Congratulations! You passed!

TO PASS 70% or higher

Keep Learning

GRADE
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Problem Set 1

LATEST SUBMISSION GRADE

100%

1.

1 / 1 point

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find the strictly dominant strategy:

- 1) a;
- 2) b;
- 3) c;
- 4) d;
- 5) x;
- 6) y;
- 7) z

Correct

(3) c is a strictly dominant strategy.

- Because when 2 plays x or y or z, playing c always gives 1 a strictly higher payoff than playing a, b or d.
- None of the strategies is always strictly best for player 2.

2.

1 / 1 point

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find a very weakly dominant strategy that is not strictly dominant.

- 1) a;
- 4) d;
- 2) b;
- 3) c;
- 7) z
- 6) y;
- 5) x;

Correct

(6) y is a weakly dominant strategy that is not strictly dominant.

- Because when 1 plays a, b, c or d, playing y always gives 2 a weakly higher payoff than playing x or z.
- Note that it is only weakly higher when 1 plays a, as then playing x and y gives 2 the same payoff.

3.

1 / 1 point

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

When player 1 plays d, what is player 2's best response:

- a) Only x
- b) Only y
- c) Only z
- d) Both y and z

Correct

(b) only y is a best response for player 2.

When player 1 plays d, player 2 earns 3 from playing x, 4 from playing y and 0

from playing z. Thus only y is a best response.

4.

1 / 1 point

1 \ 2	x	y	z
a	1,2	2,2	5,1
b	4,1	3,5	3,3
c	5,2	4,4	7,0
d	2,3	0,4	3,0

Find all strategy profiles that form pure strategy Nash equilibria (there may be more than one, or none):

- [ ] (b, y);
- [ ] (d, y);
- [ ] (b, z);
- [ ] (a, y);
- [ ] (c, x);
- [x] (c, y);

Correct

(c, y) is the only pure strategy Nash equilibria.

- Check that no one wants to deviate.
- Note that c is the strictly dominant strategy and so is the only possible strategy for player 1 in a pure strategy Nash equilibrium.
- When player 1 plays c, playing y gives player 2 the highest payoff.

- [ ] (a, x);
- [ ] (b, x);
- [ ] (a, z);
- [ ] (d, z).
- [ ] (c, z);
- [ ] (d, x);

5. There are 2 players who have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero. This game is known as `Nash Bargaining'.

1 / 1 point

Which of the following is a strictly dominant strategy?

- a)1;
- b) 0.5;
- c) 0;

d) None of the above.

Correct
(d) is true.

- No player has any strictly dominant strategies. Any of the options given constitutes a best response to some strategy played by the other player, and so no strategy always strictly outperforms all other strategies.
- Strategies (a) and (c) are in the set of best responses of player i when player j's strategy is $s_j > 1$.
- Strategies (b) is the best response of player i when player j's strategy is $s_j = 0.5$.

6. There are 2 players who have to decide how to split one dollar. The bargaining process works as follows. Players simultaneously announce the share they would like to receive s_1 and s_2, with $0 \leq s_1, s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named and if $s_1 + s_2 > 1$, then both players fail to achieve an agreement and receive zero.

1 / 1 point

Which of the following strategy profiles is a pure strategy Nash equilibrium?

a) (0.3, 0.7);

b) (0.5, 0.5);

c) (1.0, 1.0);

d) All of the above

Correct
(d) is true.

- Check that no one wants to deviate.
- Note that when player i plays $s_i < 1$, player j's best response is $s_j = 1 - s_i$. This holds in a) and b). Thus, both players are best responding.
- .When player i plays $s_i = 1$, player j's best response can be any number as she will get 0 no matter 1. Thus c) also forms a pure strategy NE.

7. Two firms produce identical goods, with a production cost of $c > 0$ per unit.

1 / 1 point

Each firm sets a nonnegative price (p_1 and p_2).

All consumers buy from the firm with the lower price, if $p_1 \neq p_2$. Half of the consumers buy from each firm if $p_1 = p_2$.

D is the total demand.

Profit of firm i is:

- 0 if $p_i > p_j$ (no one buys from firm i);
- $D\frac{p_i - c}{2}$ if $p_i = p_j$(Half of customers buy from firm i);
- $D(p_i - c)$ if $p_i < p_j$ (All customers buy from firm i)

Find the pure strategy Nash equilibrium:

a) Both firms set $p = 0$.

b) Firm 1 sets $p = 0$, and firm 2 sets $p = c$.

c) Both firms set $p = c$.

d) No pure strategy Nash equilibrium exists.

Correct
(c) is true.

- Notice than in a) and b) at least one firm i is making negative profits since $p_i < c$ and it sells a positive quantity. Thus, firm i would prefer to deviate to $p_i > p_j$ and earn a profit of 0.
- It is easy to verify that $p_1 = p_2 = c$ is an equilibrium by checking that no firm wants to deviate:
- When $p_1 = p_2 = c$, both firms are earning null profits.
- If firm 1 increases its price above c ($p_1 > c$), it will still earn null profits.
- If firm 2 decreases its price below c ($p_1 < c$), it will earn strictly negative profits.
- In both cases, either the firm is indifferent or strictly worse off. Then, it does not have incentives to deviate given the other firm's strategy.

8.

1 / 1 point

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.

- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find all ***very weakly* dominant** strategies (click all that apply: there may be more than one, or none).

☑ a) Voter 1 voting for A.

✓ **Correct**
(a) and (d) are (very weakly) dominant strategies.

- Check (b): for voter 1, voting for candidate A always results in at least as high a payoff as voting for candidate B and indeed is sometimes strictly better (when the other players vote for different candidates).
- When voters 2 and 3 vote for B, voter 1 is indifferent between A or B (since B will win anyways).
- When either 2 or 3 (or both) vote for A, voter 1 strictly prefers to vote for A than for B.
- Check (c): for voter 2, voting for candidate B is a very weakly dominant strategy.
- When voters 1 and 3 vote for A, voter 2 is indifferent between A or B (since A will win anyways).
- When either 1 or 3 (or both) vote for B, voter 2 strictly prefers to vote for B than for A.
- (b) and (c) can't be very weakly dominant strategies, since they sometimes do worse than the other strategy.

☐ b) Voter 1 voting for B.

☐ c) Voter 2 (or 3) voting for A.

☑ d) Voter 2 (or 3) voting for B.

✓ **Correct**
(a) and (d) are (very weakly) dominant strategies.

- Check (b): for voter 1, voting for candidate A always results in at least as high a payoff as voting for candidate B and indeed is sometimes strictly better (when the other players vote for different candidates).
- When voters 2 and 3 vote for B, voter 1 is indifferent between A or B (since B will win anyways).
- When either 2 or 3 (or both) vote for A, voter 1 strictly prefers to vote for A than for B.
- Check (c): for voter 2, voting for candidate B is a very weakly dominant strategy.
- When voters 1 and 3 vote for A, voter 2 is indifferent between A or B (since A will win anyways).
- When either 1 or 3 (or both) vote for B, voter 2 strictly prefers to vote for B than for A.
- (b) and (c) can't be very weakly dominant strategies, since they sometimes do worse than the other strategy.

9. **1 / 1 point**

- Three voters vote over two candidates (A and B), and each voter has two pure strategies: vote for A and vote for B.
- When A wins, voter 1 gets a payoff of 1, and 2 and 3 get payoffs of 0; when B wins, 1 gets 0 and 2 and 3 get 1. Thus, 1 prefers A, and 2 and 3 prefer B.
- The candidate getting 2 or more votes is the winner (majority rule).

Find **all** pure strategy Nash equilibria (click all that apply)? Hint: there are three.

☑ a) 1 voting for A, and 2 and 3 voting for B.

✓ **Correct**
(a), (b) and (c) are pure strategy Nash equilibria.

- It is easy to verify that (a), (b) and (c) are equilibria by checking that no voter wants to deviate:
- When all voters vote for the same candidate, no single voter has any incentives to deviate because his/her individual vote can't modify the outcome of the election.
- In (a), voter 1 is indifferent between candidates A and B, and voters 2 and 3 are best responding to the strategies played by the remaining voters (if voter 2 votes for A, candidate A wins; if voter 2 votes for B, candidate B wins).
- (d) is not an equilibrium, since voter 2 has incentives to deviate and vote for candidate B.

☑ b) All voting for A.

Correct

(a), (b) and (c) are pure strategy Nash equilibria.

- It is easy to verify that (a), (b) and (c) are equilibria by checking that no voter wants to deviate:
- When all voters vote for the same candidate, no single voter has any incentives to deviate because his/her individual vote can't modify the outcome of the election.
- In (a), voter 1 is indifferent between candidates A and B, and voters 2 and 3 are best responding to the strategies played by the remaining voters (if voter 2 votes for A, candidate A wins; if voter 2 votes for B, candidate B wins).
- (d) is not an equilibrium, since voter 2 has incentives to deviate and vote for candidate B.

- [x] c) All voting for B.

Correct

(a), (b) and (c) are pure strategy Nash equilibria.

- It is easy to verify that (a), (b) and (c) are equilibria by checking that no voter wants to deviate:
- When all voters vote for the same candidate, no single voter has any incentives to deviate because his/her individual vote can't modify the outcome of the election.
- In (a), voter 1 is indifferent between candidates A and B, and voters 2 and 3 are best responding to the strategies played by the remaining voters (if voter 2 votes for A, candidate A wins; if voter 2 votes for B, candidate B wins).
- (d) is not an equilibrium, since voter 2 has incentives to deviate and vote for candidate B.

- [ ] d) 1 and 2 voting for A, and 3 voting for B.