Midterm 2

Math 112

Winter 2021

You have 50 minutes to complete this exam (plus 10 minutes to account for the time it takes to scan and upload it). **You may use a scientific calculator, but no other resources.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

1. (32 points) Multiple choice. You don't need to show your work.

a) (8 points) The slope of a line that passes through the origin and a point on the unit circle with angle θ is

A) $\sin(\theta)$.

B) $\tan(\theta)$.

C) $\sec(\theta)$.

D) $\cos(\theta)$.

b) (8 points) The angle measure of $\frac{\pi}{5}$ in degrees is

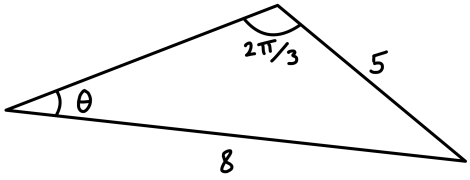
A) $36°$.

B) $50°$.

C) $72°$.

D) $100°$.

c) (8 points) To find θ in the following triangle in **one step**, we must use



A) The Pythagorean theorem.

B) The tangent function.

C) The Law of Cosines.

D) The Law of Sines.

d) (8 points) The function $f(x) = 3\sin(2(x+2))$ has

A) Amplitude 3, period 2, and midline 2.

B) Amplitude 3, period π, and midline 0.

C) Amplitude 3, period π, and midline 2.

D) Amplitude 3, period 2, and midline 0.

2. (32 points) Miscellaneous questions. These don't make sense as full-length problems, so the four parts here are unrelated to one another.

a) (8 points) Find an exact value for $\tan\left(\frac{2\pi}{3}\right)$. Show all your work — if you use a reference angle, you must draw a picture.

b) (8 points) Find the exact value of $\sin(105°)$. Show all your work.

c) Write the equation of a sinusoidal function $f(x)$ with amplitude 2, midline $-\sqrt{3}$, and period 2π, such that $f(0) = 0$.

d) Find all solutions to the equation $\cos(\theta) = \frac{\sqrt{3}}{2}$. Show all your work.

3. (32 points) Let $g(\theta) = 1 - \tan(3\theta)$.

a) (12 points) Find $g\left(\frac{\pi}{4}\right)$. Leave your answer in exact form, and show all your work — specifically, how you calculate the tangent.

b) (8 points) Sketch a graph of g. Label at least three points.

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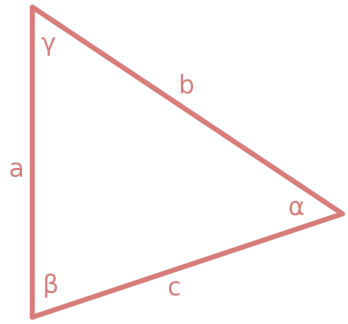
c) (12 points) For what values of θ is $g(\theta) = 0$? List all of the values, and express your answers in radians (in exact value).

4. (32 points) Consider the following triangle with sides a, b, and c, and angles α, β, and γ.



a) (12 points) Given that $a = 4$, $b = 5$, and $\gamma = 60°$, find c. Leave your answer in exact form.

b) (12 points) Use your answer to part a) to find α.

c) (8 points) Now find β.

d) (Extra credit — 8 points) Find the area of this triangle. (Hint: pick one side to be the base, then draw a line perpendicular to that base that reaches to the opposite vertex to split the triangle into two right triangles. Then use trig functions to find the length of that line.)