Name: ____________________ Homework 3 | Math 342 | Cruz Godar

Due Wednesday of Week 4 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 3

In problems 1–3, find the indicated derivative.

1. $\frac{\partial}{\partial t}\left[2ty + \sin(t)\right].$
2. $\frac{\partial}{\partial y}\left[\sin(xy^x)\right].$
3. $\frac{\partial}{\partial x}\left[\sin(xy^x)\right].$

4. Using the multivariable chain rule, find $\frac{df(x,y)}{dt}$, where $f(x,y) = 2x^2 + \sec(xy^2)$, $x(t) = t$, and $y(t) = 5t^2$.

In problems 5–7, find the indicated integral. Make sure to express the constant as a function of the correct variable.

5. $\int f(x,y)\,\mathrm{d}x$ for $f(x,y) = 2x\cos(y-x)$.
6. $\int f(x,y)\,\mathrm{d}y$ for f as in the previous problem.
7. $\int g(x,y)\,\mathrm{d}y$ for $g(x,y) = e^{x^2}$.

In problems 8–14, solve the the given DE.

8. $2y + 1 + (2x+1)y' = 0$, $y(1) = 1$.

9. $1-\sin(t+y)+y'(-\sin(t+y))=0$, $y(0)=0$.

10. $\sin(y)y' - te^t\cos(y)=0$.

11. $y' = -\dfrac{yx^{y-1}}{x^y\log(x)}$, $y(2)=1$.

12. $ty'+y+t^{-2}=0$, $y(2)=2$.

13. $(10t+t^2)-2\sin(y)y'=0$, $y(0)=1$.

14.

$$\sec^2(x)\sec(y)+\left(\tan(x)\tan(y)\sec(y)+\frac{1}{y}\right)y'=0.$$

</div>2px"></p>In problems 6–7, express $\vec{v}$ as a linear combination of the $\vec{u}_i$ or show it's impossible.

6. $\vec{v}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\vec{u_1}=\begin{bmatrix}1\\2\\2\end{bmatrix}$, $\vec{u_2}=\begin{bmatrix}2\\-1\\0\end{bmatrix}$, $\vec{u_3}=\begin{bmatrix}3\\1\\1\end{bmatrix}$.

7. $\vec{v}=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $\vec{u_1}=\begin{bmatrix}-1\\3\\0\end{bmatrix}$, $\vec{u_2}=\begin{bmatrix}1\\1\\1\end{bmatrix}$, $\vec{u_3}=\begin{bmatrix}2\\6\\3\end{bmatrix}$.

8. Let $\vec{w_1}=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $\vec{w_2}=\begin{bmatrix}2\\0\\-1\end{bmatrix}$, $\vec{w_3}=\begin{bmatrix}0\\2\\1\end{bmatrix}$.

a) Show that $\vec{w_1}$, $\vec{w_2}$, and $\vec{w_3}$ are linearly dependent.

b) Show that just $\vec{w_1}$ and $\vec{w_2}$ on their own are linearly independent. (Hint: You should be able to modify the last step in the previous part to get this result without starting over).

c) Using the previous two parts, write a sentence explaining why

$$\text{span}\{\vec{w_1},\vec{w_2}\}=\text{span}\{\vec{w_1},\vec{w_2},\vec{w_3}\}.$$

a) The span of $\vec{w_1}$ and $\vec{w_2}$ is the set consisting of all $\vec{w} = c_1\vec{w_1} + c_2\vec{w_2}$. Renaming $c_1 = u$ and $c_2 = v$, write the generic vector $\vec{w}$ in the span. Your answer should depend on u and v.

b) Math3D is a capable 3D grapher that can handle parametric surfaces. Open the linked example and replace the span expression with the one you found in the previous part — if all went well, you should see the three vectors lying *in* that plane. This plane is the span, and the fact that the three vectors are contained in a two-dimensional surface is their linear dependence.

Section 4

9. Linear transformations are related to typical linear functions like $y = mx + b$, but they're not quite the same. For example, the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 3$ is a linear function, but not a linear transformation. Pick two inputs a and b and show that $f(a + b) \neq f(a) + f(b)$.

In problems 10–12, find the matrix for the linear transformation T and use it to evaluate $T(\vec{v})$.

10. $T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $T\left(\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$.

11. $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

12. $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 2$, $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = 7$, and $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

13. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x + 3y \\ x + 2y \\ 3x + z \end{bmatrix}.$$

a) Express T as a 3×3 matrix A.

b) Find A^{-1}.

c) Write a linear transformation S whose matrix is A^{-1}.

d) Since $A^{-1}A = I$ and matrix multiplication is equivalent to function composition, we should expect $S \circ T = id$, the identity function. Evaluate $S \circ T$ as functions by using the output of T as the input to S, and show this is in fact the case.