Analysis H – Hahn / Hlasek / Tantod
AtPS Quiz 2, 2023-24
NO CALCULATORS

21.5
25 points

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Period: 6

1. Simplify each expression to a single Fibonacci number. Show your work to receive full credit. [3 pts each]

a) $2(F_1 + F_4 + F_7 + F_{10} + \cdots + F_{334}) =$

$= F_1 + F_1 + F_4 + F_4 + F_7 + F_7 + \ldots + F_{334} + F_{334}$

$= F_3 + F_4 + F_4 + F_7 + F_7 + \ldots + F_{334} + F_{334}$

F_5 F_6 F_8 F_9 ...

$F_{335} + F_{334}$

$= \boxed{F_{336}}$ ✓

$2F_1 = F_3$

0, 1, 1, 2, 3, 5, 8, 13

b) $F_{17} + 4F_{18} + 6F_{19} + 4F_{20} + F_{21} =$

$3F_{18} + 1F_{19}$ $\quad 2F_{19} + 4F_{21}$

$= 3F_{18} + 3F_{19} + 5F_{21}$

$= 3F_{20} + 5F_{21}$

$= 2F_{21} + 3F_{22}$

$= F_{22} + 2F_{23}$

$= F_{23} + F_{24}$

$= \boxed{F_{25}}$ ✓

-0.5 2. Evaluate ("evaluate" means "give the value of". Your answer should be a single number). Show the work that you used to arrive at your answer. [3 pts]

$r<1: \left(\frac{1}{2}\right)^\infty \to 0$

$$\sum_{5}^{\infty} 1024\left(\frac{1}{2}\right)^n = \frac{a_1(1-r^\infty)}{1-r} = \frac{1024\left(\frac{1}{2^5}\right)\left(1-\left(\frac{1}{2}\right)^\infty\right)}{1-\frac{1}{2}} = \frac{2^5(1)}{\frac{1}{2}} = 2^6 = 64$$

-1 3. Write this in Sigma Notation: [3 pts] $\quad 72 + 66 + 60 + 54 + \ldots - 594 - 600$

$$\sum_{n=1}^{113} (72-6(n-1))$$

$d=-6 \quad a_1 = 72 \quad a_n = 72 - 6(n-1)$

$-600 = 72 - 6(n-1)$

$-672 = -6(n-1)$

$72 - 6(88) = 6\cdot 12 - 6(88)$

$= -6 \cdot 100$

$= -600$

$112 = n-1$

$n = 113$

$600 - 72 = 528$

$528 / 6 = 88$

-2 4. Solve for x in terms of n and k, simplified as much as possible. Leave your answer in factored form (no need to multiply it out), and you can also leave factorials in your answer, if necessary. [2 pts]

$$\frac{(n-1)!}{(k-1)!(n-k)!} \cdot \frac{n!}{(k+1)!(n-k-1)!} = \frac{(n+1)!}{k!(n-k+1)!} \cdot \frac{(n-1)!}{k!(n-k-1)!} \cdot x$$

$\binom{n-1}{k-1} \cdot \binom{n}{k+1} = \binom{n+1}{k}\binom{n-1}{k} \cdot x$

$$x = \frac{\binom{n-1}{k-1}\cdot\binom{n}{k+1}}{\binom{n+1}{k}\binom{n-1}{k}} = \frac{\dfrac{(n-1)!\, n!}{(k-1)!(n-k)!(k+1)!(n-k-1)!}}{\dfrac{(n+1)!\,(n-1)!}{k!(n-k+1)!\,k!(n-k-1)!}}$$

?

$\binom{n+1}{k-1}\binom{n-1}{k}$

$\binom{n}{k-1}\binom{n}{k}\binom{n}{k+1}$

$\binom{n+1}{k}\binom{n+1}{k+1}$

$x = \frac{\binom{n}{k+1}\binom{n+1}{k+1}}{\binom{n+1}{k}}$

-3.5

5. Consider the summation: $S = \frac{4}{5!} + \frac{5}{6!} + \frac{6}{7!} + \cdots + \frac{102}{103!}$

We can use **telescoping** to write S as a compact expression if we replace the numerators like this:

$$S = \frac{5-1}{5!} + \frac{6-1}{6!} + \frac{7-1}{7!} + \cdots + \frac{103-1}{103!}$$

Continue simplifying to write S as a compact expression. [3 pts]

$S = \frac{5}{5!} - \frac{1}{5!} + \frac{6}{6!} - \frac{1}{6!} + \frac{7}{7!} - \frac{1}{7!} + \ldots + \frac{103}{103!} - \frac{1}{103!}$

$= \frac{1}{4!} - \cancel{\frac{1}{5!}} + \cancel{\frac{1}{5!}} - \cancel{\frac{1}{6!}} + \cancel{\frac{1}{6!}} - \cancel{\frac{1}{7!}} + \ldots + \cancel{\frac{1}{102!}} - \frac{1}{103!} = \frac{1}{4!} - \frac{1}{103!}$ ✓ $= \boxed{\frac{103! - 4!}{103! \cdot 4!}}$ ✓

6. Evaluate each (each answer should be a single number). [1 pt each]

a) $\binom{20}{3} = \frac{20!}{3!(20-3)!}$

$= \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1}$

$= 60 \cdot 19$

$= 1140$ ✓

19
60
00
114
1140

b) $\binom{3}{-4} = 0$ ✓

-2 -1 0 1 2 3

0 1

- - - 0 1 1

0 1 2 1

0 1 3 3 1

c) $\binom{-2}{7} = \frac{(-2)(-3)(-4)(-5)\cdots(-8)}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= -8$ ✓

d) $\binom{12}{15} = \frac{12(11)(10)\cdots(-2)}{15!}$

$= 0$ ✓

7. Prove using Mathematical Induction: [4 pts] "$11^n - 6$ is divisible by 5 for all values of n > 0"

Positive integer

base case n=1

$11^1 - 6 = 5$ divisible by 5 ✓

Assume for n=k

$11^k - 6$ = is divisible by 5

Prove for n=k+1

$11^{k+1} - 6 = 11 \cdot 11^k - 6$

$= \underbrace{11^k - 6}_{\text{divisible by 5 (Assumed)}} + \underbrace{10 \cdot 11^k}_{\text{divisible by 5}}$

QED by Mathematical Induction

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