Final Practice Worksheet

Math 252

This worksheet is optional. If you would like to take it for practice and get detailed feedback, which I strongly recommend doing, write your answers on a separate sheet of paper and scan and email that to me no later than Saturday night. I'll grade it and get it back to you as soon as I can.

Exercise 1:

a) Given a continuous function f, what is an antiderivative of f?

b) What does it mean for $\int_0^\infty f(x)\, dx$ to converge?

Exercise 2: A thin rope of length 1 meter has linear mass density of $\rho(x) = x^2 e^{5x}$ milligrams per meter, x meters from the left endpoint of the rope. What is the mass of the rope? Include units.

Exercise 3: Is $y = e^{3x}$ a solution to the differential equation $\frac{y'}{y} = x + y - e^{3x}$?

Exercise 4: A mug of tea is initially at $210°F$ and is placed in a room at $72°F$. The temperature in the mug is initially declining at a rate of $4°F$ per minute. Find the temperature of the mug after 15 minutes.

Hint: think carefully about how to turn the initial cooling rate into an equation.

Exercise 5: Find the general solution to the differential equation $y'(t) = \dfrac{\ln|t|}{ty^4}$.

Exercise 6: Find the arc length of the curve $f(x) = \sqrt{1-x^2}$ on $\left[0, \dfrac{\sqrt{3}}{2}\right]$.

Exercise 7: Find the average value of $\dfrac{1}{\sqrt{1-x^2}}$ on $\left[0, \frac{1}{2}\right]$.

Exercise 8: Compute $\displaystyle\int_0^\infty xe^{-x^2}\,dx$. Show all your work and use good notation.

Exercise 9: (Adapted from a bonus problem) In a branch of math called Number Theory, a famous theorem states that for a fixed large number x, the probability of a random positive integer less than or equal to x being a prime number is approximately $\dfrac{1}{\ln(x)}$. Therefore, to count the number of primes less than or equal to x, a good approximation is $\displaystyle\int_2^x \frac{1}{\ln(t)}\,dt$. Unfortunately, there isn't an exact solution for this integral, so we just call it $\mathrm{Li}(x)$.

a) Show that

$$\mathrm{Li}(x) = \frac{x}{\ln(x)} - \frac{2}{\ln(2)} + \int_2^x \frac{1}{\ln^2(t)}\,dt.$$

b) The term $\int_2^x \frac{1}{\ln^2(t)}\,dt$ is called the error term because it is small compared to $\mathrm{Li}(x)$. Specifically,

$$\lim_{x\to\infty} \frac{\int_2^x \frac{1}{\ln^2(t)}\,dt}{\mathrm{Li}(x)} = 0.$$

Show this (hint: use L'Hôpital).