Midterm 2

Math 252

Spring 2021

You have 50 minutes to complete this exam and upload it to Canvas. **You may use a scientific calculator, but no other resources.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Multiple choice. You don't need to show any work.

a) (8 points) Suppose $y = f(x)$, and that the graph of f is rotated about the x-axis. Then

A) the shell method integrates with respect to y and the disk method with respect to x.

B) the shell method integrates with respect to x and the disk method also with respect to x.

C) the shell method integrates with respect to x and the disk method with respect to y.

D) the shell method integrates with respect to y and the disk method also with respect to y.

b) (8 points) The area between two functions can be negative...

A) when one function goes below the x-axis.

B) when both functions are to the left of the y-axis.

C) when the functions cross over each other.

D) never.

c) (8 points) If a lamina is given by $f(x)$ on $[a, b]$ with $f(x) \geq 0$, then it is always true that

A) $a \leq \overline{x} \leq b$.

B) $\overline{y} \geq 0$.

C) both.

D) neither.

d) (8 points) True or false: there is a number L such that for any continuous function f on $[0, 1]$, the arc length of f is less than L.

e) (4 points extra credit) Justify your answer to part d): if you answered true, then find L. If you answered false, then given any number L, find a function on $[0, 1]$ with arc length of at least L.

2. (32 points) Short answer.

a) (8 points) Let f and g be continuous functions on $[1,6]$ such that $f(x) \geq g(x)$ on $[1,3]$ and $g(x) \geq f(x)$ on $[3,6]$. What is the area between f and g on $[1,6]$?

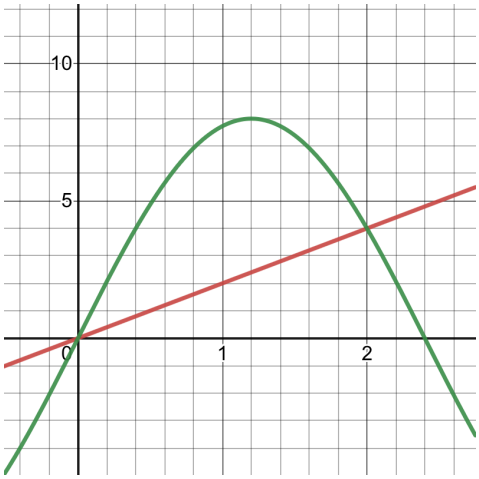
b) (8 points) Let $f(x) = 3x^2$. Set up the integrals to find the volume of the solid given by rotating the graph of f on $[0,3]$ about the x-axis, using **both** the disk and shell methods. Don't solve either of the integrals.

c) (8 points) Let L be a lamina bounded above by $f(x)$ on $[1,2]$. Write the three integrals necessary to calculate $\overline{x}$ and $\overline{y}$.

d) (8 points) Evaluate $\int xe^{2x}\, dx$.

The rest of the problems require setting up and solving integrals. Half the credit is for the set-up and half for the solving.

3. (32 points) Consider the region given in the graph below, bounded by $f(x) = 8\sin\left(\frac{5\pi}{12}x\right)$ above and $g(x) = 2x$ below. These functions intersect at $(0, 0)$ and $(2, 4)$.



a) (16 points) Find the volume of the solid of revolution given by rotating the region about the y-axis.

b) (16 points) Suppose the region is a lamina with density $\rho = 1$. Find the center of mass.

4. (16 points) Evaluate $\int_1^2 x^2 \ln(x)\, dx$.