1.3 Affordability: the budget constraint

Let me emphasize that in our discussion above we did not referred to prices or income. We were talking ONLY about the consumer's tastes. When one dreams, one does not need to feel constrained by things like the impossibility of flying or of traveling in time. We can perfectly think of whether we would prefer flying with Bezos and giving up the Bugatti that Branson offers us in exchange (the bundle $(1, 0)$) or we would rather go for the Bugatti (the bundle $(0, 1)$) ... even though neither B. nor B. are likely to approach us with those offers (and we probably don't have the money to buy either bundle).

That settled, let us move now to the land of possibility. In fact, let us now forget the consumer's (dreams and) tastes. Let us instead turn to what are the bundles that the consumer could possibly afford.

In a market economy like the one we live in, many goods can be acquired at a **price** (prices which consumers cannot affect: that is, are given, from their point of view). Also, many of us have (or earn) some **income**, which is an amount of money, and we mainly acquire goods or services by using that income to pay the prices of those goods and services. This is not a complete picture of the situation faced by a consumer, but we are going to take that as a sufficiently good description. So, we postulate that a consumer has some given amount of money, y, and faces some prices, p_1, p_2 of goods 1 and 2 respectively. She does not decide (for the time being) on any of those numbers. That is, we assume that our consumer is **competitive**. Also, we are going to keep the analysis simple by ignoring the existence of time. That is, we are going to analyze a model that is **static**. So, you will not see issues like the decision of saving for the future, or borrowing, etc.

Then, a consumer can afford a particular bundle (x_1, x_2) if, but only if, the amount of dollars needed to buy it is not higher than her income:

$$p_1 \times x_1 + p_2 \times x_2 \leq y.$$

That is, if the bundle satisfies that constraint, which we call **budget constraint**. We may also call the set of all bundles that satisfy that constraint **budget set** (these are only names which are good to know, but are useless unless we understand what they mean). So, the budget set is the choice set for the consumer. The feasible consumption plans for the consumer are the bundles that satisfy the budget constraint.

In Figure 6 we represent one such budget constraint (and set). The bundles on the line are bundles that cost exactly y dollars. Bundles below the line cost less that y dollars (right?). So the bundles on or under the line (and in the positive quadrant) constitute the budget set. The consumer can afford any of those bundles.

Make sure that you understand the info displayed in the figure. First, the intercept on the vertical axis: that is a bundle that (costs all the consumer's income, y, and) contains no good 1. Therefore, it contains y/p_2 units of good 2: the amount of good 2 that can be purchased if all the income y is spent on it. Similarly for the intercept with the horizontal axis. Also, the slope of the budget constraint is (negative) p_1/p_2. Why?

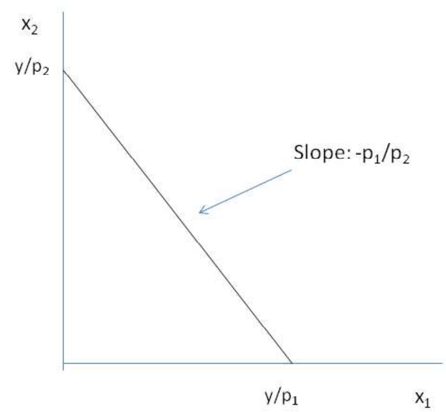

Figure 6

You can compute that slope by computing the equation of a straight line that passes through the points $(0, y/p_2)$ and $(y/p_1, 0)$, as we have seen that the budget constraint does. (I encourage you to practice your basic algebra skills by doing so.) But reasoning in economic terms (that is, taking into account what that mathematical expression represents) is even more interesting. Indeed, suppose that a bundle (x_1, x_2) is on the consumer´s budget constraint (i.e., costs exactly y dollars). If the consumer reduces by one unit the amount of x_1 that she buys, how much extra good x_2 could she buy? (Note why the negative sign: the consumer "reduces" one and "increases" the other, so we are talking about changes that go in opposite directions, the negative of the "slope."). The answer: by reducing the amount of good 1 by one unit she saves p_1 dollars, which spent in good 2 buy p_1/p_2 (extra) units of good 2.

Read that paragraph again: what we have obtained in the second part of it can be put as follows: p_1/p_2 is the rate at which the consumer **may** trade good 2 for good 1in the market. It is the exchange rate of the two goods that is **feasible** (for all consumers). This will be important to understand later results.

As we have already mentioned, we will analyze the problem faced by a consumer with some income y who can purchase any amount of good i at a price per unit of p_i, where we are using i here as an index (the name of the good). However, obtaining the budget constraint (or the set of all bundles that the consumer could purchase) in other cases is not difficult. I encourage you to try obtaining (graphically or analytically) budget constraints in other cases. For instance, if:

a) The consumer's income is \$10, the prices of each good is \$1, but the consumer is not allow to buy more than 2 units of good 1.

b) The consumer owns 2 apples and 3 oranges. There is a market where she can buy or sell either of the two at a price of \$1 per unit.

c) The consumer's income is \$10, the price of good 1 is \$1, and the price of good 2 is \$2 if she buys less than 2 units, and after that each additional unit costs \$1.

1.4 Consumer choice

We have finished defining the elements that we need to formalize our (basic) model of consumer behavior. Indeed, in such model, a consumer is an individual with tastes or **preferences** $\succsim$ (which we will always assumed that can be represented by some mapping –utility function– u), who is endowed with certain **income** of \$$y$, and has access to markets for all goods, $i = 1, 2, ..., n$ (we usually assume that $n = 2$, i.e., there are only 2 goods) at **prices** p_i.

How will such consumer behave? That is, how does our model picture this consumer to choose what to consume? Simple: we will postulate that the consumer will choose **the bundle (a list of quantities of all goods) that she likes the most (has the highest label which we call utility) among the ones that she can afford (are in the budget set).** That is, among all bundles (x_1, x_2) such that

$$p_1 \times x_1 + p_2 \times x_2 \leq y,$$

the consumer will choose the one that ranks highest in her preferences. Or, if preferences can be represented with the labels u, the one at which $u(x_1, x_2)$ is highest. In more compact mathematical language, the consumer will behave so as to:

$$\begin{aligned} &\max_{x_1, x_2} u(x_1, x_2) \\ &\text{s.t.} \quad p_1 \times x_1 + p_2 \times x_2 \leq y. \end{aligned} \tag{5}$$

Let me explain this language. We will use it often, but remember, it is just that: language, a shorthand for more convoluted utterances. The first line means that we are searching for two values, x_1, x_2 that "**maximize**" (that is what Max_{x_1,x_2} means) the function $u(\cdot, \cdot)$; that is, the pair x_1, x_2 at which $u(x_1, x_2)$ is highest. In the second line, the initials s.t. stand for "subject to", and what follows describes the set of feasible choices. That is, the whole expression indeed means that among all bundles (x_1, x_2) affordable with income y given prices p_1 and p_2, we are searching for the one(s) at which the value of the function $u(\cdot, \cdot)$ is highest. That is, the one(s) with highest label, and so the one that the consumer prefers.

So, after all our preparatory work, we can say that, in a nut shell, our **model of consumer behavior** boils down to that: we predict the consumer to purchase the bundle (x_1, x_2) that **solves** problem (5).

1.4.1 Solving with rudimentary tools: the discrete case

Let us see what this means by returning to our discrete example of non divisible goods represented in Table 1 and Table 2, and assuming that $y = 7$, $p_1 = 2$ and $p_2 = 3$. Figure 7 represents this example.

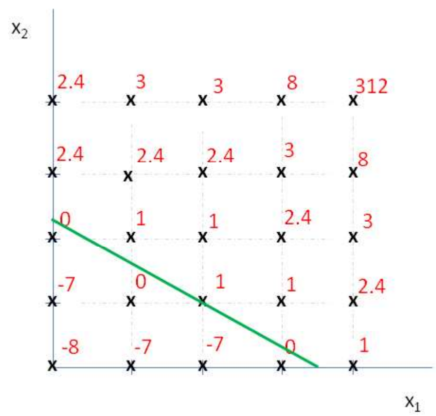

Figure 7

We have added the label that our utility function in Table 2 assigns to each bundle. Check that indeed the label (in red, above and to the right of the bundle) of each bundle is the one assigned in Table 2. Also, the green line represents all the quantities of the two goods that would exhaust the income. For instance, with \$7 one could pay $2,333$ times \$3 (buy $2,333$ units of good 2 if divisible, and so at most 2 units when it is not divisible). Or pay exactly the cost of 2 units of good 1 (at \$2) and 1 unit of good 2 (at \$3). Thus, in the case of non divisible goods that we are discussing, the bundles that satisfy the budget constraint are $\{(0,0),(0,1),(0,2),(1,0),(1,1),(2,0),(2,1),(3,0)\}$. (All the bundles that lie on or below the green line, the budget constraint.) Among those, the one that has the highest label is the bundle $(2,1)$, and so that is the solution to problem (5). That is the prediction that our model makes for this case: this consumer would choose to buy 2 units of good 1 and 1 unit of good 2 when her income is \$7 and the prices are $p_1 = 2$ and $p_2 = 3$.

1.4.2 A little fancier: graphical analysis

Let's get back to the continuous (divisible) case, since typically that case is easier to handle (thanks, calculus!) and also easier to think about. We cannot write labels for all bundles, since there are too many of them. (Can you imagine Figure 7 for this case? You would see nothing there!) But we can draw a few indifference curves, which will contain enough information about the labels: remember, all points in an indifference curve have the same label, and curves farther from the origin sport higher labels!

Thus, consider the consumer whose preferences are sketched in Figure 3, and suppose that her income and the prices of the goods are such that the green line in Figure 8 represents the budget constraint that she faces.

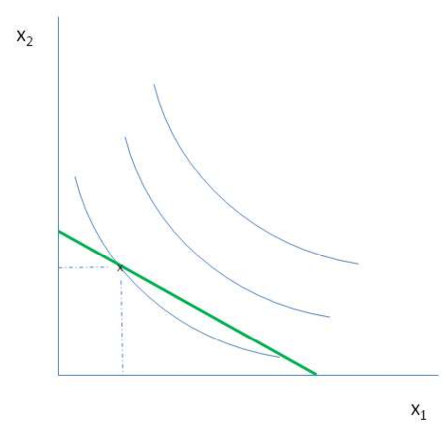

Figure 8

Would the bundle marked with an x in the figure be the best choice for that consumer? Try to understand that the answer is negative by looking at all the bundles that lie above the indifference curve on which that bundle is, but below (or on) the budget constraint. Each of those bundles is affordable (costs less or equal than the consumer's income), and lie further from the origin than the indifference curve that contains the bundle marked with an **x**. Remember that each bundle is on an indifference curve (all bundles are ranked, if preferences satisfy completeness), and so each of those bundles lie on an indifference curve north-east of the one on with our bundle lies. Therefore, every bundle in that sort of lens-shaped area is an affordable bundle that the consumer prefers strictly to the bundle marked with a **x**.

So, the best choice for the consumer must lie on an indifference curve that does not "cut" the budget line, but is **not beyond** the budget line! That is, the indifference curve on which the optimal bundle is "touches" the budget line (since the bundle must be affordable) but does not cross it (otherwise, there are affordable bundles that the consumer prefers). Or, in other words, at the optimal solution (bundle), the indifference curve to which that bundle belongs is **tangent** to the budget line. Figure 9 shows that bundle for this case, marked with a star, $*$.

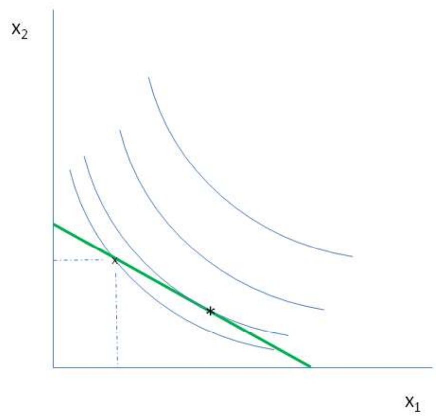

Figure 9

Let us understand this as a solution to the trading possibilities for the consumer. As we have discussed, the (negative of the) slope of the budget constraint is simply the ratio of prices of the two goods, p_1/p_2. As we have also discussed, that is the rate at which the consumer **can** trade good 2 for good 1 in the market. That is, by lowering the purchase/consumption of good 1 by one unit, she saves p_2 dollars that then she can use to buy p_1/p_2 more units of good 2. On the other hand, the slope of the indifference curve at a bundle ($*$ in this case), which we call MRS, is the rate at which the consumer is **willing** to trade these goods. Why do these two slopes need to be equal at the optimal consumption bundle for the consumer? Remember: as long as the consumer gets an amount of at least MRS of good 2 per unit of good 1, the consumer is happy to engage in a (small) trade of good 1 for good 2. And, for the same reason, as long as she has to give up less than MRS units of good 2 per unit of good 1, she is happy (prefers to) engage in a (small) trade of good 2 for good 1. Therefore, as long as MRS is not equal (exceeds or is smaller) to the ratio of prices, the consumer is better off engaging in some possible trade in the market. If the consumer prefers to (engage in a feasible) trade, then the bundle she currently has is not the best of the ones she can obtain. Unless, of course, we are talking about a bundle with 0 amount of one good!

We will come back to this last point later, but what we have learnt is that a bundle with positive amounts of both goods **cannot be** the best the consumer can get unless the MRS at that bundle equals the ratio of prices. Or, put in other words, at the best bundle, (if it contains positive amounts of both goods) the MRS at that bundle must be equal to the ratio of prices. The indifference curve must be tangent (have the same slope) to the budget constraint.