Name: ________________________ Homework 8 | Math 342 | Cruz Godar

Due Wednesday of Week 9 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 9

In problems 1–3, find the change of basis matrix B that converts from the basis $\mathcal{B}$for the vector space V to the standard basis, and use it to find $[\vec{v}]_{\mathcal{B}}$ for the given vector $\vec{v}$.

1. V is the subspace of $\mathbb{R}[x]$ of polynomials with degree at most 3, $\mathcal{B} = \{1, x + x^2 + x^3, x^3 - x, x^3 + 2x^2\}$, and $\vec{v} = 2 + 7x - x^2$.

2. $V = M_{2\times 2}(\mathbb{R})$,

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right\},$$

and $\vec{v} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$.

3. $V = \mathcal{L}(\mathbb{R}^4, \mathbb{R})$, $\mathcal{B} = \{T_1, T_2, T_3, T_4\}$, where

$$T_1\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = 2x+y-z \qquad T_2\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = x-w \qquad T_3\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = y+z \qquad T_4\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = x+y+z+w,$$

and $\vec{v} : \mathbb{R}^4 \to \mathbb{R}$ is defined by $\vec{v}\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = 2x - 2y + 4z + 2w.$

In problems 4–6, find bases for V, $\ker T$ and $\operatorname{image} T$, and verify that the fundamental theorem of linear algebra correctly relates the three.

4. $T : V \to \mathbb{R}$, where V is the subspace of $\mathbb{R}[x]$ of polynomials with degree at most 3, is defined by $T(a + bx + cx^2 + dx^3) = d - c$.

5. $T : M_{2\times 3}(\mathbb{R}) \to \mathbb{R}^3$ is defined by

$$T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

6. $T : \mathbb{R}^2 \to \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = S_{x,y}$, where

$$S_{x,y}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 0 \end{bmatrix}$$
$$S_{x,y}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ y - x \end{bmatrix}.$$