

Name: _____

Homework 2 | Math 256 | Cruz Godar

Due Wednesday of Week 3 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 2

In problems 1–10, solve the DE. Whenever possible, fully evaluate all integrals.

1. $y' + 2y = e^{-2t}$, $y(0) = -1$.

2. $y' + \frac{y}{t} = \sin(t)$, $y(\pi) = 0$.

3. $xy' = \sqrt{y^2x^2 - y^2}$. Hint: during the integration, you'll want to set $x = \sec(\theta)$ and use trig sub.

4. $y' = y + t^2e^t$.

5. $y' = \frac{t}{1+y}$, $y(0) = -2$.

6. $y' + ty^2 + ty + 2y^2 + 2y = 0$.

7. $ty' + (t+1)y = t+2$, $y(1) = 2$.

8. $\cos^2(t)y' = 1 + y$.

9. $y' = t + \frac{t}{y}$.

10. $y' = e^{x+y}$, $y(0) = 0$.

11. You invest \$1000 into an account which compounds interest **continuously** — in other words, your balance is constantly increasing at a rate proportional to its current value. If the continuous interest rate is 0.2% per year, how much money will be in the account after 10 years?

12. The population $P(t)$ of a colony of bacteria after t hours (in thousands) follows the equation $P' + P = 2^{-t} + 1$. After enough time, what will the population settle down to?
13. The function y satisfies $y' = y \log(x)$ with $y(1) = \frac{1}{e}$.
- a) Express y'' in terms of x, y , and y' .
 - b) Solve the original DE, find y'' in terms of x only, and verify that your answer to part a) was correct.
14. Solve the DE $y' + xy = x$ in two ways: first with integrating factors and then with separation of variables. Make sure your answers are the same!
15. True or false: if a DE is separable, then its direction field is symmetric about the origin — i.e. the slope at (x, y) is the same as the slope at $(-x, -y)$. If it's true, briefly explain why, and if it's false, give a counterexample.
16. A tank is filled with 1 L of water. We begin pouring in a salt solution with a concentration of $2 \frac{\text{g}}{\text{mL}}$ at a rate of $10 \frac{\text{mL}}{\text{s}}$, and also begin draining the tank at a rate of $10 \frac{\text{mL}}{\text{s}}$. What will the concentration of salt be after an hour?
17. A tank is filled with 100 L of uncolored paint. We begin pouring in a solution of red paint, which contains $0.5 \frac{\text{kg}}{\text{L}}$ of dye, at a rate of $1 \frac{\text{L}}{\text{s}}$. We also begin draining the well-mixed paint from the bottom of the tank at a rate of $1 \frac{\text{L}}{\text{s}}$. How long will it take for the paint in the tank to have half the concentration of dye that the incoming red paint does?