

No Calculation Section:

1. [4 pts] Solve for r in terms of other constants in this equation. $k \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} r & s \\ t & u \end{bmatrix}$

Eigen go for that! _____

Period: _____

2. Does there exist a 2×3 matrix M that satisfies the equation below? If so, find it. If not clearly justify mathematically why it does not exist. [4]

$$\begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 2 \end{bmatrix} M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer y/n _____

Justification:

3. A certain transformation matrix T has eigenvalues $\lambda = -2$ (with eigenvector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$) and $\lambda = 3$ (with eigenvector $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$). What is $T \begin{bmatrix} 6 \\ 10 \end{bmatrix}$? [3 points]

4. Prove algebraically that the following matrix has no real eigenvalues. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ [3]

5. Find the eigenvalues for the matrix below along with the corresponding set of eigenvectors. Clearly display your answers. $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ [5]

6. Find the inverse of matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ using either of the methods we learned in class. [5]

7. In the fine city of Sunnyvale (where the weather always transitions from Sunny to Sunny) there are 3 Starbucks within a mile of downtown, let's call them A,B and C. The following 3×3 matrix M captures their average number of coffees sold by size (Grande, Tall, and Venti) during the morning commute hours. You can call this M

$$A \begin{bmatrix} G & T & V \\ 100 & 180 & 60 \\ 120 & 150 & 100 \\ 30 & 70 & 110 \end{bmatrix} = M$$

If the price of a Grande is \$1.85, a Tall is \$2.10 and a Venti is \$2.45 we can create a price matrix.

- a) Circle the product below that would yield meaningful information: [3]

$$\begin{bmatrix} 1.85 & 2.10 & 2.45 \end{bmatrix} \begin{bmatrix} 100 & 180 & 60 \\ 120 & 150 & 100 \\ 30 & 70 & 110 \end{bmatrix} \quad OR \quad \begin{bmatrix} 100 & 180 & 60 \\ 120 & 150 & 100 \\ 30 & 70 & 110 \end{bmatrix} \begin{bmatrix} 1.85 \\ 2.10 \\ 2.45 \end{bmatrix}$$

- b) Write a sentence explaining what your product (from above) would represent. YOU DO NOT NEED TO ACTUALLY MULTIPLY!! [3]

8. Let M be a random 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = M$

a) True/False: $M(M^T) = M^T(M)$ for any M _____ [3]

b) True/False: If $\det M = 7$, then the equation $M \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ has exactly 1 solution. _____ [3]

c) If $M = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ find M^{700}

_____ [3]

9. Professor Eigentdoit has made an error in his Gauss Jordan Elimination below! Find his error, fix it, and finish the problem correctly (making changes along the way) to find the true solution using G-J. [5]

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{array} \right] \rightarrow R_2 - 2R_1 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 4 & 0 & 5 & 2 \end{array} \right] \rightarrow R_3 - 4R_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -4 & 1 & -18 \end{array} \right] \rightarrow$$

$$R_3 + 4R_2 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{array} \right] \rightarrow \div 13 \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array} \right] \rightarrow R_1 - R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array} \right] \rightarrow R_1 + 2R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array} \right] \rightarrow R_2 - 3R_3 \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

Correct final answer: _____

10. Circle all matrices below which are **regular transition matrices** [5]

a. $\begin{bmatrix} .3 & .4 & .3 \\ .2 & .6 & .2 \\ .1 & .1 & .8 \end{bmatrix}$

b. $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} .4 & -.6 \\ .6 & .4 \end{bmatrix}$

e. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. Circle all matrices below that would generate **absorbing Markov Chains**: [5]

a.
$$\begin{bmatrix} .2 & .3 & .5 \\ 1 & 0 & 0 \\ .5 & .2 & .3 \end{bmatrix}$$

b.
$$\begin{bmatrix} .1 & .3 & .6 \\ 0 & .2 & .8 \\ 0 & 0 & 1 \end{bmatrix}$$

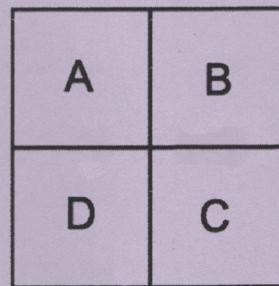
c.
$$\begin{bmatrix} 0 & .5 & .5 & 0 \\ .2 & 0 & 0 & .8 \\ .3 & 0 & 0 & .7 \\ 0 & .2 & .8 & 0 \end{bmatrix}$$

d.
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ .3 & .5 & .2 & 0 \\ .5 & .2 & .3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12. A rat is roaming through a square maze with four rooms. There is one door between rooms A and B, 3 doors between B and C, 5 doors between C and D, and one door between A and D. A step in the Markov process consists of moving from one room to a different room. **The rat must change rooms each move. Each door in a given room is equally likely to be taken.**

- a) Draw the correct number of doors between rooms on the diagram to the right. [2]

- b) Write the transition matrix which represents the probabilities of the rat moving from room to room in the maze: [3]



- c) Roxie the Rat is in room C right now. Given that she takes exactly 1 step, what is the probability she will end up in room D? [2]

- d) Roxie the Rat is in C right now. Given that she takes exactly 2 steps, what is the probability she will end up back in room C? NO need to simplify. [3]

- e) In the long run, Roxie's chances of being in a room is directly proportional to the number of doors in that room. Write the number of doors per room as a 1x4 matrix and then state the probability of the rat ending up in room B in the long run, given the rat started in any room. [3]

Matrix: _____

probability: _____

Calculator Section:

I hope _____ doesn't get Mark'ed Ov.

1. Consider the transformation matrix for a Markov chain:

$$M = \begin{bmatrix} 0 & 1 & 0 \\ .5 & 0 & .5 \\ 0 & 1 & 0 \end{bmatrix}$$

M is a unique transition matrix in that it is neither absorbing or regular.

- a) Suppose the initial distribution vector of the Markov Chain is: [.1 .6 .3]

Explore what would happen over time to the distribution, considering consecutive "big" powers like M^{200} and M^{201} . Write a sentence summarizing your findings: [4]

- b) Surprisingly there **does** exist an equilibrium distribution for this Markov Chain! Set up and solve a system of equations to find it. Show your work and clearly state your answer. [5]

FLIP OVER FOR THE THRILLING LAST PROBLEM!!!!

2. You are playing a game with a spinner which lands red 70% of the time and blue 30% of the time. You win as soon as you spin **three reds in a row**. The transition matrix is given below.

a) Partition the matrix into canonical form, and label the sub matrices Q, R, I and O.

$$\begin{array}{cccc} & 0 & r & rr & rrr \\ \begin{matrix} 0 \\ r \\ rr \\ rrr \end{matrix} & \left[\begin{matrix} .3 & .7 & 0 & 0 \\ .3 & 0 & .7 & 0 \\ .3 & 0 & 0 & .7 \\ 0 & 0 & 0 & 1 \end{matrix} \right] \end{array}$$

b) Compute the Fundamental Matrix F, showing how you used your calculator to arrive at your answer. [4]

c) Explain what entry $F_{3,2}$ means in your fundamental matrix. [3]

d) Suppose the game is structured so that it costs \$1 for every time you hit the spinner. And for a win in the game (3 reds in a row), the prize is \$10. Suppose Dr. Eigendoit decides to start playing the game (at state 0), and he has \$100 in his pocket. He spins until he wins the \$10, and then stops. How much money do you expect he has now?