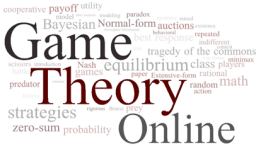


Stochastic Games

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Stochastic Games

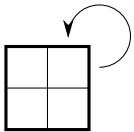
Introduction

- What if we didn't always repeat back to the same stage game?
- A stochastic game is a generalization of repeated games
  - agents repeatedly play games from a set of normal-form games
  - the game played at any iteration depends on the previous game played and on the actions taken by all agents in that game

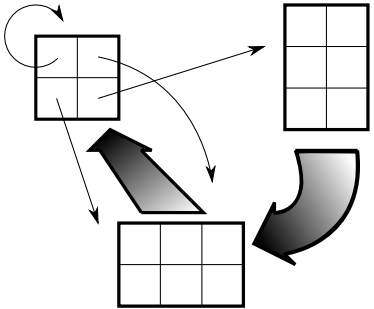
Stochastic Games

Visualization

Repeated Game



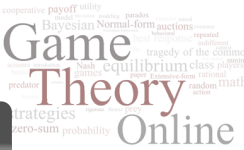
Stochastic Game



An informal visualization of the difference between repeated and stochastic games.

Stochastic Games

Formal Definition



Definition

A stochastic game is a tuple (Q, N, A, P, R), where

- Q is a finite set of states,
- N is a finite set of n players,
- $A = A_1 \times \cdots \times A_n$, where A_i is a finite set of actions available to player i,
- $P : Q \times A \times Q \to [0, 1]$ is the transition probability function; $P(q, a, \hat{q})$ is the probability of transitioning from state q to state $\hat{q}$ after joint action a, and
- $R = r_1, \ldots, r_n$, where $r_i : Q \times A \to \mathbb{R}$ is a real-valued payoff function for player i.

Stochastic Games

Remarks

- This definition assumes strategy space is the same in all games
  - otherwise just more notation
- Also generalizes MDP (Markov Decision Process)
  - i.e. MDP is a single-agent stochastic game

Stochasitc Games

Analysis

Can do analysis as with repeated games.

- limit average reward
- future discount reward