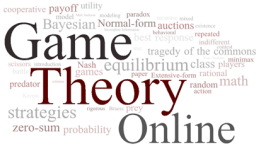


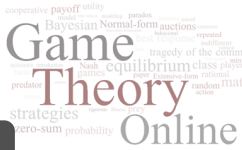
Imperfect Information Extensive Form: Definition, Strategies

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Intro

- So far, we've allowed players to choose an action at every choice node.
  - This implies that players know the node they are in and all the prior choices, including those of other agents.
  - We may want to model agents needing to act with partial or no knowledge of the actions taken by others, or even themselves.
- Imperfect information extensive-form games:
  - each player's choice nodes partitioned into information sets
  - agents cannot distinguish between choice nodes in the same information set.

Formal definition

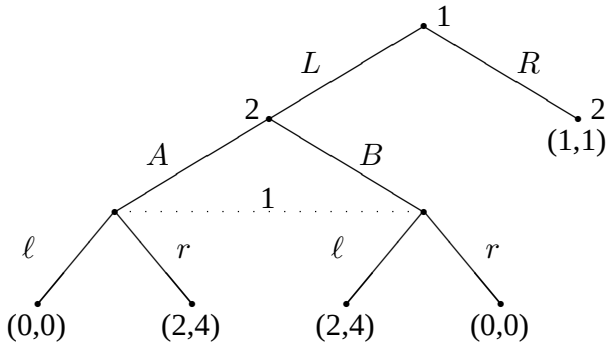


Definition

An imperfect-information game (in extensive form) is a tuple $(N, A, H, Z, \chi, \rho, \sigma, u, I)$, where

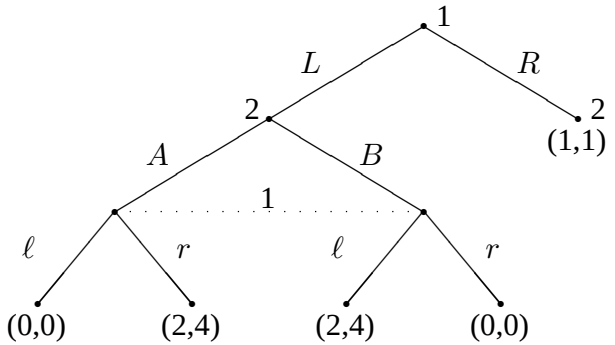
- $(N, A, H, Z, \chi, \rho, \sigma, u)$ is a perfect-information extensive-form game, and
- $I = (I_1, \ldots, I_n)$, where $I_i = (I_{i,1}, \ldots, I_{i,k_i})$ is an equivalence relation on (that is, a partition of) $\{h \in H : \rho(h) = i\}$ with the property that $\chi(h) = \chi(h')$ and $\rho(h) = \rho(h')$ whenever there exists a j for which $h \in I_{i,j}$ and $h' \in I_{i,j}$.

Strategies



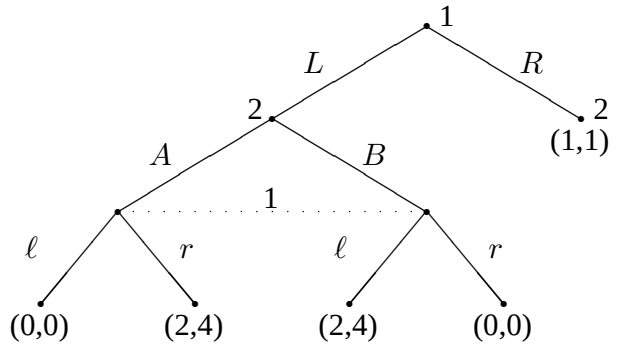
- What are the equivalence classes for each player?

Strategies



- What are the equivalence classes for each player?
- How should we define the pure strategies for each player?

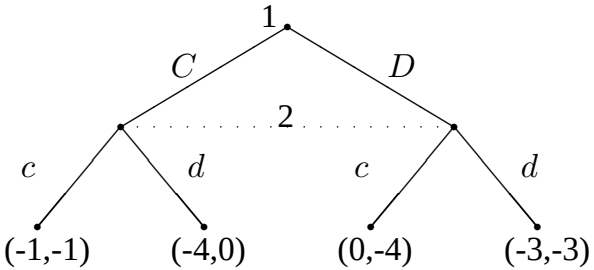
Strategies



- What are the equivalence classes for each player?
- How should we define the pure strategies for each player?
  - choice of an action in each equivalence class.
- Formally, the pure strategies of player i consist of the cross product $\prod_{I_{i,j} \in I_i} \chi(I_{i,j})$.

Normal-form games

- We can represent any normal form game.



- It would be the same if we put player 2 at the root node.

Induced Normal Form

- Same as before: enumerate pure strategies for all agents
- Mixed strategies are just mixtures over the pure strategies.
- Nash equilibria are also preserved.

Induced Normal Form

- Same as before: enumerate pure strategies for all agents
- Mixed strategies are just mixtures over the pure strategies.
- Nash equilibria are also preserved.
- We've now defined two mappings: NF $\rightarrow$ IIEF and IIEF $\rightarrow$ NF.
  - what happens if we apply each mapping in turn?
  - we might not end up with the same game, but we do get one with the same strategy spaces and equilibria.