Name: _____ Homework 8 | Math 256 | Cruz Godar

Due Wednesday of Week 9 at the start of class

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

Section 10

In problems 1–5, find the determinant of the given matrix.

1. $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$.

2. $\mathbf{B} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$.

3. $\mathbf{C} = \begin{bmatrix} 19 & -4 & 8 \\ -8 & 5 & -10 \\ -1 & -2 & 4 \end{bmatrix}$.

4. $\mathbf{D} = \begin{bmatrix} 2 & 2 & -2 \\ -3 & 7 & 3 \\ -5 & 5 & 5 \end{bmatrix}$.

5. $\mathbf{E} = \begin{bmatrix} 5 & 6 & 4 & -4 \\ 3 & 8 & -2 & 2 \\ 3 & -3 & 9 & 2 \\ 0 & 0 & 0 & 11 \end{bmatrix}$.

In problems 6–10, find the eigenvalues and eigenvectors of the given matrix, and verify that the product of the eigenvalues is the determinant you found before.

6. $\mathbf{A}$ from problem 1.

7. $\mathbf{B}$ from problem 2.

8. $\mathbf{C}$ from problem 3.

9. $\mathbf{D}$ from problem 4.

10. $\mathbf{E}$ from problem 5. Hint: you may it useful to know that $(x-11)^3 = x^3 - 33x^2 + 363x - 1331$.

11. With $\mathbf{A}$ from problem 1, find the following:

a) $\mathbf{A}^{100} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

b) $\mathbf{A}^{100} \begin{bmatrix} -2 \\ -6 \end{bmatrix}$.

c) $\mathbf{A}^{100} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$.

12. A remarkable property of the determinant is that it's **multiplicative**: for any two $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$, $\det(\mathbf{AB}) = (\det \mathbf{A})(\det \mathbf{B})$. Use this property to answer the following questions.

a) What is $\det \mathbf{I}$? Hint: $\mathbf{IA} = \mathbf{A}$ for any matrix $\mathbf{A}$ for which the product makes sense.

b) Let $\mathbf{A}$ be an invertible matrix. What is $\det\left(\mathbf{A}^{-1}\right)$ in terms of $\det \mathbf{A}$?

13. Suppose $\mathbf{A}$ is an invertible matrix with eigenvectors $\mathbf{v_1}, ..., \mathbf{v_n}$ with corresponding eigenvalues $\lambda_1, ..., \lambda_n$.

a) For $\mathbf{A}$ to be invertible, none of the λ_i can be zero. Why is this?

b) What are the eigenvectors and eigenvalues of $\mathbf{A}^{-1}$? Hint: start with $\mathbf{Av_i} = \lambda_i \mathbf{v_i}$ and find a way to introduce $\mathbf{A}^{-1}$ into the equation.