

Name: \_\_\_\_\_

Homework 4 | Math 256 | Cruz Godar

*Due Wednesday of Week 5 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

## Section 4

In problems 1–5, solve the DE and verify that you've found the general solution with the Wronskian.

1.  $y'' + 2y' - 3y = 0$ .

2.  $y'' + 3y' = 0$ .

3.  $4y'' - 9y = 0$ .

4.  $y'' - y' - y = 0$ .

5.  $y'' = y'$ .

In problems 6–8, solve the initial value problem.

6.  $y'' + y' - 20y = 0$ ,  $y(0) = 9$ ,  $y'(0) = 18$ .

7.  $2y'' + 6y' = 20y$ ,  $y(0) = 14$ ,  $y'(0) = 0$ .

8.  $y'' - y = 0$ ,  $y(0) = 4$ ,  $y'(0) = 2$ .

9. Consider the DE  $2t^2y'' - ty' + y = 0$ .

a) Show that a solution is  $y = c_1t + c_2\sqrt{t}$ .

b) Compute  $W \begin{bmatrix} t, \sqrt{t} \end{bmatrix}$ . For which value  $t = t_0$  is it zero?

c) Why is  $t = t_0$  a problem?

10. Let  $y = a(t) + b(t)i$  be a solution to

$$y'' + p(t)y' + q(t)y = 0,$$

where  $a$ ,  $b$ ,  $p$ , and  $q$  are all real-valued functions. Show that both  $a$  and  $b$  must be solutions to  $y'' + p(t)y' + q(t)y = 0$  themselves.

11. Consider the DE  $y'' - yy' = 0$ .

a) Show that both  $y = \tan\left(\frac{x}{2}\right)$  and  $y = 2 \tan(x + 1)$  are solutions.

b) Show that

$$y = c_1 \tan\left(\frac{x}{2}\right) + 2c_2 \tan(x + 1)$$

is *not* a solution in general. Why does this not contradict the results of section 4?