Final Exam

Math 105

Summer 2020

You have 110 minutes to complete this exam **(this includes the time it takes to scan and upload it)**. You may not use a calculator, your notes, the textbook, or any other resources. Write your solutions on a separate sheet of paper, and when you're finished, first check your work if there is time remaining, then scan it and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (8 points) You draw a card off the top of a randomly-shuffled 52-card deck. If it's a spade, you win four dollars, and if it's not, you lose one dollar. What is the expected value of drawing a card? Show all your work.

2. (8 points) From a 52-card deck, how many 5-card hands containing exactly three aces are there, where we don't consider a reordering to be a new hand? Show all your work. (*Hint: split the problem into choosing the aces and choosing the non-aces, and then figure out how to combine those two results*).

3. (8 points) Using your answer from the previous problem, what is the probability of dealing a 5-card hand with exactly three aces off the top of a randomly shuffled 52-card deck? Give your answer as a fraction — you don't need to simplify any arithmetic, but there shouldn't be any non-arithmetic symbols in your final answer. Show all your work.

4. (8 points) Consider the following argument:

1. All cats are yellow animals.
2. Some yellow animals are squares.

Conclusion: some cats are squares.

Is this argument valid or invalid? Justify your answer with a Venn diagram.

5. (8 points) A handful of people downtown receive parking tickets of different severities — some red and some yellow. If 20 people got red tickets and 15 got yellow tickets, and 28 got tickets overall, how many people got both red and yellow tickets?

6. (8 points) A zoo contains three major regions: desert, forest, and plains. In an effort to better represent natural biomes, all three overlap and blend into one another. At noon, the numbers of people in each environment is recorded. There are:

- 75 people in the desert
- 68 people in the plains
- 142 people in the forest
- 19 people in both the desert and plains
- 28 people in both the forest and plains
- 21 people in the forest and plains, but not the desert
- 48 people in the desert only

Find the number of people in:

- the forest only
- the plains only
- all three biomes at once

Show all your work.

7. (8 points) You roll two 6-sided dice. If A is the event of rolling a 5 on the first die and B is the event of rolling a 5 on the second, then A and B are:

i. Mutually exclusive
ii. Independent
iii. Both
iv. Neither

Pick one and explain your reasoning.

8. (8 points) Construct the truth table for $(\sim p \vee q) \wedge r$.

9. (8 points) How many orderings of 6 objects are possible? Give your answer as a single, simplified number.

10. (8 points) Describe the set $A = \{x \in \mathbb{R} \mid x^2 = 2\}$ in words.

Bonus (2 points): Write A as a list of elements (e.g. $\{1, 2, 3\}$) instead of in set-builder notation.

11. (8 points) Simplify $\sim (p \wedge q)$ and write it as a sentence, leaving p and q as variables. There shouldn't be any parentheses in your answer.

12. (8 points) You draw a card off the top of a randomly-shuffled 52-card desk. If A is the event of the card being a spade and B is the event of the card being the king of spades, find $p(B \mid A)$. Show all your work.