Name: ______________________ Homework 6 | Math 341 | Cruz Godar

Due Wednesday of Week 7 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 7

In problems 1–5, determine if the set X is a subspace of the vector space V. If it is, show that X is closed under addition and scalar multiplication and contains the zero vector, and if not, give an example showing one of those three fails.

1. X is the set of vectors of the form $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ for real numbers x and y, and $V = \mathbb{R}^3$.

2. $X = \operatorname{span}\{\cos(x), \sin(x)\}$, and $V = C^0(\mathbb{R})$.

3. X is the set of matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, and $V = M_{2\times 2}(\mathbb{R})$.

4. X is the set of linear transformations $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $V = \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$.

5. X is the set of linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$ with

$$\ker T = \operatorname{span}\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\},$$

and $V = \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$.

In problems 6–10, determine if the given function T is a linear transformation. If it is, show that T splits across addition and scalar multiplication, and if it is not, give an example showing one of those two things fails. If T is a linear transformation, also find $\ker T$ and write it as a span of vectors.

6. $T : \mathbb{R}^3 \to \mathbb{R}^2$, defined by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2x-y-z \end{bmatrix}.$$

7. $T : \mathbb{R}[x] \to \mathbb{R}$ given by $T(p(x)) = p''(x)$.

8. $T : \mathbb{R}^{\mathbb{R}} \to \mathbb{R}$ given by $T(f) = f(0)$. In this problem, just describe the kernel in words rather than as a span.

9. $T : M_{2\times 2}(\mathbb{R}) \to \mathbb{R}$ given by $T(A) = \det A$.

10. $T : \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2) \to \mathbb{R}^2$ given by $T(S) = S\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$.