Final Exam

Math 243

Summer 2021

You have 2 hours to complete this exam and scan and upload it to Canvas. **Show all your work. You may use a scientific calculator or a graphing one, and you may use your notes and textbook, but not the internet or other people.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE C ***t*** **distribution critical values**

Degrees of freedom	Confidence level C											
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
One-sided P	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
Two-sided P	.50	.40	.30	.20	.10	.05	.04	.02	.01	.005	.002	.001

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) When we increase the sample size n of a sample, why are we more confident that a sample mean $\overline{x}$ is a better approximation of the population mean μ?

A) Because the standard deviation of the sample data (i.e. s) becomes smaller.

B) Because the standard deviation of the sample means becomes smaller.

C) Because the mean of the sample means becomes closer to the population mean.

D) Because the z-score or t-score used to calculate the confidence interval increases.

b) (8 points) In a room full of people, 10% are wearing red shirts and 50% are wearing glasses. Among the red-shirts, only 20% are wearing glasses. What percent of the people wearing glasses are also wearing red shirts?

A) 4%.

B) 5%.

C) 40%.

D) 60%.

c) (8 points) If events A and B are independent, then $P(A \text{ and } B) =$

A) $P(A)P(B)$.

B) $P(A \mid B)$.

C) $P(A) + P(B)$.

D) $P(A \text{ or } B)$.

d) (8 points) Suppose that given a sample mean $\overline{x} = 2.5$, we find that a 95% confidence interval is 2.5 ± 1. This means that

A) There is a 95% chance that future sample means $\overline{x}$ will lie between 1.5 and 3.5.

B) There is a 95% chance that the population mean μ will lie between 1.5 and 3.5.

C) There is a 95% chance that a single individual's statistic x will lie between 1.5 and 3.5.

2. (64 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) In a standard Normal distribution, what is the minimum score in the 95th percentile?

b) (8 points) Draw a box plot of the following data, marking outliers as circles.

$$3, 4, -1, 5, 12, 0, 0, 2, 3, -1, -4, -5, -1$$

c) (8 points) We want to study a statistic across the entire adult US population. State a reason why it is difficult to take a true SRS of this population.

d) (8 points) You draw two cards off the top of a 52-card deck. What is the probability that both cards are spades? (Hint: are these events independent?)

e) (8 points) Suppose we take a sample of size n from a population whose distribution is **not** normal, with mean μ and standard deviation σ. For large n, what is the approximate distribution of $\overline{x}$?

f) (8 points) If we want to halve the size of a confidence interval, we need to multiply the sample size by a factor of what?

g) (8 points) Suppose we perform a hypothesis test on a subset of a population, where the subset has some unknown mean μ. The null hypothesis is that $\mu = 10$ and the alternative hypothesis is that $\mu > 10$. We use $\alpha = .05$ and find a p-value of .075. Do we reject the null hypothesis? Write a sentence or two explaining why and what this means.

h) (8 points) Suppose we take a sample of size n of a population and compute the proportion $\hat{p}$ of individuals in the sample with a certain property. We know that for large n, the distribution of $\hat{p}$ is roughly Normal. However, this is certainly not true when n is small. Give a reason why it is impossible for the distribution of $\hat{p}$ to be Normal when n is small.

3. (32 points) The distribution of the number of hours of sleep per night among college students is found to be approximately Normally distributed, with mean 6.5 hours and standard deviation 1 hour.

a) (8 points) What range contains approximately 99.7% of all college students?

b) (8 points) What is the probability that two randomly-selected college students both get 8 hours of sleep per night?

c) (8 points) You claim the average sleep times of fourth-year students is higher than that of all college students. To test this, you take a sample of 25 fourth-year students and find that they have an average of 7.5 hours per night. With a significance level of .05, is there sufficient evidence to support your claim?

d) (8 points) With the sample you took in part b), suppose the sample standard deviation of the 25 students is $s = .5$. What is the approximate probability that the mean sleep time of fourth year students lies within .3 hours (i.e. 18 minutes) of 7.5?

4. (32 points) Suppose two classes — class 1 and class 2 — are being offered at UO. Both have the same course ID (e.g. Math 243), but differ in instructor and (of course) enrolled students. Both classes are large lectures, so we can treat their students as being populations.

a) (8 points) Suppose a proportion of p_1 of the students in class 1 are passing and a proportion of p_2 of the students in class 2 are passing. If we sample 30 students from class 1 and 20 from class 2, and find the sample proportions $\hat{p_1}$ and $\hat{p_2}$, what are the individual distributions of $\hat{p_1}$ and $\hat{p_2}$, separately?

b) (8 points) What is the distribution of $\hat{p}_1 - \hat{p}_2$?

c) (8 points) This particular sample has $\hat{p}_1 = .75$ and $\hat{p}_2 = .7$. Give a 86% confidence interval for the actual value of $p_1 - p_2$.

d) (8 points) You want to assess the claim that the class 2 is harder than class 1. To do this, you let the null hypothesis be that $p_1 = p_2$ and the alternative hypothesis be that $p_1 > p_2$. With a significance level of .1, is the claim supported?