

Homework 6

Math 112

Due February 26th at the start of class

Textbook Exercises

3.4: 3.4.1B, 3.4.2B, 3.4.3B, 3.4.6B, 3.4.7B, 3.4.10B, 3.4.11B, 3.4.12B

3.5: 3.5.1B, 3.5.2B, 3.5.3B, 3.5.8B, 3.5.9B

Exercise 1: Most electric appliances use *alternating current*, a type in which the voltage moves between A volts and $-A$ volts, where A is a constant that depends on the strength of the power. The current alternates from A to $-A$ and back again at 60 times per second.

- a) At time $t = 0$, you plug in an appliance that uses alternating current. Write a function $V(t)$ for the voltage after t seconds, given that the initial voltage is zero and increasing. You may leave A as an unknown in your answer, but you should solve for all other constants.
- b) If the voltage after a millisecond is 58.9 volts, find A .
- c) At what times t will the voltage be zero?

Exercise 2: We know the values of the trig functions for π , $\frac{\pi}{2}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$ — but what about $\frac{\pi}{5}$? We can find it exactly, but it's going to take some work. In this exercise we'll find an exact value for $\cos\left(\frac{\pi}{5}\right)$.

- a) First, we'll derive the *triple-angle formula* for cosine. We know $\cos(3\theta) = \cos(\theta + 2\theta)$, and we have the sum formula for the cosine of a sum. Use it, then simplify $\cos(2\theta)$ and $\sin(2\theta)$ with the double-angle formulas.

- b) Using the fact that cosine is an even function and has period 2π , show that $\cos\left(\frac{4\pi}{5}\right) = \cos\left(\frac{6\pi}{5}\right)$.

- c) Let $\alpha = \frac{2\pi}{5}$. By part b), $\cos(2\alpha) = \cos(3\alpha)$. Expand the left side with the double-angle formula and the right with the triple-angle formula you found in part a).

- d) Your answer to part c) should involve one or two copies of $\sin^2(\alpha)$. Since $\sin^2(\alpha) + \cos^2(\alpha) = 1$, $\sin^2(\alpha) = 1 - \cos^2(\alpha)$. Use this to replace every instance of $\sin^2(\alpha)$ so that your equation only involves cosines.

- e) Let $x = \cos(\alpha)$ — you should now have a cubic equation. Move all the terms to one side and factor out an $(x - 1)$. Now deal with the two factors separately to solve for x .

- f) You just solved for $x = \cos(\alpha) = \cos\left(\frac{2\pi}{5}\right)$, but you got three answers. Only one of them makes sense, though — figure out which one and why.

- g) Finally, use the half-angle formula on $\cos\left(\frac{2\pi}{5}\right)$ to find $\cos\left(\frac{\pi}{5}\right)$.