Name: ______________________ Homework 8 | Math 342 | Cruz Godar

Due Wednesday of Week 10 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–5, do the following:

a) Find the eigenvalues of A.

b) Find the corresponding eigenvectors and generalized eigenvectors of A.

c) Determine if A is diagonalizable. If it is, write $A = BDB^{-1}$ for a diagonal matrix D, and if not, write $A = BJB^{-1}$ for a matrix J in Jordan normal form (you don't need to invert B).

d) If A is diagonalizable, determine if there is an orthonormal basis of eigenvectors; if so, find it.

e) If the eigenvalues of A are distinct, find the general solution to the system of differential equations $\vec{x}' = A\vec{x}$.

1. $A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$.

2. $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 3 \end{bmatrix}$.

3. $A = \begin{bmatrix} -3 & -1 & -3 \\ -8 & -3 & -8 \\ 4 & 1 & 3 \end{bmatrix}$.

4. $A = \begin{bmatrix} -9 & -10 & -10 \\ 4 & 4 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

5. $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -3 & 1 \end{bmatrix}$.

In problems 6–8, do the following:

a) Find a singular value decomposition $A = U\Sigma V^T$.

b) Find a least-squares solution to $A\vec{x} = \vec{b}$.

c) Determine if the least-squares solution is unique.

6. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

7. $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

8. $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

In problems 9–12, do the following:

a) Find an orthonormal basis for the given inner product space X, and then extend it to an orthonormal basis for V.

b) Find the orthogonal decomposition of the vector $\vec{v}$ as $\vec{v} = \vec{x} + \vec{x}'$ for $\vec{x} \in X$ and $\vec{x}' \in X^{\perp}$.

9. $V = \mathbb{R}^3$ with $\langle \vec{v}, \vec{w} \rangle = \vec{v} \bullet \vec{w}$, $X = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \right\}$, and $\vec{v} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$.

10. $V = \operatorname{span}\left\{1, x, x^2\right\}$ with $\langle p, q\rangle = \sum_{n=0}^{2} p(n)q(n)$, X is the subspace of polynomials p with $p'(0) = 0$, and $\vec{v} = 1 + 2x + x^2$.

11. (optional) $V = \operatorname{span}\{1, \sin(x), \cos(x)\}$ with $\langle f, g\rangle = \int_0^1 f(x)g(x)\,\mathrm{d}x$, X is the subspace of functions f with $f'(0) = 0$, and $\vec{v} = \sin(x) + 2\cos(x) - 1$.

12. (optional; more recommended than the previous problem) $V = \operatorname{span}\{1, \cos(x)\}$ with $\langle f, g\rangle = \int_0^{\pi/2} f(x)g(x)\,\mathrm{d}x$, $X = \operatorname{span}\{1\}$, and $\vec{v} = \cos(x) - 2$.