1/22

Math Analysis H – Deggeller, Gleason, Hahn Timothy Hecher is stronger than a determined ant.
Matrices Quiz 1 Period: 6
No Calculators

$\frac{28}{30}$

$$A = \begin{bmatrix} 3 & -2 & 6 \\ -4 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 4 & 1 \\ -2 & 1 & 6 \\ 2 & -1 & 0 \end{bmatrix}$$

1. Given the matrices named above, find each of the following (or write "not possible").

a) 3A

$$3A = \begin{bmatrix} 9 & -6 & 18 \\ -12 & 3 & 6 \end{bmatrix}$$

b) $A^T B$

$$A^T = \begin{bmatrix} 3 & -4 \\ -2 & 1 \\ 6 & 2 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 3 & -4 \\ -2 & 1 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix} = \boxed{\text{not } \cancel{\text{possible}}}$$

2x3 2x2

-2

c) B^2

$$B^2 = \begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (-3)(-3)+4(2) & (-3)4+4(5) \\ 2(-3)+5(2) & 2(4)+5(5) \end{bmatrix}$$

$$= \begin{bmatrix} 9+8 & -12+20 \\ -6+10 & 8+25 \end{bmatrix} = \boxed{\begin{bmatrix} 17 & 8 \\ 4 & 33 \end{bmatrix}}$$

d) Matrix D, such that $B + D = I$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{D = \begin{bmatrix} 4 & -4 \\ -2 & -4 \end{bmatrix}}$$

e) BC

2x2, 3x3

$\boxed{\text{not possible}}$

$$\left[\begin{array}{cc|cc} -3 & 4 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

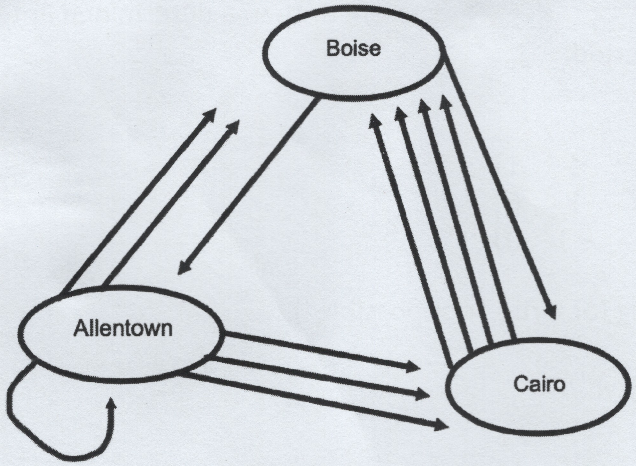
f) B^{-1}

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-23}\begin{bmatrix} 5 & -4 \\ -2 & -3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -5/23 & 4/23 \\ 2/23 & 3/23 \end{bmatrix}}$$

$$\begin{bmatrix} -5/23 & 4/23 \\ 2/23 & 3/23 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} \frac{15+8}{23} & 0 \\ 0 & \frac{8+15}{23} \end{bmatrix}$$

-2



$$\begin{bmatrix} 6 & -2 & -5 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. The diagram above shows train routes between 3 cities. How many ways could you travel from Allentown to Cairo if you wanted to ride exactly 3 trains? Show your work, which should include matrices.

$$T = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 14 & 5 \\ 1 & 6 & 3 \\ 4 & 0 & 4 \end{bmatrix} = T^2$$

$$T^3 = \begin{bmatrix} 3 & 14 & 5 \\ 1 & 6 & 3 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix} = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 17 & 26 & 23 \\ 7 & 14 & 9 \\ 4 & 24 & 12 \end{bmatrix} \end{array}$$

$A \to C = \boxed{23}$ ways

3. Solve the system of equations using either Gauss-Jordan Elimination or Matrix Inverses.

$$\begin{cases} x + 2y + 3z = 1 \\ y - z = 2 \\ x + 2y + 4z = 3 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\boxed{x = -13,\ y = 4,\ z = 2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 & -2 & -5 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 - 4 - 15 \\ -1 + 2 + 3 \\ -1 + 0 + 3 \end{bmatrix} = \begin{bmatrix} -13 \\ 4 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 1 & 2 & 4 \end{bmatrix} \to \underbrace{\begin{bmatrix} 6 & 1 & -1 \\ 2 & 1 & 0 \\ -5 & -1 & 1 \end{bmatrix}}_{\text{Minors}} \to \underbrace{\begin{bmatrix} 6 & -1 & -1 \\ -2 & 1 & 0 \\ -5 & 1 & 1 \end{bmatrix}}_{\text{Cofactor + Minor}} \to \underbrace{\begin{bmatrix} 6 & -2 & -5 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}}_{\text{Transposed}} \to \frac{1}{1}$$

$\det A = 6(1) - 1(2) - 1(3) = 1$