Midterm 2

Math 112

Spring 2020

You have 50 minutes to complete this exam (plus 10 minutes to account for the time it takes to scan and upload it). **You may use a scientific calculator, but no other resources.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

1. (32 points) Let $g(\theta) = \tan(2\theta) - 1$.

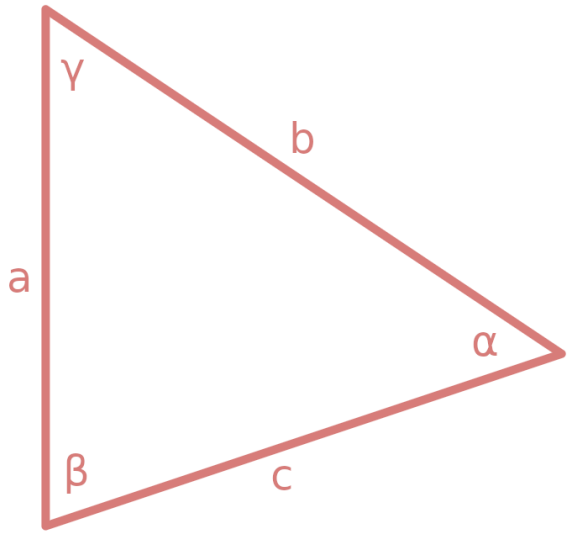
a) (8 points) Find $g\left(\frac{\pi}{6}\right)$. Leave your answer in exact form, and show all your work — specifically, how you calculate the tangent.

b) (8 points) Sketch a graph of g. Label at least three points.

c) (8 points) For what values of θ is $g(\theta) = -1$? List all of the values, and express your answers in radians.

d) (8 points) For what values of θ is $g(\theta) = 0$? List all of the values, and express your answers in radians.

2. (32 points) Consider the following triangle with sides a, b, and c, and angles α, β, and γ.



a) (8 points) Given that $a = 3$, $b = 3.61$, and $\gamma = 56.3°$, find c.

b) (8 points) Use your answer to part a) to find α.

c) Now find β.

d) (8 points) Find the area of this triangle. (Hint: pick one side to be the base, then draw a line perpendicular to that base that reaches to the opposite vertex to form a right triangle. Then use trig functions to find the length of that line.)

3. (32 points) Miscellaneous questions: these don't make sense as full-length problems, so the four parts here are unrelated to one another.

a) (8 points) Find an exact value for $\sec\left(\frac{2\pi}{3}\right)$. Show all your work.

b) (8 points) Find an exact value for $\sin(75°)$. Show all your work.

c) Write the equation of a sinusoidal function $f(x)$ with amplitude 2, midline $-\sqrt{3}$, and period 2π, such that $f(0) = 0$.

d) Sketch a graph of $\arctan(x)$. Label at least three points.