Final Exam

Math 112

Winter 2021

You have 2 hours to complete this exam, scan it, and upload it to Canvas. You may use a scientific calculator, but no other resources. When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it. There are 192 points possible on the exam, and 3 points will be deducted for each minute late.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

1. (64 points) Multiple choice. You don't need to show your work.

a) (8 points) Suppose θ is an angle in quadrant II. Which of the following is true?

  A) $\arcsin(\sin(\theta)) = \theta$.

  B) $\arccos(\cos(\theta)) = \theta$.

  C) $\arctan(\tan(\theta)) = \theta$.

  D) All of the above.

b) (8 points) In which scenario could we apply the Law of Sines?

  A) We know two sides and an angle and want to find a third side.

  B) We know a side and two angles and want to find a second side.

  C) We know all three angles and want to find a side.

  D) We know all three sides and want to find an angle.

c) (8 points) Let $\vec{v}$ be a 2-dimensional vector. Which of the following can $\vec{v}$ **not** have?

  A) Negative magnitude.

  B) Negative angle.

  C) Negative $\vec{i}$ component.

  D) Negative $\vec{j}$ component.

d) (8 points) The definition of the tangent function is

  A) $\tan(\theta) = \dfrac{1}{\sin(\theta)}$.

  B) $\tan(\theta) = \cos(\pi - \theta)$.

  C) $\tan(\theta) = \sin(\theta)^2 + \cos(\theta)^2$

  D) $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$.

e) (8 points) The measure of $\dfrac{13\pi}{12}$ in degrees is

    A) 3.403°.

    B) 195°.

    C) 62°.

    D) .0594°.

f) (8 points) If the dot product of two vectors is zero and both vectors have nonzero magnitude, then the angle θ between the two vectors satisfies

    A) $\theta = 0°$.

    B) $0° < \theta < 90°$.

    C) $\theta = 90°$.

    D) $90° < \theta < 180°$.

g) (8 points) Which of the following is equal to $\sin(\theta)$ for any angle θ?

    A) $2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$.

    B) $\tan(\cos(\theta))$.

    C) $\sin(\sqrt{1-\theta^2})$.

    D) $\arcsin(\sin(\theta))$.

h) (8 points) The function $y = 3(x+1)^5$ is a transformation of $y = x^5$ by

    A) A vertical stretch and horizontal stretch.

    B) A vertical stretch and horizontal shift.

    C) A vertical shift and horizontal stretch.

    D) A vertical shift and horizontal shift.

2. (32 points) Consider a circle of radius 2 centered at the origin.

a) (8 points) What are the coordinates of a point on this circle with angle θ counter-clockwise from the positive x-axis?

b) (8 points) Find the coordinates of a point with angle $150°$ counter-clockwise from the positive x-axis. Simplify your answer and leave it in exact form, and show your work — specifically, any reference angles you use.

c) (8 points) If the circle were centered at $(1, -1)$, what would the coordinates of this point be?

d) (8 points) This point and the point on the circle with angle $0°$ are the endpoints of an arc of the circle. Sketch this arc and find its length. (Hint: the center of the circle no longer matters.)

3. A 2-dimensional vector $\vec{v}$ has magnitude 2 and angle $\dfrac{7\pi}{6}$ counter-clockwise from the positive x-axis.

a) (8 points) Find the unit vector decomposition for $\vec{v}$, showing all your work.

b) (8 points) Vector $\vec{w}$ has unit vector decomposition $\vec{w} = 2\vec{i} - 3\vec{j}$. Find $\|\vec{w}\|$ and the angle $\vec{w}$ makes with the positive x-axis.

c) Find $\vec{v} \bullet \vec{w}$.

d) (8 points) What is the angle between $\vec{v}$ and $\vec{w}$?

4. (32 points) Let $f(x)$ be a sinusoidal function with amplitude 2, midline 1, and period 3.

a) (12 points) Assuming there is no horizontal shift, find a formula for f.

b) (12 points) If $f(1) = 0$ and f is increasing there, find a formula for f.

c) (8 points) Sketch a graph of f, labeling at least three points.

5. (32 points) Let $f(x) = x^2$.

a) (8 points) Sketch a graph of f, labeling at least three points.

b) (8 points) Let $g(x) = 2x^2 + 1$. g is a transformation of f — list the transformation(s) you'd need to apply to f to get g, and then sketch a graph of g, labeling the points that correspond to the ones from part a).

c) (8 points) Let $h(x) = 2(4(x+3))^2 + 1$. h is a transformation of g — again, list the transformation(s) applied to g to get h, then sketch a graph of h, labeling the points that correspond to the ones from part a).

d) (8 points) If we want to apply a vertical stretch to h to make a new function k such that $k(1) = 1$, what must $k(x)$ be?