Summary Notes for Week 6: Bayesian Games

Bayesian Games

Situation whereby players have incomplete information about one another (eg. Player 1 is not sure whether Player 2 is a weak or strong player -- and this information determines the payoffs player 2 gets for each action he decides to take).

Example 1: A Modified Prisoner's Dilemma Game

With probability λ, player 2 has the normal preferences as before (type I), while with probability $(1-\lambda)$, player 2 hates to rat on his accomplice and pays a psychic penalty equal to 6 years in prison for confessing (type II).

λ

	C	D
C	5, 5	0, 8
D	8, 0	1, 1

Type I

$1-\lambda$

	C	D
C	5, 5	0, 2
D	8, 0	1, -5

Type II

Image taken from http://www.eecs.harvard.edu/cs286r/courses/fall08/files/lecture5.pdf

In order to solve, first try to see which strategy strictly dominates when the player is type A VS when the player is type B. Playing D is a dominant strategy for type I player 2; player C is a dominant strategy for type II player 2. Hence, player 2 will always play D when type I and always play C when type II. This leaves player 1 with a more concrete expected utility when he plays either C or D.

- Player 1's expected utility by playing C is $\lambda \times 0 + (1-\lambda) \times 5 = 5 - 5\lambda$.
- Player 1's expected utility by playing D is $\lambda \times 1 + (1-\lambda) \times 8 = 8 - 7\lambda$

C: 5 - 5λ; D = 8 - 7λ

Equating C to D, 5-5λ = 8 - 7λ => λ=3/2.

In order for C to provide a larger utility, 5 - 5λ > 8 - 7λ and hence λ should be > 3/2. If λ < 3/2, D provides larger utility. If λ = 3/2, then player 1 is indifferent to player 2's type, i.e. both are Nash

equilibria. However, since $0 < \lambda < 1$ (λ is a probability), D will always provide a larger utility to player 1, hence player 1 will always choose D regardless of player 2's type.
Thus, (D, (D if type I, C if type II)) is a Bayesian Nash Equilibria of the game where D is always the strategy of player 1 and D is the strategy of player 2 if type I, and C is the strategy of player 2 if type II.

<u>Calculating</u>

$Good$	$Shoot$	Not
$Shoot$	$-3, -1$	$-1, -2$
Not	$-2, -1$	$0, 0$

Bad	$Shoot$	Not
$Shoot$	$0, 0$	$2, -2$
Not	$-2, -1$	$-1, 1$

Player 1 knows whether he is good or bad, but player 1 does not.
If player 1 is good, then not shooting strictly dominates shooting. If player 1 is bad, then shooting strictly dominates not shooting. Given that player 1 has a probability p of being good and probability 1-p of being bad, player 2's expected utility of shooting = -p + 0(1-p) = -p. Player 2's expected utility of not shooting = 0p - 2(1-p) = 2p - 2
Equating, -p = 2p - 2 => 3p = 2, p = 2/3. Hence, if p >= 2/3, player 2's expected utility of not shooting > player 2's expected utility of shooting. So if probability of player 1 being good >= 2/3, then player 2 should not shoot, and vice versa.