

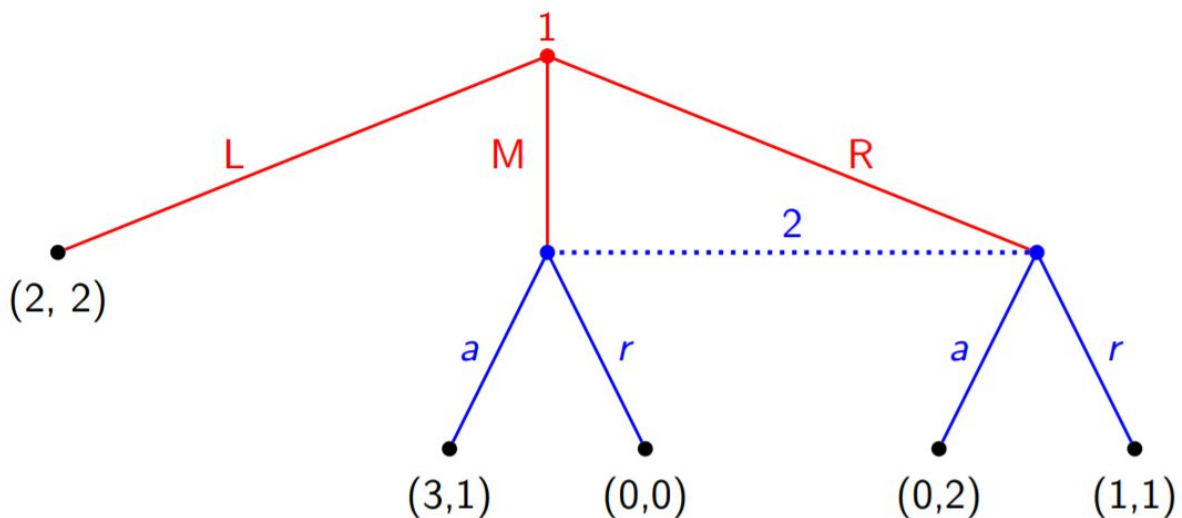
## Summary Notes for Week 4: Extensive-Form Games

### Extensive Form Games

Extensive form games are used to describe sequential games where each player takes an action following one another.

Perfect information: each player can see the decision taken by the previous player, and the rules of the game and each player's payoffs are common knowledge. All players know the game structure. Each player, when making any decision, is perfectly informed of all the events that have previously occurred.

Imperfect information: While all players know the game structure, each player, when making any decision, may not be perfectly informed about some (or all) of the events that have already occurred.



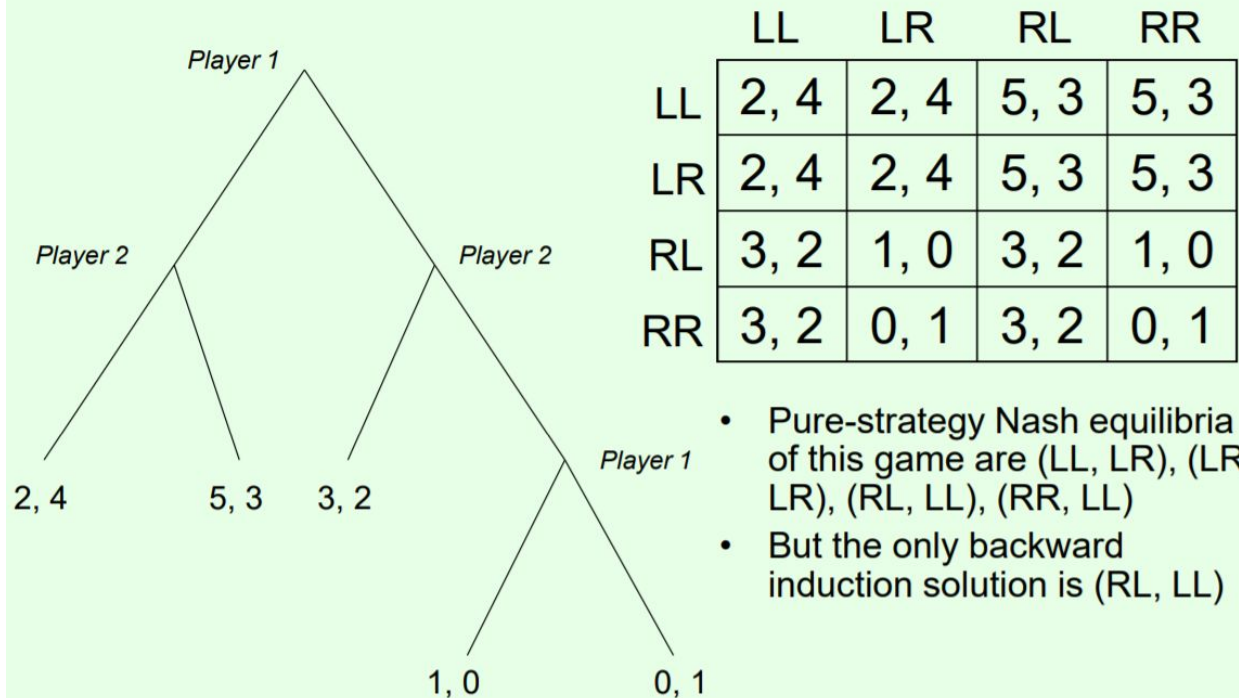
Imperfect information results when the player 2 only responds with the same possible set of actions no matter the previous action taken by player 1. (in this case, no matter whether player 1 chooses M or R, player 2 can respond with either a or r, so they are equivalent to player 2).

Subgame Perfect Equilibria is a subset of the set of Nash Equilibria

Nash Equilibria is a strategy profile such that no player can increase their payoff by changing their strategy, holding fixed the strategies (not the actions) of the other players. When players are indifferent about the other player choosing either A or B, then that also satisfies Nash Equilibria because the player cannot increase their payoff anyway.

In order to calculate Nash Equilibria, first convert tree diagram to a normal form.

# Converting the first game to normal form



- Pure-strategy Nash equilibria of this game are (LL, LR), (LR, LR), (RL, LL), (RR, LL)
- But the only backward induction solution is (RL, LL)

Image taken from [https://www2.cs.duke.edu/courses/fall18/compsci590.2/extensive\\_form.pdf](https://www2.cs.duke.edu/courses/fall18/compsci590.2/extensive_form.pdf)

Pure-strategy Nash equilibria are (LL, LR): given player 1 has chosen LL already, then player 2 should choose either LL or LR. In this case, LL for player 2 is not a Nash equilibrium if player 2 chose LL, then player 1 would choose RL / RR instead and not LL. Given LL is chosen by player 1, player 2 will choose LR. and so on.

Subgame Perfect Equilibria can be found using backward induction: starting from the last move by the last player.

In the above example, player 1 will choose Left because it gets a higher payoff. Player 2 knows player 1 will choose left, so player 2 would choose left since it gets a higher payoff ( $2 > 0$ ).

Player 1, knowing player 2 would choose left and thus he would get 3, would choose right. This is because, although choosing left might get him to (5, 3) if player 2 chooses left, player 1 knows player 2 will choose left instead because player 2 will get ( $4 > 3$ ) from choosing left. Thus, player 1 chooses right on his second move so he can get ( $3 > 2$ ) when player 2 chooses left in the right node.

For player 2, in the left node, he would choose left since ( $4 < 3$ ), whereas in the right node, he will choose left since  $2 > 0$  (since he knows if he chose right, player 1 will choose left instead of right. (Not that this matters, because player 2 chooses left strictly dominates choosing right).