Analysis 2023/24
Hahn/Hlasek/Tantod

22.5
27 pts

${}_iC_u$ to take this quiz: Michi Tanaka

Date 9/13/23 period 6

No calculators. **Leave answers in factorial, exponent, or "choose" form.**

1. The 8 letters of the word REMEMBER are arranged in a line.

   a) Find the number of different arrangements if there are no restrictions. As a hint: one possible arrangement is BEEEMMRR [2 pts]

   $\frac{8!}{3!2!2!}$ ✓

   b) Find the number of different arrangements which start and finish with the letter M. [2 pts]

   M _____ M $\frac{6!}{3!2!}$ ✓

2. A. committee of 6 animals is to be chosen from 8 different frogs and 5 different bunnies. In how many ways can the committee be selected if...

   a) ...there must be more frogs than bunnies on the committee? (all frogs and no bunnies is ok) [2 pts]

   at least 4 frogs, no more than 2 bunnies

   $P(A \cup B) = P(A) + P(B)$

   $\binom{8}{4}\binom{5}{2} + \binom{8}{5}\binom{5}{1} + \binom{8}{6}\binom{5}{0}$ ✓

   b) ...the committee consists of 3 frogs and 3 bunnies, but two particular bunnies refuse to be on the committee together? [2 pts]

   $\binom{8}{3}\cdot\binom{5}{3} - 3$ ~~$\binom{5}{2}\cdot\binom{3}{1}$~~ $\binom{5}{3} - \binom{5}{2}\binom{3}{1}$ $\binom{8}{3} \times \left(\binom{5}{3} - \binom{3}{1}\right)$ ✓

   refuse ↑ others ↑ bunny combo

3. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated. [2 pts]

   $\frac{6}{1} \cdot \frac{8}{8} \cdot \frac{7}{7} \cdot \frac{5}{2}$ $8 \cdot 7 \cdot 2 = 112$ ✓

4. If ${}_nP_r = 840$ and ${}_nC_r = 35$, then solve for r (your answer is a single number). [3 pts]

   $\frac{n!}{(n-r)!} = 840$ $\frac{n!}{r!(n-r)!} = 35$

   $\frac{840}{r!} = 35$ $r! = \frac{840}{35}$ $r! = 24$ $r = 4$ ✓

   ~~5~~ ~~840~~ 168 $7\overline{)168}$ 24

5. My son's baseball team plays 11 games in the regular season. How many ways are there for the team to win 6 games, lose 3 and tie 2? (they only care about final standings, and not who they beat) [2pts]

   WWWWWW LLL TT

   $\frac{11!}{6!3!2!}$ ways ✓

-0

6. Tomorrow, 3 fathers are taking their 3 daughters (each father has 1 daughter) to the movies.

a) They want to sit in a row of 6 seats. If each father is sitting next to his own daughter, in how many ways can they be seated? [2pts]

-1

FFF DDD

F,DF,DF,D ×2

$3! \cdot 2^3$

b) They still want to sit in a row of 6 seats. How many ways can they be seated if all the daughters sit together? [2pts]

FFF DDD FDDDFF DDDFFF ...

~~4!·3~~

$4! \cdot 3!$ ✓

c) Now they want to sit in a row having 8 seats. With no other restrictions, in how many ways can they all be seated? [2pts]

-1

empty vs people

~~n!~~ $8!$

7. In the expansion of $(a - 3b)^{16}$, the sum of the 9th and 10th term is zero. Find the value of $\frac{a}{b}$.

-0.5 (your answer should be a number) [3 pts]

$\binom{16}{8}a^8(-3b)^8 + \binom{16}{9}a^7(-3b)^9 = 0$

$\binom{16}{8}a^8 3^8 b^8 - \binom{16}{9}a^7 3^9 b^9 = 0$

$\binom{16}{8}a^8 3^8 b^8 = \binom{16}{9}a^7 3^9 b^9$

$\frac{9}{8}a = 3b \quad \frac{9a}{8b} = 3$

$\frac{24}{9} = \frac{a}{b}$ / $\frac{24}{9} = \frac{a}{8}$ simplify $\frac{8}{3}$

$\frac{16!}{8!(8!)} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= 13 \cdot 10 \cdot 9$

$= 1170$

$\frac{\frac{16!}{8!(8!)}}{\frac{16!}{9!(7!)}} = \frac{16!}{8!8!} \cdot \frac{9!7!}{16!} = \frac{9}{8}$

8. If all permutations of the letters of the word MANGA are arranged in the order as in a dictionary. What is the 49th word? [3pts]

-2

A _ _ _ _ 24

G _ _ _ 18

M _ _ _ 18

N

A < G M N

G < M N

M < G N

N < G M

6

49 = 24 + 25

= 24 + 18 + 7

~~MAANG~~

MGAAN

M < A G N A

A < G N A

G < N A

N < A

1

$\binom{8}{3}$ $\binom{5}{3}$

4 frogs $\binom{8}{4}\binom{5}{2}$ +

5 frogs $\binom{8}{5}\binom{5}{1}$ +

6 frogs $\binom{8}{6}\binom{5}{0}$ +

-4.5