Final Exam

Math 112

Spring 2020

You have 2 hours to complete this exam (this includes the 10 minutes accounting for the time it takes to scan and upload it). You may use a scientific calculator, but no other resources. When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

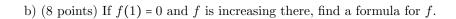
$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

1. (32 points) Consider a circle of radius 2 centered at the origin.
a) (8 points) What are the coordinates of a point on this circle with angle θ counter-clockwise from the positive x -axis?
b) (8 points) Find the coordinates of a point with angle 150° counter-clockwise from the positive x-axis. Simplify your answer and leave it in exact form, and show your work — specifically, any reference angles you use.
c) If the circle were centered at $(1,-1)$, what would the coordinates of this point be?
d) (8 points) This point and the point on the circle with angle 0° are the endpoints of an arc of the circle. Sketch this arc and find its length.

2. A 2-dimensional vector \vec{v} has magnitude 2 and angle $\frac{7\pi}{6}$ from the positive x-axis.	
a) (8 points) Find the unit vector decomposition for \vec{v} , showing all your work.	
b) (8 points) Vector \vec{w} has unit vector decomposition $\vec{w} = 2\vec{i} - 3\vec{j}$. Find $ \vec{w} $ and the angle \vec{w} makes with the positive \vec{w} -axis.	1V6
c) Find $\vec{v} \bullet \vec{w}$.	

d) (8 points) What is the angle between \vec{v} and \vec{w} ?

3.	32 points) Let $f(x)$ be a sinusoidal function with amplitude 2, midline 1, and period 3	
a)	8 points) Assuming there is no horizontal shift, find a formula for f .	



c) With the value of h you found in part b), find all values of x that make f(x) = 0.

d) Sketch a graph of f, labeling at least three points.

4.	(32)	points)	Let .	f(x)	$=x^2$

a) (8 points) Sketch a graph of f, labeling three different points.

b) (8 points) Let $g(x) = 2x^2 + 1$. g is a transformation of f—list the transformation(s) you'd need to apply to f to get g, and then sketch a graph of g, labeling the points that correspond to the ones from part a).

c) (8 points) Let $h(x) = 2(4(x+3))^2 + 1$. h is a transformation of g — again, list the transformation(s), then sketch a graph of h, labeling the points that correspond to the ones from part a).

d) (8 points) If we want to apply a vertical stretch to h to make a new function k such that k(1) = 1, what must k(x) be?

5. (32 points your work.	s) Miscellaneous, multiple-ch	noice questions. For each, circle the	correct answer. You don't need to show
a) (8 points)	Suppose θ is an angle in qu	adrant II. Which of the following i	s true?
	$\arcsin(\sin(\theta)) = \theta$	$\arccos(\cos(\theta)) = \theta$	$\arctan(\tan(\theta)) = \theta$
h) (8 points)	In which scenario can we a	only the Law of Sines?	
b) (o points)	in which seehano can we a	ppry the naw of offices.	
	i. We know two	sides and an angle and want to fir	nd a third side.
		ide and two angles and want to fin	
		now all three angles and want to fin	
	iv. we kir	ow all three sides and want to find	an angle.
c) (8 points)	Let \vec{v} be a 2-dimensional ve	ector. Which of the following can \vec{v}	not have?
		i. Negative angle	
		ii. Negative magnitude	
		iii. Negative \vec{i} component	
		iv. Negative \vec{j} component	
d) (8 points)	If the dot product of two ve	ectors is negative, then the angle b	etween the two vectors must be
		i. Acute	

ii. Rightiii. Obtuse