Quiz 4 Solutions:

(1) $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$E_1 = \{7\}$

$E_2 = \{2, 4, 6, 8\}$

$E_3 = \{3, 4, 5, 6\}$

(2) $p(E_1) = \frac{1}{8}$ $\quad o(E_1) = 1:7$

$p(E_2) = \frac{4}{8} = \frac{1}{2}$ $\quad o(E_2) = 4:4$

$p(E_3) = \frac{4}{8} = \frac{1}{2}$ $\quad o(E_3) = 4:4$

Let E be the event of rolling a 3 on the 8-sided die and F the event of rolling a 3 on the 4-sided one. Then we want

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$n(E) = 4$ $\quad P(E) = 4/32 = 1/8$

$n(F) = 8$ $\quad P(F) = 8/32 = 1/4$

$n(E \cap F) = 1$ $\quad P(E \cap F) = 1/32$

$n(S) = 32$

$$P(E \cup F) = 1/8 + 1/4 - 1/32 = 11/32.$$

3.7: Independence

Def: Events A and B are <u>independent</u> if $p(A|B) = p(A)$. What this means is that B taking place has no effect on the chance that A will take place.

Ex: if you toss two coins, the result of the second toss doesn't depend on the result of the first, so they're independent. In symbols, if A is the event of getting heads on the first toss and B is the event of getting heads on the second, then

$p(A) = 1/2$

$p(B) = 1/2$

$p(B|A) = 1/2$

Since $p(B|A) = p(B)$, A and B are independent.

Ex: If A is the event of drawing a heart off the top of a 52-card and B is the event of the card underneath it also being a heart, then

$p(A) = 13/52 = 1/4$ ✗

$p(A|B) = 12/51$

$\Rightarrow$ A and B are dependent.

Comment: Independent vs Mutually exclusive

Independent means $P(A|B) = P(A)$.

Mutually exclusive means $P(A \cap B) = 0$.

Ex: Are the events A and B independent or mutually exclusive or neither, where A is having freckles and B is having red hair.

Since it's possible to have both freckles and red hair at the same time, A and B are mutually exclusive.

But having one makes you more likely to have the other, so A and B are dependent.

Theorem (Product rule for independent events)

If A and B are independent, then

$$P(A \cap B) = P(A) \cdot P(B).$$

Ex: If A is the event of rolling a 3 on an 8-sided die and B is the event of rolling a 3 on a 4-sided die, then

$P(A \cap B) = \frac{1}{8} \cdot \frac{1}{4} = \frac{1}{32}$ because

$P(A) = \frac{1}{8}$

$P(B) = \frac{1}{4}$

A and B are independent.

The Final

- 12 - 1:50 on Friday
- 1.5x midterm length (expect ~12 Qs)
- No outside resources (including a calculator)
- I'll post a list of topics
- Tue, Wed, Thu are open for questions, so come with questions ready
- office hours Wed + Fri as usual