Final Exam

Math 112

Spring 2020

You have 2 hours to complete this exam **(this includes the 10 minutes accounting for the time it takes to scan and upload it)**. You may use a scientific calculator, but no other resources. When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

1. (32 points) Consider a circle of radius 2 centered at the origin.

a) (8 points) What are the coordinates of a point on this circle with angle θ counter-clockwise from the positive x-axis?

b) (8 points) Find the coordinates of a point with angle $150°$ counter-clockwise from the positive x-axis. Simplify your answer and leave it in exact form, and show your work — specifically, any reference angles you use.

c) If the circle were centered at $(1, -1)$, what would the coordinates of this point be?

d) (8 points) This point and the point on the circle with angle $0°$ are the endpoints of an arc of the circle. Sketch this arc and find its length.

2. A 2-dimensional vector $\vec{v}$ has magnitude 2 and angle $\frac{7\pi}{6}$ from the positive x-axis.

a) (8 points) Find the unit vector decomposition for $\vec{v}$, showing all your work.

b) (8 points) Vector $\vec{w}$ has unit vector decomposition $\vec{w} = 2\vec{i} - 3\vec{j}$. Find $||\vec{w}||$ and the angle $\vec{w}$ makes with the positive x-axis.

c) Find $\vec{v} \bullet \vec{w}$.

d) (8 points) What is the angle between $\vec{v}$ and $\vec{w}$?

3. (32 points) Let $f(x)$ be a sinusoidal function with amplitude 2, midline 1, and period 3.

a) (8 points) Assuming there is no horizontal shift, find a formula for f.

b) (8 points) If $f(1) = 0$ and f is increasing there, find a formula for f.

c) With the value of h you found in part b), find all values of x that make $f(x) = 0$.

d) Sketch a graph of f, labeling at least three points.

4. (32 points) Let $f(x) = x^2$.

a) (8 points) Sketch a graph of f, labeling three different points.

b) (8 points) Let $g(x) = 2x^2 + 1$. g is a transformation of f — list the transformation(s) you'd need to apply to f to get g, and then sketch a graph of g, labeling the points that correspond to the ones from part a).

c) (8 points) Let $h(x) = 2(4(x + 3))^2 + 1$. h is a transformation of g — again, list the transformation(s), then sketch a graph of h, labeling the points that correspond to the ones from part a).

d) (8 points) If we want to apply a vertical stretch to h to make a new function k such that $k(1) = 1$, what must $k(x)$ be?

5. (32 points) Miscellaneous, multiple-choice questions. For each, circle the correct answer. You don't need to show your work.

a) (8 points) Suppose θ is an angle in quadrant II. Which of the following is true?

$\arcsin(\sin(\theta)) = \theta$ $\arccos(\cos(\theta)) = \theta$ $\arctan(\tan(\theta)) = \theta$

b) (8 points) In which scenario can we apply the Law of Sines?

i. We know two sides and an angle and want to find a third side.
ii. We know a side and two angles and want to find a second side.
iii. We know all three angles and want to find a side.
iv. We know all three sides and want to find an angle.

c) (8 points) Let $\vec{v}$ be a 2-dimensional vector. Which of the following can $\vec{v}$ **not** have?

i. Negative angle
ii. Negative magnitude
iii. Negative $\vec{i}$ component
iv. Negative $\vec{j}$ component

d) (8 points) If the dot product of two vectors is negative, then the angle between the two vectors must be

i. Acute
ii. Right
iii. Obtuse