Name: ______________________ Homework 4 | Math 253 | Cruz Godar

Due Wednesday of Week 5 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–8, determine if the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + \ln(n)}$.

2. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2 - k - 1}$.

3. $\displaystyle\sum_{m=3}^{\infty} \frac{m}{m^2 - 8}$.

4. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2 + 2}$.

5. $\displaystyle\sum_{m=1}^{\infty} \frac{\sin(m) + 1}{m^{1.5}}$.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + \ln(n)}}$ (Hint: factor out an n from the root).

7. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln(n) \ln(\ln(n))}$.

8. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln^k(n)}{n^2}$ for $k \geq 1$ any positive integer. Here, $\ln^k(n) = (\ln(n))^k$. (Hint: use the limit comparison test with $\frac{1}{n^{1.5}}$.)

9. Give an example of a divergent series $\sum_{n=1}^{\infty} a_n$ so that $a_n \leq \frac{1}{n^2}$ for all n. Why does this not contradict the comparison test?

10. If $a_n \geq 0$ and $\displaystyle\lim_{n\to\infty} \frac{a_n}{1/n} = 0$, can $\sum_{n=1}^{\infty} a_n$ still diverge?