Coalitional Game Theory: Definitions

Game Theory Course:
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Introduction

- Our focus is on what groups of agents, rather than individual agents, can achieve.
- Given a set of agents, a coalitional game defines how well each group (or *coalition*) of agents can do for itself.
- We are not concerned with:
  - how the agents make individual choices within a coalition;
  - how they coordinate;
- ...instead, we take the payoffs to a coalition as given.

Definition

- Transferable utility assumption:
  - payoffs may be redistributed among a coalition's members.
  - satisfied whenever payoffs are dispensed in a universal *currency*.
  - each coalition can be assigned a single value as its payoff.

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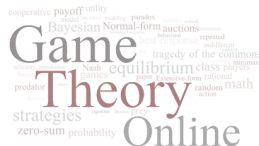
Definition (Coalitional game with transferable utility)

A **coalitional game with transferable utility** is a pair (N, v), where

- N is a finite set of players, indexed by i; and
- $v : 2^N \to \mathbb{R}$ associates with each coalition $S \subseteq N$ a real-valued payoff $v(S)$ that the coalition's members can distribute among themselves. We assume that $v(\emptyset) = 0$.
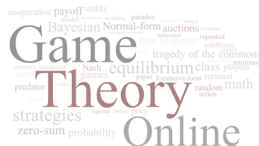
Using Coalitional Game Theory

Questions we use coalitional game theory to answer:

1. Which coalition will form?
2. How should that coalition divide its payoff among its members?

The answer to (1) is often "the grand coalition" (all agents in N) though this can depend on having made the right choice about (2).

Superaddaitive games

Definition (Superadditive game)

A game $G = (N, v)$ is superadditive if for all $S, T \subset N$, if $S \cap T = \emptyset$, then $v(S \cup T) \geq v(S) + v(T)$.

- Superadditivity is justified when coalitions can always work without interfering with one another
  - the value of two coalitions will be no less than the sum of their individual values.
  - implies that the grand coalition has the highest payoff

Analyzing coalitional games

1. Which coalition will form?
   - we'll consider cases where the answer is the grand coalition
   - makes sense for superadditive games

2. How should the coalition divide its payoff?
   - in order to be fair
   - in order to be stable