Name: _____ Homework 5 | Math 253 | Cruz Godar

Due Wednesday of Week 6 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–12, determine if the series converges absolutely, converges conditionally, or diverges. For alternating series that converge, estimate the series to within 0.1 of its actual value.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$.

2. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^{k+1}}{\ln(\ln(k))}$.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$.

4. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+1}{3n+2}\right)^n$.

5. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$.

6. $\displaystyle\sum_{n=2}^{\infty} \frac{2^n}{\ln(n)}$.

7. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.

8. $\displaystyle\sum_{i=1}^{\infty} \frac{\sin(i)}{i\sqrt{i}}$.

9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$.

10. $\displaystyle\sum_{m=1}^{\infty} \frac{(-1)^m \sin\left(\frac{1}{m}\right)}{m}$.

11. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(1 - n^{1/n}\right)$.

12. $\displaystyle\sum_{k=1}^{\infty}\left(k^{1/k}-1\right)^{k}$.

The ratio and root tests look pretty similar to one another — let's see if we can find a relationship between the two.

13. Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a series with positive terms that converges via the ratio test: i.e. $\displaystyle\lim_{n\to\infty}\frac{a_{n+1}}{a_n}<1$. Therefore, there is a value $r<1$ so that whenever $n\geq N$ for some N, $\frac{a_{n+1}}{a_n}\leq r$.

a) Show that when $n\geq N$, $a_n\leq a_N r^{n-N}$.

b) Raise both sides of the inequality to the power of $\frac{1}{n}$ to get $a_n^{1/n}\leq a_N^{1/n} r^{1-\frac{N}{n}}$. Now take the limit of both sides as $n\to\infty$.

c) What can we conclude about the relationship between ratio test and the root test?

14. Let $a_n=\dfrac{1}{2^{n+(-1)^{n+1}}}$ and consider the sum

$$\sum_{n=1}^{\infty} a_n=\frac{1}{2^1}+\frac{1}{2^0}+\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^5}+\frac{1}{2^4}+\frac{1}{2^7}+\frac{1}{2^6}+\cdots.$$

a) What does the ratio test say about this series? (Hint: treat the cases when n is even and odd separately.)

b) What does the root test say?

c) What can we conclude about the relationship between ratio test and the root test?