You have n chairs that are to seat mathematicians and physicists. But no two physicists should be seated next to each other. If n = 3, these are some valid seatings: MMP, MPM. But PPM is not a valid seating.
What are the number of valid seatings given n?
Find a recurrence relation f(n) to give the number of valid seatings for any n.
Show your work

Solution:

For n = 2
We can have the following combination of seating

P	M

M	P

M	M

So for 2 seats we can have 3 ways for seating -> F(2) = 3

For n = 3
We have the following combinations of seating

M	M	M

M	M	P

M	P	M

P	M	P

P	M	M

So for 3 seats we can have 5 ways for seating -> F(3) = 5

For $n = 4$
We have the following combinations of seating

M	M	M	M
M	M	M	P
M	M	P	M
M	P	M	M
P	M	M	M
M	P	M	P
P	M	P	M
P	M	M	P

So for 4 seats we can have 8 ways for seating -> $F(4) = 8$

If we continue to do this we will get 13, 21 …..

So this leads that every solution is the sum of the previous 2 numbers this will lead us to recurrence equation of

$$F(n) = F(n-1) + F(n-2)$$

Where it's valid as $n >= 2$ and $F(0) = 1$ $F(1) = 2$

$$F(n) = \begin{cases} 1 & n = 0 \\ 2 & n = 1 \\ F(n-1) + F(n-2) & n >= 2 \end{cases}$$