

1. For this problem, consider the 3 matrices below.

$$A = \begin{bmatrix} 3 & 1 & -4 \\ -2 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & -2 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 2 \\ 6 & -4 \end{bmatrix}$$

a) Of the 6 potential products AB, BA, BC, CB, AC and CA only 2 are possible. State which 2, and then do the actual multiplication. [4 each]

#1 BA

#2 CB

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & -2 & 5 \end{bmatrix} \begin{bmatrix} 3 & 1 & -4 \\ -2 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 3 & -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -6 \\ 18 & -1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 13 & -4 & 3 \\ -6 & 8 & -26 \end{bmatrix}$$

c) Can any of the 3 matrices A, B or C be added together? no Explain your answer below. [2]  
they do not have the same dimensions

b) if you were allowed to transpose exactly ONE of the matrices above, state a different product that would be possible, using **proper notation**. No need to actually do the multiplication. [2]

answer:  $B^T C$

2. Represent the following system of equations **using matrices**. Then solve the system using matrices, clearly showing your use of an inverse matrix. [5]

$$\begin{aligned} 7x + 2y &= 3 \\ 6x - 4y &= 14 \end{aligned}$$

$$\begin{bmatrix} 7 & 2 \\ 6 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 14 \end{bmatrix}$$

$\uparrow$   
A

$$\det(A) = -40$$

$$\text{inv}(A)$$

$$= \begin{bmatrix} \frac{1}{10} & \frac{1}{20} \\ -\frac{3}{20} & -\frac{7}{40} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{10} & \frac{1}{20} \\ -\frac{3}{20} & -\frac{7}{40} \end{bmatrix} \begin{bmatrix} 3 \\ 14 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Answer: (1, -2)



3. Insureco has three traveling insurance salespeople: Ally, Bob and Carlos. They sell 3 different types of insurance, Travel, Life, and Home. Travel insurance costs \$100, Life insurance costs \$500 and Health insurance costs \$600. Ally sold 5 Travel, 4 Life, and 4 Health last week. Bob sold 3/3/3 respectively. Carlos sold 6/0/2 respectively. Create two matrices, one of which is a  $3 \times 3$ , that when multiplied together (show order), would yield a  $3 \times 1$  matrix consisting of each salesperson's total sales in \$ last week. [4]

$$\begin{bmatrix} 5 & 4 & 4 \\ 3 & 3 & 3 \\ 6 & 0 & 2 \end{bmatrix} \begin{bmatrix} 100 \\ 500 \\ 600 \end{bmatrix} = \begin{bmatrix} 4900 \\ 3600 \\ 1800 \end{bmatrix}$$

4. Solve the system using Gauss-Jordan Elimination clearly showing and labeling each step. (6 pts)

$$\begin{cases} x + y - z = 6 \\ x + z = 1 \\ -2y - 3z = 4 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 1 & 0 & 1 & 1 \\ 0 & -2 & -3 & 4 \end{array} \right]$$

$$R_1 \Rightarrow R_1 - R_2$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & -2 & 5 \\ 1 & 0 & 1 & 1 \\ 0 & -2 & -3 & 4 \end{array} \right]$$

$$R_1 \Rightarrow 2 \cdot R_1$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & -4 & 10 \\ 1 & 0 & 1 & 1 \\ 0 & -2 & -3 & 4 \end{array} \right]$$

$$R_3 \Rightarrow R_1 + R_3$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & -4 & 10 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & -7 & 14 \end{array} \right]$$

$$R_3 \Rightarrow \frac{R_3}{-7}$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & -4 & 10 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$R_1 \Rightarrow R_1 + 4R_3$$

$$\left[ \begin{array}{ccc|c} 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$R_1 \Rightarrow \frac{R_1}{2}$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$R_2 \Rightarrow R_2 - R_3$$

$$\left[ \begin{array}{ccc|c} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$R_1 \Leftrightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$(x, y, z)$$

$$= \boxed{(3, 1, -2)}$$