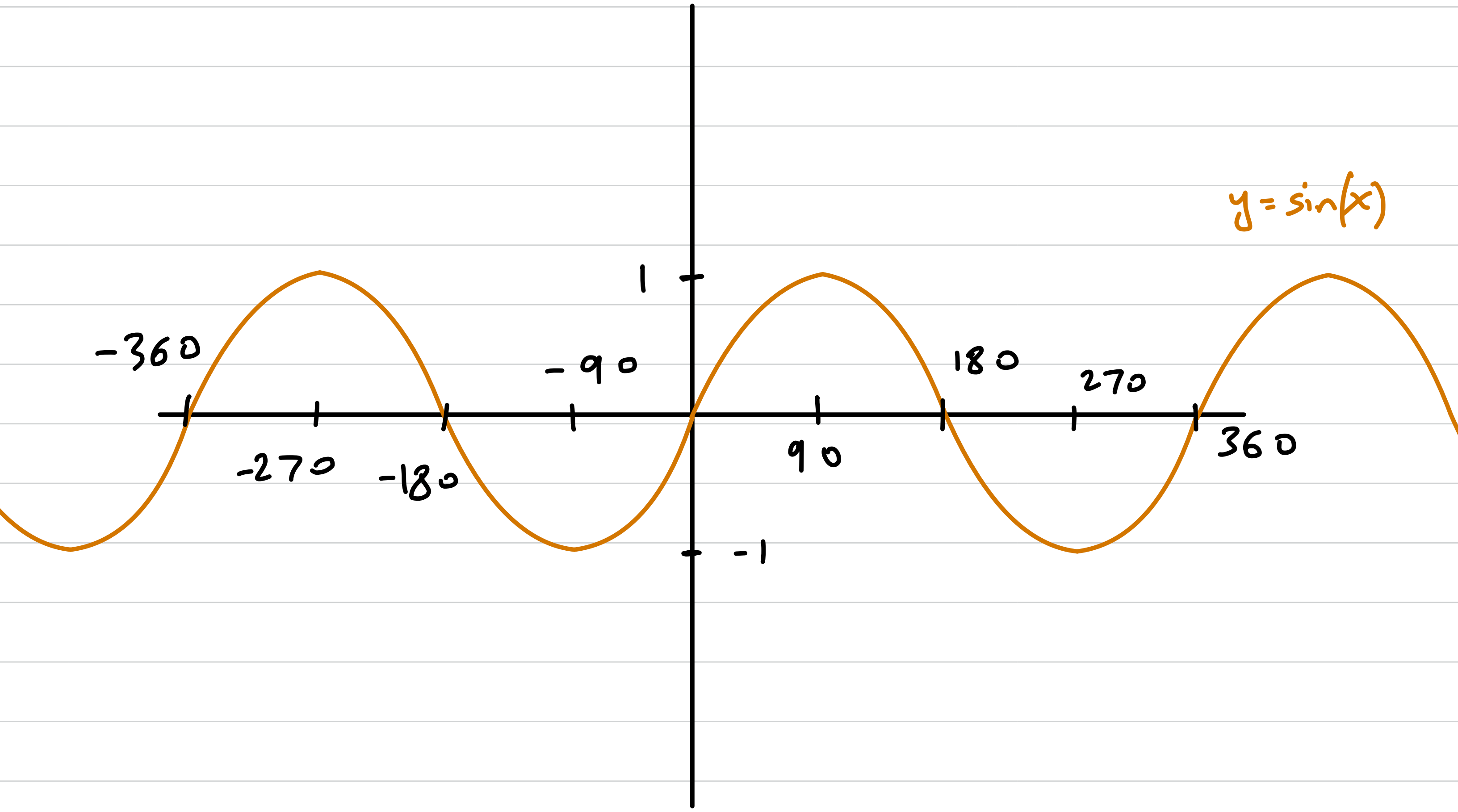
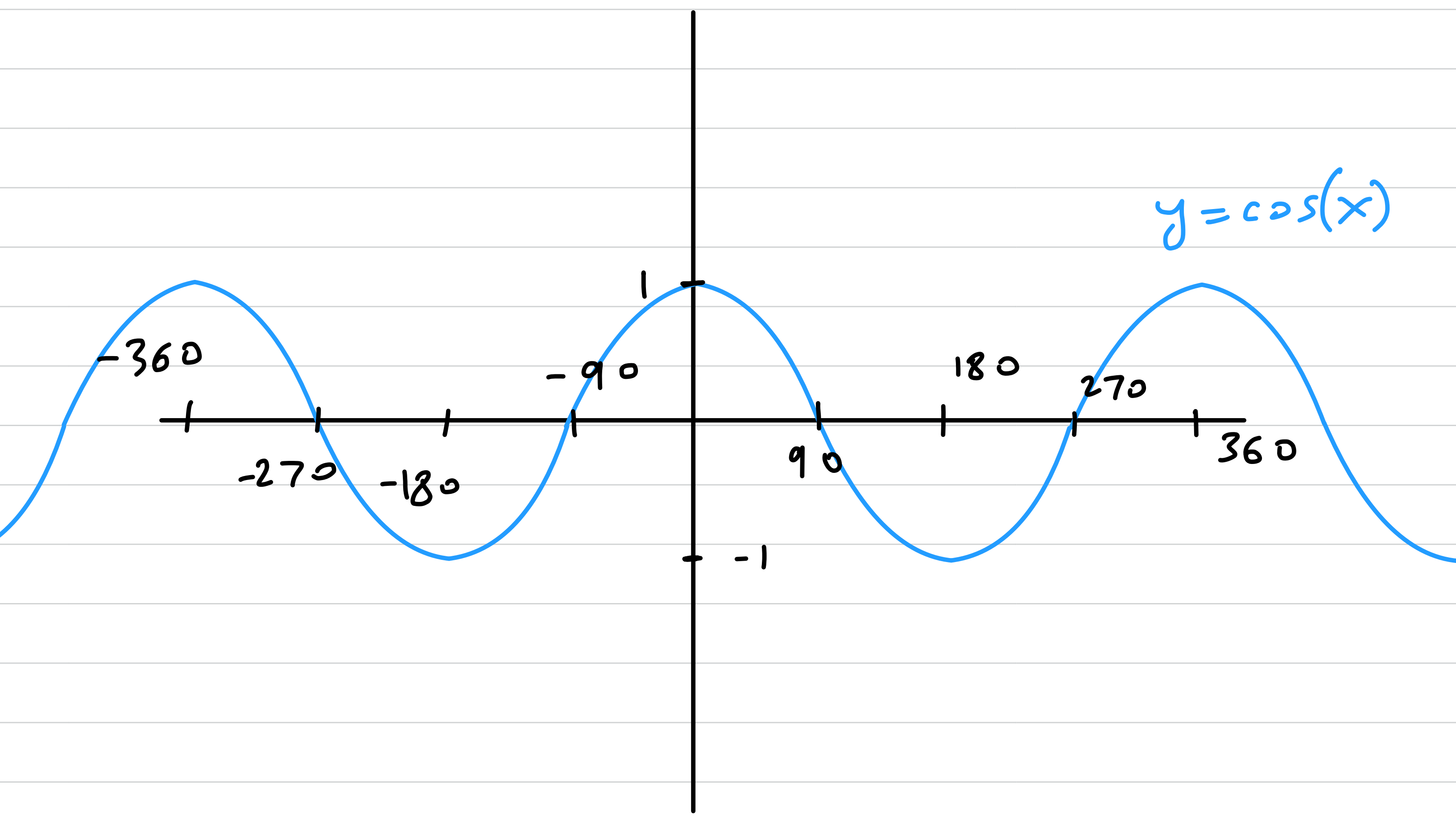

y = sin(x)
1
-1
-360
-270
-180
-90
90
180
270
360

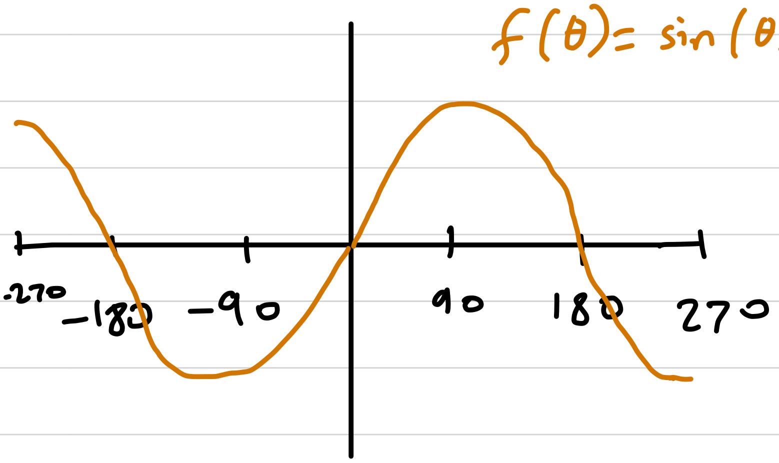

y = cos(x)
1
-1
-360
-270
-180
-90
90
180
270
360

Ex: Graph $f(\theta) = -2\sin(2\theta) + 5$.

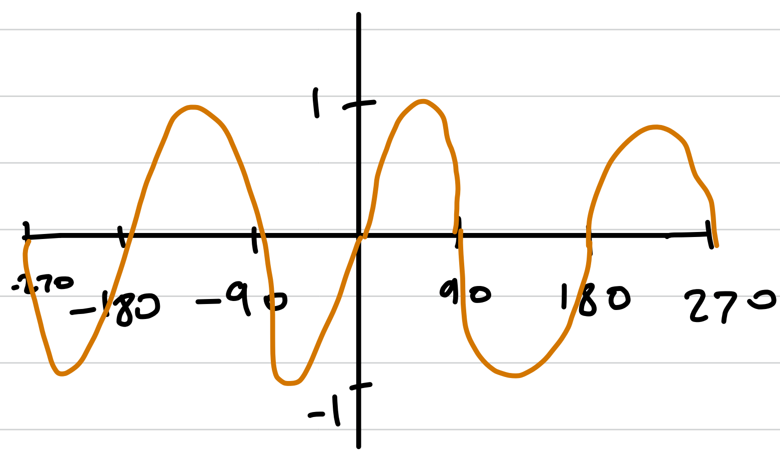
Parent function: $\sin(\theta)$

- Horizontal stretch by a factor of $\frac{1}{2}$
- Vertical stretch by a factor of 2
- Vertical reflection
- Vertical shift 5 up

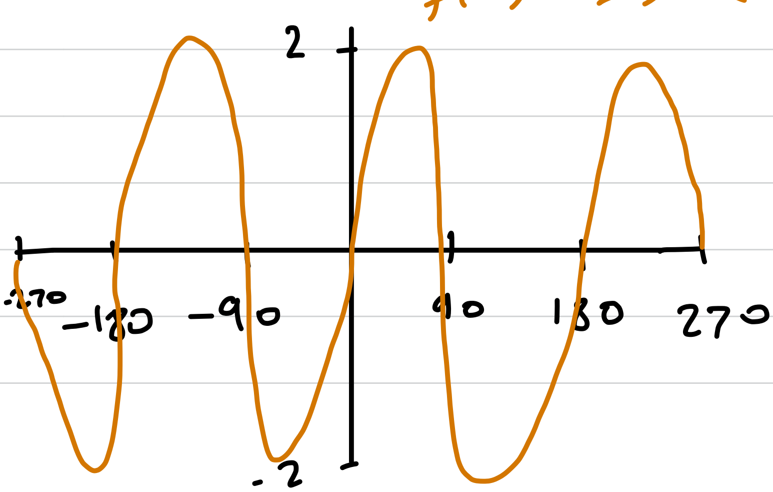
$f(\theta) = \sin(\theta)$



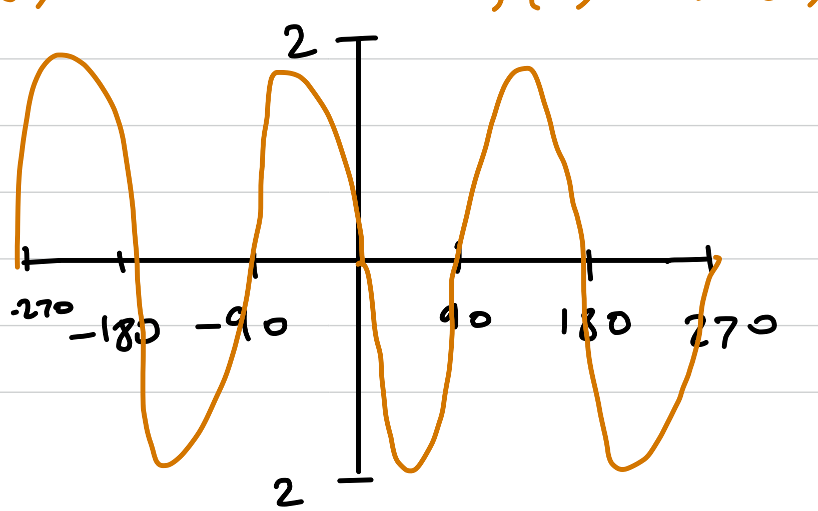
$f(\theta) = \sin(2\theta)$

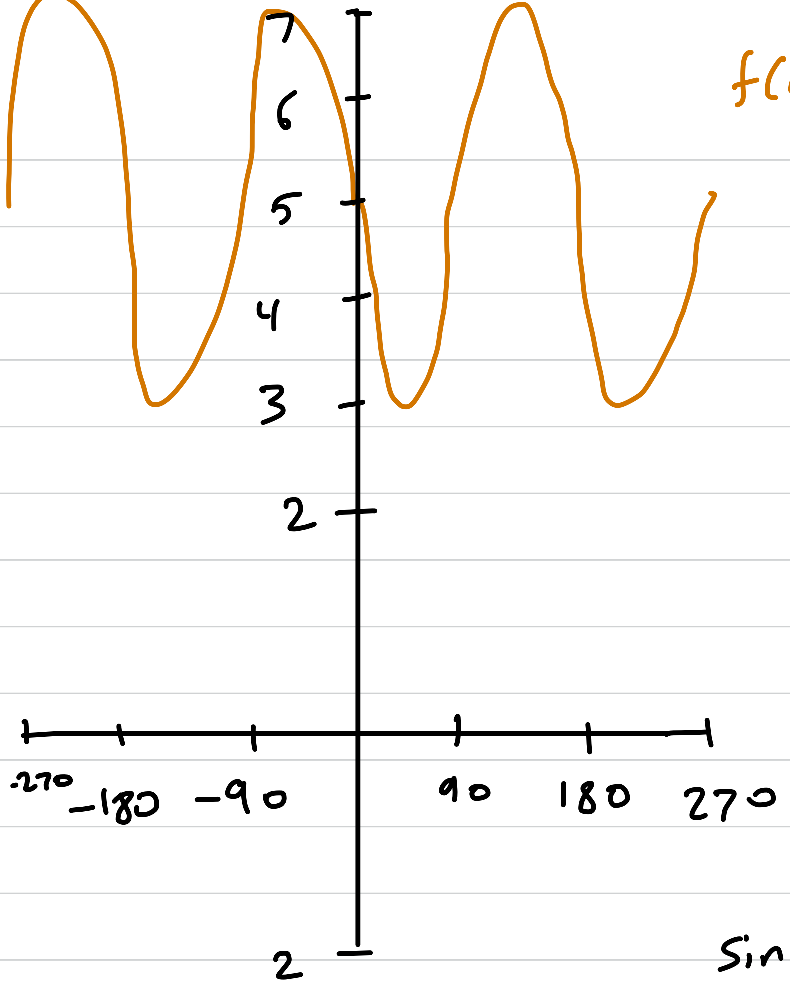


$f(\theta) = 2\sin(2\theta)$



$f(\theta) = -2\sin(2\theta)$



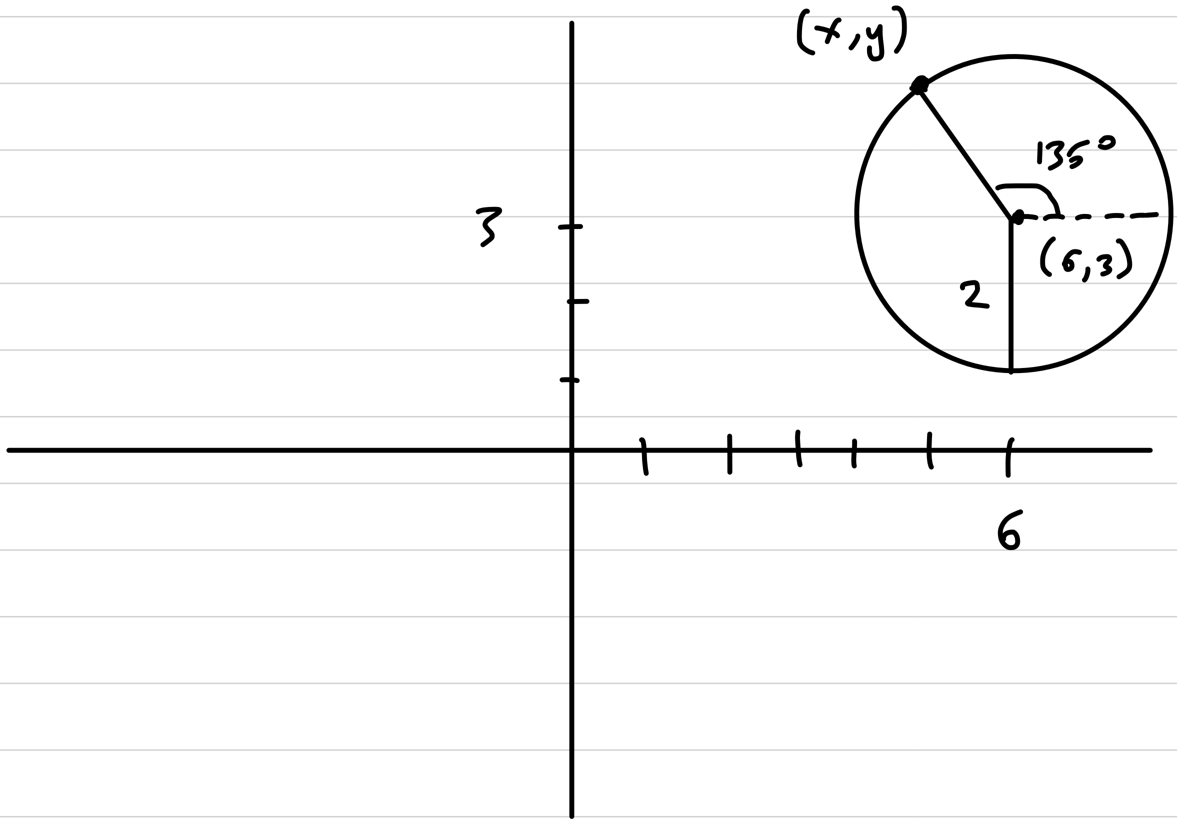


$f(\theta) = -2\sin(2\theta) + 5$

sin cos

Prop: Let $f(\theta) = r\cos(\theta) + h$ and $g(\theta) = r\sin(\theta) + k$. Then $(f(\theta), g(\theta))$ are the coordinates of the point on the circle of radius r centered at (h, k) that has angle θ.

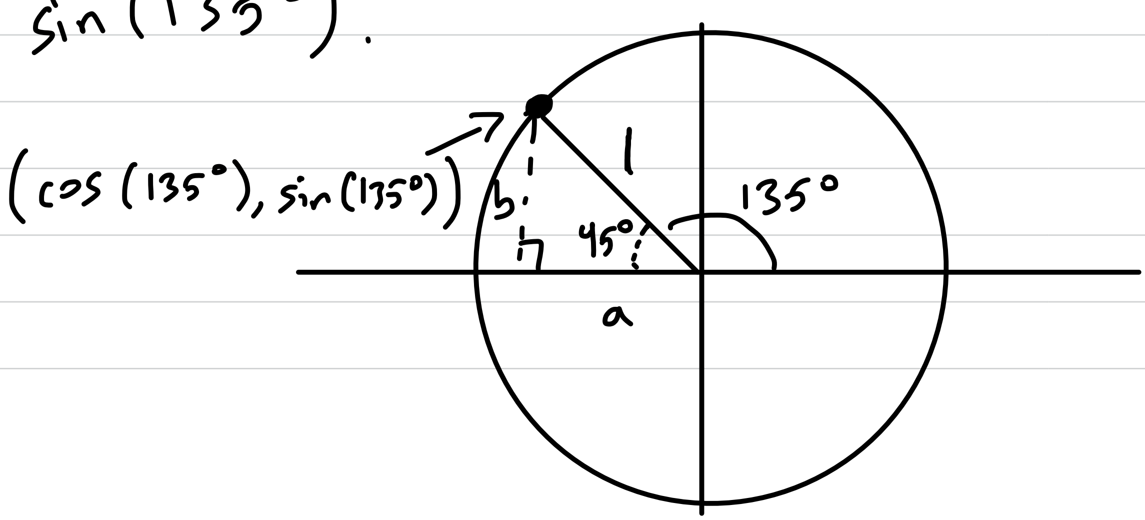
Ex:

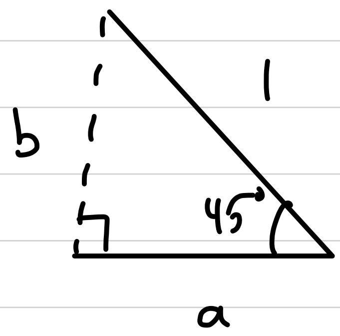


Find (x, y).

by prop $\begin{cases} x = 2\cos(135^\circ) + 6 \\ y = 2\sin(135^\circ) + 3 \end{cases}$

Now we need to find $\cos(135^\circ)$ and $\sin(135^\circ)$.





$\cos(45°) = a/1 = a$

$\sin(45°) = b/1 = b$

$45°$ is a special angle: $\cos(45°) = \frac{\sqrt{2}}{2}$ and $\sin(45°) = \sqrt{2}/2$.

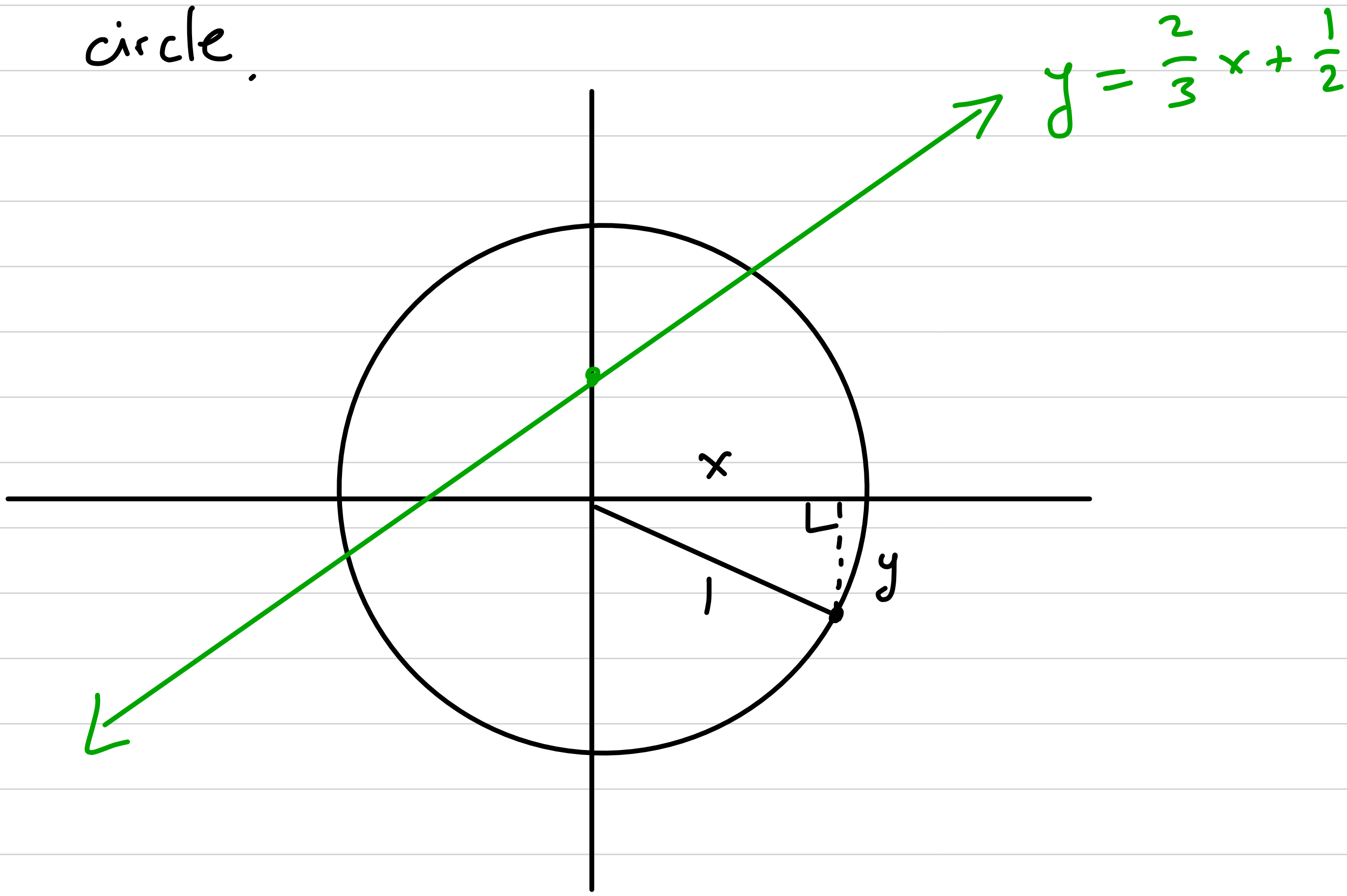
$a = \frac{\sqrt{2}}{2}$ $\quad b = \frac{\sqrt{2}}{2}$

$\cos(135°) = -\frac{\sqrt{2}}{2}$ $\quad \sin(135°) = \frac{\sqrt{2}}{2}$

$x = 2\left(-\frac{\sqrt{2}}{2}\right) + 6$ $\quad y = 2\left(\frac{\sqrt{2}}{2}\right) + 3$

$x = 6 - \sqrt{2}$ $\quad y = 3 + \sqrt{2}$

Ex: Find the points where the line $y = \frac{2}{3}x + \frac{1}{2}$ intersects the unit circle.



We know that any point (x, y) on th unit circle satisfies $x^2 + y^2 = 1$. That means we have a system of equations:

$$\begin{cases} y = \frac{2}{3}x + \frac{1}{2} \\ x^2 + y^2 = 1 \end{cases}$$

$$x^2 + \left(\frac{2}{3}x + \frac{1}{2}\right)^2 = 1$$

$$x^2 + \frac{4}{9}x^2 + \frac{2}{3}x + \frac{1}{4} = 1$$

$$13/9\, x^2 + 2/3\, x - 3/4 = 0$$

$$x = \frac{-2/3 \pm \sqrt{4/9 - 4\left(\frac{13}{9}\right)\left(-\frac{3}{4}\right)}}{2\left(\frac{13}{9}\right)}$$

$$x = -\frac{3}{13} + \frac{3}{26}\sqrt{43} \qquad y = -\frac{2}{13} + \frac{2}{26}\sqrt{43} + \frac{1}{2}$$

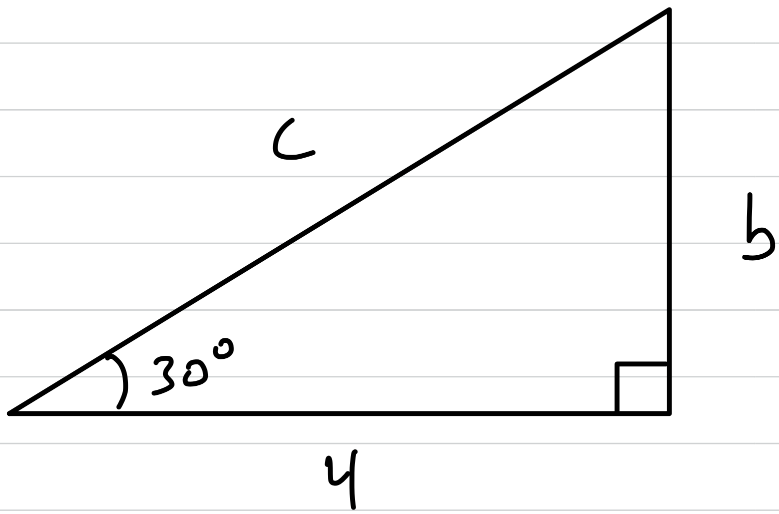
$$x = -\frac{3}{13} - \frac{3}{26}\sqrt{43} \qquad y = -\frac{2}{13} - \frac{2}{26}\sqrt{43} + \frac{1}{2}$$

$$\Rightarrow \left(-\frac{3}{13} + \frac{3}{26}\sqrt{43},\ -\frac{2}{13} + \frac{2}{26}\sqrt{43} + \frac{1}{2}\right)$$

$$\left(-\frac{3}{13} - \frac{3}{26}\sqrt{43},\ -\frac{2}{13} - \frac{2}{26}\sqrt{43} + \frac{1}{2}\right)$$

The Tangent Function

Ex: Find b:



We know that $\sin(30°) = \frac{b}{c}$ and $\cos(30°) = \frac{4}{c}$, but neither one of these equations lets us solve for b. But,

$$\frac{1}{\sqrt{3}} = \frac{1/2}{\sqrt{3}/2} = \frac{\sin(30°)}{\cos(30°)} = \frac{b/c}{4/c} = \frac{b}{\cancel{c}} \cdot \frac{\cancel{c}}{4} = \frac{b}{4}$$

$$b = \frac{4}{\sqrt{3}}$$

Def: The tangent of an angle θ is

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}.$$

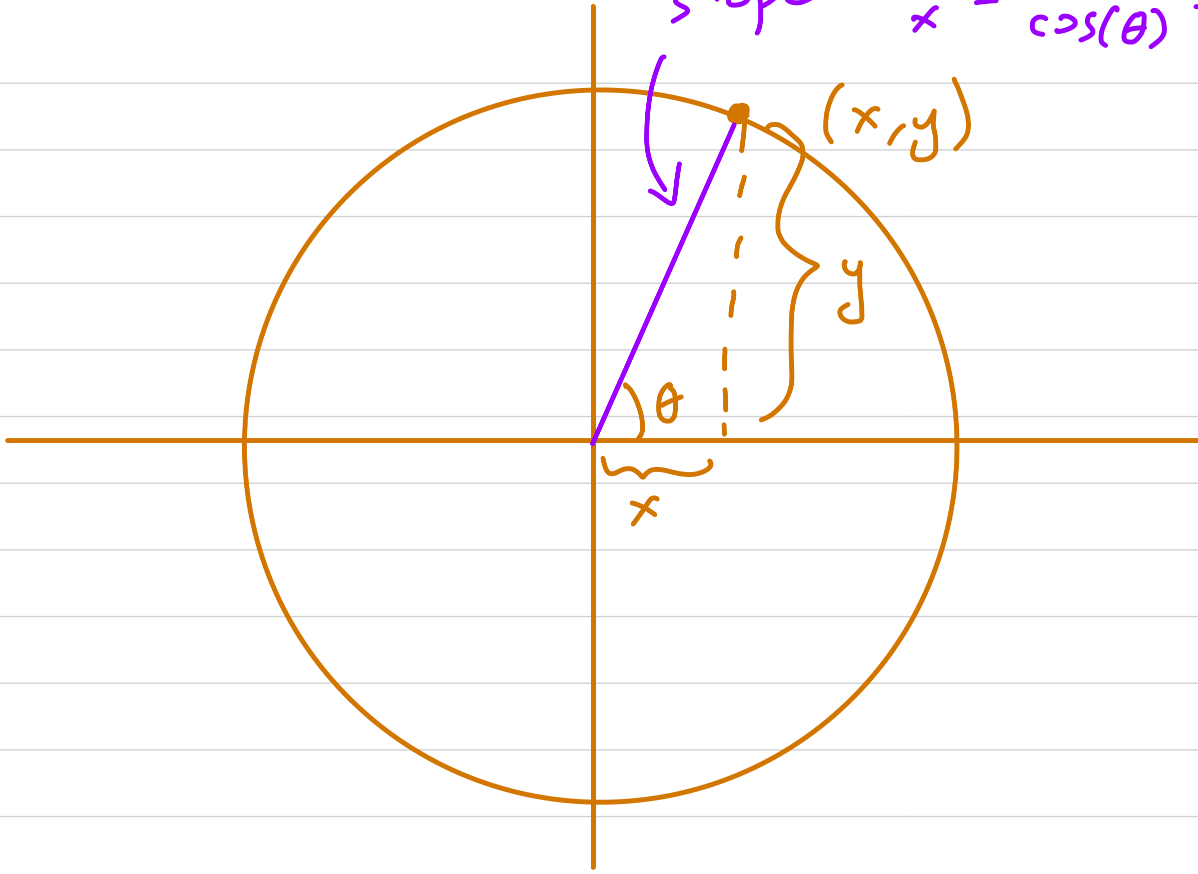
Prop. (1) In a right triangle, $\tan(\theta) = \frac{\text{opp}}{\text{adj}}$.

(2) We have the following special angles for $\tan(\theta)$:

θ	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(\theta)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(\theta)$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined!

(3) In a unit circle, $\tan(\theta)$ is the slope of the line from the origin through the point with angle θ.

$$\text{slope} = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$



Midterm 1 : Friday

50 minutes

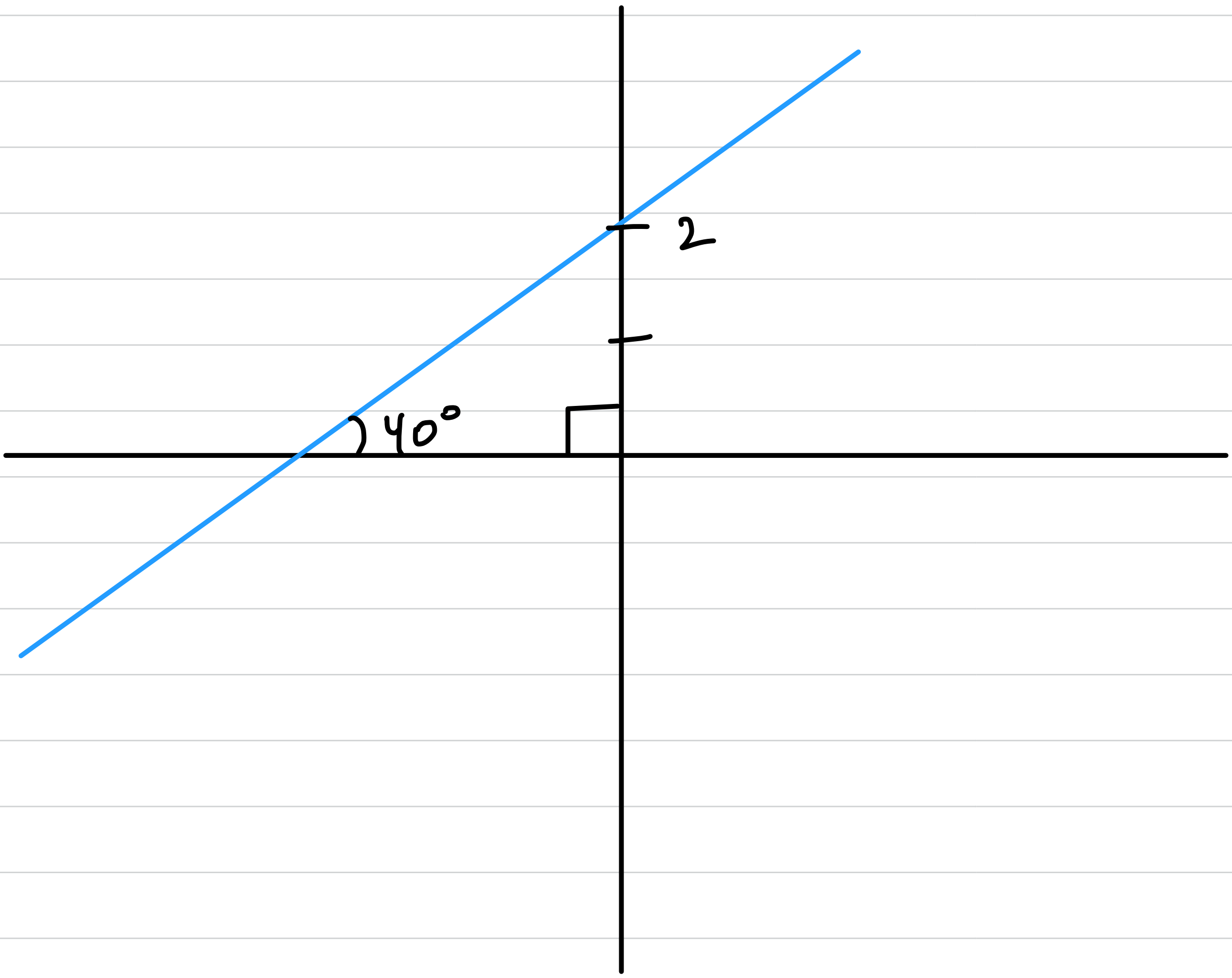
Cameras on

Through 2.4.

Closed book/notes/internet/people

Scientific calculator allowed, but not a graphing one

Ex: Find the slope of the blue line:

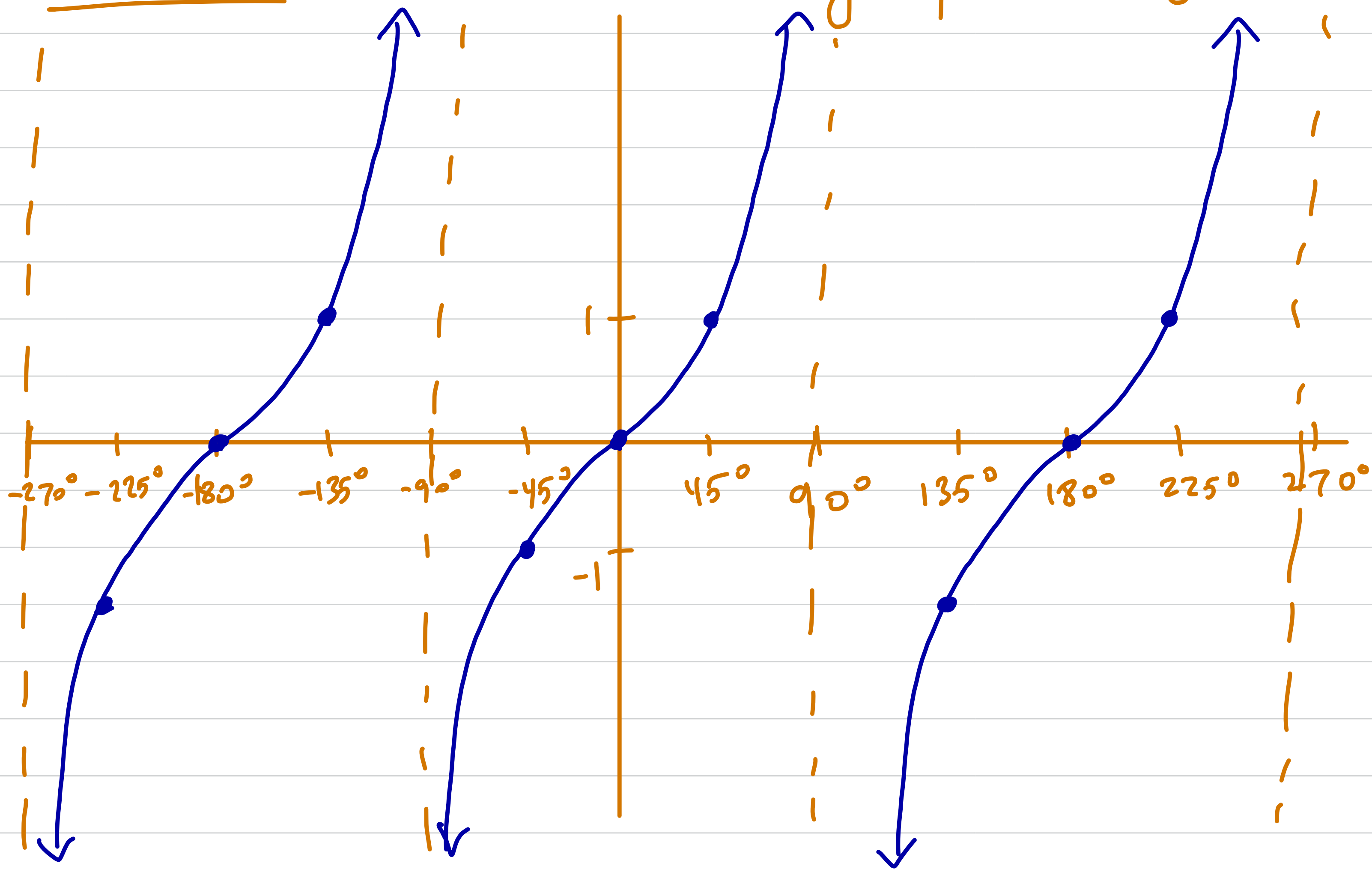


The slope is $\tan(40°) = .8391$.

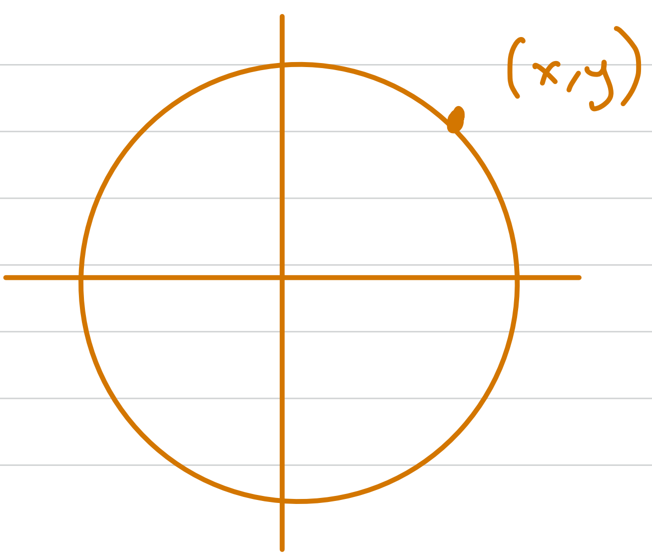
So the equation of the line is

$y = .8391x + 2$.

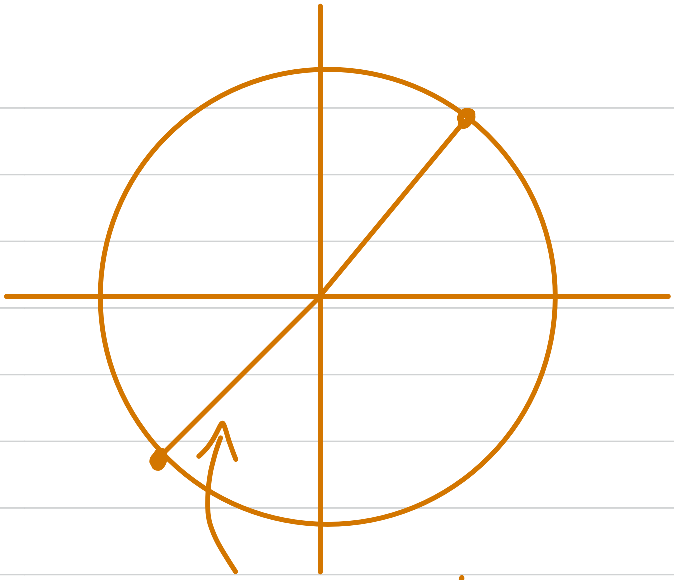
Theorem: This is the graph of $y = \tan(x)$.



It's periodic with period 180° (not 360°)



x and y only repeat themselves by going a full 360° around the circle. ⇒ period of sin and cos is 360°



same slope by going around $180°$

$\Rightarrow$ period of tan is $180°$.

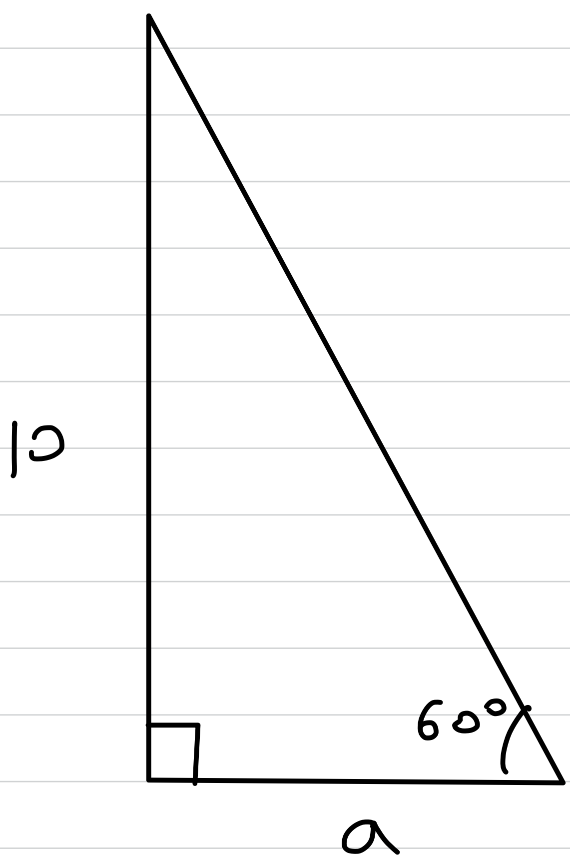
tan is an odd function.

$\tan(x)$ has asymptotes at $x = 180°n + 90°$ for any integer n.

$\tan(x)$ has roots at $x = 180°n$ for any integer n.

Since $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, the roots occur at the roots of sin and the asymptotes occur at the roots of cos.

Ex: A ladder is leaning up against a wall. It reaches 10 feet up the wall and makes an angle of $60°$ with the ground. Find the distance from the wall to the base of the ladder without finding the length of the ladder.



$$\tan(60°) = \frac{10}{a}$$

$$\underset{\sqrt{3}}{\shortparallel}$$

$$a = \frac{10}{\sqrt{3}}.$$