

Name: \_\_\_\_\_

Homework 7 | Math 342 | Cruz Godar

*Due Wednesday of Week 8 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 7

1. Let  $A$  be a real  $m \times n$  matrix. Explain why  $A^T A$  is guaranteed to be diagonalizable.
2. Let  $A$  be a symmetric (i.e.  $A^T = A$ ),  $n \times n$ , real matrix with all positive eigenvalues and define  $\langle \vec{v}, \vec{w} \rangle = \vec{v}^T A \vec{w}$  for  $\vec{v}, \vec{w} \in \mathbb{R}^n$ .
  - a) Show from the formula that this is symmetric — that  $\langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$ .
  - b) Show that it's also bilinear — that  $\langle c\vec{u} + \vec{v}, \vec{w} \rangle = c \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$ .
  - c) Setting  $\vec{v} = \vec{w} = \vec{0}$ , we have that  $\langle \vec{0}, \vec{0} \rangle = 0$ . Explain why for any nonzero vector  $\vec{v} \in \mathbb{R}^n$ ,  $\langle \vec{v}, \vec{v} \rangle > 0$ .

## Section 8

In problems 3–5, write the matrix as  $A = BJB^{-1}$  for a matrix  $J$  in Jordan normal form.

3.  $A = \begin{bmatrix} 0 & -5 & -2 \\ 2 & 6 & 1 \\ -2 & -3 & 2 \end{bmatrix}.$

4.  $A = \begin{bmatrix} -3 & -1 & -2 \\ -1 & -1 & -1 \\ 2 & 1 & 1 \end{bmatrix}.$

5.  $A = \begin{bmatrix} 12 & 4 & 4 \\ -24 & -8 & -8 \\ -3 & -1 & 0 \end{bmatrix}.$