Name: [YOUR NAME HERE] Homework 1 | Math 253 | Cruz Godar

*Due Wednesday of Week 2 at the start of class*

Complete the following problems and submit them as a pdf to Canvas. 8 points are awarded for thoroughly attempting every problem, and I'll select three problems to grade on correctness for 4 points each. Enough work should be shown that there is no question about the mathematical process used to obtain your answers.

In problems 1–6, write the first five terms of the sequence and find an explicit formula for the $n$th term of the sequence if it is not already given.

1. $a_n = \left(\frac{1}{2}\right)^n$.

2. $b_n = \cos\left(\frac{\pi}{2}n\right)$.

3. $c_1 = 1$ and $c_n = nc_{n-1}$ for $n \geq 2$.

4. $d_1 = \frac{1}{5}$, $d_2 = \frac{1}{5}$, and $d_n = d_{n-1}d_{n-2}$ for $n \geq 3$. (You may state your explicit formula in terms of another sequence).

5. $e_1 = 6$, $e_2 = 2$, and $(e_n)$ is an arithmetic sequence.

6. $f_1 = 6$, $f_2 = 2$, and $(f_n)$ is a geometric sequence.

7. For each of the sequences in problems 1–6, find the limit if it exists. If it does exist, find a positive integer $N$ so that all terms of the sequence with index at least $N$ are within $\varepsilon = 0.1$ of the limit.

In problems 8–12, determine if the sequence converges and find the limit of the sequence if it does. Justify your answers.

8. $a_n = \frac{3n^3 - 2n^2}{n^3 + 1}$.

9. $b_n = \frac{(-1)^n}{\sqrt{n}}$.

10. $c_n = \tan(n)$.

11. $d_n = \frac{2^n}{n!}$.

12. $e_n = \dfrac{n^n}{n!}$.

13. Give an example of a sequence that is monotone increasing that does not converge, and a sequence that is bounded below but does not converge.

14. If a sequence is not bounded above, can it converge? Explain.

15. If a sequence $a_n$ has infinitely many positive terms *and* infinitely many negative terms, can it still converge? If so, are there restrictions on what it can converge to?

16. Suppose $(a_n)$ is a sequence of rational numbers with $(a_n) \to a$. Is $a$ necessarily a rational number?

17. Let $a_n$ be a sequence and let $b_n = |a_{n+1} - a_n|$. If $(b_n) \to 0$, does $(a_n)$ have to converge?