Analysis H – Hahn / Hlasek / Tantod
Unit 5: Growth Quiz

31
35 points

Exponentially Savvy Student: Michi Tanaka
Period: 6

No Calculator Section
Simplify answers fully

1. Evaluate if $\log x = 2$, $\log y = 4$, and $\log z = 7$ [2 pts each]

a) $\log\left(\frac{x^3}{yz^2}\right)$

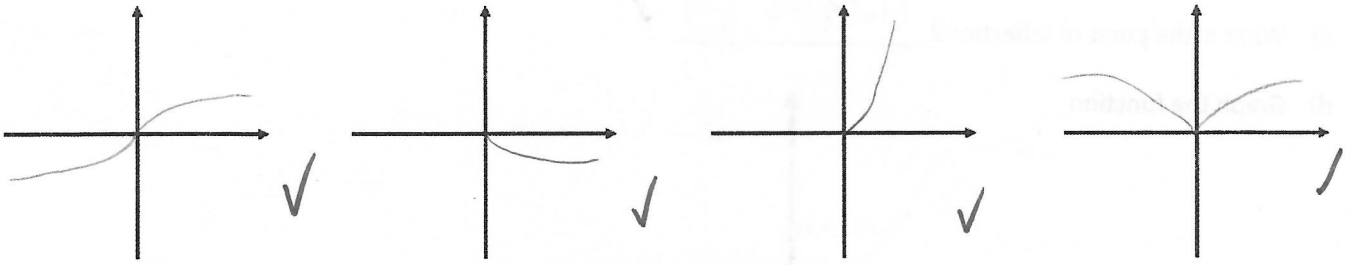
$= \log x^3 - \log yz^2$
$= 3\log x - (\log y + \log z^2)$
$= 3\log x - (\log y + 2\log z)$
$= 3(2) - 4 - 2(7) = \boxed{-12}$ ✓

b) $\log\sqrt[3]{x^9y^6z^2}$

$= \frac{1}{3}\log x^9y^6z^2$
$= \frac{1}{3}(\log x^9 + \log y^6 + \log z^2)$
$= \frac{1}{3}(9\log x + 6\log y + 2\log z)$
$= \frac{1}{3}(18 + 24 + 14) = \frac{1}{3}(56) = \boxed{\frac{56}{3}}$ ✓

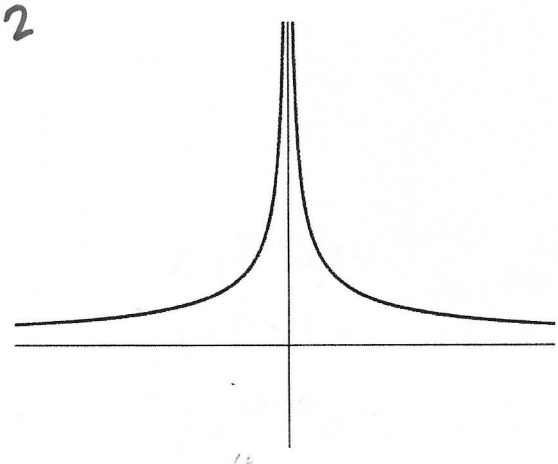
2. Graph each power function (thumbnail sketch is ok- just grading on curvature and quadrants) [2 pts each]

a) $y = x^{\frac{3}{5}}$ b) $y = -2x^{\frac{7}{8}}$ c) $y = x^{\frac{5}{4}}$ d) $y = |x|^{\frac{3}{4}}$



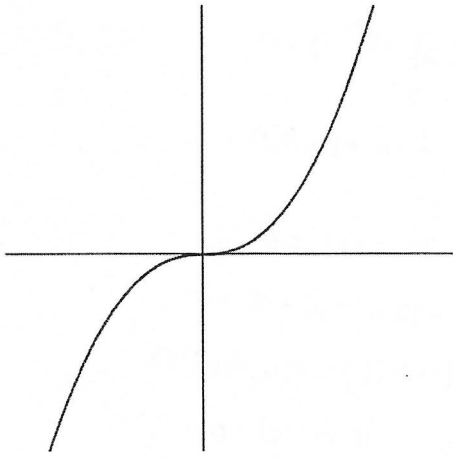
3. Write a possible equation for each power function [2 pts each]

a)



$y = x^{-\frac{3}{4}}$ ✗ $-\frac{4}{3}$

b)



$y = x^3$ ✓

-2

4. Solve for x. Show all your work to receive full credit. [3 pts each]

a) $4^x - 33 \cdot 2^{x-1} + 8 = 0$

$4^x - 33 \cdot 2^{x-1} = -8$

$4^x - 33 \cdot 2 \cdot 2^{x-1} = -8$

$4^x\left(1 - 33 \cdot 2 \cdot \frac{1}{2^x}\right) = -8$

$4^x = \frac{-8}{1-33}$

$4^x = \frac{-8}{-32}$

$4^x = \frac{1}{4}$

$x = -1$

b) $\log_3 x + \log_3(x+1) = \log_3 2 + \log_3(x+3)$

$\log_3(x(x+1)) = \log_3(2(x+3))$

$x(x+1) = 2(x+3)$

$x^2 + x = 2x + 6$

$x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x = -2, 3$

$x = 3$ ✓ $\log_3 -2$ ← not possible

$x^y = z$

$\log_x z = y$

$z > 0$

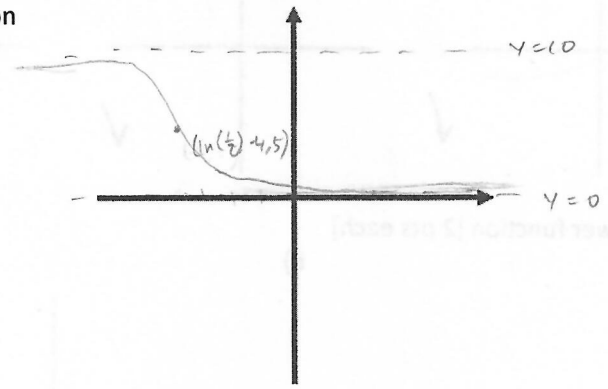
5. Consider the function: $y = \frac{10}{1+2e^{(x+4)}}$ [1 pt per blank, 2 for the graph]

a) What is the y-intercept? $y = \frac{10}{1+2e^4}$ ✓

b) What are the equations of the asymptotes? $y = 10$ ✓ and $y = 0$ ✓

c) What is the point of inflection? $(\ln(\frac{1}{2}) - 4, 5)$ ✓

d) Graph the function



$4^x - 33 \cdot 2^{x-1} + 8 = 0$

$2^{2x} - \frac{33}{2} \cdot 2^x + 8 = 0$

let $u = 2^x$

$u^2 - \frac{33}{2}u + 8 = 0$

$2u^2 - 33u + 16 = 0$

$2u^2 - 32u - u + 16 = 0$

$2u(u-16) - 1(u-16) = 0$

$(2u-1)(u-16) = 0$

$u = \frac{1}{2}, 16$

$2^x = \frac{1}{2}, 16$

$x = -1, 4$

$y = \frac{10}{1+2e^{(4)}}$

$1 + 2e^{(x+4)} = 2$

$2e^{(x+4)} = 1$

$e^{(x+4)} = \frac{1}{2}$

$\ln\frac{1}{2} = x + 4$

$\ln(\frac{1}{2}) - 4 = x$

-2

Calculator section

Invested Student Michi Tanaka
Per 6

Potentially useful formulas:

$$FV = C\left[\frac{(1+i)^n - 1}{i}\right] \qquad PV = C\left[\frac{1-(1+i)^{-n}}{i}\right]$$

1. At age 30, after working for a few years, you get together with your twin brother Jay to talk about saving and investing. You have decided that you should start saving some of the money you are earning each month. You decide to set aside $500 at the end of each month. You find an investment fund that will pay you 6% per year compounded monthly. You try to convince your brother to do the same, but he goes all YOLO and tells you he plans to live La Vida Loca. If you invest $500 per month in an annuity that pays 6% compounded monthly, how much will your investment account be worth in 30 years (when you turn 60)? [3 points]

$$FV = 500\left(\frac{\left(1+\frac{0.06}{12}\right)^{12\cdot 30} - 1}{\frac{0.06}{12}}\right)$$

$$= 500\left(\frac{12\left(\left(1+\frac{0.06}{12}\right)^{360} - 1\right)}{0.06}\right)$$

$$= 502257.52$$

$\$502,257.52$ ✓

2. After settling in Seattle, you decide to buy a house. The size of house you want in a nice neighborhood with good schools costs about $500,000. You have saved up $100,000 for the down payment. You will borrow the other $400,000 by taking out a mortgage from the local bank. The mortgage is a 30 year fixed rate mortgage, where the interest rate is 4% per year. You will make equal monthly payments (this is an annuity). How much is the monthly payment amount? [4 pts]

$$PV = C\left(\frac{1-(1+\frac{r}{n})^{-nt}}{\frac{r}{n}}\right)$$

$$400000 = C\left(\frac{1-(1+\frac{0.04}{12})^{-12\cdot 30}}{\frac{0.04}{12}}\right)$$

$$C = \frac{400000}{\left(\frac{1-(1+\frac{0.04}{12})^{-12\cdot 30}}{\frac{0.04}{12}}\right)}$$

$$= 1909.66$$ ✓

$\$1909.66$ per month

-0