

Name: \_\_\_\_\_

Homework 3 | Math 342 | Cruz Godar

*Due Wednesday of Week 4 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

### Section 3

In problems 1–3, find the general solution to the system of differential equations.

1.

$$\begin{aligned}x_1' &= 2x_1 - x_2 \\x_2' &= 3x_1 - 2x_2.\end{aligned}$$

For this problem, also sketch a vector field.

2.

$$\begin{aligned}x_1' &= 19x_1 - 4x_2 + 8x_3 \\x_2' &= -8x_1 + 5x_2 - 10x_3 \\x_3' &= -x_1 - 2x_2 + 4x_3.\end{aligned}$$

3.

$$\begin{aligned}x' &= 2x + 2y - 2z \\y' &= -3x + 7y + 3z \\z' &= -5x + 5y + 5z.\end{aligned}$$

4. Two tanks are set up in a cyclical cascade. Tank 1 initially contains 100 gallons of water and 10 pounds of sugar, and tank 2 initially contains 50 gallons of water and no sugar. At time  $t = 0$ , two valves are opened — the well-mixed solution in tank 1 flows into tank 2 at a rate of 5 gallons per second, and the well-mixed solution in tank 2 flows *back* into tank 1 at 5 gallons per second. After one minute, what is the concentration of sugar in each tank?

5. Let  $f(\lambda) = \lambda^2 + a\lambda + b$  be a polynomial. The **companion matrix** to  $f$  is the  $2 \times 2$  matrix

$$C(f) = \begin{bmatrix} 0 & -b \\ 1 & -a \end{bmatrix}.$$

Show that the characteristic polynomial  $\chi_{C(f)}$  is  $f$ .

6. Using companion matrices, create  $2 \times 2$  matrices with real entries and the following eigenvalues:

a)  $\lambda_1 = 1, \quad \lambda_2 = 1.$

b)  $\lambda_1 = 1, \quad \lambda_2 = -1.$

c)  $\lambda_1 = -1, \quad \lambda_2 = -1.$

d)  $\lambda_1 = 1 + 2i, \quad \lambda_2 = 1 - 2i.$

e)  $\lambda_1 = -1 + 2i, \quad \lambda_2 = -1 - 2i.$

f)  $\lambda_1 = i, \quad \lambda_2 = -i.$

7. For each of the six matrices in the previous problem, sketch a vector field for  $-2 \leq x \leq 2$  and  $-2 \leq y \leq 2$ .

Check your answer with the vector fields applet — for example, to plot

$$x' = 2x - y$$

$$y' = x + 4y,$$

enter `<code>(2x - y, x + 4y)</code>` in the box and hit generate.