

Name: \_\_\_\_\_

## Homework 8 | Math 253 | Cruz Godar

*Due Wednesday of Week 10 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–5, find a Taylor series for  $f(x)$  centered at  $x = a$  and determine its interval of convergence.

1.  $f(x) = \cos(x)$ ,  $a = 0$ .
2.  $f(x) = \sin(2x)$ ,  $a = \frac{\pi}{2}$ .
3.  $f(x) = \ln(-x)$ ,  $a = -1$ .
4.  $f(x) = \frac{1}{x}$ ,  $a = 1$ .
5.  $f(x) = \sqrt[3]{x+1}$ ,  $a = 1$ .

In problems 6–9, compute the value of the series and justify your answer.

6.  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ .
7.  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ .
8.  $\sum_{n=0}^{\infty} (n+1) \left(\frac{1}{2}\right)^n$ .
9.  $\sum_{n=0}^{\infty} \frac{(-4)^n}{(2n)!}$ .

In problems 10–13, approximate the value to within 0.01 using Taylor series.

10.  $\cos(2)$ .

11.  $\frac{1}{e^2}$ .

12.  $\ln\left(\frac{1}{2}\right)$ .

13.  $\sqrt[4]{\frac{1}{2}}$ .

14. Using the fact that  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ , evaluate  $\int_0^1 \frac{\ln(1+x)}{x}$ .

15. Find a series solution to  $\frac{1}{x} + y'' = \frac{y}{x}$ .

16. The function  $\sin(x)$  has zeroes at exactly  $x = \pi n$  for integers  $n$ , and this allows us to write  $\sin(x)$  as

$$\sin(x) = x \left(1 - \frac{x}{\pi}\right) \left(1 + \frac{x}{\pi}\right) \left(1 - \frac{x}{2\pi}\right) \left(1 + \frac{x}{2\pi}\right) \cdots$$

Note that this isn't true for functions in general! There can often be a lot more going on with a function than we can determine by its zeros.

a) Divide by  $x$  to find an expression for  $\frac{\sin(x)}{x}$ .

b) Use the difference of squares formula  $(a-b)(a+b) = a^2 - b^2$  to group each two consecutive factors in the product.

c) Expand  $\frac{\sin(x)}{x}$  as a Maclaurin series. What is the coefficient of  $x^2$ ?

d) In the grouped product, what is the coefficient of  $x^2$ ? You'll likely need to express it as a sum.

e) Set the two sides equal to one another. What have you shown?