Analysis H – Hahn/Hlasek/Tantod
Unit 6: Matrices – Quiz 1
NO Calculator! [30 pts]

30/30

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~~Keanu Reeves~~ ^ dominates the matrix

Per: 7

1. Suppose that A is 3 x 5 matrix and B is a 5 x 4 matrix and C is a 2 x 3 matrix. Is the product CAB possible? If so state the dimensions of the resultant matrix. If not, state why it is impossible. [2 pts]

CAB: C is 2x3, A is 3x5, B is 5x4

AB : 3x4

CAB has dimensions 2x4

$$A = \begin{bmatrix} -2 & 4 \\ 6 & 5 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 0 & 3 \\ -2 & 1 & 4 \end{bmatrix} \qquad C = \begin{bmatrix} -1 & -3 \\ 1 & 4 \end{bmatrix}$$

For questions 2-5, reference matrices A, B, and C above. If the operation is not possible, write "not possible". [2 points each]

2. 3C – B

$3\begin{bmatrix} -1 & -3 \\ 1 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 3 \\ -2 & 1 & 4 \end{bmatrix} = \boxed{\text{not possible}}$

3. AC^T

$= \begin{bmatrix} -2 & 4 \\ 6 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -3 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -10 & 14 \\ -21 & 26 \\ 1 & -1 \end{bmatrix}}$

4. AB $= \begin{bmatrix} -2 & 4 \\ 6 & 5 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 5 & 0 & 3 \\ -2 & 1 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} -18 & 4 & 10 \\ 20 & 5 & 38 \\ -5 & 0 & -3 \end{bmatrix}}$

5. C^{-1}

$= \frac{1}{\det C} \cdot \begin{bmatrix} 4 & 3 \\ -1 & -1 \end{bmatrix} = \boxed{\begin{bmatrix} -4 & -3 \\ 1 & 1 \end{bmatrix}}$

-0

6. Consider the matrix $A = \begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}$. Find two <u>different</u> matrices B such that AB=BA. **<u>In total between your two answers for B, you may not have more than three entries be "0".</u>** [3]

$$A = \begin{bmatrix} 4 & 0 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{12}\begin{bmatrix} 3 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ 1/6 & 1/3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{or} \quad B = \begin{bmatrix} 1/4 & 0 \\ 1/6 & 1/3 \end{bmatrix}$$

7. Brian is trying to find the inverse of $A = \begin{bmatrix} 2 & -4 \\ -8 & 5 \end{bmatrix}$ using Gauss Jordan elimination, but he made a mistake somewhere. Find and CIRCLE his mistake. Then correctly complete his work to find A^{-1}. [4pts]

Should be 22

$$\begin{bmatrix} 2 & -4 & 1 & 0 \\ -8 & 5 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -4 & 1 & 0 \\ 0 & -11 & 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -44 & 11 & 0 \\ 0 & 44 & -16 & -4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2 & 0 & -5 & -4 \\ 0 & 44 & -16 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -5/2 & -4/2 \\ 0 & 1 & -4/11 & -1/11 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 22 & -44 & 11 & 0 \\ 0 & 44 & -16 & -4 \end{array}\right] \xrightarrow[R_2/44]{R_1+R_2} \begin{bmatrix} 22 & 0 & -5 & -4 \\ 0 & 1 & -4/11 & -1/11 \end{bmatrix} \xrightarrow{R_1/22} \left[\begin{array}{cc|cc} 1 & 0 & -5/22 & -2/11 \\ 0 & 1 & -4/11 & -1/11 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -5/22 & -2/11 \\ -4/11 & -1/11 \end{bmatrix}}$$

8. Solve the system of equations using inverse matrices. Show all your work, correctly labeling each matrix along the way. [*4 pts*]

$$\begin{cases} 2x + 5y = -2 \\ x + 2y = -3 \end{cases}$$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix};\ V = \begin{bmatrix} -2 \\ -3 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \end{bmatrix};\ A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$$

$$A^{-1}(AX) = A^{-1}V$$

$$X = A^{-1}V$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix} \Rightarrow \boxed{\begin{matrix} x = -11 \\ y = 4 \end{matrix}}$$

-0

9. Find the inverse of the following matrix (using any method). Show all your work. $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 0 \end{bmatrix}$. [4 pts]

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} 9 & 12 & -3 \\ -6 & -8 & +4 \\ -3 & 2 & -1 \end{bmatrix}^T = \frac{1}{12+2(-3)}\begin{bmatrix} 9 & -6 & 3 \\ 12 & -8 & 2 \\ -3 & +4 & -1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 3/2 & -1 & 1/2 \\ 2 & -4/3 & 1/3 \\ -1/2 & +2/3 & -1/6 \end{bmatrix}}$$

10. Solve the system of equations using Gauss-Jordan Elimination. Clearly show your steps. [5 pts]

$$\begin{cases} x + y + z = 5 \\ y + 3z = -2 \\ 4x + 5z = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 4 & 0 & 5 & 2 \end{array}\right] \xrightarrow[R_3-4R_1]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & -4 & 1 & -18 \end{array}\right] \xrightarrow[R_3+4R_2]{} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{array}\right] \xrightarrow[R_3/13]{} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow[R_2-3R_3]{R_1+2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \Rightarrow \boxed{\begin{aligned} x &= 3 \\ y &= 4 \\ z &= -2 \end{aligned}}$$

-0