Summary Notes for Week 1: Introduction and Overview

Table 1: A Prisoners' Dilemma Game

		Player 2	
		C	D
Player 1	C	-1, -1	-3, 0
	D	0, -3	-2, -2

Definitions

Dominant Strategies

A dominant strategy for a player is one that produces the highest payoff of any strategy available for every possible action by the other players.

$$u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i})$$

i.e. the utility of choosing action a_i for agent *i* will be greater than or equal to choosing to any other strategy a'_i

Weakly dominant: greater than or equal

Strictly dominant: greater than only

eg. for the prisoners' dilemma, defecting is always the strictly dominant strategy for both because regardless of what the other player chooses, this player choosing to defect always yields a higher payoff (Player 1 chooses cooperate, Player 2: 0 (defect) > -1 (cooperate); Player 1 chooses defect, Player 2: -2 (defect) > -3 (cooperate))

Nash Equilibrium

A pure Nash equilibrium is a strategy profile in which no player would benefit by deviating, given that all other players don't deviate

Table 6: A "hawk-dove" game

		Player 2 Hawk	Dove
Player 1	Hawk	0, 0	3, 1
	Dove	1, 3	2, 2

Here there are two pure strategy equilibria, (Hawk, Dove) and (Dove, Hawk). Players are in a potential conflict and can be either aggressive like a hawk or timid like a dove. If they both act like hawks, then the outcome is destructive and costly for both players with payoffs of 0 for both. If they each act like doves, then the outcome is peaceful and each gets a payoff of 2. However, if the other player acts like a dove, then a player would prefer to act like a hawk and take advantage of the other player, receiving a payoff of 3. If the other player is playing a hawk strategy, then it is best to play a dove strategy and at least survive rather than to be hawkish and end in mutual destruction.

The difference between Nash Equilibrium & dominant strategy is that a pure strategy Nash equilibrium only requires that the action taken by each agent be best against the actual equilibrium actions taken by the other players, and not necessarily against all possible actions of the other players.

<u>Pareto Optimality</u>
One thing is Pareto dominant compared to another if it leaves at least one participant better off and no-one worse off.
An outcome o* is Pareto-optimal if there is no other outcome that Pareto-dominates it; i.e. no other outcome that results in at least one participant being better off while no-one is worse off.

- A game can have more than one pareto-optimal outcome when neither outcome pareto-dominates another (eg. the payoff for every action is the same at 1 for every player, so no outcome pareto-dominates the other)
- Every game has at least one pareto-optimal outcome because if an outcome is not pareto-optimal, another outcome must pareto-dominate it. This means there must be a cycle of pareto-dominance where A > B > C > D > A > B > C > D > ... This is not possible because in order for something to be pareto-dominant, it must be strictly better for some agent.

- Idea: sometimes, one outcome o is at least as good for every agent as another outcome o', and there is some agent who strictly prefers o to o'
  - in this case, it seems reasonable to say that o is better than o'
  - we say that o Pareto-dominates o'.

Definition (Pareto Optimality)

An outcome o^* is Pareto-optimal if there is no other outcome that Pareto-dominates it.

eg. for the prisoner's dilemma, {C, C} and {C, D} are pareto-optimal outcomes because no other outcome pareto-dominates it. {D, D} is not pareto-optimal because {C, C} dominates it.

Consider the game:

Player 1\ Player 2	Left	Right
Left	3,3	1,1
Right	1,4	1,1

{3, 3} and {1, 4} are pareto-optimal because no other outcome pareto-dominates it. {1, 1} is not pareto-dominant because {3, 3} dominates it (3 > 1) and {1, 4} dominates it (1=1, 4>1)