Midterm 1

Math 252

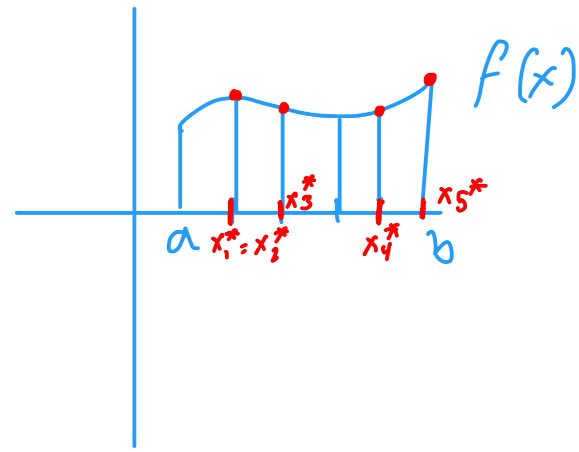
Spring 2021

You have 50 minutes to complete this exam and scan and upload it to Canvas. **Show all your work. You may use a scientific calculator, but not a graphing one.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) Let f be a positive function on an interval $[a,b]$. The upper Riemann sum of f on $[a,b]$ with 5 subintervals chooses x_i^* to be

max value



A) the left endpoint on each subinterval.

B) the right endpoint on each subinterval.

C) the x in the subinterval that has the largest value of $f(x)$.

D) the x in the subinterval that has the smallest value of $f(x)$.

b) (8 points) What is $\int \frac{1}{\sqrt{1-x^2}}\,dx$?

A) $\tan^{-1}(x) + C$.

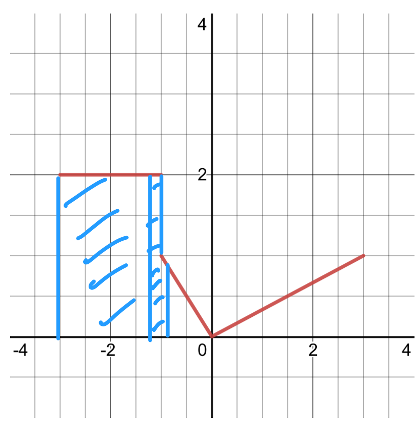
B) $\sin^{-1}(x) + C$.

C) $\cos^{-1}(x) + C$.

D) $\cancel{\tan}^{-1}(x) + C$.

$\sec^{-1}(x) + C$

c) (8 points)



Let $f(t)$ be defined by the previous graph. Then $\int_{-2}^{1} f(t)\,dt$ is

A) positive.

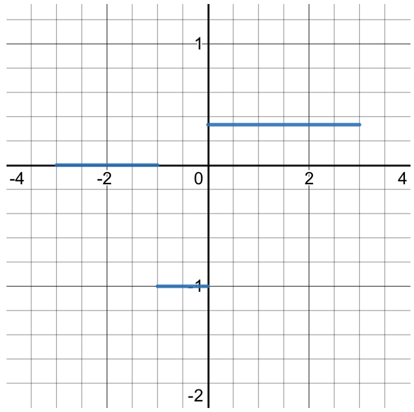
B) negative.

C) zero.

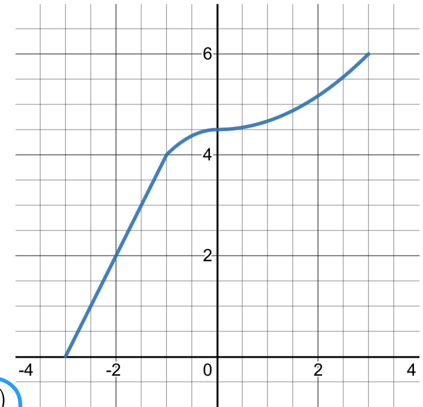
D) undefined.

area under $f(t)$ between $t=-3$ and $t=x$

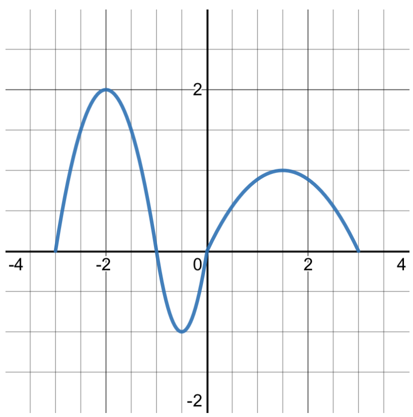
d) (8 points) With f defined from the same graph as before, let $g(x) = \int_{-3}^{x} f(t)\, dt$. Which of the following could possibly be a graph of g?



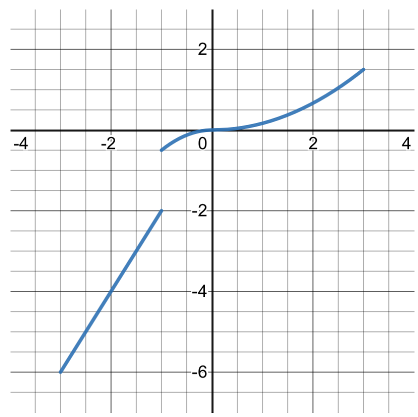
A)



B)



C)



D)

2. (32 points) Short-answer. Explain your reasoning and/or show your work for each question.

a) (8 points) Evaluate $1 + 2 + 3 + \cdots + 99 + 100$.

$$= \sum_{i=1}^{100} i = \frac{100(100+1)}{2} = \frac{100 \cdot 101}{2} = 5050$$

b) (8 points) Evaluate $\int (x^2 + \ln(x))\, dx$.

$$= \int x^2\, dx + \int \ln(x)\, dx$$

$$= \frac{x^3}{3} + x \ln(x) - x + C$$

c) (8 points) Evaluate $\int_2^6 \frac{1}{r}\, dr$.

$\int \frac{1}{r} dr = \ln(r) + C$ b/c $\frac{d}{dr}[\ln(r)] = \frac{1}{r}$, so

$$\int_2^6 \frac{1}{r} dr = [\ln(r)]\Big|_2^6 = \ln(6) - \ln(2)$$

$$\left(= \ln\left(\frac{6}{2}\right) = \ln(3) \right)$$

d) (8 points) Evaluate $\int \sin(t)\cos(t)\sin(\cos(2t))\, dt$. (Hint: $\sin(2t) = 2\sin(t)\cos(t)$)

composition: $\sin(\cos(2t))$

try: $u = \cos(2t)$

$du = -\sin(2t) \cdot 2\, dt$

$du = -2 \cdot 2 \sin(t) \cos(t)\, dt$

$$-\frac{1}{4}\,du = \sin(t)\cos(t)\,dt$$

$$\int -\frac{1}{4}\sin(u)\,du$$

$$-\frac{1}{4}\left(-\cos(u) + C\right)$$

$$\frac{1}{4}\cos(\cos(2t)) + C$$

Calc I: $\frac{d}{dt}[s(t)] = v(t)$ and $\frac{d}{dt}[v(t)] = a(t)$

3. (32 points) Let $v(t) = 2t + 2t^2$ be the velocity of a particle at time t.

a) (8 points) Find $a(t)$, the acceleration of the particle at time t.

$v'(t) = a(t)$, so $a(t) = \frac{d}{dt}[2t + 2t^2]$

$= 2 + 4t.$

b) (8 points) Find $s(t)$, the position of the particle at time t, given that $s(3) = 2$.

$$s(t) = \int v(t)\,dt = \int 2t + 2t^2\,dt = t^2 + \frac{2t^3}{3} + C$$

$$2 = 3^2 + \frac{2\cdot 3^3}{3} + C$$

$$2 = 9 + 18 + C$$

$$C = -25$$

$$s(t) = t^2 + \frac{2t^3}{3} - 25.$$

c) (8 points) Sketch graphs of $s(t)$, $v(t)$, and $a(t)$ on $[0, 5]$.

d) (8 points) What is the average position of the particle on $[0, 5]$?

$$= \frac{1}{5-0}\int_0^5 s(t)\,dt = \frac{1}{5}\left[\frac{t^3}{3} + \frac{2t^4}{12} - 25t\right]\Big|_0^5$$

e) (4 points extra credit) Let $e(x)$ be the average position of the particle on $[0, x]$. Find $e(x)$ and sketch a graph.

$$= \frac{1}{x-0}\int_0^x s(t)\,dt = \frac{1}{x}\left[\frac{t^3}{3} + \frac{2t^4}{12} - 25t\right]\Big|_0^x$$

$$= \frac{x^2}{3} + \frac{2x^3}{12} - 25$$

Alternate method for 2b: use FTC

$$s(x) - s(3) = \int_3^x r(t)\,dt$$

$$s(x) - 2 = \left[t^2 + 2\frac{t^3}{3}\right]\Big|_3^x$$

$$= x^2 + \frac{2}{3}x^3 - 3^2 + 2\cdot\frac{3^3}{3}$$

$$= x^2 + \frac{2}{3}x^3 - 27$$

$$s(x) = x^2 + \frac{2}{3}x^3 - 25$$