Report on the outcomes of a Short-Term Scientific Mission[1]

Action number: CA20111

Grantee name: Elena Di Lavore

Details of the STSM

Title: Bidimensional Markov Categories

Start and end date: 03/03/2024 to 16/03/2024

Description of the work carried out during the STSM

Description of the activities carried out during the STSM. Any deviations from the initial working plan shall also be described in this section.

(max. 500 words)

Grantee enters max 500 word summary here.

Partial Markov categories are a minimal semantic universe for probabilistic programs with updating; it has been jointly developed by the host, Mario Román, and me. The goal of this STSM was to extend this previous work by constructing a type theory for partial Markov categories and exploring the addition of a two dimensional structure.

The main activity of this academic visit has been to work, closely with the host, on expanding previous work and preparing it for a journal submission. The additions to the previous results include a type theory and do-notation for partial Markov categories.

We also explored new directions for future research on partial Markov categories. We explored connections with effectuses [Jacobs, 2015] and proposed a 2-dimensional structure similar to that of cartesian bicategories of relations [Carboni and Walters, 1987]. The main example of partial Markov category, the Kleisli category of the finitary subdistribution monad, carries this posetal 2-dimensional structure.

Finally, I have given a talk about partial Markov categories at the seminar OASIS at the computer science department of the University of Oxford.

Description of the STSM main achievements and planned follow-up activities

[1] This report is submitted by the grantee to the Action MC for approval and for claiming payment of the awarded grant. The Grant Awarding Coordinator coordinates the evaluation of this report on behalf of the Action MC and instructs the GH for payment of the Grant.

Description and assessment of whether the STSM achieved its planned goals and expected outcomes, including specific contribution to Action objective and deliverables, or publications resulting from the STSM. Agreed plans for future follow-up collaborations shall also be described in this section.

(max. 500 words)

Grantee enters max 500 word summary here.

One of the goals of this STSM was to define a type theory for partial Markov categories, which we have accomplished. We also formalised a do-notation for these structures which is useful for implementing the theory in functional programming languages. We will include these results in the journal article that we are planning to submit by the end of the year.

The other goal was to define a bicategorical analogue of partial Markov categories, which would serve as syntax for deriving bounds on the behaviour of probabilistic programs. We have proposed a definition of Markov bicategory of relations, in analogy with cartesian bicategories of relations. Conditionals play an important role in this definition as they allow the preorder to respect the categorical structure. An example is the Kleisli category of the finitary subdistribution monad, where the order on morphisms is pointwise. This ordering allows us to express statements like "the probability that the outcome of a probabilistic process is unsafe is less than a given bound". These results will also be included in the journal article that we are preparing. This part of the research still needs to be developed further, but we have made significant progress during the visit. We would like to find more relational-flavoured models of this structure. It is possible that more research developments will come out of these results that might deserve publication on their own.

We can conclude that this STSM achieved its goals and contributed to the Action objectives by developing a type theory for partial Markov categories. We plan to submit an article on partial Markov categories including the results found during this STSM to a relevant computer science journal (e.g. LMCS). We will keep working on this research thread, which might lead to another submission to a relevant theoretical computer science conference (e.g. ETAPS, LiCS).