

Name: _____

Homework 1 | Math 341 | Cruz Godar

Due Wednesday of Week 2 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 1

In problems 1–3, write the system in the form $A\vec{x} = \vec{b}$ for a matrix A and vector \vec{b} of constants and a vector \vec{x} of variables.

1.

$$x_1 = 2$$

$$2x_1 - x_2 = 3.$$

2.

$$2x_3 - x_2 = 0$$

$$x_1 = x_3 - x_2 + 1$$

3.

$$x + y - z = x$$

$$x + 2y - 1 = 2z$$

$$x - z + 1 = y$$

In problems 4–8, evaluate the product.

$$4. \begin{bmatrix} 3 & 0 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

$$5. \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

$$6. \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}.$$

$$7. \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

$$8. \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}.$$

9. Let A be an $n \times n$ matrix with entries a_{ij} .

- a) For the products AI and IA to make sense, what dimension must I have?
- b) The i th row of A is $\begin{bmatrix} a_{i1} & a_{i2} & \cdots & a_{in} \end{bmatrix}$. If the j th column of I is denoted \vec{e}_j , what is the entry in row i and column j of AI ? Your answer should be in terms of i and j .
- c) What does part b) imply AI is equal to? Why does this make sense in the context of function composition?

10. Let A be an $m \times n$ matrix with entries a_{ij} .

- a) There is a row vector \vec{x} and a column vector \vec{y} such that $\vec{x}A\vec{y} = a_{11}$. What are they?
- b) What about vectors \vec{x} and \vec{y} such that $\vec{x}A\vec{y} = a_{ij}$? Your answer should be in terms of i and j .

11. State whether each part is true or false. If true, briefly justify why, and if false, provide a small counterexample.

- a) A system with 3 equations and 2 unknowns always has at least one solution.
- b) If the product AB is defined, then A and B have the same number of rows.
- c) If A is a 2×2 matrix so that $A\vec{x} = \vec{x}$ for *every* vector \vec{x} , then $A = I_2$. Hint: try plugging in specific values of x_1 and x_2 , like 0 and 1.

In problems 12–14, we’ll show that many of the nice properties of multiplication of numbers don’t hold for matrices.

12. With real numbers x and y , it must be the case that $xy = yx$, but this isn’t true with matrices. Define matrices A and B by

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Compute AB and BA and show that they’re different.

13. With real numbers x , y , and z where $x \neq 0$ and $xy = xz$, it’s always the case that $y = z$, but this also isn’t true for matrices. Define matrices A , B , and C by

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 5 & 4 \\ 1 & 3 \end{bmatrix}$$

Show that $AB = AC$, despite $B \neq C$.

14. With real numbers x and y where $xy = 0$, either $x = 0$ or $y = 0$. Unfortunately, this also isn’t the case for matrices. Define A and B by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}.$$

Show that $AB = 0$ (the matrix of all zeros), even though both A and B are nonzero.