Analysis H – Hahn/Tantod
Unit 6: Matrices – Quiz 1
NO Calculator! [24 pts]

24/24

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~~Keanu Reeves~~ ^ dominates the matrix
Per: 3

$$A = \begin{bmatrix} -2 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

For questions 1-4, reference the matrices above. If the operation is not possible, write "not possible" [2 points each]

1. 3B - A

$$3\begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 9 & 0 \\ 6 & -3 & 15 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 1 \\ 3 & -1 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & 9 & -1 \\ 3 & -2 & 11 \end{bmatrix}}$$

2. $A^T C$

$$A^T = \begin{bmatrix} -2 & 3 \\ 0 & -1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 11 \\ -1 & -3 \\ 6 & 11 \end{bmatrix}}$$

3 · 2   2 2

3. AB

not possible (2 · 3 and 2 · 3) inner numbers don't match

4. C^{-1}

$$\det C = 7 \quad C^{-1} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{bmatrix}}$$

5. Find the value of k for which $\begin{bmatrix} 4 & 2 \\ k & 3 \end{bmatrix}$ has no inverse. [*2pts*]

no inverse if det = 0 $12 - 2k = 0$ $\boxed{k = 6}$

6. Solve the system of equations using inverse matrices. [*4 pts*]

$$\begin{cases} 5x - 2y = 8 \\ 2x - 3y = 1 \end{cases}$$

$$A = \begin{bmatrix} 5 & -2 \\ 2 & -3 \end{bmatrix} \qquad \det A = -11$$

$$A^{-1} = \frac{-1}{11}\begin{bmatrix} -3 & 2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} \frac{3}{11} & \frac{-2}{11} \\ \frac{2}{11} & \frac{-5}{11} \end{bmatrix}$$

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{11} & \frac{-2}{11} \\ \frac{2}{11} & \frac{-5}{11} \end{bmatrix} \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \boxed{(2, 1)}$$

$x = 2$
$y = 1$

-0

7. Solve the system of equations using Gauss-Jordan Elimination. Clearly show your steps. [6]

$$\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array}\right] \begin{array}{c} \longrightarrow \\ -3\,\mathrm{I} \\ -2\,\mathrm{I} \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 2 & -3 & -11 \end{array}\right] \begin{array}{c} \\ \longrightarrow \\ -2\,\mathrm{II} \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & -2 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{c} \\ +2\,\mathrm{III} \longrightarrow \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{c} -\mathrm{III} \\ \longrightarrow \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{c} +2\,\mathrm{II} \\ \longrightarrow \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \boxed{(2, -1, 3)}$$

$x = 2$

$y = -1$

$z = 3$

8. [4 points] Matrix G is a 2x2 matrix. Find G such that: $G\begin{bmatrix} 5 & -9 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -1 & 7 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 5 & -9 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -1 & 7 \end{bmatrix}$$

$5a - 3b = 5$

$-9a + 8b = 4$

$45a - 27b = 45$

$-45a + 40b = 20$

$13b = 65$

$b = 5$

$a = 4$

$5c - 3d = -1$

$-9c + 8d = 7$

$45c - 27d = -9$

$-45c + 40d = 35$

$13d = 26$

$d = 2$

$c = 1$

$$\boxed{G = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}}$$

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