Statistics is the science of data, and is used to evaluate claims.

Ex: I make 80% of free throws I shoot.

Chapter 1: Picturing Distributions with Graphs

Def: An individual is an object described by data.

Ex: Person, city, animal, company.

Def: A variable is a characteristic of an individual.

Ex: Age, population, species, profit.

Ex: We randomly select 4 people in the US and ask them to report their age and gender. We also ask them what state they're living in.

State	Age	Reported Gender
Kentucky	61	Female
Florida	27	Female
Wisconsin	27	Male
California	33	Female

catagorical quantative catagorical

4 individuals and 3 variables measured for each individual

Def: A variable is quantitative if it takes numerical values and arithmetic makes sense.

Def: A variable is catagorical if it is not quantative.

Now we ask for zip codes

State	Age	Reported Gender	Zip
Kentucky	61	Female	41375
Florida	27	Female	93402
Wisconsin	27	Male	97403
California	33	Female	49102

catagorical!

Ex: A study classifies bison in Yellowstone as young or adult. State the individuals, variables, and the type of variable.

Bison, age, catagorical

Def: The distribution of a variable is the information of both its possible values and how often they occur.

Ex:

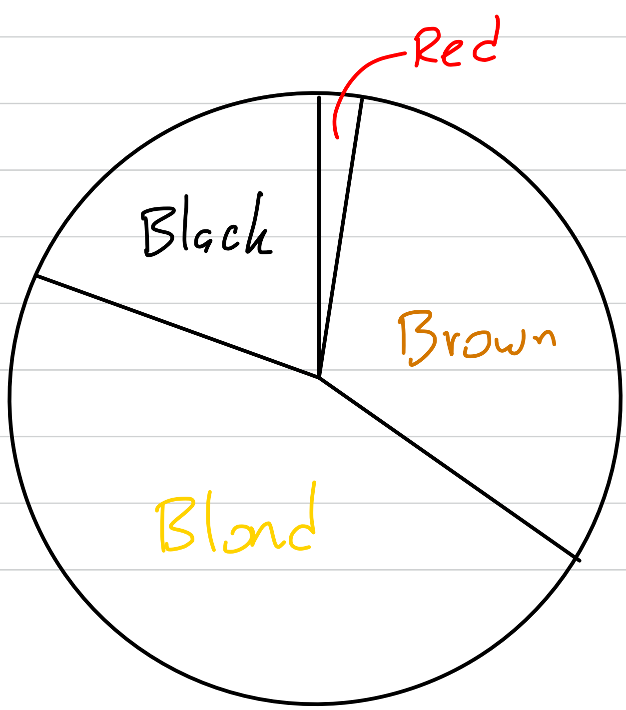
Student ID	Hair color
003	Red
005	Brown
035	Brown
089	Black

not a distribution

Hair color	% of students w/ this color
Red	2 %
Brown	35 %
Blond	43 %
Black	20 %

distribution

Pie chart



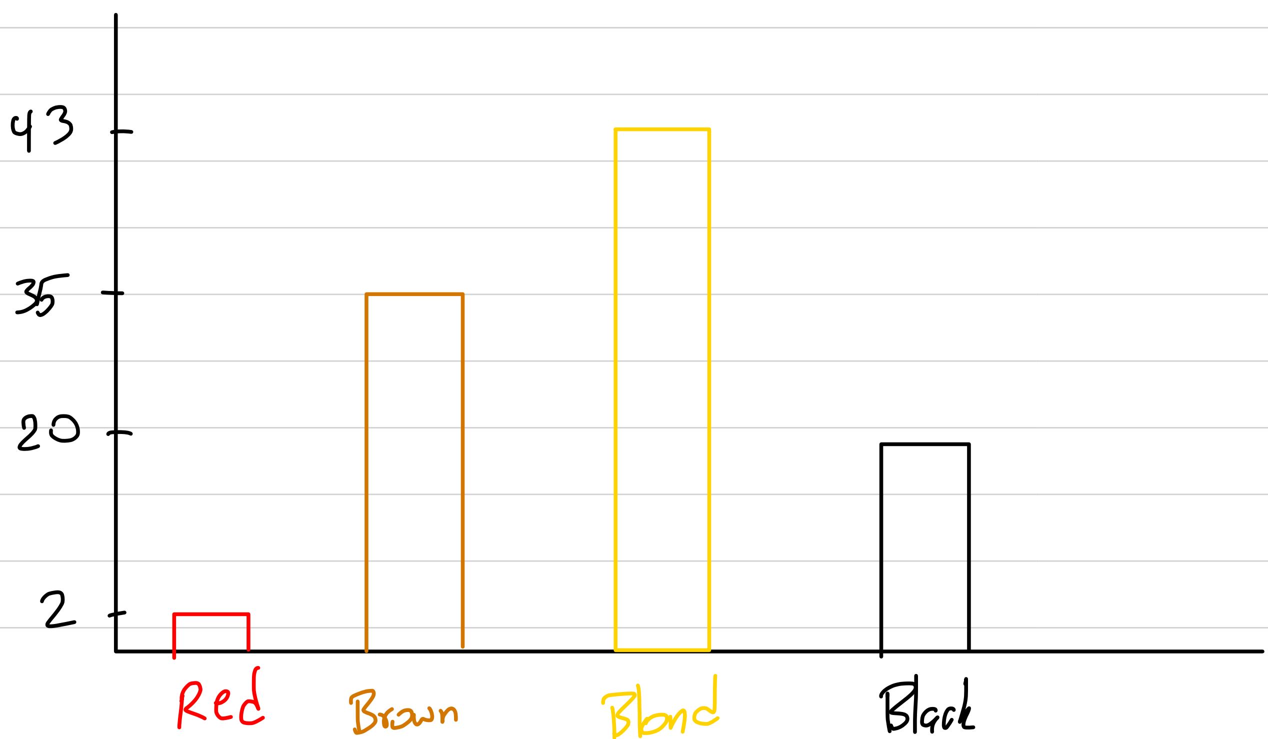
Comment: Only use pie charts when the values the variable can take are mutually exclusive — i.e. every individual has at most one value. Hair color is mutually exclusive since you can have at most one. A survey asking which types of soda you'd had in the past month would not be mutually exclusive since you could have had more than one type.

	% pop
Sprite	30 %
Dr. Pepper	25 %

this doesn't reflect the people who have had both

Hair color	% of students w/ this color
Red	2 %
Brown	35 %
Blond	43 %
Black	20 %

Bar graph:



Ex

Music source	% of 12-24 year olds who have used it
Radio	72
YouTube	77
iTunes	47

Don't use a pie chart, because the different music sources aren't mutually exclusive!

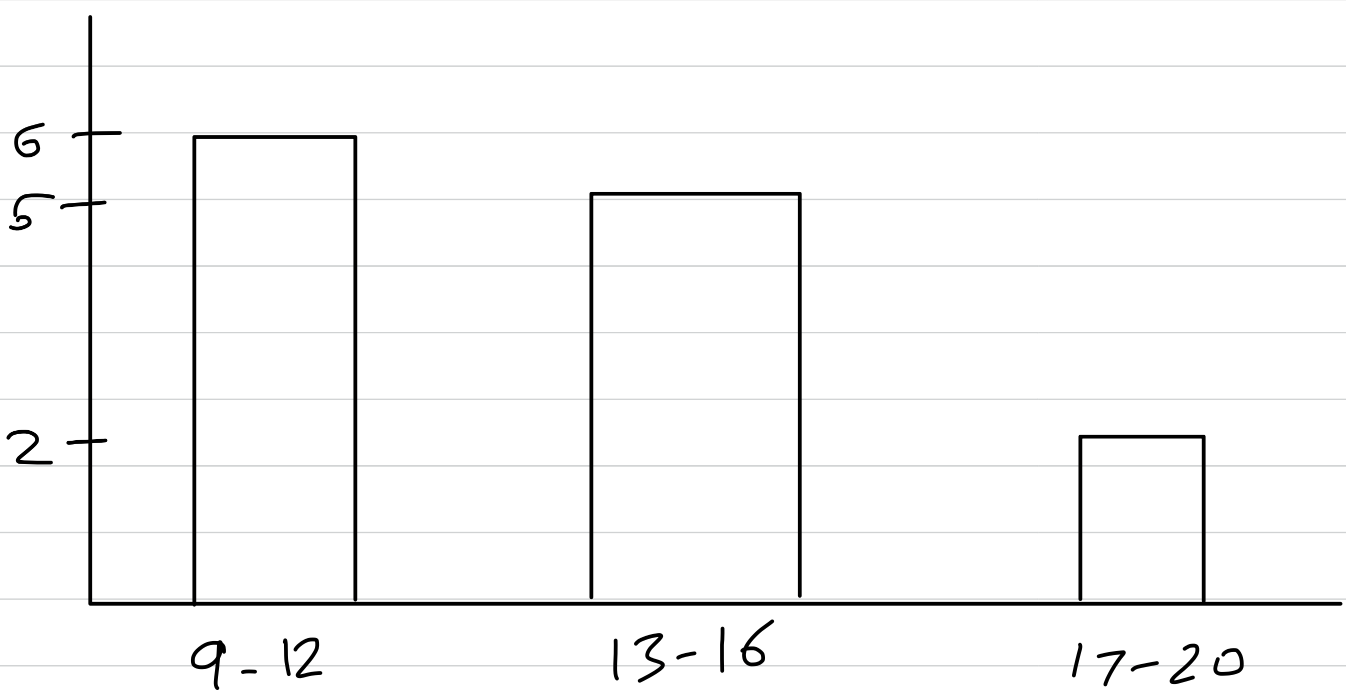
Histograms: when given a sample of individuals, you can make a histogram by dividing the data into ranges (called classes) and counting the number of individuals in each class. Then we make a bar graph of the result. This roughly approximates the distribution

Ex: We get a set of ages:

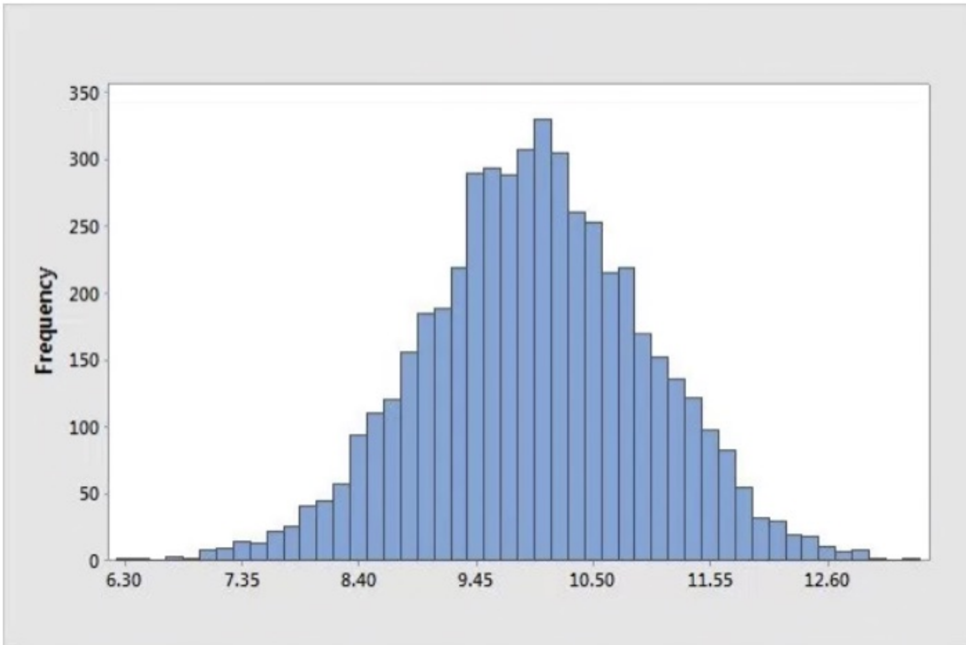
9 10 10 11 12 12 14 15 15

16 16 18 20

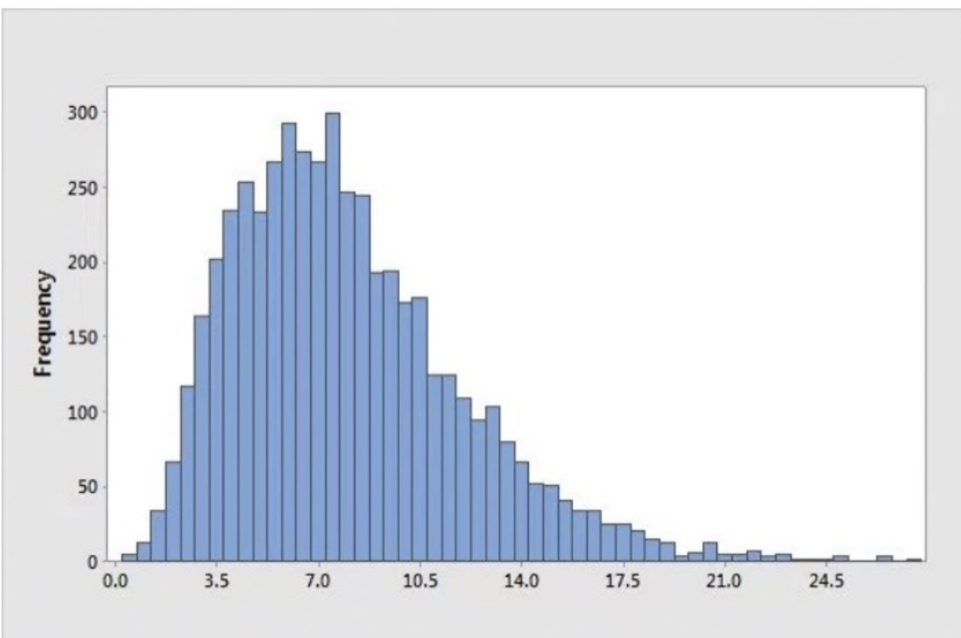
Classes: $\underbrace{9-12}_{6}$, $\underbrace{13-16}_{5}$, $\underbrace{17-20}_{2}$



A **symmetric** distribution.
Ex: Heights of young women, Lengths of bird bills

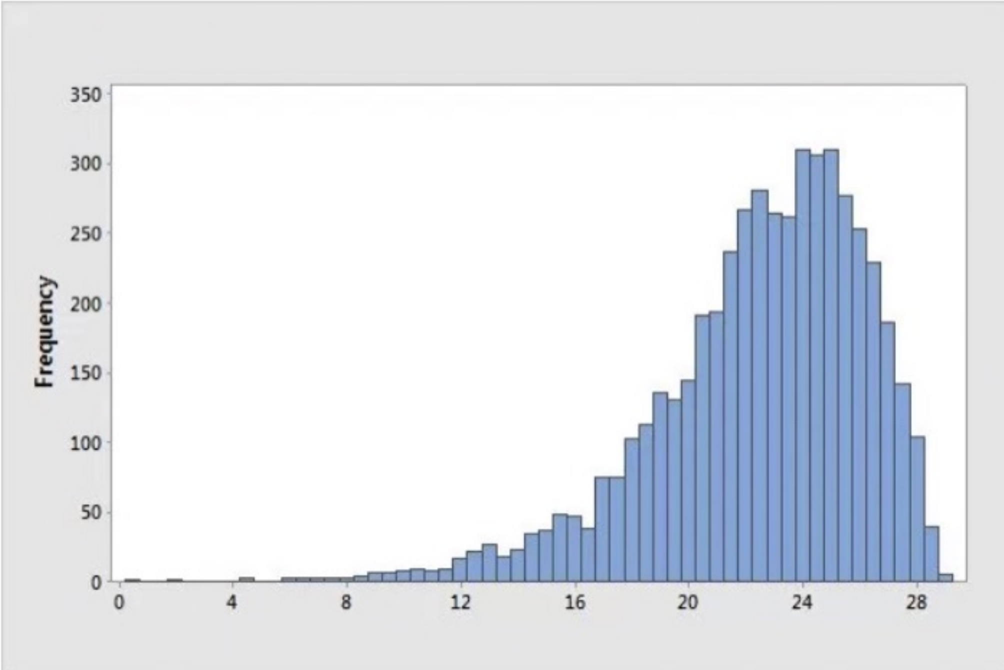


Right-skewed

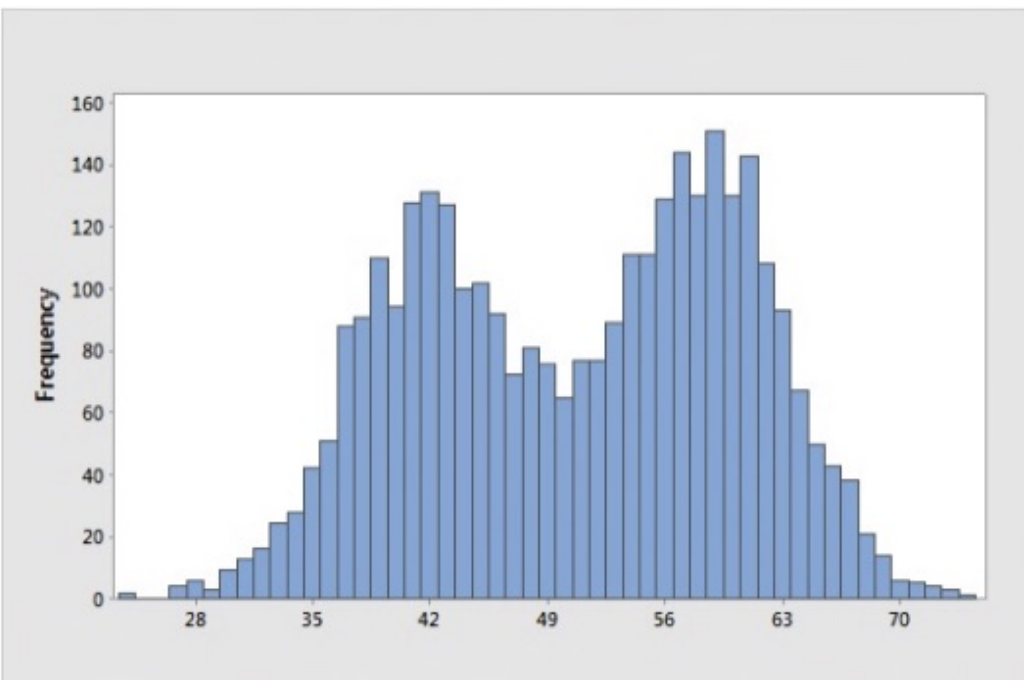


Ex: incomes

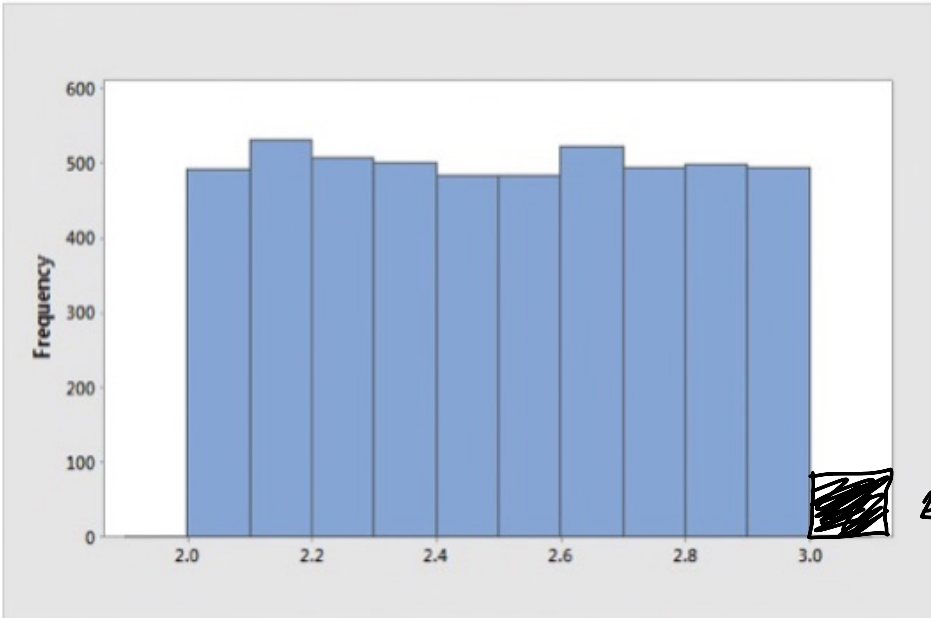
A **left-skewed** distribution.
Ex: Grades on an easy test



A **bimodal** distribution.
Ex: Exam scores when one group studied and another didn't



An **approximately uniform** distribution.
Ex: Rolling a die



outlier

Def: The center of the distribution is the mean or median. The variability is roughly how spread out the distribution is. Outliers are individuals who don't fit the pattern.

Def: Given a $\overbrace{\text{set of data}}^{\text{quantative}}$, we can form a <u>stem-and-leaf</u> plot: take all of the numbers and split them into the last digit and all the other digits. Then write the second piece (i.e. the prefix) and all the final digits with that prefix.

Ex: 9 10 10 11 12 12 14 15 15 16 16 18 20

0	9
1	0 0 1 2 2 4 5 5 6 6 8
2	0

Ex: 5, 13, 18, 32, 91   40, 45, 19, 60

0	5
1	3 8
3	2
9	1

correct

Webwork + Textbook:

	0	5
9	1	38
	2	
	3	2
05	4	
	5	
0	6	
	7	
	8	
	9	1

Comment: we can also split the stems:

0	9
1	0 0 1 2 2 4 5 5 6 6 8
2	0

||

0	9
0	
1	0 0 1 2 2
1	4 5 5 6 6 8
2	0
2	

Chapter 2: Describing Distributions with Numbers

Ex: A list of travel times to work in North Carolina:

30, 20, 10, 40, 25, 20, 10, 60,

15, 40, 5, 30, 12, 10, 10

How to calculate center? One way is taking the average.

Def Given a set of data $x_1, \cdots, x_n$, the mean of the data is

$$\bar{x} = \frac{x_1 + \cdots + x_n}{n}.$$

Ex: $$\bar{x} = \frac{30 + 20 + 10 + \cdots + 12 + 10 + 10}{15} = 22.5$$

15 samples

Ex: 5, 10, 15, 200 ← Right-skewed

$$\bar{x} = \frac{5+10+15+200}{4} = \frac{230}{4} = 57.5$$

Comment: In a skewed distribution, the mean is drawn toward the skew (i.e. the tail). We say the mean is not a resistant measure of center.

Def: Let $x_1, \dots, x_n$ be a set of data. The median is M, defined by:

① if n is odd, then M is the data point such that as many x_i are greater than M as are less than M

② if n is even, M is the average of the two numbers with

as many x_i greater than them
as there are x_i less than them

Ex: 30, 20, 10, 40, 25, 20, 10, 60, 15, 40, 5, 30, 12, 10, 10

First arrange from smallest to largest

$\underbrace{5, 10, 10, 10, 10, 12, 15,}_{7}\ \boxed{20,}\ \underbrace{20, 25, 30, 30, 40, 40, 60}_{7}$

median

15 data points, which is odd, so we want the number "in the middle"

Ex: 5, $\boxed{10, 15}$, 200

average is $\frac{10+15}{2} = 12.5$

median : 12.5

Comment: The median *is* a resistant measure of center.

Ex: you roll a die. If you roll a 1-5, you get nothing. If you roll a 6, you get \$100. What should you expect to get on average from rolling 6 times?

0 0 0 0 0 100

median : 0

mean : $\frac{100}{6}$ ← this is better for our purposes!

How do we measure variability?

Start small: min and max

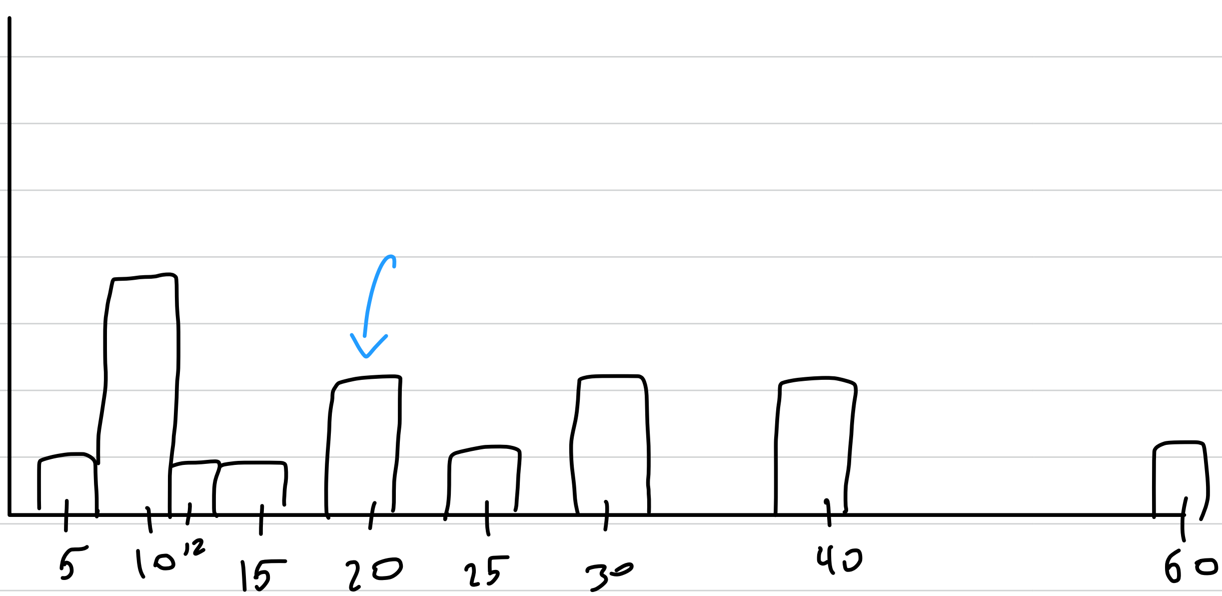
Ex. $\boxed{5}$, 10, 10, 10, 10, 12, 15, $\boxed{20}$, 20, 25, 30, 30,
40, 40, $\boxed{60}$

5, 60

Better: min, median, max

5, 20, 60

gap indicates that
this is a right-skewed distribution



Def: The first and third quartiles, Q_1 and Q_3, are the medians of the two halves of the data, not including the median of the whole data.

$$\boxed{5},\ 10,\ 10,\ \boxed{10},\ 10,\ 12,\ 15,\ \boxed{20},\ 20,\ 25,\ 30,\ \boxed{30},\ 40,\ 40,\ \boxed{60}$$

(you could say that $Q_2 = 20$)

$Q_1 = 10$

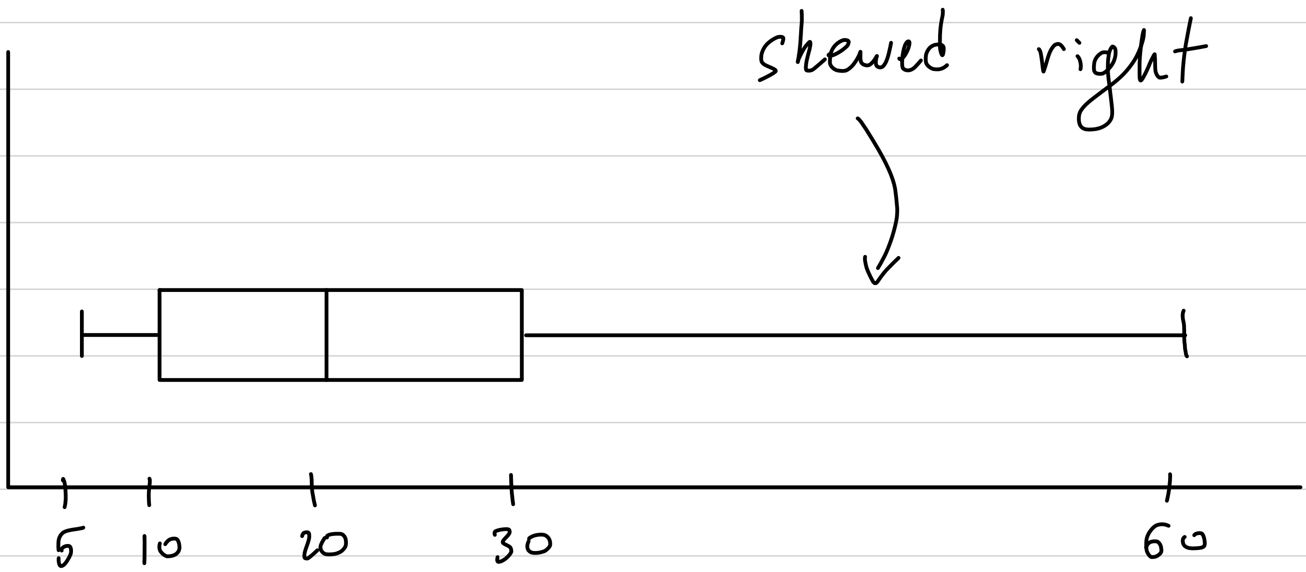
$Q_3 = 30$

Def: The 5-number summary of a set of data is min, Q_1, median, Q_3, max

Ex: 5, 10, 20, 30, 60

All 4 gaps have the same # of data points.

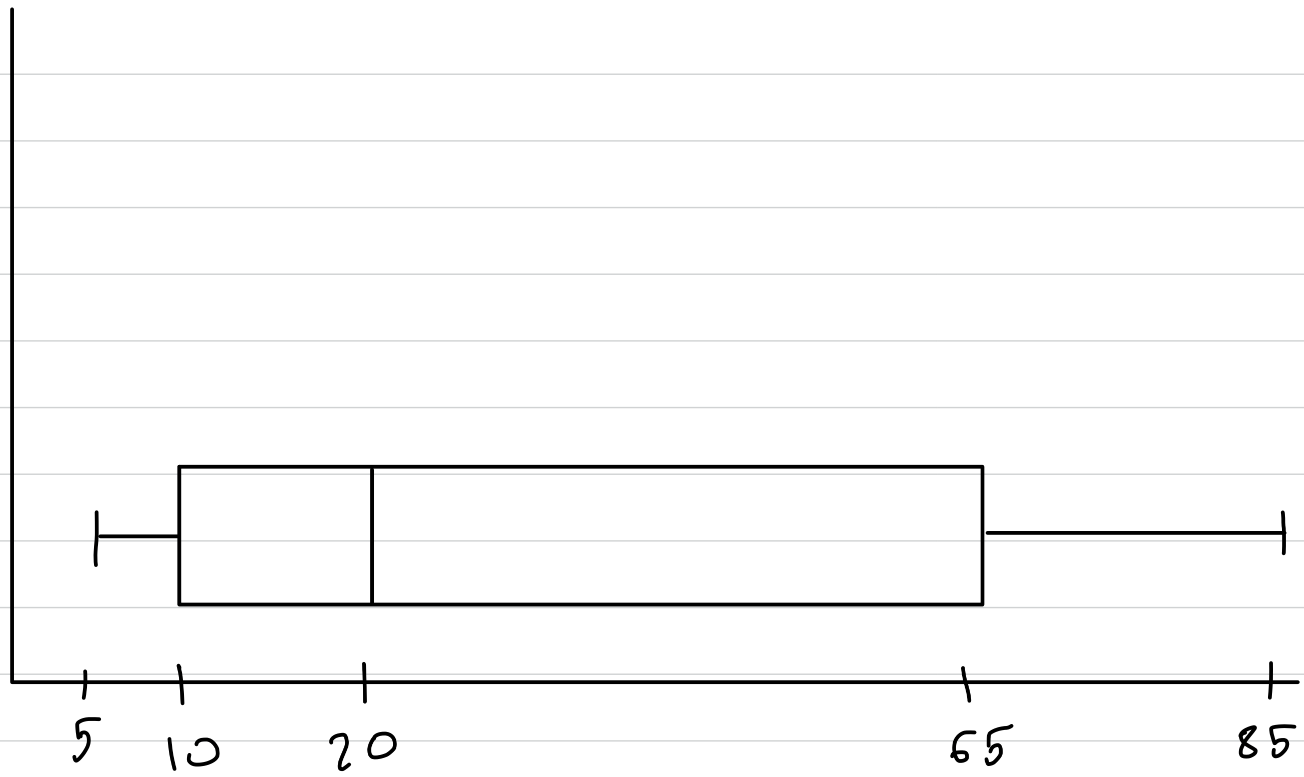
Box plots:



Ex: Draw a box plot of

10, 30, 5, 85, 65, 20, 10.

5, 10, 10, 20, 30, 65, 85



Def: The interquartile range, or IQR, is given by $IQR = Q_3 - Q_1$

Def: An outlier in a data set is any point more than 1.5 IQR above Q_3 or below Q_1.

<u>Ex</u>: 10, 30, 5, 1000, 65, 20, 10.

5-num: 5, 10, 20, 65, 1000

$Q_1 = 10$
$Q_3 = 65$

$IQR = 65 - 10 = 55$

$1.5\,IQR = 82.5$

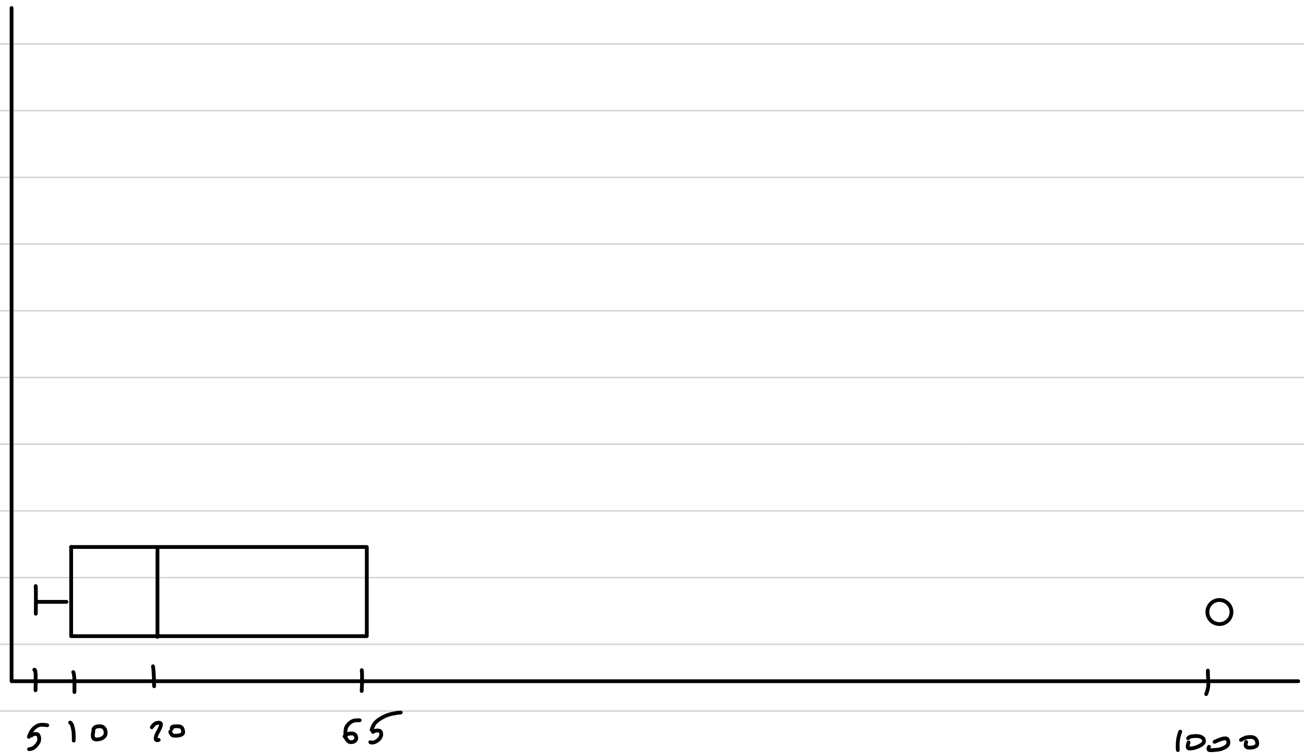
$Q_3 + 82.5 = 147.25$

$Q_1 - 82.5 = -72.5$

outliers are anything <u>not</u> between -72.5 and 147.25. So 1000 is an outlier.

Represent outliers by modifying the

box-plot: make the whiskers only reach the non-outliers.



The 5-num summary is a resistant measure of variability (but it's a little lacking)

How do we get a nonresistant measure of variability? Naive approach: take average distance to the mean

$x_1, \dots, x_n$ mean: $\bar{x}$

$$\frac{(x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})}{n}$$

This actually always is zero!

$$= \frac{x_1 + x_2 + \cdots + x_n - n\bar{x}}{n}$$

$$= \underbrace{\frac{x_1 + x_2 + \cdots + x_n}{n}}_{\bar{x}} - \underbrace{\frac{n\bar{x}}{n}}_{\bar{x}}$$

$$= 0$$

We can fix this issue by making the

distance to the mean always be positive:

Def: Let $x_1, \cdots, x_n$ be data with mean $\bar{x}$. The <u>variance</u> is

$$s^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{\underbrace{n-1}}$$

Bessel's correction

(eliminates bias)

don't worry about this yet

Def: The <u>standard deviation</u> is s.

Ex: SAT math scores at Georgia

southern high school

490 580 450 570 650

x_1 x_2 x_3 x_4 x_5

$$\bar{x} = \frac{490+580+450+570+650}{5} = 548$$

$$s^2 = \frac{(490-548)^2 + (580-548)^2 + (450-548)^2 + (570-548)^2 + (650-548)^2}{5-1}$$

$= 6220$

$s = 78.87$ ← standard deviation

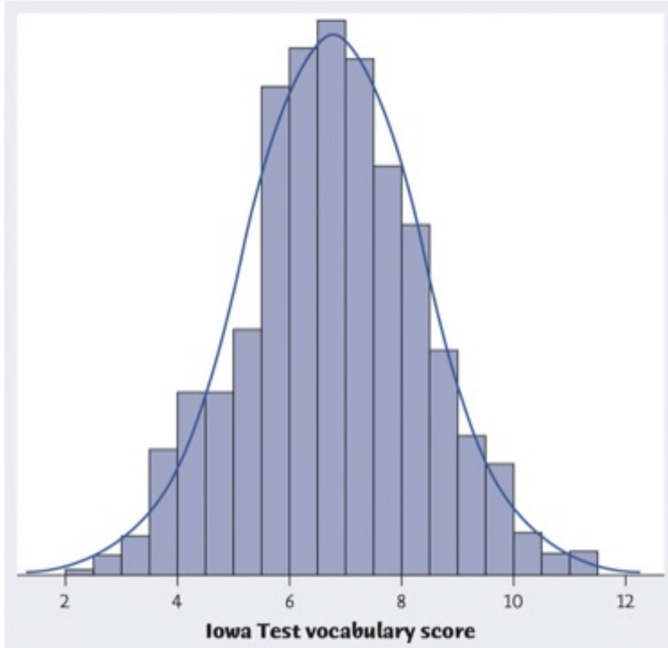
think of as: the average distance to mean among this data is 78.87

s is a good measure of variability, but only when $\bar{x}$ is a good measure of center.

Note: $\bar{x}$ and s do <u>not</u> give a complete description of data

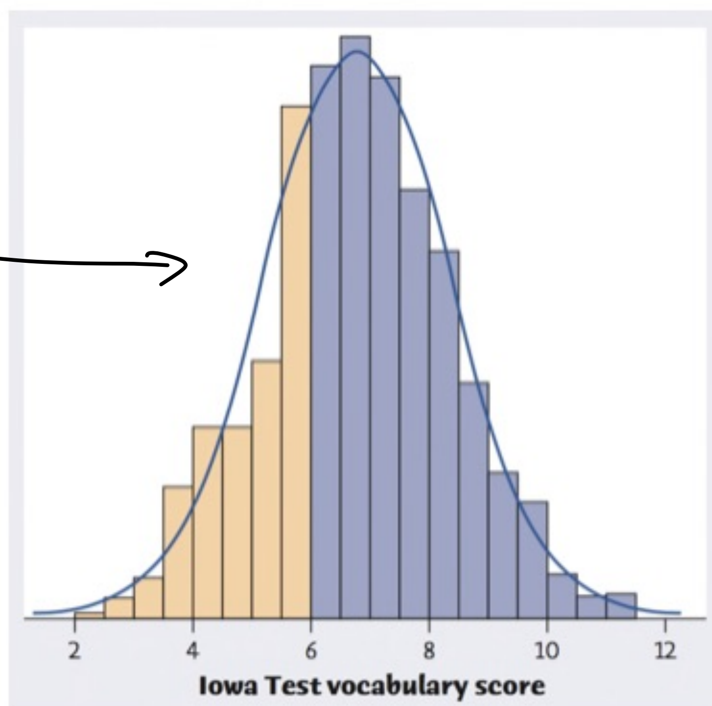
Chapter 3: Normal Distributions

<u>Ex</u> 947 students in Gary, IN took the Iowa test. Here is a histogram of their scores. The histogram is roughly symmetric, has no large gaps or outliers



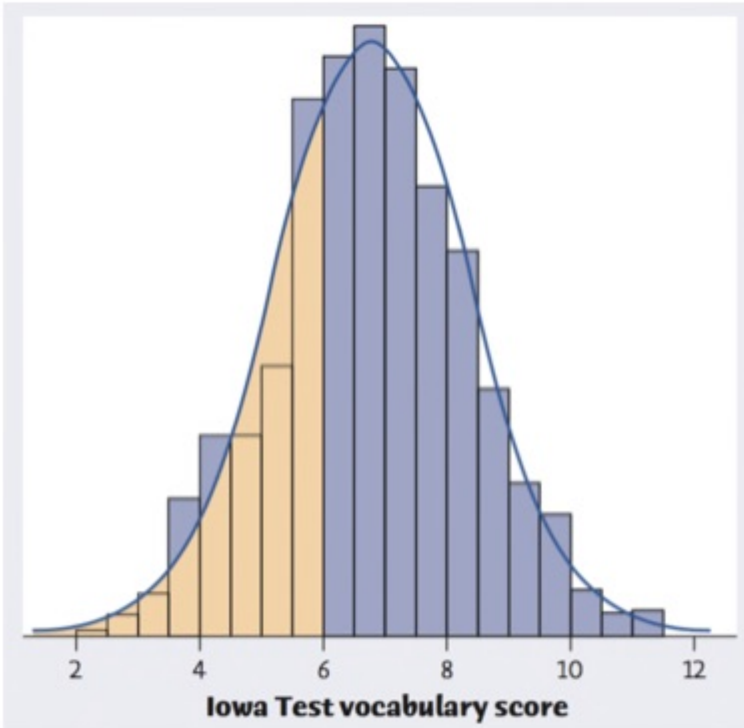
what is the proportion of students who scored 6 or lower?

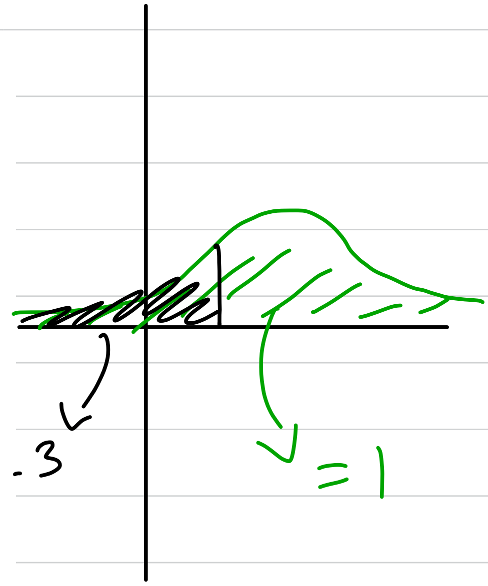
curve: approximate distribution



bars: observation

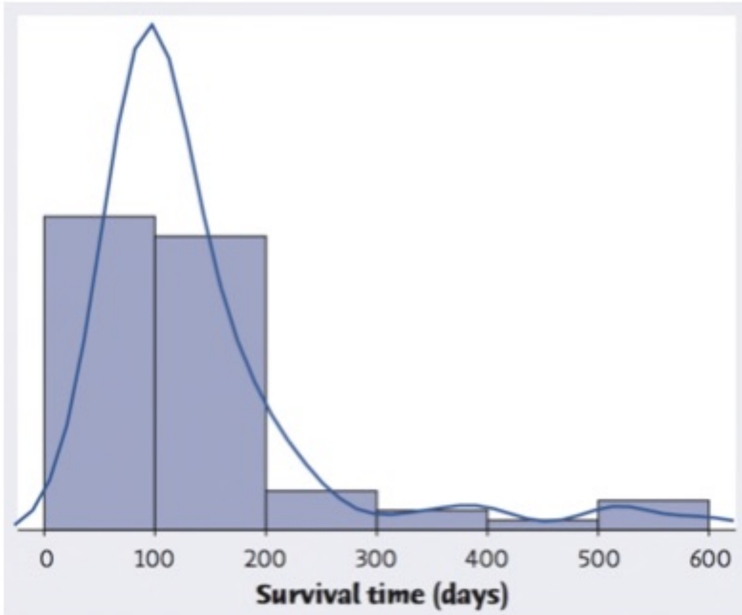
We want to find this proportion via the bell curve and not the histogram. Want the area under the curve less than 6





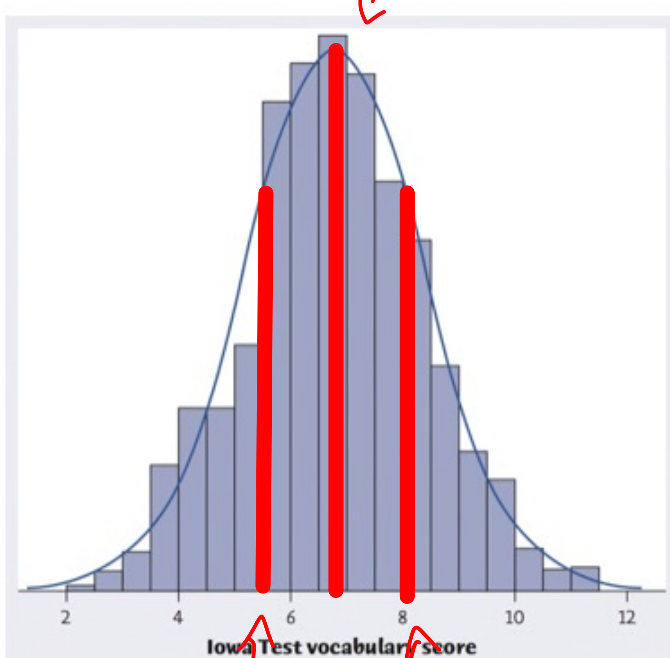
If we rescale the bell curve to have area 1, then the orange area will already be the proportion we want.

This defines something called a density curve: it's positive and has area 1.
They come in many shapes: here's one that approximates a skewed dist:



Median + quartiles of density curves: just split the area into quarters.

median



no max b/c there is an asymptote at 0

Q_1 Q_3

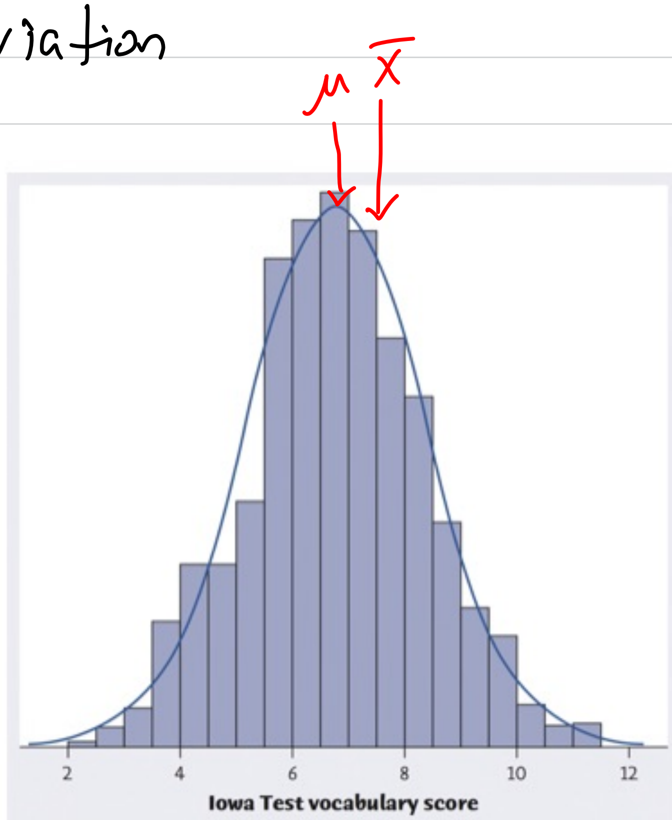
Think of the mean as a weighted average:

It's the "balance point" of the density curve

For symmetric distributions, mean = median

Notational convention:

Observation (sample)	Distribution (population)
$\bar{x}$: mean	μ (mu) : mean
s : standard deviation	σ (sigma) : std dev



It turns out that normal distributions are completely determined by μ and σ.

Def: A normal distribution is one whose density curve is symmetric, single-peaked, and bell-shaped.

Eyeball σ: it's where the curve changes concavity: imagine skiing down the curve. The point when the slope stops getting steeper is the inflection point, and it's where σ is.