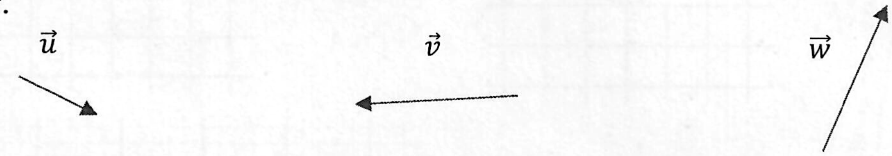
Analysis H 2023-2024 – Hahn, Hlasek, Tantod
Unit 4 – Vectors and Parametrics, Quiz 1
No Calculators

32 / 34 points

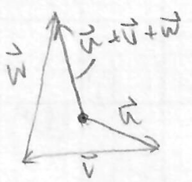
Not vexed by vectors: Michi Tanaka
Period: 6

For all numerical answers on this quiz – simplify as much as possible, but leave your answers in exact form (with square roots if applicable). Rationalize all denominators, if applicable.

1. Given the vectors below, draw each. In your drawings make sure to label each vector so that it's **clear** which is your answer.

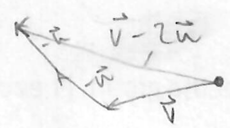


a) Draw $\vec{u}+\vec{v}+\vec{w}$ [2 pts]



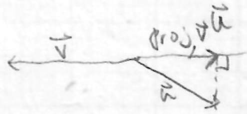
✓

b) Draw $\vec{v}-2\vec{u}$ [2 pts]



✓

c) Draw $proj_{\vec{v}}\vec{u}$ [2 pts]



✓

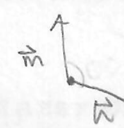
d) Draw vector $\vec{m}$ such that $\vec{m}\cdot\vec{u}$ is negative. [2 pts]

$\vec{m}\cdot\vec{u}$ is negative when

$\vec{m}\cdot\vec{u} = |m||u|\cos\theta$

+ + ← negative

$\frac{\pi}{2} < \theta < \frac{3\pi}{2}$



✓

2. Given: $\vec{a} = \langle 3,2,-1\rangle$ and $\vec{b} = \langle -2,1,5\rangle$,

a) Find $\vec{a}+\vec{b}$ [2 pts]

$\vec{a}+\vec{b} = \langle 1,3,4\rangle$ ✓

b) Find $\vec{a}\cdot\vec{b}$ [2 pts]

$\vec{a}\cdot\vec{b} = -6+2-5$
$= -9$ ✓

c) Find $proj_{\vec{a}}\vec{b}$. [3 pts]

$proj_{\vec{a}}\vec{b} = \frac{a\cdot b}{|a|^2}\vec{a} = \frac{-9}{(\sqrt{9+4+1})^2}\cdot\langle 3,2,-1\rangle$

$= \frac{-9}{14}\langle 3,2,-1\rangle = \boxed{\left\langle -\frac{27}{14}, -\frac{9}{7}, \frac{9}{14}\right\rangle}$ ✓

3. Given the 3D points P(4, 6, -2) and Q(1, -7, 5),

a) Write a vector equation of line PQ. [3 pts]

$\langle x,y,z\rangle = \langle 4,6,-2\rangle + t\langle -3,-13,7\rangle$ ✓

$\begin{array}{r} 42 \\ -26 \\ \hline 16 \end{array}$ $\begin{array}{r} 13 \\ \times 4 \\ \hline 52 \\ -18 \\ \hline 34 \end{array}$

b) Use your vector equation from part (a) to find the point where line PQ intersects the x-z plane. [1 pt]

$y=0$

$\langle x,0,z\rangle = \langle 4,6,-2\rangle + t\langle -3,-13,7\rangle$

$0 = 6-13t$
$-6 = -13t$
$t = \frac{6}{13}$

$x: 4-3\left(\frac{6}{13}\right) = \frac{34}{13}$

$z = -2+7\left(\frac{6}{13}\right) = \frac{16}{13}$

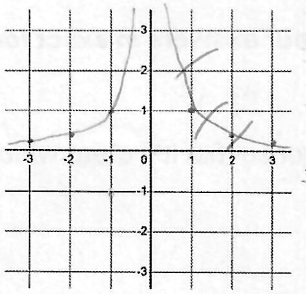
$\boxed{\left(\frac{34}{13}, 0, \frac{16}{13}\right)}$ ✓

-0

4. Graph each set of parametric equations. [2 pts each]

a) $x(t) = -\sqrt{t}\ ;\ y(t) = \frac{1}{t}$ $\quad$ b) $x(t) = \cos(t)\ ;\ y(t) = -\sin(t)$ $\quad$ c) $x(t) = 2\cos(t)\ ;\ y(t) = \sin(t)$

-1

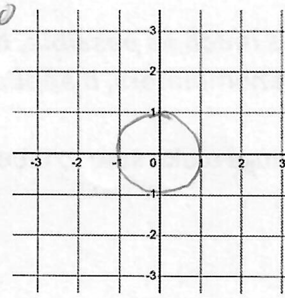


$x = \sqrt{t}$ $t \geq 0$

$x^2 = t$

$y = \frac{1}{x^2}$

x	y
-3	$\frac{1}{9}$
-2	$\frac{1}{4}$
-1	1
0	x
1	1
2	$\frac{1}{4}$
3	$\frac{1}{9}$



$(x = \cos t)^2$

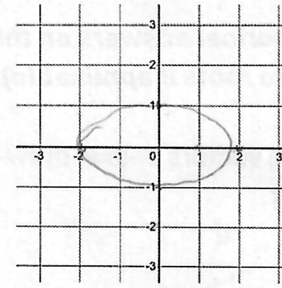
$(y = -\sin t)^2$

$\cos^2\theta + \sin^2\theta = 1$

$x^2 = \cos^2 t$

$y^2 = \sin^2 t$

$x^2 + y^2 = 1$



$(x = 2\cos t)^2$

$(y = \sin t)^2$

$x^2 = 4\cos^2 t$

$\frac{x^2}{4}$

$y^2 = \sin^2 t$

$\frac{x^2}{4} + y^2 = 1$

5. Consider the parametric equations:

$x(t) = 4\sin(\pi t)$

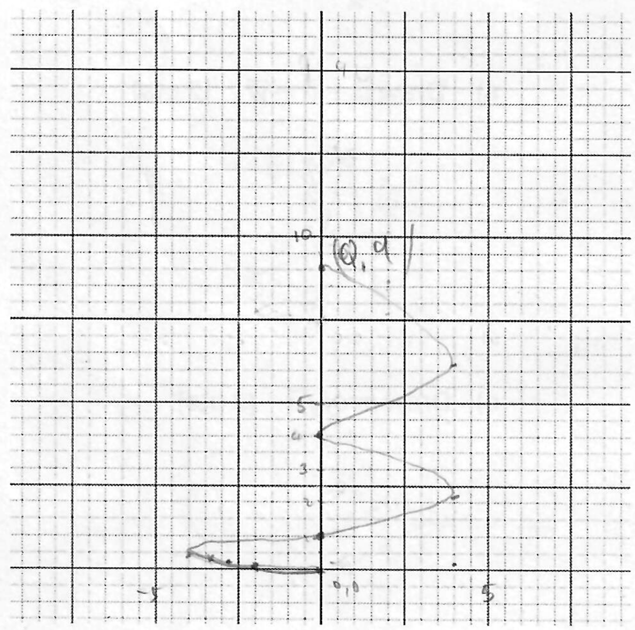
$y(t) = t^2$

$t: [-1,\ 3]$

t	x	y
$\frac{1}{2}$	4	$\frac{1}{4}$
$\frac{3}{2}$	4	$\frac{9}{4}$
$\frac{5}{2}$	4	$\frac{25}{4}$

t	x	y
-1	0	1
0	0	0
1	0	1
2	0	4
3	0	9

t	x	y
$-\frac{1}{3}$	$-2\sqrt{3}$	$\frac{1}{9}$
$-\frac{1}{2}$	-4	$\frac{1}{4}$
$-\frac{1}{4}$	$-2\sqrt{2}$	$\frac{1}{16}$
$-\frac{1}{6}$	-2	$\frac{1}{36}$



a) State the domain and range of the graph. [1 each]

Domain: $[-4, 4]$ ✓

Range: $[0, 9]$ ✓

b) Graph the equations on the xy axis to the right. **Clearly label numbers on your axes**, and choose an appropriate scale so that you can fit the entire graph into the grid provided. [3 pts]

$-1\frac{1}{2}$

c) Eliminate the parameter to write **x as a function of y.** [2 pts]

$x = 4\sin(\pi \pm \sqrt{y})$ ✓

y	x
0	0
1	
4	
9	

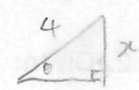
$x = 4\sin(\pi t)$

$\frac{x}{4} = \sin(\pi t)$

$\sin^{-1}\left(\frac{x}{4}\right) = \pi t$

$t = \frac{\sin^{-1}\left(\frac{x}{4}\right)}{\pi}$

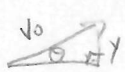
$y = \left(\frac{\sin^{-1}\left(\frac{x}{4}\right)}{\pi}\right)^2$



6. Andy is trying to solve the following math problem:

"Brock Purdy throws the football at an angle of 20 degrees with the horizontal, with a velocity of 30 feet per second. If the ball is released at 5 feet high, what is the height of the football at 0.8 seconds after it is released?"

In Andy's work, he writes: $\boldsymbol{height = -16(0.8)^2 + 30(0.8) + 5}$



Andy's work contains an error. **Circle** his mistake, and then write the number and/or expression that should replace his incorrect element. [2 pts]

$30\sin 20°(0.8)$ ✓

$y(t) = -16(t)^2 + V_0\sin(\theta)t + h_0$

$y(0.8) = -16(0.8)^2 + 30\sin(20°)0.8 + 5$

-2