Summary Notes for Week 5: Repeated Games

Repeated Games

As opposed to one-shot games, repeated games introduce a new series of incentives: the possibility of cooperating means that we may decide to compromise in order to carry on receiving a payoff over time, knowing that if we do not uphold our end of the deal, our opponent may decide not to either.

Repeated games provide different payoffs at each repetition, with discounted rewards with a discount factor raised to the power of the round of the game.

Given an infinite sequence of payoffs $r_1, r_2, \ldots$ for player i and discount factor β with $0 < \beta < 1$, i's future discounted reward is

$$\sum_{j=1}^{\infty} \beta^j r_j.$$

Strategies:

A trigger strategy essentially threatens other players with a “worse,” punishment, action if they deviate from an implicitly agreed action profile.

- Tit-for-tat: Start out cooperating. If the opponent defected, defect in the next round. Then go back to cooperation.
- Grim Trigger Strategy: Start out cooperating. If the opponent ever defects, defect forever.

Calculating payoffs:

- There are two "statuses" that player i might be in during any period: "normal" and "revenge";
- In a normal status player i cooperates;
- In a revenge status player i defects;
- From a normal status, player i switches to the revenge status in the next period only if the other player defects in this period;
- From a revenge status player i automatically switches back to the normal status in the next period regardless of the other player's action in this period.

Consider an infinitely repeated game so that with probability p that the game continues to the next period and with probability $(1-p)$ it ends.

	Cooperate (C)	Defect (D)
Cooperate (C)	4,4	0,5
Defect (D)	5,0	1,1

Payoff for player 2 from always cooperating when player 1 uses this tit for tat strategy and begins in a normal status: 4 + 4p + $4p^2$ + ... (since players will always cooperate) = 4/(1-p) (sum to infinity)
Payoff for player 2 from always defecting when 1 begins in a normal status: first, player 2 gets a 5 because 1 cooperates. Because player 2 defects, player 1 switches to revenge status in round 2, resulting in (1, 1) for both. Player 1 again chooses to defect, but since player 2 was in revenge status, player 2 would switch back to normal status in round 3 regardless of player 2's actions, so player 1 earns 5 again. This process repeats in a cylic fashion so payoff for player 2 is 5 + p + $5p^2$ + p^3 + ...

Calculating threshold for discount factor:

Player 1 / 2	Cooperate	Defect
Cooperate	3, 3	0, 5
Defect	5, 0	1, 1

Given the strategies of both players to cooperate and then defect forever if the other defects,

- Cooperate: $3 + \beta 3 + \beta^2 3 + \beta^3 3 ... = \frac{3}{1-\beta}$
- Defect: $5 + \beta 1 + \beta^2 1 + \beta^3 1 ... = 5 + \beta \frac{1}{1-\beta}$
- Difference: $-2 + \beta 2 + \beta^2 2 + \beta^3 2 ... = \beta \frac{2}{1-\beta} - 2$
- Difference is nonnegative if $\beta \frac{2}{1-\beta} - 2 \geq 0$ or $\beta \geq (1-\beta)$, so $\beta \geq 1/2$

Thus in order for both players to want to cooperate, β should be more than or equal to 1/2.