Name: ______________________ Homework 7 | Math 342 | Cruz Godar

Due Wednesday of Week 8 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 7

1. Let A be a real $m \times n$ matrix. Explain why $A^T A$ is guaranteed to be diagonalizable.

2. Let A be a symmetric (i.e. $A^T = A$), $n \times n$, real matrix with all positive eigenvalues and define $\langle \vec{v}, \vec{w} \rangle = \vec{v}^T A \vec{w}$ for $\vec{v}, \vec{w} \in \mathbb{R}^n$.

   a) Show from the formula that this is symmetric — that $\langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$.

   b) Show that it's also bilinear — that $\langle c\vec{u} + \vec{v}, \vec{w} \rangle = c \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$.

   c) Setting $\vec{v} = \vec{w} = \vec{0}$, we have that $\left\langle \vec{0}, \vec{0} \right\rangle = 0$. Explain why for any nonzero vector $\vec{v} \in \mathbb{R}^n$, $\langle \vec{v}, \vec{v} \rangle > 0$.

Section 8

In problems 3–5, write the matrix as $A = BJB^{-1}$ for a matrix J in Jordan normal form.

3. $A = \begin{bmatrix} 0 & -5 & -2 \\ 2 & 6 & 1 \\ -2 & -3 & 2 \end{bmatrix}.$

4. $A = \begin{bmatrix} -3 & -1 & -2 \\ -1 & -1 & -1 \\ 2 & 1 & 1 \end{bmatrix}.$

5. $A = \begin{bmatrix} 12 & 4 & 4 \\ -24 & -8 & -8 \\ -3 & -1 & 0 \end{bmatrix}.$