2.4 Elasticities

Often, we want to answer questions like: is the consumer's demand of a good **very** responsive to its price? Does the consumer's consumption of one good increase by **much** when her income increases? One measure of the responsiveness of a mapping (demand is, after all, a mapping) to one variable is the (partial) derivative. That won't do in our case.

Indeed, suppose that I tell you that an increase in the price of a good results in a reduction (per dollar increased) of 20 (units of the good). Is that a lot? If I tell you that for some other good an increase in its price results in a reduction (per dollar increased) of 2 units, would you conclude that the demand of the first good is more responsive to its price than the demand of the second good is to its price? What if I tell you that the first good is phone calls and the second good is cars. That is, if the price of a phone call increases by one dollar the consumer makes 20 fewer calls (say, in 10 years) and if the price of a car goes up by that same dollar the consumer buys 2 fewer cars (in those 10 years). Probably now you think that the consumer's demand for cars is extremely sensitive, and her demand for calls extremely insensitive to changes in their respective prices.

Why? Most likely you have thought that the increase in one dollar in the price of phone calls is a large increase (proportionally to its price, I mean) and the reduction in the amount of calls not that large (proportionally to the quantity usually purchased), whereas in the case of cars the opposite is true. If that is so, you came to the same conclusion as economists: the derivative measures absolute changes which, for our purposes, –that is, for comparing sensitives of goods that come in very different prices and quantities– is a big problem.

So, how about measuring sensitivities with normalized prices and quantities? For instance, how about talking about percentage changes? In our example, we may say: as a result of a 1,000% increase in its price (if the price of phone calls was 10 cents) the demand for calls decreased by 1% (if the consumer was making 2,000 calls in those 10 years). Also, as a result of a 0.005% in the price of a car (say, $20,000), the consumer's demand of cars decreased by 66% (if she used to change cars every three years or so). Put in this terms, the consumer's demand for cars seems clearly much more sensitive to changes in prices than her demand for phone calls. Indeed, now we are comparing things that are comparable.

A measure like that is what we call **elasticity**. We may define the elasticity of a consumer's demand of a good with respect to changes in its price, the price of other goods, or the consumer's income. Indeed, all those variables affect the demand, as we have seen.

2.4.1 Income elasticity

This is simply the percentage change of demand divided by the percentage change of income. That is, informally, for good 1 and representing "increase" with the symbol Δ as is common,

$$\frac{\frac{\Delta D_1(p_1,p_2,y)}{D_1(p_1,p_2,y)} \times 100}{\frac{\Delta y}{y} \times 100}.$$

The first thing to notice is that the 100 in numerator and denominator cancel. That is, we said "percentage", but we could have said "per unit". The second thing to note is that what is dividing in the numerator may be written as multiplying in the denominator, and vice versa. Thus, the expression above is the same as

$$\frac{\Delta D_1(p_1,p_2,y)}{\Delta y} \times \frac{y}{D_1(p_1,p_2,y)}.$$

The third thing to note is that this expression is a little bit, well, ambiguous. Indeed, beginning at some value for income y (and prices p_1 and p_2), the value of $\frac{y}{D_1(p_1,p_2,y)}$ is clear, but the other part of the expression may depend on the **size** of the change in y that we are talking about. This is the same issue that we discussed when we were talking about willingness to trade and so MRS. If you remember, we decided that whenever we talked about changes we would mean a tiny little change, a change arbitrarily close to zero. If we do so here (and we will), then whenever we talk about elasticity we mean a tiny little change in (in this case) income. That is, the **income elasticity** (of the demand of good 1 when prices are p_1 and p_2 and income is y) is

$$\lim_{\Delta y \to 0} \frac{\Delta D_1(p_1,p_2,y)}{\Delta y} \times \frac{y}{D_1(p_1,p_2,y)}.$$

That fancy object, $\lim_{\Delta y \to 0} \frac{\Delta D_1(p_1,p_2,y)}{\Delta y}$ is nothing but what we call (partial) derivative, isn't it? So, income elasticity (of the demand of good 1 when prices are p_1 and p_2 and income is y) is simply

$$\frac{\partial D_1(p_1,p_2,y)}{\partial y} \times \frac{y}{D_1(p_1,p_2,y)}.$$

Previously, we have defined what a normal or inferior good was. Remember: the difference was the sign of $\frac{\partial D_1(p_1,p_2,y)}{\partial y}$: when it is positive, we say that the good is normal, and when it is negative, we say that the good is inferior. Then, another way of saying this is: the good is inferior if the income elasticity of demand is negative. If it is positive, then the good is normal.

We may also make a difference between the case when the income elasticity is above 1 and the case when it is below 1. Let us understand what a lower-than-1 income elasticity means: a one percent increase in income leads the consumer to increase consumption of the good by less than one percent. Therefore, (since prices have not change) the consumer spends on the good a **lower share** of her income than before the increase of income. That is, as the consumer gets richer she (perhaps buys more of the good, if the elasticity is still positive but) spends a smaller proportion of her income in the good. Vice versa: if the income elasticity of demand of the good is larger the 1 for the consumer, she (certainly buys more of the good but also) spends a higher proportion of her income in the good as she gets richer. This is why we call goods with an income elasticity (for the consumer) larger than one **luxury goods**.

In any case, the general insight that we get is that income elasticity measures how the income share spent on the good behaves as the consumer´s income changes.

2.4.2 Cross-price elasticity

We can also measure the responsiveness of the demand of a good to changes in the price of another good by defining what we call cross-price elasticity:

$$\frac{\partial D_1(p_1,p_2,y)}{\partial p_2} \times \frac{p_2}{D_1(p_1,p_2,y)}.$$

As with income elasticity, this new concept measures the percentage increse in the demand of good 1 per percentage increase in the price of good 2 (for a tiny change in the latter). Obviously, the first term in the expression, $\frac{\partial D_1(p_1,p_2,y)}{\partial p_2}$ is positive if the goods are substitutes and negative if they are complement. Thus, this measure is an important one in determining if two goods(perhaps produced by two different firms, and so not "exactly" the same, say, two brands of soda) could be considered "the same" in practical terms. If a slight increase in the price of one (in percentage terms) results in a large percentual increase in the demand of the other (which indicates that consumers substitute the latter for the former in large numbers), then we may consider the two products to be the same "good" to all effects. Thus, this measure is important in "defining" what a market is, and so analyzing competition.

2.4.3 Price elasticity

Finally, we may define the elasticity of the demand of a good with respect to its own price. (Almost) As before, this measure is simply:

$$-\frac{\partial D_1(p_1, p_2, y)}{\partial p_1} \times \frac{p_1}{D_1(p_1, p_2, y)}.$$

Note the negative sign. It is common (although it is not done in the textbook) to add that sign. Why? The reason is simply that the sign of $\frac{\partial D_1(p_1,p_2,y)}{\partial p_1}$ is (usually) negative. Thus, in (virtually) all cases, the price elasticity would be negative without the sign. So, why carry the sign? (You may use whatever convention: everybody will understand you.)

Note what a price elasticity larger (or smaller) than 1 means: if the price increases by 1%, the demand decreases by more (less) than 1%. Then the **spending** on that good, which is nothing but price times quantity, will be lower after the price increase. That is, if a consumer's price elasticity of demand for one good is larger (smaller) than one, the spending in that good is lower (higher) the higher the price of the good. We will get some results from this observation in future chapters.

Finally, note that we have been referring to *a* consumer's demand all this time. However, in the future, when we talk about *market* demand, these measures may be defined also for that demand, with similar implications.

2.5 Analyzing the effects of price changes on demand

When the price of one good changes, the quantity that the consumer demands of that good will typically change. That effect is what $\frac{\partial D_1(p_1,p_2,y)}{\partial p_1}$ measures for the case of good 1. However, we may characterize (say) an increase of p_1 as actually incorporating two changes for the consumer. First, some bundles that were previously affordable (most likely including the bundle that she chose before) are not affordable anymore. That is, in a very intuitive sense, the consumer is now "**poorer**". Second, the price of the good, good 1 in this case, is now higher **relative** to the prices of other goods, e.g., good 2. That is, the amount of good 2 that she needs to **trade** in order to get an additional unit of good 1 (i.e., p_1/p_2) is now higher.

In fact, we could very precisely decompose the effect of the increase in the price, $\frac{\partial D_1(p_1,p_2,y)}{\partial p_1}$, in the part that is due to the consumer's lower **real income** and the part that is due to the higher **relative price** of good 1.

Le us begin by **isolating** the (notional) effect of **only** the increase in the relative price of good 1on the quantity demanded of that good by the consumer. That is, let us compute what would happen to that quantity demanded if the price of good 1 increased **but** the consumer was not poorer. That is, **if** we gave the consumer sufficient income to **compensate** for the effect that the increase in the price of good 1 has on her purchasing power.

The first question to answer is, what do we mean by the same purchasing power? A simple (but unsatisfactory) answer could be that the consumer can still (and just so) afford the bundle that she was choosing before. OK, let's investigate what that means. In Figure 17 we have drawn an example. Suppose the darker green line is the original budget constraint, with the consumer's income and initial prices. We have identified the choice at those prices and that income. (Remember: a bundle where the budget constraint is tangent to the indifference curve at that bundle.) Now the price of good 1 increases: suppose the lighter green line is the new budget constraint after that increase. Note that the bundle that the consumer was choosing before is not affordable now. At the new prices (the new slope of the budget line), we would need to provide the consumer with more income so that the old bundle is again affordable. The red line represents the budget constraint for the consumer if we do so.

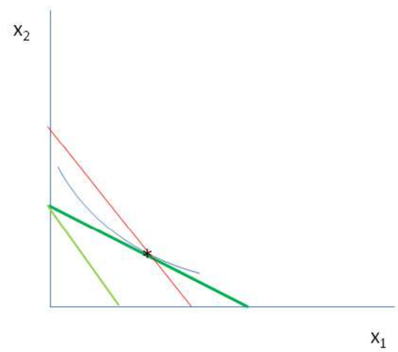

Figure 17

But notice that now the indifference curve at the original bundle would not be tangent to this new, **compensated** budget constraint. That is, if we did increase the consumer´s income in this manner, then the consumer would not consume the old bundle since then the consumer could afford bundles that she actually **prefers** to the original bundle! Don´t you think that this would be overshooting, in compensating the consumer?

If you do, then you would conclude that maintaining the consumer's purchasing power requires **less** income compensation than that. In fact, look at Figure 18. We have represented the same change in prices but we have compensated the consumer with less income: just enough to be able to afford (at the new prices) not the original bundle, but **some** bundle that is **just as good as** that. Don´t you think that this is enough? The consumer (choosing optimally, as our model predicts) is just *as happy* after the change in prices as before it, if that increase in price is compensated as in Figure 18.

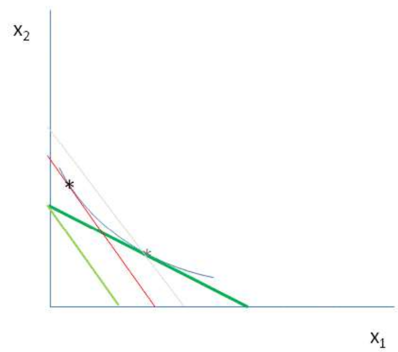

Figure 18

OK, now we know what we mean by "same purchasing power": the ability to achieve the same level of satisfaction (say, the same of "utility", the label). Then, the effect of the change in relative price of good 1 (in isolation) on the quantity that the consumer demands of that good is simply the effect on the demand of good 1 of a change in budget constraint from the dark green to the red in Figure 18. We call this effect (of an increase in the price of good 1 on the demand of good 1) the **substitution effect**: because good 1 is now relatively more expensive than good 2, and even though the consumer is as "rich" as before (in the sense discussed above), the consumer *substitutes* consumption of good 2 for consumption of good 1.

Figure 19 represents the size of this substitution effect (on the demand of good 1) for the example we were discussing.

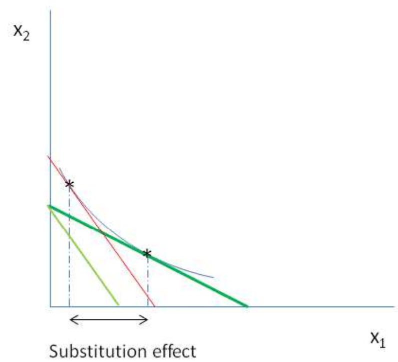

Figure 19

An increase in the price of good 1, when compensated with enough additional income so that the consumer can buy the cheapest bundle at the new prices which is just as good as the original, best bundle, leads the consumer to reduce the consumption of good 1 by a quantity equal to the double arrow in the figure. This effect is always negative, i.e., the substitution effect of a price increase is always to reduce the consumption of the good.

Let us now turn to the second effect of the increase in the price of good 1: the reduction in "real income", or purchasing power (since the consumer is not compensated). What would be the effect of this reduction in "real income" on the demand of good 1? The answer is simple: we were trying to decompose the effect of the increase in price in two parts. We know the total effect, the sum: simply the change in the demand $D_1(p_1, p_2, y)$ as a consequence of the increase in p_1. (For infinitesimal changes in p_1, and per unit of change, $\frac{\partial D_1(p_1,p_2,y)}{\partial p_1}$.) We also know one of these parts, the substitution effect. Therefore, the difference between the total effect and the substitution effect must correspond to the reduction in purchasing power.

We call this second part, or effect, corresponding to the reduction of real income (yes, you guessed) **income effect**. In Figure 20 we have added the representaiton of this effect for the example we have been discussing. If the change in the price had been compensated (if the consumer was not poorer now), the consumer would have chosen the bundle in the left in Figure 19. But because the consumer is not compensated, and so now her budget constraint is not the red one but the light green one, the consumer chooses instead the bundle represented in Figure 20. The difference in the demand of good 1 in those two bundles is the income effect.

Pay attention to where the arrows point in Figure 20. The total effect, in red, is the difference between the initial bundle (before any price change) and the demand at the new price (with no change of income, of course). We have obtained the substitution effect in Figure 19, and it is represented in Figure 20 in grey. Then, the rest, that is, the total effect minus the substitution effect, represented in orange, is the income effect. Indeed, this is how the consumption of good 1 changes because the consumer (at the new prices, and since she is not compensated with additional income) is now *poorer* in real terms.

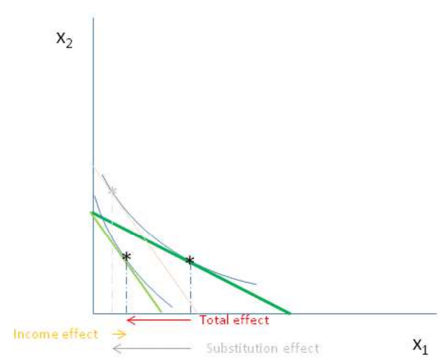

Figure 20

Note that in this example the income effect and the substitution effect point in opposite directions. That is, in this case the reduction of real "income" (from red to light green) causes the consumer to buy **more** of good 1. That is, good 1 is an inferior good for the consumer: because she is poorer, she buys more of that good.

Thus, the substitution effect of an increase in the price is always negative, but the income effect is negative for normal goods and positive for inferior goods. Make sure that you understand this simple point.

2.5.1 Computing income and substitution effects with calculus

Let us obtain the two effects using calculus. As you have probably guessed by now, the real difficulty is to compute the "income complement" that the consumer needs in order to attain, at the new prices, the purchasing power that she had before the price increase. That is, to compute how much income the red budget line represents.

So, let us tackle that issue. What we are looking for is the minimum possible spending that allows the consumer to (spending her dollars as our model predicts) purchase a bundle with the same label (utility) as the one she was purchasing before the increase in prices. That is, with the same label as the solution to our problem (5): $x_1(p_1, p_2, y)$ and $x_2(p_1, p_2, y)$. What is that label? Well, simply $u(x_1(p_1, p_2, y), x_2(p_1, p_2, y))$.

For example, if the consumer's preferences can be represented with the utility function $u(x_1, x_2) = x_1 \cdot x_2$ and the prices and income (initially) are, respectively, $p_1 = 5$, $p_2 = 10$, $y = 100$, then $x_1(5, 10, 100) = 10$ and $x_2(5, 10, 100) = 5$. (As an exercise, you can heck that indeed this is the case.) Thus, the label we are talking about is $u(10, 5) = 50$. Now suppose that the price of good 1 goes up to $p_1 = 10$. We want to know how much income the consumer needs to be able to buy a bundle with label 50.

We can put this in other words: among the bundles with label 50, that is, such that $u(x_1, x_2) = x_1 \cdot x_2 = 50$ (a constraint), we want to find the one that costs the least (an objective). The cost of that bundle will be $10x_1 + 10x_2$. That is, the bundle we are talking about is the one that solves:

$$\begin{aligned} &\min_{x_1, x_2} \{10x_1 + 10x_2\} \\ &\text{s.t. } x_1 \cdot x_2 = 50. \end{aligned}$$

In this case, the solution is $x_1 = x_2 = \sqrt{50}$ (approx., 7), and that bundle costs $10 \times \sqrt{50} + 10 \times \sqrt{50} = 20\sqrt{50}$ (approx., 140). Thus, the consumer would need \$40 extra to have the same "real income". (Note that in order to buy the old bundle, $(10, 5)$ the consumer would now need more: \$150, but, as we have discussed, with that income and the new prices the consumer would actually buy a bundle with higher label. Check that out.)

Thus, in general, computing the necessary monetary income that leaves the consumer unaffected in front of an increase in one price involves solving:

$$\min_{x_1, x_2} \{p_1 x_1 + p_2 x_2\}$$
$$\text{s.t. } u(x_1, x_2) = \overline{u}.$$

for the new prices p_1, p_2 where $\overline{u}$ is the label (utility) the consumer was attaining before the change (that is, with her actual monetary income and the old prices).

Now we know how to compute how much income the consumer needs to be compensated for the change in the price of one good.[2] With this in hand, we know that, if after the change in the price of good 1 the consumer was compensated so that her income was as calculated above, all the change in the consumption in good 1 would be due to the fact that now good 1 is **more expensive relative** to good 2, so that the consumer substitutes good 2 for good 1. In the example, the consumer reduces consumption of good 1 from 10 units to $\sqrt{50}$ units. That is, reduces her consumption of good 1 by (approx.) 3 units even though her "real income" is unchanged. That is what we call **substitution effect**, and is represented in Figure 19.

When no compensation happens (that is, as nobody actually gives the consumer the extra \$40), the consumer's choice actually solves (5) with the new prices and the same income, in our example, with prices $p_1 = 10$ and $p_2 = 10$. You can solve that problem to see that, with her old and uncompensated income and the new prices, the consumer will choose the bundle $x_1 = 5$, $x_2 = 5$. That is, the total effect of the change in the price of good 1 is a reduction of 5 units of consumption. Therefore, the **income effect** (the consequence of the actual reduction in "real income", when the effect of the increase in relative prices has been taken care of) is a reduction in $\sqrt{50} - 5$ (approx. 2 units).

[2] In fact, you should note that there is nothing special about the fact that we consider only the change of one price. If all or many prices had changed, we could have used the same reasoning to obtain how many dollars the consumer would need to be compensated for the change.

2.6 Aggregate demand

Typically, we are interested in analyzing outcomes in the market. A "**market**" is simply an intellectual construct that represents the aggreagation and interactions of (perhaps) many agents' decisions to buy and to sell **one good**. We have analyzed so far one of these agent's (a consumer) decision to buy the good. In a market, several (perhaps many) of these consumers will trade. Different consumers will typically have different preferences and different income, and so facing the same price(s) will make different decisions. Thus, for each of these consumers there will be a mapping from prices of the good (given other prices and consumers' incomes) to the quantity they demand of the good. That is, each consumer's decisions in the market can be characterized by her **individual** demand curve (function).

How about the market as a whole? That is, is there a mapping from prices of the good to total (**aggregate**) quantity demanded? Is there an aggregate demand curve (function)? If consumers "mind only their own business", it is simple to see that the answer is in the positive: aggregate demand at each price will simply be the sum of the demands of all the individual consumers at that price. Thus, obtaining that mapping is as simple as obtaining the demand function for each consumer, then add all of them up.

Note that we **add the quantity** demanded by all consumers for each price. That is, graphically, we add demand curves **horizontally**. For instance, if there are two consumers, Ms. A and Mr. B, and they have a demand function for good 1 given by $x_1^A(p_1)$ and $x_1^B(p_1)$, respectively, then the market demand function for good 1 will be simply $x_1(p_1) = x_1^A(p_1) + x_1^B(p_1)$.

Remark: as you know, the demand function of each consumer also depends on other prices (the price of good 2 for instance) and their income. Usually, when we look at one market in particular, as we will mostly do from now on, we keep those other prices and incomes fixed, and so we don't write them in the list of variables. Also, we typically represent the demand function with the letter D instead of x. Thus, when we write $D_1^A(p_1)$ we simply mean $x_1^A(p_1)$, and likewise, we write $D_1(p_1)$ for aggregate demand. Moreover, if there is no need to specify the good, we even drop the subscript 1. Just keep in mind that we are just talking about those demand functions that we obtained from solving our problem (5). Finally, when we **invert** the demand function (individual or aggregate) we typically use the notation $P(q)$. That is, $P(q)$ represents the price (p_1, if we are talking about good 1) at which the consumer/market demand an amount q, that is, at which $D(p_1) = q$.

Remark: be careful (in the future) not to aggregate individual demands vertically. That is, not to aggregate for each quantity the price that each consumer would need to see in the market for that consumer to buy that quantity. (That makes not much sense, does it?) In other words, in our example above, the inverse demand function is **not** $P(q) = P^A(q) + P^B(q)$.