$(1^{35}$ to $2^{10})$ COMPLETED

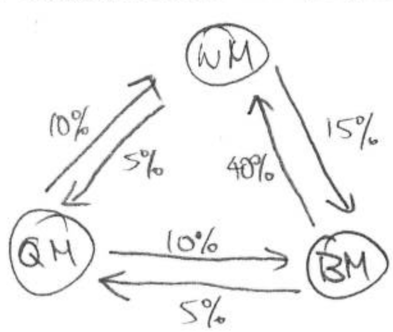
26/36

Analysis Honors – Deggeller / Hahn ____ is off the chaaaaaain!
Markov Chains Quiz [31points] **Calculator OK** Date 3/11/14 Per F

***** You may use your calculator to do operations, but you must show your work to receive credit. Write down any matrix that you input into your calculator, as well as what the calculator returns to you, before you interpret the matrices to arrive at your answers.***

1. Consider the following situation:
Three different Marts serve a community. During the year, Window Mart is expected to retain 80% of its customers, lose 5% of its customers to Q-Mart, and lose 15% of its customers to Bullseye Mart. Q- Mart is expected to retain 80% of its customers and lose 10% to each of Window Mart and Bullseye Mart. Bullseye Mart is expected to retain 55% of its customers, lose 40% to Window Mart and lose 5% to Q-Mart.

a) Construct a transition **diagram** [3]



b) Construct a transition **matrix** [2]

$$\begin{bmatrix} 0.8 & 0.05 & 0.15 \\ 0.1 & 0.8 & 0.1 \\ 0.4 & 0.05 & 0.55 \end{bmatrix}$$

c) Algebraically find the long range prediction (Equilibrium Vector). Show the set up of your system to support the algebraic approach to your solution, along with an explanation of how you used your calculator to solve for the answers (saying "I used the calculator" is not enough). [4]

$$\begin{bmatrix} x & y & z \end{bmatrix}\begin{bmatrix} 0.8 & 0.05 & 0.15 \\ 0.1 & 0.8 & 0.1 \\ 0.4 & 0.05 & 0.55 \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \frac{17}{30} & \frac{1}{5} & \frac{7}{30} \end{bmatrix}}$$

$0.8x+0.1y+0.4z=x \rightarrow -0.2x+0.1y+0.4z=0$
$0.05x+0.8y+0.05z=y \rightarrow 0.05x-0.2y+0.05z=0$
$x+y+z=1 \longrightarrow x+y+z=1$

You put the system of equations as a matrix based on its coefficients:
$$\left[\begin{array}{ccc|c} -0.2 & 0.1 & 0.4 & 0 \\ 0.5 & -0.2 & 0.05 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$$
Use the function rref of the matrix to find what x, y and z equal.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{17}{30} \\ 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & \frac{7}{30} \end{array}\right]$$

-1

2. Find 2 eigenvalues for the following matrix, and one eigenvector for each eigenvalue: $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \lambda\begin{bmatrix} x \\ y \end{bmatrix}$

$2x+3y=\lambda x$ let $x=1$

$x+4y=\lambda y$

$2+3y=\lambda$

$1+4y=\lambda y$

$1+4y=2y+3y^2$

$3y^2-2y-1=0$

$(y-1)(3y+1)=0$

$y=1, -\frac{1}{3}$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -\frac{1}{3} \end{bmatrix}$

5

Eigenvalue 1: ~~1~~ -2 Eigenvector 1: ~~$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$~~

Eigenvalue 2: ~~$-\frac{1}{3}$~~ -2 Eigenvector 2: ~~$\begin{bmatrix} -3 \\ 1 \end{bmatrix}$~~

3. Jake and Emily are playing a game where Emily must roll a 4 three times in a row in order to win.

a) Make a matrix to illustrate the transitions in rolling a 4 or not 4 each time. Label the outside of the matrix with the different states and what they represent.

~~$\begin{array}{c|cc} & 4 & \text{NOT 4} \\ \hline 4 & \frac{1}{6} & \frac{5}{6} \\ \text{NOT4} & \frac{1}{6} & \frac{5}{6} \end{array}$~~

-3

0: NOT 4
1: 4
2: LAST 2 WAS 4
3: WIN

$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & \frac{5}{6} & \frac{1}{6} & 0 & 0 \\ 1 & \frac{5}{6} & 0 & \frac{1}{6} & 0 \\ 2 & \frac{5}{6} & 0 & 0 & \frac{1}{6} \\ 3 & 0 & 0 & 0 & 1 \end{array}$

b) Jake says that he will allow Emily 200 rolls to get her three 4's in a row. Is this a fair number? Why or why not? Use mathematics (and show all your work) to justify your answer.

No, because there is a $\frac{1}{216}$ chance of rolling 3 in a row. 200 is less than 216.

-3

-10

4. Here's a messed up board game for you: The game board has 5 spaces.
You win by reaching either space 2 or 5.
If you're on space 1, you have a 20% chance of moving to space 2 and a 30% chance of moving to space 3.
If you're on space 3, you have a 50% chance of moving to space 1 and a 30% chance of moving to space 4.
If you're on space 4, you have a 10% chance of moving to space 1, 40% to go to space 2, and 40% to go to 5.
To clarify, if you don't move from one space to another, you stay on the space you're on.

a) Write the transition matrix for this situation in Canonical Form. [3]

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0.5 & 0.2 & 0.5 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0.5 & 0 & 0.2 & 0.3 & 0 \\ 4 & 0.1 & 0.4 & 0 & 0.1 & 0.4 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array} \longrightarrow \begin{array}{c|ccc|cc} & 1 & 3 & 4 & 2 & 5 \\ \hline 1 & 0.5 & 0.3 & 0 & 0.2 & 0 \\ 3 & 0.5 & 0.2 & 0.3 & 0 & 0 \\ 4 & 0.1 & 0 & 0.1 & 0.4 & 0.4 \\ \hline 2 & 0 & 0 & 0 & 1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 1 \end{array}$$

b) If you start on space 3, how many times do you expect to land on space 4 in the course of one game? [3]

$$(I-Q)^{-1} = \begin{array}{c} 1 \\ 3 \\ 4 \end{array}\begin{bmatrix} 3.33 & 1.25 & 0.4166 \\ 2.22 & 2.0833 & 0.6944 \\ 0.37037 & 0.1388 & 1.1574 \end{bmatrix}$$

$\boxed{0.6944 \text{ times}}$

c) If you start on space 4, what is the probability that you will end on space 5? [3]

$$FR = \begin{bmatrix} 0.833 & 0.1666 \\ 0.722 & 0.2777 \\ 0.537037 & 0.462963 \end{bmatrix}$$

$\boxed{\frac{25}{54}}$

-0