

Question - 1

SCORE: 5 points

Question 1

By induction show that $5^n - 1$ is divisible by 4 for all values of $n \geq 1$

Question - 2

SCORE: 5 points

Stirling's approximation

What is Stirling's approximation for $\lg(n!)$ in tilde notation?

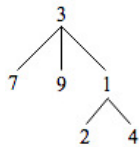
- ☐ $\sim \ln(n)$
- ☐ $\sim \lg(n)$
- ☐ $\sim n \ln(n)$
- ☒ $\sim n \lg(n)$

Question - 3

SCORE: 5 points

Question 3

The operations $\text{union}(1, 2)$, $\text{union}(4, 2)$, $\text{union}(3, 7)$, $\text{union}(3, 9)$, $\text{union}(3, 4)$ result in the tree:



Which algorithm was used?

- ☐ Quick union
- ☒ Weighted quick union
- ☐ Quick union with path compression
- ☐ Weighted quick union with path compression

Question - 4

SCORE: 5 points

Question 4

What is the final id array if the quick-find algorithm is performed with the sequence $\text{union}(3, 1)$, $\text{union}(4, 2)$, $\text{union}(3, 4)$, $\text{union}(7, 8)$, $\text{union}(9, 0)$, $\text{union}(0, 1)$, $\text{union}(8, 2)$?

- ☐ [9, 9, 9, 9, 9, 5, 6, 9, 9, 9]
- ☐ [7, 7, 7, 7, 7, 5, 6, 7, 7, 7]

☒ [2, 2, 2, 2, 2, 5, 6, 2, 2, 2]☐ [8, 8, 8, 8, 8, 5, 6, 8, 8, 8]

Question - 5

Weighted quick union

SCORE: 30 points

Your task is to implement the find and union methods for the weighted quick union algorithm