

HW2:

① a) $5n^3 + 2n^2 + 3n = O(n^3)$

$$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3$$
$$5n^3 + 2n^2 + 3n \leq 7n^3$$

$$c = 7$$
$$g(n) = n^3$$

b) $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

$$\sqrt{7n^2} \leq \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2}$$
$$\sqrt{7}n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$$

$$c_1 = \sqrt{7} \quad c_2 = 3$$
$$g(n) = n$$

c) $\mathcal{O}(n) = \mathcal{O}(f(n))$

$$e(n) = \mathcal{O}(g(n))$$

$$\mathcal{O}(n) \leq c_1 f(n)$$

$$e(n) \leq c_2 g(n)$$

$$\mathcal{O}(n) e(n) \leq c_1 c_2 f(n) g(n)$$

$$\mathcal{O}(n) e(n) \leq c f(n) g(n)$$

$$\mathcal{O}(n) e(n) = \mathcal{O}(f(n) g(n))$$

② a)

```
def example1(lst):  
    """Return the sum of the prefix sums of sequence S."""  
    n = len(lst)  
    total = 0  
    for j in range(n):  
        for k in range(1+j):  
            total += lst[k]  
    return total
```

$$\left. \begin{matrix} \\ \\ \end{matrix} \right\}^n \quad \left. \begin{matrix} \\ \\ \end{matrix} \right\}^n$$

$$\Theta(n^2)$$

b)

```
def example2(lst):
    """Return the sum of the prefix sums of sequence S."""
    n = len(lst)
    prefix = 0
    total = 0
    for j in range(n):
        prefix += lst[j]
        total += prefix
    return total
```

] n $\Theta(n)$

c)

```
def example3(n):
    i = 1
    sum = 0
    while (i <  $n^2$ ):
        i *= 2
        sum += i
    return sum
```

] $\log_2(n^2) = 2\log n$ $\Theta(\log n)$

d)

```
def example4(n):
    i = n
    sum = 0
    while (i > 1):
        for j in range(i):
            sum += i*j
        i //= 2
    return sum
```

] n] $\log(n)$ $\Theta(n \log n)$