Analysis H – Hahn/Hlasek/Tantod
Unit 6: Matrices – Quiz 1
NO Calculator!

30
30 pts

Mrs. Hlasek loves juggling tricks, Mr. Hahn loves card tricks, and Mich. Tanaka loves matrix. Period: 6

For questions 1-3, reference matrices A, B, and C below. If the operation is not possible, write "not possible". [2 pts each]

$$A = \begin{bmatrix} 3 & -1 & -3 \\ 2 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 0 \\ 5 & 1 \\ -1 & -2 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 \\ -4 & -3 \end{bmatrix}$$

1. $A^T - 2B$

3x2 3x2 ✓

$$\begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -3 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 5 & 1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -11 & -2 \\ -1 & 8 \end{bmatrix} \checkmark$$

$$-\begin{bmatrix} 4 & 0 \\ 10 & 2 \\ -2 & -4 \end{bmatrix}$$

2. A^2

2x3 • 2x3 not possible ✓

3. BC

3x2 2x2 ✓

$$\begin{bmatrix} 2 & 0 \\ 5 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 1 & 7 \\ 7 & 4 \end{bmatrix} \checkmark$$

4. Suppose that K is 4 x 3 matrix and L is a 4 x 5. Is each calculation below possible? If so, state the dimensions of the resultant matrix. If not, state "not possible". [1 pt each]

a) KL

4x3 4x5

not possible ✓

b) $K^T L$

3x4 4x5

possible

↓

$K^T L$ is 3x5 ✓

c) KL^T

4x3 5x4

not possible ✓

-0

5. A square matrix A is called "*involutory*" if $A^2 = I$, where I is the identity matrix. Find two different 2 x 2 involutory matrices A. Neither solution can be the identity matrix, and your two answers cannot be inverses of each other.

In total between your two answers for A, you may not have more than two entries be "0". [3 pts]

$\cancel{A} \times A = I$ Inverse of itself

$\cancel{A^{-1}}$ A^{-1}

$A = A^{-1} I$

$A = A^{-1}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a = d$

$b = c$

$a^2 + bc = 1$

$ab + bd = 0$

$ac + dc = 0$

$bc + d^2 = 1$

$a^2 + \cancel{bc} = \cancel{bc} + d^2$

$a^2 = d^2$

$ab + bd = ac + dc$

$b\cancel{(a+d)} = c\cancel{(a+d)}$

$b = c$

$db + bd = 0$

$db = -bd$

$1 = -1$

$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}^2$

$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$A = \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$ or $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ ✓

$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ \cdot & \cdot \end{bmatrix}$ not inverse

6. Find the value of p for which $\begin{bmatrix} -6 & p \\ 3 & 2 \end{bmatrix}$ has no inverse. [2 pts]

$-6 \cdot 2 - 3p = 0$

$-12 - 3p = 0$

$-3p = 12$

$\boxed{p = -4}$ ✓

$\begin{bmatrix} -6 & -4 \\ 3 & 2 \end{bmatrix}^{-1} = \frac{1}{\det} \begin{bmatrix} 2 & 4 \\ -3 & -6 \end{bmatrix}$

$-12 - (-12) = 0$ ✓

-0

7. Kirk is trying to derive the inverse of matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ using Gauss Jordan elimination, but he made a mistake somewhere. Find and CIRCLE his mistake. Then correctly complete his work to derive $\cancel{A}^{-1}$. [4 pts]

M^{-1} check w/ normal method.

$$\begin{bmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{bmatrix} \xrightarrow{aR_2} \begin{bmatrix} a & b & 1 & 0 \\ ac & ad & 0 & a \end{bmatrix} \xrightarrow{-cR_1} \begin{bmatrix} -ac & -bc & -c & 0 \\ ac & ad & 0 & a \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} -ac & -bc & -c & 0 \\ 0 & ad-bc & -c & a \end{bmatrix} \xrightarrow{R_1/-c}$$

$$\rightarrow \begin{bmatrix} a & b & 1 & 0 \\ 0 & ad-bc & -c & a \end{bmatrix} \xrightarrow{\frac{R_2}{ad-bc}} \begin{bmatrix} a & b & 1 & 0 \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix} \xrightarrow{-bR_2} \begin{bmatrix} a & b & 1 & 0 \\ 0 & -b & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \end{bmatrix} \xrightarrow{R_1+R_2}$$

$$\rightarrow \begin{bmatrix} a & 0 & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \\ 0 & -b & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{bc}{a(ad-bc)} & \frac{-b}{ad-bc} \\ 0 & -b & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{bc}{a(ad-bc)} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\begin{bmatrix} a & b & 1 & 0 \\ 0 & -b & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} a & 0 & \frac{ad}{ad-bc} & \frac{-ab}{ad-bc} \\ 0 & -b & \frac{bc}{ad-bc} & \frac{-ab}{ad-bc} \end{bmatrix} \xrightarrow[R_2/-b]{R_1/a} \begin{bmatrix} 1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$

$$\frac{ad-bc}{ad-bc} + \frac{bc}{ad-bc}$$

$$\frac{ad}{ad-bc}$$

8. Solve the system of equations using **inverse matrices**. Show all your work, correctly labeling each matrix along the way. [4 pts]

$$\begin{cases} 2x - 5y = -25 \\ -x + 4y = 17 \end{cases}$$

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -25 \\ 17 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix}^{-1}\begin{bmatrix} -25 \\ 17 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 4 \end{bmatrix}^{-1} = \frac{1}{\begin{vmatrix} 2 & -5 \\ -1 & 4 \end{vmatrix}}\begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4/3 & 5/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 4/3 & 5/3 \\ 1/3 & 2/3 \end{bmatrix}\begin{bmatrix} -25 \\ 17 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

$$\boxed{\begin{matrix} x = -5 \\ y = 3 \end{matrix}}$$

$\frac{4}{3}(-25) + \frac{5}{3}(17)$

$\frac{-100}{3} + \frac{85}{3} = \frac{-15}{3} = -5$

$17 \times 5 = 85$; $100 - 85 = 15$

$\frac{1}{3}(-25) + \frac{2}{3}(17)$

$= \frac{-25}{3} + \frac{34}{3}$

$= \frac{9}{3} = 3$

$2(-5) - 5(3) = -10 - 15 = -25$ ✓

$-(-5) + 4(3) = 5 + 12 = 17$ ✓

-0

9. Find the inverse of the following matrix (using any method). Show all your work. $A = \begin{bmatrix} 7 & 2 & 1 \\ 0 & -3 & 1 \\ -3 & 4 & -2 \end{bmatrix}$. [4 pts]

$A^{-1} = \frac{1}{\det A} \operatorname{adj}(A)$

① $\begin{bmatrix} 2 & 3 & -9 \\ -8 & -11 & 34 \\ 5 & 7 & -21 \end{bmatrix}$ ② cofactor $\begin{bmatrix} 2 & -3 & -9 \\ 8 & -11 & -34 \\ 5 & -7 & -21 \end{bmatrix}$ ③ Transpose $\begin{bmatrix} 2 & 8 & 5 \\ -3 & -11 & -7 \\ -9 & -34 & -21 \end{bmatrix} \rightarrow \operatorname{adj} A$

$\det A = 7 \begin{vmatrix} -3 & 1 \\ 4 & -2 \end{vmatrix} - 0 + (-3) \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}$

$= 14 - 15 = -1$

$\boxed{A^{-1} = \begin{bmatrix} -2 & -8 & -5 \\ 3 & 11 & 7 \\ 9 & 34 & 21 \end{bmatrix}}$ ✓

$\begin{array}{r} 34 \\ \times 3 \\ \hline 102 \end{array}$ $\begin{array}{r} 102 \\ -18 \\ \hline 84 \end{array}$

$A^{-1}A = \begin{bmatrix} -2 & -8 & -5 \\ 3 & 11 & 7 \\ 9 & 34 & 21 \end{bmatrix} \begin{bmatrix} 7 & 2 & 1 \\ 0 & -3 & 1 \\ -3 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

10. Write the system as a 3x4 matrix, and then solve the system using **Gauss-Jordan Elimination**. Clearly show your steps. [4 pts]

$$\begin{cases} x + y = 0 \\ -3y - z = 5 \\ -3x + 2y + z = -9 \end{cases}$$

$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & 5 \\ -3 & 2 & 1 & -9 \end{bmatrix} \xrightarrow[3R_1 + R_3]{} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & -1 & 5 \\ 0 & 5 & 1 & -9 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & -4 \\ 0 & 5 & 1 & -9 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 5 & 1 & -9 \end{bmatrix} \xrightarrow[R_3 - 5R_2]{R_1 - R_2} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{\begin{array}{l} x = 2 \\ y = -2 \\ z = 1 \end{array}}$ ✓

$2 + (-2) = 0$ ✓
$-3(-2) - (1) = 5$ ✓
$-3(2) + 2(-2) + (1) = -9$ ✓

-0