

Name: \_\_\_\_\_

Homework 6 | Math 342 | Cruz Godar

*Due Wednesday of Week 7 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 5

In problems 1–4, find the least-squares solution  $\vec{x}'$  to the matrix equation  $A\vec{x} = \vec{b}$  and the distance between  $A\vec{x}'$  and  $\vec{b}$ . Also determine if that least-squares solution is unique.

$$1. \begin{bmatrix} 1 & 1 \\ 2 & 3 \\ 0 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

$$2. \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}.$$

$$3. \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 8 \\ 5 \\ 6 \end{bmatrix}.$$

$$4. \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -3 \end{bmatrix}.$$

5. Let  $A$  be an  $m \times n$  unitary matrix (i.e. not necessarily square). What is the least-squares solution to  $A\vec{x} = \vec{b}$  in terms of  $A$  and  $\vec{b}$ ?

## Section 6

In problems 6–8, compute  $\langle \vec{v}, \vec{w} \rangle$ ,  $\|\vec{v}\|$ ,  $\|\vec{w}\|$ , and the angle between  $\vec{v}$  and  $\vec{w}$  for the given inner product  $V$ .

6.  $\vec{v} = 1 + 2x$  and  $\vec{w} = 2x + x^2 - x^3$  for  $V = \text{span}\{1, x, x^2, x^3\}$  with the inner product

$$\langle p, q \rangle = \sum_{n=0}^3 p(n)q(n).$$

7.  $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$  and  $\vec{w} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$  for  $V = \mathbb{R}^3$  with the inner product

$$\langle \vec{v}, \vec{w} \rangle = 2v_1w_1 + v_2w_2 + 50v_3w_3.$$

8.  $\vec{v}(x) = e^x$  and  $\vec{w}(x) = x$  for  $V = C[0, 1]$  with the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x) \, dx.$$

9. Let  $V = C[0, 1]$ . For each of the following possible inner product formulas, give an example showing that it does not define an inner product.

a)  $\langle f, g \rangle = \int_0^1 |f(x) + g(x)| \, dx.$

b)  $\langle f, g \rangle = \int_0^1 (f(x)g(x))^2 \, dx.$

c)  $\langle f, g \rangle = \int_0^1 f(g(x)) \, dx.$

d)  $\langle f, g \rangle = \int_{1/4}^{3/4} f(x)g(x) \, dx.$