Analysis H 23/24
Hahn/ Hlasek/ Tantod
Unit 1: AtPS, Quiz 1
NO CALCULATORS

$\frac{19.5}{24 \text{ points}}$ 21.4

Triangle expert named Michi Tanaka
Period 6

Odd Number Triangle (Reminder: In the Odd Number Triangle, the row with [3 5] is the 2nd row.)

1. Write "true" or "false" for each statement. (1 pt each)

   a) The median of any row of the odd number triangle is a cube number. false square

   b) The sum of all the terms in the first n rows of the odd number triangle is $\left(\frac{n(n+1)}{2}\right)^2$. True

   c) The sum of any two consecutive triangular numbers is a square number. true

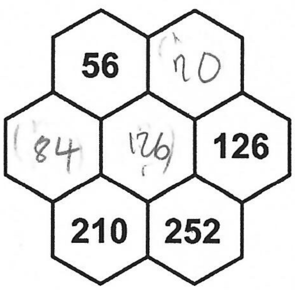
1
3 5
7 9 11
13 15 17 19

$\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$
$= \frac{n^2+n+(n+1)(n+2)}{2}$
$= \frac{2n^2+4n+2}{2}$
$= n^2+2n+1$

$\frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r+1)!} = (n+1)^2$

Pascal's Triangle

2. The following flower is a portion of Pascal's Triangle. Find all the three missing numbers. (2 pts)



252
−126
126
252 − 126
= 126

210
−126
84

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35

$\binom{2}{0}-\binom{2}{1}+\binom{2}{2} = 1-2+1$
$= 2-2 = 0$

$\binom{3}{0}-\binom{3}{1}+\binom{3}{2}-\binom{3}{3} = 1-3+3-1$
$= 0$

3. Simplify each expression below as a single term or a single binomial coefficient. (2 pts each)

   a) $\binom{20}{20}-\binom{20}{19}+\binom{20}{18}-\binom{20}{17}+\cdots+\binom{20}{2}-\binom{20}{1}+\binom{20}{0} =$ 0

   add every other − every other

   $\left(\binom{20}{20}+\binom{20}{18}+\binom{20}{16}+\ldots+\binom{20}{0}\right)-\left(\binom{20}{1}+\binom{20}{19}+\ldots+\binom{20}{1}\right)$
   $= 2^{19}-2^{19} = 0$

   b) $\binom{k}{0}+\binom{k+1}{1}+\binom{k+2}{2}+\cdots+\binom{n}{n-k} =$ $\binom{n+1}{n-k}$

Fibonacci Numbers

−0.5 4. $F_n = (F_{61})^2 + (F_k)^2$. Solve for n and k. No proof or work shown is needed for this question. (1 pt)

$F_n = F_{61}F_{61} + F_kF_k$ $k=60$ ✓ $n = 60+60 = \cancel{120}$ $a+b+\underline{1}$ 121

5. Justify the following identity with a clear explanation. (3 pts)
$$2(F_4+F_7+F_{10}+F_{13}+F_{16}+F_{19}) = F_2+F_3+F_4+F_5+\cdots+F_{18}+F_{19}$$

$F_4+F_4+F_7+F_7+F_{10}+F_{10}+\ldots+F_{19}+F_{19} = F_2+F_3+F_4+F_5+F_6+\ldots+F_{18}+F_{19}$

$F_2+F_3+F_4+F_5+F_6+F_7 \ldots + F_{19}$

break one of the two duplicates into components.

−0.5

Sequences and Series

6. Given that $a_2 = \frac{2}{49}$ and $a_6 = 98$, find the sum of the finite geometric series $\sum_{n=1}^{8} a_n$. Leave your answer as a numerical expression without sigma notation. (3 pts)

$\frac{a_6}{a_2} = \frac{98}{\frac{2}{49}}$ $= \frac{98 \cdot 49}{2}$ $= 49^2$

$\frac{a_6}{a_2} = r^4$

$49^2 = r^4$

$r = 7$

$a_n = a_1 (7)^{n-1}$

$a_2 = \frac{2}{49} = a_1 (7)^1$

$\frac{2}{49} = a_1 7$

$\frac{2}{7^3} = a_1$

$a_8 = \frac{2}{7^3}(7)^7$

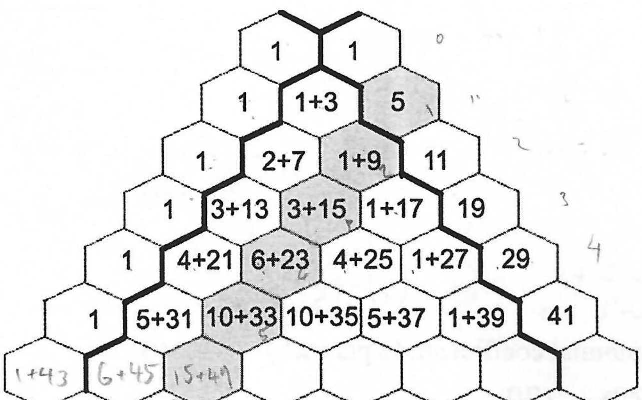
$= 2 \cdot 7^4$

$S_8 = \left(\frac{2}{7^3}\right)\left(\frac{1-r^8}{1-r}\right)$

$= \left(\frac{2}{7^3}\right)\left(\frac{1-7^8}{-6}\right)$

The Fun Problem! ☺

For questions 7 and 8, refer to the array of numbers, created by overlapping Pascal's Triangle and the Odd Number Triangle by adding their terms.



7. The highlighted diagonal forms a sequence such that $a_1 = 5$, $a_2 = 1 + 9 = 10$, $a_3 = 3 + 15 = 18$, $a_4 = 6 + 23 = 29$, $a_5 = 10 + 33 = 43$.

-1 a) Find a_6. (1 pt)

$a_6 = 15 + 4\cancel{7} = 62$ $15 + 45 = 60$

-2 b) Find a formula for a_n in terms of n. (3 pts)

$a_n = \frac{(n+1)(n)}{2} + 2^n + 2n + 1$

$= \frac{n^2 - n}{2} + 2^n \cancel{+ 2n + 1}$

$0, 1, 3, 6, 10, 15, \ldots$ Δ num $\frac{n(n-1)}{2}$ starts at 0

$+$

$5, 9, 15, 23, 33$ $= 2^n + 2n + 1$

$n = 1 \; 2 \; 3 \; 4 \; 5$

8. The sum of row 0 is $1 + 1 = 2$. The sum of row 1 is $1 + 4 + 5 = 10$. The sum of row 2 is $1 + 9 + 10 + 11 = 31$.

-1 a) Find the sum of the 4-th row. (1 pt)

$1 + 4 + 21 + 6 + 23 + 4 + 25 + 1 + 27 + 29 = \cancel{147}$ 141

b) Find a formula for the sum of the n-th row in terms of n. (3 pts)

$S_n = 2^n + (n+1)^3$

$S_1 = 2^1 + 2^3$

$= 2 + 8$

$= 10$

-4