Homework 6

Math 112

Due November 17th at 11:59 PM

Textbook Exercises

3.4: 3.4.1B, 3.4.2B, 3.4.3B, 3.4.6B, 3.4.7B, 3.4.10B, 3.4.11B, 3.4.12B

3.5: 3.5.1B, 3.5.2B, 3.5.3B, 3.5.8B, 3.5.9B

Exercise 1: Most electric appliances use *alternating current*, a type in which the voltage moves between A volts and $-A$ volts, where A is a constant that depends on the strength of the power. The current alternates from A to $-A$ and back again at 60 times per second.

a) At time $t = 0$, you plug in an appliance that uses alternating current. Write a function $V(t)$ for the voltage after t seconds, given that the initial voltage is zero and increasing. You may leave A as an unknown in your answer, but you should solve for all other constants.

b) If the voltage after a millisecond is 58.9 volts, find A.

c) At what times t will the voltage be zero?

Exercise 2: (Extra Credit) We know the values of the trig functions for π, $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$ — but what about $\frac{\pi}{5}$? We can find it exactly, but it's going to take some work. In this exercise we'll find an exact value for $\cos\left(\frac{\pi}{5}\right)$. The goals for most of the parts are provided — your job is to carefully fill in the details to get to them.

a) First, we'll derive the *triple-angle formula* for cosine. We know $\cos(3\theta) = \cos(\theta + 2\theta)$, and we have the sum formula for the cosine of a sum. Use it, then simplify $\cos(2\theta)$ and $\sin(2\theta)$ with the double-angle formulas.

Goal: $\cos(3\theta) = \cos^3(\theta) - 3\sin^2(\theta)\cos(\theta)$.

b) Using the fact that cosine is an even function and has period 2π, show that $\cos\left(\frac{4\pi}{5}\right) = \cos\left(\frac{6\pi}{5}\right)$.

c) Let $\alpha = \frac{2\pi}{5}$. By part b), $\cos(2\alpha) = \cos(3\alpha)$. Expand the left side with the double-angle formula and the right with the triple-angle formula you found in part a).

Goal: $\cos^2(\alpha) - \sin^2(\alpha) = \cos^3(\alpha) - 3\sin^2(\alpha)\cos(\alpha)$.

d) The answer to part c) involves $\sin^2(\alpha)$. Since $\sin^2(\alpha) + \cos^2(\alpha) = 1$, $\sin^2(\alpha) = 1 - \cos^2(\alpha)$. Use this to replace every instance of $\sin^2(\alpha)$ so that your equation only involves cosines, and simplify.

Goal: $2\cos^2(\alpha) - 1 = 4\cos^3(\alpha) - 3\cos(\alpha)$.

e) Let $x = \cos(\alpha)$ — you should now have a cubic equation. Move all the terms to one side and factor out an $(x - 1)$. Now deal with the two factors separately to solve for x.

Goal: $x = 1$ *or* $x = \frac{-1 \pm \sqrt{5}}{4}$.

f) You just solved for $x = \cos(\alpha) = \cos\left(\frac{2\pi}{5}\right)$, but you got three answers. Only one of them makes sense, though — figure out which one and why.

g) Finally, use the half-angle formula on $\cos\left(\frac{2\pi}{5}\right)$ to find $\cos\left(\frac{\pi}{5}\right)$.

Goal: $\cos\left(\frac{\pi}{5}\right) = \sqrt{\frac{3 + \sqrt{5}}{8}}$.