Name: ________________________ Homework 1 | Math 341 | Cruz Godar

Due Wednesday of Week 2 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 1

In problems 1–3, write the system in the form $A\vec{x} = \vec{b}$ for a matrix A and vector $\vec{b}$ of constants and a vector $\vec{x}$ of variables.

1.

$$\begin{aligned} x_1 &= 2 \\ 2x_1 - x_2 &= 3. \end{aligned}$$

2.

$$\begin{aligned} 2x_3 - x_2 &= 0 \\ x_1 &= x_3 - x_2 + 1 \end{aligned}$$

3.

$$\begin{aligned} x + y - z &= x \\ x + 2y - 1 &= 2z \\ x - z + 1 &= y \end{aligned}$$

In problems 4–8, evaluate the product.

4. $\begin{bmatrix} 3 & 0 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

5. $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

6. $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.

7. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$.

8. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$.

9. Let A be an $n \times n$ matrix with entries a_{ij}.

a) For the products AI and IA to make sense, what dimension must I have?

b) The ith row of A is $\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$. If the jth column of I is denoted $\vec{e_j}$, what is the entry in row i and column j of AI? Your answer should be in terms of i and j.

c) What does part b) imply AI is equal to? Why does this make sense in the context of function composition?

10. Let A be an $m \times n$ matrix with entries a_{ij}.

a) There is a row vector $\vec{x}$ and a column vector $\vec{y}$ such that $\vec{x}A\vec{y} = a_{11}$. What are they?

b) What about vectors $\vec{x}$ and $\vec{y}$ such that $\vec{x}A\vec{y} = a_{ij}$? Your answer should be in terms of i and j.

11. State whether each part is true or false. If true, briefly justify why, and if false, provide a small counterexample.

a) A system with 3 equations and 2 unknowns always has at least one solution.

b) If the product AB is defined, then A and B have the same number of rows.

c) If A is a 2×2 matrix so that $A\vec{x} = \vec{x}$ for *every* vector $\vec{x}$, then $A = I_2$. Hint: try plugging in specific values of x_1 and x_2, like 0 and 1.

In problems 12–14, we'll show that many of the nice properties of multiplication of numbers don't hold for matrices.

12. With real numbers x and y, it must be the case that $xy = yx$, but this isn't true with matrices. Define matrices A and B by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Compute AB and BA and show that they're different.

13. With real numbers x, y, and z where $x \neq 0$ and $xy = xz$, it's always the case that $y = z$, but this also isn't true for matrices. Define matrices A, B, and C by

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$$

Show that $AB = AC$, despite $B \neq C$.

14. With real numbers x and y where $xy = 0$, either $x = 0$ or $y = 0$. Unfortunately, this also isn't the case for matrices. Define A and B by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}.$$

Show that $AB = 0$ (the matrix of all zeros), even though both A and B are nonzero.