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GRADE
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Problem Set 4

LATEST SUBMISSION GRADE

100%

1.
- Two players have to share 50 coins (of equal value).
- Players' payoffs are the number of coins they each get
- First, player 1 splits the coins into 2 piles.
- Second, player 2 chooses one pile for him/herself and gives the other pile to player 1

1 / 1 point

What is agent 1's strategy in a backward induction solution?

- ○ d) Splitting coins into 1/49.
- ◉ a) Splitting coins into 25/25.
- ○ c) Splitting coins into 15/35.
- ○ b) Splitting coins into 0/50.

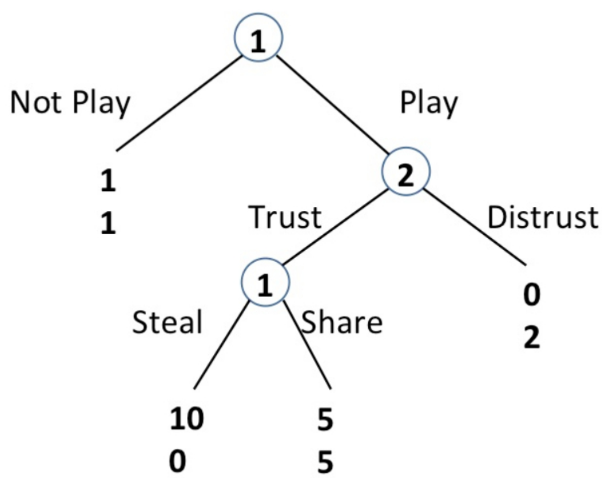
Correct

(a) is true.

- When there are two piles for 2 to choose, 2 prefers to have the pile with more coins and gives 1 the pile with fewer coins.
- Thus, player 1 will never get more than 25 coins from any split. Thus, when 1 splits the coins, it is best to split them into 25/25 as any other split will lead to fewer coins for player 1.

2.

1 / 1 point



Find all of the pure strategy **Nash** Equilibria of this game. There can be more than one equilibrium. [Here ((Not Play,Steal),(Trust)) indicates that player 1 chooses Not Play at the first decision node and Steal at the second decision node, and 2 chooses Trust at his unique decision node.]

- ☐ a) ((Play, Share), (Trust))
- ☑ b) ((Not play, Share), (Distrust))

Correct

(b) and (c) are correct.

- To see that (b) and (c) form two Nash Equilibria we have to check that no player wants to deviate:
- Given that player 1 decides Not to play, player 2 is indifferent between Trust and Distrust since his/her node is not reached.
- Given that player 2 decides to Distrust, player 1 prefers to Not play and get a payoff of 1 rather than Play and getting a payoff of 0.
- No player has an incentive to deviate.
- (d) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Not

play, Steal) and getting a payoff of 1.

- (d) cannot be a NE since 1 would prefer to deviate to (Not play, Steal) and get 1 instead of (Play, Steal) and getting a payoff of 0.
- (a) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Play, Share) and getting a payoff of 5.

☑ c) ((Not play, Steal),(Distrust))

✓ **Correct**
(b) and (c) are correct.

- To see that (b) and (c) form two Nash Equilibria we have to check that no player wants to deviate:
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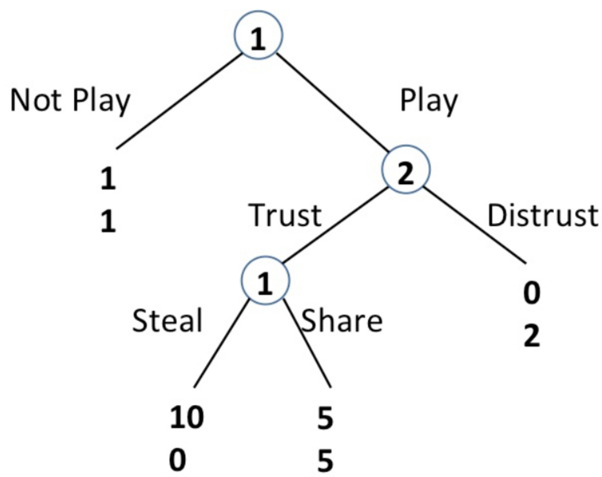
- No player has an incentive to deviate.
- (d) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Not play, Steal) and getting a payoff of 1.
- (d) cannot be a NE since 1 would prefer to deviate to (Not play, Steal) and get 1 instead of (Play, Steal) and getting a payoff of 0.
- (a) cannot be a NE since 1 would prefer to deviate to (Play, Steal) and get 10 instead of (Play, Share) and getting a payoff of 5.

☐ d) ((Not play, Steal), (Trust))

☐ e) ((Play, Steal), (Distrust))

3. Which is the Subgame Perfect Equilibrium of this game? [Here ((Not Play,Steal),(Trust)) indicates that player 1 chooses Not Play at the first decision node and Steal at the second decision node, and 2 chooses Trust at his unique decision node.]

1 / 1 point



◉ a) ((Not play, Steal),(Distrust))

○ b) ((Not play, Share), (Distrust))

○ c) ((Not play, Steal), (Trust))

○ d) ((Play, Steal), (Distrust))

○ e) ((Play, Share), (Trust))

✓ **Correct**
(a) is correct.

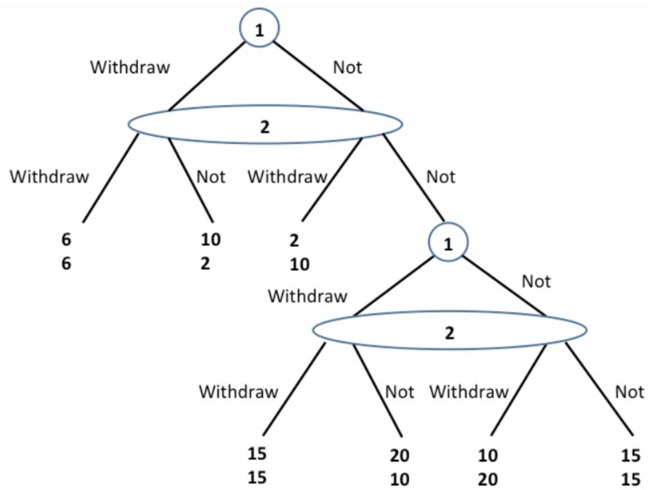
- Recall that the set of Subgame Perfect Equilibria is a subset of the set of Nash Equilibria (and sometimes a strict subset). Thus, given the answer to the previous question, we only need to check if (a) and (b) are SPNE.
- Checking (a):
- If 2 plays Trust, 1 prefers to Steal;
- Given this, if 2 plays Trust he/she receives 0 and if 2 plays Distrust he/she receives 2. Player 2 will choose Distrust.
- Since player 2 plays Distrust, 1 prefers Not Play (payoff 1) to Play (payoff 0).
- ((Not Play, Steal), (Distrust)) is SPNE.

- Checking (b):
- ((Not Play, Share), (Distrust)) cannot be a SPNE since 1 is not best responding in his/her second decision node (his/her best response is Steal).

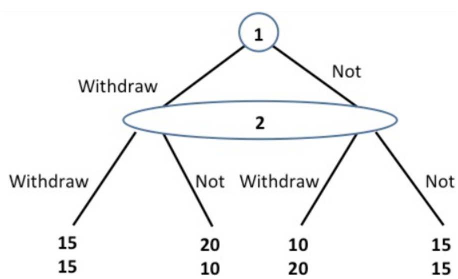
4.
- There are 2 investors. Each has deposited $10 in the same bank.
- The bank invested both deposits in a single long-term project.
- If the bank wants to end the project before its completion, a total of $12 can be recovered (out of the $20 invested).
- If the bank waits until the project is completed, it will receive a total of $30.
- Investors can withdraw money from their bank accounts at only 2 periods: before the project is completed and after.

1 / 1 point

The extensive from representation of the game between both investors is depicted below:



In order to find the subgame perfect Nash equilibrium of the whole game first focus on the subgame that starts with investor 1's second decision node:



Which is a pure strategy Nash equilibrium of this subgame?

- a) (Withdraw, Withdraw)
- b) (Withdraw, Not)
- c) (Not, Withdraw)
- d) (Not, Not)

Correct

(a) is correct.

- It is easy to check that Withdraw is a strictly dominant strategy for both players.
- Therefore, they have to play Withdraw in equilibrium.

5.
- There are 2 investors. Each has deposited $10 in the same bank.
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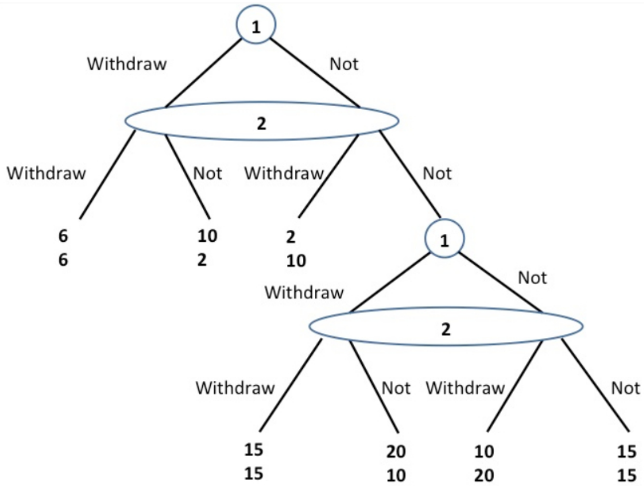

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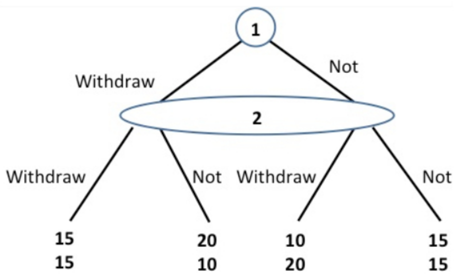

What are the subgame perfect Nash equilibria of the whole game? There might be more than one. [Hint: ((Withdraw, Not),(Not, Withdraw)) are the first and second investors' strategies in their first and second decision nodes, respectively. So, ((Withdraw, Not),(Not, Withdraw)) indicates that the first investor withdraws at her first decision node but not at her second, while the second invest does not withdraw at his first decision node but does at his second decision node.]

- [x] a) ((Not, Withdraw), (Not, Withdraw))

Correct

(a) and (d) are correct.

- In a subgame perfect Nash equilibrium both investors must play Withdraw in the subgame analyzed above. Then, the payoff that they expect to receive when neither of them Withdraws in the first period is (15,15).
- Replacing the payoff of (Not, Not) in the first period to (15,15) simplifies the whole game to a one-period game. This simplified game has two equilibria: one in which both investors withdraw in the first period, and another in which both investors wait and withdraw the money in the second period.
- Verify that the best response to the other investor withdrawing, is also to withdraw.
- Verify that the best response to the other investor not withdrawing, is also not to withdraw.
- (b) and (c) are not subgame perfect Nash equilibrium because either of the investors has incentives to deviate in his/her first decision node.

- [ ] b) ((Not, Withdraw), (Withdraw, Withdraw))

- [ ] c) ((Withdraw, Withdraw), (Not, Withdraw))

- [x] d) ((Withdraw, Withdraw), (Withdraw, Withdraw))

Correct

(a) and (d) are correct.

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6.
- Five pirates have obtained 100 gold coins and have to divide up the loot. The pirates are all extremely intelligent, treacherous and selfish (especially the captain) each wanting to maximize the number of coins that he gets.
- It is always the captain who proposes a distribution of the loot. All pirates vote on the proposal, and if half the crew or more go "Aye", the loot is divided as proposed.
- If the captain fails to obtain support of at least half his crew (which includes himself), all pirates turn against him and make him walk the plank. The pirates then start over again with the next most senior pirate as captain (the pirates have a strict order of seniority denoted by A, B, C, D and E).
- Pirates' preferences are ordered in the following way. First of all, each pirate wants to survive. Second, given survival, each pirate wants to maximize the number of gold coins he receives. Finally, each pirate would prefer to throw another overboard in the case of indifference.

1 / 1 point

What is the maximum number of coins that the original captain gets to keep across all subgame perfect equilibria of this game?

(Hint, work by backward induction to reason what the split will be if three captains have been forced to walk the plank and there are only two pirates left. Just one vote is enough to approve the split among the two pirates. Then use that to solve for what happens when two have walked the plank and there are three pirates left, and so forth.)

- a) 100;
- b) 0;
- c) 50;
- d) 98;

Correct

(d) is correct.

- If three captains have walked the plan and only pirates D and E remain, captain D will offer a split $D = 100$ and $E = 0$ since just one vote is enough to approve a split among two pirates.
- If two captains have walked the plan and only pirates C, D and E remain, captain C will offer a split $C = 99$, $D = 0$ and $E = 1$ since $E = 1$ is larger than what E would get if he rejects the proposal (remember that D would offer $E = 0$). Thus, pirate E approves the split.
- If pirates B, C, D and E remain, captain B will offer a split $B = 99$, $C = 0$, $D = 1$ and $E = 0$ since only two votes are needed to approve a split among four pirates. Pirate D approves the split because otherwise he would receive 0.
- In the last case, pirate B would have not preferred to offer 1 coin to E. The reason is that when indifferent, pirate E would prefer to throw pirate B overboard.
- If all 5 pirates remain, in order to obtain enough votes, pirate A will offer a split $A = 98$, $B = 0$, $C = 1$, $D = 0$ and $E = 1$.