6.6 Long-run and short-run

In the previous sections we were assuming that the firm was able to choose the amounts of all inputs that it used to produce its output. This is usually the case when a firm is first considering to operate, before actually starting operations. Also, this is the case when the firm is already operating but is considering its scale for the (relatively) distant future. *Distant* here means a future far enough so that no current contract (with providers of inputs, in particular) is binding anymore, and all inputs (equipment, etc.) needed for a substantive change in scale may be procured.

When making decisions for a nearer future, however, some inputs may be better understood as predetermined. For instance, if conditions in the market change, but the change is considered transitory, not permanent, the firm may wish to reduce/increase output for as long as the transitory conditions last. Consequently, it will consider changing the amount of inputs it uses. Some of them may be easy to change (the amount of gas that the power plant uses may be reduced if less energy is to be produced), but others may be impossible (to all effects) to change (it makes no sense to even consider reducing the size of the turbine) for only a transitory period, or on short notice.

We call **long-run** the **planning horizon** where the amount used of **all** inputs can (sensibly) be changed, and **short-run** the planning horizon where the amount of **some** inputs cannot be altered.

Thus, what we have been discussing in the previous sections can be considered the firm's cost minimization problem in the long run. Also, note that the fact that the amount of one input cannot be changed (in the short-run) simply reduces the combinations of inputs that the firm may consider. Obviously, it does not affect what can be produced with the combinations of inputs that **can** be procured. So, in terms of problem (11), if we consider the same problem in the short run and if, for instance, K is fixed in that planning horizon, say $K = 100$, the problem can be written as (11), except that the firm cannot chose K and instead $K = 100$. In our example with $f(K,L) = K \times L$, the problem in the short run is

$$\begin{aligned} &\min_{L} \ w \times L + r \times 100 \\ &\text{s.t. } 100 \times L = q. \end{aligned} \tag{14}$$

With only two inputs the problem is trivial, isn't it?: the firm can only choose one input, L, but there is only one combination of inputs with $K = 100$

that allows to produce q units: $(K, L) = (100, \frac{q}{10})$. In other words, in such case there is no possibility of input substitution: the firm needs $L = \frac{q}{100}$ to produce q units. Thus, the conditional demand of L in the **short run** is $L(w, r, q) = \frac{q}{100}$ (independent of prices of inputs) and the cost function is $C(q, w, r) = w \times \frac{q}{10} + r \times 100$.

However, in general, there are more than three inputs and, even in the short run, there is the possibility of substituting one (variable) input for another. For instance, imagine that energy E (with price s) is also used in producing the good, and the production function is $f(K, L, E) = K \times L \times E$. In that case, the problem with $K = 100$ is

$$\begin{aligned} &\min_{L,E} \; w \times L + s \times E + r \times 100 \\ &\text{s.t. } 100 \times L \times E = q. \end{aligned}$$

You can solve this problem as an exercise and obtain the conditional demands for L and E as $L_{SR}(w, r, s, q) = \left(\frac{q}{100}\frac{s}{w}\right)^{\frac{1}{2}}$ and $E_{SR}(w, r, s, q) = \left(\frac{q}{100}\frac{w}{s}\right)^{\frac{1}{2}}$, so that

$$C_{SR}(q, w, r, s) = 100 \times r + 2(w \times s)^{\frac{1}{2}} q^{\frac{1}{2}}. \tag{15}$$

We have included a subindex, SR to indicate that we are talking about the short run. (In fact, we could also specify in the subindex that we refer to the short run **given that** $K = 100$!)

6.7 Fixed costs and variable costs

Look carefully at expression (15). Remember that prices r, w, and s are something the firm will not decide. Contrary, q will be chosen (we will study that issue later) by the firm. Thus, the first part of the expression, $100 \times r$, is a cost that the firm does not affect with its decisions (in the short run), and in particular is independent of q. Even if $q = 0$ (the firm did not produce any output), that cost would be incurred: it is the cost of the $K = 100$ that the firm cannot reduce in the short run (perhaps the payments linked to the lease of its 100 square meters facility,...).

We call this component of the cost **fixed cost**, FC. Note that this fixed cost was absent in the long-run: if the firm decides to produce $q = 0$ there is no need to buy any inputs! The rest of costs, $2(w \times s)^{\frac{1}{2}} q^{\frac{1}{2}}$, change when q changes, and so we call this component the **variable cost**. Ignoring the prices of inputs (for compactness), we can represent these costs as $VC(q)$. (That is, in the long run, all costs are variable.) Obviously, $C_{SR}(q) = FC + VC(q)$, as in (15).

Thus, in the short run we may distinguish between fixed costs and variable costs. Yet, remember that this is simply a way of classifying expenditures related to the same problem, our problem (11) when we fix the amount of those inputs that cannot be changed in the short run.

Likewise, we may talk about average variable cost: $AVC(q) = \frac{VC(q)}{q}$; average fixed cost: $AFC(q) = \frac{FC}{q}$, and average total cost, $AC_{SR}(q) = \frac{C_{SR}(q)}{q}$. Finally, if we want to measure how the cost changes in the short run as we increase the output, i.e., the short-run marginal cost, this is simply $MC_{SR}(q) = \frac{\partial C_{SR}(q)}{\partial q}$. There is nothing new here other than introducing some notation and vocabulary.

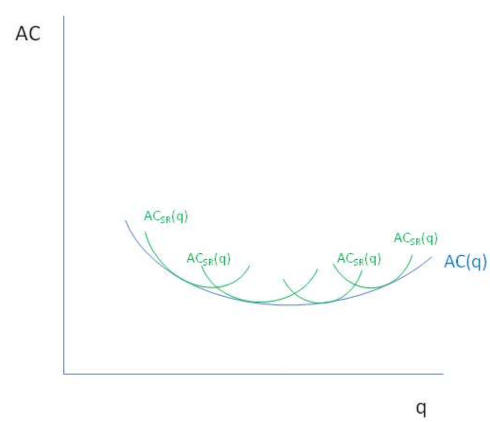
6.8 Relationship between long- and short-run costs

Suppose that you are the CEO of a new firm and have decided to produce q units. Your team has obtained that the least expensive way to produce this output is by using $K = 100$ and $L = 50$, so you decide to install (purchase) $K = 100$ and hire $L = 50$. The following week, after operations have begun, you analyze the market and decide that you want to keep producing q units, the same output. Your team now comes up to tell you that, given that in the short run K cannot be modified (i.e., you have to do with $K = 100$) the least expensive way of producing those q units is now... yes, using $K = 100$ and $L = 50$. Right? That is, the long-run cost of q units, $C(q)$, is the same as the short-run cost of q units, $C_{SR}(q)$, given that you have installed $K = 100$. That is, given that what you already have is what, if you could choose, you would choose anyway.

However, the one week later the situation has changed and now you want to produce a different amount of output, q'. You revisit your team's computations when the firm was first launched and realize that for that amount of output you would have chosen $K' \neq 100$. That sounds quite possible, right? After all, the long-run conditional demand of K, $K(q, r, w, ...)$, was a function of q and different values of q would probably mean different values of K. However, you cannot change K. That is, for this week you will have to produce with a combination of inputs **that is not** the least expensive among the ones that allow you to produce q'. It will have to be the least expensive one among those that allow you to produce q' **and** have $K = 100$. That is, for q' and given that $K = 100$, $C_{SR}(q') > C(q')$. (Obviously, that same inequality applies to the average costs.)

For any q' other than our original q, that will be the case. Therefore, a straightforward conclusion is that typically, given any fixed input K, $C(q) = C_{SR}(q)$ for **one** value of q (the value at which conditional demand $K(q, ...)$ coincides with the installed value of K), and for any other value of q, $C(q) < C_{SR}(q)$.

29



$_30.pdf$

Figure 29

In Figure 29 we have represented the long-run average cost function for a firm and some short-run average cost functions. (Note that there is a whole family of short-run cost functions, one for each level of the fixed inputs, K in our case.) Each of the short-run average cost curves "touches" the long-run average cost curve at one point and is above it everywhere else.

(We could have drawn the marginal cost curves, both in the long run and in the short run (the latter, for each value of the fixed inputs). As for any relationship between averages and new values in the population, each marginal cost curve would cut the corresponding average cost curve at its minimum.)

6.9 Envelope (a little advanced)

The discussion in the previous section could be presented in a slightly different way. Suppose that the CEO's team, in order to save some time in the future, decided that, before the CEO finished choosing what the long run q should be, and then what q in each week (given whatever the K the CEO ends up installing), they will write a code to compute all the possible problems they may face in the future. Also, let us return to the three input case, not to make it too trivial.

This is how they would go, perhaps. Given any preset $\overline{K}$ that the CEO may choose, let us solve

$$\begin{aligned} &\min_{L,E} \left\{w \times L + s \times E + r \times \overline{K}\right\} \\ &\text{s.t.} \quad f(L, E, \overline{K}) = q \end{aligned}$$

for any value of q (given the prices s, w, r). Again, note that K may take any arbitrary level, just as q: the CEO may come up with any plan q for the week, and you want to cover also any possible case of K you may find yourself constrained to use. Obviously, from that code the team would get a **solution** L, E **for each** pair of values q and $\overline{K}$: $L(q; \overline{K})$ and $E(q; \overline{K})$ (again ignoring w, r and s that we keep unchanged). For the corresponding value of $\overline{K}$, these are simply the short-run conditional demands of L and E. Also, for **each** value of $\overline{K}$, the short-run cost function would simply be

$$C_{SR}(q; \overline{K}) = w \times L(q; \overline{K}) + s \times E(q; \overline{K}) + r \times \overline{K}.$$

We have drawn some of these (in average form) in Figure 29.

Simple, isn't it? After all, it is a code, so we can run it as many times as we want for different (q's and) K's. Now, here is the beauty of it, how the team is going to save time: When finally the CEO decides how much the firm should produce (long-run decision), that is, the value of q, we only need to run a little subroutine:

$$\min_{K} C_{SR}(q; K).$$

From there we will obtain the value of K that minimizes the cost of producing q units when we ot is also possible to choose K as well as every other input.

That is, we obtain the (long-run conditional demands of inputs and so) the long-run cost function! Indeed, convince yourself that what you obtain, i.e., the solution K to this last problem, and then the demands $L(q;K), E(q;K)$ of the other two inputs, is the least expensive combination of inputs that allows you to produce q units.

Exercise: You may want to check that. So, suppose that $w = r = s = 1$, and $f(L,E,K) = L \times E \times K$. Obtain the long-run cost function both ways, solving (11), and solving that two-step method just described. Check that you obtain the same results.

What you have just seen is simply an illustration of the result known in calculus as the envelope theorem.