[redacted] may trick S. Arty Best

Per: [redacted] Date: 1/17/14

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Analysis H – Deggeller / Hahn
Unit 6 Quest – Matrix Basics
NO CALCULATOR (60 pts)

1. $A=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ If A – 3B = I, find B (4 pts)

$3B = \begin{bmatrix} 0 & 2 & 3 \\ 4 & 4 & 6 \\ 7 & 8 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 0 & \frac{2}{3} & 1 \\ \frac{4}{3} & \frac{4}{3} & 2 \\ \frac{7}{3} & \frac{8}{3} & \frac{8}{3} \end{bmatrix}$

1x4 4x1

2. $A=\begin{bmatrix} -2 & 3 & 1 & 6 \end{bmatrix}$ and $B=\begin{bmatrix} 3 \\ 0 \\ -4 \\ 5 \end{bmatrix}$. Find AB and BA (if they exist). (2 pts each)

AB = $\begin{bmatrix} 20 \end{bmatrix}$ BA = $\begin{bmatrix} -6 & 9 & 3 & 18 \\ 0 & 0 & 0 & 0 \\ 8 & -12 & -4 & -24 \\ -10 & 15 & 5 & 30 \end{bmatrix}$

3. A is a 4 x 8 matrix, and C is a 9 x 12 matrix. ABC = D. What are the dimensions of B and D? (2 pts ea)

4x8
8x9
9x12

Dimensions of B: 8x9 Dimensions of D: 4x12

4. Solve the system using inverse matrices. Show all your work, including the inverse matrix, to receive credit. (5 pts)

$\begin{cases} 4x-3y=8 \\ 3x-2y=7 \end{cases}$

$A^{-1}B$ determinant: 1 inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ = $\frac{1}{\det}\begin{bmatrix} a & -b \\ -c & a \end{bmatrix}$

$\begin{bmatrix} 4 & -3 \\ 3 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \frac{1}{1}\begin{bmatrix} -2 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

x=5, y=4

-0

5. Solve for a, b, c, and d. (2 pts per variable)

$$\begin{bmatrix} 4 & -1 \\ 7b & 0 \end{bmatrix}\begin{bmatrix} a & -9 \\ 0 & 5 \end{bmatrix}=\begin{bmatrix} 18 & (-c^2-16) \\ d & 5 \end{bmatrix}$$

$4a=18$ $\boxed{a=\frac{9}{2}}$

$(4)(-9)+(5)(-1)=-c^2-16$ $\boxed{c=5}$

$7ab=d$ $\boxed{d=-\frac{5}{2}}$

$-63b=5$ $\boxed{b=-\frac{5}{63}}$

6. A matrix is **singular** if it has no inverse. Given matrix $S=\begin{bmatrix} 5 & 2 & 1 \\ 0 & 3 & x \\ -4 & 8 & 2 \end{bmatrix}$ is singular, solve for x. (4 pts)

$5(6-8x)-2(0+4x)+1(0+12)=0$

$30-40x-8x+12=0$

$48x=42$

$\boxed{x=\frac{7}{8}}$

7. Neo is trying to solve a 3x3 system using Gauss-Jordan Elimination. His partial work is shown below.

$$\begin{bmatrix} 0 & 2 & 5 & -4 \\ 1 & 1 & 1 & 6 \\ 2 & 5 & -1 & 27 \end{bmatrix}\rightarrow\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 0 & 3 & -3 & 15 \end{bmatrix}\rightarrow\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 2 & 5 & -4 \\ 0 & 1 & -1 & 5 \\ 0 & 2 & -2 & 10 \end{bmatrix}\rightarrow\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 3 & -14 \\ 0 & 1 & -1 & 5 \end{bmatrix}$$

a) One of his numbers is wrong. Circle his mistake, and change it to a correct number. (3 pts)

It should be 7.

b) Starting from the corrected entry, finish solving the system using GJE, and state the solution. (5)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 7 & -14 \\ 0 & 1 & -1 & 5 \end{array}\right]\rightarrow\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & -1 & 5 \end{array}\right]\rightarrow\left[\begin{array}{ccc|c} 1 & 1 & 0 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]\rightarrow\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

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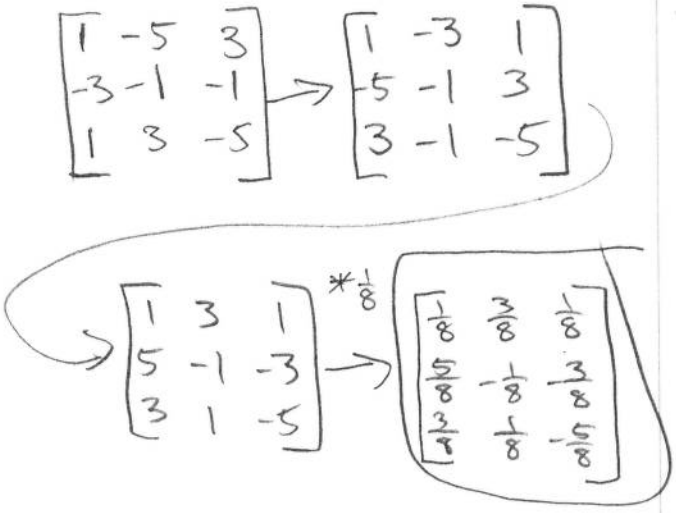
8. Find the inverse of $\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$, using any method. (6)



9. A and B are 4x4 matrices (but are not the same matrix), and both A and B have inverses. Circle "ALWAYS", "SOMETIMES" or "NEVER" for each of the statements. (2 each)

a) $A+B=B+A$ ALWAYS or SOMETIMES or NEVER

b) $AB=BA$ ALWAYS or SOMETIMES or NEVER

c) $A^{-1}AB=B$ ALWAYS or SOMETIMES or NEVER

d) $\det A=0$ ALWAYS or SOMETIMES or NEVER

e) $BI=B^{-1}$ ALWAYS or SOMETIMES or NEVER

f) $AB+B=(A+I)B$ ALWAYS or SOMETIMES or NEVER

10. Find the production matrix for the following input-output and demand matrix. (5 pts)

$$A=\begin{bmatrix} .5 & .4 \\ .25 & .2 \end{bmatrix},\ D=\begin{bmatrix} 2 \\ 4 \end{bmatrix} \qquad (I-A)^{-1}D$$

$$\begin{bmatrix} 0.5 & -0.4 \\ -0.25 & 0.8 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 4 \end{bmatrix}=\begin{bmatrix} 0.8 & 0.4 \\ 0.25 & 0.5 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix}=\begin{bmatrix} 3.2 \\ 2.5 \end{bmatrix}$$

$$\begin{bmatrix} 3.2 \\ 2.5 \end{bmatrix}-\begin{bmatrix} 2 \\ 4 \end{bmatrix}=\begin{bmatrix} 1.2 \\ -1.5 \end{bmatrix}$$ -2

-2