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GRADE
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Problem Set 5

LATEST SUBMISSION GRADE

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1. Two players play the following normal form game.

1 / 1 point

1 \ 2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Which is the pure strategy Nash equilibrium of this stage game (if it is played only once)?

- a) (Left, Left);
- b) (Left, Middle);
- c) (Left, Right);
- d) (Middle, Left);
- e) (Middle, Middle);
- f) (Middle, Right);
- g) (Right, Left);
- h) (Right, Middle);
- i) (Right, Right).

Correct

(i) is the unique Nash equilibrium of the stage game.

- (Right, Right) is a Nash equilibrium of the stage game because Right is the best response when the other player is playing Right.
- It is also the unique Nash equilibrium. To see this, check that in all other cases at least one player has an incentive to deviate.

2. Two players play the following normal form game.

1 / 1 point

1 \ 2	Left	Middle	Right
Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Suppose that the game is repeated for two periods. What is the outcome from the subgame perfect Nash equilibrium of the whole game:

- a) (Left, Left) is played in both periods.
- b) (Right, Right) is played in both periods.
- c) (Middle, Middle) is played in the first period, followed by (Left, Left)
- d) (Middle, Middle) is played in the first period, followed by (Right, Right)

Correct

(b) is true.

- The stage game has a unique Nash equilibrium.
- In the second period, (Right, Right) must be played regardless of the outcome obtained in the first period.
- Then, it is optimal for both players to maximize the current payoff at the first period and play (Right, Right).

3. Two players play the following normal form game.

1 / 1 point

1 \ 2	Left	Middle	Right

Left	4,2	3,3	1,2
Middle	3,3	5,5	2,6
Right	2,1	6,2	3,3

Suppose that there is a probability p that the game continues next period and a probability $(1-p)$ that it ends. What is the threshold p^* such that when $p \geq p^*$ (Middle, Middle) is sustainable as a subgame perfect equilibrium by grim trigger strategies, but when $p < p^*$ playing Middle in all periods is not a best response? [Here the grim strategy is: play Middle if the play in all previous periods was (Middle, Middle); play Right otherwise.]

- a) 1/2;
- b) 1/3;
- c) 1/4;
- d) 2/5.

Correct

(b) is true.

- In the infinitely repeated game supporting (Middle, Middle):
- Suppose player 1 uses the grim trigger strategy.
- If player 2 deviates to the best response Right, player 2 earns 6-5=1 more in the current period, but loses 2 from all following periods, which is 2p/(1-p) in total.
- Thus in order to support (Middle, Middle), the threshold is 1=2p/(1-p), which is p = 1/3.
- It is easy to check that the threshold is the same for player 1.

4. Consider the following game: **1 / 1 point**

1 \ 2	Left	Middle	Right
Left	1,1	5,0	0,0
Middle	0,5	4,4	0,0
Right	0,0	0,0	3,3

Which are the pure strategy Nash equilibria of this stage game? There can be more than one.

- [ ] a) (Left, Right);
- [x] b) (Left, Left);

Correct

(b) and (f) are pure strategy Nash equilibria of the stage game.

- (Left, Left) and (Right, Right) are Nash equilibria of the stage game because Right is the best response when the other player is playing Right, and Left is the best response when the other player is playing Left.
- There are no other pure strategy Nash equilibria. To see this, check that in all other cases at least one player has an incentive to deviate.

- [ ] c) (Left, Middle);
- [ ] d) (Middle, Right);
- [ ] e) (Middle, Left);
- [x] f) (Right, Right).

Correct

(b) and (f) are pure strategy Nash equilibria of the stage game.

- (Left, Left) and (Right, Right) are Nash equilibria of the stage game because Right is the best response when the other player is playing Right, and Left is the best response when the other player is playing Left.
- There are no other pure strategy Nash equilibria. To see this, check that in all other cases at least one player has an incentive to deviate.

- [ ] g) (Right, Middle);
- [ ] h) (Right, Left);
- [ ] i) (Middle, Middle);

5. Consider the following game: **1 / 1 point**

1 \ 2	Left	Middle	Right
Left	1,1	5,0	0,0
Middle	0,5	4,4	0,0
Right	0,0	0,0	3,3

Suppose that the game is repeated for two periods. Which of the following outcomes could occur in some subgame perfect equilibrium? (There might be more than one).

☑ a) (Middle, Middle) is played in the first period, followed by (Right, Right)

✓ **Correct**
(a), (b) and (c) are all correct.

- Recall that playing a Nash equilibrium of the stage game in each period forms a subgame perfect Nash equilibrium of the whole game. Then, (b) and (c) are subgame perfect Nash equilibria.
- Outcome (a) can be obtained when both players play the following strategy:
- Play Middle in the first period.
- If outcome in first period was (Middle, Middle) play Right in the second period; otherwise play Left.
- It is easy to check that this grim strategy forms a subgame perfect Nash equilibrium:
- Suppose that player 1 plays this strategy.
- If player 2 plays the same strategy, he/she will receive a total payoff of $4+3=7$ (assume no discounting).
- If player 2 deviates to (Left, Right), he/she will receive a total payoff of $5+1=6$ (which is lower than the payoff of following the grim strategy).

☑ b) (Left, Left) is played in both periods.

✓ **Correct**
(a), (b) and (c) are all correct.

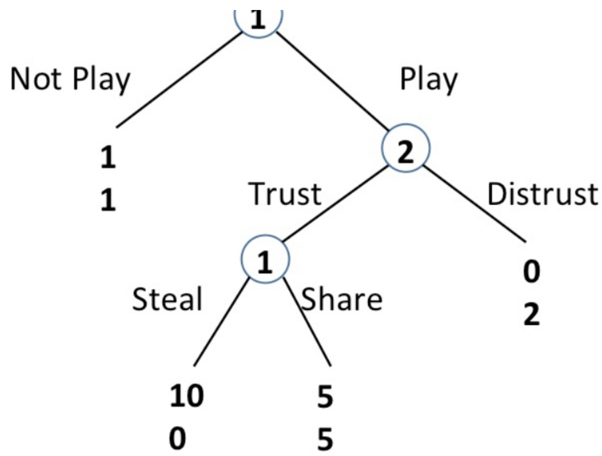
- Recall that playing a Nash equilibrium of the stage game in each period forms a subgame perfect Nash equilibrium of the whole game. Then, (b) and (c) are subgame perfect Nash equilibria.
- Outcome (a) can be obtained when both players play the following strategy:
- Play Middle in the first period.
- If outcome in first period was (Middle, Middle) play Right in the second period; otherwise play Left.
- It is easy to check that this grim strategy forms a subgame perfect Nash equilibrium:
- Suppose that player 1 plays this strategy.
- If player 2 plays the same strategy, he/she will receive a total payoff of $4+3=7$ (assume no discounting).
- If player 2 deviates to (Left, Right), he/she will receive a total payoff of $5+1=6$ (which is lower than the payoff of following the grim strategy).

☑ c) (Right, Right) is played in both periods.

✓ **Correct**
(a), (b) and (c) are all correct.

- Recall that playing a Nash equilibrium of the stage game in each period forms a subgame perfect Nash equilibrium of the whole game. Then, (b) and (c) are subgame perfect Nash equilibria.
- Outcome (a) can be obtained when both players play the following strategy:
- Play Middle in the first period.
- If outcome in first period was (Middle, Middle) play Right in the second period; otherwise play Left.
- It is easy to check that this grim strategy forms a subgame perfect Nash equilibrium:
- Suppose that player 1 plays this strategy.
- If player 2 plays the same strategy, he/she will receive a total payoff of $4+3=7$ (assume no discounting).
- If player 2 deviates to (Left, Right), he/she will receive a total payoff of $5+1=6$ (which is lower than the payoff of following the grim strategy).

6. Consider the following trust game: **1 / 1 point**



There is a probability p that the game continues next period and a probability $(1-p)$ that it ends. The game is repeated indefinitely. Which statement is true? [Grim trigger in (c) and (d) is player 1 playing Not play and player 2 playing Distrust forever after a deviation from ((Play,Share), (Trust)).]

- a) There exists a pure strategy Nash equilibrium in the one-shot game with player 2 playing Trust.
- b) There exists a pure strategy subgame perfect equilibrium with player 2 playing Trust in any period in the finitely repeated game.
- c) ((Play,Share), (Trust)) is sustainable as a subgame perfect equilibrium by grim trigger in the indefinitely repeated game with a probability of continuation of $p \geq 5/9$.

Correct

(c) is true.

- There are only two Nash equilibria in the one-shot game: ((Not play, Steal),(Distrust)) and ((Not play, Share), (Distrust)). Both require player 2 playing Distrust.
- Since both equilibria lead to the same payoff for both players, there can't exist a subgame perfect equilibrium in the finely repeated game in which player 2 plays Trust in any period (verify this by backward induction).
- In the infinitely repeated game supporting ((Play,Share),(Trust)):
- Suppose player 2 uses the grim trigger strategy: start playing Trust and play Distrust forever after a deviation from ((Play,Share), (Trust)).
- If player 1 deviates and plays (Play, Steal), player 1 earns $10-5=5$ more in the current period, but loses 4 from all following periods, which is $4p/(1-p)$ in total.
- Thus in order to support ((Play,Share), (Trust)), the threshold is $5=4p/(1-p)$, which is $p=5/9$.
- Note that given player 1's strategy, player 2 has no incentive to deviate for any value of p.

7. In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

1 / 1 point

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an infinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

True or False:

When player 1 uses the above-described "tit for tat" strategy and starts the first period in a revenge status (thus plays defect for sure), in any infinite payoff maximizing strategy, player 2 plays defect in the first period

- True.
- False.

Correct

True.

- If player 1 uses "tit for tat" strategy and starts in a revenge status, the payoff in the first period is higher for player 2 from defection than cooperation.
- Moreover, the action played by 2 in the first period when 1 begins in revenge status doesn't affect the remaining periods since 1 switches to normal status in the second period regardless of what player 2 does in the first period.

8. In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

1 / 1 point

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an infinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

What is the payoff for player 2 from always cooperating when player 1 uses this tit for tat strategy and begins in a normal status? How about always defecting when 1 begins in a normal status?

- a) $4+4p+4p^2+4p^3+\ldots; 5+p+p^2+p^3+\ldots$
- b) $4+4p+4p^2+4p^3+\ldots; 5+p+5p^2+p^3+\ldots$
- c) $5+4p+4p^2+4p^3+\ldots; 4+4p+4p^2+4p^3+\ldots$
- d) $5+4p+4p^2+4p^3+\ldots; 5+p+p^2+p^3+\ldots$

Correct

(b) is true.

- If 2 always cooperates, then 1 stays `normal' and cooperates always as well, and the payoff to each player is 4 in each period.
- If 2 always defects, then 1 is normal in odd periods and switches to revenge in even periods (because 2 defects). 1 cooperates in odd periods and defects in even periods, thus 2 earns 5 in odd periods and 1 in even periods.

9. In an infinitely repeated Prisoner's Dilemma, a version of what is known as a "tit for tat" strategy of a player i is described as follows:

1 / 1 point

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an infinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

What is the threshold p^* such that when $p \geq p^*$ always cooperating by player 2 is a best response to player 1 playing tit for tat and starting in a normal status, but when $p < p^*$ always cooperating is not a best response?

- a) 1/2
- b) 1/3
- c) 1/4
- d) 1/5

Correct

(b) is true.

- From part (2), in order to sustain cooperation, we need $4+4p+4p^2+4p^3+\ldots \geq 5+p+5p^2+p^3+\ldots$, which is $4+4p \geq 5+p$, thus $p \geq 1/3$.
- p* = 1/3.
- Note that this just checks always cooperating against always defecting. However, you can easily check that if player 2 wants to defect in the first period,then s/he should also do so in the second period (our answer from part (1)). Then the third period looks just like we are starting the game over, so player 2 would want to defect again...