MONASH University

MONASH BUSINESS SCHOOL

Tidy Time Series & Forecasting in R

9. Dynamic regression

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Outline

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Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \dots, x_{k,t}$.
- In regression, we assume that ε_t was WN.
- Now we want to allow ε_t to be autocorrelated.

Regression with ARIMA errors

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- y_t modeled as function of k explanatory variables $x_{1,t}, \ldots, x_{k,t}$.
- In regression, we assume that ε_t was WN.
- Now we want to allow ε_t to be autocorrelated.

Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where ε_t is white noise.

Residuals and errors

Example: η_t = ARIMA(1,1,1)

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Residuals and errors

Example: η_t = ARIMA(1,1,1)

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$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

- Be careful in distinguishing η_t from ε_t.
- Only the errors ε_t are assumed to be white noise.
- In ordinary regression, η_t is assumed to be white noise and so $\eta_t = \varepsilon_t$.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
3. *p*-values for coefficients usually too small ("spurious regression").
4. AIC of fitted models misleading.

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4. AIC of fitted models misleading.

- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood similar to minimizing $\sum \varepsilon_t^2$.

Stationarity

Regression with ARMA errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$

where η_t is an ARMA process.

- All variables in the model must be stationary.
- If we estimate the model while any of these are non-stationary, the estimated coefficients can be incorrect.
- Difference variables until all stationary.
- If necessary, apply same differencing to all variables.

Stationarity

Model with ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
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Equivalent to model with ARIMA(1,0,1) errors

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t,$$
$$(1 - \phi_1 B)\eta'_t = (1 + \theta_1 B)\varepsilon_t,$$

where $y'_t = y_t - y_{t-1}$, $x'_{t,i} = x_{t,i} - x_{t-1,i}$ and $\eta'_t = \eta_t - \eta_{t-1}$.

Regression with ARIMA errors

Any regression with an ARIMA error can be rewritten as a regression with an ARMA error by differencing all variables with the same differencing operator as in the ARIMA model.

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Original data

$$y_t = \beta_0 + \beta_1 x_{1,t} + \dots + \beta_k x_{k,t} + \eta_t$$
$$\text{where} \quad \phi(B)(1 - B)^d \eta_t = \theta(B)\varepsilon_t$$

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After differencing all variables

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$
$$\text{where} \quad \phi(B)\eta_t = \theta(B)\varepsilon_t$$
$$\text{and} \quad y'_t = (1-B)^d y_t$$

Model selection

- Fit regression model with automatically selected ARIMA errors. (R will take care of differencing before estimation.)
- Check that ε_t series looks like white noise.

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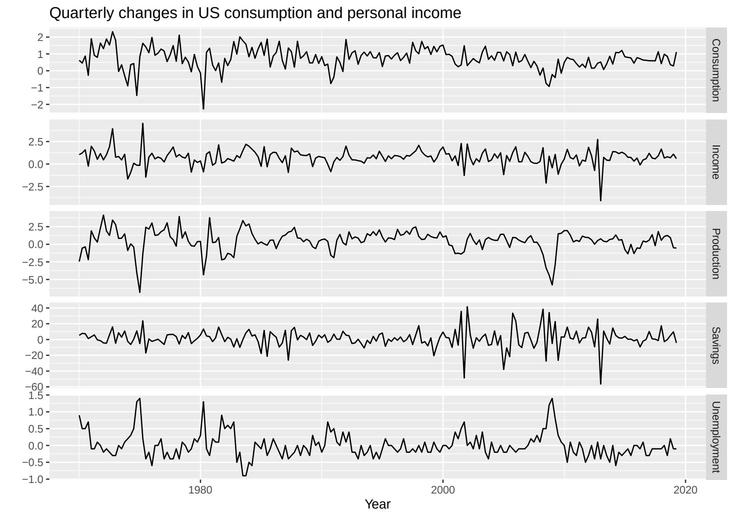
Selecting predictors

- AICc can be calculated for final model.
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AICc value.

US personal consumption and income



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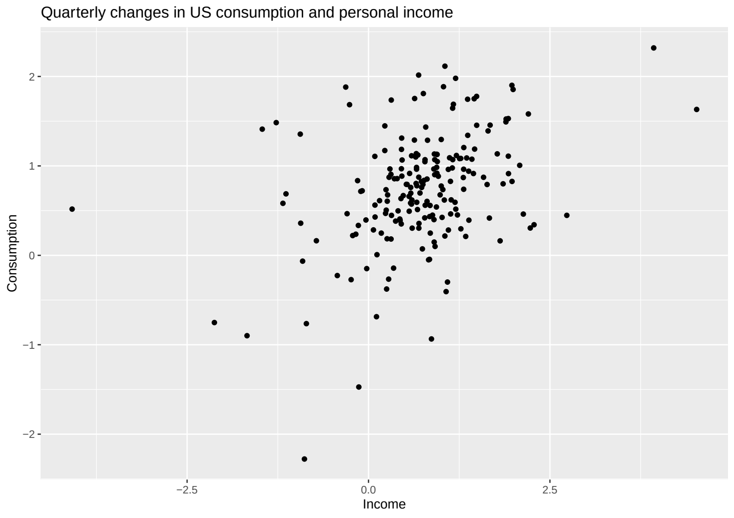

US personal consumption and income

- No need for transformations or further differencing.
- Increase in income does not necessarily translate into instant increase in consumption (e.g., after the loss of a job, it may take a few months for expenses to be reduced to allow for the new circumstances). We will ignore this for now.

US personal consumption and income

```r
fit <- us_change %>% model(ARIMA(Consumption ~ Income))
report(fit)
```

```
## Series: Consumption
## Model: LM w/ ARIMA(1,0,2) errors
##
## Coefficients:
##          ar1      ma1     ma2  Income  intercept
##       0.7070  -0.6172  0.2066  0.1976     0.5949
## s.e.  0.1068   0.1218  0.0741  0.0462     0.0850
##
## sigma^2 estimated as 0.3113:  log likelihood=-163.04
## AIC=338.07   AICc=338.51   BIC=357.8
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```
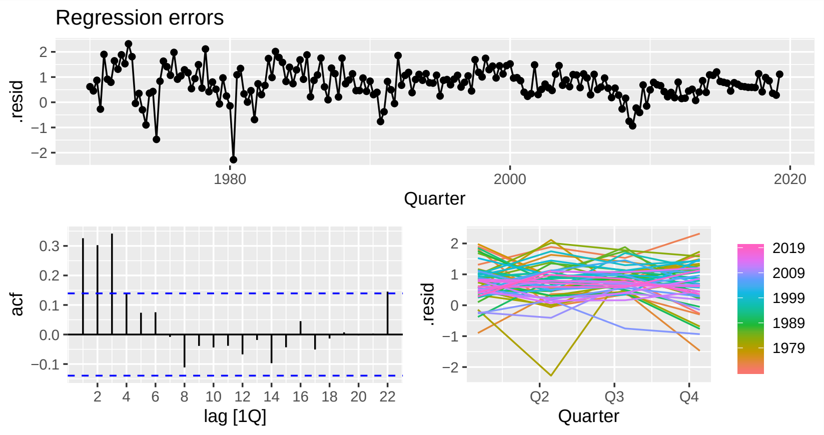

Write down the equations for the fitted model.

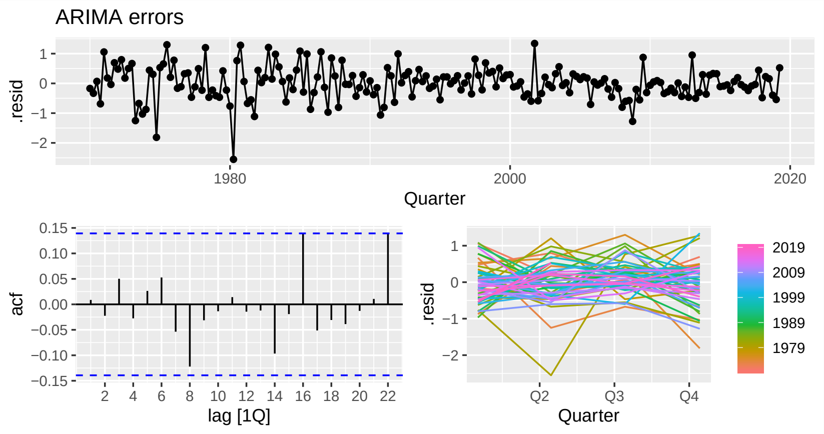
US personal consumption and income

```r
residuals(fit, type='regression') %>%
  gg_tsdisplay(.resid) + ggtitle("Regression errors")
```



US personal consumption and income

```r
residuals(fit, type='response') %>%
  gg_tsdisplay(.resid) + ggtitle("ARIMA errors")
```



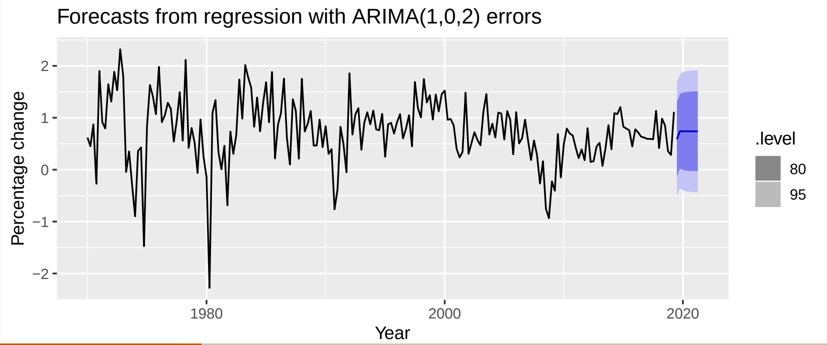
US personal consumption and income

```
augment(fit) %>%
  features(.resid, ljung_box, dof = 5, lag = 12)
```

```
## # A tibble: 1 x 3
##   .model                              lb_stat lb_pvalue
##   <chr>                                 <dbl>     <dbl>
## 1 ARIMA(Consumption ~ Income)            5.54     0.595
```

US personal consumption and income

```r
us_change_future <- new_data(us_change, 8) %>%
  mutate(Income = mean(us_change$Income))
forecast(fit, new_data = us_change_future) %>%
  autoplot(us_change) +
  labs(x = "Year", y = "Percentage change",
       title = "Forecasts from regression with ARIMA(1,0,2) errors")
```



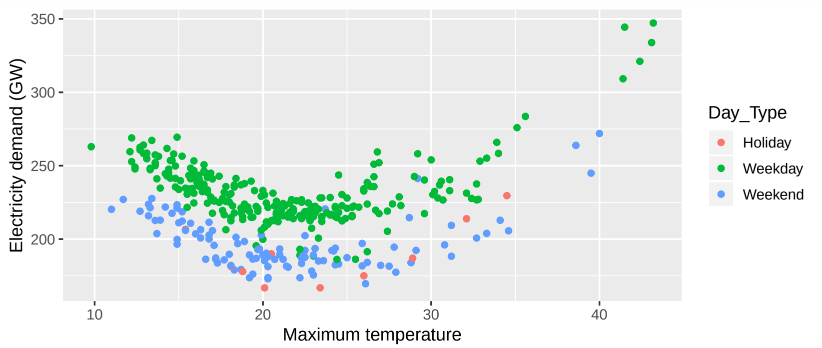
Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.

Daily electricity demand

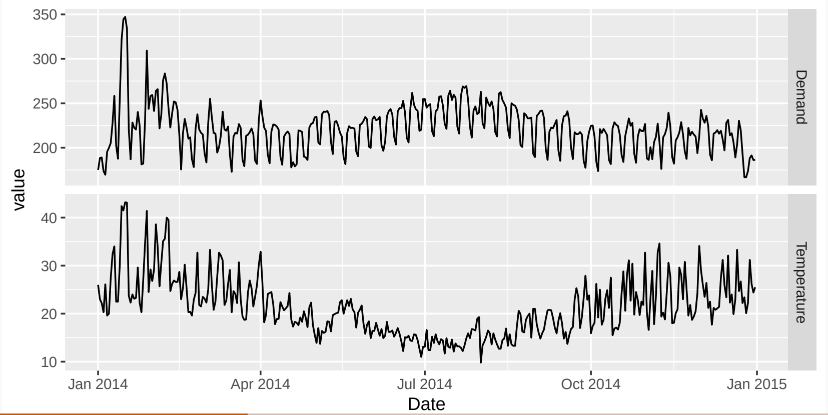
Model daily electricity demand as a function of temperature using quadratic regression with ARMA errors.

```
vic_elec_daily %>%
  ggplot(aes(x=Temperature, y=Demand, colour=Day_Type)) +
  geom_point() +
  labs(x = "Maximum temperature", y = "Electricity demand (GW)")
```



Daily electricity demand

```r
vic_elec_daily %>%
  gather("var", "value", Demand, Temperature) %>%
  ggplot(aes(x = Date, y = value)) + geom_line() +
  facet_grid(vars(var), scales = "free_y")
```



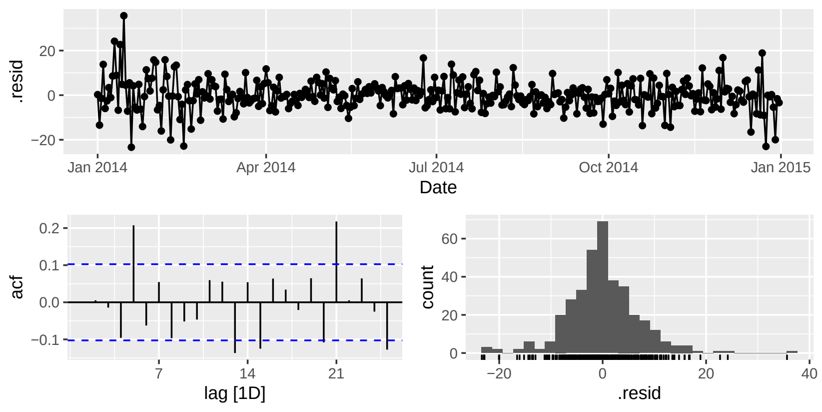
Daily electricity demand

```r
fit <- vic_elec_daily %>%
  model(ARIMA(Demand ~ Temperature + I(Temperature^2) +
                (Day_Type=="Weekday")))
report(fit)
```

```
## Series: Demand
## Model: LM w/ ARIMA(2,1,2)(0,0,2)[7] errors
##
## Coefficients:
##          ar1      ar2      ma1     ma2    sma1    sma2  Temperature
##       1.1521  -0.2750  -1.3851  0.4071  0.1589  0.3103      -
7.9467
## s.e.  0.6265   0.4812   0.6082  0.5804  0.0591  0.0538       0.4920
##       I(Temperature^2)  Day_Type == "Weekday"TRUE
##                 0.1865                    31.8245
## s.e.            0.0097                     1.0189
##
## sigma^2 estimated as 48.82:  log likelihood=-1220.48
## AIC=2460.96   AICc=2461.58   BIC=2499.93
```

Daily electricity demand

```r
augment(fit) %>%
  gg_tsdisplay(.resid, plot_type = "histogram")
```



Daily electricity demand

```
augment(fit) %>%
  features(.resid, ljung_box, dof = 8, lag = 14)
```

```
## # A tibble: 1 x 3
##   .model                                                        lb_stat
##   <chr>                                                           <dbl>
## 1 "ARIMA(Demand ~ Temperature + I(Temperature^2) + (Day_Type ~    38.1
6
```

Daily electricity demand

```r
# Forecast one day ahead
vic_next_day <- new_data(vic_elec_daily, 1) %>%
  mutate(Temperature = 26, Day_Type = "Holiday")
forecast(fit, vic_next_day)
```

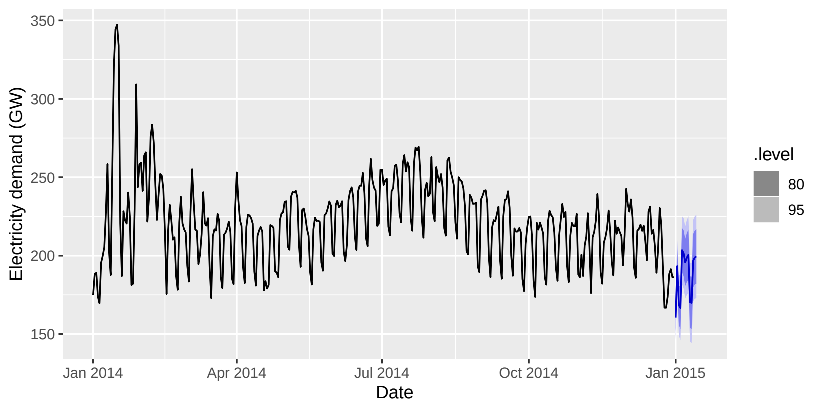
```
## # A fable: 1 x 6 [1D]
## # Key:     .model [1]
##   .model                     Date       Demand .distribution Tem
##   <chr>                      <date>      <dbl> <dist>
## 1 "ARIMA(Demand ~ Temperat~ 2015-01-01   161. N(161, 49)
```

Daily electricity demand

```r
vic_elec_future <- new_data(vic_elec_daily, 14) %>%
  mutate(
    Temperature = 26,
    Holiday = c(TRUE, rep(FALSE, 13)),
    Day_Type = case_when(
      Holiday ~ "Holiday",
      wday(Date) %in% 2:6 ~ "Weekday",
      TRUE ~ "Weekend"
    )
  )
```

Daily electricity demand

```r
forecast(fit, vic_elec_future) %>%
  autoplot(vic_elec_daily) + ylab("Electricity demand (GW)")
```



Outline

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

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Stochastic trend

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where η_t is ARIMA process with $d \geq 1$.

Stochastic & deterministic trends

Deterministic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

where η_t is ARMA process.

Stochastic trend

$$y_t = \beta_0 + \beta_1 t + \eta_t$$

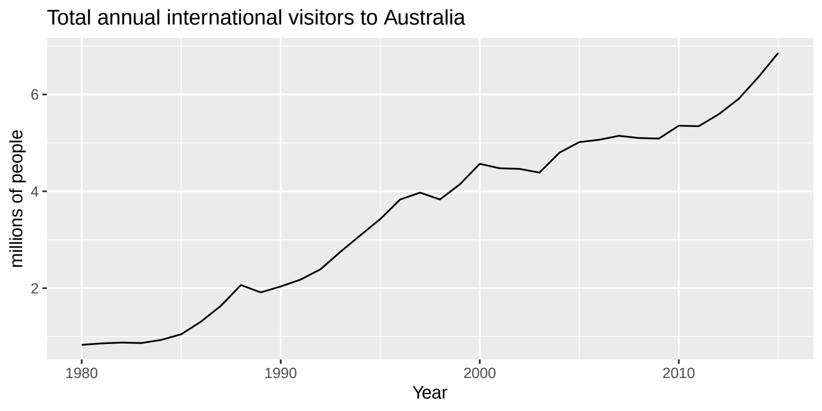
where η_t is ARIMA process with $d \geq 1$.

Difference both sides until η_t is stationary:

$$y'_t = \beta_1 + \eta'_t$$

where η'_t is ARMA process.

International visitors



International visitors

Deterministic trend

```r
fit_deterministic <- aus_visitors %>%
  model(Deterministic = ARIMA(value ~ trend() + pdq(d = 0)))
report(fit_deterministic)
```

```
## Series: value
## Model: LM w/ ARIMA(2,0,0) errors
##
## Coefficients:
##          ar1      ar2  trend()  intercept
##       1.1127  -0.3805   0.1710     0.4156
## s.e.  0.1600   0.1585   0.0088     0.1897
##
## sigma^2 estimated as 0.02979:  log likelihood=13.6
## AIC=-17.2   AICc=-15.2   BIC=-9.28
```

International visitors

Deterministic trend

```
fit_deterministic <- aus_visitors %>%
  model(Deterministic = ARIMA(value ~ trend() + pdq(d = 0)))
report(fit_deterministic)
```

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## Series: value
## Model: LM w/ ARIMA(2,0,0) errors
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## sigma^2 estimated as 0.02979:  log likelihood=13.6
## AIC=-17.2   AICc=-15.2   BIC=-9.28
```

$$y_t = 0.42 + 0.17t + \eta_t$$
$$\eta_t = 1.11\eta_{t-1} - 0.38\eta_{t-2} + \varepsilon_t$$
$$\varepsilon_t \sim \text{NID}(0, 0.0298).$$

International visitors

Stochastic trend

```r
fit_stochastic <- aus_visitors %>%
  model(Stochastic = ARIMA(value ~ pdq(d=1)))
report(fit_stochastic)
```

```
## Series: value
## Model: ARIMA(0,1,1) w/ drift
##
## Coefficients:
##          ma1  constant
##       0.3006    0.1735
## s.e.  0.1647    0.0390
##
## sigma^2 estimated as 0.03376:  log likelihood=10.62
## AIC=-15.24   AICc=-14.46   BIC=-10.57
```

International visitors

Stochastic trend

```r
fit_stochastic <- aus_visitors %>%
  model(Stochastic = ARIMA(value ~ pdq(d=1)))
report(fit_stochastic)
```

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## Series: value
## Model: ARIMA(0,1,1) w/ drift
##
## Coefficients:
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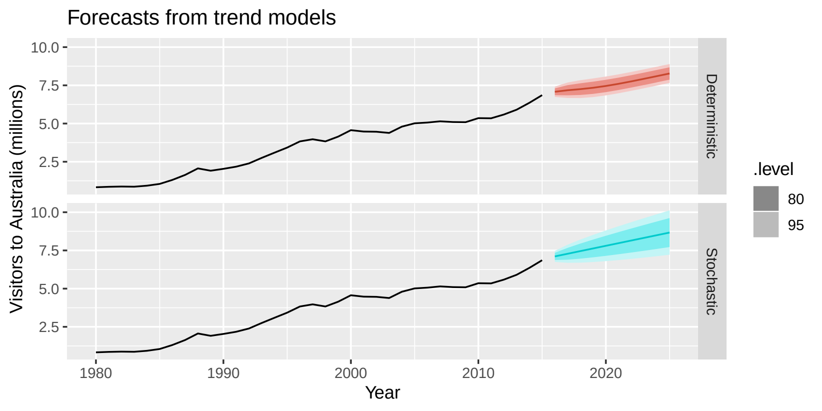
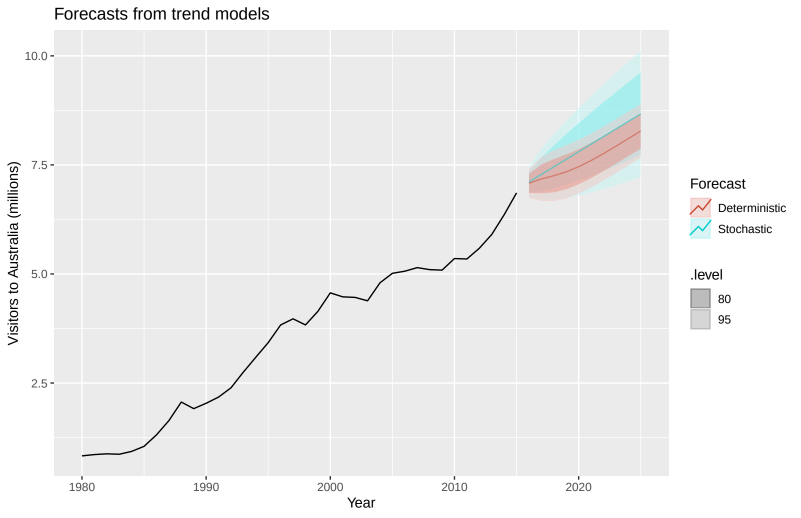
##       0.3006    0.1735
## s.e.  0.1647    0.0390
##
## sigma^2 estimated as 0.03376:  log likelihood=10.62
## AIC=-15.24   AICc=-14.46   BIC=-10.57
```

$$
\begin{aligned}
y_t - y_{t-1} &= 0.17 + \varepsilon_t \\
y_t &= y_0 + 0.17t + \eta_t \\
\eta_t &= \eta_{t-1} + 0.30\varepsilon_{t-1} + \varepsilon_t
\end{aligned}
$$

International visitors



International visitors



Forecasting with trend

- Point forecasts are almost identical, but prediction intervals differ.
- Stochastic trends have much wider prediction intervals because the errors are non-stationary.
- Be careful of forecasting with deterministic trends too far ahead.

Outline

Dynamic harmonic regression

Combine Fourier terms with ARIMA errors

Advantages

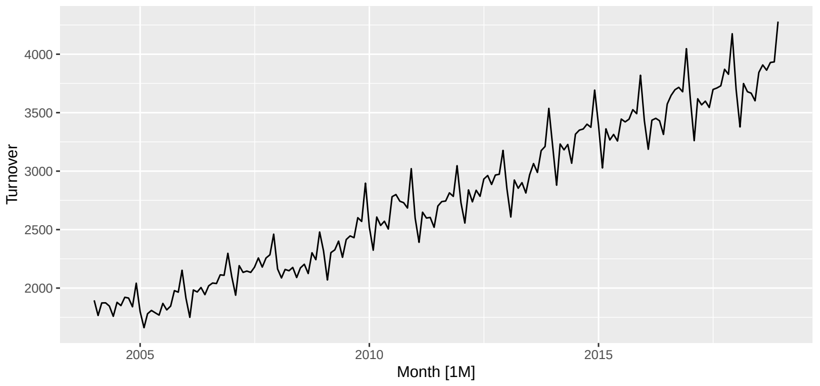
- it allows any length seasonality;
- for data with more than one seasonal period, you can include Fourier terms of different frequencies;
- the seasonal pattern is smooth for small values of K (but more wiggly seasonality can be handled by increasing K);
- the short-term dynamics are easily handled with a simple ARMA error.

Disadvantages

- seasonality is assumed to be fixed

Eating-out expenditure

```
aus_cafe <- aus_retail %>% filter(
    Industry == "Cafes, restaurants and takeaway food services",
    year(Month) %in% 2004:2018
  ) %>% summarise(Turnover = sum(Turnover))
aus_cafe %>% autoplot(Turnover)
```

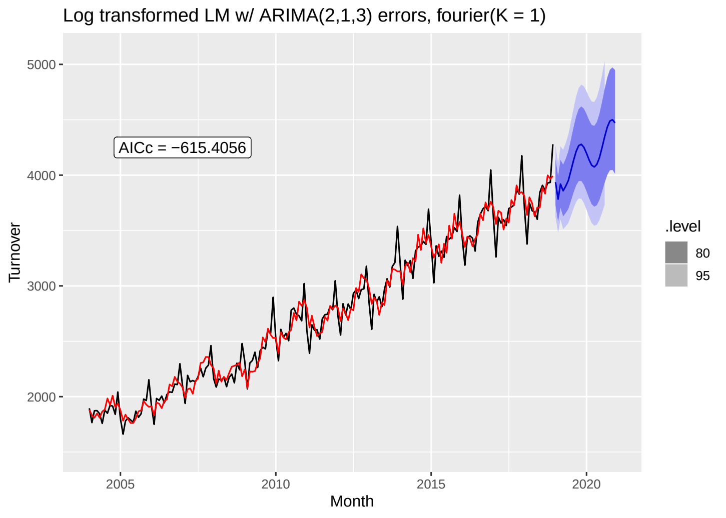


Eating-out expenditure

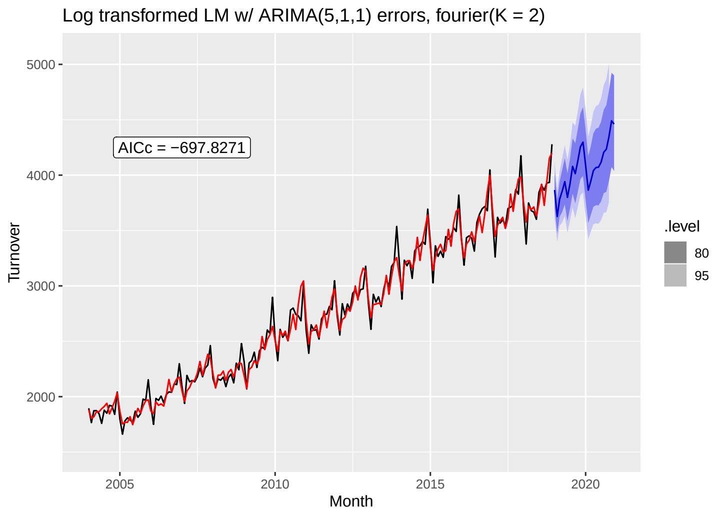
```
fit <- aus_cafe %>% model(
    K = 1 = ARIMA(log(Turnover) ~ fourier(K = 1) + PDQ(0,0,0)),
    K = 2 = ARIMA(log(Turnover) ~ fourier(K = 2) + PDQ(0,0,0)),
    K = 3 = ARIMA(log(Turnover) ~ fourier(K = 3) + PDQ(0,0,0)),
    K = 4 = ARIMA(log(Turnover) ~ fourier(K = 4) + PDQ(0,0,0)),
    K = 5 = ARIMA(log(Turnover) ~ fourier(K = 5) + PDQ(0,0,0)),
    K = 6 = ARIMA(log(Turnover) ~ fourier(K = 6) + PDQ(0,0,0)))
glance(fit)
```

.model	sigma2	log_lik	AIC	AICc	BIC
K = 1	0.0017471	317.2353	-616.4707	-615.4056	-587.7842
K = 2	0.0010732	361.8533	-699.7066	-697.8271	-661.4579
K = 3	0.0007609	393.6062	-763.2125	-761.3329	-724.9638
K = 4	0.0005386	426.7839	-821.5678	-818.2098	-770.5697
K = 5	0.0003173	473.7344	-919.4688	-916.9078	-874.8454
K = 6	0.0003163	474.0307	-920.0614	-917.5004	-875.4380

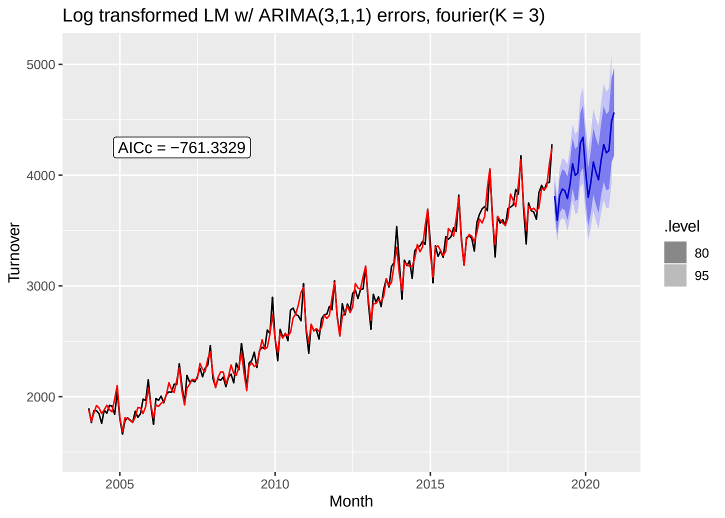
Eating-out expenditure



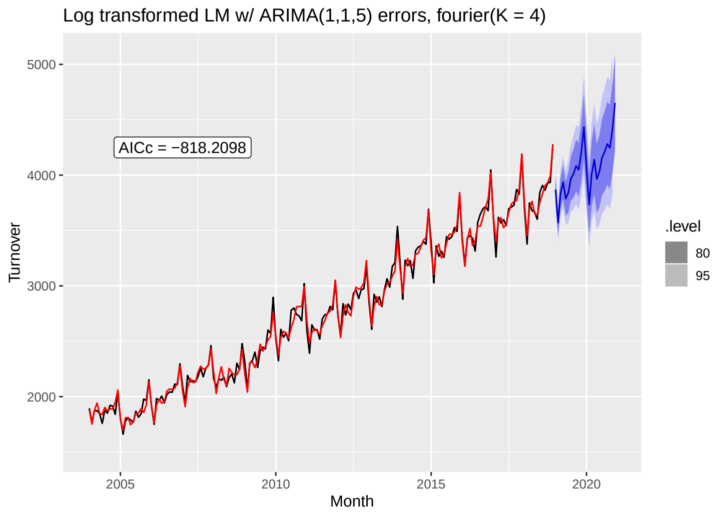
Eating-out expenditure



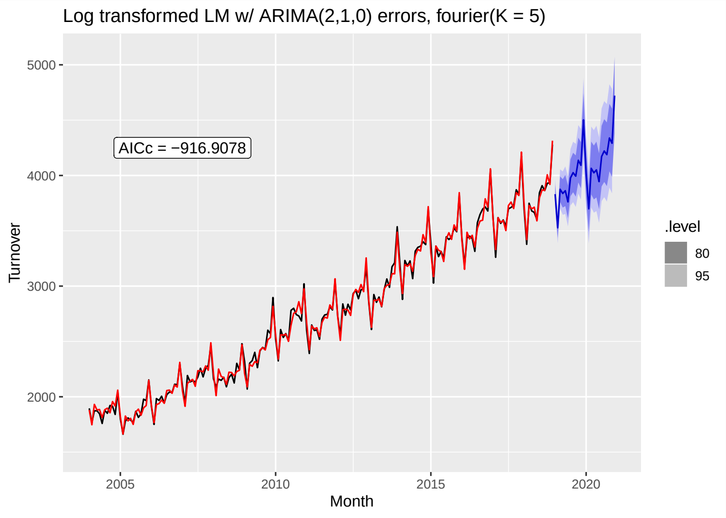
Eating-out expenditure



Eating-out expenditure



Eating-out expenditure



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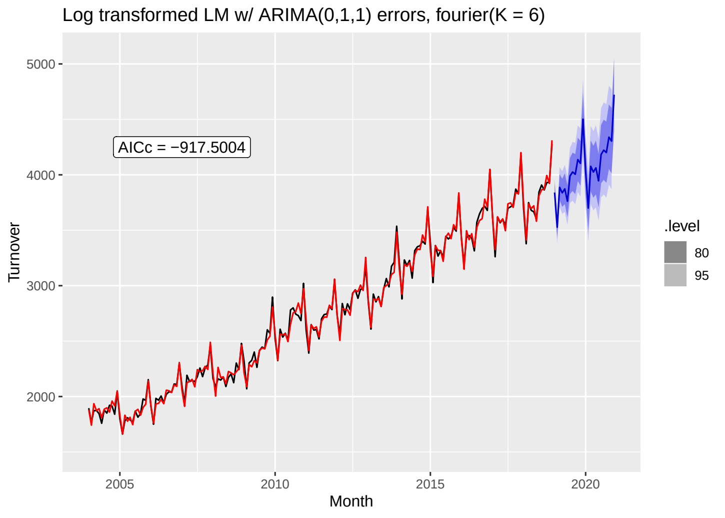


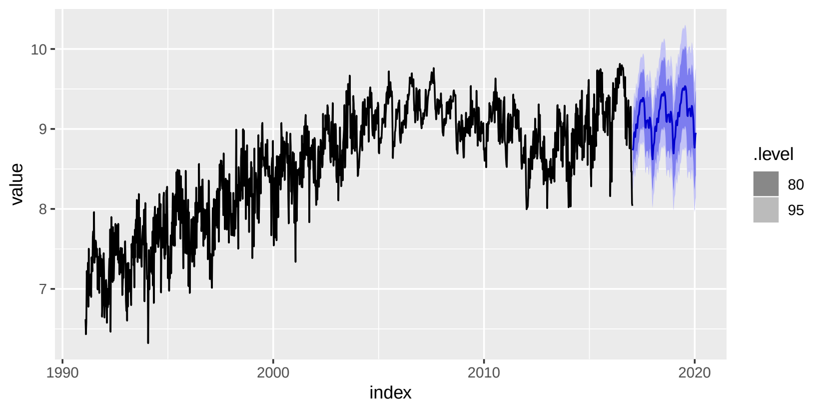
Example: weekly gasoline products

```r
gasoline <- as_tsibble(fpp2::gasoline)
fit <- gasoline %>% model(ARIMA(value ~ fourier(K = 13) + PDQ(0,0,0)))
report(fit)
```

```
## Series: value
## Model: LM w/ ARIMA(0,1,1) errors
##
## Coefficients:
##             ma1  fourier(K = 13)C1_52  fourier(K = 13)S1_52  fourier(K = 13)C2_52
##         -0.8934               -0.1121               -0.2300                0.0420
## s.e.     0.0132                0.0123                0.0122                0.0099
##         fourier(K = 13)S2_52  fourier(K = 13)C3_52  fourier(K = 13)S3_52
##                       0.0317                0.0832                0.0346
## s.e.                  0.0099                0.0094                0.0094
##         fourier(K = 13)C4_52  fourier(K = 13)S4_52  fourier(K = 13)C5_52
##                       0.0185                0.0398               -0.0315
## s.e.                  0.0092                0.0092                0.0091
##         fourier(K = 13)S5_52  fourier(K = 13)C6_52  fourier(K = 13)S6_52
##                       0.0009               -0.0522                 0.000
## s.e.                  0.0091                0.0090                 0.009
##         fourier(K = 13)C7_52  fourier(K = 13)S7_52  fourier(K = 13)C8_52
##                      -0.0173                0.0053                0.0075
## s.e.                  0.0090                0.0090                0.0090
##         fourier(K = 13)S8_52  fourier(K = 13)C9_52  fourier(K = 13)S9_52
```

Example: weekly gasoline products

```r
forecast(fit, h = "3 years") %>%
  autoplot(gasoline)
```



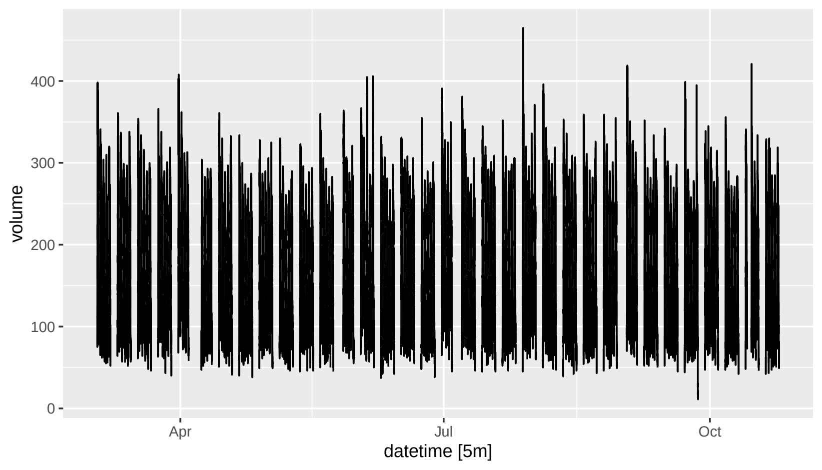
5-minute call centre volume

```
(calls <- readr::read_tsv("http://robjhyndman.com/data/callcenter.txt") %>
  gather("date", "volume", -X1) %>% transmute(
    time = X1, date = as.Date(date, format = "%d/%m/%Y"),
    datetime = as_datetime(date) + time, volume) %>%
  as_tsibble(index = datetime))
```

```
## # A tsibble: 27,716 x 4 [5m] <UTC>
##     time   date       datetime            volume
##     <time> <date>     <dttm>               <dbl>
##   1 07:00  2003-03-03 2003-03-03 07:00:00    111
##   2 07:05  2003-03-03 2003-03-03 07:05:00    113
##   3 07:10  2003-03-03 2003-03-03 07:10:00     76
##   4 07:15  2003-03-03 2003-03-03 07:15:00     82
##   5 07:20  2003-03-03 2003-03-03 07:20:00     91
##   6 07:25  2003-03-03 2003-03-03 07:25:00     87
##   7 07:30  2003-03-03 2003-03-03 07:30:00     75
##   8 07:35  2003-03-03 2003-03-03 07:35:00     89
##   9 07:40  2003-03-03 2003-03-03 07:40:00     99
## 10 07:45  2003-03-03 2003-03-03 07:45:00    125
```

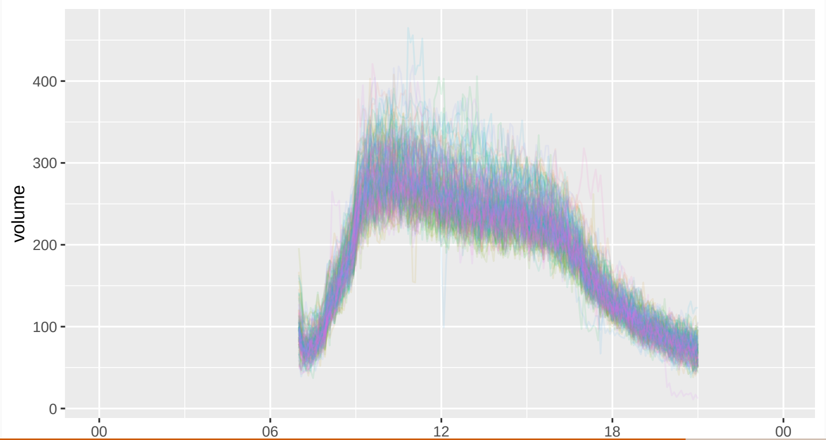
5-minute call centre volume

```r
calls %>% fill_gaps() %>% autoplot(volume)
```



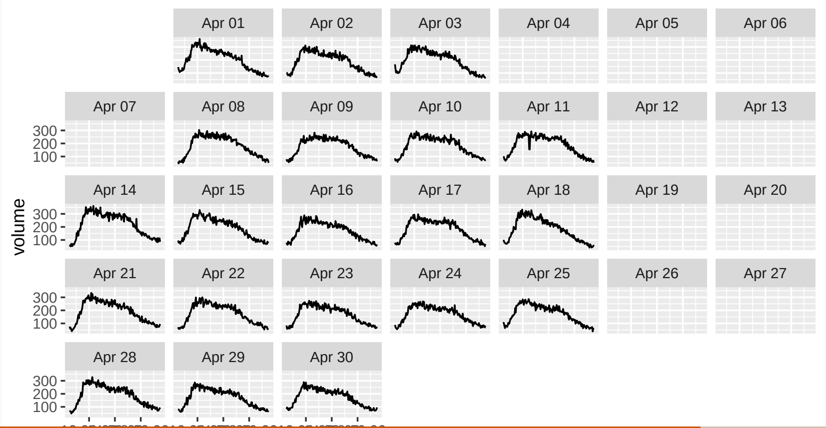
5-minute call centre volume

```
calls %>% fill_gaps() %>%
  gg_season(volume, period = "day", alpha = 0.1) +
  guides(colour = FALSE)
```



5-minute call centre volume

```r
library(sugrrants)
calls %>% filter(month(date, label = TRUE) == "Apr") %>%
  ggplot(aes(x = time, y = volume)) +
  geom_line() + facet_calendar(date)
```



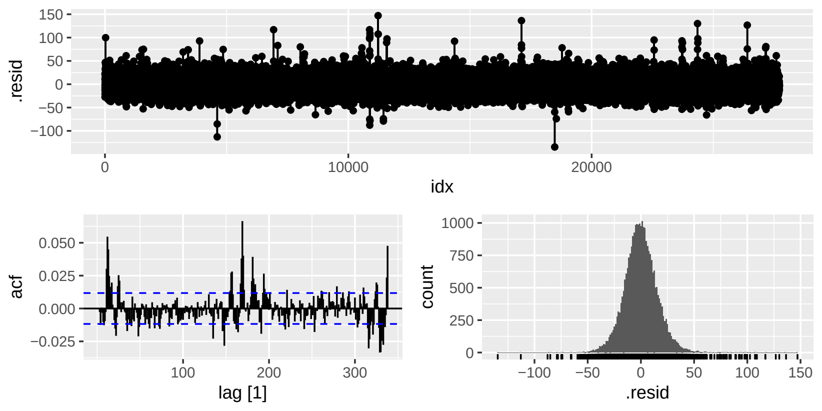
5-minute call centre volume

```r
calls_mdl <- calls %>%
  mutate(idx = row_number()) %>%
  update_tsibble(index = idx)
fit <- calls_mdl %>%
  model(ARIMA(volume ~ fourier(169, K = 10) + pdq(d=0) + PDQ(0,0,0)))
report(fit)
```

```
## Series: volume
## Model: LM w/ ARIMA(1,0,3) errors
##
## Coefficients:
##          ar1      ma1      ma2      ma3  fourier(169, K = 10)C1_169
##       0.9894  -0.7383  -0.0333  -0.0282                    -79.0702
## s.e.  0.0010   0.0061   0.0075   0.0060                      0.7001
##       fourier(169, K = 10)S1_169  fourier(169, K = 10)C2_169
##                          55.2985                    -32.3615
## s.e.                      0.7006                      0.3784
##       fourier(169, K = 10)S2_169  fourier(169, K = 10)C3_169
##                          13.7417                     -9.3180
## s.e.                      0.3786                      0.2725
##       fourier(169, K = 10)S3_169  fourier(169, K = 10)C4_169
##                         -13.6446                     -2.7913
## s.e.                      0.2726                      0.2230
##       fourier(169, K = 10)S4_169  fourier(169, K = 10)C5_169
```

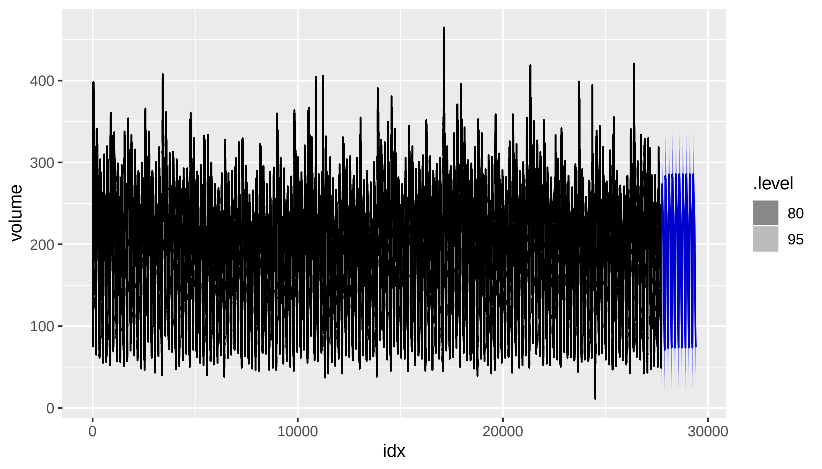
5-minute call centre volume

```r
augment(fit) %>%
  gg_tsdisplay(.resid, plot_type = "histogram", lag_max = 338)
```



5-minute call centre volume

```r
fit %>% forecast(h = 1690) %>%
  autoplot(calls_mdl)
```



Outline

Lagged predictors

Sometimes a change in x_t does not affect y_t instantaneously

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- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

Lagged predictors

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- y_t = sales, x_t = advertising.
- y_t = stream flow, x_t = rainfall.
- y_t = size of herd, x_t = breeding stock.

- These are dynamic systems with input (x_t) and output (y_t).
- x_t is often a leading indicator.
- There can be multiple predictors.

Lagged predictors

The model include present and past values of predictor: $x_t, x_{t-1}, x_{t-2}, \dots$.

$$y_t = a + \nu_0 x_t + \nu_1 x_{t-1} + \cdots + \nu_k x_{t-k} + \eta_t$$

where η_t is an ARIMA process.

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where η_t is an ARIMA process.

Rewrite model as

$$\begin{aligned} y_t &= a + (\nu_0 + \nu_1 B + \nu_2 B^2 + \cdots + \nu_k B^k) x_t + \eta_t \\ &= a + \nu(B) x_t + \eta_t. \end{aligned}$$

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The model include present and past values of predictor: $x_t, x_{t-1}, x_{t-2}, \dots$.

$$y_t = a + \nu_0 x_t + \nu_1 x_{t-1} + \cdots + \nu_k x_{t-k} + \eta_t$$

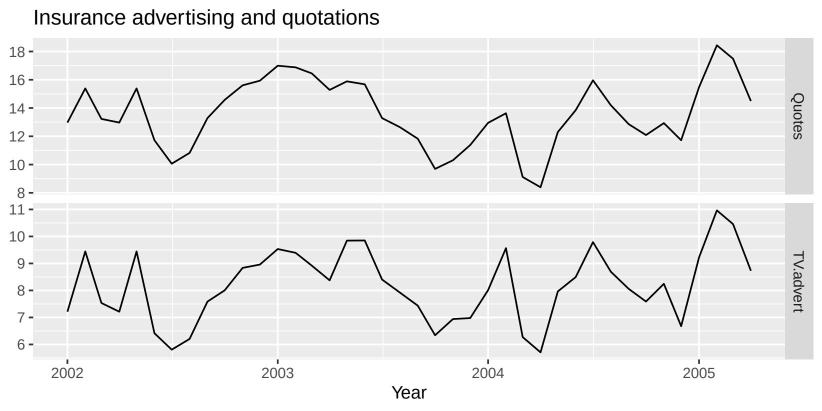
where η_t is an ARIMA process.

Rewrite model as

$$\begin{aligned} y_t &= a + (\nu_0 + \nu_1 B + \nu_2 B^2 + \cdots + \nu_k B^k) x_t + \eta_t \\ &= a + \nu(B) x_t + \eta_t. \end{aligned}$$

- $\nu(B)$ is called a *transfer function* since it describes how change in x_t is transferred to y_t.
- x can influence y, but y is not allowed to influence x.

Example: Insurance quotes and TV adverts



Example: Insurance quotes and TV adverts

```r
fit <- insurance %>%
  # Restrict data so models use same fitting period
  mutate(Quotes = c(NA,NA,NA,Quotes[4:40])) %>%
  # Estimate models
  model(
    ARIMA(Quotes ~ pdq(d = 0) + TV.advert),
    ARIMA(Quotes ~ pdq(d = 0) + TV.advert + lag(TV.advert)),
    ARIMA(Quotes ~ pdq(d = 0) + TV.advert + lag(TV.advert) +
            lag(TV.advert, 2)),
    ARIMA(Quotes ~ pdq(d = 0) + TV.advert + lag(TV.advert) +
            lag(TV.advert, 2) + lag(TV.advert, 3))
  )
```

Example: Insurance quotes and TV adverts

```r
glance(fit)
```

Lag order	sigma2	log_lik	AIC	AICc	BIC
0	0.2649757	-28.28210	66.56420	68.32890	75.00859
1	0.2094368	-24.04404	58.08809	59.85279	66.53249
2	0.2150429	-24.01627	60.03254	62.57799	70.16581
3	0.2056454	-22.15731	60.31461	64.95977	73.82565

Example: Insurance quotes and TV adverts

```r
fit <- insurance %>%
  model(ARIMA(Quotes ~ pdq(3, 0, 0) + TV.advert + lag(TV.advert)))
report(fit)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(3,0,0) errors
##
## Coefficients:
##          ar1      ar2     ar3  TV.advert  lag(TV.advert)  intercept
##       1.4117  -0.9317  0.3591     1.2564          0.1625     2.0393
## s.e.  0.1698   0.2545  0.1592     0.0667          0.0591     0.9931
##
## sigma^2 estimated as 0.2165:  log likelihood=-23.89
## AIC=61.78   AICc=65.28   BIC=73.6
```

Example: Insurance quotes and TV adverts

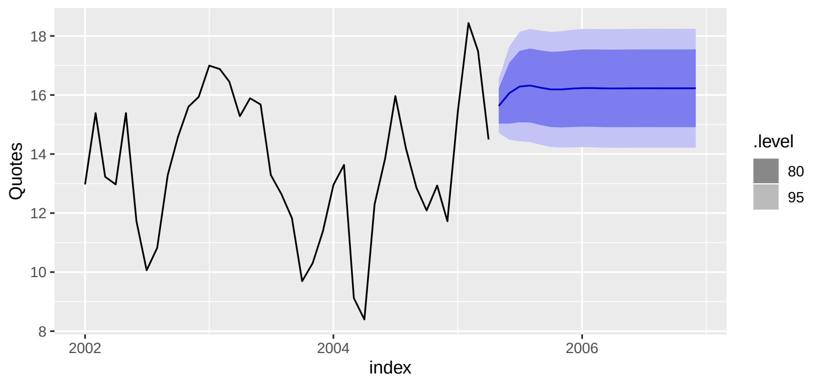
```r
fit <- insurance %>%
  model(ARIMA(Quotes ~ pdq(3, 0, 0) + TV.advert + lag(TV.advert)))
report(fit)
```

```
## Series: Quotes
## Model: LM w/ ARIMA(3,0,0) errors
##
## Coefficients:
##          ar1      ar2     ar3  TV.advert  lag(TV.advert)  intercept
##       1.4117  -0.9317  0.3591     1.2564          0.1625     2.0393
## s.e.  0.1698   0.2545  0.1592     0.0667          0.0591     0.9931
##
## sigma^2 estimated as 0.2165:  log likelihood=-23.89
## AIC=61.78   AICc=65.28   BIC=73.6
```

$$
\begin{aligned}
y_t &= 2.04 + 1.26x_t + 0.16x_{t-1} + \eta_t, \\
\eta_t &= 1.41\eta_{t-1} - 0.93\eta_{t-2} + 0.36\eta_{t-3} + \varepsilon_t,
\end{aligned}
$$

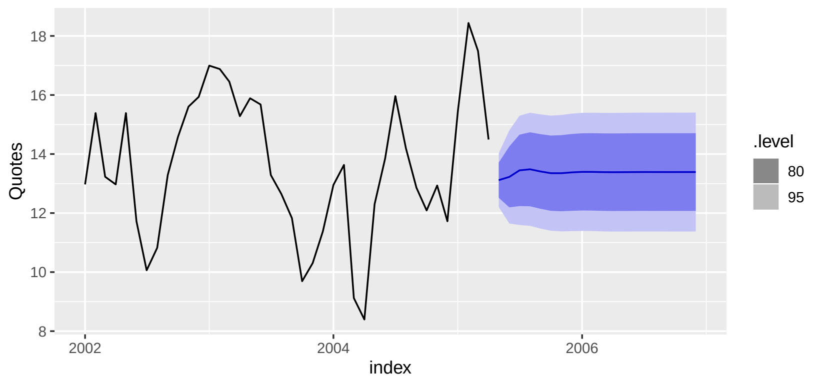
Example: Insurance quotes and TV adverts

```r
advert_a <- new_data(insurance, 20) %>%
  mutate(TV.advert = 10)
forecast(fit, advert_a) %>% autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_b <- new_data(insurance, 20) %>%
  mutate(TV.advert = 8)
forecast(fit, advert_b) %>% autoplot(insurance)
```



Example: Insurance quotes and TV adverts

```
advert_c <- new_data(insurance, 20) %>%
  mutate(TV.advert = 6)
forecast(fit, advert_c) %>% autoplot(insurance)
```