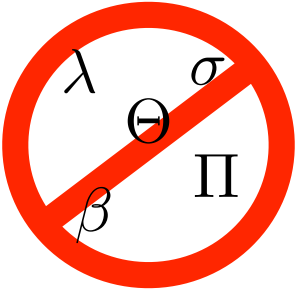
Pragmatics of Efficient Proof Generation

Stephan Schulz




```c
void ClausePushDerivation(Clause_p clause, DerivationCode op,
                          void* arg1, void* arg2)
{
   assert(clause);
   assert(op);

   CLAUSE_ENSURE_DERIVATION(clause);
   assert(DCOpHasCnfArg1(op)||DCOpHasFofArg1(op)||!arg1);
   assert(DCOpHasCnfArg2(op)||DCOpHasFofArg2(op)||!arg2);
   assert(DCOpHasCnfArg1(op)||!DCOpHasCnfArg2(op));

   PStackPushInt(clause->derivation, op);
   if(arg1)
   {
      PStackPushP(clause->derivation, arg1);
      if(arg2)
      {
         PStackPushP(clause->derivation, arg2);
      }
   }
}
```

DHBW Duale Hochschule Baden-Württemberg Stuttgart

Introduction

Refutational Theorem Proving

$$\{A_1, A_2, \ldots, A_n\} \models C$$

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$$\{A_1, A_2, \ldots, A_n\} \models C$$

iff

$$\{A_1, A_2, \ldots, A_n, \neg C\} \text{ is unsatisfiable}$$

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$\mathrm{cnf}(\{A_1, A_2, A_n, \neg C\})$ is unsatisfiable

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$$\mathrm{cnf}(\{A_1, A_2, A_n, \neg C\}) \vdash^{*} \square$$

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Clausification

iff

$$\mathrm{cnf}(\{A_1, A_2, A_n, \neg C\}) \vdash^{*} \Box$$

Refutation/
Saturation

Ideal: Proofs as Sequences of Proof Steps

- A derivation is a list of steps
- Each step carries a clause/formula
- Each step is either...
  - Assumed (e.g. axioms, conjecture)
  - Logically derived from earlier steps
- A proof is a derivation that either...
  - derives the conjecture
  - derives a contradiction from the negated conjecture

Good mental model!

Reality: Proofs as Sequences of Proof Steps

- Initial clauses/formulas
  - Axioms/Conjectures/Hypotheses
  - Justified by assumption
- Derived clauses/formulas
  - Justified by reference to (topologically) preceding steps
  - Defined logical relationship to predecessors
    - Most frequent case: theorem of predecessors
    - Exceptions: Skolemization, negation of conjecture, ...
- (Introduced definitions)
  - Don't affect satisfiability/provability
  - Justified by definition

Example

```
fof(c_0_0, conjecture, (?[X3]:(human(X3)&X3!=john)), file('humen.p', someone_not_john)).
fof(c_0_1, axiom, (?[X3]:(human(X3)&grade(X3)=a)), file('humen.p', someone_got_an_a)).
fof(c_0_2, axiom, (grade(john)=f), file('humen.p', john_failed)).
fof(c_0_3, axiom, (a!=f), file('humen.p', distinct_grades)).
fof(c_0_4, negated_conjecture, (~(?[X3]:(human(X3)&X3!=john))),
    inference(assume_negation,[status(cth)],[c_0_0])).
fof(c_0_5, negated_conjecture, (![X4]:(~human(X4)|X4=john)),
    inference(variable_rename,[status(thm)],[inference(fof_nnf,[status(thm)],[c_0_4])])).
fof(c_0_6, plain, ((human(esk1_0)&grade(esk1_0)=a)),
    inference(skolemize,[status(esa)],[inference(variable_rename,[status(thm)],[c_0_1])])).
cnf(c_0_7,negated_conjecture,(X1=john|~human(X1)),
    inference(split_conjunct,[status(thm)],[c_0_5])).
cnf(c_0_8,plain,(human(esk1_0)),
    inference(split_conjunct,[status(thm)],[c_0_6])).
cnf(c_0_9,plain,(grade(esk1_0)=a),
    inference(split_conjunct,[status(thm)],[c_0_6])).
cnf(c_0_10,negated_conjecture,(esk1_0=john),
    inference(spm,[status(thm)],[c_0_7, c_0_8])).
cnf(c_0_11,plain,(grade(john)=f),
    inference(split_conjunct,[status(thm)],[c_0_2])).
cnf(c_0_12,plain,(a!=f),
    inference(split_conjunct,[status(thm)],[c_0_3])).
cnf(c_0_13,plain,($false),
    inference(sr,[status(thm)],[inference(rw,[status(thm)],
    [inference(rw,[status(thm)],[c_0_9, c_0_10]), c_0_11]), c_0_12]), ['proof']).
```

```
cnf(c_0_13,
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(cnf, fof, tff, ...)

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(axiom,lemma,
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Optional "useful
information" (this step
concludes the proof)

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Inference rule (sr: Simplify-reflect, rw: Rewriting, pm: Paramodulation, ...)

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"Useful information": logical status (formula is theorem of premises)

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Names of the premises

```
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```
c_0_9:  grade(esk1_0)=a
c_0_10: esk1_0=john
c_0_11:grade(john)=f
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Innermost inference: Rewrite c_0_9 with c_0_10

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Intermediate inference: Rewrite the result of the innermost inference with c_0_11

f=a

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c_0_12: a!=f
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Innermost inference: Rewrite c_0_9 with c_0_10

grade(john)=a

Outermost (final) inference: Cut off a literal from the result of the intermediate inference with c_0_12

Intermediate inference: Rewrite the result of the innermost inference with c_0_11

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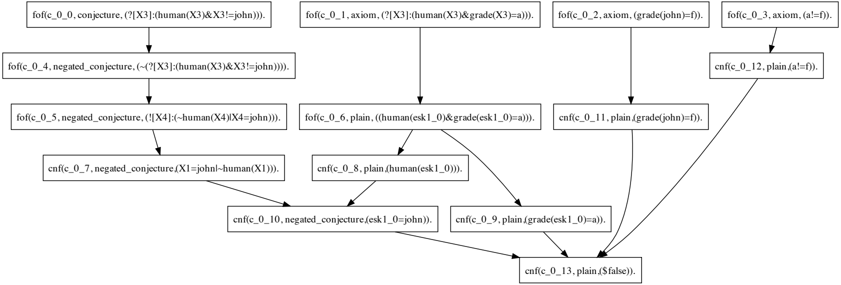
Innermost inference: Rewrite c_0_9 with c_0_10

grade(john)=a

Proofs as Graphs



Proof Generation and Representation

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Clausification

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Refutation

Clausification and Saturation

- Clausification
  - Terminating
  - (Usually) deterministic
  - (Usually) non-destructive
  - Sometimes done by external tool
- Saturation
  - Many degrees of freedom
  - Arbitrary search time
  - Generating inferences
    - Create new clauses
    - Necessary for completeness
  - Simplifying inferences
    - Modify/remove existing clauses
    - Necessary for performance

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- Recording clausification is straightforward
  - . . . but not always done
- Efficiently recording saturation is difficult
  - . . . some settle for inefficient

Deduction vs. Simplification

- Superposition $$\frac{s \simeq t \vee S \quad u \not\simeq v \vee R}{\sigma(u[p \leftarrow t] \not\simeq v \vee S \vee R)}$$

  if $\sigma = mgu(u|_p, s)$, $[\ldots]$

- Rewriting $$\frac{s \simeq t \quad u \not\simeq v \vee R}{s \simeq t \quad u[p \leftarrow \sigma(t)] \not\simeq v \vee R}$$

  if $u|_p = \sigma(s)$ and $\sigma(s) > \sigma(t)$

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- Generating inferences *create* new clauses

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- Generating inferences *create* new clauses
- Simplifiying inferences *replace* or remove clauses

Clauses vs. Clause Objects

Clauses	Clause Objects
Logical abstraction	Real data structures
Purely logical object	Can carry meta-information and history
Immutable (changing a clause creates a completely new clause)	Mutable, changes are frequent
Used in theoretical inference systems	Implemented in actual provers

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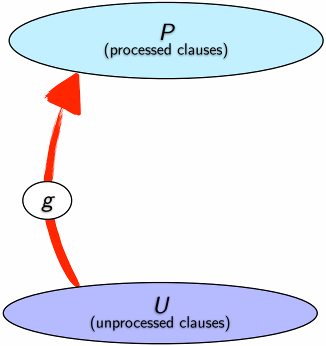
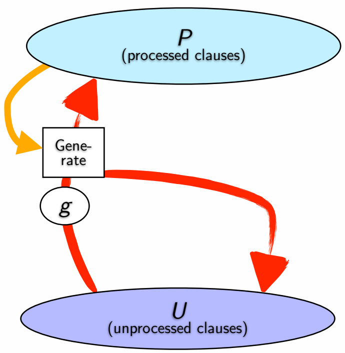
Clause objects represent a sequence of clauses over time (and possibly their history)

The Given-Clause Algorithm



- Aim: Move everything from U to P

The Given-Clause Algorithm



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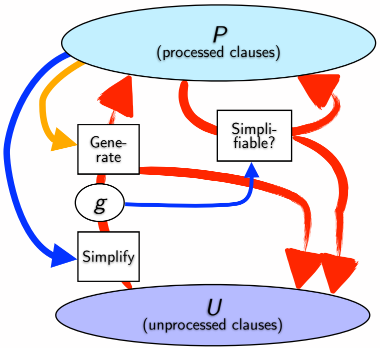

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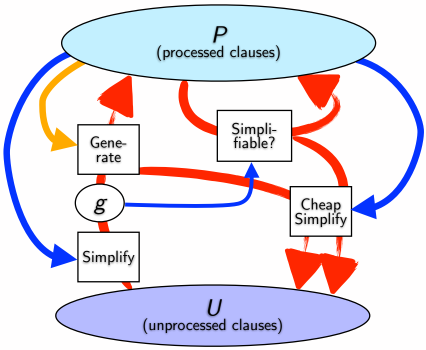

- Aim: Move everything from U to P
- Invariant: All generating inferences with premises from P have been performed
- Invariant: P is interreduced
- Clauses added to U are simplified with respect to P

Typical Clause Object Lifecycle

- Generating inference *creates* a new clause object
  - Usually paramodulation (but may be equality factoring, equality resolution, ...)
- Simplifying inferences *modifies* the clause object
  - Multiple rewrite steps
  - Possibly literal cutting, trivial literal removal, ...
  - This modifies the existing clause object...
  - $\approx$10 modifications per clause on average (varies wildly)
- Deleting inference *removes* clause, but not (always) clause object
  - Subsumption
  - Tautology deletion
  - Typically $\approx$90% of all clauses

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90% of clauses eventually deleted, 9 modified versions
$\Longrightarrow$ 99% of (logical) clauses are not persistent

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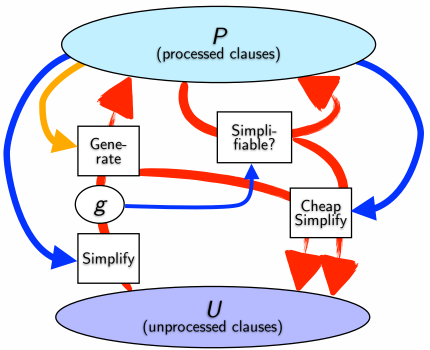
Storing all clauses is too expensive, but
we don't know a-priori which clauses are needed!

Optimized Proof Object Construction



- Observation: Only clauses in *P* are premises!

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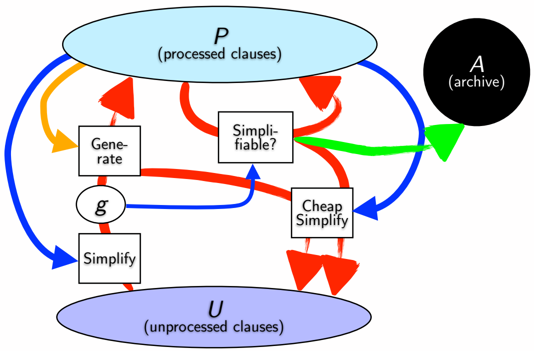

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  - Simplified *P*-clauses are archived
  - Clauses record their history
    - Inference rules
    - *P*-clauses involved

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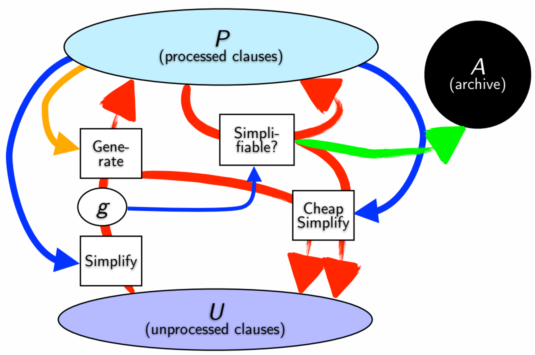

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- Proof extraction
  - Track parent relation
  - Topological sort
  - Print proof

Real Clauses (excerpt)

```
typedef struct clause_cell
{
   long               ident;       /* Hopefully unique ident for
                                      all clauses created during
                                      proof run */
   ...
   Eqn_p              literals;    /* List of literals */
   ...
   FormulaProperties  properties;/* Anything we want to note at
                                      the clause? */
   ...
   ClauseInfo_p       info;        /* Currently about source in
                                      input, NULL for derived
                                      clauses */
   PStack_p           derivation;/* Derivation of the clause for
                                      proof reconstruction. */
   ...
}ClauseCell, *Clause_p;
```

Internal Representation

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              [inference(rw,[status(thm)],
                         [inference(rw,[status(thm)],
                                    [c_0_9, c_0_10]),
                          c_0_11]),
               c_0_12]),
    ['proof']).
```

13	NULL	…	NULL	
ident	literals	properties	info	derivation

Internal Representation

```
cnf(c_0_13,
    plain,
    ($false),
    inference(sr,[status(thm)],
              [inference(rw,[status(thm)],
                         [inference(rw,[status(thm)],
                                    [c_0_9, c_0_10]),
                          c_0_11]),
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    ['proof']).
```

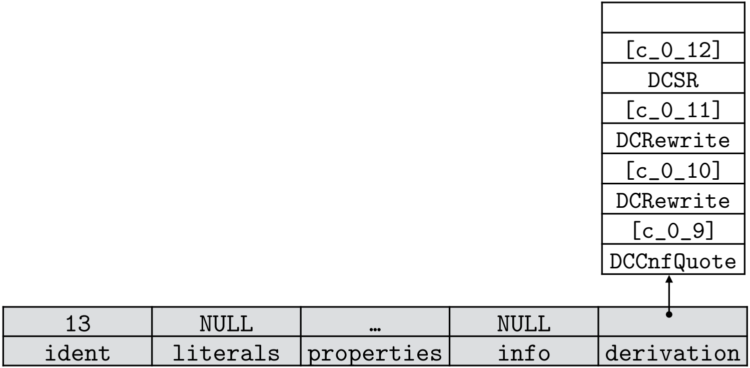

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```

...clause 12	[c_0_12]
...and looses a literal by cutting with...	DCSR
...clause 11...	[c_0_11]
...and further rewritten with...	DCRewrite
...with clause number 10	[c_0_10]
It was modified by rewriting...	DCRewrite
...of clause number 9	[c_0_9]
Original clause is copy...	DCCnfQuote

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Encoding Operations (except)

```
typedef enum
{
    DONop,
    DOQuote,
    DORewrite,
    DOApplyDef,
    DOSR,
    ...
    DONegateConjecture,
    DOFofSimplify,
    DOFNNF,
    DOShiftQuantors,
    DOSkolemize,
    ...
    DOParamod,
    DOSimParamod,
    DOOrderedFactor,
    ...
}OpCode;
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```

```
typedef enum
{
    Arg1Fof = 1<<8,
    Arg1Cnf = 1<<9,
    Arg1Num = 1<<10,
    Arg2Fof = 1<<11,
    ...
}ArgDesc;
```

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```

```
typedef enum
{
    DCNop        = DONop,
    DCCnfQuote=  DOQuote|Arg1Cnf,
    DCFofQuote=  DOQuote|Arg1Fof,
    ...
    DCRewrite    = DORewrite|Arg1Cnf,
    ...
    DCParamod    = DOParamod|Arg1Cnf|Arg2Cnf,
    ...
}
```

Consistent Encodings

- DerivationCodes inspired by machine instructions
  - Operation (OpCode)
  - Arguments
- Supports flexible processing
  - Proof *structure* is independent of exact inferences

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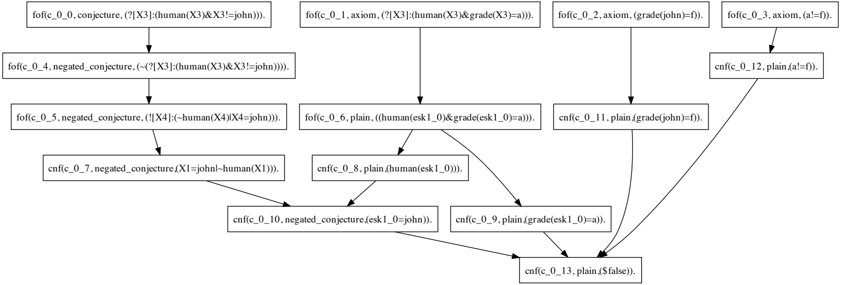
Proofs as Graphs



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Derivation stacks allow a compact and efficient (in data and code) representation of clause history.

Challenges

Unambiguous Inferences

- Complete inference records
  - Add inference positions (more arguments to OpCodes)
  - Add unifiers (if neccessary, e.g. HO)
  - ...
- Complete clausification records
  - Clause simplification as rewriting (?)
  - Mini-scoping as rewriting (?)
  - Step-by-step skolemization

Theoretically managable, but practically difficult – especially retroactively

Proof Expansion

- Calculus level expansion
  - Explicit results of each inference
  - Good for semantic proof checking
  - Good for understanding the structure of the proof
  - Potentially good for machine learning
- Primitive inferences
  - Convert inferences into primitive operations
  - For superposition:
    - Instantiation
    - Lazy conditional term replacement
    - Deleting trivial and duplicated literals
  - Uniform proof format for different provers/calculi
  - Uniform post-processing (proof checking, proof presentation, . . .)

Conclusion

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- Efficient proof generation is non-trivial, but possible
- Clause objects and derivation stacks can represent internal inference sequences compactly
- TPTP v3 is a useful and used standard for external proof representation
- Proof objects are useful for trust building and learning
- Use of proof objects is still in its infancy - we need more tools

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Efficient and flexible proof representation is a key to useful proof output.

Ceterum Censeo...

- Bug reports for E should include:
  - The exact command line leading to the bug
  - All input files needed to reproduce the bug
  - A description of what seems wrong
  - **The output of `eprover --version`**