Problem Set 7
Graded Quiz • 18 min

Due Jun 8, 2:59 PM +08

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Problem Set 7

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1.
- Three players together can obtain 1 to share, any two players can obtain 0.8, and one player by herself can obtain zero.
- Then, $N = 3$ and $v(1) = v(2) = v(3) = 0$, $v(1, 2) = v(2, 3) = v(3, 1) = 0.8$, $v(1, 2, 3) = 1$.

1 / 1 point

Which allocation is in the core of this coalitional game?

○ a) (0,0,0);

○ b) (0.4, 0.4, 0);

○ c) (1/3, 1/3, 1/3);

● d) The core is empty;

✓ **Correct**

(d) is true.

- By definition, the core of this game is formed by a triplet $(x_1, x_2, x_3) \in R^3_+$ that satisfies:
- $x_i + x_j \geq 0.8$ for $i \neq j$
- $x_1 + x_2 + x_3 \geq 1$
- There is no triplet (x_1, x_2, x_3) that satisfies all inequalities. Then, the core is empty.

2.

1 / 1 point

- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i, j) = 1$ if i, j are a pair of a buyer and seller).

If the number of buyers and sellers is $B = 2$ and $S = 1$, respectively, which allocations are in the core? [There might be more than one]

☑ a) Each seller receives 1 and each buyer receives 0.

✓ **Correct**

(a) is true.

- By definition, the core of this game is formed by a vector of payoffs to buyers ($b1$ and $b2$) and to the seller (s) $(x_{b1}, x_{b2}, x_s) \in R^3_+$ that satisfies:
- $x_{b1} + x_{b2} \geq 0$;
- $x_{bi} + x_s \geq 1$ for $i = 1, 2$;
- $x_{b1} + x_{b2} + x_s \geq 1$;
- and the feasibility constraint $x_{b1} + x_{b2} + x_s \leq 1$.
- It is easy to verify that allocation (a) is the only one that satisfies the set of inequalities.

☐ b) Each seller receives 0 and each buyer receives 1.

☐ c) Each seller receives 1/2 and each buyer receives 1/2.

3.
- There is a market for an indivisible good with B buyers and S sellers.
- Each seller has only one unit of the good and has a reservation price of 0.
- Each buyer wants to buy only one unit of the good and has a reservation price of 1.
- Thus $v(C) = min(B_C, S_C)$ where B_C and S_C are the number of buyers and sellers in coalition C (and so, for instance, $v(i) = 0$ for any single player, and $v(i, j) = 1$ if i, j are a pair of a buyer and seller).

1 / 1 point

Now assume that we increase the number of sellers so that $B = 2$ and $S = 2$. Which allocations are in the core? [There might be more than one]

☑ a) Each seller receives 1 and each buyer receives 0.

✓ **Correct**

All are in the core.

- Again, the core of this game is formed by a vector of payoffs to buyers and sellers $(x_{b1}, x_{b2}, x_{s1}, x_{s2}) \in R^4_+$ that satisfies:
- $x_{b1} + x_{b2} \geq 0;$
- $x_{s1} + x_{s2} \geq 0;$
- $x_{bi} + x_{sj} \geq 1$ for $i = 1, 2$ and $j = 1, 2;$
- $x_{b1} + x_{b2} + x_{s1} + x_{s2} \geq 2;$
- and the feasibility constraint $x_{b1} + x_{b2} + x_{s1} + x_{s2} \leq 2.$
- It is easy to verify that allocations (a), (b) and (c) satisfy the set of inequalities.
- In fact, any split of the surplus that gives α to all sellers and $1 - \alpha$ to all buyers (with $\alpha \in [0, 1]$) is in the core. That is, any split of the surplus is possible; the only restriction imposed by the increase in competition (i.e., increase in the number of sellers) is that all pairs must receive the same share of the surplus.

☑ b) Each seller receives 0 and each buyer receives 1.

✓ **Correct**

All are in the core.

- Again, the core of this game is formed by a vector of payoffs to buyers and sellers $(x_{b1}, x_{b2}, x_{s1}, x_{s2}) \in R^4_+$ that satisfies:
- $x_{b1} + x_{b2} \geq 0;$
- $x_{s1} + x_{s2} \geq 0;$
- $x_{bi} + x_{sj} \geq 1$ for $i = 1, 2$ and $j = 1, 2;$
- $x_{b1} + x_{b2} + x_{s1} + x_{s2} \geq 2;$
- and the feasibility constraint $x_{b1} + x_{b2} + x_{s1} + x_{s2} \leq 2.$
- It is easy to verify that allocations (a), (b) and (c) satisfy the set of inequalities.
- In fact, any split of the surplus that gives α to all sellers and $1 - \alpha$ to all buyers (with $\alpha \in [0, 1]$) is in the core. That is, any split of the surplus is possible; the only restriction imposed by the increase in competition (i.e., increase in the number of sellers) is that all pairs must receive the same share of the surplus.

☑ c) Each seller receives 1/2 and each buyer receives 1/2.

✓ **Correct**

All are in the core.

- Again, the core of this game is formed by a vector of payoffs to buyers and sellers $(x_{b1}, x_{b2}, x_{s1}, x_{s2}) \in R^4_+$ that satisfies:
- $x_{b1} + x_{b2} \geq 0;$
- $x_{s1} + x_{s2} \geq 0;$
- $x_{bi} + x_{sj} \geq 1$ for $i = 1, 2$ and $j = 1, 2;$
- $x_{b1} + x_{b2} + x_{s1} + x_{s2} \geq 2;$
- and the feasibility constraint $x_{b1} + x_{b2} + x_{s1} + x_{s2} \leq 2.$
- It is easy to verify that allocations (a), (b) and (c) satisfy the set of inequalities.
- In fact, any split of the surplus that gives α to all sellers and $1 - \alpha$ to all buyers (with $\alpha \in [0, 1]$) is in the core. That is, any split of the surplus is possible; the only restriction imposed by the increase in competition (i.e., increase in the number of sellers) is that all pairs must receive the same share of the surplus.

4.

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

1 / 1 point

Which allocation is in the core of this coalitional game?

○ b) (2, 2, 0);

○ d) The core is empty;

● c) (2, 2, 2);

○ a) (0,0,0);

✓ **Correct**

(c) is true.

- By definition, the core of this game is formed by a vector of payoffs to each student $(x_1, x_2, x_3) \in R^3_+$ that satisfies:
- $x_i + x_j \geq 4$ for $i \neq j$
- $x_1 + x_2 + x_3 \geq 6$
- (2,2,2) is the only option that satisfies these inequalities. Then, it belongs to the core.

5.

1 / 1 point

- The instructor of a class allows the students to collaborate and write up together a particular problem in the homework assignment.
- Points earned by a collaborating team are divided among the students in any way they agree on.
- There are exactly three students taking the course, all equally talented, and they need to decide which of them if any should collaborate.
- The problem is so hard that none of them working alone would score any points. Any two of them can score 4 points together. If all three collaborate, they can score 6 points.

What is the Shapley value of each player?

○ a) $\phi = (0, 0, 0)$

○ b) $\phi = (2, 0, 2)$

○ c) $\phi = (1/3, 1/3, 1/3)$

● d) $\phi = (2, 2, 2)$

✓ **Correct**

(d) is true.

- Use the definition of the Shapley Value to compute its value for each player.
- Another way to find the Shapley Value is to remember that:
- by the axiom of symmetry, all agents should receive the same payoff.
- the Shapley value divides the payoff to the grand coalition completely
- Then, all agents will have a Shapley value of $6/3 = 2$.

6. There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

1 / 1 point

The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

The production function satisfies:

- $F(c \cup w1) = F(c \cup w2) = 3$
- $F(c \cup w1 \cup w2) = 4$

Which allocations are in the core of this coalitional game? [There might be more than one]

☑ a) $x_c = 2$, $x_{w1} = 1$, $x_{w2} = 1$;

✓ **Correct**

(d) is true.

- It is easy to verify that allocations (a), (b) and (c) satisfy the definition of the core.
- It can be shown more generally that for any given number n of workers and any increasing and concave production function f, the core of this coalitional game is defined by:
- $x_{wi} \leq f(c \cup w1 \ldots \cup wn) - f(c \cup w1 \ldots \cup w(n-1))$
- $x_c + \sum_{i=1}^{n} x_{w1} \leq f(c \cup w1 \cup \ldots \cup wn)$
- Intuitively, the first equation requires each worker to receive less than the marginal product of the n^{th} worker. If this condition would not hold for worker i, then the rest of the workers and the capitalist could abandon him and get a higher value for the new coalition.
- The second condition is a feasibility condition (the sum of payoffs of the grand coalition is not greater than the resources available).

☑ b) $x_c = 2.5$, $x_{w1} = 0.5$, $x_{w2} = 1$;

✓ **Correct**

(d) is true.

- It is easy to verify that allocations (a), (b) and (c) satisfy the definition of the core.
- It can be shown more generally that for any given number n of workers and any increasing and concave production function f, the core of this coalitional game is defined by:
- $x_{wi} \leq f(c \cup w1 \ldots \cup wn) - f(c \cup w1 \ldots \cup w(n-1))$
- $x_c + \sum_{i=1}^{n} x_{w1} \leq f(c \cup w1 \cup \ldots \cup wn)$
- Intuitively, the first equation requires each worker to receive less than the marginal product of the n^{th} worker. If this condition would not hold for worker i, then the rest of the workers and the capitalist could abandon him and get a higher value for the new coalition.
- The second condition is a feasibility condition (the sum of payoffs of the grand coalition is not greater than the resources available).

☑ c) $x_c = 4, x_{w1} = 0, x_{w2} = 0$;

✓ **Correct**
(d) is true.

- It is easy to verify that allocations (a), (b) and (c) satisfy the definition of the core.
- It can be shown more generally that for any given number n of workers and any increasing and concave production function f, the core of this coalitional game is defined by:
- $x_{wi} \leq f(c \cup w1 \ldots \cup wn) - f(c \cup w1 \ldots \cup w(n-1))$
- $x_c + \sum_{i=1}^{n} x_{w1} \leq f(c \cup w1 \cup \ldots \cup wn)$
- Intuitively, the first equation requires each worker to receive less than the marginal product of the n^{th} worker. If this condition would not hold for worker i, then the rest of the workers and the capitalist could abandon him and get a higher value for the new coalition.
- The second condition is a feasibility condition (the sum of payoffs of the grand coalition is not greater than the resources available).

7. There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

1 / 1 point

The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

The production function satisfies:

- $F(c \cup w1) = F(c \cup w2) = 3$
- $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of the capitalist?

○ a) 3;

○ b) 4;

● c) 7/3;

○ d) 7;

✓ **Correct**
(c) is true.

- Use the definition of the Shapley Value to compute its value for the capitalist.

8. There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

1 / 1 point

The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

The production function satisfies:

- $F(c \cup w1) = F(c \cup w2) = 3$
- $F(c \cup w1 \cup w2) = 4$

What is the Shapley value of each worker?

○ a) 1;

● b) 5/6;

○ c) 3/4;

○ d) 1/2;

✓ **Correct**
(b) is true.

- Use the definition of the Shapley Value to compute its value for each worker.
- Another way to find the Shapley Value is to remember that:

- by the axiom of symmetry, all workers should receive the same payoff
- the Shapley value divides the payoff to the grand coalition completely
- Then, all agents will have a Shapley value of $(F(c \cup w1 \cup w2) - 7/3)/2 = (4 - 7/3)/2 = 5/6$. (Where 7/3 is the Shapley value of the capitalist)

9. There is a single capitalist (c) and a group of 2 workers ($w1$ and $w2$).

1 / 1 point

The production function is such that total output is 0 if the firm (coalition) is composed only of the capitalist or of the workers (a coalition between the capitalist and a worker is required to produce positive output).

The production function satisfies:

- $F(c \cup w1) = F(c \cup w2) = 3$
- $F(c \cup w1 \cup w2) = 4$

True or False: If there was an additional 3^{rd} worker that is completely useless (i.e., his marginal contribution is 0 in every coalition), then the sum of the Shapley Values of the capitalist and the first two workers will remain unchanged.

● a) True;

○ b) False;

✓ **Correct**

(a) is correct.

- The Shapley Value satisfies the Dummy player Axiom:
- if i is a dummy player, then he/she must have a Shapley Value of 0
- Since the 3^{rd} worker is a Dummy player (check the definition), his/her Shapley Value must be 0.
- Thus, the statement is true because the Shapley Value divides the payoff of the grand coalition completely.