Name: _____ Homework 6 | Math 253 | Cruz Godar

Due Friday of Week 7 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–6, find the interval and radius of convergence of the power series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$.

2. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$.

3. $\displaystyle\sum_{n=1}^{\infty} nx^n$.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n}$.

5. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n!}$.

6. $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$.

7. Find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^3 x^n}{(3n)!}$, but not the interval (i.e. you don't need to check the endpoints).

In problems 8–15, expand each function $f(x)$ as a power series and find its interval and radius of convergence.

8. $\dfrac{1}{1-x}$.

9. $\dfrac{1}{1-x^3}$.

10. $\dfrac{1}{1-2x}$.

11. $\dfrac{1}{x}$.

12. $\dfrac{x^4}{1-x^2}$.

13. $\dfrac{1}{2-x}$.

14. $\dfrac{1}{x}$.

15. $\dfrac{x}{1-(1-x)}$. Does your result make sense?

16. Give examples of power series with intervals of convergence of

a) $(-1, 1]$.

b) $[-4, 0]$.

c) Only $x = 5$ and no other numbers.

17. If a power series converges at $x = 1$, does it also have to converge at $x = 0$? Why or why not?

18. Is it possible to express a function as a power series in more than one way? For example, can we express $\dfrac{1}{1-x}$ as a power series centered at $x = \frac{1}{2}$? What happens to the interval of convergence if so? What about as a series centered at $x = 1$?