examples: 1.3.2, 1.3.3, 1.3.6, 1.3.8, 1.3.10, 1.3.14, 1.3.15

practice problems: 1-8

additional exercises: 3-11

-> exercises

1.3.2 -> $\neg P \vee Q$

P	Q	¬P	¬P∨Q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

1.3.3 ->

P -> you get more doubles than any other player

Q -> you will lose

R -> you must have bought the most properties

P	Q	R	P→Q	Q→R	(P→Q)∨(Q→R)
T	T	T	T	T	T
T	F	T	F	T	T
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	F	F	T	T	T

1.3.6 -> it will not rain or snow and it will not rain and it will not snow

P -> rain

Q -> snow

P	Q	P∨Q	¬(P∨Q)	¬P∧¬Q
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

-> logically equiv

1.3.8 -> prove $\neg(P \to Q) \equiv P \wedge \neg Q$ with no truth tables

step 1. rewrite $\neg(P \to Q)$ to $\neg(\neg P \vee Q)$ because $P \to Q \equiv \neg P \vee Q$

step 2. apply de morgans laws to get $\neg\neg P \wedge \neg Q$

step 3. use double negation to get $P \wedge \neg Q$

1.3.10 -> are $(P \vee Q) \to R \equiv (P \to R) \vee (Q \to R)$

P	Q	R	P∨Q	P→R	Q→R	(P∨Q)→R	(P→R)∨(Q→R)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

-> no not logically equiv

1.3.14 -> is $\dfrac{\begin{array}{l}P \to Q \\ \neg P \to Q\end{array}}{\therefore Q}$ valid?

P	Q	P→Q	¬P→Q
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	F

-> is a valid deduction rule
↳ because in the rows P→Q & ¬P→Q are true Q is also true making it valid

1.3.15 -> is $\dfrac{\begin{array}{l}P \to R \\ Q \to R \\ R\end{array}}{\therefore P \vee Q}$ valid?

P	Q	R	P→R	Q→R	P∨Q
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

-> it is invalid because of row 7

-> Practice problems

1. truth table $(P \wedge Q) \to (P \vee Q)$

P	Q	P∧Q	P∨Q	(P∧Q)→(P∨Q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

2. truth table $\neg Q \vee (Q \to P)$

P	Q	¬Q	Q→P	¬Q∨(Q→P)
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	T

c. that p is false and q is true

3. truth table $\neg P \wedge (Q \rightarrow R)$

P	Q	R	¬P	Q→R	¬P∧(Q→R)
T	T	T	F	T	F
T	T	F	F	F	F
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

4. are $p \rightarrow (Q \vee R) \equiv (p \rightarrow Q) \vee (p \rightarrow R)$

P	Q	R	Q∨R	P→(Q∨R)	P→Q	P→R	(P→Q)∨(P→R)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

→ logically equiv

5. is $\dfrac{P \rightarrow Q \quad \neg Q}{\therefore \neg P}$ valid?

P	Q	P→Q	¬Q	¬P
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

→ valid!

6. is $\dfrac{P \rightarrow (Q \vee R) \quad \neg(P \rightarrow Q)}{\therefore R}$

P	Q	R	P→(Q∨R)	¬(P→Q)
T	T	T	T	F
T	T	F	T	F

$\neg(P \rightarrow Q) \equiv P \wedge \neg Q$

T	F	T	T	T	→ is valid!
T	F	F	F	T	
F	T	T	T	F	
F	T	F	T	F	
F	F	T	T	F	
F	F	F	T	F	

7. is $\dfrac{(P \wedge Q) \rightarrow R \quad \neg P \vee \neg Q}{\therefore \neg R}$ valid

P	Q	R	$(P \wedge Q) \rightarrow R$	$\neg P \vee \neg Q$	$\neg R$
T	T	T	T	F	F
T	T	F	F	F	T
T	F	T	T	T	F
T	F	F	T	T	T
F	T	T	T	T	F
F	T	F	T	T	T
F	F	T	T	T	F
F	F	F	T	T	T

is invalid

8. is $\dfrac{P \rightarrow Q \quad P \wedge \neg Q}{\therefore R}$ valid

P	Q	R	$P \rightarrow Q$	$P \wedge \neg Q$
T	T	T	T	F
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	T	F

→ not valid because the premises are never both true in the same row

→ additional exercises

3. if its your birthday or there will be cake, then there will be cake

a. P → it is your birthday

Q → there will be cake

$(P \vee Q) \rightarrow Q$

b

P	Q	$P \vee Q$	$(P \vee Q) \rightarrow Q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

c. cannot conclude anything

d. for $(P \vee Q) \rightarrow Q$ to be true when $Q = F$, $P \vee Q$ must $= F$. if $P \vee Q = T$ while $Q = F$, the whole implication would $= F$
   - ↳ for $P \vee Q = F$ P must $= F$
     - ↳ therefor, it is NOT your birthday

e. its your birthday, but there is no cake

4. Geoff poshington

a. P → pepperoni
   Q → sausage
   R → quail
   S → ricotta cheese

$$((P \vee Q) \wedge (Q \rightarrow R) \wedge (\neg P \vee R) \rightarrow S))$$

P	Q	R	S	P∨Q	Q→R	(P∨R)→S	((P∨Q)∧(Q→R)∧(¬P∨R)→S)
T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F
T	T	F	T	T	F	T	F
T	F	T	F	T	T	F	F
T	T	F	F	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F
F	T	F	T	T	F	T	F
F	F	T	T	F	T	T	F
F	F	T	F	F	T	F	F
F	T	F	F	T	F	T	F
F	F	F	T	F	T	T	F
F	F	F	F	F	T	T	F

→ he is a liar
↳ or we can't tell

5. $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$

1. $\neg(P \rightarrow Q) = P \wedge \neg Q$ because of negation of implication or:
   1. $\neg(P \rightarrow Q) \Rightarrow \neg(\neg P \vee Q)$ → implications are disjunctions
   2. $\neg(\neg P \rightarrow Q) \Rightarrow \neg\neg P \wedge \neg Q$ → de morgans laws
   3. $\neg\neg P \wedge \neg Q \Rightarrow P \wedge \neg Q$ → double negation

6. simplify

a. $\neg(\neg P \vee \neg Q)$

$\neg\neg P \wedge \neg\neg Q$

$P \wedge Q$

b. $(\neg P \vee \neg Q) \rightarrow \neg\neg Q \vee \neg R$

$(\neg P \vee \neg Q) \rightarrow Q \vee \neg R$

$\neg((\neg P \vee \neg Q) \vee Q \vee \neg R)$

$\neg\neg P \wedge \neg\neg Q \vee Q \vee \neg R$

$(P \wedge Q) \vee (Q \vee \neg R)$

c. $\neg((\neg P \vee \neg Q) \vee \underbrace{\neg(R \wedge \neg R)}_{\neg \text{False}})$

$\neg \text{False}$ = true

$\neg(\underbrace{(P \rightarrow \neg Q) \vee \text{true}}_{\text{anything or true is true}})$

$\neg(\text{true})$

final: false

d. P → sam is a man

Q → chris is a woman

$\neg((\neg P \rightarrow Q) \wedge \neg Q)$

$\neg((\neg\neg P \vee \neg Q) \wedge \neg Q)$

$\neg(P \vee \neg Q) \vee \neg\neg Q$

$(\neg P \wedge \neg Q) \vee Q$

7. simplify

a. $\neg((\neg P \wedge Q) \vee (\neg R \wedge S))$

$(\neg(\neg P \wedge Q) \wedge \neg(\neg R \wedge S))$

$(P \vee \neg Q) \wedge (R \vee \neg S)$

b. $\neg(\neg P \rightarrow \neg Q) \vee \neg(\neg Q \rightarrow R)$

$\neg(P \vee \neg Q) \vee \neg(Q \vee R)$

$(\neg P \wedge Q) \vee (\neg Q \wedge \neg R)$

c. a →

P	Q	R	S	$(\neg P \wedge Q)$	$\neg(R \vee \neg S)$	$\neg((\neg P \wedge Q) \vee \neg(R \vee \neg S))$	$(P \vee \neg Q) \wedge (R \vee \neg S)$
T	T	T	T	F	F	T	T
T	T	T	F	F	F	T	T
T	T	F	T	F	T	F	F
T	F	T	T	F	F	T	T
T	F	T	F	F	F	T	T
T	T	F	F	F	F	T	T
T	F	F	T	F	T	F	F
T	F	F	F	F	F	T	T
F	T	T	T	T	F	F	F
F	T	T	F	T	F	F	F
F	T	F	T	T	T	F	F
F	F	T	T	F	F	T	T
F	F	T	F	F	F	T	T
F	T	F	F	T	F	F	F
F	F	F	T	F	T	F	F
F	F	F	F	F	F	T	T

b →

P	Q	R	$(\neg P \rightarrow \neg Q)$	$(\neg Q \rightarrow R)$	$\neg((\neg P \rightarrow \neg Q) \wedge (\neg Q \rightarrow R))$	$(\neg P \wedge Q) \vee (\neg Q \wedge \neg R)$ ↘
T	T	T	T	T	F	F
T	T	F	T	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	F	F
F	F	F	T	F	T	T

8. if a number is triangular or square, then it is not prime

T → # is triangular

S → # is square

P → # is prime

a.

T	S	P	(T ∨ S)	(T ∨ S) → ¬P
T	T	T	T	F
T	T	F	T	T
T	F	T	T	F
T	F	F	T	T
F	T	T	T	F
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

b. the # would need to be either square or triangular & # must be prime

c. the number 6657 can not be triangular or square

9. tommy flanagan

P = popcorn

Q = raisins

R = cucumber sandwiches

S = soda

T = tea

(P ∨ Q), (R → S), (¬S ∨ ¬T)

↓

¬(P ∨ Q) ¬(R → S) ¬(¬S ∨ ¬T)

(¬P ∧ ¬Q), (R ∧ ¬S), (S ∧ T)

↓ neither popcorn or raisins

↓ had cucumber sandwich but no soda

↓ did not have tea or soda?

10. is $\dfrac{\begin{array}{l}P \to Q \\ Q \to R\end{array}}{\therefore P \to R}$ valid?

P	Q	R	P → Q	Q → R	P → R
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

→ valid

11. P ≡ Q using P ↔ Q

→ normal

P	Q	P ↔ Q
T	T	T
T	F	F
F	T	F
F	F	T

- when P ↔ Q is a tautology

P	Q	P ↔ Q
T	T	T
T	T	T
F	F	T
F	F	T

→ if P ≡ Q, then P ↔ Q is a tautology