1. **Vocabulary**

(a) (10 points) Let $f(x)$ be a continuous function on the interval $[a, b]$. State both versions of the **Fundamental Theorem of Calculus** (it doesn't matter which order you state them in):

i.

ii.

(b) (5 points) The parts of this problem can be answered very briefly. No words are necessary. In the expression $\int_1^4 s^3 + s^2\, ds$...

i. What is the **integrand**?

ii. What are the **limits of integration**?

iii. What is the **variable of integration**?

2. (4 points each) True or false? For each statement, *circle* the word "true" if the statement is true and "false" otherwise. No work is necessary, but you can show work for some partial credit if you get the answer wrong.

   (a) If $f(x)$ is a continuous function on $[a, b]$, then the upper Riemann sum with 5 rectangles for $f(x)$ on $[a, b]$ is greater than or equal to $\int_a^b f(x)$

   (b) If $f'(x) = A(x)$, then $A(x) - \pi$ is an antiderivative of $f(x)$

   (c) Let $a < b < c$. If $\displaystyle\int_a^c f(x) > 0$, then $\displaystyle\int_a^b f(x) > 0$

   (d) If an object travels a total distance of 10 meters, then its total displacement could be 8 meters.

3. (5 points) Briefly explain why the expression $\int_{-3}^{3} t^2 + 1$ does not make sense.

4. (17 points) Below is the graph of a function, $f(x)$.

[xmin = -4, xmax = 4, axis x line = center, xtick = -4,...,4, ymin = -4, ymax = 4, axis y line = center, ytick = -4,...,4, axis equal, grid = both, scale = 1] [thick, domain = -5:5]-x/2 + 2;

(a) On the graph, draw the rectangles which correspond to the left Riemann sum with 3 rectangles for $f(x)$ on the interval $[-2, 4]$.

(b) Compute the left Riemann sum with three rectangles to approximate $\int_{-2}^{4} f(x)$. You can use your picture in part (a) to help.

(c) Compute $\displaystyle\int_{-2}^{4} f(x)$ exactly.

(d) Compute $\displaystyle\int_{-2}^{4} -5f(x) + 3x^2$. You can use your answer from part (c) to help.

5. (6 points) On the below axes, draw the graph of a function $g(t)$ with the following properties:

   (a) $g(t)$ is continuous on $[0, 4]$

   (b) $\int_0^2 g(t) > 0$

   (c) $\int_0^4 g(t) < 0$

   [xmin = 0, xmax = 4, axis x line = center, ymin = -2, ymax = 2, axis y line = center, axis equal, grid = both]

6. (14 points) A particle is moving along an axis with velocity function $v(t) = \frac{\ln(t+e)}{t+e}$ (e here is the usual constant $e \approx 2.71...$). Find the position function, $s(t)$, of the particle assuming that $s(0) = 3$.

7. (10 points) Compute $\dfrac{d}{dx}\displaystyle\int_4^{e^{3x}} \frac{t^4-2}{t^2+2t+1}$

8. (13 points) Give two different functions which are antiderivatives of $\sin^3(\pi x)\cos(\pi x)$.

9. (19 points) You have been counting the number of leaves on your favorite tree for the last 4 years and you find that t years after 2017, the number of leaves on the tree is approximately

$$L(t) = \frac{16}{4+t^2} \quad thousandleaves$$

What is the average number of leaves your tree will have between the years 2019 and 2023?

- Show your work when integrating. Don't just appeal to a formula from the book.
- Include units in your answer.