

Name: \_\_\_\_\_

Homework 1 | Math 341 | Cruz Godar

*Due Wednesday of Week 2 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 1

In problems 1–3, write the system in the form  $A\vec{x} = \vec{b}$  for a matrix  $A$  and vector  $\vec{b}$  of constants and a vector  $\vec{x}$  of variables.

1.

$$x_1 = 2$$

$$2x_1 - x_2 = 3.$$

2.

$$2x_3 - x_2 = 0$$

$$x_1 = x_3 - x_2 + 1$$

3.

$$x + y - z = x$$

$$x + 2y - 1 = 2z$$

$$x - z + 1 = y$$

In problems 4–8, evaluate the product.

$$4. \begin{bmatrix} 3 & 0 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$5. \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

$$6. \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}.$$

$$7. \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

$$8. \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}.$$

9. Let  $A$  be an  $n \times n$  matrix with entries  $a_{ij}$ .

- a) For the products  $AI$  and  $IA$  to make sense, what dimension must  $I$  have?
- b) The  $i$ th row of  $A$  is  $\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$ . If the  $j$ th column of  $I$  is denoted  $\vec{e}_j$ , what is the entry in row  $i$  and column  $j$  of  $AI$ ? Your answer should be in terms of  $i$  and  $j$ .
- c) What does part b) imply  $AI$  is equal to? Why does this make sense in the context of function composition?

10. Let  $A$  be an  $m \times n$  matrix with entries  $a_{ij}$ .

- a) There is a row vector  $\vec{x}$  and a column vector  $\vec{y}$  such that  $\vec{x}A\vec{y} = a_{11}$ . What are they?
- b) What about vectors  $\vec{x}$  and  $\vec{y}$  such that  $\vec{x}A\vec{y} = a_{ij}$ ? Your answer should be in terms of  $i$  and  $j$ .

11. State whether each part is true or false. If true, briefly justify why, and if false, provide a small counterexample.

- a) A system with 3 equations and 2 unknowns always has at least one solution.
- b) If the product  $AB$  is defined, then  $A$  and  $B$  have the same number of rows.
- c) If  $A$  is a  $2 \times 2$  matrix so that  $A\vec{x} = \vec{x}$  for *every* vector  $\vec{x}$ , then  $A = I_2$ . Hint: try plugging in specific values of  $x_1$  and  $x_2$ , like 0 and 1.

In problems 12–14, we'll show that many of the nice properties of multiplication of numbers don't hold for matrices.

12. With real numbers  $x$  and  $y$ , it must be the case that  $xy = yx$ , but this isn't true with matrices. Define matrices  $A$  and  $B$  by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Compute  $AB$  and  $BA$  and show that they're different.

13. With real numbers  $x$ ,  $y$ , and  $z$  where  $x \neq 0$  and  $xy = xz$ , it's always the case that  $y = z$ , but this also isn't true for matrices. Define matrices  $A$ ,  $B$ , and  $C$  by

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$$

Show that  $AB = AC$ , despite  $B \neq C$ .

14. With real numbers  $x$  and  $y$  where  $xy = 0$ , either  $x = 0$  or  $y = 0$ . Unfortunately, this also isn't the case for matrices. Define  $A$  and  $B$  by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}.$$

Show that  $AB = 0$  (the matrix of all zeros), even though both  $A$  and  $B$  are nonzero.