Formalizing Perfect Information Extensive Form Games

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Introduction

- The normal form game representation does not incorporate any notion of sequence, or time, of the actions of the players
- The extensive form is an alternative representation that makes the temporal structure explicit.
- Two variants:
  - perfect information extensive-form games
  - imperfect-information extensive-form games

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N is a set of n players

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A is a (single) set of actions

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
  - Choice nodes: H is a set of non-terminal choice nodes

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
  - Choice nodes: H
  - Action function: $\chi : H \to 2^A$ assigns to each choice node a set of possible actions

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
    - Choice nodes: H
    - Action function: $\chi : H \to 2^A$
    - Player function: $\rho : H \to N$ assigns to each non-terminal node h a player $i \in N$ who chooses an action at h

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N

- Actions: A

- Choice nodes and labels for these nodes:
  - Choice nodes: H
  - Action function: $\chi : H \to 2^A$
  - Player function: $\rho : H \to N$

- Terminal nodes: Z is a set of terminal nodes, disjoint from H

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
    - Choice nodes: H
    - Action function: $\chi : H \to 2^A$
    - Player function: $\rho : H \to N$
- Terminal nodes: Z
- Successor function: $\sigma : H \times A \to H \cup Z$ maps a choice node and an action to a new choice node or terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$
    - Choice nodes form a tree: nodes encode history

Definition

A (finite) perfect-information game (in extensive form) is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$, where:

- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
  - Choice nodes: H
  - Action function: $\chi : H \to 2^A$
  - Player function: $\rho : H \to N$
- Terminal nodes: Z
- Successor function: $\sigma : H \times A \to H \cup Z$
- Utility function: $u = (u_1, \ldots, u_n)$; $u_i : Z \to \mathbb{R}$ is a utility function for player i on the terminal nodes Z

Example: the sharing game