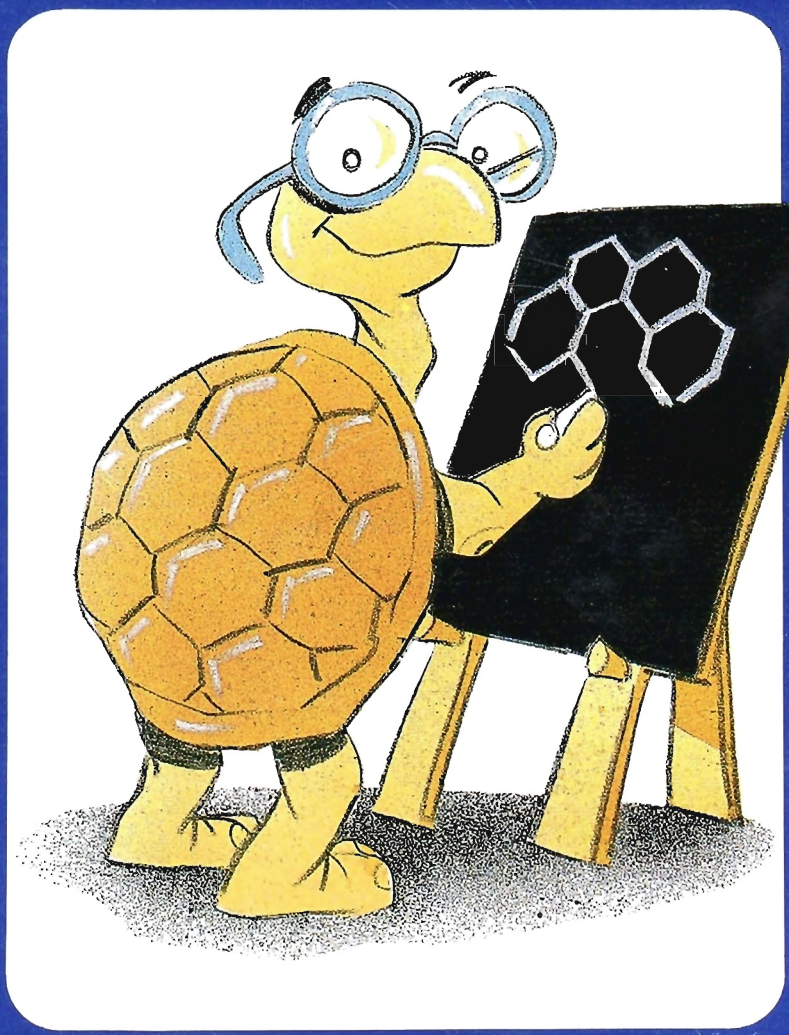
turtle confusion

Logo Puzzles and Riddles

BARRY NEWELL



Curriculum Development Centre
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Australia

TURTLE CONFUSION

LOGO PUZZLES AND RIDDLES

Barry Newell

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FOREWORD

This delightful publication is the outcome of some inspired thinking and dedicated work by Dr Barry Newell. Because it offers challenging and engaging problems - such as the way to solve the puzzles, the meaning of riddles, the buried clues that help to confirm "theories" and the underlying and embedded mathematics - it will require a little patience and plenty of time for discussion, sharing and thinking. Students, parents and teachers can enjoy the excitement of working together and solving problems in much the same way as we do in everyday life.

Those who read like to read more about the ideas behind this book should read *Turtles Speak Mathematics* also published by the Curriculum Development Centre.

Brent Corish
Director

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PROLOGUE

During the last five years the Turtle and I have become firm friends. Of course, we have had our differences of opinion; he with his commitment to research in mathematics and science and me with a teacher's view of the same subjects. But, on the whole, we have learnt a great deal from each other and have enjoyed ourselves in the process.

Some little time ago, as I was looking through the notes that I had made during those years, I couldn't help feeling that our more lively discussions might have value to a wider audience. There was our debate concerning the Terrible Turtle Traps, there was The Great Circular Argument, and several others that were equally memorable. But one discussion in particular caught my eye as being eminently suitable to test my hypothesis; for want of a better name we had called it *Turtle Confusion.*

My initial notes are somewhat sketchy but, as far as I can recollect, we had been talking about the shortcomings of mathematics education. The Turtle had maintained that the major difficulty was that mathematics was not taught in a context that was meaningful to the students ... students could not possibly make sense out of isolated mathematical topics and they certainly would not value something that seemed to have no practical use ... students needed to see mathematics as a tool that would help them to solve interesting problems.

Then I said that it was all very well to talk about meeting mathematics in a 'personally meaningful context', but had he ever tried to introduce mathematical ideas that way ... how could a working teacher find the time to devise problems that related to the student's lives?

The Turtle looked amused and asked if I'd forgotten about his classroom experiences with Logo. He said that actually Logo was a good example because it provided both access to important mathematical ideas **and** excellent reasons to learn those ideas. He stated that 'meaningful problems' were not only those that related to everyday life ... they could be problems that derived their appeal from their aesthetics or from their entertainment value ... problems could be fun and still present a genuine challenge to the students.

I must have looked skeptical because, without any prompting from me, he went right on to say that it wouldn't be hard to produce a set of problems that met his criteria ... problems that were fun, that involved worthwhile mathematics, and that would challenge students ... and teachers and parents for that matter.

I remember that I then said, "Rubbish, I don't believe that you can do it!"

The Turtle had grinned and said, "Well, EBN, you'll just have to wait and see!"

I saw very little of the Turtle for the next three months; he buried himself in the library for hours on end and, if I did happen to be in the vicinity, he closed up his notebook and just looked at me and chuckled.

But let me now take up the narrative in full; my notes are quite detailed when it comes to subsequent events ...

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INTRODUCTION

I was working in the study when the Turtle bustled in and thumped a manuscript on the desk.

"There!" he said, with obvious satisfaction. "Finished!"

"About time too," I said, "perhaps life can get back to normal now."

"Aren't you even going to look at it?" said the Turtle, looking a little miffed.

"Oh, I guess so," I said, reaching for the manuscript and yawning, "I need a short break."

"Glad to be of service," said the Turtle over his shoulder, " ... it should keep you busy for a good five minutes. Call me if you want another break ... I'll probably make it a leg!"

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After about an hour I went along to the kitchen with the manuscript in my hand. The Turtle looked up expectantly and said, "Well?"

"Well, what?" I said, taking a sandwich from the heap that he had assembled.

"What do you **think**?" he said.

"I like it," I said, "I really enjoyed reading it ... but it's not what I expected. I thought that you were writing something about Logo and mathematics and problem solving. I see some shapes that you can draw using Logo, but I don't see much problem solving or maths. What are you trying to do?"

"It's a long story," the Turtle began, " ... "

" I've got a bit longer," I said, looking at my watch, "can you tell me about it in a few words?"

"I can't do it in just a few words."

"Try," I said, taking a bite of my sandwich.

"Well," said the Turtle, looking uncomfortable. "They **are** all there ... the Logo, and the mathematics, and the problem solving are all there ... even if it's not obvious.

"Uh-huh," I said, taking another bite of sandwich.

"Look at it this way," said the Turtle. "There are 'puzzles' ... the geometrical shapes ... you have to figure out a sequence of Logo commands that will make the screen turtle draw each shape ... you need to think mathematically to do that. And there are 'riddles' ... stories and poems with clues hidden in them ... the clues are mostly about the connections between the puzzles. That's mathematics too."

"Connections?" I said. "Do you mean that the puzzles are all related to each other?

"Yes," said the Turtle, his enthusiasm beginning to return.

"Ah-ha," I said, putting my sandwich down. "So the booklet is really a connected set of puzzles and riddles, and you need to use good problem solving methods and mathematical thinking to figure out what it's all about."

"Right! You can regard the whole booklet as one big problem."

I looked at the Turtle for a moment: "Do you want to rephrase that?"

"Really, EBN, you know what I mean," he said.

"Sorry ... I do know what you mean," I said. "Now ... I have another question. Some of this stuff looks fairly challenging at a first reading ... who've you written it for?"

"For high-school students and teachers and parents. There are some easy ways into the booklet, for example, you can start off with the simplest Logo shapes and the easiest riddles, but there are also some riddles that will challenge adults."

"Why would you want to do that?" I said. "Why have such a big range of difficulty?"

"Well," said the Turtle, "I believe that it is important that teachers and parents become deeply involved in the problem-solving process themselves. If that is to happen then there must be riddles that will genuinely challenge adults."

"But how can the students handle that?" I said. "Surely they can't make much progress with problems that are hard for adults."

"You might be surprised," said the Turtle, with a twinkle in his eye. "Most students will be able to hold their own with their teachers and parents ... if they're given half a chance. That's why the booklet is designed so that **no one** knows the answers ... it's particularly important that the teachers and parents aren't sure of the solutions to the riddles."

"I was going to ask you about that," I said, riffling the manuscript, "where **are** the answers?"

The Turtle tapped his forehead.

"What good is that?" I said, "how do people know when they're right?"

"They have to take a scientific approach," said the Turtle. "They have to construct their own theories and then test them out. They have to look for consistency between their answers to the different puzzles and riddles. They have to work together and discuss their ideas. In a few cases they may never be really sure that they are on the right track. Just like real life."

"This is serious!" I said, frowning. "You'd better tell me about it in a bit more detail."

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The Turtle and I were out by the fish pond in the garden. We were lying back in comfortable deck chairs with a bowl set on a low table between us. The Turtle leant over and filled his cup with what he called 'traditional celebratory punch'.

"Well, where do you want me to begin?" he said, settling back in his chair.

"From the beginning," I said. "Start by telling me about what's in the booklet."

"Well, " said the Turtle after a short pause. "First, there is a set of forty Logo puzzles."

"Exactly what is a Logo puzzle?"

"Well, I draw a shape, or a pattern, and then I say to you , '*Here you are, EBN, get the turtle to draw this shape*', but I don't tell you what Logo commands to use."

"So, a Logo puzzle is a certain shape or pattern, and the solution is a set of Logo commands that will cause the turtle to draw the required lines?"

"Yes," said the Turtle.

"Fair enough," I said. "But there will always be more than one set of commands that will produce each shape. How do you know when you have the right answer?"

"Any set of commands that cause the turtle to trace the shape is **a** right answer," said the Turtle. "There is no such thing as **the** right answer. Some answers are simpler that others, and you could argue that the simplest solutions are the best, but **any** set of commands that works is acceptable."

"Go on," I said.

"Second," said the Turtle, "there is a set of forty riddles ... the stories and the poems."

"I don't fully understand the connection between the riddles and the puzzles," I said. "Surely you don't need to solve the riddles to be able to solve the puzzles."

"No, you don't have to solve the riddles first," said the Turtle. "In fact, you can do it the other way 'round ... you solve the puzzles mathematically, or by trial-and-error, or any way that you like, and then you can use your discoveries about the puzzles to help you to solve the riddles."

"So, what's the point of the riddles?"

"Well, they add another dimension," said the Turtle, "they give clues and hints to help you think about the solutions to the puzzles, about the relationships between the puzzles, and about the underlying mathematical principles. Also,

they represent a challenge in their own right ... they call on a wide variety of material; for example, there are ideas from geography, history, mythology, physics, chemistry, language, literature and, of course, mathematics and Logo."

"Why do you have riddles that involve things outside of Logo and mathematics?" I said.

"That's another long story ... but the main idea is to set up a problem solving situation that is like real life. Real problems involve many factors, and their solutions are often not at all obvious ... I wanted the booklet to be like that."

"In other words, you want people to get used to the idea that there are problem solving methods that don't involve mathematics."

"Exactly!" said the Turtle; he got up and took my cup to refill it.

"Thanks," I said, as he handed me the cup. "One thing is still worrying me ... how do you know that the level of difficulty is right? When I read through the manuscript I get the impression that the Logo puzzles might prove too easy for anyone advanced enough to solve the riddles."

"Yes. That's a good question," said the Turtle. He sat down again and stared off across the garden. After a while he said: "Well, the Logo puzzles are not meant to be too difficult; they need to be accessible to beginners. But, even more important, the puzzles are intended to be solved as a complete set ... the puzzles are related to each other in various ways, and an attempt to understand those relationships, and to discuss them, will reveal a lot of the underlying mathematics. This property of the Logo puzzles, their relationships, is not so cut-and-dried and not so easy."

"Are you saying that the actual activity of writing down the Logo commands is not important?"

"No!" said the Turtle, "not for a minute! For a beginner the problem of writing the Logo commands will be challenge enough ... and it is a worthwhile challenge; there is a lot of mathematical thinking and experimenting needed before one can write down a workable set of Logo commands."

"That's reasonable," I said, "I guess the fact that you are working with the screen turtle means that you can easily see when your solution is right and when it is wrong."

"Yes," said the Turtle, "that's very important ... Logo gives unusually good feedback."

"But, what about the riddles? I still don't see how you can know if your solutions to the riddles are even approximately correct," I said, wrinkling my brow.

"By comparing notes, by discussion, by making sure that the proposed solutions to the riddles match the properties of the Logo puzzles. Also, there are often several clues in each riddle, and they allow some cross-checking."

"Ah-ha! So there is information built into the riddles that can be used to test your solutions?"

"Yes. In most cases it will be obvious when you are on the right track . When you propose a new solution your view of the problem changes and, if your new view-point is right, other clues will then pop out at you."

"Sounds like rabbits out of a hat," I said.

"Yes," said the Turtle with a smile. "Magic."

"Well," I said, getting up and starting to fold up my deck chair, "it sounds pretty good. What about resources? Not everyone will have computers available."

"Yes," said the Turtle, "that's true. But you can use the booklet without a computer; for example, you can write down the Logo commands, and then have a friend 'play turtle' and follow your instructions ... or you can plot the moves out on paper."

"But surely it is better to use a computer," I said. "That's the easiest way to find out if your solution works."

"That's right ... but solving the Logo puzzles is not the only activity in the booklet, so you could easily run a class with only one or two computers. Of course, you do need other resources ... to solve the riddles you need a good dictionary, an atlas, an encyclopaedia, and a modest library of classical literature."

The Turtle picked up the empty punch bowl and started off across the lawn. Then he stopped and turned back to me. "Why don't you work through the booklet yourself?" he said, "I'd value your comments."

"Well," I said, gathering up the cups, "I just might do that ... but would you mind trying it in one of my classes first? I would like to see all of this in action before I decide."

"Happy to oblige," said the Turtle, leading the way into the house.

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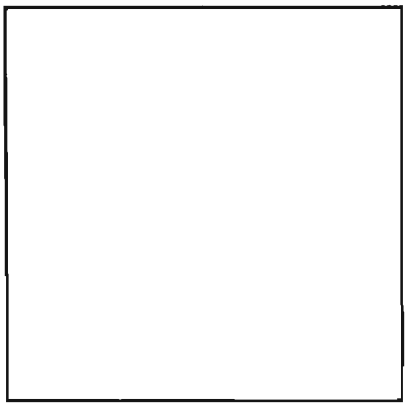
PUZZLE 1



"I'll sit somewhere down the back," I said to the Turtle as we entered the class-room. He put his books on the table, pushed his glasses up on the bridge of his nose, and surveyed the room.

"Let us begin," he said, in a formal voice. "We will call this puzzle 'Square One'. You might think that drawing a square is easy, but make sure that you really know how the commands work. Also, even if you can make the turtle draw a square, there is a good chance that you still have some rough spots in your thinking about squares."

"But what is there to know?" said Myrtle from the front row, "a square is a square is a square is a square."

The class laughed with delight and the Turtle chuckled. He began to relax.

"Well," he said, "if you don't understand what I mean, then try a couple of exercises. See if you can tell a friend what a square is ... or tell the turtle to draw squares of different sizes, and then think about what parts of the commands you had to change and what parts you didn't have to change."

Everyone started to type commands at their keyboards and green squares began to appear on the computer screens. The Turtle turned to the black-board and wrote in a clear, round hand:

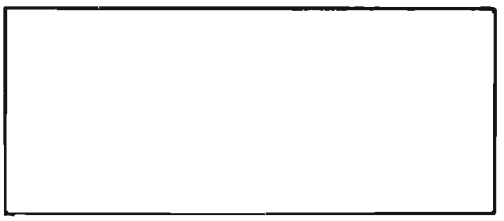
There was a young student named Myrtle,
Who tried to converse with the Turtle,
She later said, "Guys,
The scales fall from your eyes,
When you clear the homology hurdle."

"There," he said, dusting off his hands, "that might help you figure out what I am talking about."

Silence ... they all stared at the board; then some of them stared at each other.

"They probably think they have a nut case here," I thought.

PUZZLE 2



I put down my coffee mug and looked at the Turtle. "I thought that your performance was OK the other day ... and the kids seemed to enjoy discussing that ghastly limerick." I paused for effect. "Well ... anyway, you have convinced me ... despite my doubts I will have a go at your problems.

"You're on!" said the Turtle, jumping up from his chair and beaming.

"Where do I start?" I asked.

"With the second puzzle in my booklet," said the Turtle. "If you can solve Puzzle 1 then you should have no problems with this one. For an extra challenge, try drawing the rectangle with the REPEAT command."

"No worries," I said heading down the hall toward the study.

"Wait a minute," the Turtle called after me, "you need the whole story. When you have solved the puzzle you should think about the ways in which a rectangle is different from a square ... and the ways that they are the same."

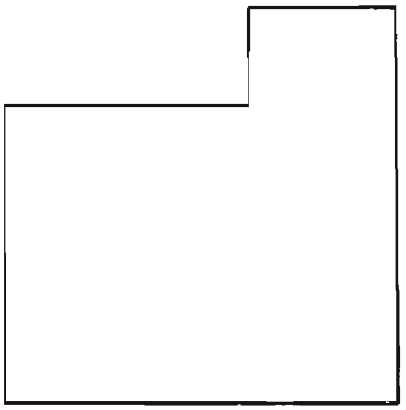
"Is that it? Can I go now?" I said sweetly.

The Turtle handed me a sheet of paper. "Read this riddle," he said.

> A butterfly floated delicately down and alighted on a branch near a hungry caterpillar. The butterfly, peering down its proboscis at the caterpillar, said, "Move aside, you uncouth beast."
>
> The caterpillar looked up from its munching and said, in a rather muffled, mouth-fullish way, "You might think that you are special, My Lady, but consider your origins. Also, I ask that you remember my versatility; can you, My Lady, ... BURP, pardon me ... perform the Big Change?"

I looked up and caught the Turtle's eye. "Do they all have riddles?" I said. He nodded.

PUZZLE 3



The Turtle opened up his booklet at the third puzzle and looked at it for a while.

"Perhaps," he said, "it might help if you regarded this shape as a rectangle with a chunk bitten out of the corner."

"How can that possibly help?" I said as I settled down in front of the computer.

"Well, at least it would allow you to know what I meant when I said 'Which corner has a bite bitten out of it?', or if I said 'See if you can draw this shape with the bite taken out of another corner,' or if I said 'Can you draw this shape with a rectangular bite, or a square bite, or more than one bite,' or ....."

"I get the point," I said, "Let's get on with it."

"Really, EBN, you can be difficult sometimes; we do need to look at all the possibilities." The Turtle regained his composure and looked at the puzzle again.

"You know," he said, "some of these puzzles can be solved in a very neat way and some of them can't."

"Oh?" I said. "What do you mean by 'neat'?"

"Well, you can sometimes find a very simple way to write down the commands. Sometimes there is a pattern in the commands, sometimes you can find a way to use very few commands ... it depends on the puzzle. Sometimes there isn't a neat solution, but you can usually learn something by looking for one."

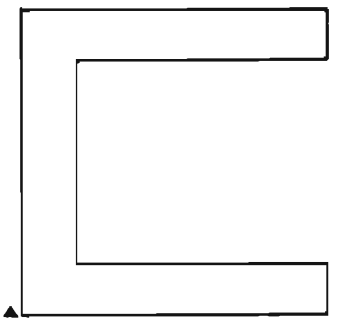
"I get the point," I said.

"By the way, I nearly forgot," said the Turtle. "What do you call the tracks left by a Logo Turtle?"

I considered the matter for a moment. "I don't know," I said, "What do you call the tracks left by a Logo Turtle?"

"The pattern of tidy feet."

PUZZLE 4



The small triangle indicates the corner at which the turtle should start and end when the Hollow C is drawn. Make sure that the turtle is pointing in the same direction at the beginning and end.

"This is my famous 'Hollow C Puzzle'," said the Turtle with obvious pride, "I usually get the class to do it three different ways ... I actually specify which commands they can use."

"Go on," I said.

"First," said the Turtle, holding up one finger, "I ask them to produce the shape using any commands that they like ... that helps them get the properties of the puzzle into their minds."

"Makes sense," I said.

"Second," he held up another finger, "I ask them to draw the shape using only the commands FORWARD and RIGHT ... of course, they can use any input numbers that they want with these two commands."

"And ... Third?" I said, holding up a finger.

"Third, I ask them to draw the puzzle using only three commands ... FORWARD, BACK, and RIGHT 90. In this case they must use 90, and only 90, with the RIGHT command."

"Very ingenious," I said. "But what's the point?"

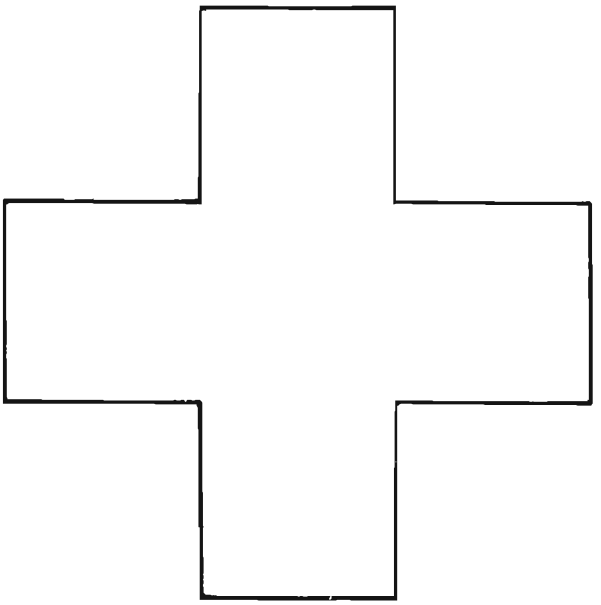
"Well," said the Turtle, "having to use restricted sets of commands helps people discover new things about Logo and mathematics ... different ways to use commands ... try it for yourself. Oh, and think about this when you have solved the puzzle." He handed me the riddle sheet.

> Said Senator Seemore, "I'm feeling ambivalent,
> I've come to believe all parties equivalent,
> Even those at extremes of political persuasion,
> Tend to create the same State of the Nation."

"Who is Senator Seemore?" I said.

"An old friend," said the Turtle with a touch of reverence in his voice, "Someone I look up to."

PUZZLE 5



The Turtle looked at the booklet lying open on my study desk. "How are you progressing?" he asked.

"I'd hardly call it progressing," I said, not looking up.

"Can I help?" he said. He seemed to respond well to admissions of defeat. "What's the problem?"

"Well," I said, "I worked out a set of commands to draw the cross, but they were not very satisfying; lots of commands ... a whole string of commands."

"There's nothing wrong with that," said the Turtle, "that's a perfectly good solution."

"Yes, I suppose so," I said, "it did work. But, I was thinking about what you said the other day ... you know, about looking for simple solutions. I can't find any."

"Let me make two comments," said the Turtle. He was sounding positively fatherly. "First, you don't **have** to find neat solutions; any commands that work are a worthwhile result. Second, if you really do want to find a simple solution consider the following riddle:

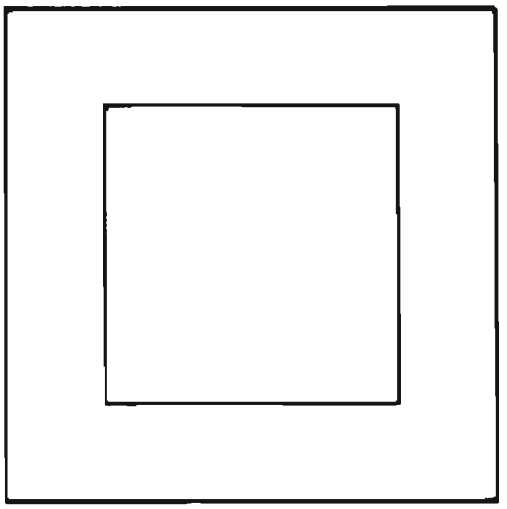
What does this puzzle have in common with:

A barque, but not a schooner?
A steak with three veg., but not a can of beans?
***The Four Seasons,* but not *Yellow Submarine* ?**
A reef knot, but not a turk's head?"

The Turtle paused to draw breath and then suddenly stepped back in alarm. "What are you looking at me like that for?" he said.

"I don't think you want to know," I said.

PUZZLE 6



"I suppose there is a very simple way to solve this puzzle," I said from the depths of my armchair.

"Yes," said the Turtle from the depths of his armchair, " do you want a clue?"

"You've had worse ideas," I said.

The Turtle thought for a while, and then he said, "How's this?

> **Have you ever thought about the way a pizza,**
> **As you wrap yourself around each tasty wedge,**
> **Is eaten in the manner cakes and sweets are,**
> **We consume them from the middle to the edge."**

"Sweets are ... pizza? I don't think you've had worse poems," I said, wrinkling my nose.

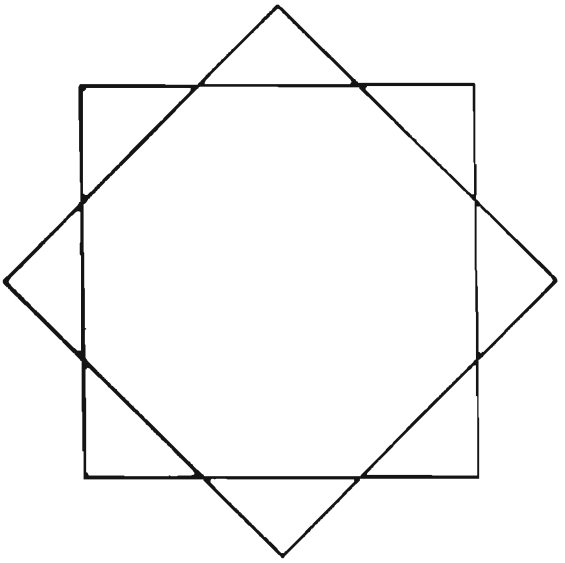
"Good rhymes are hard to find," said the Turtle defensively. "In any case, that should help you find a simple solution to this puzzle. Once you have done it you should see if you can write a procedure that tells the turtle how to draw your basic shapes."

"Oh, no," I said, "Not more complexity! What's a procedure?"

"Nothing to it," said the Turtle, struggling up out of his armchair to hand me a copy of *Teaching with Logo* by Molly and Dan Watt. "Read this."

"Gee, thanks," I said.

PUZZLE 7



The Turtle, in the grip of one of his periodic detective phases, was re-reading *The Adventures of Sherlock Holmes*. I was staring at Puzzle 7, but my mind was a blank.

After a while I said: "How about a riddle to help me solve this puzzle."

The Turtle laid his book on the table and considered my request for a moment. "No." he said. "This one's too easy. Especially after you solved Puzzle 6 so elegantly."

"Well, that's the one that convinced me that your riddles are worth having ... not that I can say the same for the poetry, or whatever you call it, that you use to express them."

"I tell you what I will do," said the Turtle, ignoring my compliment, "I will give you a pointer to a harder puzzle that is related to this one."

"What's a **pointer**?" I said.

"A pointer is a riddle that has a single number as its solution; the number is the sequence-number of another puzzle in this booklet. In some cases the related puzzle will be a harder puzzle, in some cases it will be an easier puzzle."

"Seems clear enough," I said apprehensively.

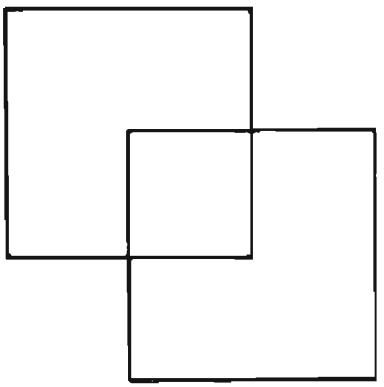
The Turtle paused for a moment. "How's this?" he said.

> **"If they explode enough uranium devices,**
> **Our planet will become somewhat exciting,**
> **We may end up with glowing herbs and spices,**
> **And bones that give a soft internal lighting."**

I looked puzzled.

"It's elementary, my dear EBN," said the Turtle, picking up his book.

PUZZLE 8



The Turtle said to me the other day, "You know, EBN, I think that this puzzle will prove difficult to solve in an elegant way. You should have no trouble throwing together a lot of commands to draw the shape, but I am not sure that you will be able to discover the Super Neat Solution."

"You do seem to place a lot of emphasis on "neat solutions", I said, "are they really all that important?"

"I've always thought so," said the Turtle, scratching his head. "It depends a bit on 'where you're at' ... if you are struggling to understand how to tell the turtle to draw **anything**, then you should not have to worry about elegant solutions, but if you have got to the stage where you can control the turtle with some ease, then the next challenge is to do it with style. I really believe that you can improve your understanding a lot by trying to do the same thing in a different, simpler, way."

It was quite a speech. I sat and thought about it for a while.

The Turtle went on, "If you **do** want to find a simple solution, then you have to think about the puzzle in the right way. You have to try and see it with different eyes."

"Of course," I said, "That's obvious."

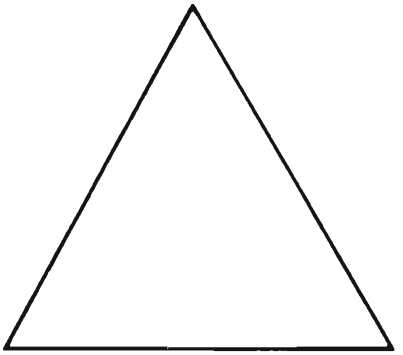
"Maybe," said the Turtle, with a gentle smile, "But can you do it?"

"Well, ...," I began, "Um, ..."

"Look at it this way," said the Turtle, his smile becoming even more tolerant, "You are better off thinking of a *llama* on a *llano*, than you are contemplating the Cyclops's barbershop quartet."

"Great," I said.

PUZZLE 9



"Before you start on this puzzle," said the Turtle with some urgency, "I should warn you about a short-coming of many introductory Logo books."

"Uh-huh," I grunted, yawning.

"No. This is serious," said the Turtle.

"I'm quivering with eagerness," I said scratching my back against the door frame.

The Turtle's mouth tightened a little, but he continued: "A lot of books tell the reader how to draw a triangle. Now, some things are better learnt with guidance, and some things you can discover for yourself ... by far the best way to learn how to draw a triangle is to do it by yourself. There is a chance for very important discovery learning here."

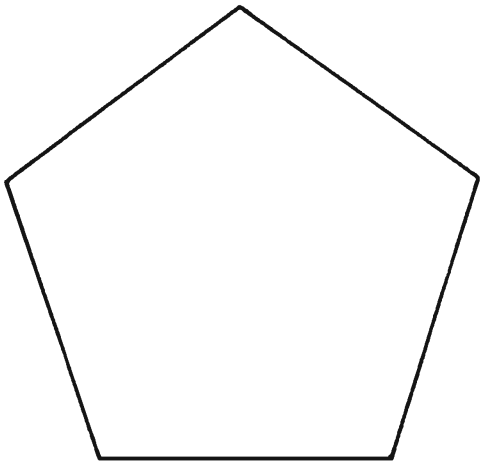
"OK," I said, "Leave me to it!"

The Turtle turned to leave the study. In the doorway he paused and turned back. "You might," he said, "learn something of value about the process of telling the turtle how to draw, and about geometry, by thinking about the following poem:

> **The bugs are all dead, you feel like you're Lister,**
> **You've a puzzle for Clio, one more for each sister,**
> **And then you find out, from a Flatlander sentry,**
> **That you and the Turtle have views supplementary.**"

I groaned. "Please leave," I said.

PUZZLE 10



The Turtle came into the kitchen with his booklet in his hand. "You know, EBN," he said with some excitement, "the next three puzzles, numbers 10, 11, and 12, tell part of a very interesting story. In many ways the hidden story is more important than the puzzles themselves. "

"Tell away," I said as I broke another egg into the frying pan.

"Well," said the Turtle. "I think that you should do a little playing around with this story by yourself. You know, another chance for discovery learning."

"Oh, no!" I said, "not more discovery learning."

"Sometimes it is a good approach," said the Turtle firmly, "and this is one of them. I won't give you any clues to the puzzles, I'm sure that you can solve them for yourself by now, but I will give you some clues to the hidden story. It's a geometry story."

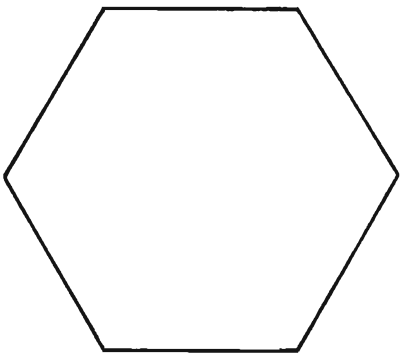
"OK, I'll give it a go," I said, "I must admit that I enjoyed that triangle business. Once I got over the initial shock. I begin to see that I could get to be quite good at this turtle geometry business."

"Good," said the Turtle as he settled down to demolish his usual breakfast of six eggs. He looked at the plate for a moment and then said, "Actually, there are six puzzles that are part of the hidden story. Here is a clue to help you locate two more of them:

> **'Don't put on airs,' the Turtle said, 'It's early in the day,**
> **You have yet to solve this puzzle in a periodic way,**
> **That relates two other puzzles to the puzzle pentagon,**
> **Just as hydrogen and flourine integrate to give neon.'** "

"Sounds like a lot of hot air to me," I said, putting some more sauce on my eggs.

PUZZLE 11



I leant back in my swivel chair and said, "What did you mean when you said that there was a story hidden in these puzzles?"

The Turtle put down the book he was reading and thought for a moment.

"Well," he said, "Puzzles 10, 11, and 12, and three others, are related in a special way. There are geometrical ideas that apply in the same way to all of them. Once you know the general ideas, then you will be able to see each of the six puzzles as a special case. And you will also be able to solve a wide range of geometrical problems using the same general ideas."

"Yes, that really is exciting," I said. "I had no problem instructing the turtle to draw the Puzzle 11 shape, but I still don't have any ideas about the hidden story."

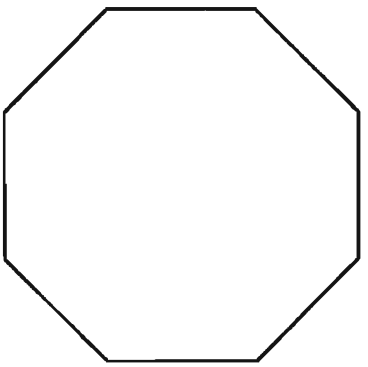
The Turtle looked at me speculatively, with his eye-brows raised, and then he said, "Read the riddle."

I opened the booklet at Puzzle 11 and read:

> **You must, to solve the puzzles, lead our turtle in the dance,**
> **If you talk the Logo lingo he will pirouette and prance,**
> **But remember, as you labour, as you seek ideas *obscura*,**
> **That his trips are all appealing to *Farancia abacura*.**

I looked up at the Turtle and sighed. "That's good advice," I said.

PUZZLE 12



The Turtle and I were in his laboratory when he looked up and said, "By the way, how are you going with the Logo Puzzles?"

"Not bad," I said, "I am up to Puzzle 12."

"Ah, yes," said the Turtle, with a smile, "that is the third of the related puzzles. At least, it is the third of the ones that I have told you about. When you have worked out a way to tell the turtle how to draw an octogon, then you might want to think about the hidden story again."

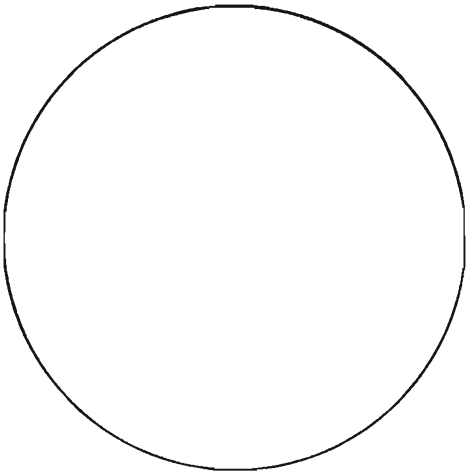
"I'm not making much progress. Sometimes it seems as though it's going to take me centuries to understand this stuff," I said.

"Yes. I suppose that I'd better give you a pointer to the sixth puzzle," said the Turtle as he picked up a magnifying glass and leant forward to examine a small heap of black powder that he had placed in a crucible. He paused for a moment and then said: "You would do well to remember, if you want to be sure of bringing home the bacon, that you must always be objective and pay attention to observational facts."

"Is that it?" I asked.

The Turtle held up the magnifying glass and looked at me with a monstrous eye. "Yes," he said.

PUZZLE 13



The Turtle and I were sitting in the canteen at the school. After a while I said, "This is my unlucky day. I am up to Puzzle 13."

The Turtle munched his alfalfa sandwich. "What's the problem?"

"So far all the puzzles have involved straight lines. To draw a circle I have to work out how to tell the turtle to draw curved lines."

"That's right," said the Turtle.

"Well, I don't know how," I said with a touch of exasperation. He was being particularly unhelpful today.

"Have you tried Playing Turtle," said the Turtle.

"What, and run the risk of ending up like you?" I said.

It was the Turtle's turn to look exasperated. "That was uncalled for," he said sharply. "I meant that you could pretend to be the turtle; you walk in a circle and you notice what you have to do."

"I'm sorry," I said. "I guess I'm starting to take all of this too seriously."

The Turtle munched on his sandwich again. Suddenly he said, "I've just had a profound thought."

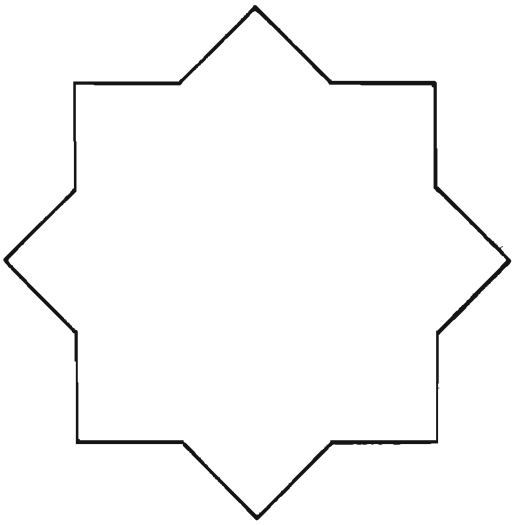
"Do tell," I said.

"Well, it occurred to me that there are more seconds than hours in a day."

I glanced around to make sure that we could not be overheard. "There seems to be no limit to your insight," I said. "Are you **always** so profound?"

"Invariably," said the Turtle.

PUZZLE 14



"Midnight snack!" the Turtle called, and I got up from the computer and went along to the kitchen.

"Well, Puzzle 14 certainly is a snap," I said as I buttered a piece of toast.

The Turtle placed a plate of kippers on the table and sat down opposite me. "Did you know that there are three distinct ways of solving that puzzle," he said reaching for a kipper.

"Oh, no," I said, "Just when I was feeling that I had it licked."

"Sleep on it," he said reaching for another kipper.

The Turtle stumbled into my bedroom and sat down suddenly on the end of my bed. "By George!" he said, clutching at his stomach.

"Waa's matta?" I struggled awake and switched on the light to look at the time - 2 o'clock!

"I have the most incredibly powerful tummy ache."

"Kippers. How many kippers did you have for your midnight snack?"

"One," said the Turtle experimentally. I just looked at him.

"Two," said the Turtle. I kept looking.

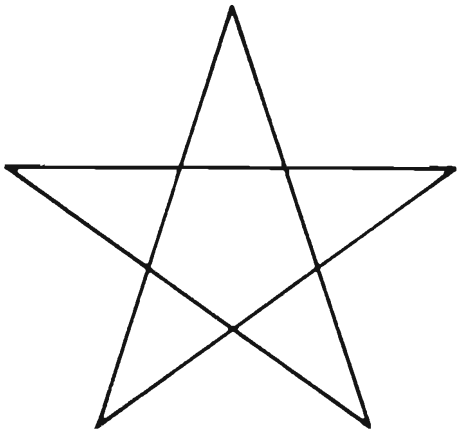
"Three," said the Turtle.

"**And** the rest ... I counted at least seven," I rolled over and pulled the blanket up over my head.

"No," said the Turtle firmly. "Three. Boole's Rules."

But I was already asleep.

PUZZLE 15



The Turtle threw his gardening gloves down on the bench and looked at the patterns that I was scribbling on a pad. "You know, EBN," he said, "that shape is a favourite of witches. It is called a 'pentacle' or 'pentagram' ."

"Oh," I said.

"Did you know that it is closely related to another shape that you have already drawn?"

"No," I said.

"Are you going to figure out what the other shape is?"

"What do I use? Witchcraft?"

"No, double-think," said the Turtle chuckling. "Seriously though, this riddle might help."

"It might not!" I muttered under my breath.

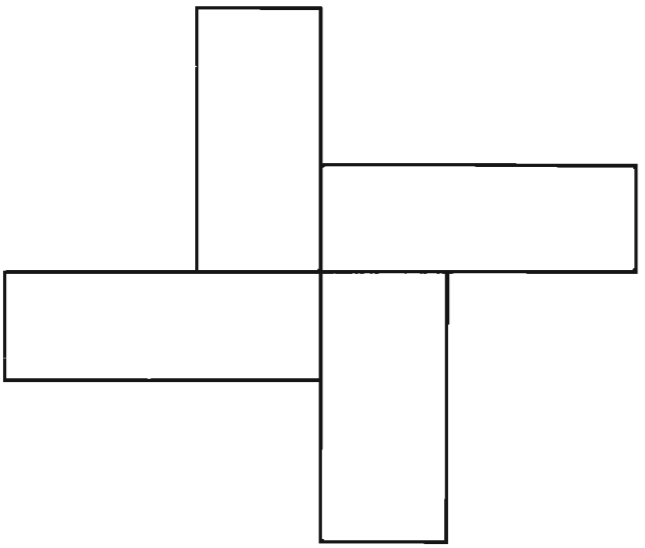
The Turtle took a deep breath and recited:

> **"The pentacle is useful for casting spells and kind,**
> **It will lead you on the path in the garden of your mind,**
> **You may wish to feed your turtle a little witch's brew,**
> **Kippers and tomato sauce will help to see him through."**

He paused. "Any questions?"

"Yes. How do you make voodoo dolls?" I said. "And where are the pins?".

PUZZLE 16



"How is it all going?" said the Turtle as he came into the study.

"OK," I said. "I am trying to write all of my solutions as procedures now ... it sure saves a lot of time."

"Yes," said the Turtle. "Let's see ... Puzzle 16 ... have you found out that there is one very good way of describing patterns like this to the turtle?"

"I am not sure," I said, "Do you mean a 'neat solution'?"

"Not really ... but, yes," said the Turtle helpfully.

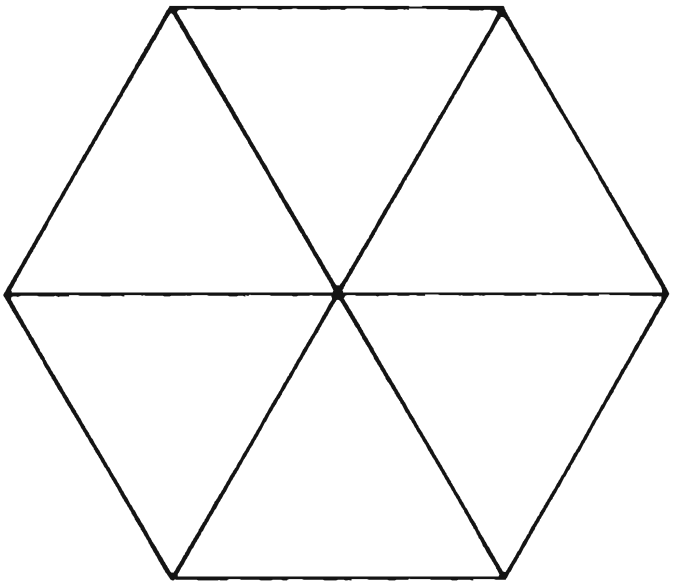
I must have looked pained. He left.

"Muffins up!" I called down the hall the next morning, and the Turtle came along in his bathrobe to join me for breakfast.

"I was just reading in the newspaper," he said, "that medical researchers say that it is better to eat a meal mouthful by mouthful, rather than trying to ingest it all at once." He reached over, took a muffin, stuffed it into his mouth, and swallowed it in one gulp.

I stared at him in amazement. He was looking decidedly peculiar, with watery eyes and a dribble of melted butter running down his chin.

"They are probably right," I said.



PUZZLE 17

The Turtle was busy laying new tiles in our bathroom. I went along with some afternoon tea for him.

"Tell me," he said, as I poured the tea, "does it strike you that we are engaged on similar activities this afternoon."

"Yes," I said, " we are both drinking tea and eating muffins."

"No, not that," he said seriously (indicating that I was loosing my touch), "I mean that we are both creating patterns."

"Oh. Yes, that too," I said.

The Turtle gave me a brief, but searching, glance. He could detect nothing but pure essence of earnestness. Nevertheless, he changed the subject.

"I believe that the hardest part of tiling is putting the small pieces in at the end of the row," he said, casting a critical eye over his handiwork.

"I agree," I said, "you really need to think carefully about the shapes, and how they fit in, and about ..." I paused for a moment ... **had** he changed the subject? I gave him a brief, but searching, glance, but could detect nothing but pure essence of earnestness. Nevertheless, I changed the subject.

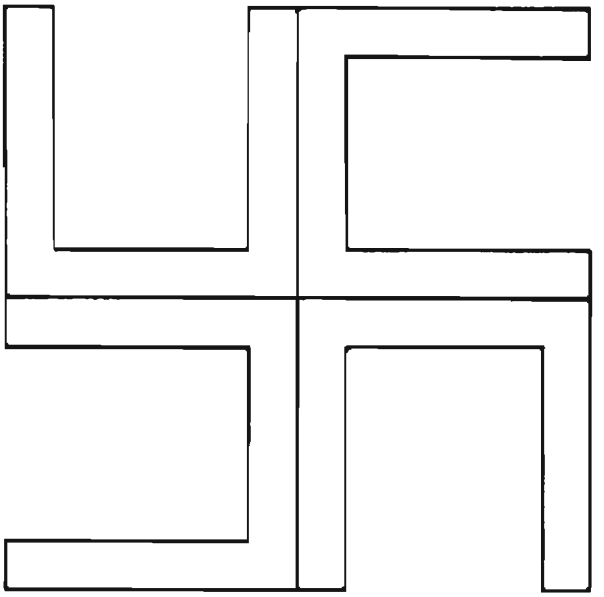
"Have another muffin," I said, holding the plate out to him.

"Thank you," he said, picking one up delicately and taking a small bite.

"You know," he said, looking sideways at me, "It just occurred to me that the number of bites it takes to eat a muffin depends inversely on the size of one's mouth."

I took a big bite of muffin. "Delicious," I said, changing the subject.

PUZZLE 18



The Turtle glanced at the screen, where the turtle was busily drawing a pattern, and said, "Well, you seem to have come a long way in just a few weeks."

"Yes," I said, "I was a bit apprehensive at first, but I have really begun to enjoy myself. It helps to be able to control the turtle well."

"That is one of the essentials," said the Turtle, "A turtle out of control is more of a hinderance than a help."

I gave him a long, hard look. He had the decency to blush.

He looked again at the screen and said, "Do you want a pointer to another related puzzle?"

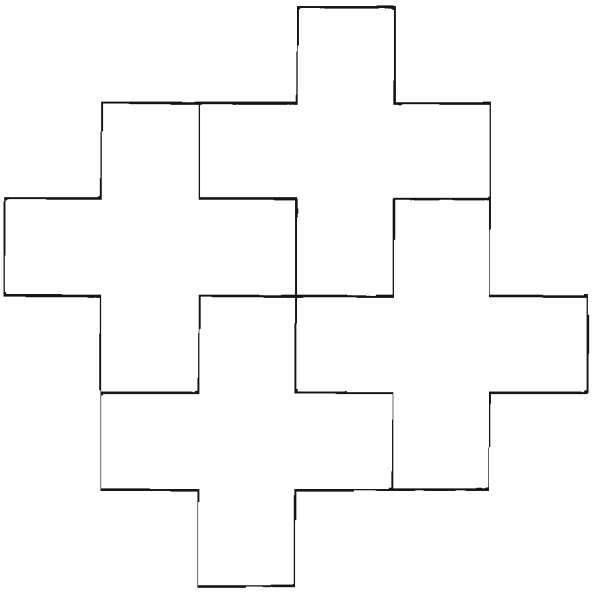
"Yes," I said, "but tell me why you think it is worthwhile giving clues to other puzzles?"

"It's all part of the game," he said. "Sometimes you will solve the Logo puzzle first, and that will help with the riddle; sometimes you will solve the riddle first, and that will help with the puzzle. I think that having the riddles makes it more likely that anyone trying to solve the puzzles will also think about other, connected ideas."

"Sometimes it seems a bit like 'I spy with my little eye'," I said. "But I guess it is not just a guessing game."

"No, it is not." The Turtle dug around in his pocket and pulled out a piece of crumpled paper. He took a deep breath and recited:

> **"Tweedsmuir was a lively old fellow,**
> **He could make his friends quiver like jello,**
> **He said, 'Goodness sake,**
> **The way ye bairns quake,**
> **I'll take steps to be much more mellow'."**



PUZZLE 19

I threw another log on the fire and for a while we just watched the flames dancing. The Turtle turned back to his book (he was making a concerted effort to read some classical works ... his latest motto was '*a chapter a day keeps the cobwebs away*'), and I continued to think about the Logo puzzle that I was trying to solve.

"Would you mind if I interrupted your reading for a moment?" I said after a few minutes.

The Turtle looked up and I thought that I caught a flash of something like gratitude in his eyes - perhaps I imagined it.

"Not at all," he said. "What can I do for you?"

"I am currently working on Puzzle 19," I said, "I wouldn't mind a pointer to a related puzzle."

"Humm," said the Turtle, "You realise that the rules of the game prevent me from telling you whether the related puzzle is an easier one or a harder one. Or even whether or not the riddle is a complete red herring?"

"Yes," I said, "Lay it on me!"

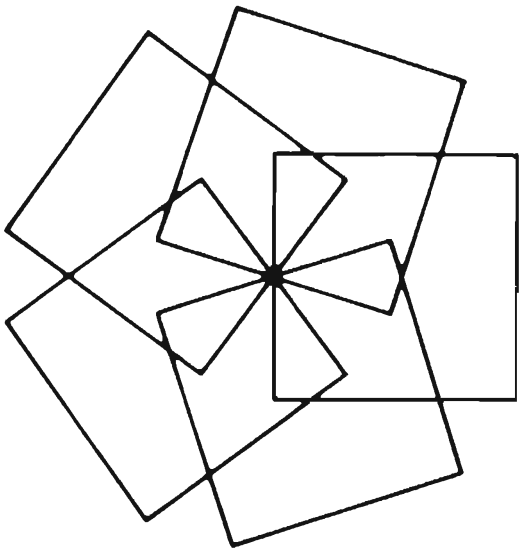
He thought for a while and then scribbled a few lines on a piece of paper. He passed the paper across to me and I read:

Finn and Co. played on Jim's gaoler's fears,
They practically drove him to tears,
The poor blighter said,
'I wisht I was dead!'
When he heard witch's words in his ears.

"Well?" said the Turtle.

"I guess I can't complain," I said, " I asked for it!"

PUZZLE 20



"You should try to draw some general conclusions about patterns after you have completed this puzzle," said the Turtle. He had laid aside his leather-bound classic and was toasting muffins on the fire. I was attempting to write down a workable procedure for Puzzle 20.

"Uhuh," I said.

"These puzzles are designed to help build your understanding of patterns ... patterns are the basis of mathematics, of science, even of life ....."

"The Universe, and everything," I said.

"Well. Yes. Something like that."

"Douglas Adams would have loved you," I said.

"Who?" said the Turtle.

"Let's change the subject," I said, "How about a riddle that gives me a clue to your deep thoughts."

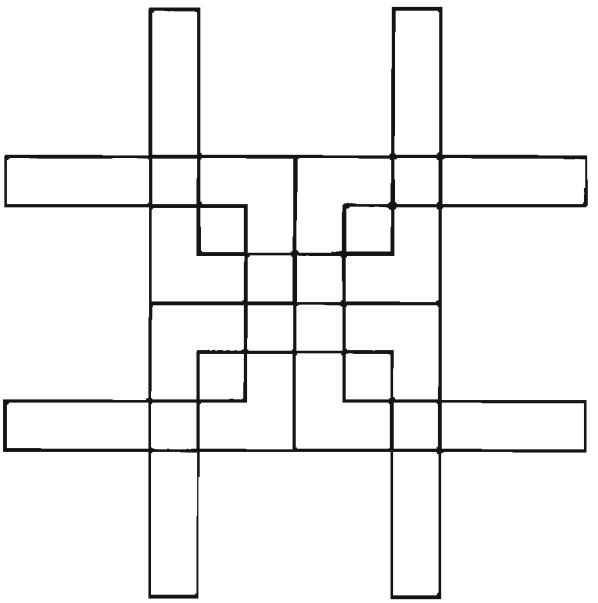
The Turtle settled back in his chair and held his toes out closer to the fire. "Sometimes," he said, "I am not sure that you understand what I am talking about."

"That's an understatement," I said.

"You need nimble mental feet," said the Turtle, warming to the task, "You need to recognise that the argument can turn about this point or that, or neither, or that ..."

"The result is mostly the same," I said, "Turtle Confusion."

PUZZLE 21



"Let's go for a walk down by the lake," I said. The Turtle put down his palette and stretched.

"An excellent idea," he said, " I need a break. Just give me a moment to clean my brushes."

Ten minutes later we were strolling down the path to the lake. The Turtle was sniffing the air with relish. "You don't realise how much paint vapour you inhale when you are working indoors," he said.

"Now that I have your attention," I said. "Would you mind giving me a pointer to a puzzle that is related to Puzzle 21?"

"Not at all," he said. "Ah, yes. Pointers ... they symbolise for me the deep links between language and mathematics ... the riddle is a play on words, but the solution is a number ..."

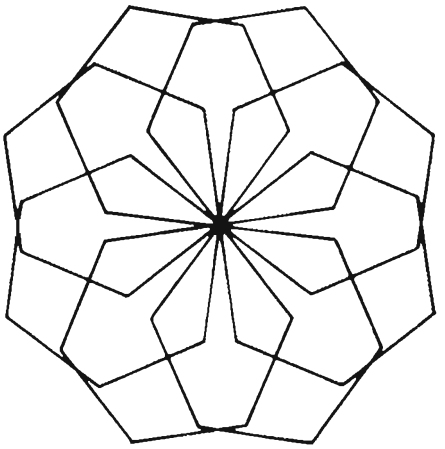
"Paint vapours?" I asked.

"What? Where was I?" the Turtle said. "Oh, yes, a pointer ... well, how about this:

Joe Kesselring's brew was no pie,
But the mixture caught everyone's eye,
The magic, it's true,
Came from mixing a clue,
With wings of neuropterous fly."

"Paint vapours," I said.

PUZZLE 22



"I've been meaning to ask you a question," I said to the Turtle as he came into the study. "Why are there so many puzzles in your booklet?"

"Why so many?" said the Turtle. "Because I wanted to give everyone a chance to discover things about the mathematics of patterns ... and that requires a lot of puzzles."

"But some of them look very much alike," I said.

"Aha! That is exactly the point! You have already started to make discoveries!"

"What do you mean?"

"Well, you have looked at the patterns and you have noticed that some of them are alike. But you couldn't do that unless there **were** a few puzzles that were alike, could you? Now, let me ask you a question ... some patterns are alike, but are they **exactly** alike?"

"No," I said, "they have some features in common, but there are also some differences."

"Right!" said the Turtle. "Would you agree that you can't possibly find out what "same" means if you don't have puzzles of the same type **and** puzzles of different types?"

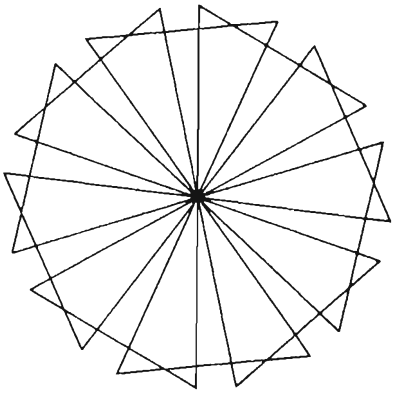
"I see what you mean," I said, "so you need quite a few puzzles to show several different types of patterns, especially when each type needs to be shown more than once."

"*Correcto*!" said the Turtle. "Oh, while you are thinking about such matters, I meant to give you this pointer to a puzzle that is related to Puzzle 22."

The Turtle handed me a sheet of paper:

I met Liz one day last November,
Her shape is not hard to remember,
Her measurements grow,
From her head to her toe,
Of the class "pyramid" she's a member.

Liz loves to sing songs with a beat,
Her diction's surprisingly neat,
But a fourth is her range,
The timbre is strange,
And the sounds seem to come from her feet.



PUZZLE 23

The Turtle and I were sitting by the pond. Every now and then the Turtle would throw a small stone into the water. We watched the patterns created by the ripples.

"You know," said the Turtle, "those patterns remind me of some of the Logo problems that you have been doing."

"I should have thought that you didn't need any reminding," I said. "Actually they remind me to get some aspirin the next time we go shopping."

"That's remarkable," said the Turtle, "Why did they remind you of aspirin?"

"There are bound to be fish with headaches in that pond," I said.

The Turtle chuckled for a while, and then he said, "As you go through these latest puzzles are you stopping to think about the solutions, and about what you have learnt about describing patterns?"

"Yes," I said, "although I sometimes think that your clues are a bit too enigmatic."

"I know what you mean," said the Turtle, "but I am trying to strike the right balance between giving too much help, and so leaving nothing for discovery, and not giving enough help."

"Oh, I think that you succeed in general," I said. "I mean, in not giving enough help."

The Turtle lay back on the picnic rug, with his hands behind his head, and gazed up through the leaves. After a while he sat up.

"Why?" he said, looking at me over the top of his glasses, "Can't you fit a square peg in a round hole?"

"Because the corners stick out," I said.

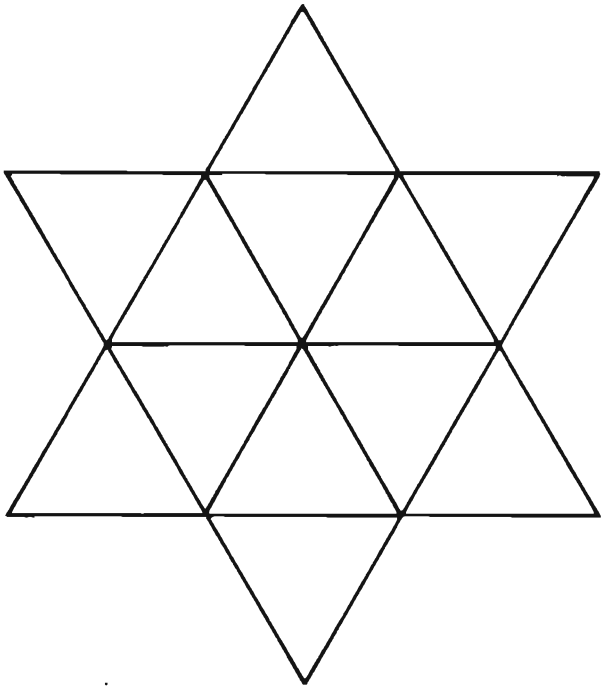
"But surely if the peg is small enough it will fit."

"I see what you mean!" I said, getting excited, "and a round peg won't fit if it is too big! .....

Oh, Tesselation! ... I am starting to talk like a turtle!"

"A passable student," said the Turtle.

PUZZLE 24



The Turtle was sitting on the sofa working on a large patchwork quilt. I watched him for a while.

"I hardly slept a wink last night," I said, "I kept thinking about how patterns can be the same and yet be different."

"Congratulations, my boy," said the Turtle expansively, "you are in the grip of a genuine research problem."

"What do you mean?" I said weakly.

"Well, you seem to have got to the point where you want to know how to group patterns together. In other words, you want to classify them - you are trying to understand how the shapes in the patterns are organised. That is the kind of problem that has driven all sorts of people to distraction down through the ages. Keep at it and you will develop some real insight into the mathematics of patterns."

"Or a bad dose of mental repetition strain injury," I said with real insight.

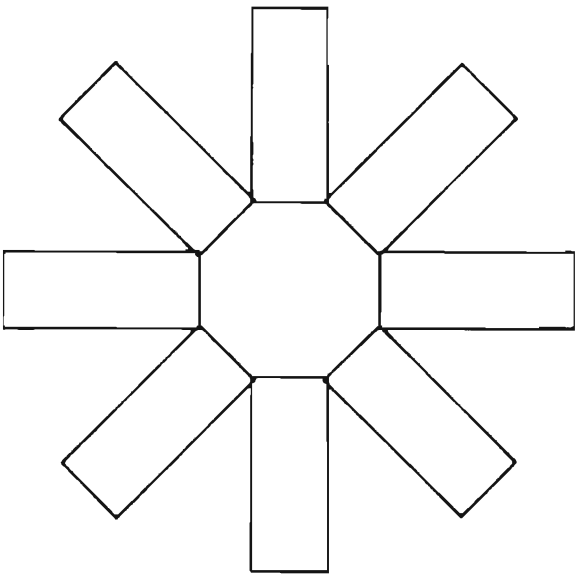
The Turtle worked on.

"Sometimes!" I said suddenly, "You can't see the trees for the woods!"

"What?" said the Turtle, sticking the needle into his finger tip, "Drat!"

"Sorry," I said, "Just practising turtle insight."

"Seems more like a terminal case of blurred vision to me," said the Turtle, sucking his finger.



PUZZLE 25

The Turtle was looking over my shoulder as I struggled on with Logo Puzzle 25. I tried a few commands at random, and he watched without comment; though he did fidget a bit ... well, actually ... he was pretty jumpy for someone who believes in exploratory learning.

"Do you want me to translate for you?" he said after a while.

"No thanks," I said, "I want to do it by myself. Besides, I thoroughly understand your extremely simple language."

The Turtle turned away, a little abruptly I thought, and walked to the door.

"Where are you going?" I asked.

"To join the Displaced Turtle's Rehabilitation Movement," said the Turtle over his shoulder.

"Come back here," I said firmly, "I'm sorry if I offended you."

The Turtle stopped and came back slowly. "That's OK," he said.

"I guess I'm getting a bit touchy because I'm confused," I said, "It's one thing to solve the puzzles and it's another thing to think about the organisation of patterns."

"Do you want some help?" said the Turtle.

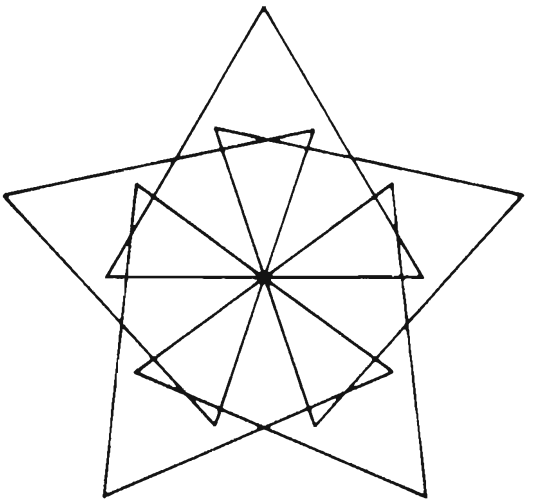
"Yes," I said, "as long as your charges are reasonable."

"Well, think about Puzzle 25 and Puzzle 8," said the Turtle, charging right on, "how are they the same, how are they different?"

"That's help?" I said.

"Yes," said the Turtle, "that's help. Take it or leave it. No Charge!"

PUZZLE 26



"Tell me a bit more about the riddles," I said to the Turtle as we floated around the swimming pool on our Li-Los.

"What do you mean?" said the Turtle, "do you want me to give you the solutions?"

"No, of course not," I said. "I mean, tell me about the type of information that is buried in the riddles."

"Well," said the Turtle, paddling his Li-Lo closer to mine, "there are basically three types of riddle - the first type gives ideas about how to solve the puzzles, the second type gives clues to ideas about the properties of patterns, and the third type gives pointers to other puzzles that are related to the one being solved."

"What do you mean by 'related'?"

"I mean that some identical arrangements of Logo commands will appear in the solutions to both puzzles," said the Turtle.

"Fair enough," I said. "Will you always tell me what type of information is hidden in the riddle?"

"I have always identified pointers so far," said the Turtle contemplatively, "but I could change that ... in fact, I **will** change that; I won't give any such hints from now on."

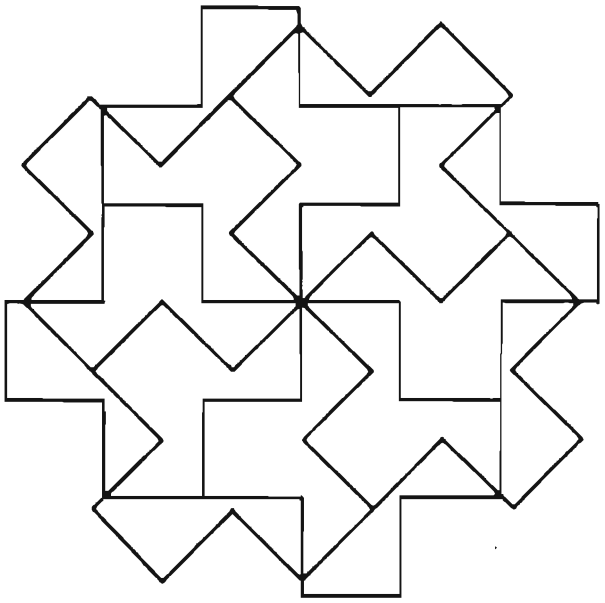
"Rats!" I said with feeling, "I should have kept my mouth shut."

"Always a good policy in a swimming pool," said the Turtle, with a chuckle. "Here is your next riddle:

> **Said Constable Bill on his beat,**
> **'Interpreting crime is a treat,**
> **Provided you know,**
> **The ideas that grow,**
> **As motive and method compete.' ...**

"Criminal," I said.

PUZZLE 27



The Turtle hummed a little tune to himself as he sat measuring the distance between Aden and Dhubri on a large globe of the world ... I was working on the problem of sorting the Logo Puzzles into groups with similar properties, and my mutterings were growing steadily louder.

"The world is wonderful," said the Turtle, giving the globe a gentle spin, "so many interesting places."

"Those two puzzles are almost identical, but they are still different," I said.

"Different communities with different cultures and different view-points ... "

"Perhaps if I took a different view-point, perhaps if I counted the number of pointy bits ... "

"But all with a common bond of humanity ."

"Another idea would be to look at what the puzzles have in common ... "

"But ... I need a theme for my next trip ... perhaps Famous Carnivals of the World ... Yes! That would be excellent!"

"This is terrible! I don't know why I bother with Logo at all! I give up!"

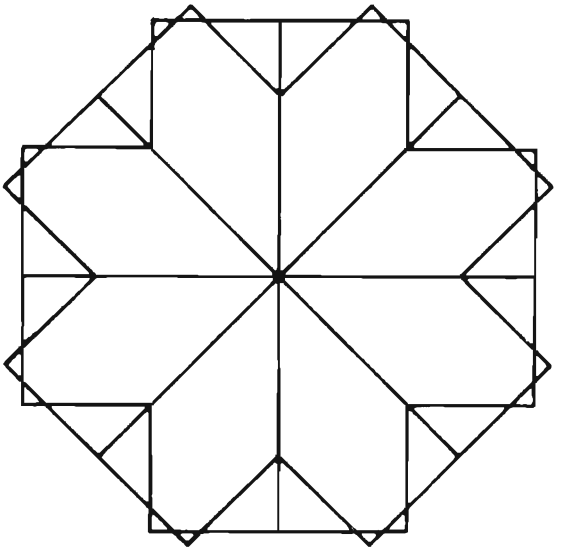
The Turtle turned to me and said, with a twinkle in his eye, "Really, EBN, the search for solutions to these puzzles should be driven by something like love."

"More like hate," I said shortly.

"Aaah, yes," said the Turtle. "Love. That's it. Love with a capital L ... L as in Lili."

He turned back to the globe and left me glaring at his shell.

PUZZLE 28



"You know, EBN," said the Turtle, "you seem to have missed the fact that all the puzzles from Puzzle 16 to Puzzle 40 have at least one property in common."

"You mean that they were all drawn by you?"

"Two properties in common."

"You mean that they are all hard to solve?"

"Three properties in common."

"You mean that they all have riddles attached?"

"Four properties in common."

"You mean that they all ... "

"EBN! You know that I mean a geometrical property in common."

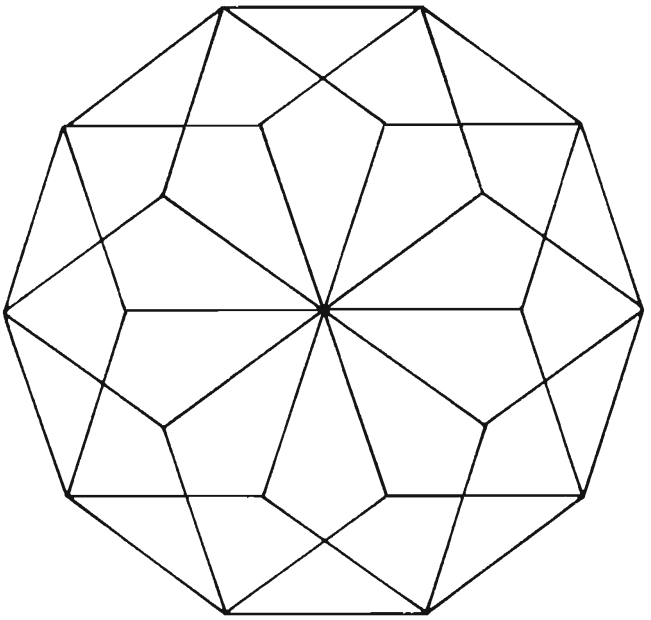
"Just what I was about to say when I was interrupted," I said innocently, "but thanks for the clue."

The Turtle glowered at me; then, without a word, he handed me a sheet of paper and stalked off.

I looked at the paper:

Thalia enjoys a good giggle,
And poetry's high on her list,
But give her a role in a riddle,
And neither of them will be missed.

"Thanks for the additional clues," I called down the hall.



PUZZLE 29

We were sitting in front of the living-room fire. The Turtle was reading his 'good literature' again, and I was writing poetry ... well, I was writing. After a while I looked up and said:

"Ahem ... here is my latest offering ...

They tell me that man learned to count
By looking at his fingers,
T'was done by people long ago
Their legacy still lingers.
But if I had made the counting rules
I think it would be fun,
To have the numbers go like this;
One Two Three Four Thumb.

... Well? What do you think of that?"

The Turtle laid the leather-bound volume reverently on his lap and looked across at me.

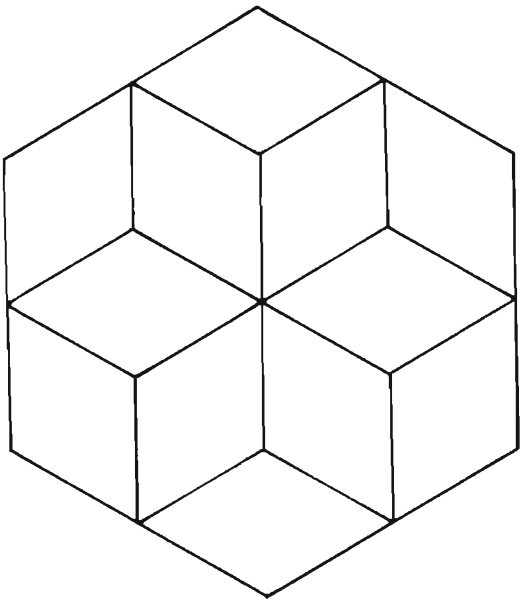
Then he raised his book again. "This was written more than one hundred and fifty years ago," he said. "Think of that."

Taking a deep breath he read aloud:

"Vex not thou the poet's mind
With thy shallow wit:
Vex not thou the poet's mind;
For thou canst not fathom it.
Clear and bright it should be ever,
Flowing like a crystal river;
Bright as light, and clear as wind."

I sat in silence and thought about it.

PUZZLE 30



The Turtle and I had had an early dinner and were now out in the garden setting up for a night's observing. There was a small amount of very thin cirrus high in the south, but otherwise the sky was clear. The last faint glow of the twilight was fading in the west, and the stars were cold, hard points of light.

"Looks like the seeing will be good tonight," I said.

"Certainly does," said the Turtle, "seems to be hardly any scintillation at all."

"Yes ... and the planets are well placed. We should get a really fine view of Saturn tonight ... good seeing and the rings in an excellent aspect."

For a while we simply gazed at the sky as the night deepened. Then the Turtle took the dew-cap off his small reflector.

"What do you think of Bode's Law?" he asked as he adjusted the setting circles, "I was reading a bit about it today."

"Pretty rocky. It doesn't work for Neptune and Pluto, and the index has to be fiddled for Mercury. Still, it's an interesting correlation."

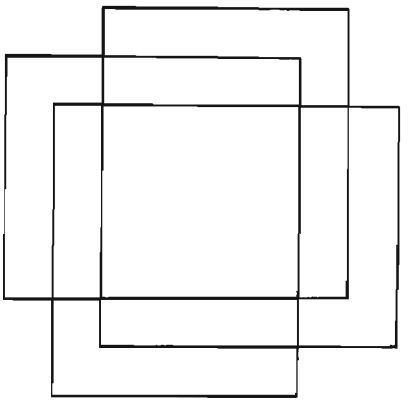
"Don't you also have to assume that the asteroids represent the remnants of the fifth planet?"

"Yes. Well, that's not stretching things too far. In fact, you can probably turn it around and use the law to predict that there should have been a planet at that distance from the Sun."

"Sounds like the product of a fertile imagination to me," said the Turtle, turning the telescope on Saturn.

"Should be right up your orbit," I said, going inside to get a warmer jacket.

PUZZLE 31



It was a beautiful day. The sunlight sparkled on a calm sea and small waves caressed the sand. The Turtle and I were wandering slowly along the beach, picking up shells and other treasures.

"I'm always impressed by the patterns that occur in nature," said the Turtle, examining the intricate network of ridges on a small shell, "makes you realise that all those Logo puzzles have some connection with the real world."

"Here's a dead fish," I said, poking the remains with my toe. An interesting aroma arose, and the Turtle hurried ahead with a rather set expression on his face.

Later on, back at the cottage, we were working on a Logo puzzle that had given me some problems. I had made sketches of possible solutions, and had asked the Turtle for one of his usual 'helpful hints'. He thought for a while but said nothing; finally he reached over and picked up a large Nautilus shell from the desk.

"Beautiful," said the Turtle, turning the shell slowly in his hands, "*Nautilus pompilius* ... leaves us turtles for dead ... leaves us for dead."

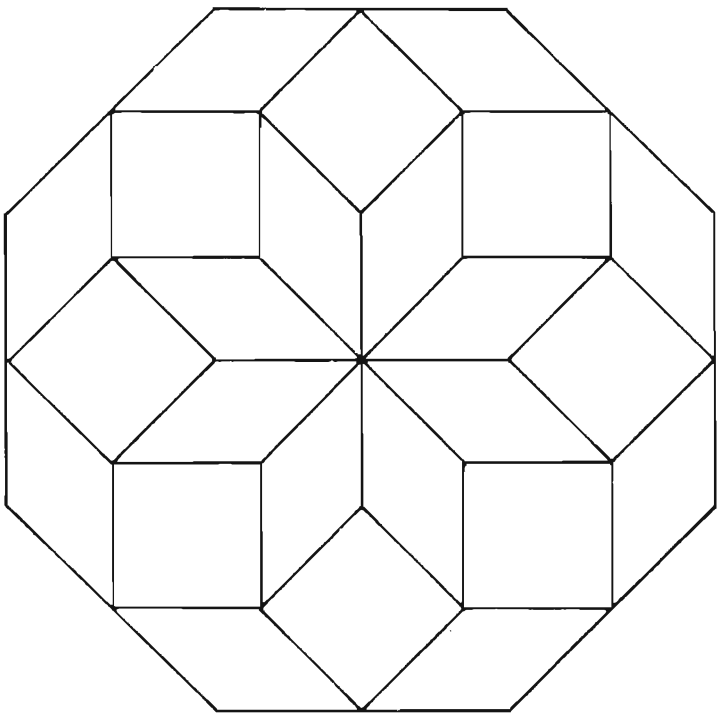
"It does do my heart good to hear you wax lyric," I said, "but I thought that we were discussing this puzzle. Aren't you helping me any more?"

"I am," said the Turtle, "I am."

"I don't see it," I said, crumpling up my page of scribbles and throwing it at the wastepaper basket. "I'm getting tired of hints ... always hints."

The Turtle walked over, picked up the ball of paper, and dropped it into the basket. "Try a little lateral thinking, my boy. Yes ... a little lateral thinking will be just the ticket."

PUZZLE 32



"Here's a clue to your next puzzle," said the Turtle as he trotted into the study.

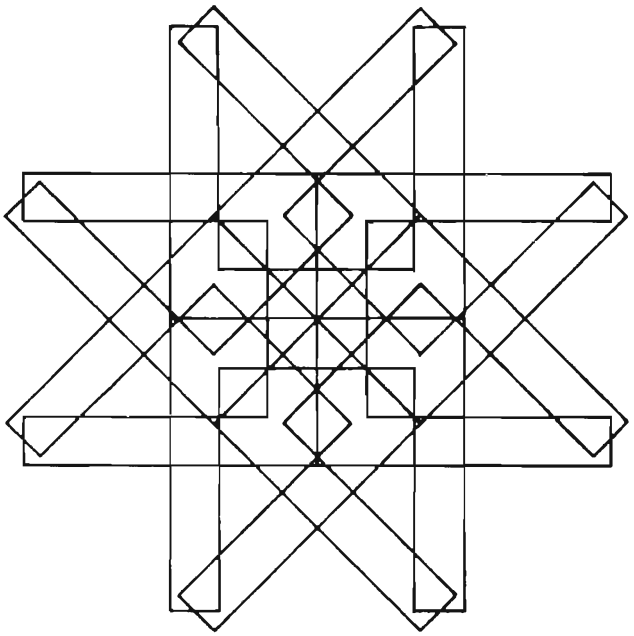
"Oh, no!" I said, eyeing the leather-bound book in his hand, "not the classics again!"

"Why not?" said the Turtle, sitting down in my most comfortable chair. "An endless source of wisdom." He opened the book at a marked page and read aloud:

> **"Primo, secundo, tertio, is a good play; and the old saying is, the third pays for all: the triplex, sir, is a good tripping measure; or the bells of Saint Bennet, sir, may put you in mind; one, two, three."**

"What am I supposed to make of that? It has all the fishy smell of a red herring."

The Turtle tapped the table a few times with his tooled leather bookmark. "Make of it what you will," he said, licking his lips.



PUZZLE 33

The postman's whistle sounded, and the Turtle bustled out to see what the day might bring. He returned with a considerable stack of mail, and for a while silence reigned, apart from soft slitting sounds and occasional rustles.

"My Goodness," said the Turtle, holding up a single sheet of light blue paper, "listen to this ..

If you have infectious sores,
Don't give them to your daughter.
Strip two miceys of their skins,
And dunk them in the water."

"Who on Earth sent you that?" I asked.

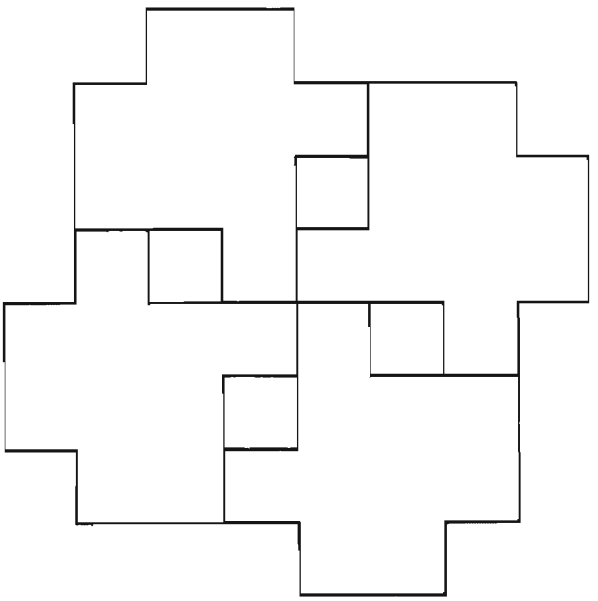
"Old Anonymous," said the Turtle, turning the paper over and looking in vain at the other side.

"Well, I've got one too," I said, leaning over and handing him a similar sheet of light blue paper. On it was a small matrix of numbers and letters:

3	**V**
8	**X**
14	**B**
15	**H**

After a while the Turtle looked at me with something approaching panic in his eyes. "Now we are **both** in trouble," he said.

PUZZLE 34



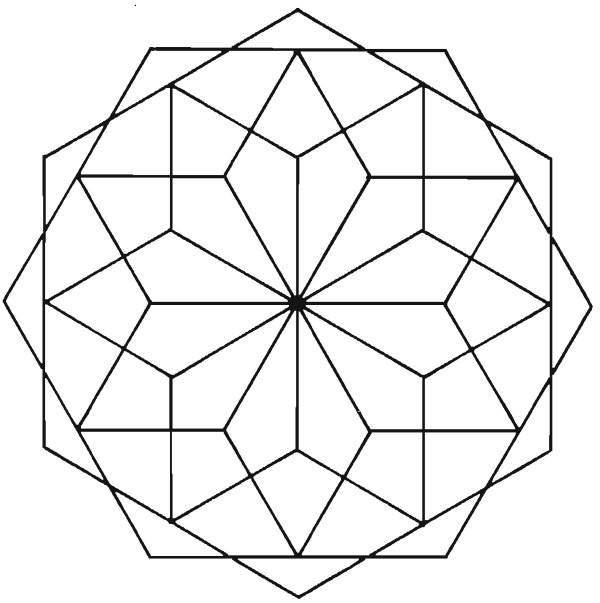
When I arrived home from my Sanskrit class, the Turtle was ensconced in the kitchen stirring some vile-looking concoction that he had assembled in our large punch bowl.

"What's that yuk?" I asked politely, as I took off my coat and hung it behind the door.

"I felt like celebrating after we solved that anonymous puzzle. I'm making a variation on our traditional bowl of punch," said the Turtle, as he dipped a cup into the liquid and sipped contemplatively.

"Looks more like sheep dip to me," I said, still polite and reaching for the cup. "How do you expect us to drink all of that before it ferments?" I took a cautious sip ... actually it wasn't too bad.

"All will be well," he said, "we have Soma's blessing."



PUZZLE 35

"You know," I said to the Turtle, "it just struck me that there is no way that I could do this puzzle without having some idea of the basic underlying patterns ... if I used two commands to draw each little line segment the list of commands would be huge."

"This calls for a celebration, my boy," said the Turtle with real pleasure on his face, "you have reached a new plane of understanding ... I could not have said it better myself." He trotted off to get the last of the punch.

"Gee, thanks," I said, when he returned, "that is high praise indeed, especially coming from you."

The Turtle looked at me sideways for a moment, but decided to let it pass.

"Well," he said, dipping a cup of punch, "what do you do next?"

"I guess I need to design some kind of sub-pattern that I can use to build the big pattern ..."

"Yes," said the Turtle, "do you want a clue?"

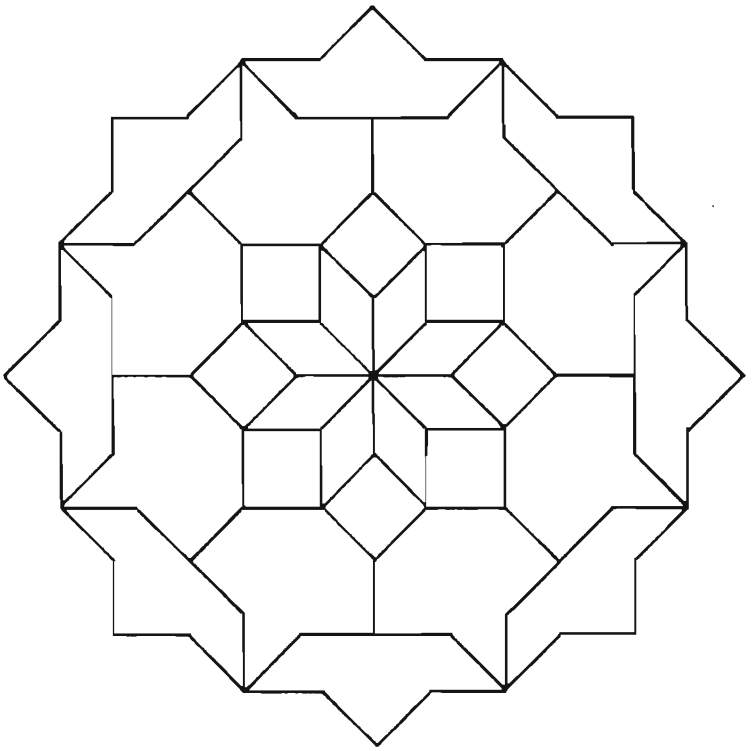
"That would indeed be comforting," I said, sipping.

The Turtle thought about it for a moment (he always seemed to think for a moment, but I am sure that he concocted his riddles in advance) and then, taking a deep breath, he recited:

> **"If you have an old sea-worthy vessel,**
> **Want something in common with fun,**
> **Islands Oxley, and Tern, and Cape Wessel,**
> **Offer freedom to play in the Sun."**

"How come I have a sinking feeling?" I said.

PUZZLE 36



I found the Turtle in the study. He was looking decidedly worried, not his usual jaunty self.

"What's the matter?" I began ... but, even as I spoke, I noticed the sheet of light blue paper lying on the desk beside an opened envelope. "Oh, no! Not another of those anonymous notes?"

"I am afraid so," said the Turtle without looking up, "although this one is not anonymous." He passed the sheet of paper across with a sigh.

> **Greetin's from Modulo Eight and 'is bunch,**
> **We 'ad Sneaky Snake and 'is wife 'ere for lunch,**
> **Together we cooked up this riddle for you,**
> **Begin with line one if you're lackin' a cue.**

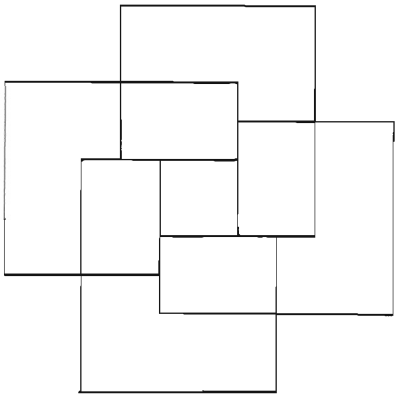
"Who is Modulo Eight?" I asked.

The Turtle looked at me wearily, "An octopus ... an old aquaintance of mine. He and two inseparable friends formed a puzzle-solving group a few years ago ... but, they have no finesse ... no respect for real insight."

I smiled at the Turtle, and then looked again at the note:

> JXUDU NJJME AUOIT EDJTU FUDTE DCU
> NBYZC LMNEY SXYJY HXMIH GYFUF YOWUF YOWUX YHXLI H
> NBYMY WIHXE YSXYJ YHXMI HUNBI OMUHX IZWUY MULME CHMGY H
> TEEMA XHVXT GLHYM AXPHK EWMXX FTQAO QMZEA RFTQI ADXP

"When do we start?" I asked. I didn't expect an answer; the Turtle likes to be on top in the puzzle solving business.



PUZZLE 37

I burst into the laboratory and the Turtle looked up startled. "What's wrong?" he said, his face turning a lighter shade of green.

"I've got it!" I shouted, "I've got it! I've got it!"

"What've you got?" said the Turtle, "are you ill?"

"No," I said, calming down a little, "I suddenly understand part of what you've been hinting at."

"Oh," said the Turtle, sinking down on a chair. He gradually regained his normal colour. After a while he said: "Don't do that to me. I've just used up a year's supply of adrenalin."

"Sorry," I said.

"OK ... I'm pleased," said the Turtle. "Tell me about your insight."

"Well," I said, a bit subdued, "it occurred to me that using procedures, **and** looking for neat solutions, **and** trying to understand the organisation of patterns are **all** just different aspects of the same thing."

The Turtle jumped up suddenly from his chair and rushed at me. I stepped back in alarm, fear clutching at my throat as the word 'hydrophobia' flashed through my mind. The Turtle slid to a halt in front of me and shouted, "You've got it!"

"Oh," I said after a while. Then I started to laugh. "Serves me right," I said, "I'll endeavour to keep my enthusiasm under control in future."

"Don't do that, my boy," said the Turtle, beaming, "enthusiasm is the spice of life. I really am pleased that you have begun to see the light."

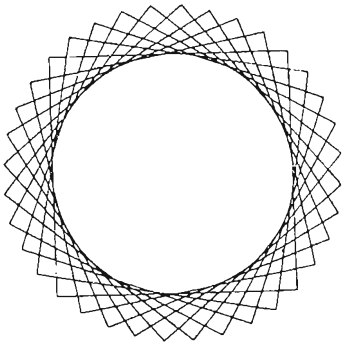
"In that case," I said, "how about a hint or two about Puzzle 37?"

The Turtle didn't hesitate; it must have been all that adrenalin:

> **"Andromeda stands, quintessence distilled,**
> **Cassiopeia and Cepheus rejected,**
> **She has taken up arms in Messier's Guild,**
> **Perseus, her love, lies neglected."**

"Can turtles get hydrophobia?" I asked.

PUZZLE 38



"You know, EBN," said the Turtle, "I should mention another reason why the riddles are important."

"You mean in addition to driving everyone to distraction?" I said.

"Can't you be serious for a few minutes?" said the Turtle tartly.

"Sorry," I said, repressing a smile. "Please continue."

"Well," said the Turtle, "the Logo puzzles offer people a chance to use mathematical reasoning to solve interesting problems, but the challenge is not over once the pattern has been drawn. The search for general principles, for the underlying relationships and organisation, should help people develop a deeper understanding of the mathematics involved, and it should also help them to develop better problem-solving skills."

"So ... what's that got to do with the riddles?"

"Well," said the Turtle, "apart from giving clues and hints, the riddles require a style of problem solving that is close to that needed in scientific and mathematical research. So thinking about the solutions to the riddles can help people develop a research approach. And that's also the best way to approach the Logo puzzles themselves."

"What do you mean by saying that the style of problem solving is close to that used in research?"

"I mean that a research problem is not cut and dried," said the Turtle. "One can't be sure that one is on the right path, or even if there is a right path. It is often necessary to try a number of promising leads before the best one is found."

"And often the answers can't be checked," I said, picking up the thread.

"Right!" said the Turtle. "There are no answers in the back of the Book of Life."

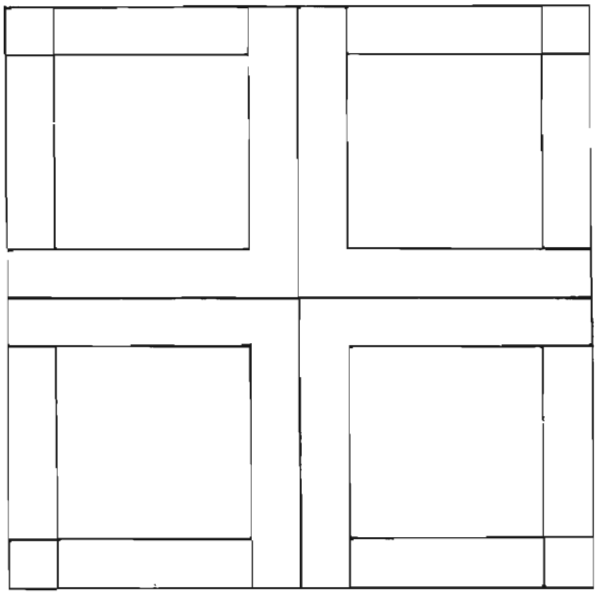
"But the solutions to the different riddles must be consistent with each other, and with the solutions to the puzzles, so there must be a level of cross-checking built in to the whole booklet," I said.

"Just like in real life," said the Turtle. "For example, consider the following riddle:

Not West Point, but wetter.
Not Worcester, but wester.
Not Wabuk, but walled."

"Just like real life," I said.

PUZZLE 39



I was working in the study when the Turtle came in with an enigmatic smile on his face. Without a word he handed me a small sheet of light blue paper. On the paper was written a group of letters

YHWH

and it was signed, '*From He Who Came From Across The River*'.

"YHWH? Is this from Modulo Eight?" I asked.

"No," said the Turtle, "it is from me."

"From you! What's all this stuff about coming from across the river?"

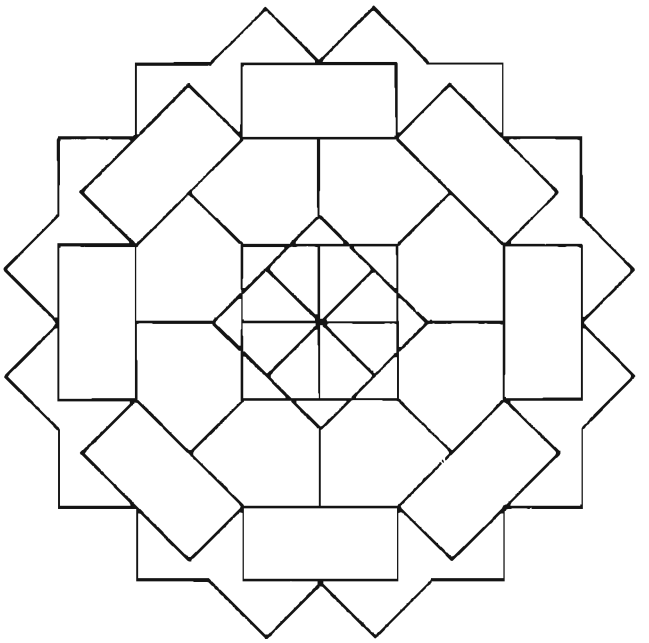
"Oh, I was just over the river at Good Old Dave's getting some fish and chips," he said, "I wrote it while I was waiting."

"Must have taken a lot of writing," I said. The Turtle smiled.

The smell of fish and chips percolated down the hall and into the study.

"Let's eat," I said, dropping the paper on the desk and leading the way to the kitchen.

PUZZLE 40



The Turtle poked his head around the study door and said, "How's it going?"

"Not good," I said. "This last puzzle is giving me all sorts of problems."

The Turtle looked at the screen, then at my notes, and then shook his head. "I thought that you had developed a fine approach to looking for sub-patterns."

"My fine approach seems to have abandoned me," I said.

"Look," said the Turtle, "what really counts is sticking to it when the going gets tough."

"That's easy to say," I said, "but it's not easy to do."

"Look," said the Turtle again, "this puzzle is related to two others in the booklet ... surely you can take comfort from the fact that there is a pointer to one of these other puzzles in the riddle below."

"That's really comforting," I said, "Thank you."

"Don't mention it," said the Turtle.

"I wouldn't dream of telling a soul," I said in a whisper.

"Well," said the Turtle, "The Last Conundrum."

"Yes ... I'm really quite sad that there are no more."

"Oh!" the Turtle looked surprised. "Have you solved this puzzle already?"

"No," I said.

"Then it's not all over **yet**," said the Turtle.

FURTHER STEPS

The Turtle and I were out by the fish pond studying and working our way through another bowl of celebratory punch. There was some excellent cheese and a loaf of Italian bread on the table.

After a while I leant back in my chair, stretched, and said: "Well ... it certainly feels good to have made it through your booklet. It was well worth it."

"I'm glad that you feel like that," said the Turtle, looking up from his notes and smiling.

"Actually, I was going to ask you if you could tell me a little more about the connections between the Logo puzzles and mathematics ... I was not always sure when I was learning mathematical ideas."

"I'd be glad to," said the Turtle, "but why don't you first read my discussion paper *Turtles Speak Mathematics* ... that will start you off on the right foot."

"Good idea," I said. "Is that the one published by the Curriculum Development Centre, Canberra, Australia (1988)?"

"Yes," said theTurtle.

"While we're at it ... you might as well recommend a book or two that I could read ... about Logo and the types of problems that we were working on. Any suggestions?"

The Turtle took a sip from his cup and thought for a moment: "Yes, I could do that now that you have worked through the puzzles ... "

"Are you suggesting that I couldn't understand any other book until I'd worked through yours?"

"No! Of course not!" said the Turtle. "I just believe that most of the introductory Logo books tell you too much about problems that you can solve yourself. But, once you've had a go at the kind of problems in my booklet, then there are a lot of good books that you would benefit from."

"Name some," I said.

"Well, you would certainly enjoy *Mindstorms*. It will give you a good insight into the possibilities offered by Logo, and it will also help you understand the way that mathematics is 'built into' my booklet. Actually, I've got a copy here." He reached into his brief-case, pulled out a well-thumbed paperback, and passed it across to me.

"Thanks," I said, getting out my note-book, "Let's see ... Seymour Papert ... The Harvester Press, 1980."

"Yes," said the Turtle, wistfully, "1980 was a vintage year for turtles. There are also a number of good books published by Addison-Wesley ... in particular, you might look at the first two of David Thornburg's books, *Discovering Apple Logo* (1983) and *Exploring Logo Without a Computer* (1984) ... and I know

that you like the style of Molly and Dan Watt's book, *Teaching with Logo* (1986) ... Yes, you can depend on Addison-Wesley turtles."

"Can you recommend a book that will help me learn about the more advanced features of Logo itself ... you know, recursion and lists and ... "

"Sure," said the Turtle, scratching his head, "Hal Ableson's book *Logo for the Apple II* (Byte/McGraw-Hill; 1982) remains one of the best, and Donald Martin, Stephen Prata, and Marijane Paulsen have written a good reference book called *Apple Logo Programming Primer* (SAMS; 1984), and there are ..."

"Hold it!" I said, "Slow down! I'm getting writer's cramp!"

" ... exciting advanced books by David Thornburg, *Beyond Turtle Graphics* (Addison-Wesley; 1986), and Brian Harvey, *Computer Science Logo Style* (MIT Press; 1985) ... and if you want to pursue the possibilities of Logo and mathematics, right up to General Relativity, then you'd get a lot out of *Turtle Geometry* by Hal Ableson and Andy diSessa (MIT Press; 1980)."

"Thanks," I said, still writing fast, "that should keep me going for a while."

"Yes," said the Turtle, "that'll keep you out of my hair."

I gave him a long look as I put away my note-book: "You can't possibly mean that."

"Of course not," said the Turtle, a touch of fondness creeping into his voice. "Talking with you helps me to realise how far I have advanced."

"I refuse to rise to the bait," I said, throwing some bread crumbs into the fish pond.

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HOW TO USE THIS BOOKLET

This booklet is designed to be shared. That is, it is intended to support a 'collaborative problem solving' process wherein the problems are tackled by two, or more, people working together and sharing ideas and insights.

If you are working through the material by yourself then, clearly, you are free to proceed in any way that you wish. If, however, you are working as part of a 'problem-solving group', then there are a number of constraints that you should try to observe in order that the experience can be as valuable as possible for all members of the group. This is particularly true if you are the group 'leader'.

Leaders can be teachers, parents, or younger persons. Often people with a range of ages will be involved in the groups; for example, parents and their children, teenagers and their younger siblings, teachers and their classes. It is of fundamental importance, in such cases, that the older, more experienced, members of the group are sensitive to the needs of the younger members.

I consider that the leader of a problem-solving group has very clearly defined responsibilities. As group leader you should try to:

(a) Ensure that the members of the group are allowed ample time.

They need time to become familiar with the booklet before they embark on a serious attempt to solve the puzzles and riddles. If possible each member of the class, or group, should have their own copy. They should be encouraged to take the booklet home and to read through it at their leisure; to think about the puzzles in their spare-time; to discuss the riddles with their parents.

They need time to think about each problem that they encounter. They need time to formulate ideas about possible meanings. They need time to test out their ideas. They need time to discuss their ideas with their peers and leaders. They need time to consider each other's ideas.

(b) Allow group members to experience some confusion.

The material in this booklet is intended to be somewhat confusing at first sight. In real life we cannot avoid being confused every now and then. A feeling of confusion is an indication that our understanding of a given situation is inadequate ... in other words, confusion signals a chance to learn.

Adults should recognise the danger of shielding young people from feelings of confusion. Life does not serve up its problems in a carefully graded sequence of little packages, each of which can be solved by 'the average student' in a normal class period. We owe it to our students to allow them to feel confused sometimes; to allow them to learn how to cope with, and even benefit from, confusion; to allow them to develop responses, other than panic, to situations where they are initially 'out of their depth'.

(c) Keep the problem-solving sessions relaxed.

The assimilation of new ideas takes time; even though a given puzzle may have been solved, there may be benefit in looking at the solution again a few days later ... can a better (simpler) approach be found? It is important to allow the problems to percolate around and around in people's minds. Under no circumstance should you require a class to complete one puzzle per class session. Keep a light-hearted approach ... try to balance serious intent with fun.

(d) Allow each member of the group to approach the puzzles and riddles in the order they prefer.

The sequence of Logo puzzles is, overall, a sequence of increasing difficulty. There is some advantage, if you are a beginner, in working through the puzzles in order. But the order is not particularly important - the material is designed to be restructured by each reader. If possible, set up the group sessions so that individuals can re-order the material in any way that seems appropriate to them.

It is a good idea to attempt all the puzzles and riddles eventually - there are many inter-connections between various parts of the booklet. This means also that you should keep the booklet intact ... if you separate parts out you will destroy some of the connections and some of the clues.

(e) Ensure that all members of the group know how to operate the computer, if one is used, and that they know enough Logo to get started.

Because I wished to produce the booklet in a form that could be used with any implemention of Logo I have not included machine-specific details. Therefore, I have not discussed the Logo language itself, nor procedures for operating computers.

These omissions should not be a limitation ... there are many books that cover these topics (see, for example, the short bibliography given in the last section), and there is a steadily growing expertise among teachers, students, and parents. Furthermore, the Logo puzzles presented here can be drawn using a very small subset of the Logo commands. In LCSI Apple Logo these are:

FORWARD (FD)	BACK (BK)	RIGHT (RT)	LEFT (LT)
PENDOWN (PD)	PENUP (PU)	REPEAT	

The commands needed to create and edit procedures including

TO	END	EDIT

Plus the housekeeping command

CLEARSCREEN (CS).

(f) Make sure that the following additional resources are available to the problem-solvers:

A LARGE DICTIONARY
AN ATLAS
A GOOD ENCYCLOPAEDIA
A MODEST LIBRARY OF CLASSICAL LITERATURE

- the riddles are designed so that the process of solving them can become a 'community' project. Group members should be encouraged to share their resources.

I have assumed that teachers in areas other than mathematics and computing will be seen as a resource, and will be invited to share in the problem-solving activities; in some cases their skills and knowledge will be necessary.

(g) **Avoid imposing your own solutions.**

You can easily lead other members of the group to your solutions without meaning to ... whenever you lead a discussion, or help with a difficulty, your answers will be conditioned by your theories about the solutions to the puzzles and the meaning of the riddles. Remember, **always**, that your theories may be wrong!

(h) **Approach the activities with an open mind.**

I have written this booklet not only for students, but for teachers and parents as well. I hope that teachers and parents will see in it an opportunity to extend their own learning. I firmly believe that the best classrooms, and the best homes, are those where adults and children are learning together.

Make sure that you can admit to not knowing the answers - then you will be free to participate in the group process. Above all - enjoy the adventure yourself!

ACKNOWLEDGEMENTS

The approach to problem solving presented in this booklet has grown over a number of years. During that time I have absorbed ideas from many sources - from the research literature and monographs in several fields, from my scientific and educational colleagues, and from the parents, teachers, and children with whom I have worked.

I first read *Mindstorms*, by Seymour Papert, early in 1983. His ideas, and those of other members of the Logo community, continue to excite and challenge me. Some of the connections with Papert's work are outlined in the discussion paper entitled "Turtles Speak Mathematics", published by the Curriculum Development Centre, Canberra, Australia (1988).

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THE AUTHOR

Dr. Barry Newell is an astronomer who is active in science and mathematics education. He is committed to the task of working with teachers and parents to improve children's scientific and mathematical experiences. He has extensive experience with computers in education and has led many Logo in-service workshops. Dr. Newell is married with six children.

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