Midterm 2 Practice

Math 111

Exercise 1: The function $P(r) = \dfrac{100r^2 - 50r - 50}{r^2 + 100}$, is an (inaccurate) model for the percent of customers who will prefer Starbucks to all other coffee shops when Starbucks spends r million dollars on marketing.

a) Find and interpret $P(5)$.

$P(5) = 17.6$, so if Starbucks spends 5 million on advertising, then 17.6% of people will see Starbucks as their favorite coffee shop.

b) What is the mathematical domain of P?

We can't have $r^2 + 10 = 0$, so $r^2 \neq -10$. No real r squares to be -10, so all rs work. Thus the domain is $(-\infty, \infty)$.

c) What is the practical domain? Note that $P(r)$ is not always positive.

We certainly can't spend a negative amount of money, so $r \geq 0$. Also, we need $P(r) \geq 0$, so either both the numberator and denominator are positive or they're both negative. The denominator is always positive, since $r^2 \geq 0$ for all r, and so we need the numerator to be positive too. If $100r^2 - 50r - 50 = 0$, then $r = -\frac{1}{2}$ or $r = 1$. This tells us that the graph switches sign at $r = 1$, so we need $r \geq 1$. Thus the practical domain is $[1, \infty)$.

d) What is the behavior of P as $r \longrightarrow \infty$? As $r \longrightarrow -\infty$? Interpret each or explain why it's not meaningful.

As $r \longrightarrow \infty$, $P(r) \longrightarrow 100$, so with enough ads, everyone will think Starbucks is the best. The behavior as $r \longrightarrow -\infty$ isn't meaningful, since we can't spend a negative amount of money.

Exercise 2: A heater requires $H(t) = (90 - t)^2$ Watts of power to keep my office at a constant 90 degrees when the outside temperature is t degrees. Then I open the window, and the room starts to cool down. Its temperature m minutes after I open the window is $T(m) = 90 - \sqrt{2m}$.

a) How can we combine H and T to create a function that gives the power the heater uses, m minutes after

the window is opened? Find and simplify a formula.

$(H \circ T)(m) = 2m$.

b) What is the mathematical and practical domain of the function from part a)?

The mathematical domain looks like it's just $(-\infty, \infty)$, but we actually need to take the overlap of that with the inner function, T. Its domain is $[0, \infty)$, and so the entire function's domain is $[0, \infty)$. For the practical domain, it only makes sense to plug in a positive amount of time, to we also have $[0, \infty)$.

c) H is *almost* the inverse function to T. What should the actual inverse be?

If H were actually the inverse, then $(H \circ T)(m) = m$. In order to account for that factor of 2, we can just put a factor of $\frac{1}{2}$ on the outside of H: $T^{-1}(t) = \frac{1}{2}(90 - t)^2$.