

You have n chairs that are to seat mathematicians and physicists. But no two physicists should be seated next to each other. If $n = 3$, these are some valid seatings: MMP, MPM. But PPM is not a valid seating.

What are the number of valid seatings given n ?

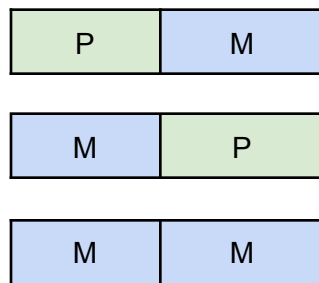
Find a recurrence relation $f(n)$ to give the number of valid seatings for any n .

Show your work

Solution:

For $n = 2$

We can have the following combination of seating

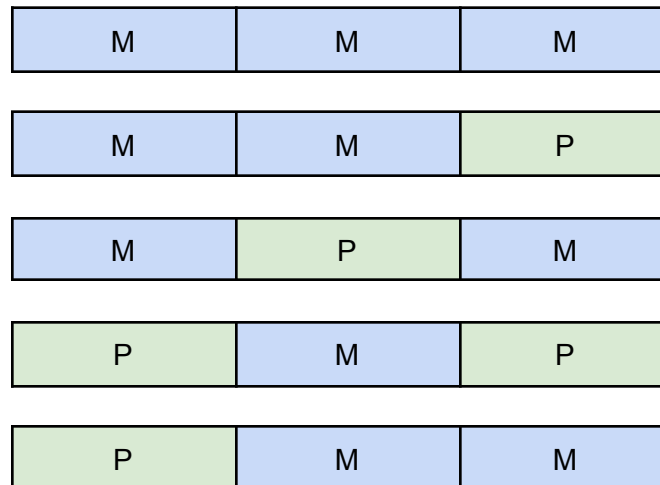


So for 2 seats we can have 3 ways for seating ->

$$F(2) = 3$$

For $n = 3$

We have the following combinations of seating



So for 3 seats we can have 5 ways for seating ->

$$F(3) = 5$$

For $n = 4$

We have the following combinations of seating

M	M	M	M
M	M	M	P
M	M	P	M
M	P	M	M
P	M	M	M
M	P	M	P
P	M	P	M
P	M	M	P

So for 4 seats we can have 8 ways for seating ->

$$F(4) = 8$$

If we continue to do this we will get 13, 21

So this leads that every solution is the sum of the previous 2 numbers this will lead us to recurrence equation of

$$F(n) = F(n-1) + F(n-2)$$

Where it's valid as $n \geq 2$ and $F(0) = 1$ $F(1) = 2$

$$F(n) = 1$$

$$n = 0$$

$$F(n) = 2$$

$$n = 1$$

$$F(n) = F(n-1) + F(n-2)$$

$$n \geq 2$$