

Formula

1. Least Square

The diagram illustrates the components of the linear regression equation $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$. Labels with arrows point to each term: Y_i is the Dependent Variable, β_0 is the Population Y intercept, β_1 is the Population Slope Coefficient, X_i is the Independent Variable, and ϵ_i is the Random Error term. Below the equation, two blue brackets group the terms: the first bracket under $\beta_0 + \beta_1 X_i$ is labeled 'Linear component', and the second bracket under ϵ_i is labeled 'Random Error component'.

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$$

Dependent Variable

Population Y intercept

Population Slope Coefficient

Independent Variable

Random Error term

Linear component

Random Error component

Linear Regression

- Gradient Descendant
- Least Square
- Linear Equation
- SVD
- QR Decomposition
- Monte Carlo Method

$$Y = mx + c$$

Least Square

$$\text{var}(x) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

$$\text{cov}(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

$$\beta = \frac{\text{cov}(x, y)}{\text{var}(x)}$$

$$\beta = \frac{22.65}{23.2} = 0.9762931034482758$$

$$\alpha = \bar{y} - \beta \bar{x}$$

\bar{x} is the mean of x , x_i is the value of x for the i th training instance, and n is the number of training instances.

Linear Equation

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

Table 1

x	y
1	1
2	3
3	3
4	2
5	5

2. Logistic Regression




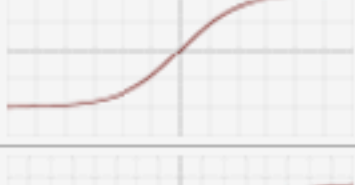




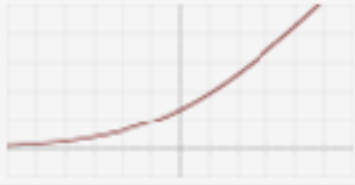
Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) [2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU) [3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$

Table 1

X1	X2	Y
2.7810836	2.550537003	0
1.465489372	2.362125076	0
3.396561688	4.400293529	0
1.38807019	1.850220317	0
3.06407232	3.005305973	0
7.627531214	2.759262235	1
5.332441248	2.088626775	1
6.922596716	1.77106367	1
8.675418651	-0.242068655	1
7.673756466	3.508563011	1

3. KNN

$$\text{Euclidean Distance} = \sqrt{(X_H - H_1)^2 + (X_W - W_1)^2}$$

Where

X_H : Observation value of variable Height |

H_1 : Centroid value of Cluster 1 for variable Height

X_W : Observation Value of variable Weight

W_1 : Centroid value of cluster 1 for variable Weight

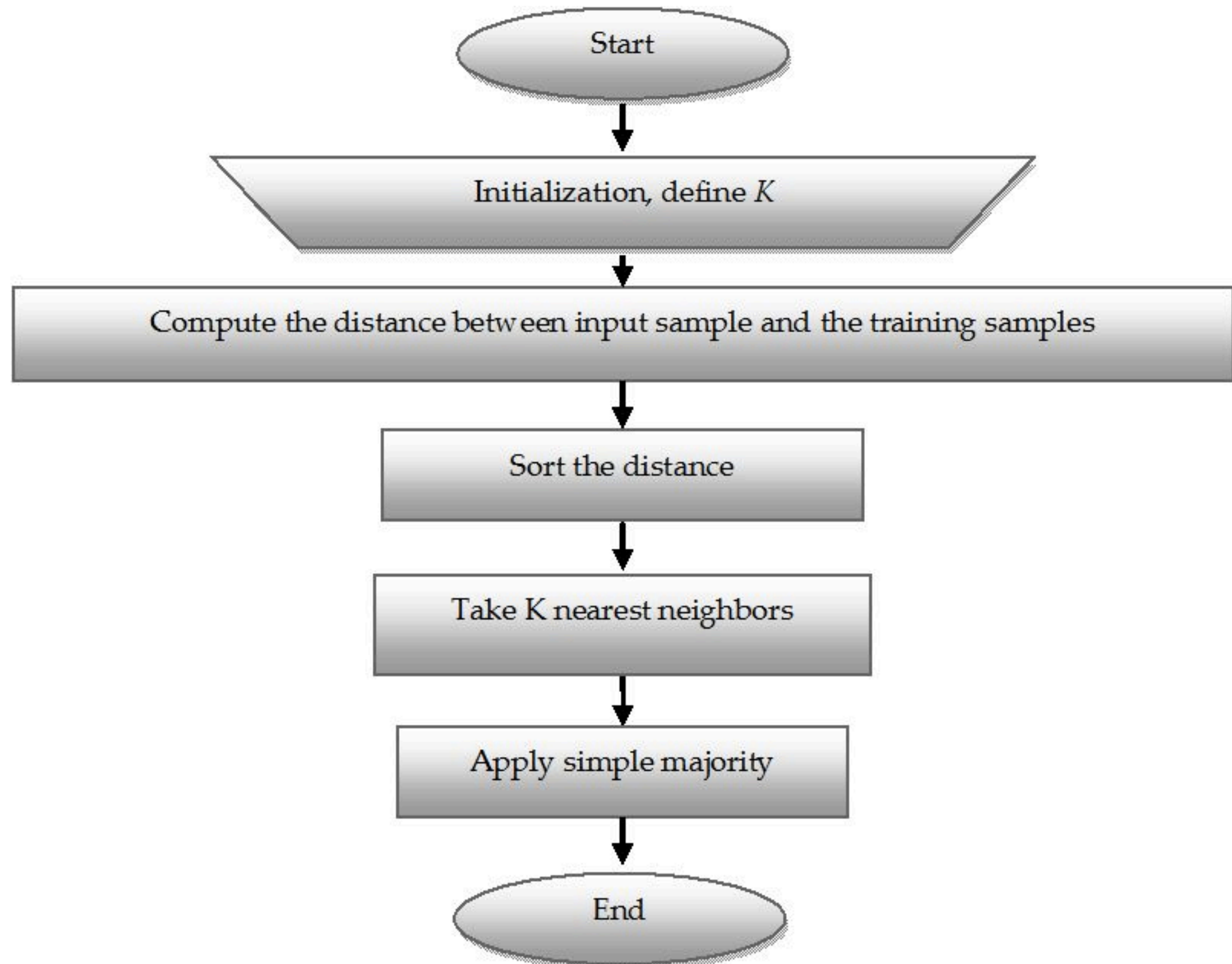


Table 1

X1	X2	Y
3.393533211	2.331273381	0
3.110073483	1.781539638	0
1.343808831	3.368360954	0
3.582294042	4.67917911	0
2.280362439	2.866990263	0
7.423436942	4.696522875	1
5.745051997	3.533989803	1
9.172168622	2.511101045	1
7.792783481	3.424088941	1
7.939820817	0.791637231	1

4. Naive Bayes

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

Chills	Runny Nose	Headache	Fever	Flu?
Yes	No	Moderate	Yes	No
Yes	Yes	No	No	Yes
Yes	No	Strong	Yes	Yes
No	Yes	Moderate	Yes	Yes
No	No	No	No	No
No	Yes	Strong	Yes	Yes
No	Yes	Strong	No	No
Yes	Yes	Moderate	Yes	Yes

What about the next patient? Symptoms:

Chills	Runny Nose	Headache	Fever	Flu?
Yes	No	Moderate	No	?

Condition	Probability	Condition	Probability
$P(\text{Flu}=\text{Yes})$	0.625	$P(\text{Flu}=\text{No})$	0.375
$P(\text{Chills}=\text{Yes} \text{Flu}=\text{Yes})$	0,6	$P(\text{Chills}=\text{Yes} \text{Flu}=\text{No})$	0.333
$P(\text{Chills}=\text{No} \text{Flu}=\text{Yes})$	0,4	$P(\text{Chills}=\text{No} \text{Flu}=\text{No})$	0.666
$P(\text{Runny Nose}=\text{Yes} \text{Flu}=\text{Yes})$	0,8	$P(\text{Runny Nose}=\text{Yes} \text{Flu}=\text{No})$	0.333
$P(\text{Runny Nose}=\text{No} \text{Flu}=\text{Yes})$	0,2	$P(\text{Runny Nose}=\text{No} \text{Flu}=\text{No})$	0.666
$P(\text{Headache}=\text{Moderate} \text{Flu}=\text{Yes})$	0,4	$P(\text{Headache}=\text{Moderate} \text{Flu}=\text{No})$	0.333
$P(\text{Headache}=\text{No} \text{Flu}=\text{Yes})$	0,2	$P(\text{Headache}=\text{No} \text{Flu}=\text{No})$	0.333
$P(\text{Headache}=\text{Strong} \text{Flu}=\text{Yes})$	0,4	$P(\text{Headache}=\text{Strong} \text{Flu}=\text{No})$	0.333
$P(\text{Temperature}=\text{Yes} \text{Flu}=\text{Yes})$	0,8	$P(\text{Temperature}=\text{Yes} \text{Flu}=\text{No})$	0.333
$P(\text{Temperature}=\text{No} \text{Flu}=\text{Yes})$	0,2	$P(\text{Temperature}=\text{No} \text{Flu}=\text{No})$	0.666

Chills	Runny Nose	Headache	Fever	Flu?
Yes	No	Moderate	No	?

$P(\text{Flu}=\text{Yes})P(\text{Chills}=\text{Yes}|\text{Flu}=\text{Yes})P(\text{Runny Nose}=\text{No}|\text{Flu}=\text{Yes})P(\text{Headache}=\text{Moderate}|\text{Flu}=\text{Yes})P(\text{Temperature}=\text{No}|\text{Flu}=\text{Yes}) = 0.006$

$P(\text{Flu}=\text{No})P(\text{Chills}=\text{Yes}|\text{Flu}=\text{No})P(\text{Runny Nose}=\text{No}|\text{Flu}=\text{No})P(\text{Headache}=\text{Moderate}|\text{Flu}=\text{No})P(\text{Temperature}=\text{No}|\text{Flu}=\text{No}) = 0.0185$

Table 1

Weather	Car	Class
sunny	working	go-out
rainy	broken	go-out
sunny	working	go-out
sunny	working	go-out
sunny	working	go-out
rainy	broken	stay-home
rainy	broken	stay-home
sunny	working	stay-home
sunny	broken	stay-home
rainy	broken	stay-home

Table 1

Chills	Runny Nose	Headache	Fever	Flu?
Yes	No	Moderate	Yes	No
Yes	Yes	No	No	Yes
Yes	No	Strong	Yes	Yes
No	Yes	Moderate	Yes	Yes
No	No	No	No	No
No	Yes	Strong	Yes	Yes
No	Yes	Strong	No	No
Yes	Yes	Moderate	Yes	Yes

5. K Means

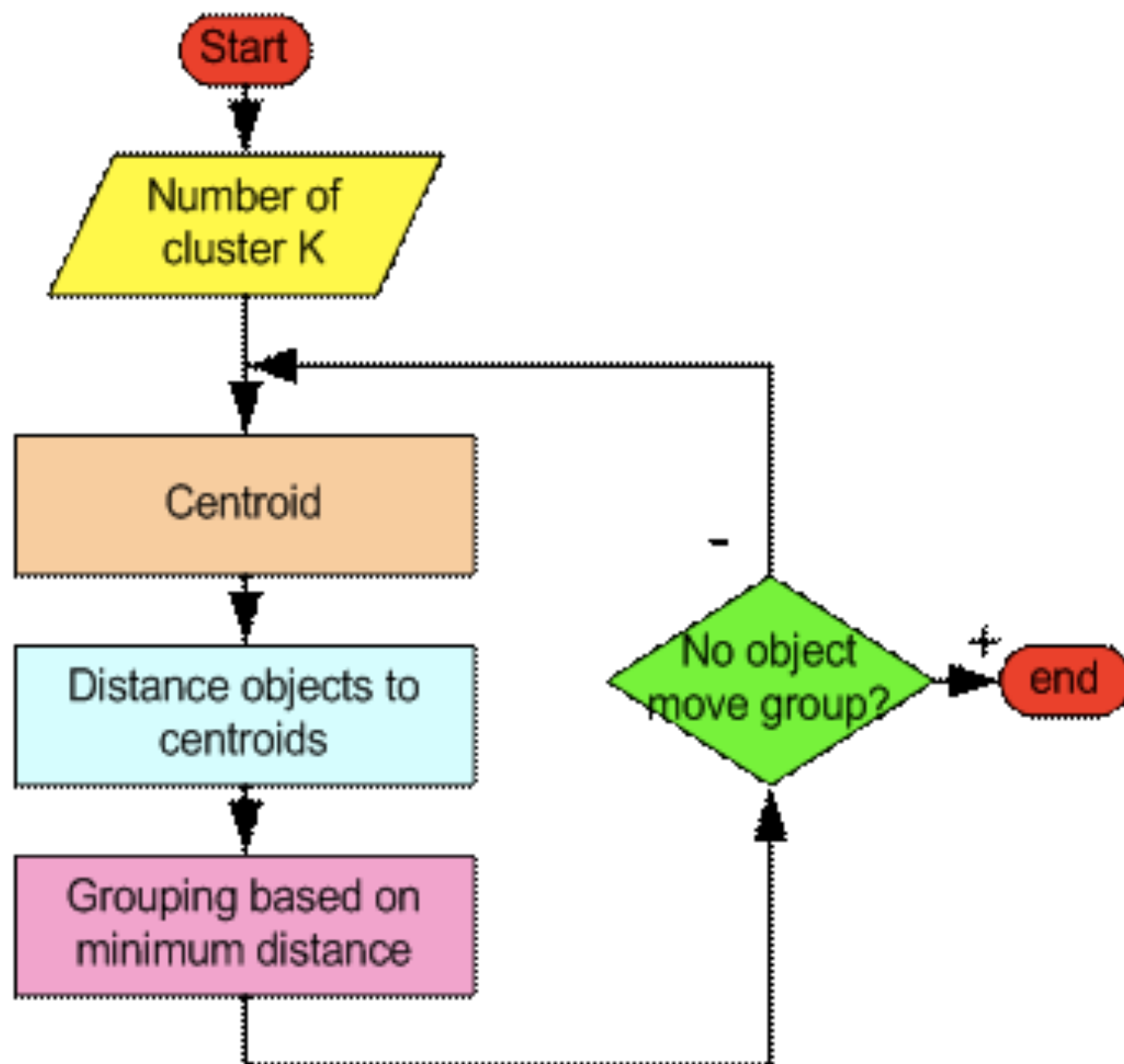
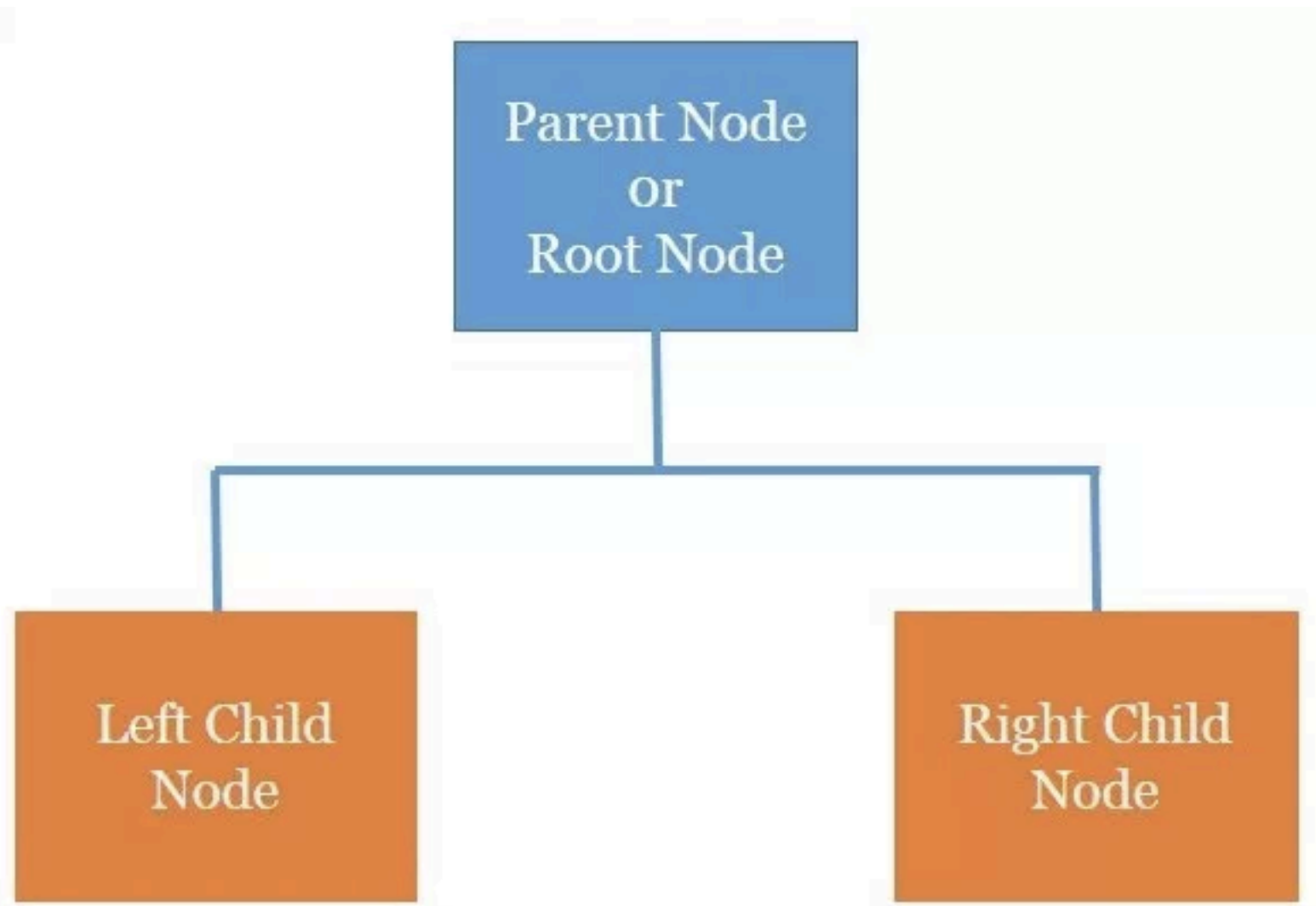


Table 1

Height	Weight
185	72
170	56
168	60
179	68
182	72
188	77
180	71
180	70
183	84
180	88
180	67
177	76

6. Decision Tree

Decision Tree



L : Left child node for the root node

R : Right child node for the root node

$$P_R = \frac{\text{Number of records in right child node}}{\text{Total number of records}}$$

$$P_R = \frac{\text{Number of records in right child node}}{\text{Total number of records}}$$

$$P(k|L) = \frac{\text{Number of class k records in left child node}}{\text{Number of records in left child node}}$$

$$P(k|R) = \frac{\text{Number of class k records in right child node}}{\text{Number of records in right child node}}$$

Activity in the Last Quarter	Number of Solicited Customers	Campaign Results		Success Rate
		Responded (<i>r</i>)	Not Responded (<i>nr</i>)	
low	40000	720	39280	1.8%
medium	30000	1380	28620	4.6%
high	30000	2100	27900	7.0%

As you know, CART decision tree algorithm splits the root node into just two child nodes. Hence for this data, CART can form three combinations of binary trees as shown in the table below.

Left Node	Right Node	P _L	P _R	P(k L) = a	P(k R) = b	Ψ(Large Piece)	Ψ(Pick Cherries)	Goodness of Split
						2P _L P _R	Σ(a-b)	
Low	Medium+High	0.4	0.6	r: 0.018	r: 0.058	0.48	0.080	0.0384
				nr: 0.982	nr: 0.942			
Low+Medium	High	0.7	0.3	r: 0.030	r: 0.070	0.42	0.080	0.0336
				nr: 0.970	nr: 0.930			
Low+high	Medium	0.7	0.3	r: 0.040	r: 0.046	0.42	0.011	0.0048
				nr: 0.960	nr: 0.954			

$$P_L = \frac{\# \text{ customers in Low}}{\text{All the customers}} = \frac{40000}{100000} = 0.4$$

$$P_R = \frac{\# \text{ customers in Medium} + \text{High}}{\text{All the customers}} = \frac{60000}{100000} = 0.6$$

$$\Psi(\text{Large Piece}) = 2P_LP_R = 2 \times 0.4 \times 0.6 = 0.48$$

Remember, r represents responded and nr represents not-responded customers for our campaign's example.

$$\text{r: } P(k|L) = \frac{\# \text{ customers responded in Low}}{\text{Total number of customers in Low}} = \frac{720}{40000} = 0.018$$

$$\text{nr: } P(k|L) = \frac{\# \text{ customers not responded in Low}}{\text{Total number of customers in Low}} = \frac{39280}{40000} = 0.982$$

$$\Psi(\text{Pick Cherries}) = |P(r|L) - P(r|R)| + |P(nr|L) - P(nr|R)|$$

$$\therefore \Psi(\text{Pick Cherries}) = |0.018 - 0.058| + |0.982 - 0.942| = 0.080$$

$$\text{Goodness of split} = \Psi(\text{Large Piece}) \times \Psi(\text{Pick Cherries}) = 0.48 \times 0.080$$

$$\therefore \text{Goodness of split} = 0.0384$$

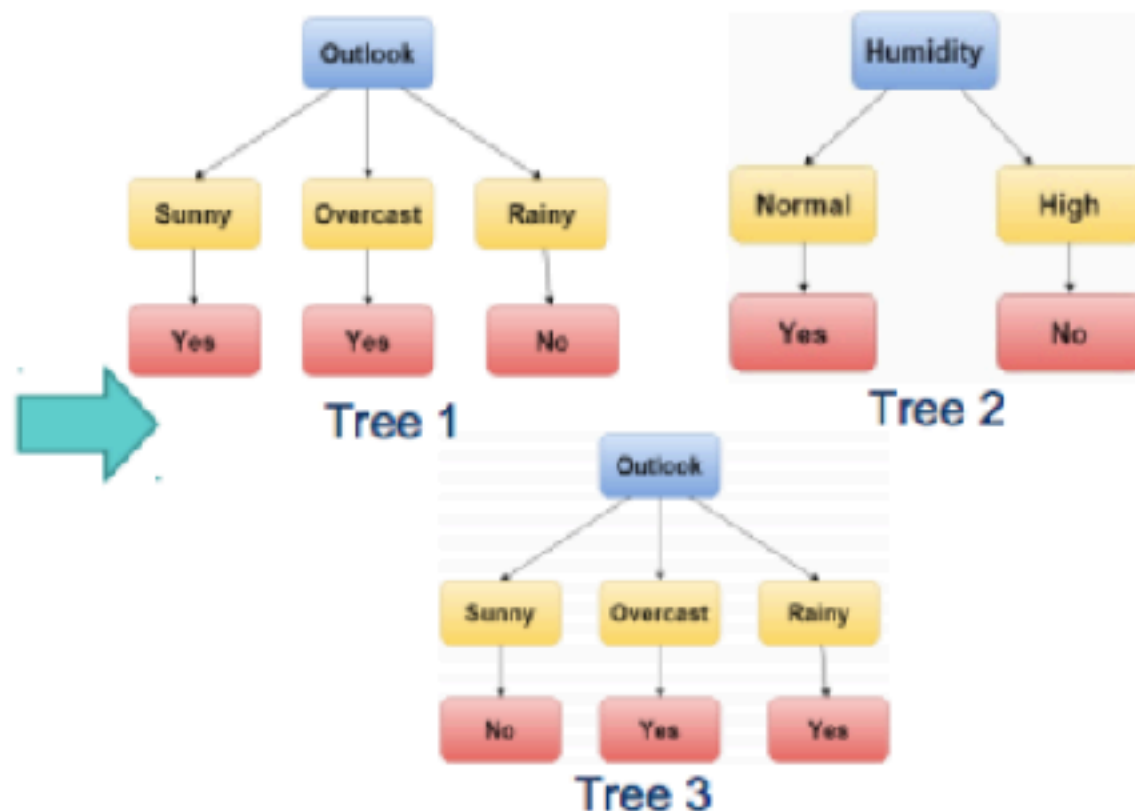
Table 1

X1	X2	Y
2.771244718	1.784783929	0
1.728571309	1.169761413	0
3.678319846	2.81281357	0
3.961043357	2.61995032	0
2.999208922	2.209014212	0
7.497545867	3.162953546	1
9.00220326	3.339047188	1
7.444542326	0.476683375	1
10.12493903	3.234550982	1
6.642287351	3.319983761	1

7. Random Forest

Outlook	Temp.	Humidity	Windy	Play Golf
Rainy	Mild	High	False	?

Predictors				Target
Outlook	Temp	Humidity	Windy	Play Golf
Rainy	Hot	High	False	No
Rainy	Hot	High	True	No
Overcast	Hot	High	False	Yes
Sunny	Mild	High	False	Yes
Sunny	Cool	Normal	False	Yes
Sunny	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Rainy	Mild	High	False	No
Rainy	Cool	Normal	False	Yes
Sunny	Mild	Normal	False	Yes
Rainy	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Sunny	Mild	High	True	No



Tree 1 : No
Tree 2 : No
Tree 3 : Yes

Yes : 1
No : 2

Result : No

Table 1

X1	X2	Y
2.771244718	1.784783929	0
1.728571309	1.169761413	0
3.678319846	2.81281357	0
3.961043357	2.61995032	0
2.999208922	2.209014212	0
7.497545867	3.162953546	1
9.00220326	3.339047188	1
7.444542326	0.476683375	1
10.12493903	3.234550982	1
6.642287351	3.319983761	1

8. NN

Neural Networks

Color Guided Matrix Multiplication for a Binary Classification Task with N = 4

Input Layer

bias X1 X2

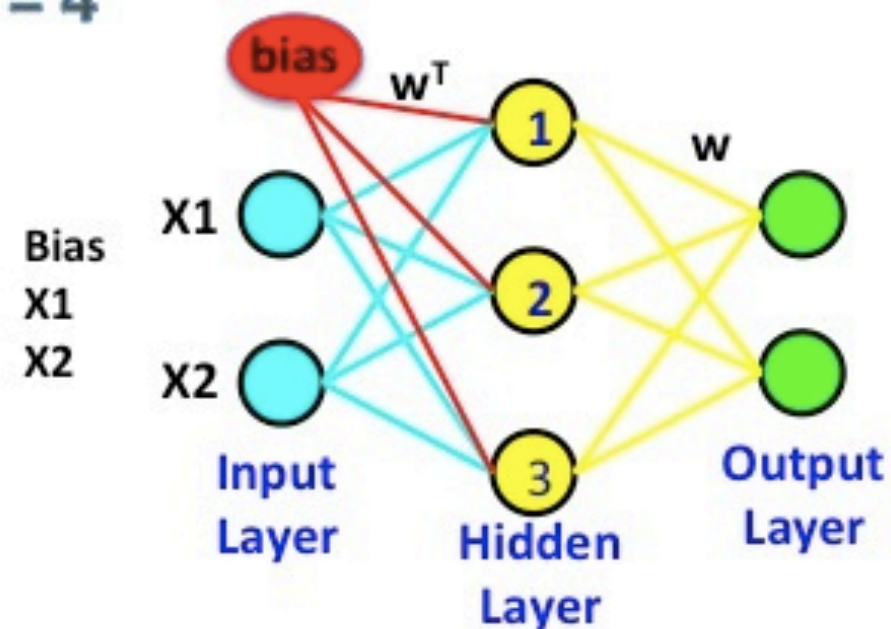
$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} .5 & .5 & .5 \\ .5 & .5 & .5 \\ .5 & .5 & .5 \end{bmatrix} =$$

Weights w^T (transposed)

Go to Hidden Nodes

1 2 3

3 x 3



Hidden Layer

Bias Node 1 Node 2 Node 3

$$= \begin{bmatrix} 1 & 1 & 1 \\ .5 & .5 & .5 \\ .5 & .5 & .5 \\ 1 & 1 & 1 \end{bmatrix} \cdot \frac{1}{1 + e^{-(wx+b)}}$$

3 x 2

Weights

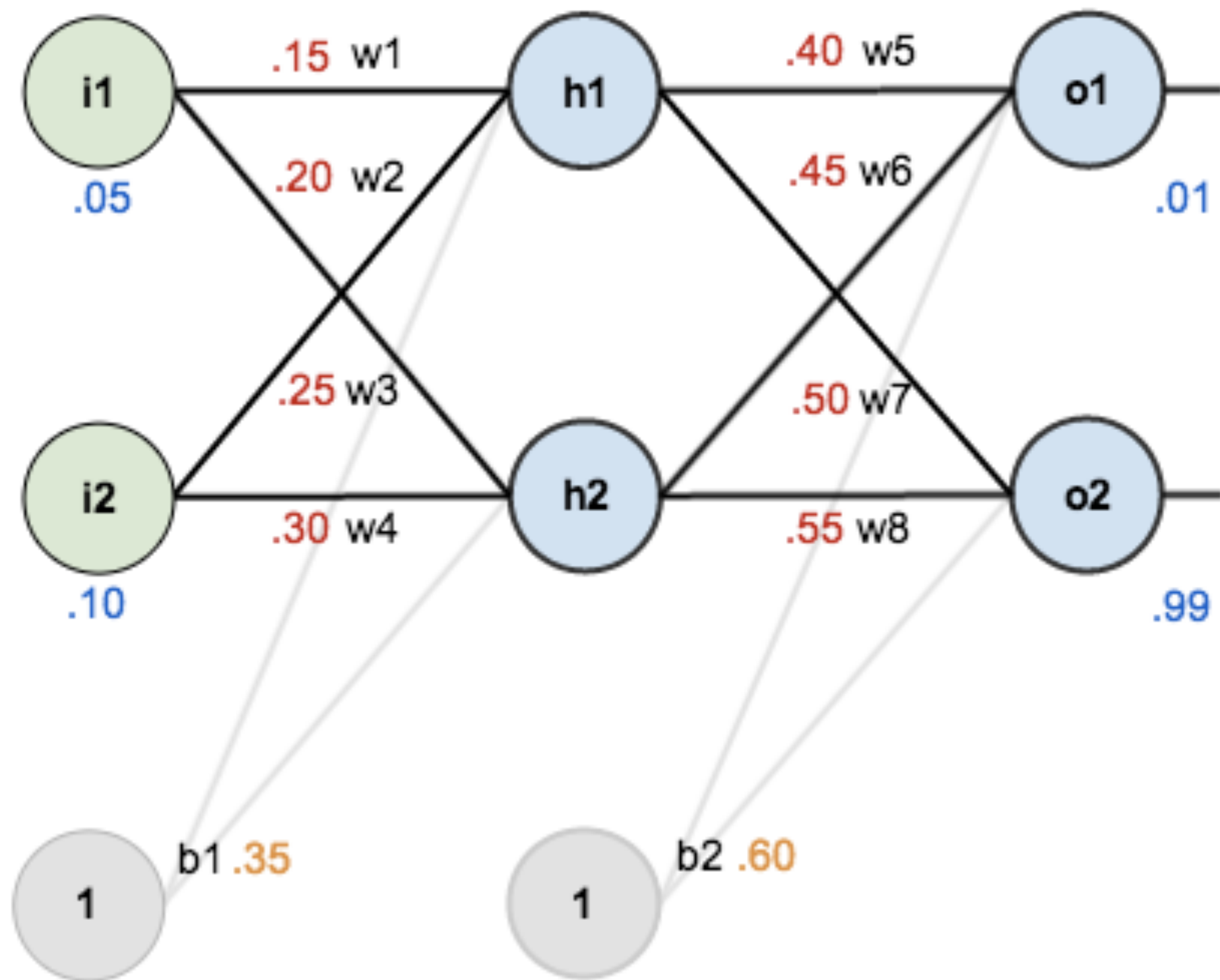
$$= \begin{bmatrix} .2 & .1 \\ .4 & .1 \\ .4 & .1 \end{bmatrix} \cdot \begin{bmatrix} 1 & .3 \\ .5 & .15 \\ .5 & .15 \\ 1 & .3 \end{bmatrix} = \frac{1}{1 + e^{-(wx+b)}}$$

Output Layer

Sigmoid Function

Output

$$= \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$



$$net_{h1} = w_1 * i_1 + w_2 * i_2 + b_1 * 1$$

$$net_{h1} = 0.15 * 0.05 + 0.2 * 0.1 + 0.35 * 1 = 0.3775$$

$$out_{h1} = \frac{1}{1+e^{-net_{h1}}} = \frac{1}{1+e^{-0.3775}} = 0.593269992$$

$$out_{h2} = 0.596884378$$

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$net_{o1} = 0.4 * 0.593269992 + 0.45 * 0.596884378 + 0.6 * 1 = 1.105905967$$

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}} = \frac{1}{1+e^{-1.105905967}} = 0.75136507$$

$$out_{o2} = 0.772928465$$

$\frac{1}{2}$

is included so that exponent is cancelled when we differentiate later on.

$$E_{total} = \sum \frac{1}{2}(target - output)^2$$

$$E_{o1} = \frac{1}{2}(target_{o1} - out_{o1})^2 = \frac{1}{2}(0.01 - 0.75136507)^2 = 0.274811083$$

$$E_{o2} = 0.023560026$$

$$E_{total} = E_{o1} + E_{o2} = 0.274811083 + 0.023560026 = 0.298371109$$

$$E_{total} = \frac{1}{2}(target_{o1} - out_{o1})^2 + \frac{1}{2}(target_{o2} - out_{o2})^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = 2 * \frac{1}{2}(target_{o1} - out_{o1})^{2-1} * -1 + 0$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1}) = -(0.01 - 0.75136507) = 0.74136507$$

$$out_{o1} = \frac{1}{1+e^{-net_{o1}}}$$

$$\frac{\partial out_{o1}}{\partial net_{o1}} = out_{o1}(1 - out_{o1}) = 0.75136507(1 - 0.75136507) = 0.186815602$$

$$net_{o1} = w_5 * out_{h1} + w_6 * out_{h2} + b_2 * 1$$

$$\frac{\partial net_{o1}}{\partial w_5} = 1 * out_{h1} * w_5^{(1-1)} + 0 + 0 = out_{h1} = 0.593269992$$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out_{o1}} * \frac{\partial out_{o1}}{\partial net_{o1}} * \frac{\partial net_{o1}}{\partial w_5}$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.74136507 * 0.186815602 * 0.593269992 = 0.082167041$$

9. SVM

$$B0 + (B1 \times X1) + (B2 \times X2) = 0$$

If the output value is greater than 1 it suggests that the training pattern was not a support vector.

$$b = \left(1 - \frac{1}{t}\right) \times b$$

If the output is less than 1 then it is assumed that the training instance is a support vector and must be updated to better explain the data.

$$b = \left(1 - \frac{1}{t}\right) \times b + \frac{1}{\text{lambda} \times t} \times (y \times x)$$

$$B1 = 0.0$$

$$B2 = 0.0$$

$$X1 = 2.327868056, X2 = 2.458016525, Y = 1$$

$$\text{output} = Y * (B1 * X1) + (B2 * X2)$$

$$\text{output} = -1 * (0.0 * 2.327868056) + (0.0 * 2.458016525)$$

$$\text{output} = 0.0 \quad // \text{The output is less than 1.0}$$

$$b = \left(1 - \frac{1}{t}\right) \times b + \frac{1}{\text{lambda} \times t} \times (y \times x)$$

$$B1 = (1 - 1/1) * 0.0 + 1/(0.45 * 1) * (1 * 2.327868056)$$

$$B1 = 5.173040124$$

$$B1 = (1 - 1/1) * 0.0 + 1/(0.45 * 1) * (1 * 2.458016525)$$

$$B2 = 5.462258944$$

$$B1 = -5.173040124$$

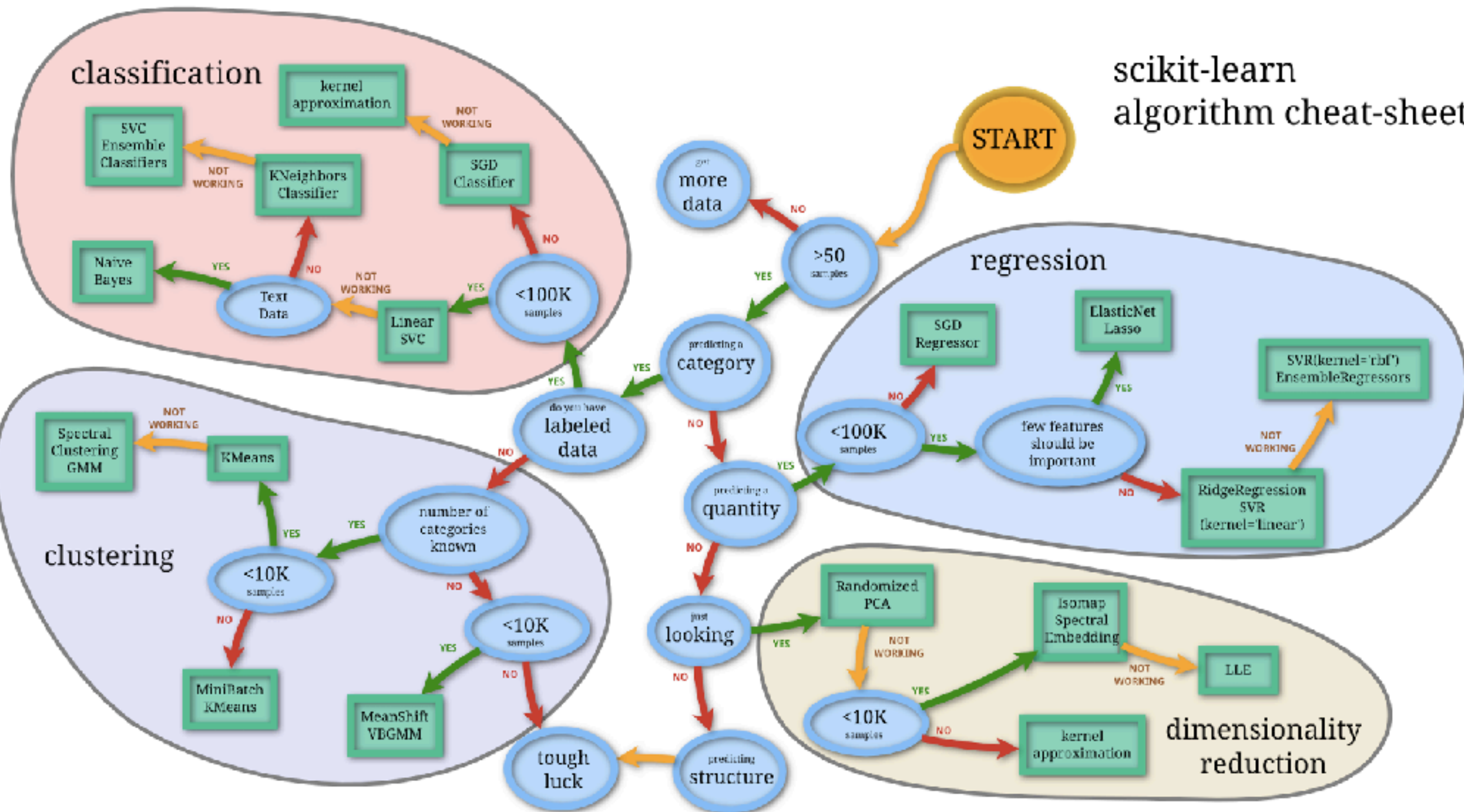
$$B2 = -5.462258944$$

$$X1 = 2.327868056, X2 = 2.458016525, Y = 1$$

Table 1

X1	X2	Y
2.327868056	2.458016525	-1
3.032830419	3.170770366	-1
4.485465382	3.696728111	-1
3.684815246	3.846846973	-1
2.283558563	1.853215997	-1
7.807521179	3.290132136	1
6.132998136	2.140563087	1
7.514829366	2.107056961	1
5.502385039	1.404002608	1
7.432932365	4.236232628	1

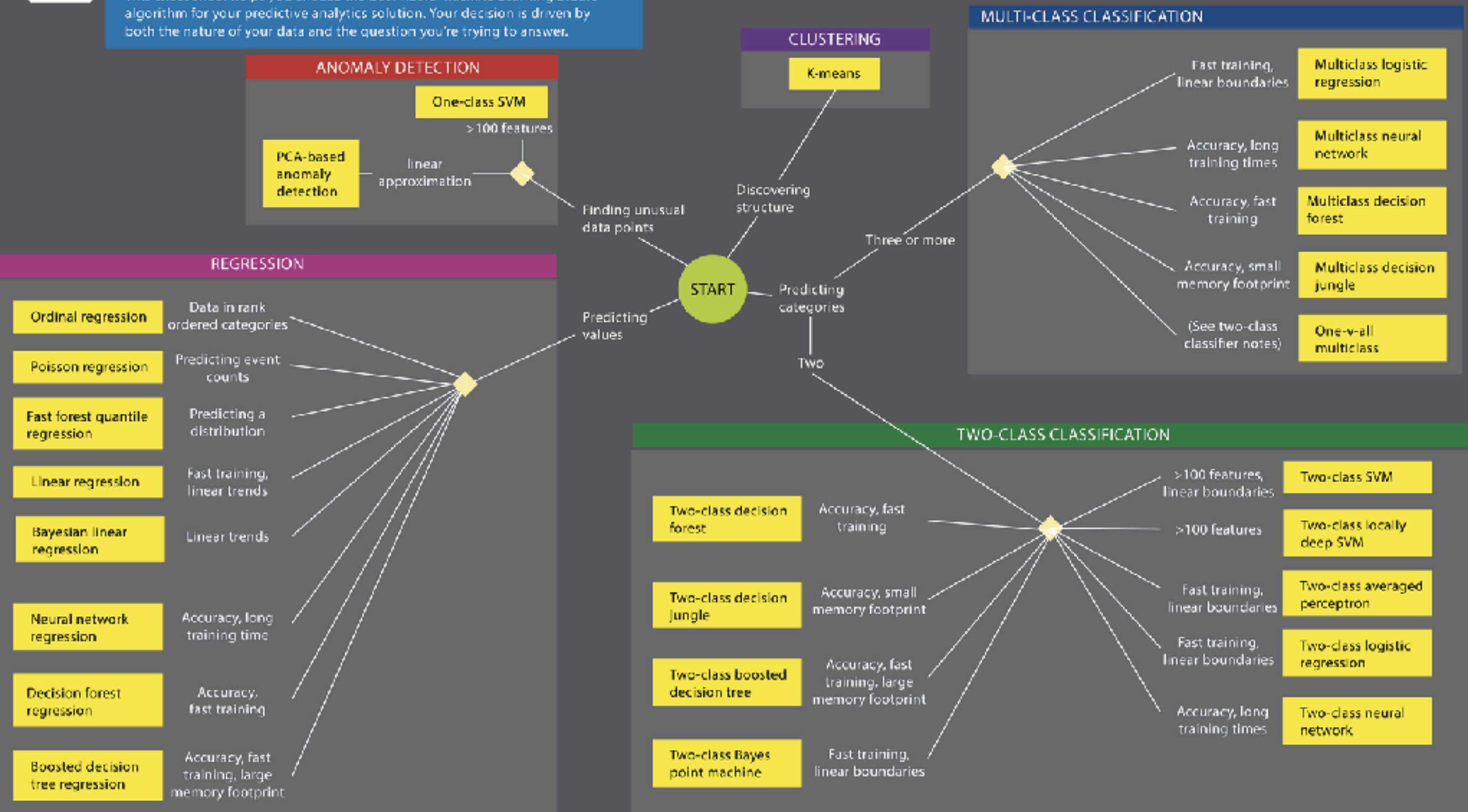
scikit-learn algorithm cheat-sheet



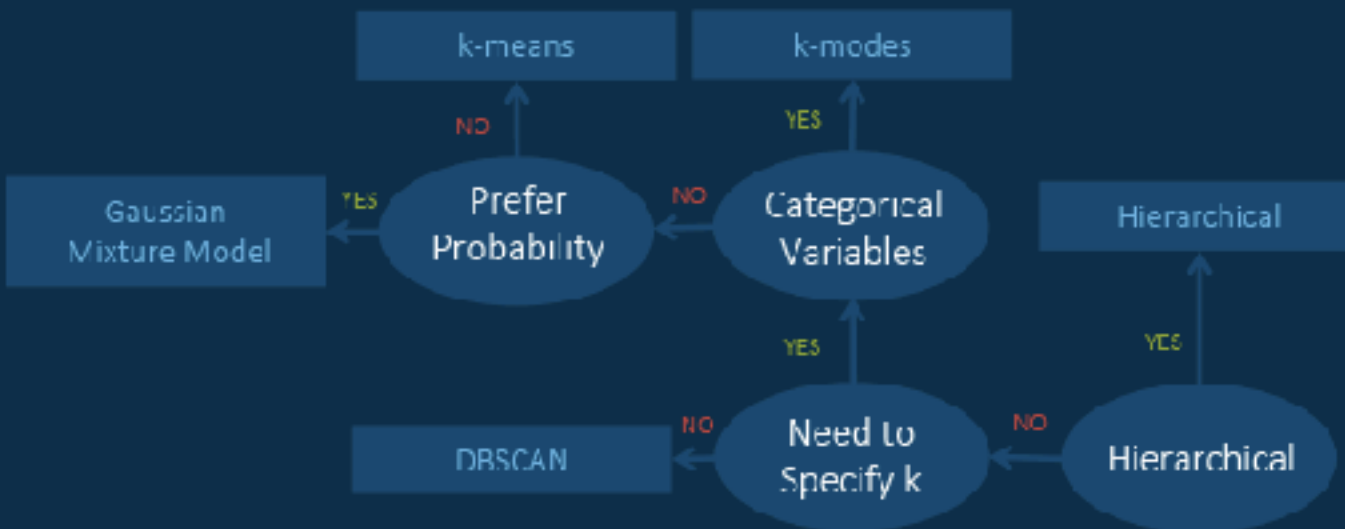


Microsoft Azure Machine Learning: Algorithm Cheat Sheet

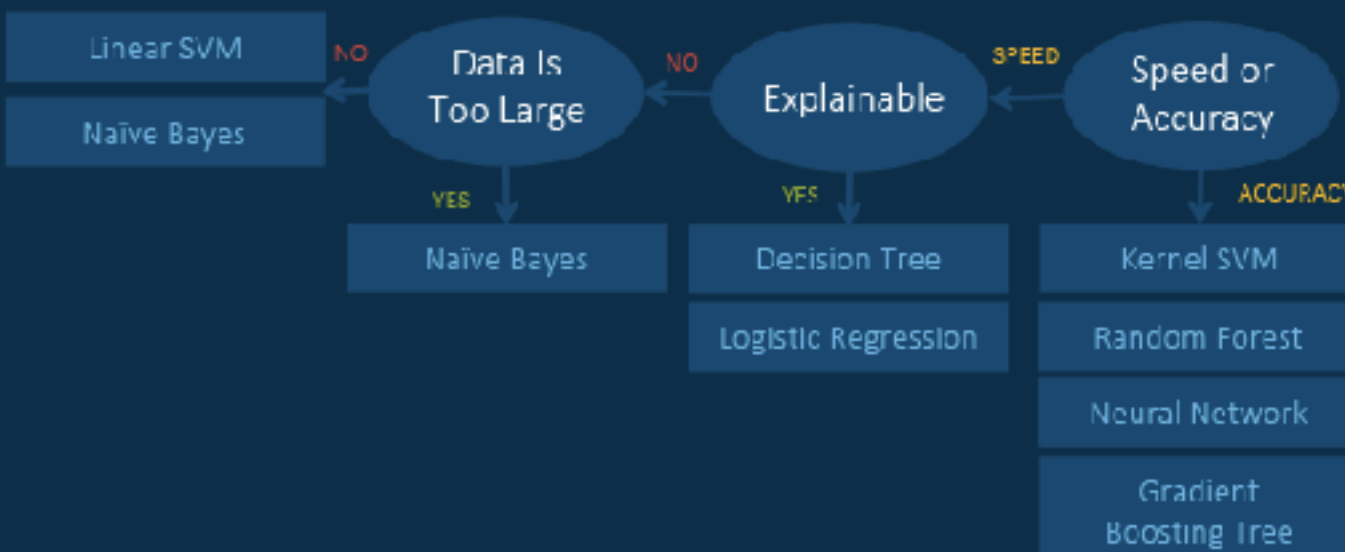
This cheat sheet helps you choose the best Azure Machine Learning Studio algorithm for your predictive analytics solution. Your decision is driven by both the nature of your data and the question you're trying to answer.



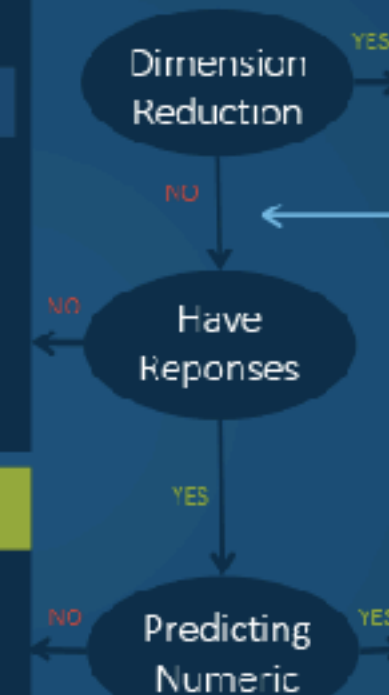
Machine Learning Algorithms Cheat Sheet



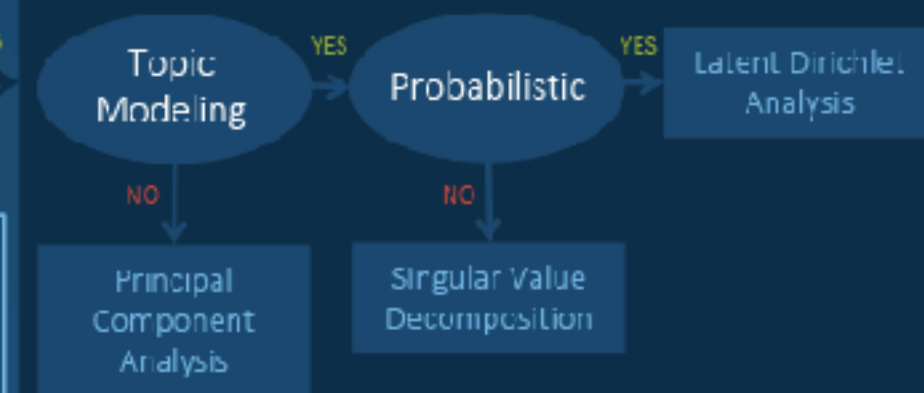
Supervised Learning: Classification



START



Unsupervised Learning: Dimension Reduction



Supervised Learning: Regression



Microsoft Azure Machine Learning: Algorithm Cheat Sheet

