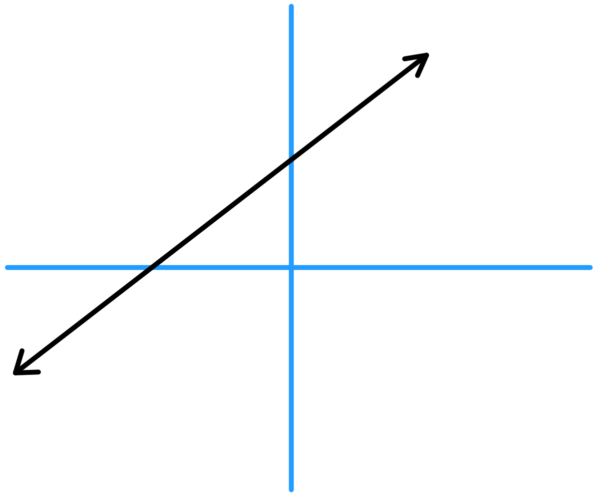
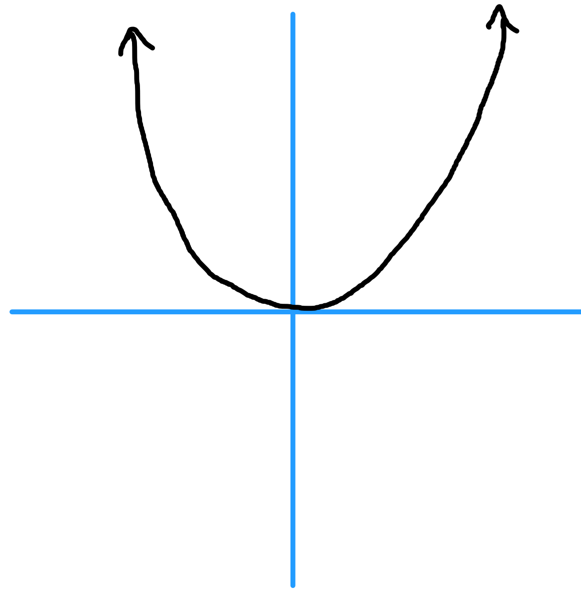
Comment: Recall from 111 or precalc:

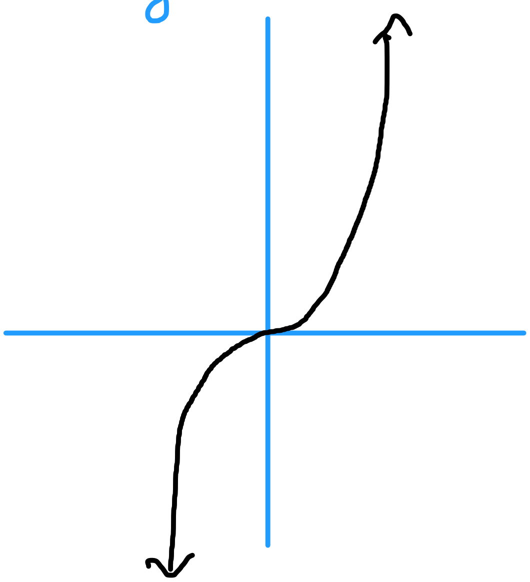
$y = mx + b$



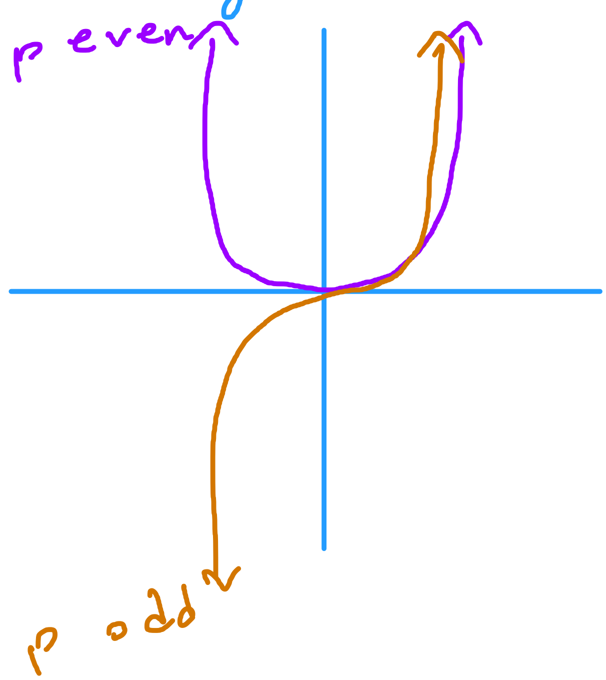
$y = x^2$



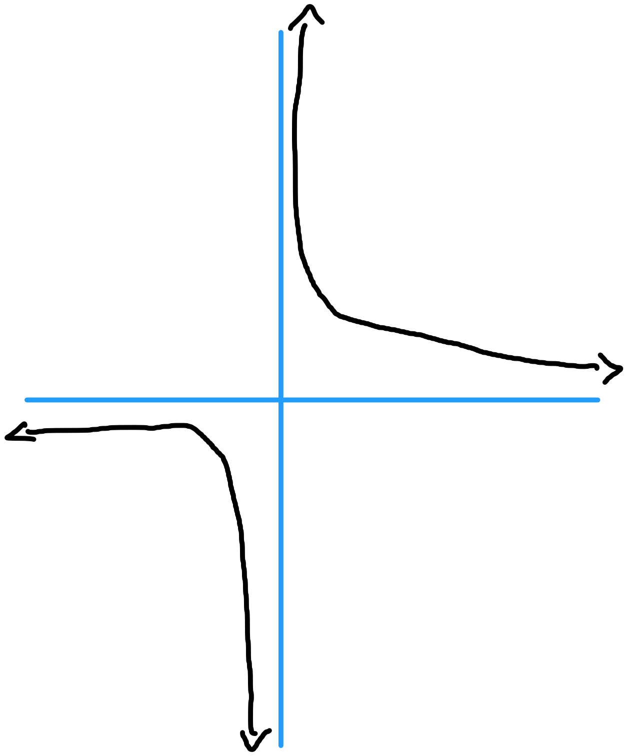
$y = x^3$



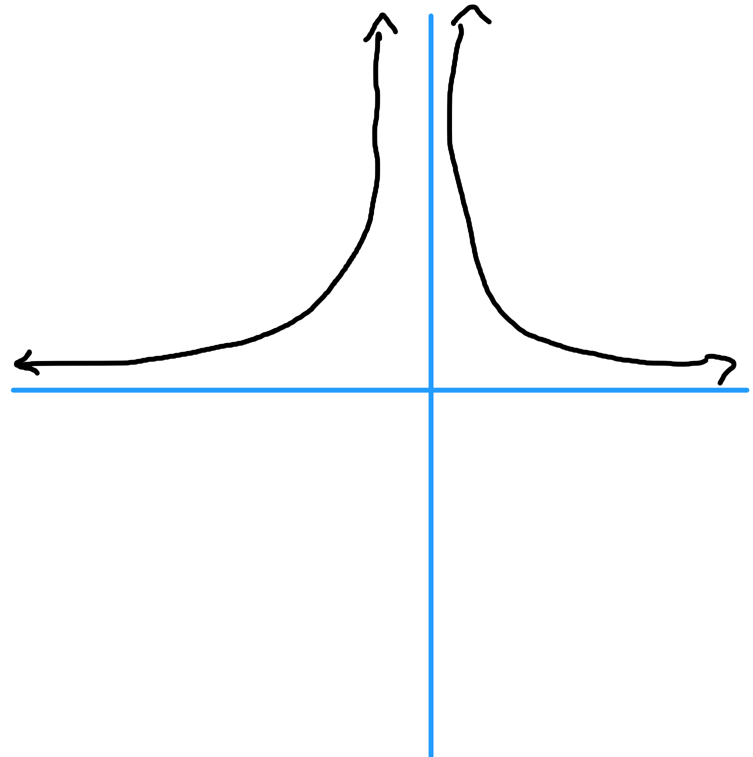
$y = x^p$



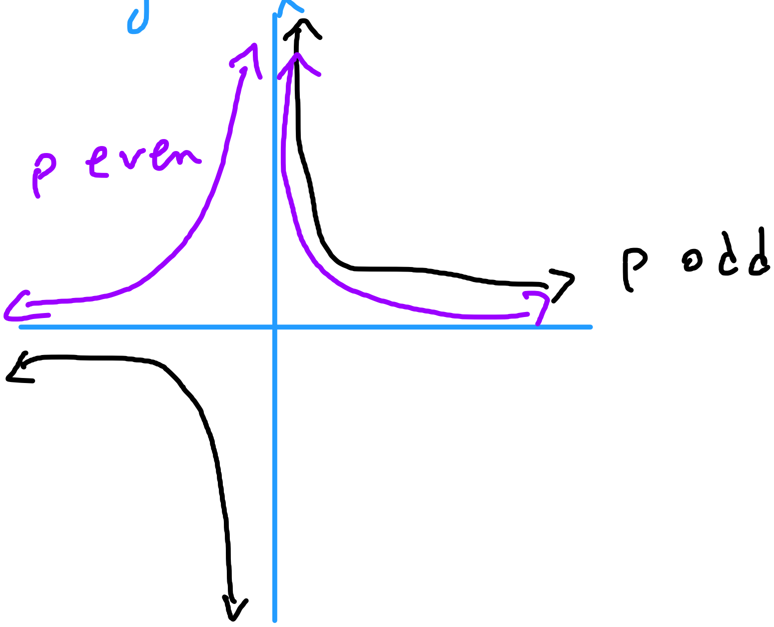
$y = \frac{1}{x}$



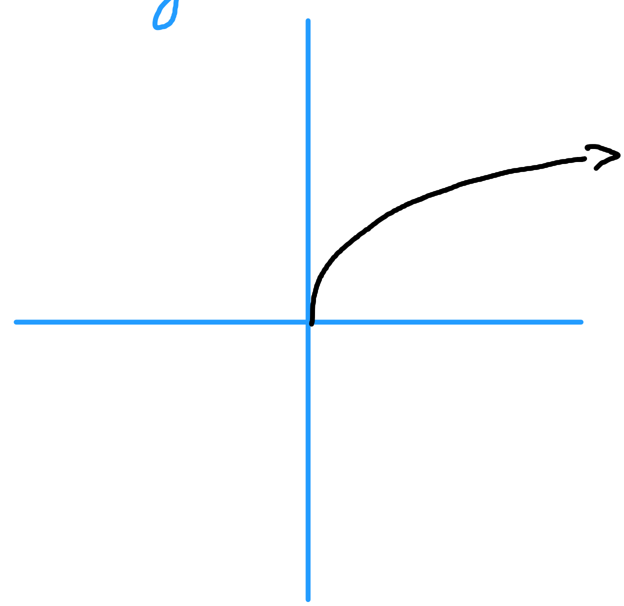
$y = \frac{1}{x^2}$



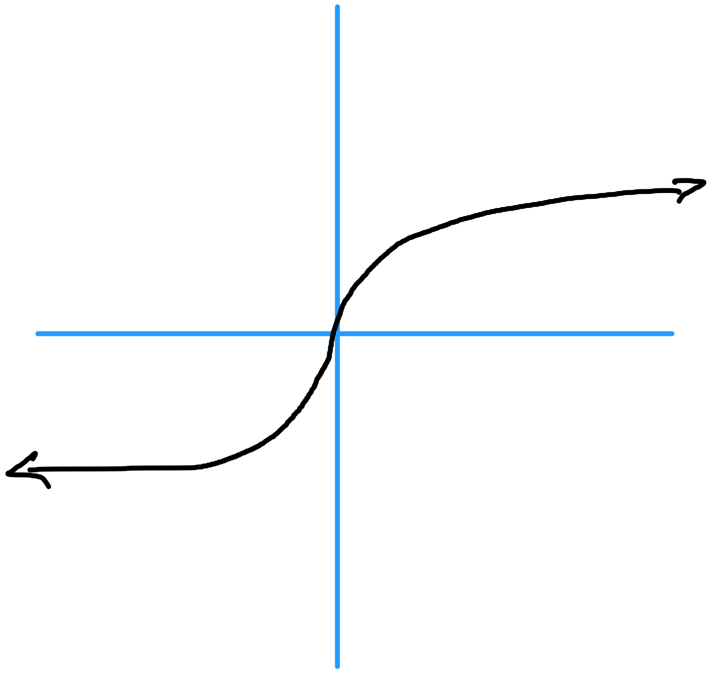
$y = \frac{1}{x^p}$



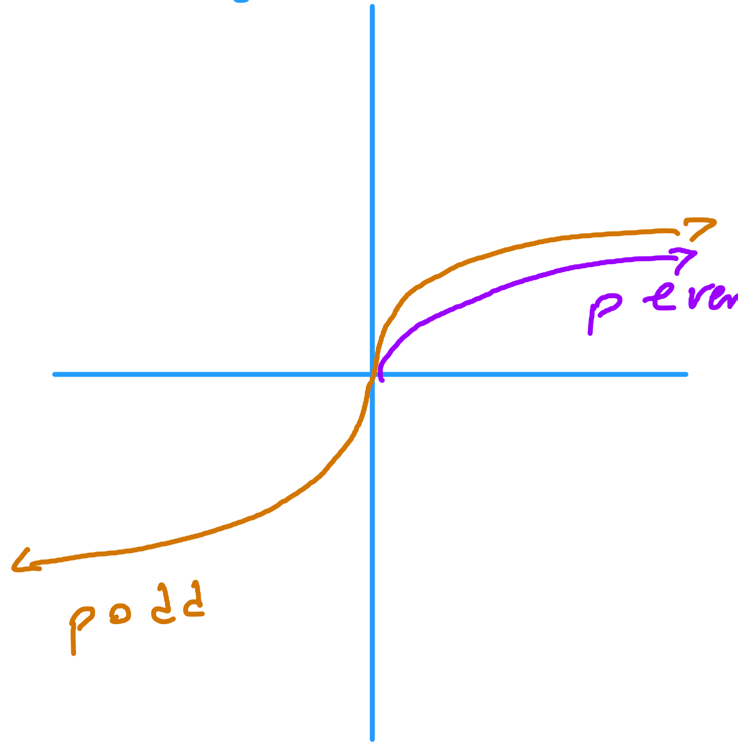
$y = \sqrt{x} = x^{1/2}$



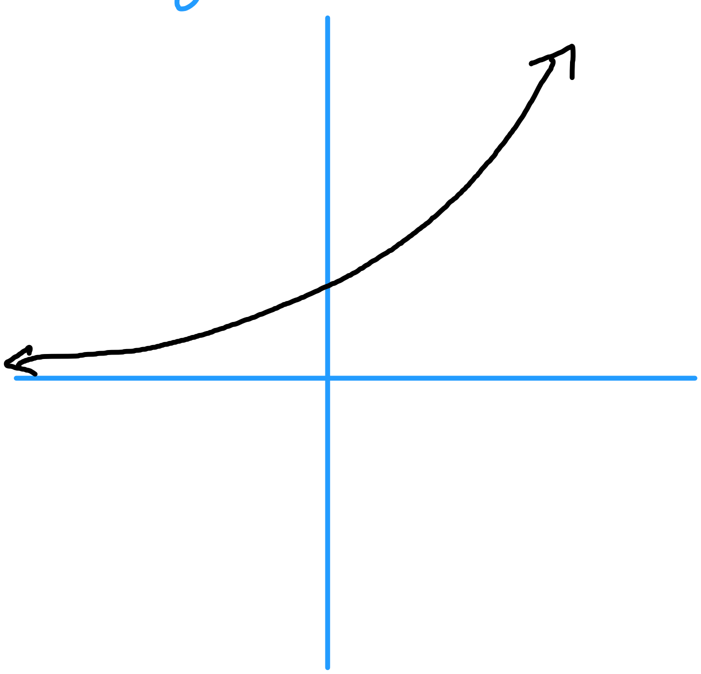
$y = \sqrt[3]{x} = x^{1/3}$



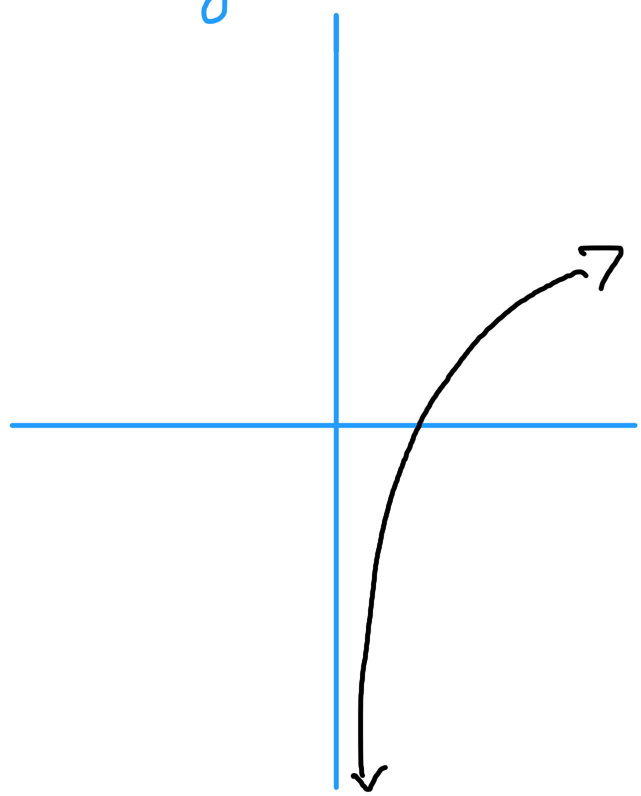
$y = x^{1/p}$



$y = e^x$



$y = \ln x$



__Comment__: These are called __elementary__ functions. There are more than these, but these ~~are~~ most of the building blocks.

__Def__: A function f is **even** if for all x in the domain, $f(-x) = f(x)$. It's **odd** if for all x in the domain, $f(-x) = -f(x)$. A function's even- or odd-ness is called its **parity**. As with regular numbers, most functions are neither even nor odd.

Ex: $f(x) = x$ is odd, since $f(-x) = -x = -f(x)$.

Ex: Is $f(x) = x^2$ even? Is it odd?

$f(-x) = (-x)^2 = (-x)(-x) = x^2$, so

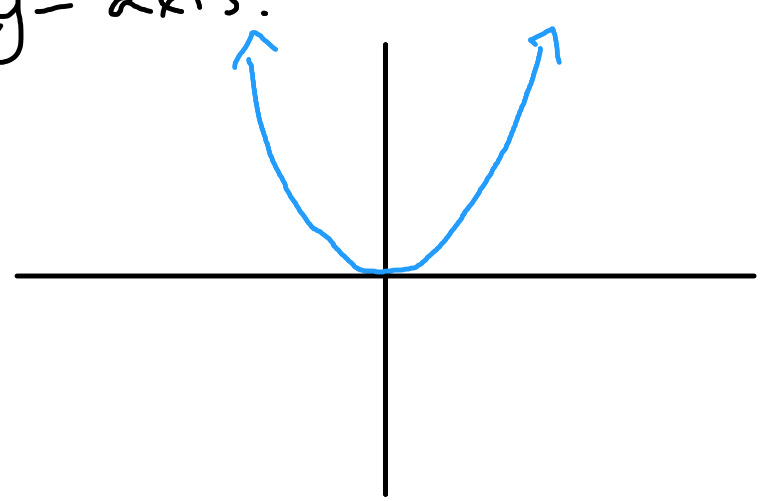
$f(-x) = f(x)$. Therefore, f is even.

Comment: $f(x) = x^p$ is an even function if p is even, and an odd function if p is odd.

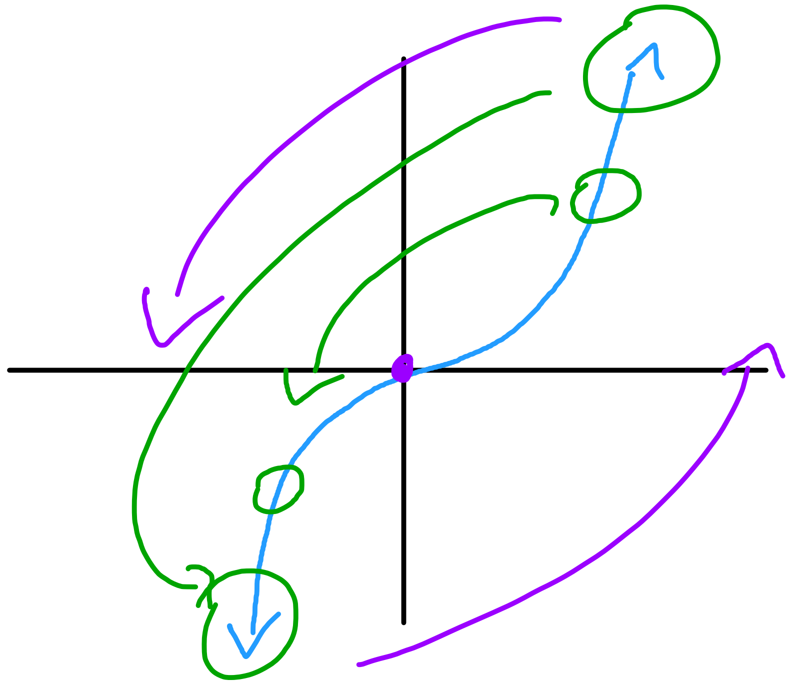
Ex: If $p = 1$, $f(x) = x^1 = x$ is odd. So is x^3, x^5, $\dots$. Similarly, x^2, x^4, $\dots$ are all even functions.

Prop: Even functions have graphs that are symmetric about the y-axis, and odd functions have graphs that are rotationally symmetric about the origin (when rotated 180°).

Ex: The graph of $f(x) = x^2$ is the same when reflected about the y-axis.



Ex: The graph of $f(x) = x^3$ is the same when rotated 180° about the origin.



Ex: Is $g(x) = xe^{-x^2}$ an even function? Is it an odd function?

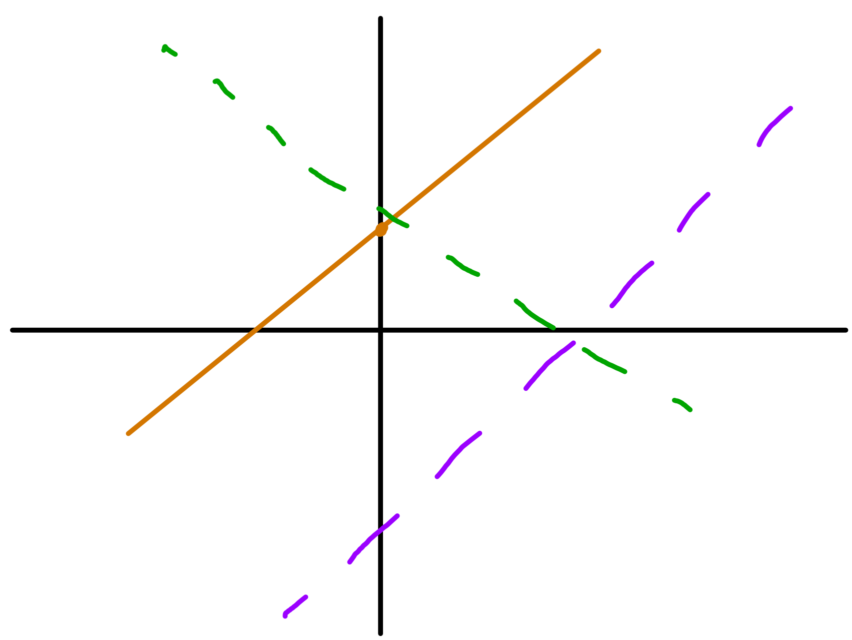
We need to check if $g(-x) = g(x)$ and if $g(-x) = -g(x)$.

$$g(-x) = (-x)e^{-(-x)^2} = -xe^{-x^2} = -\underbrace{\left(xe^{-x^2}\right)}_{g(x)}$$

$= -g(x)$, so g is odd.

Ex: The function $h(x) = x+1$ is neither even nor odd: $h(-x) = (-x)+1 = -x+1$ $\neq h(x)$ and $h(-x) \neq -h(x) = -x-1$.

Visually, the graph of h isn't symmetric either about the y-axis or the origin.

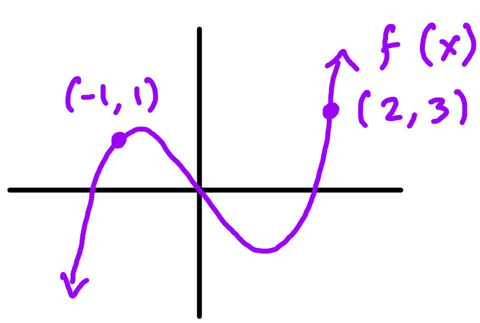


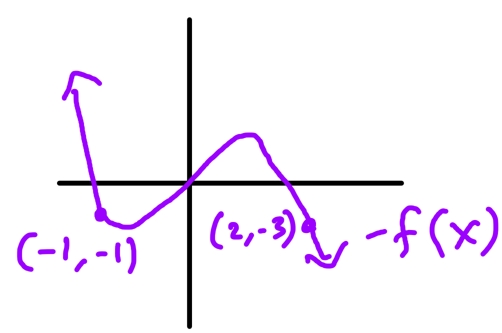
Vertical Transformations

Comment: If $h(t)$ measures your height in inches t years after you're born, how can we modify h to give an output in centimeters?

Prop: Let f be a function. The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

Ex





Theorem: Let f be a function and let $c>0$.

The graph of $y = c \cdot f(x)$ is graph of $y = f(x)$ vertically stretched by a factor of c.

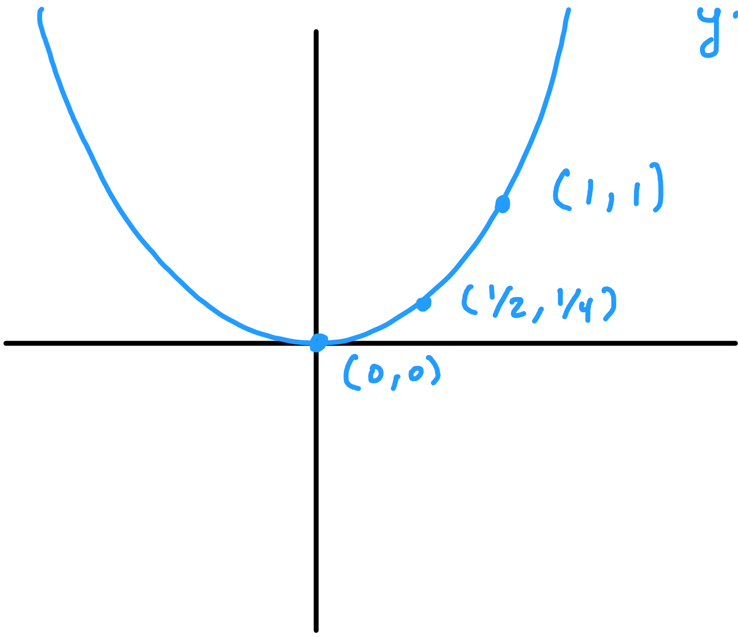
Ex: Graph $y = 3\boxed{x^2}$.

First, find the parent function.

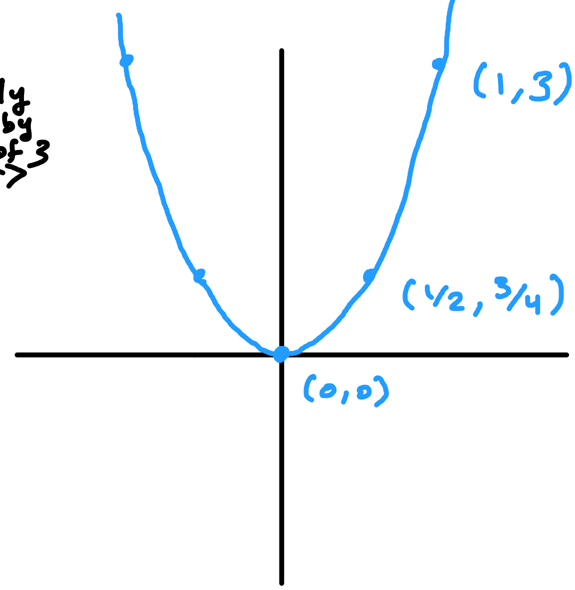
Here, it's $y = x^2$, since we're just multiplying it by a constant (so here $c=3$).

Now we'll graph $y = x^2$ and then vertically stretch it by a factor of 3.

$y = x^2$

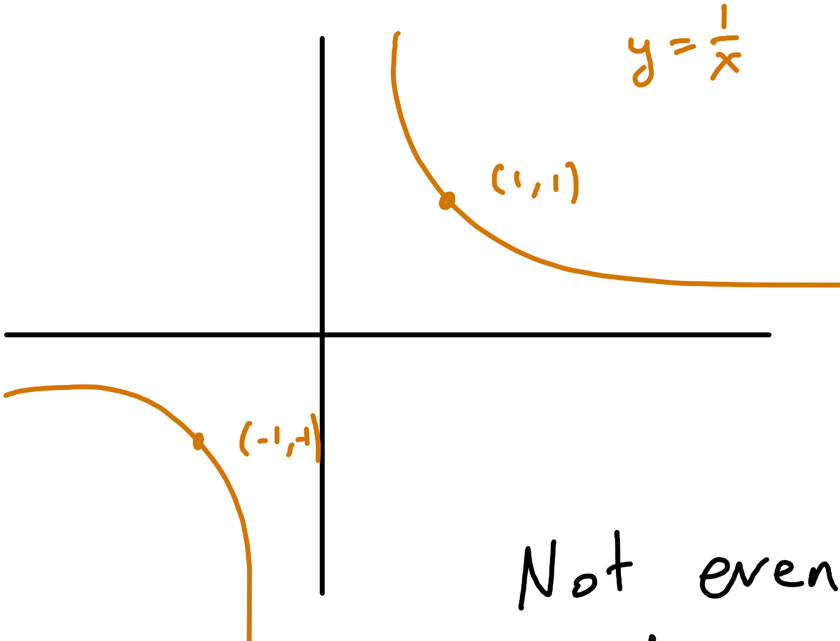


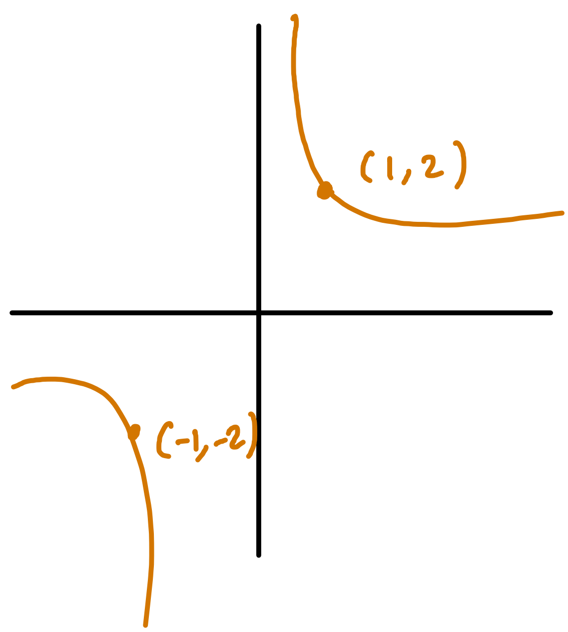
vertically stretch by a factor of 3



<u>Ex</u> Graph $y = 2\left(1/x\right)$ by first graphing $y = 1/x$, then determine from the graph if this function is even, odd, both, or neither.

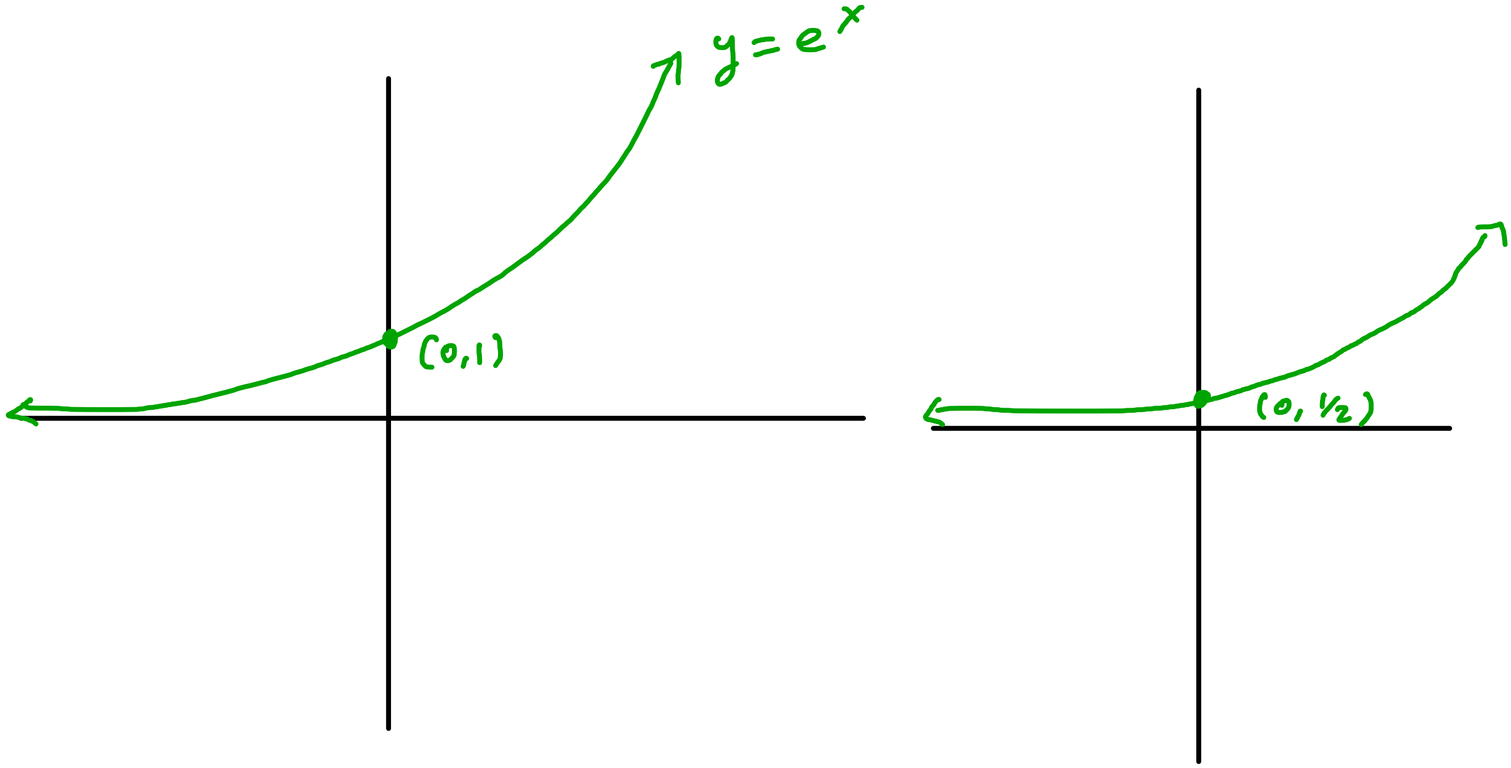
$y = \frac{1}{x}$





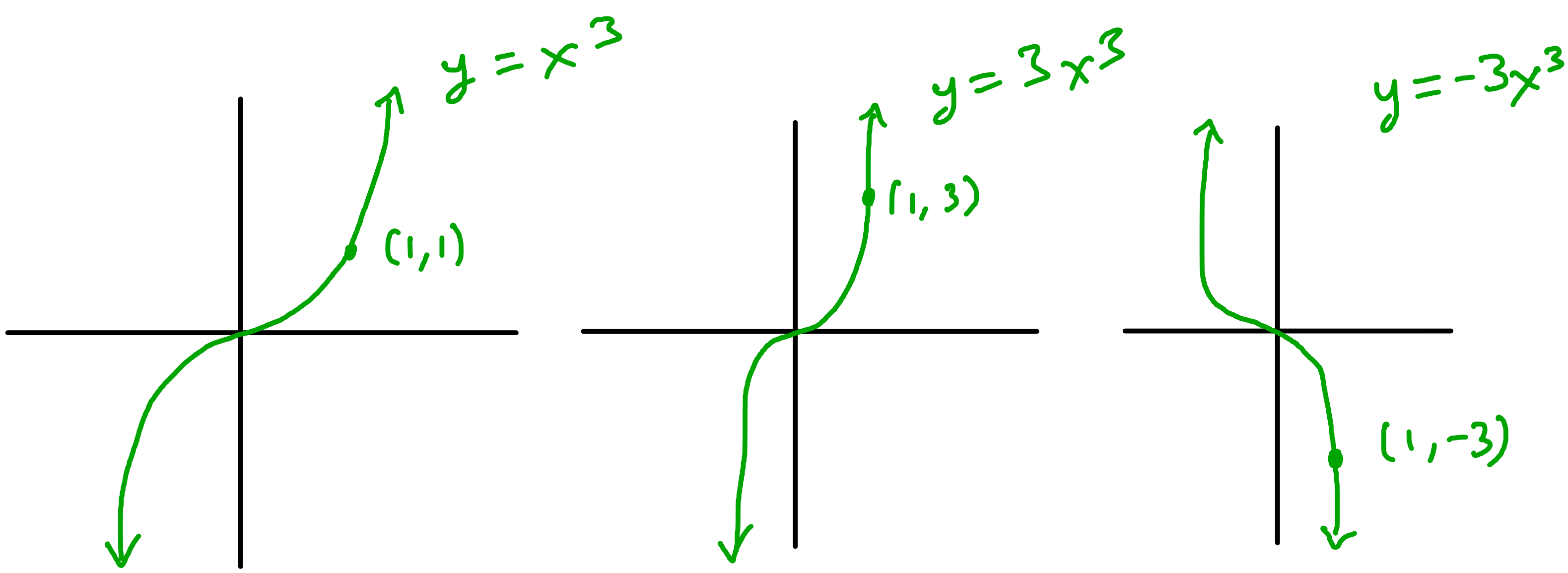
Not even,
but it is odd!

Ex: Graph $y = \frac{1}{2} e^x$.



vertical stretches by a factor less than 1 (but still positive) are <u>squishes</u>.

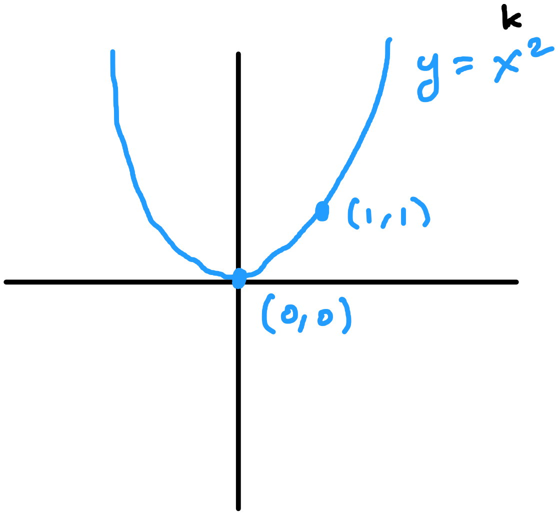
Ex: Graph $y = -3x^3$.

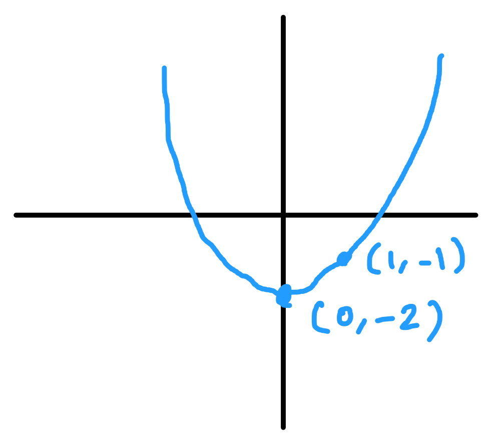


Comment: when working with multiple transformations, start at the value of x in the function equation and work outward (following order of operations).

Theorem: Let f be a function and k be any real number. The graph of $y = f(x) + k$ is the graph of $y = f(x)$ shifted vertically by k units.

Ex If $f(x) = x^2 \underbrace{-2}_{k}$, then



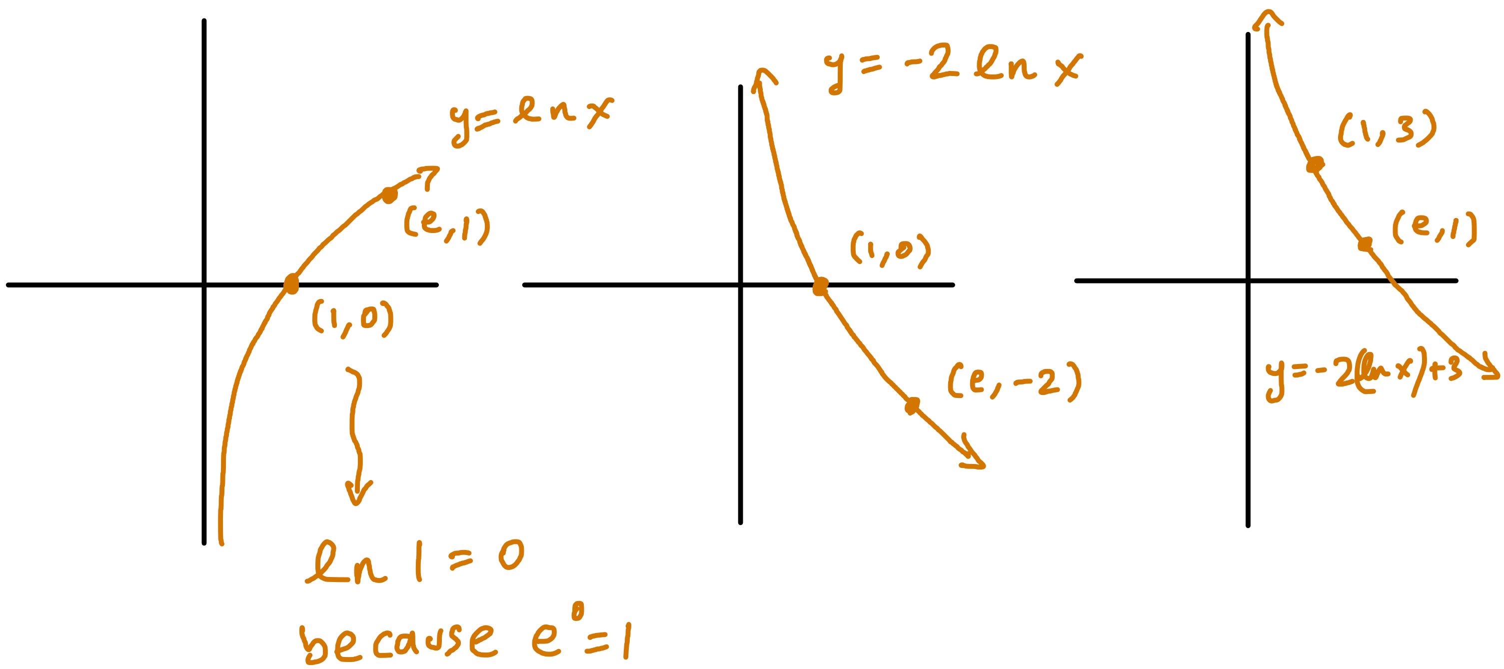


Ex: Graph $y = -2(\ln x) + 3$

First graph $y = \ln x$

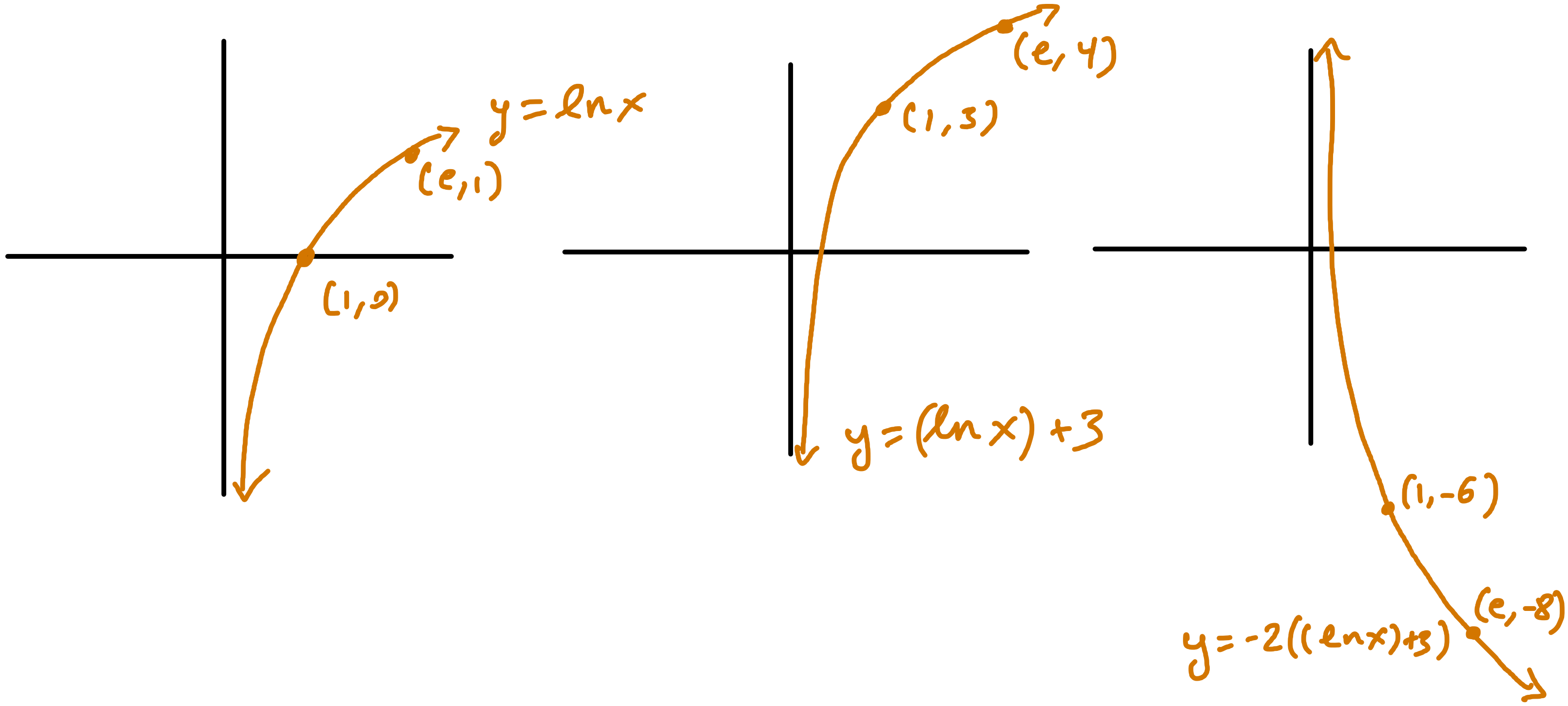
vertically stretch by a factor of -2

vertically shift by 3



Ex: Graph $y = -2\left((\ln x) + 3\right)$

- Graph $y = \ln x$
- Graph $y = (\ln x) + 3$
- Graph $y = -2\left((\ln x) + 3\right)$



Recall: $e = 2.718\ldots$

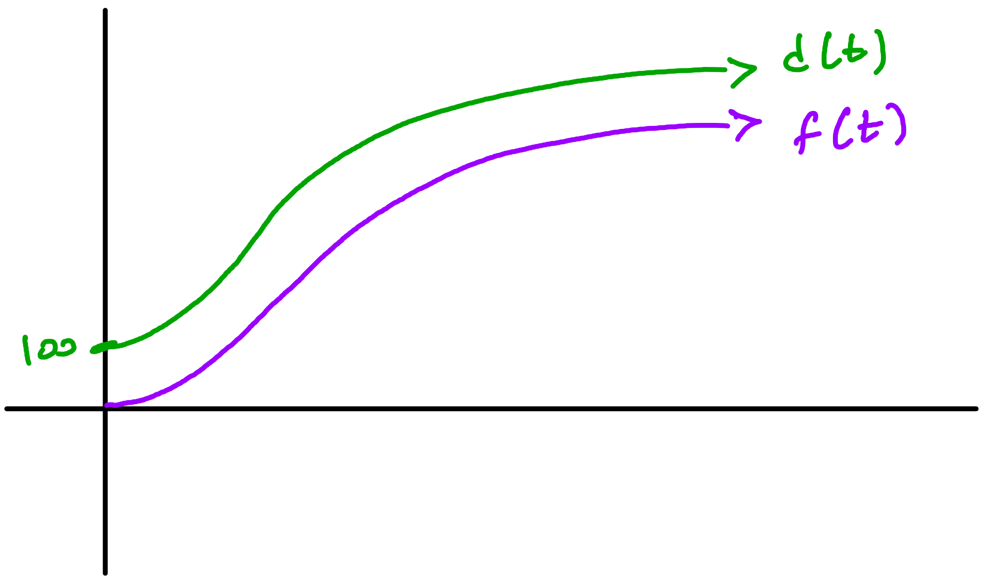
$\ln x = \log_e(x) = \#$ you raise e to in order to get x.

$\ln 1 = 0$ since $e^0 = 1$

$\ln e = 1$ since $e^1 = e$

Comment: Vertical transformations are used to rescale the output of a function. Use them when the function itself works fine, but its output need modification (e.g. if outputs inches instead of cm).

Ex: A scientist observes the population of deer in a park. It is 100 at some point in time, and t years later, it is given by $d(t)$:



What does the function $f(t) = d(t) - 100$ represent?

$f(0) = 0$

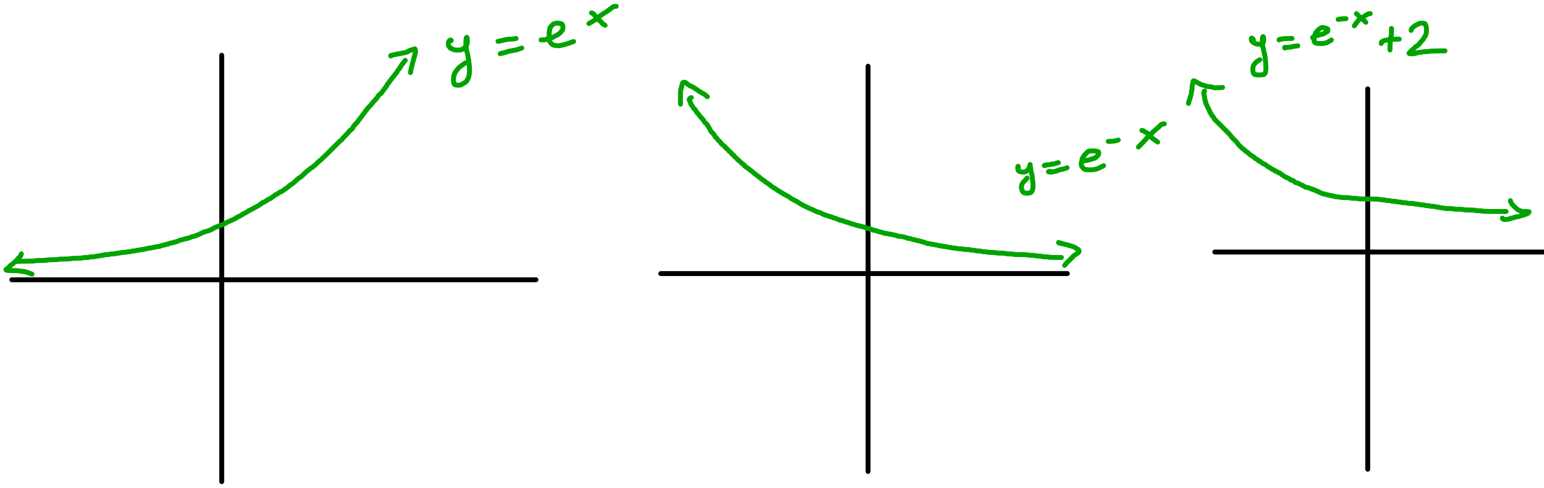
$f(t)$ is the number of new deer in the population after t years.

Horizontal Transformations

Comment: We apply vertical transformations to the outside of functions. We'll get horizontal transformations by rescaling the input.

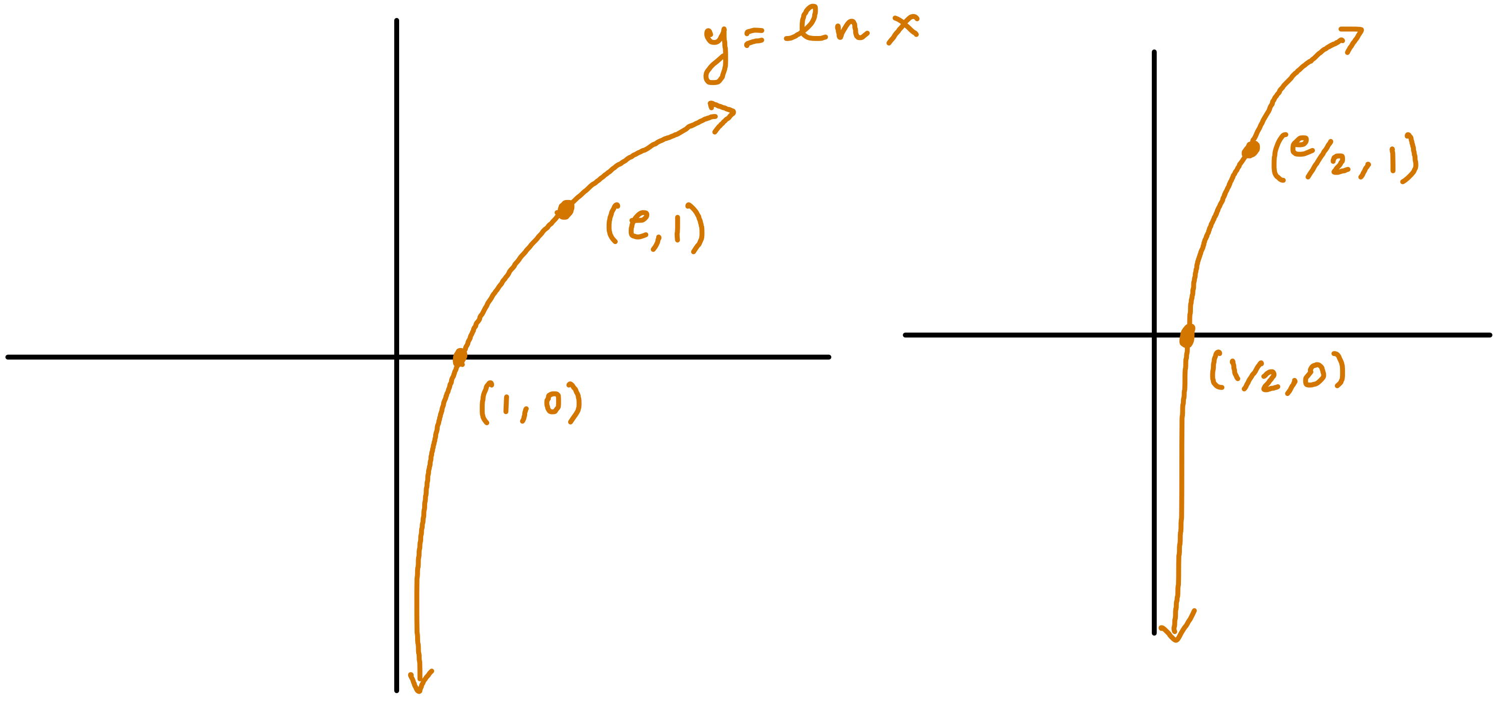
Prop: Let f be a function. The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

Ex: Graph $y = e^{-x}$. Then graph $y = e^{-x} + 2$.



Prop: Let f be a function and let $c>0$. The graph of $y=f(cx)$ is the graph of $y=f(x)$ stretched horizontally by a factor of $1/c$.

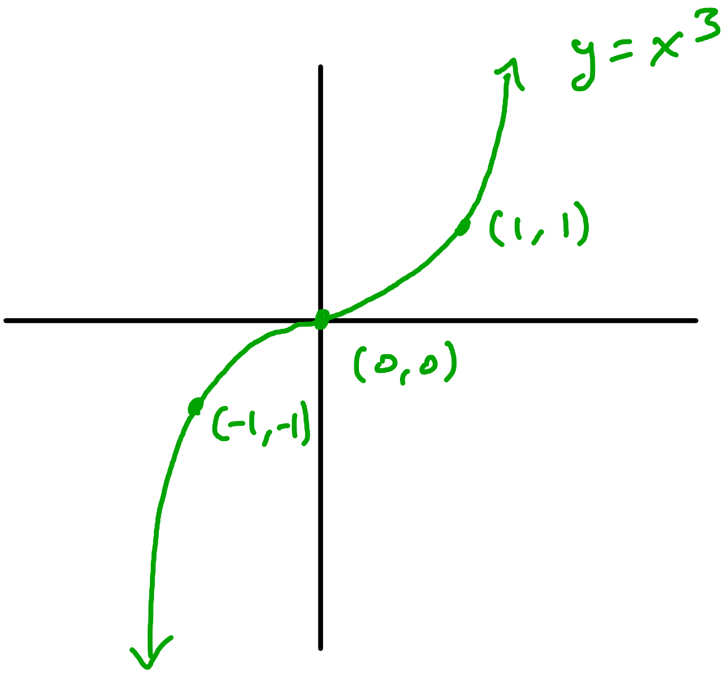
Ex: $\ln(2x)$

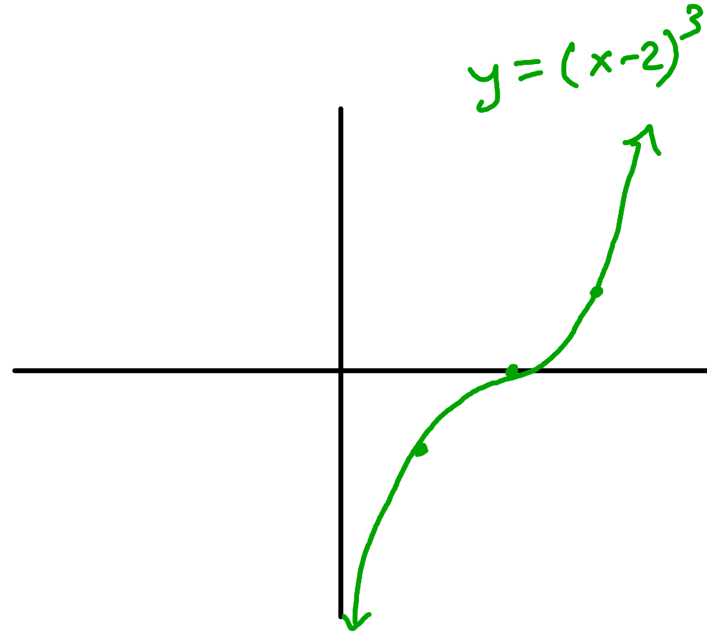


Comment: Why? Well, the point on the graph of $\ln(\underbrace{2}_{=c}x)$ that corresponds to $x = 1/3$ has the same y-value as the point on the graph of $\ln x$ that corresponds to $x = 2/3$. In a sense, the factor of 2 accelerates the x-values, making each of them twice as powerful.

Prop: Let f be a function and k be any real number. The graph of $y = f(x-k)$ is the graph of $y = f(x)$, shifted horizontally by k units.

Ex: $y=(x-2)^3$





Comment: Again, subtracting k is "slowing down" the x-value. To find the point that used to be at $x=0$, you now need to go to $x=2$.

Comment: Recall that the formula for a parabola with vertex (h, k) is $y = a(x-h)^2 + k$.

This is taking $y = x^2$, shifting to the right by h units, stretching vertically by a factor of a, then shifting vertically by k units.