Final Exam

Math 252

Spring 2021

You have 2 hours to complete this exam, scan it, and upload it to Canvas. You may use a scientific calculator, but no other resources. When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it. There are 192 points possible on the exam, and 4 points will be deducted for each minute late.

Formulas

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\sin^2(\theta) = 1 - \cos^2(\theta)$
- $\cos^2(\theta) = 1 - \sin^2(\theta)$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\sec^2(\theta) - 1 = \tan^2(\theta)$
- $\sin^2(\theta) = \dfrac{1 - \cos(2\theta)}{2}$
- $\cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2}$
- $\sin(\theta) = \dfrac{\text{opp}}{\text{hyp}}$
- $\cos(\theta) = \dfrac{\text{adj}}{\text{hyp}}$
- $\tan(\theta) = \dfrac{\text{opp}}{\text{adj}}$
- $\sec(\theta) = \dfrac{\text{hyp}}{\text{adj}}$
- $\csc(\theta) = \dfrac{\text{hyp}}{\text{opp}}$
- $\cot(\theta) = \dfrac{\text{adj}}{\text{opp}}$

Part I: Multiple Choice and Short-Answer (64 points possible)

1. (8 points) Let $f(x)$ be a continuous function on $[0,1]$ such that $f(x) > 0$ on $\left[0, \frac{1}{3}\right)$ and $f(x) < 0$ on $\left(\frac{1}{3}, 1\right]$. Which of the following is always true?

   a) $\int_0^1 f(x)\ dx > 0$.

   b) $\int_0^1 f(x)\ dx < 0$.

   c) $\int_0^1 f(x)\ dx = 0$.

   d) None of the above.

2. (8 points) Complete the formula for integration by parts.

$$\int u\ dv =$$

3. (8 points) A differential equation is **separable** if

   a) It can be written in the form $y'(x) = y(x)$.

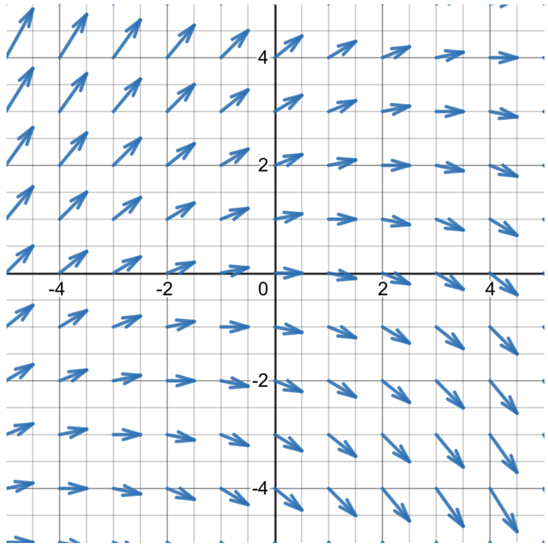
   b) It can be written in the form $y'(x) = f(x)g(y)$.

   c) It can be solved for y'.

   d) $y = x$ is a solution to the equation.

4. (8 points) Let $y = f(x)$ be a continuous function such that $f(x) > 0$ on $[a, b]$. Write the formula for the surface area of the solid of revolution given by rotating the graph of f about the x-axis.

5. (8 points) Which of the following differential equations could have generated this direction field?



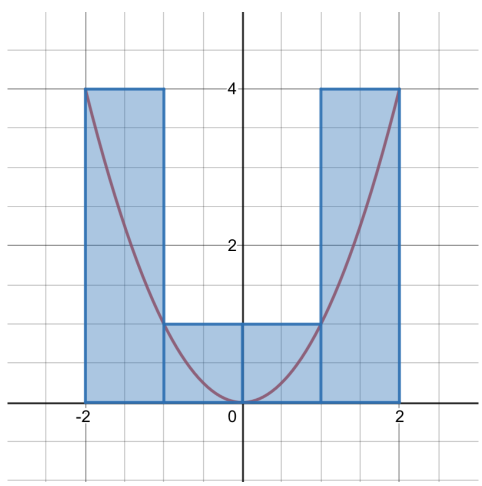
a) $y' = \sin(x)$.

b) $y' = xy$.

c) $y' = y - x$.

d) $y' = x^2$.

6. (8 points) The shaded area in the below figure is what kind of Riemann sum of $f(x) = x^2$ on $[-2, 2]$?



a) Left.

b) Right.

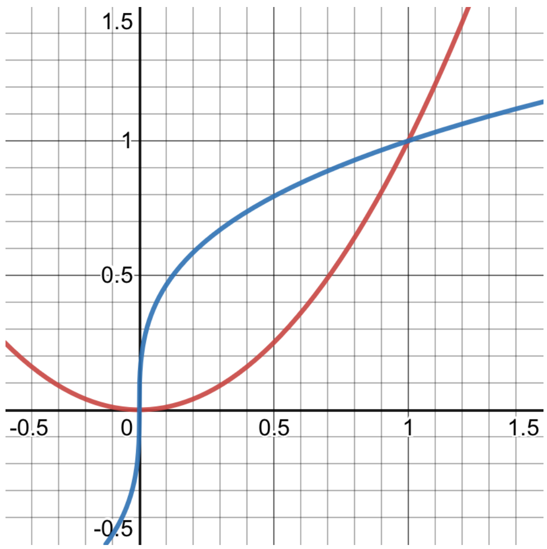
c) Upper.

d) Lower.

7. (8 points) In the following integral, what should we substitute for x and dx?

$$\int \frac{x^2}{\sqrt{x^2-4}}\,dx$$

8. (8 points) The functions $y = x^2$ and $x = y^3$ intersect at $(0,0)$ and $(1,1)$, as pictured. What is the area of the region bounded by the two curves?



a) $\int_0^1 y^3 - x^2\,dy$

b) $\int_0^1 x^2 - y^3\,dy.$

c) $\int_0^1 \sqrt{y} - y^3\,dy.$

d) $\int_0^1 y^3 - \sqrt{y}\,dy.$

Part II: Setting Things Up (64 points possible)

1. (16 points) Let $f(x) = 3x + 1$. Set up, but **do not solve**, the integral to find the volume of the solid of revolution generated by rotating the graph of f on $[1, 2]$ about the x-axis using the disk method.

2. (16 points) With f as in the previous question, set up, but **do not solve**, the integral to find the volume of the solid of revolution generated by rotating the graph of f on $[1, 2]$ about the x-axis using the disk method.

3. (16 points) A 3-meter rope hanging straight down has weight density $\rho(x) = 2x$, x meters from the bottom of the rope. Set up, but **do not solve**, the integral to find the work done by winding it all up.

4. (16 points) The graphs of $f(x) = x$ and $g(x) = 10\log(x)$ intersect at $(1.37, 1.37)$ and $(10, 10)$ and bound a region, on which $10\log(x) \geq x$. Set up, but **do not solve**, the integrals to find the center of mass of the region, given that the density is $\rho = 4$.

Part III: Integrals Proper (64 points possible)

1. (16 points) Find the solution to the differential equation $y'(t) = yt\cos(t)$, given that $y(0) = 1$.

2. (16 points) A region R is bounded above by $y = \dfrac{1}{x^2}$, below by $y = 0$, and to the left by $x = 1$. Find the area of R. You might find sketching a graph helpful.

3. (32 points) Evaluate $\displaystyle\int \frac{x-1}{(x^2+1)(x+1)^2}\,dx$.

4. (16 points extra credit) Let $f(x)$ be a continuous function on $[a,b]$ and let $A(x)$ be the average value of f on $[a,x]$. Show that the rate of change of A is $\dfrac{f(x) - A(x)}{x - a}$.