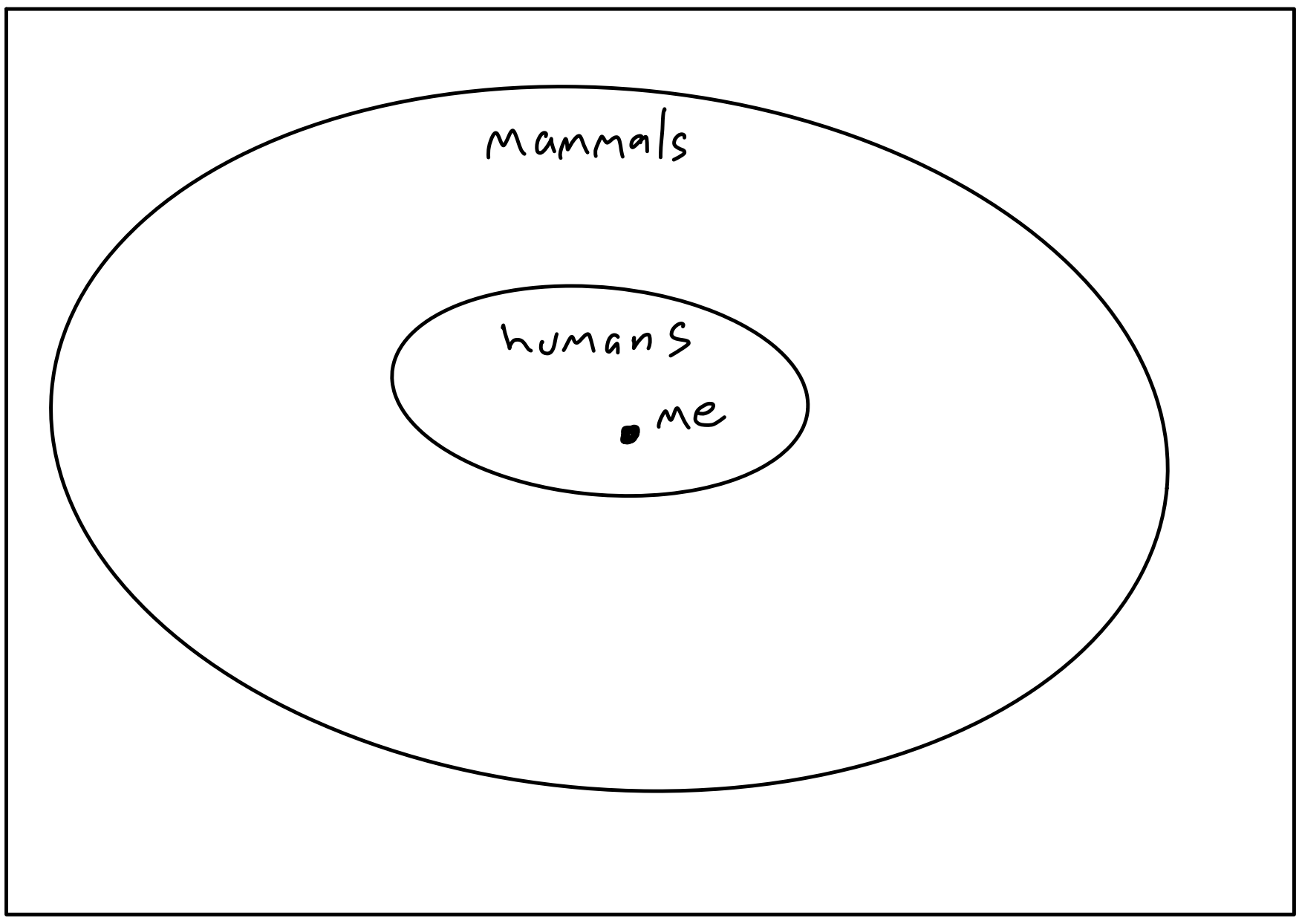
2. I am a human.

I am a mammal.

Method (Showing an argument is valid):

Draw a Venn diagram that follows all the statements and assumes nothing else. Then demonstrate that the condusion must be true

Ex: we want to draw a Venn diagram involving humans, mammals, and me.

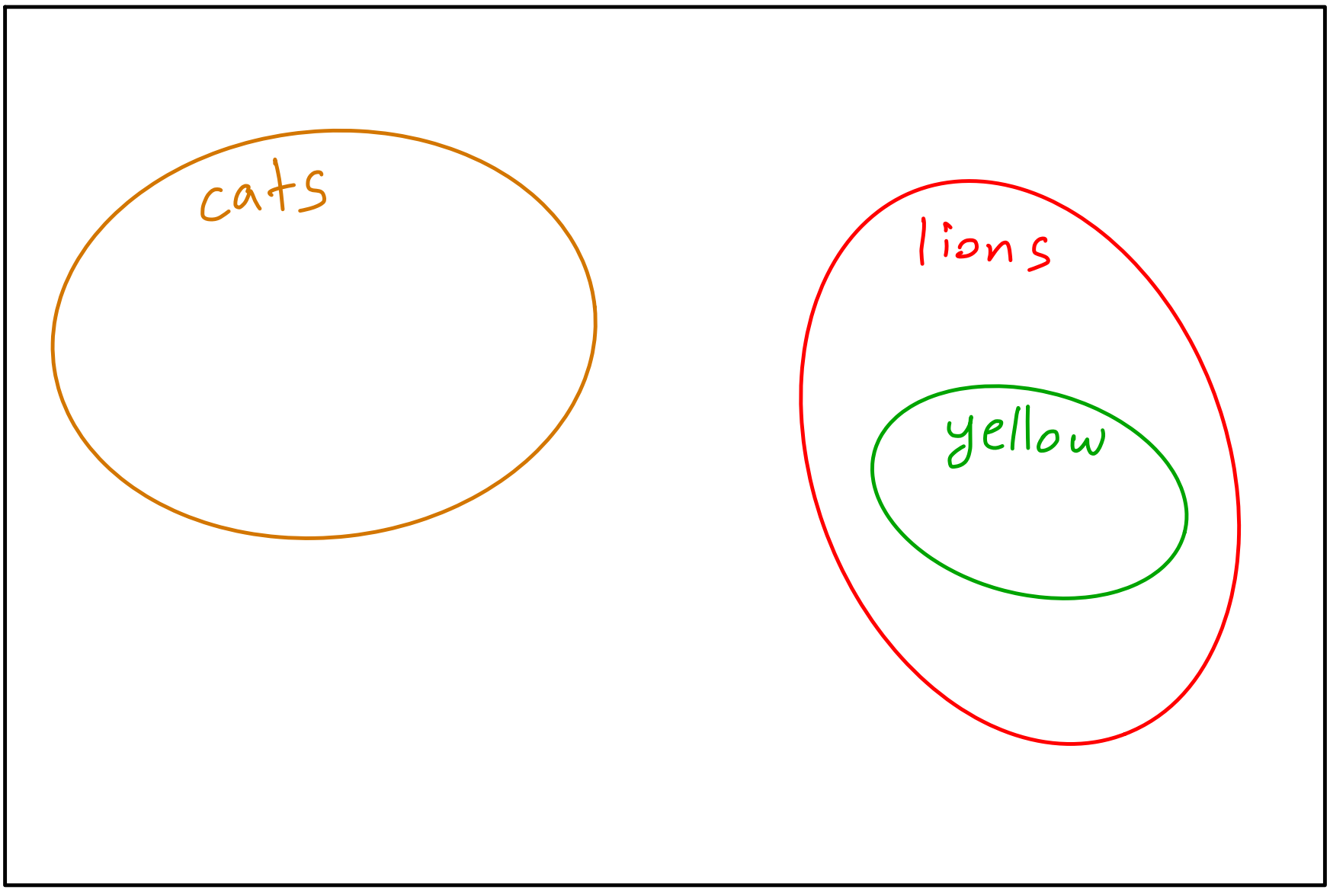


Since that dot lives inside the set of mammals, it must be the case that I am a mammal.

Ex: 1. No cats are lions.

2. All yellow animals are lions.

No cat is yellow.



Since the set of cats and the set of yellow animals don't overlap, no cat is yellow.

Comment: Venn diagrams only work when the argument uses deductive reasoning.

Method (Showing an argument is invalid):

Construct a Venn diagram that satisfies the statements but not the conclusion.

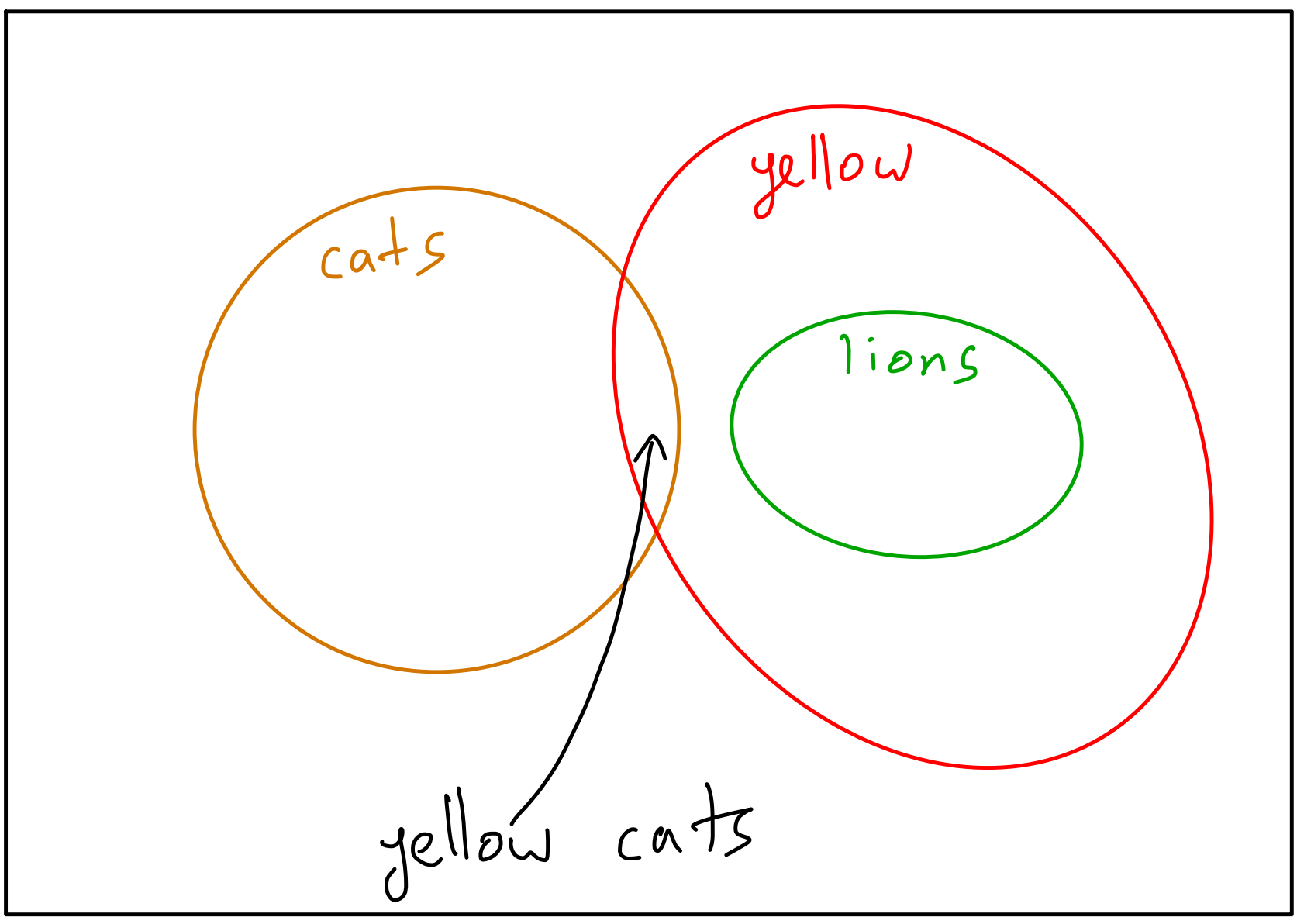
Ex: 1. No cats are lions.

2. All lions are yellow.

No cat is yellow.

To show this is invalid, we would need to draw a sitration where:

1. No cats are lions.
2. All lions are yellow.
3. Some cats are yellow.



Def: Inductive reasoning is a method to solve problems by finding a pattern in a few specific cases and conjecturing that the pattern holds in general.

Ex: 1. I got stung by a bee last month and it hurt.

2. I got stung by a bee today and it hurt.

Bee stings hurt.

Comment: We can't say for sure if an inductive argument is valid or not

general → (deductive) → specific

specific → (inductive) → general