Let G be a graph. Let S be the maximal independent set of G. The subgraph $V(G) - S$ has order $|V(G) - S|$, thus it is at most $|V(G) - S|$-colourable. The remaining vertices S are 1-colourable, since they are an independent set and are not adjacent in G. So then our graph is at most $|V(G) - S| + 1$colourable

Let G be a k-vertex-critical $(P_4+P_1, 2P_2)$-free graph. Let S be a maximum independent set in G. Let the vertices outside S, $V(G) - S$, be partitioned by A and B, where $A = \{v \in V(G) - S : |N(v) \cap S = 1\}$. Then let $B = V(G) - S - A$. Let $\forall v \in S$, $v_A = N(v) \cap A$
Claim 1: $\forall v, v' \in S$, v_A is complete to v'_A
Proof: If $a \in v_A$ and $a' \in v'_A$ such that $a \not\sim a'$, then $\{v, v', a, a'\}$ induces a $2P_2$— a contradiction.
Claim 2: $|S_A| \leq 1$
Proof: If $|S_A| \geq 2$, then let $v, v' \in S_A$ with $a \in v_A anda' \in v'_A$. From claim 1 we know that $a \sim a'$, so $\{v, v', a, a', x\}$ induces a $P_4 + P_1$.

For this graph to be k-vertex critical we can also assert that it has no comparable vertices. Which is to say, $\forall u, v \in S$, $N(u) \not\subseteq N(v)$, or every vertex in S has at least one unique neighbour compared to another vertex. Let v_1, v_2 be two vertices in $V(G) - S$ s.t. $v_1 \sim S_A$ and v_2 $\sim$some stuff in S_B. We can now assert that any element in B has no neighbours in S_A. And that S_A has no neighbours in B.

Let us find what happens when we look for a $(P_4 + P_1, K_3 + P_1, 2P_2)$-free graph. Let S be the maximum independent set. Let $A = \{v \in V(G) - S : |N(V) \cap S| = 1\}$ Let $B = V(G) - S - A$. Let $S_A = \{v \in S : N(V) \cap A \neq \emptyset\}$
Claim 3: $\forall v, v' \in S$, v_A is complete to v'_a. Proof: Assume $v_A \not\sim v'_A$. Then, $\{v, v', v_A, v'_A\}$ induces a $2P_2$, a contradiction.

Claim 4: $|S_A| \leq 1$ Proof: Assume $|S_A| \geq 2$. Then let $u, u' \in S_A$ with $a \in v_A$ and $a' \in v'_A$. From claim 1 we know that $a \sim a'$, so $\{v, v'a, a', x\}$ induces a $P_4 + P_1$ for any $x \in S - \{v, v'\}$. Note that x exists, other wise the independence number of this graph is 2.

Claim 5: $|A| \leq 1$ First lets make a new notation. Since $|S_A| \leq 1$, let us name that potential vertex v_S. Proof: Assume $|A| \geq 2$. Then we have any two vertices in A v, v'. From claim 4 we know that $|S_A| \leq 1$ and from claim 3 that v is complete to v'. Since both v, v' are adjacent to v_S, then $\{v, v', v_S, x\}$ induces a $K_3 + P_1$, where $x \in S_B$