Midterm 2

Math 112

Spring 2020

You have 50 minutes to complete this exam (plus 10 minutes to account for the time it takes to scan and upload it). **You may use a scientific calculator, but no other resources.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

1. (32 points) Let $g(\theta) = \tan(2\theta) - 1$.

a) (8 points) Find $g\left(\dfrac{\pi}{6}\right)$. Leave your answer in exact form, and show all your work — specifically, how you calculate the tangent.

$\tan(\pi/3) - 1 = \dfrac{\sin(\pi/3)}{\cos(\pi/3)} - 1 = \dfrac{\sqrt{3}/2}{1/2} - 1 = \sqrt{3} - 1.$

b) (8 points) Sketch a graph of g. Label at least three points.

This is the graph of tangent, horizontally stretched by a factor of $\dfrac{1}{2}$ and vertically shifted 1 unit down.

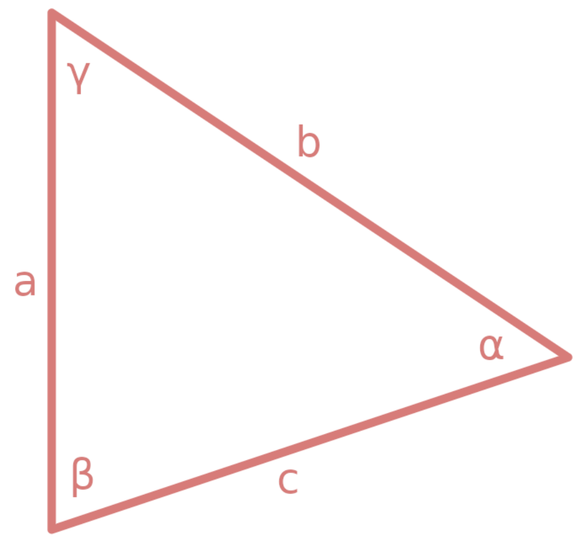
c) (8 points) For what values of θ is $g(\theta) = -1$? List all of the values, and express your answers in radians.

$\tan(2\theta) = 0$, so $2\theta = 2\pi n$ or $2\theta = \pi + 2\pi n$. Thus $\theta = \pi n$ or $\theta = \dfrac{\pi}{2} + \pi n$ for any integer n.

d) (8 points) For what values of θ is $g(\theta) = 0$? List all of the values, and express your answers in radians.

$\tan(2\theta) = 1$, so $2\theta = \frac{\pi}{4} + 2\pi n$ or $2\theta = \frac{5\pi}{4} + 2\pi n$. Thus $\theta = \frac{\pi}{8} + \pi n$ or $\theta = \frac{5\pi}{8} + \pi n$.

2. (32 points) Consider the following triangle with sides a, b, and c, and angles α, β, and γ.



a) (8 points) Given that $a = 3$, $b = 3.61$, and $\gamma = 56.3°$, find c.

By the Law of Cosines, $c^2 = 3^2 + 3.61^2 - 2 \cdot 3 \cdot 3.61 \cdot \cos(56.3°)$, so $c = 3.17$.

b) (8 points) Use your answer to part a) to find α.

By the Law of Sines, $\dfrac{\sin(\alpha)}{3} = \dfrac{\sin(56.3°)}{3.17}$, so $\alpha = \arcsin(.787) = 51.9°$.

c) Now find β.

$51.9 + \beta + 56.3 = 180$, so $\beta = 71.8°$.

d) (8 points) Find the area of this triangle. (Hint: pick one side to be the base, then draw a line perpendicular to that base that reaches to the opposite vertex to form a right triangle. Then use trig functions to find the length of that line.)

Let's pick b to be the base. Drawing a perpendicular from the opposite vertex to b, we get a right triangle with hypotenuse a. If the length of the altitude is h, then $\sin(56.3°) = \dfrac{h}{a}$, so $h = 3\sin(56.3°) = 2.5$. Thus the area of the triangle is $\dfrac{1}{2}bh = 4.51$.

3. (32 points) Miscellaneous questions: these don't make sense as full-length problems, so the four parts here are unrelated to one another.

a) (8 points) Find an exact value for $\sec\left(\frac{2\pi}{3}\right)$. Show all your work.

$\sec\left(\frac{2\pi}{3}\right) = \frac{1}{\cos\left(\frac{2\pi}{3}\right)} = \frac{1}{-1/2} = -2.$

b) (8 points) Find an exact value for $\sin(75°)$. Show all your work.

$\sin(75°) = \sin(30° + 45°) \sin(30°)\cos(45°) + \cos(30°)\sin(45°) = \frac{1+\sqrt{3}}{2\sqrt{2}}.$

c) Write the equation of a sinusoidal function $f(x)$ with amplitude 2, midline $-\sqrt{3}$, and period 2π, such that $f(0) = 0$.

We need to find h. We have $f(x) = 2\sin(x-h) - \sqrt{3}$ and we want $f(0) = 0$, so $0 = 2\sin(0-h) - \sqrt{3}$. Now $\sin(-h) = \frac{\sqrt{3}}{2}$, so $-h = \frac{\pi}{3} + 2\pi n$ or $-h = \frac{2\pi}{3} + 2\pi n$ for any integer n. Let's use the first one with $n = 0$. Then $h = -\frac{\pi}{3}$, so $f(x) = 2\sin(x + \pi/3) - \sqrt{3}$.

d) Sketch a graph of $\arctan(x)$. Label at least three points.

This is the graph of tangent on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, flipped along the line $y = x$, since arctan is the inverse of tan on that interval.