Midterm 1

Math 112

Spring 2020

You have 50 minutes to complete this exam (plus 10 minutes to account for the time it takes to scan and upload it). **You may not use a calculator of any kind.** When you're finished, first check your work if there is time remaining, then scan the exam and upload it to Canvas. If you have a question, don't hesitate to ask — I just may not be able to answer it.

1. (32 points) Let $f(x) = \dfrac{1}{x^2}$.

a) (8 points) Is f an even function? Is it an odd one? Explain your answer.

b) (8 points) Sketch a graph of f, labeling at least two points.

c) (8 points) Sketch a graph of $\dfrac{2}{(x+1)^2}$. To get credit, you must list each transformation you apply and sketch a graph after each one, as in class. Continue to label the points you labeled in part b).

d) (8 points) Apply a vertical shift of 3 units upward to the function in part c). Write the new equation of the function and sketch a graph. Again, continue to label the points you've been labeling for the past two parts.

2. (32 points) You go for a walk in a flat field. You first walk 2 miles east, then turn and walk another 1 mile north.

a) (8 points) You now walk home in a straight line. How many miles will you walk before you get home? Leave your answer in exact form.

b) (8 points) You left your house going due east and came home at an angle. What is the cosine of the angle between due east and the line you came home on? (Hint: you don't need to find the angle itself.)

c) (8 points) You want that final walk home to be 4 miles. If you still start by walking 2 miles east, but now walk x miles north, then walk home in a straight line, what must x be?

d) (8 points) Suppose you want the final walk home to be d miles, for some $d \geq 2$. Write a function $f(d)$ that gives the value of x needed in order to have a d mile walk home. (Hint: when you have an answer, plug in $d = 4$ and make sure your value of x agrees with what you found in part c)).

3. (32 points) Consider a circle of radius 2 centered at the origin.

a) (8 points) What are the coordinates of a point on this circle with angle θ (counter-clockwise) from the positive x-axis?

b) (8 points) A point on this circle has angle $150°$ from the positive x-axis. Find its coordinates, leaving them in exact form. You must explain how you calculate any trig functions to receive credit.

c) (8 points) If the circle were centered at $(-1, -2)$, what would the coordinates of this point be?

d) (8 points) The line $y = -1$ intersects the circle in part c) at two points. Find them, leaving your answer in exact form. (Hint: recall that the equation of this circle is $(x+1)^2 + (y+2)^2 = 4$.)

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