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Learning to Rank in Automatic Theorem Proving

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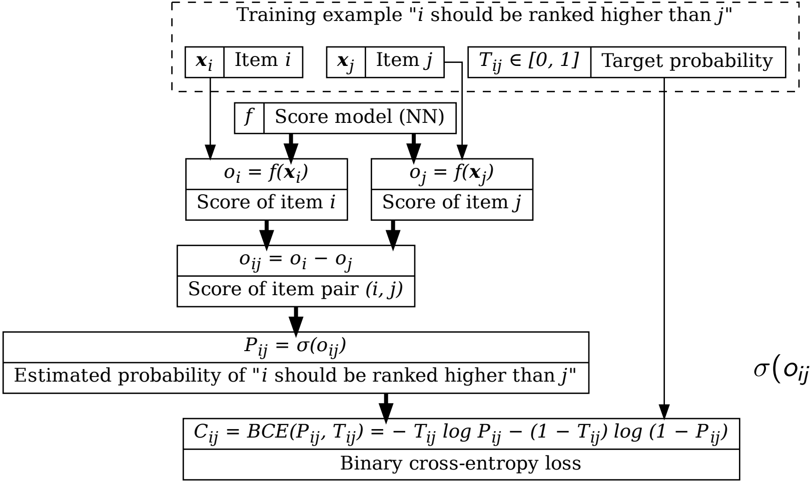
Motivation: Clause selection

- Goal: Train a *clause selection model*
  - Input: Set of clauses
  - Output (one of):
    1. Labeling of the input clauses (positive, negative)
    2. Best of the input clauses
    3. *Ranking of the input clauses*
- Training data (one of):
  1. Clauses with labels (positive, negative)
  2. Set of proof derivations. Each proof derivation is a set of clauses with labels (positive, negative).
  3. *Pairs of clauses C_+, C_- such that C_+ should be selected before C_-*

Learning to rank: Pairwise approach

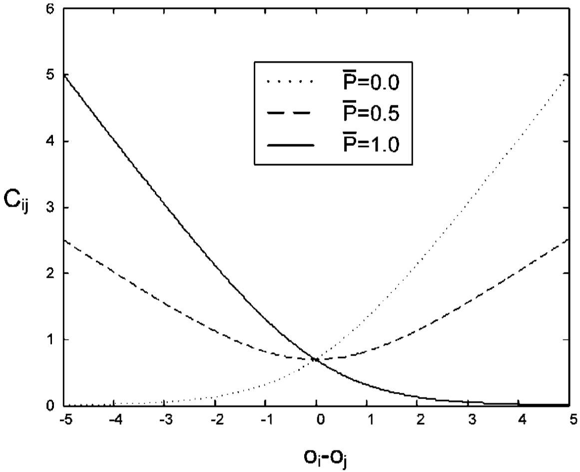
- Goal: Train a *ranking model*
  - Input: Set of items (samples, documents) D
  - Output: Ranking (permutation) over D
- Training example: Pair of items (i, j) such that i is to be ranked higher than j
- Main application domain: Recommender systems

RankNet



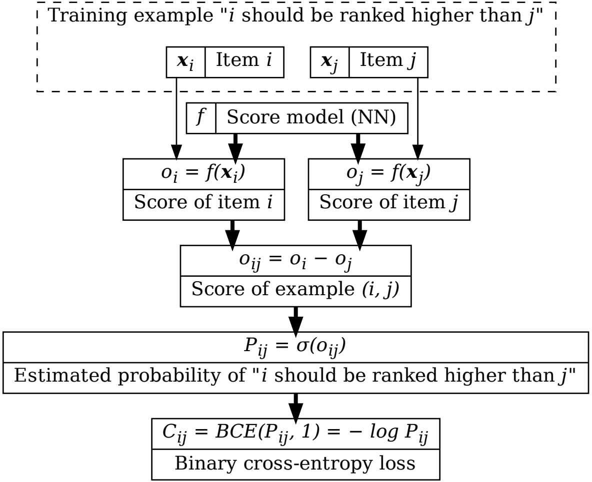
$$\sigma(o_{ij}) = \frac{e^{o_{ij}}}{1 + e^{o_{ij}}}$$

RankNet loss as a function of $o_i - o_j$



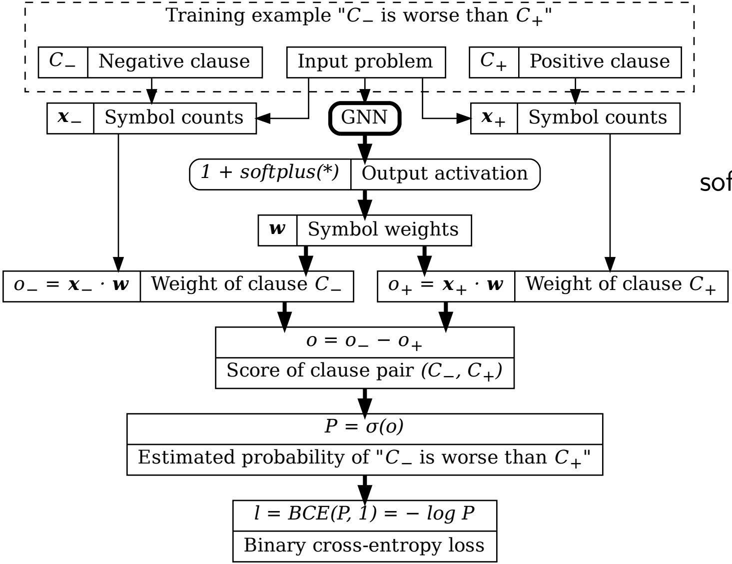
Credit: Burges et al. Learning to rank using gradient descent. ICML 2005.

RankNet with $T_{ij} = 1$



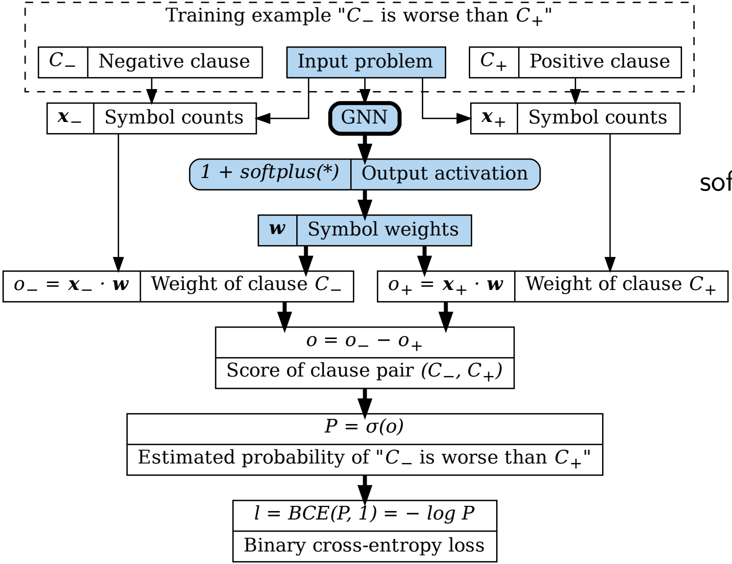
$$C_{ij} = \log(1 + e^{o_j - o_i})$$

Symbol weight recommender



$$\mathrm{softplus}(x) = \log(1 + e^x)$$

Symbol weight recommender



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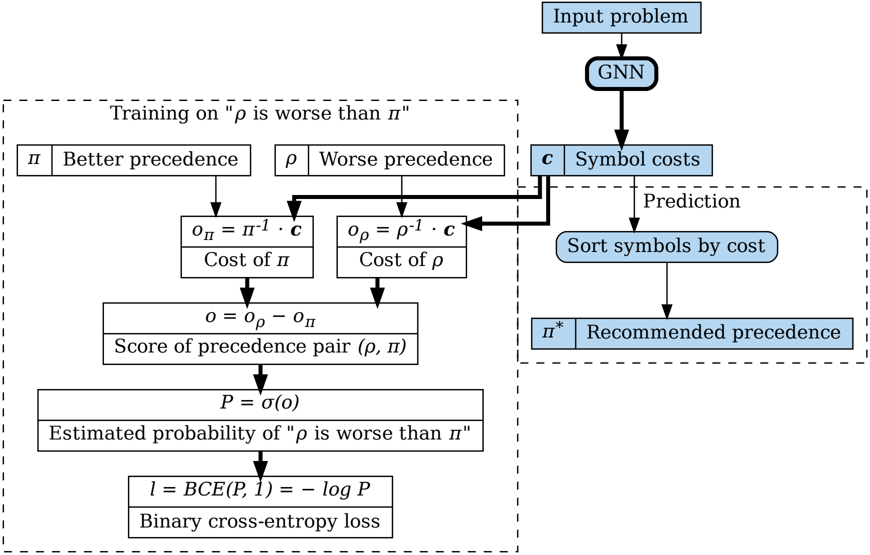
Symbol precedence recommender



Conclusion

Tasks suitable for RankNet:

- Goal: Rank a set of items or get a top-ranked item
- Training data: Ranked pairs of items

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Future work with RankNet:

- Clause selection:
  - Train a full NN clause ranking model to be queried at runtime
  - Generalize symbol counting clause weight to a RNN on term structure
  - Optimize symbol weights on problems with a common signature, use logistic regression instead of gradient descent
- Simplification ordering on terms: Train KBO symbol weight jointly with precedence
- Stress top-ranked items more when training

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Thank you for your attention!

Appendix

References

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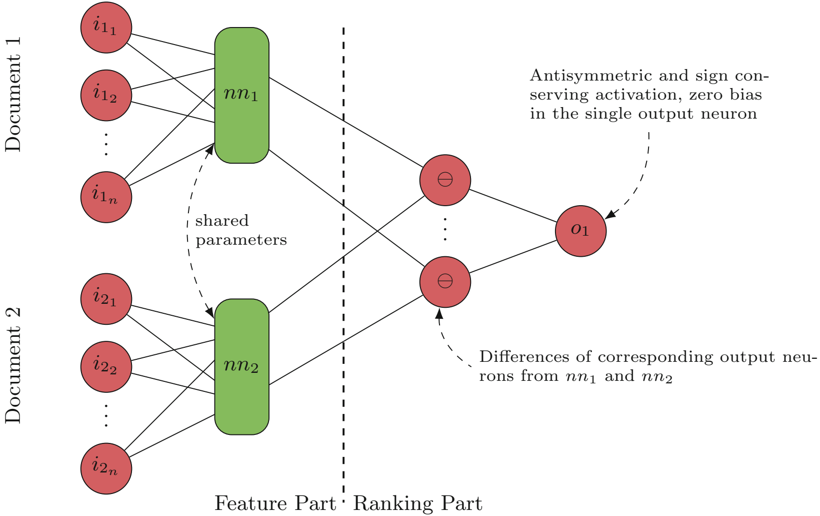
DirectRanker



Credit: Köppel et al. [4]

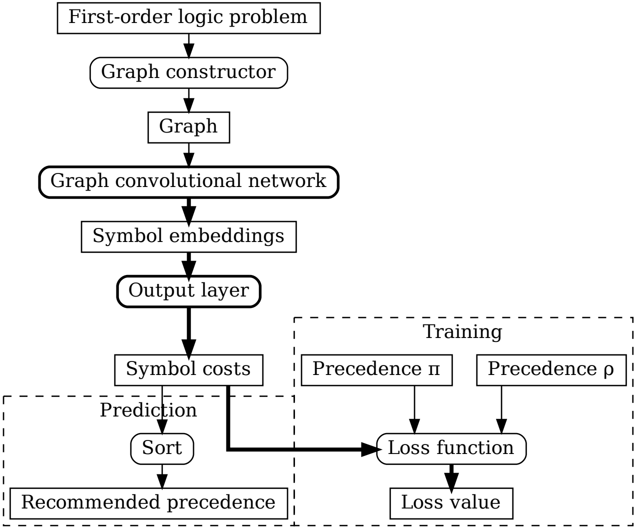
RankNet

Training example: Pair of items i, j and target probability $\bar{P}_{ij}$ of "$i \triangleright j$"
Loss function:

- $o_i = f(i)$ . . . score of item i
- $o_{ij} = o_i - o_j$ . . . score of pair of items i, j
- $P_{ij} = \sigma(o_{ij}) = \frac{e^{o_{ij}}}{1+e^{o_{ij}}}$ . . . predicted probability of "$i \triangleright j$"
- $C_{ij} = -\bar{P}_{ij} \log P_{ij} - (1 - \bar{P}_{ij}) \log(1 - P_{ij}) = -\bar{P}_{ij} o_{ij} + \log(1 + e^{o_{ij}})$ . . . binary cross-entropy loss

Properties: reflexive ($o_{ii} = 0$), antisymmetric ($o_{ij} = -o_{ji}$), transitive ($o_{ij} \geq 0 \wedge o_{jk} \geq 0 \implies o_{ik} \geq 0$)

Symbol precedence recommender: Overview



Symbol weight recommender: Overview