Name: ____________________ Homework 7 | Math 341 | Cruz Godar

Due Wednesday of Week 8 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 8

In problems 1–6, find **two different** bases $\mathcal{B}$ and $\mathcal{C}$ for the vector space V and use them to find $\dim V$. Then with the given vector $\vec{v}$, find $[\vec{v}]_{\mathcal{C}}$.

1. $V = \mathbb{R}^3$, and $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

2. $V = M_{2\times 2}(\mathbb{R})$, and $\vec{v} = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$.

3. $V = \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$, and $\vec{v} : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by $\vec{v}\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x \\ x+y \end{bmatrix}$. (Hint: your answers to the previous problem may help.)

4. V is the subspace of $\mathbb{R}^4$ of vectors $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$ satisfying $x + y - w = 0$, and $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$.

5. $V = \mathbb{R}[x]$, and $\vec{v} = (x^2 - 2)^2$.

6. $V = \operatorname{span}\{\cos(x), \sin(x)\}$, and $\vec{v} = \sin\left(x + \frac{\pi}{4}\right)$. (Hint: the sum and difference formulas for sin and cos may be helpful.)

In problems 7–9, find a matrix for the linear transformation $T : V \to W$ by choosing bases $\mathcal{B}$ for V and $\mathcal{C}$ for W. Then use the matrix to evaluate $T(\vec{v})$ for the given vector v.

7. $V = \mathbb{R}^3$ and $W = \mathbb{R}$, $T : V \to W$ is a transformation for which

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = 1 \qquad T\left(\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right) = 2 \qquad T\left(\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}\right) = -1,$$

and $\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

8. V and W are both the subspace of $\mathbb{R}[x]$ of polynomials with degree at most 2, $T : V \to W$ is a transformation for which

$$T(1) = x \qquad T(x^2 + x) = 2x \qquad T(x^2) = x^2,$$

and $\vec{v} = x^2 - x - 1$.

9. $V = M_{2\times 2}(\mathbb{R})$, $W = \mathbb{R}^2$, $T : V \to W$ is a transformation for which

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \qquad T\left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

and $\vec{v} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.