

## Report on the outcomes of a Short-Term Scientific Mission<sup>1</sup>

Action number:

Grantee name:

### **Details of the STSM**

Title:  $\omega$ -categories, syntax and models

Start and end date: 14/01/2023 to 20/01/2023

### **Description of the work carried out during the STSM**

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

I was exposed by Eric Finster to the state of research surrounding the formalisation of higher categories in type theory, both with analytical and synthetical approaches. We explored the (unpublished) literature on the perspective of directed type theories.

We reviewed recent proposals for directed type theory:

1. Licata, Daniel R., and Robert Harper. "2-Dimensional Directed Type Theory." Twenty-Seventh Conference on the Mathematical Foundations of Programming Semantics, MFPS 2011, Pittsburgh, PA, USA, May 25-28, 2011, edited by Michael W. Mislove and Joël Ouaknine, vol. 276, Elsevier, 2011, pp. 263–89, <https://doi.org/10.1016/J.ENTCS.2011.09.026>.
2. Nuyts, Andreas. Towards a Directed Homotopy Type Theory Based on 4 Kinds of Variance. 2015. KU Leuven, Master's thesis, <https://anuyts.github.io/files/mathesis.pdf>.
3. Riehl, Emily, and Michael Shulman. A Type Theory for Synthetic Infty-Categories. 2023.
4. North, Paige Randall. "Towards a Directed Homotopy Type Theory." CoRR, vol. abs/1807.10566, 2018, <http://arxiv.org/abs/1807.10566>.

### **Description of the STSM main achievements and planned follow-up activities**

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the

<sup>1</sup> This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

*(max. 500 words)*

We identified the main challenge in a systematic development of a directed type theory, intended for a synthetic study of  $(\omega)$ -category theory, that is, the lack of candidate semantics. However, this does not discourage a syntax-first investigation, as the homotopy semantics were not present in the original intent during the development of Martin-Löf style Type Theory.

Among the reviewed works, [1], [3] and [4] come with a semantic in (infinity)-categories, but lack universes and consequently univalence. Hence they cannot represent for example a type of presheaves over a given type.

Nuyts [2] presented in their master's thesis a (rather comprehensive) speculative proposal for what could be dubbed directed HoTT, but proofs of soundness or semantics were not found for this. A deeper analysis of their efforts is due.

Aside from the focus on directedness, as a more immediate course of action, we identified higher algebra as an approach to study so-called Higher Inductive Types in HoTT, in addition to their established description as homotopy types/type constructors. We suspect that developments in higher algebra could also provide great insight for the study of higher categories.