2 Demand

When we solve problem (5), our model, for some particular prices p_1 and p_2, and some particular income y, we obtain the quantity x_1 of good 1 and quantity x_2 of good 2 that the consumer will choose to purchase and consume when those are the prices and that is her income. For instance, for the consumer whose preferences could be represented with the utility function $U(x_1, x_2) = (x_1)^2 + (x_2)^2$, we obtained that she purchased $x_1 = 10$ units of good 1 and $x_2 = 0$ units of good 2 when her income was $y = 10$ and the prices were $p_1 = 1$ and $p_2 = 2$. Of course, if the price of one of the good changes, or if her income is different, we expect the consumer to make a possibly different choice. We could solve problem (5) again, this time with the new values of income and/or prices and obtain a new pair of values for x_1 and x_2. In fact, we could do this for all combinations of prices and income. By doing so, that is, by solving problem (5) for each possible vector of prices and income, (p_1, p_2, y), we would define a **mapping** from **prices** and **income** values to **purchases** of good 1, x_1, and **purchases** of good 2, x_2. That mapping for one good is what we call **demand function** for that good.

It is common to represent the mapping with the symbol D. Thus, we write $D_1(p_1, p_2, y)$ to refer to the solution x_1 to our problem (5) for prices and income (p_1, p_2, y). We are very interested in studying how this choice, this solution to (5), changes as prices and income change. This is what we will study in this chapter. Before we do, make sure that you understand the meaning of $D_1(p_1, p_2, y)$, the demand function (of good 1), and what it means this study that we are beginning now. Also, very often, we are interested only on the relationship between p_i and x_i, for $i = 1, 2$. That is, how the "quantity demanded" of one good changes as the price of that good changes. That means that we keep the prices of other goods and the income fixed (with no change). That is why often you will see expressions like $D_1(p_1)$ (or even $D(p)$, when there is no ambiguity as to what the good we talk about is, and so we do not need to mention which one it is). That simply means $D_1(p_1, p_2, y)$ for some fixed, given values of p_2 and y. The exercise in the next section will illustrate one such case.

2.1 A graphical representation

In Figure 12, we have represented the solution to problem (5) for **some** income and prices (which determine the green line, the budget constraint).

Underneath, we represent in some (other) axes **that** price of good 1 and the corresponding choice of x_1 obtained in the upper graph.

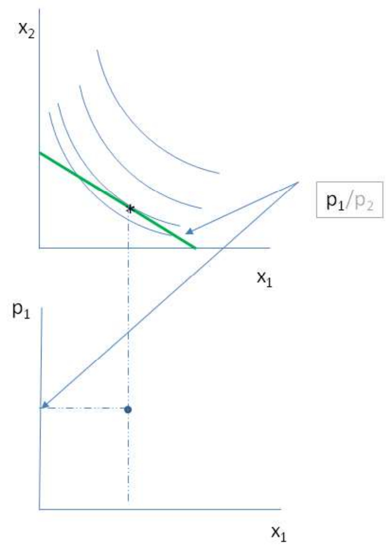

Figure 12

If we now change the price of good 1 to a new value (and keep p_2 and y unchanged), we will get a different budget line and so a possibly different choice. For instance, we can now look at a higher value of p_1 which will result in the new budget line in Figure 13.

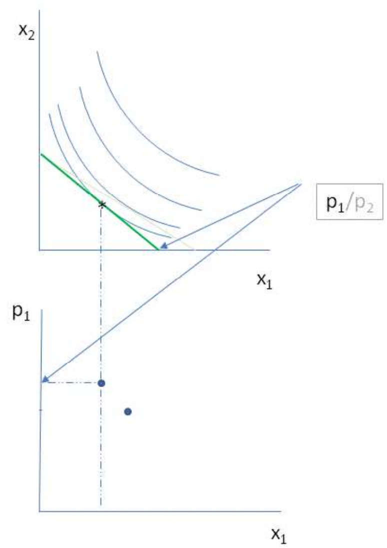

Figure 13

(Make sure you understand that indeed an increase in p_1 will have that effect on the budget line.) Then, we find a new choice x_1 (and x_2, but now we are only paying attention to good 1) corresponding to the new price p_1. That is, a new point in that mapping from price of good 1 (and price of good 2 and income) to quantity of good 1. If we repeat this exercise for every price of good 1 we get a curve, that we may call "demand curve" (or we may not call it anything, up to you), as in Figure 14.

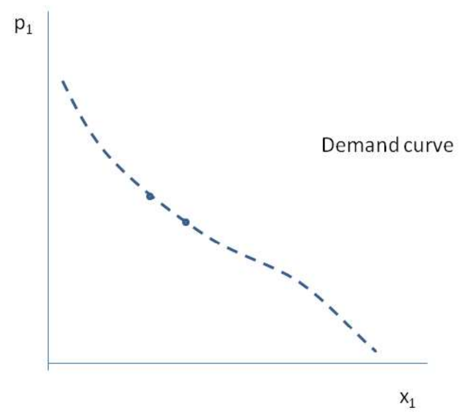

Figure 14

Remark: Make sure that you do not confuse the two graphs. In the upper parts of Figure 12 and Figure 13, we represent bundles of goods, their labels (by means of drawing indifference curves), and affordability (budget constraint). Note the dimensions in the axes: x_1 and x_2. On the contrary, in the lower parts, we represent some other relationship: quantity of good x_1 for each price p_1 (these are the dimensions in the axes). We get one from the other, but never lose sight of the labels on the axes, and therefore what the graph represents.

Thus, this graphical tool illustrates the relationship between the price of the good, p_1, and the quantity of that good demanded, x_1 (the amount of good 1 in the optimal bundle) given the other prices, p_2, and the consumer's income, y.

Now I am going to ask you to perform another thought experiment. Suppose we want to repeat this exercise for a **different value of** y. For instance, a new income $y' > y$. We would return to Figure 12, but now the green line would be **shifted outwards, parallel** to the old one, right? That is, the ratio of prices (for our initial pair of prices p_1 and p_2 used in Figure 12) would be as before, so the slope of the budget line would be the same. In any case, that new income would result in a new optimal bundle (x_1, x_2). That is, in the bottom graph in Figure 12, for that original price p_1, the quantity x_1 would be different. The same would happen for any other price p_1 (still with the old price p_2 and the new income y'). That is, for the price p_1 used in Figure 13 you would also get a new quantity x_1, etc. In other words, the demand curve in Figure 14 would **shift**, as illustrated in Figure 15.

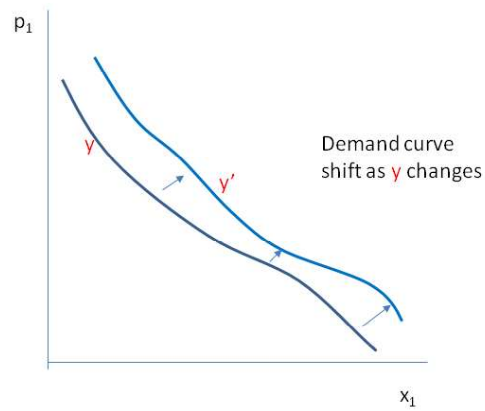

Figure 15

Remark: this is a simple point once you understand what we represent in the "demand curve": a change in income "shifts" the demand curve, as does a change in the price of the other good, good 2. A change in p_1, on the other hand, does not "**shift**" the curve: the curve represents precisely how x_1 changes **as** p_1 **changes**, so that a change of p_1 will simply take us "**along**" the curve. There is nothing mysterious about it. And, remember, all these changes are nothing but representations of the optimal choice of a bundle that we discussed in the previous chapter, what we are now calling $D_1(p_1, p_2, y)$ and $D_2(p_1, p_2, y)$.

2.2 Normal goods, inferior goods

In Figure 15, we have represented the effect of an **increase** in income as an **outward** shift of the demand curve. That is, for each price of good 1, p_1, the quantity demanded by the consumer increases as a consequence of the increase in income. You may expect this to be the case, but in fact there is nothing (in what we have assumed) that guarantees that. When a consumer gets a higher income (with no change in prices), she will obviously buy more of some goods (more is better, remember), but she may **reduce** the consumption of some goods. This possibility is represented in Figure 16. As income increases, the consumption of good 1 increases, but the consumption of good 2 decreases.

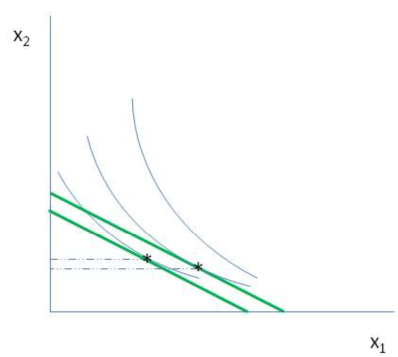

Figure 16

If you think about it, there is nothing strange here. As people get richer, they may substitute the consumption of some "higher quality" food for some "lower quality" one, buy a more expensive car model instead of the cheaper one, etc. We say that the good is **inferior** (for that consumer, for her circumstances –prices and income–) when an increase in income leads the consumer to consume less of that good (as good 2 in Figure 16):

$$\frac{\partial D_1(p_1, p_2, y)}{\partial y} < 0.$$

If, on the contrary, an increase in income leads the consumer to buy more of the good (as in the case of good 1 in Figure 16),

$$\frac{\partial D_1(p_1, p_2, y)}{\partial y} > 0.$$

we say that (for that consumer) the good is **normal**.

2.3 Substitutes and complements

Let us repeat the exercise but this time paying attention to the effect that a change in the price of another good, good 2, has on the consumer's choice of good 1. Beginning with Figure 12, suppose that the price of good 2 changes, whereas income and the price of good 1 remain the same. If p_2 increases, then the budget constraint rotates inwards around the intercept with the horizontal axis: when the consumer spends all her income in good 2 she can now buy less than before, but if she spends all her money in good 1 she can buy exactly as much as before. That is, the intercept with the horizontal axis is unaffected by the increase in p_2, but the (negative of the) slope of the budget constraint, p_1/p_2 would be smaller. In any case, for each value of p_1, the quantity demanded now will be different. That is, again, the **demand curve for good** 1 **would shift**.

Will it shift upwards or downwards? Look at Figure 13 again, but change the names of the goods. That is, now let good 2 be the one represented in the horizontal axis and good 1 be the one measured in the vertical axis. If we do so, the upper part of Figure 13 represents that change in the price p_2 (for a given price p_1) that we are talking about. Do you see this? OK, what happens with the quantity of good 1 (which in the graph we call x_2 and we measure in the vertical axis) demanded as the price of good 2 increases? (That is, what happens to the vertical component of the optimal bundle?)

It is difficult to see in that figure, but I would say it went up. That is, as the price of the good measured in the horizontal axis went up, the quantity of the good measured in the vertical axis went up as well. If so, the increase in the price of good 2 led the consumer (reduce the consumption of that good and) increase the consumption of good 1:

$$\frac{\partial D_1(p_1, p_2, y)}{\partial p_2} > 0.$$

Put in other words, the consumer substituted consumption of good 1 (the one that now is relatively less expensive) for consumption of good 2 (which now is relatively more expensive). Therefore, we may say that, for this consumer, the goods are **substitutes**.

Things may be different, though. Observe the change in Figure 16. (This time, we measure good 1 again in the horizontal axis and good 2 in the vertical axis.)

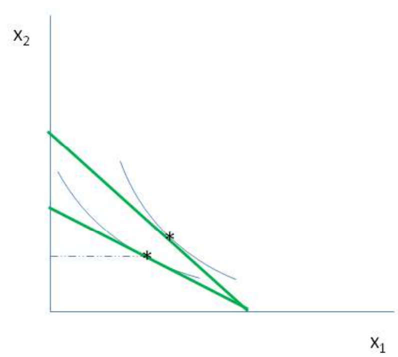

Figure 16

As you can see, as the price of good 2 increased (but p_1 and y remain the same), the consumption of both goods, and in particular the consumption of good 1, decreases as the price of good 2 goes up. That is, the consumer does not substitute one good for the other, but reduces the consumption of one good as the other becomes more expensive. Or, in other words, the demand curve of good 1 shifts down as the price of good 2 increases:

$$\frac{\partial D_1(p_1, p_2, y)}{\partial p_2} < 0.$$

We may say that, for this consumer, these two goods are (gross) **complements**.