Analysis H 23/24
Hahn/ Hlasek/ Tantod
Unit 1: AtPS, Quiz 1
NO CALCULATORS

Triangle expert named _____
Period_____

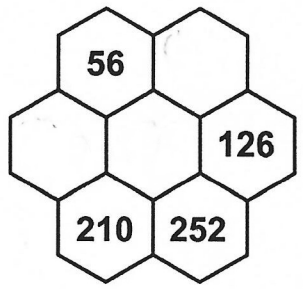
_____
24 points

Odd Number Triangle (Reminder: In the Odd Number Triangle, the row with [3 5] is the 2nd row.)

1. Write "true" or "false" for each statement. (1 pt each)
   a) The median of any row of the odd number triangle is a cube number. _____
   b) The sum of all the terms in the first n rows of the odd number triangle is $\left(\frac{n(n+1)}{2}\right)^2$. _____
   c) The sum of any two consecutive triangular numbers is a square number. _____

Pascal's Triangle

2. The following flower is a portion of Pascal's Triangle. Find all the three missing numbers. (2 pts)



3. Simplify each expression below as a single term or a single binomial coefficient. (2 pts each)
   a) $\binom{20}{20} - \binom{20}{19} + \binom{20}{18} - \binom{20}{17} + \cdots + \binom{20}{2} - \binom{20}{1} + \binom{20}{0} =$

   b) $\binom{k}{0} + \binom{k+1}{1} + \binom{k+2}{2} + \cdots + \binom{n}{n-k} =$

Fibonacci Numbers

4. $F_n = (F_{61})^2 + (F_k)^2$. Solve for n and k. No proof or work shown is needed for this question. (1 pt)

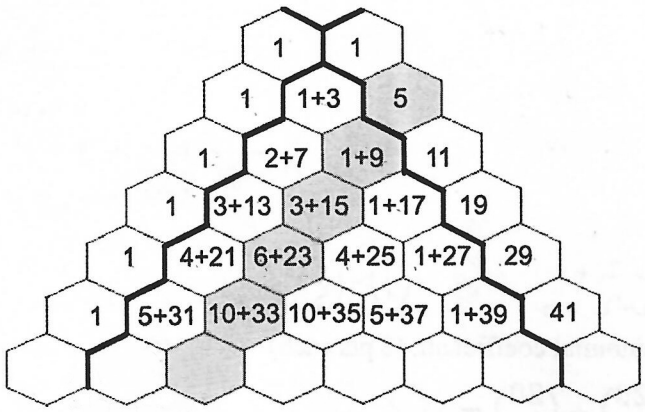
5. Justify the following identity with a clear explanation. (3 pts)
   $2(F_4 + F_7 + F_{10} + F_{13} + F_{16} + F_{19}) = F_2 + F_3 + F_4 + F_5 + \cdots + F_{18} + F_{19}$

Sequences and Series

6. Given that $a_2 = \frac{2}{49}$ and $a_6 = 98$, find the sum of the finite geometric series $\sum_{n=1}^{8} a_n$. Leave your answer as a numerical expression without sigma notation. (3 pts)

The Fun Problem! ☺

For questions 7 and 8, refer to the array of numbers, created by overlapping Pascal's Triangle and the Odd Number Triangle by adding their terms.



7. The highlighted diagonal forms a sequence such that $a_1 = 5$, $a_2 = 1 + 9 = 10$, $a_3 = 3 + 15 = 18$, $a_4 = 6 + 23 = 29$, $a_5 = 10 + 33 = 43$.
   a) Find a_6. (1 pt)

   b) Find a formula for a_n in terms of n. (3 pts)

8. The sum of row 0 is $1 + 1 = 2$. The sum of row 1 is $1 + 4 + 5 = 10$. The sum of row 2 is $1 + 9 + 10 + 11 = 31$.
   a) Find the sum of the 4-th row. (1 pt)

   b) Find a formula for the sum of the n-th row in terms of n. (3 pts)