Name: _____ Homework 4 | Math 256 | Cruz Godar

Due Wednesday of Week 5 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 4

In problems 1–5, solve the DE and verify that you've found the general solution with the Wronskian.

1. $y'' + 2y' - 3y = 0$.

2. $y'' + 3y' = 0$.

3. $4y'' - 9y = 0$.

4. $y'' - y' - y = 0$.

5. $y'' = y'$.

In problems 6–8, solve the initial value problem.

6. $y'' + y' - 20y = 0$, $y(0) = 9$, $y'(0) = 18$.

7. $2y'' + 6y' = 20y$, $y(0) = 14$, $y'(0) = 0$.

8. $y'' - y = 0$, $y(0) = 4$, $y'(0) = 2$.

9. Consider the DE $2t^2y'' - ty' + y = 0$.

   a) Show that a solution is $y = c_1t + c_2\sqrt{t}$.

   b) Compute $W\left[t, \sqrt{t}\right]$. For which value $t = t_0$ is it zero?

   c) Why is $t = t_0$ a problem?

10. Let $y = a(t) + b(t)i$ be a solution to

$$y'' + p(t)y' + q(t)y = 0,$$

where a, b, p, and q are all real-valued functions. Show that both a and b must be solutions to $y'' + p(t)y' + q(t)y = 0$ themselves.

11. Consider the DE $y'' - yy' = 0$.

a) Show that both $y = \tan\left(\frac{x}{2}\right)$ and $y = 2\tan(x+1)$ are solutions.

b) Show that

$$y = c_1 \tan\left(\frac{x}{2}\right) + 2c_2 \tan(x+1)$$

is *not* a solution in general. Why does this not contradict the results of section 4?