5 Production function

In modern economies, most productive activities are undertaken inside organizations that we may call firms. Production itself may be defined as the joint effort of humans to transform natural resources into the means to satisfy human necessities, the latter defined in a very broad sense. However, in the following pages we are going to take the narrow point of view of these organizations, i.e., firms. From their own point of view, firms acquire inputs and use them to obtain output which then they sell. In other words, they transform inputs into outputs. One of these inputs is labor (the human effort). Other inputs are usually acquired from other firms, and so they are the result of other productive processes. For the firm, each of these inputs have a price tag attached.[3]

Certainly, the organizations we call firms work in the interest of some humans, as well. Who are these people? The answer may depend on circumstances and the particular case. Here we are going to assume that these humans are the (share) owners, and (in addition) we are also going to assume that these firms are private. Finally, we are going to assume that these owners care only about the profits that the firm obtains and distribute. That is, about the difference between the revenues obtained by selling the output and the expenses incurred in acquiring inputs. Both inputs and outputs are sold or acquired in their respective markets.[4]

Before analyzing in detail the behavior (i.e., the decisions) of firms, we want to consider the productive process itself. That is, ignoring how this process of transformation of inputs into output is governed, we want to pay attention to the process, the **mapping** from inputs into output.

Can humans transform iron, wood, labor, and coal into steel? The answer is that they can now (and a few centuries ago), but could not ten thousand years ago. Can they transform steel, labor, and coal into transportation? The answer is that they could a couple of centuries ago, but not five centuries ago, before the invention of the steam engine. What determines these answers?

[3] From the point of view of society as a whole, only natural resources and labor are real inputs. A farmer may use a horse shoe acquired in the market, but that horse shoe is simply the result of previous labor that has used some minerals (iron and coal). If you think of it, every good or service boils down to these two elements: labor (by many people) and raw materials.

[4] In this sense, we speak of a (or several) labor markets, as if the human effort, measured in hours, perhaps, was one good that is purchased (and is sold by workers).

You would probably agree that, first and foremost, knowledge. Or, if you prefer, technology. The state of technology is the fundamental determinant of what output could be obtained from what inputs. Or, put in other words, how much of certain output can be obtained with a combination (**bundle**, to keep using a familiar word) of **inputs**. That is, the mapping from inputs into output that we have mentioned.

We are going to assume that the state of technology is given, and so our starting point for the analysis of firm behavior is going to be this mapping which we will call **production function**: a function that tells us for each bundle of inputs, how much output (of a particular good) can be obtained using this bundle (given the state of the technology).

Example 1 *Accounting services for retailers may be "produced" using labor, L, and computers of certain type, K. We measure the number of services produced, q, by the number of retailers served. The following table represents (some of) the production possibilities:*

L	K	q
0	1	0
1	0	1
1	1	3
2	0	2
2	1	4
1	2	4
2	2	7

Table 4

Try to identify in the example the mapping that we call production function: to the bundle $(L, K) = (1, 1)$ the production function assigns the output 3. That is, using one unit of labor and one computer it is possible to produce 3 accounting services. Note that, in the example, using both the bundle $(2, 1)$ and the bundle $(1, 2)$ it is possible to produce the same number of services, 4. That is, the technology allows for some **substitution** of K for L and vice versa. Therefore, if some firm was offering these services and wanted to produce 4 of them, it would have to decide whether to do so using two workers and one computer or one worker with two computers.

We will typically assume perfect divisibility of inputs and output, as this allows us to use calculus and so to make our life simple. Representing the mapping by f, we write $q = f(L, K)$ to represent that using bundle (L, K) of inputs it is possible to produce q units of the output. (You should understand that typically producing some output requires the use of more than two types of inputs, but the case of two inputs is rich enough to illustrate all the concepts that we will be discussing.)

5.1 Isoquants

Let's get back to the issue of input substitution. In the example above, input combinations $(2, 1)$ and $(1, 2)$ both allow to produce the same quantity of output, 4. Turning to examples with divisible inputs and outputs, suppose that $f(L, K) = L \cdot K$. Make sure to understand what this means: if you use 2 units of L and 3 units of K you can obtain 6 units of output. If you use 6 units of L and 1 units of K you can also get 6 units of output. In fact, using any combination of inputs (L, K) such that

$$L \cdot K = 6$$

you can obtain 6 units of output. We call the set of all these combination of inputs, $\{(L, K) \,|L \cdot K = 6\}$, an **isoquant**. That is, in this example, $(2, 3)$ and $(6, 1)$ are on the same isoquant. (In the example represented in the table above, $(2, 1)$ and $(1, 2)$ are on the same isoquant.) In general, an isoquant is the set of bundles of inputs that satisfy that

$$f(L, K) = Q, \tag{9}$$

for some output level Q.

We may represent isoquants much as we represented indifference curves. Remember: an indifference curve was the set of bundles of goods that had the same label (representing preferences of a consumer). An isoquant is the set all of combination of inputs that allow producing the same level of output. (Make sure you understand the similarities and differences between indifference curves and isoquants.) As such, if we represent in the horizontal axis quantities of input L and in the vertical axis quantities of input K, we can draw the isoquant "6". (Figure 25.)

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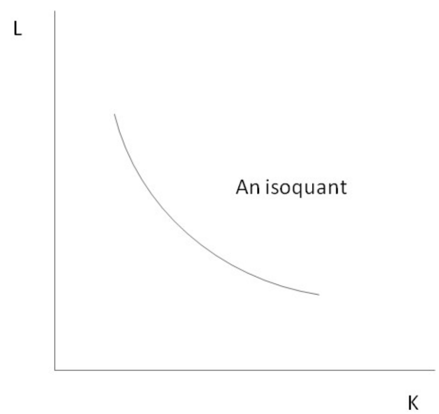

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Figure 25

Also, remember that the slope of an indifference curve at one point (at one bundle) measures the rate at which one good could be substituted for another leaving the label unchanged (leaving the consumer indifferent). Likewise, the slope of an isoquant at one point (at one input combination) measures the rate at which one input may be substituted for another without changing the output. We call (the negative of) this slope **marginal rate of technical substitution** (MRTS).

Let us compute this MRTS.

Much as when we discussed how to obtain the MRS, we note that by increasing the usage of L by an amount dL the level of output grows by $dL \times \frac{\partial f(L,K)}{\partial L}$, and by increasing the usage of K by dK, the level of output changes by $dK \times \frac{\partial f(L,K)}{\partial K}$. Therefore, changes dL and dK of the amount of inputs L and K will leave the output level unchanged (i.e., will lead us to another combination of inputs on the same isoquant) if

$$dL \times \frac{\partial f(L,K)}{\partial L} + dK \frac{\partial f(L,K)}{\partial K} = 0. \tag{10}$$

Or, solving, the increase in K (or decrease, if it is negative) per unit increase in L (the slope of the isoquant) is

$$-\frac{dK}{dL} = \frac{\frac{\partial f(L,K)}{\partial L}}{\frac{\partial f(L,K)}{\partial K}}.$$

For instance, in our example with $f(L, K) = L \cdot K$, the MRTS at the input combination $(L, K) = (3, 4)$ is $\frac{4}{3}$. That is, if we are using 3 units of L and 4 units of K, we may substitute (a tiny bit of) K for (a tiny bit of) L at a rate of $\frac{4}{3}$ units of K per unit of L, without affecting the output level.

5.2 Returns to scale

The discussion above dealt with increasing one input and reducing another (so that the level of output remained the same). We now discuss how output changes as the use of all inputs increase (or decrease). That is, how output responds to changes in the **scale** of operations. Typically, using more of all inputs results in a larger output. The question is, will output increase **proportionally** more or less than the use of inputs? That is, if we increase the amount of inputs used by a factor λ (> 1), will output increase by more or less than a factor λ. In other words, will $f(\lambda L, \lambda K)$ be larger, equal, or smaller than $\lambda f(L, K)$?

If $f(\lambda L, \lambda K) > \lambda f(L, K)$ for $\lambda > 1$, output grows by more than the increase in the use of inputs, and we say that the technology is characterized by increasing **returns to scale**. If $f(\lambda L, \lambda K) < \lambda f(L, K)$ we say that the production function is characterized by decreasing returns to scale. Finally, yes, you guessed: if $f(\lambda L, \lambda K) = \lambda f(L, K)$ we are in front of a technology characterized by constant returns to scale. You can check that our example $f(L, K) = L \cdot K$ is a production function characterized by increasing returns to scale.

Of course, a production function may present increasing returns for some levels of output (and combination of inputs) and decreasing for other levels of output. (If you consider the example in Table 4, at a combination of inputs $(1, 0)$ output is 1, and increasing the use of inputs by a factor $\lambda = 2$, that is, at a combination of inputs $(2, 0)$, output is 2, so that at that point retuns to scale are constant. However, at a combination of inputs $(1, 1)$ output is 3 and when increasing inputs by the factor $\lambda = 2$, and so using input combination $(2, 2)$, output is 7, so that at that point returns to scale are increasing.