Analysis H – Deggeller / Hahn 26/30 I'm takin' it to the Gauss! : Hannah Kim

Unit 6: Matrices – Quiz 1 Per: _____

NO Calculator! [26 pts]

$$A = \begin{bmatrix} -3 & 1 \\ 2 & 0 \\ 7 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 7 \\ -1 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 8 & -7 \\ 5 & -3 \end{bmatrix}$$

For questions 1-4, reference the matrices above. [2 points each]

1. 3B - C

$$3\begin{bmatrix} 5 & 7 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 8 & -7 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ -3 & 6 \end{bmatrix} + \begin{bmatrix} -8 & 7 \\ -5 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} 7 & 28 \\ -8 & 9 \end{bmatrix}}$$

2. A^T

$$\begin{bmatrix} -3 & 1 \\ 2 & 0 \\ 7 & -4 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} -3 & 2 & 7 \\ 1 & 0 & -4 \end{bmatrix}}$$

3. AB

$$\begin{bmatrix} -3 & 1 \\ 2 & 0 \\ 7 & -4 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -16 & -19 \\ 10 & 14 \\ 39 & 41 \end{bmatrix}}$$

4. C^{-1}

$$\begin{bmatrix} 8 & -7 \\ 5 & -3 \end{bmatrix} \Rightarrow -2\boxed{\begin{bmatrix} -3 & 7 \\ -5 & 8 \end{bmatrix}}$$

5. Is matrix multiplication associative? (a simple yes or no is sufficient)

yes

6. Is matrix multiplication commutative? (a simple yes or no is sufficient)

no

7. Solve the system of equations using inverse matrices. [4]

$$\begin{cases} 2x - 5y = 18 \\ -3x + 9y = -24 \end{cases}$$

$$\begin{bmatrix} 2 & -5 \\ -3 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ -24 \end{bmatrix} \qquad \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 18 \\ -8 \end{bmatrix}$$

(-2) not A^{-1}

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \end{bmatrix}}$$

-4

8. Solve the system of equations using Gauss-Jordan Elimination. [6]

$$\begin{cases} x - y + z = 4 \\ -x + z = -1 \\ 2x + y = 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ -1 & 0 & 1 & -1 \\ 2 & 1 & 0 & 7 \end{array}\right] \; I+III \; \left[\begin{array}{cccc} 3 & 0 & 1 & 11 \end{array}\right] \; I-II \left[\begin{array}{cccc} 4 & 0 & 0 & 12 \end{array}\right] \; \frac{I}{4}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ -1 & 0 & 1 & -1 \\ 2 & 1 & 0 & 7 \end{array}\right] \begin{array}{l} I \\ III - 2I \\ II + I \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

9. Matrix G is a 2x2 matrix. Find G such that: [4] $G\begin{bmatrix} -3 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 21 \\ 24 & -4 \end{bmatrix}$

$G = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\begin{bmatrix} -3a + 3b = -9 & 5a + b = 21 \\ -3c + 3d = 24 & 5c + d = -4 \end{bmatrix}$

$b = 21 - 5a$

$-3a + 3(21 - 5a) = -9$

$-3a + 63 - 15a = -9$

$-18a = -72$

$a = 4$

$b = 21 - 5(4)$

$b = 21 - 20$

$= 1$

$d = -5c - 4$

$-3c + 3(-5c - 4) = 24$

$-3c + -15c - 12 = 24$

$-18c = 36$

$c = -2$

$d = -5(-2) - 4$

$= 10 - 4$

$= 6$

$a = 4$

$b = 1$

$c = -2$

$d = 6$

$\begin{bmatrix} 4 & 1 \\ -2 & 6 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 21 \\ 24 & -4 \end{bmatrix}$

$G = \begin{bmatrix} 4 & 1 \\ -2 & 6 \end{bmatrix}$