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Wissen lockt. Seit 1456

Non-classical Logics in the TPTP World

Alexander Steen, University of Greifswald
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jww. G. Sutcliffe, T. Scholl, C. Benzmüller, D. Fuenmayor

Goal of this talk

Reasoning in non-classical logics (NCLs)

Relevant for many fields:

- Artificial intelligence
  - Knowledge representation
  - Multi-agent systems
  - ...
- Computer science
  - Software verification
  - Hardware verification
  - Cyber-physical systems
  - ...
- Philosophy
  - Formal ethics
  - Metaphysics
  - ...
- many more

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What about automated reasoning here?

NCL automation exists!

Modal logics formats (a specific NCL)

QMLTP: [Raths and Otten, 2011]

```
qmf(con,conjecture,
(( ! [X] : (#box : ( f(X) ) ) ) => (#box : ( ! [X] : ( f(X) ) )))).
```

LWB: [Heuerding et al., 2005]

```
1: ~((p100 & (~p101)) & (((((p101 –> p100) & box(p102 –> p101)) ...
```

MOLTAP: [van Laarhoven, 2009]

```
K_1 M_1 M_3 K_3 K_2 K_3 a –> M_1 M_3 a
```

TANCS2000: [Massacci and Donini, 2000]

```
inputformula(persat2,axiom,
    (v1 | v2 | (box r1 : v3) | (box r1 : v4))
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An anonymous (potential) user

Heterogeneous landscape for NCL ATPs

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To be fair: QMLTP is the de-facto standard for quantified modal logics

and LWB for propositional modal logic (as far as I know)

Other NCLs: Even more (system-specific) formats

- difficult to evaluate/compare within a specific logic
- syntax transformation needed
  - this might change difficulty of problems

There are standards though:

- ILTP/QMLTP syntax (only for intuitionistic/modal logic)
- DFG syntax (has some capacity for expressing different logics)
- Knowledge Interchange Format (KIF) (only first-order)
- Common Logic (CL) (first- and higher-order, XML based)
- OMDoc (representation of mathematical knowledge)
- ...

All of them: Fixing a particular semantics/logic

Some motivation and a disclaimer

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TPTP is widely accepted in (classical) theorem proving

- Why else would you be here today?

Our goal: 'gracefully' extend to non-classical

- Minimal syntactic changes
- Uniform syntax for all non-classical logics
- Consistency throughout TPTP dialects
- User-friendly syntax
  (easy reading and writing of problems)
- Developer-friendly syntax
  (easy parsing, minimal no. of cases to consider)

Important: We only provide the syntax, not the semantics!

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Typed first-order logic (TXF): Recap on TPTP syntax

```
tff(dog_decl,type, dog: $tType ).
tff(human_decl,type, human: $tType ).
tff(owner_of_decl,type, owner_of: dog > human ).
tff(bit_decl,type, bit: (dog * human * $int) > $o ).
tff(hates_decl,type, hates: (human * human) > $o ).

tff(hate_the_multi_biter_dog,axiom,
   ! [D: dog,H: human,N: $int] :
     ( ( H != owner_of(D) & bit(D,H,N) & $greater(N,1) )
    => hates(H,owner_of(D)) ) ).
```

Higher-order logic (THF): Recap on TPTP syntax

```
thf(dog_decl,type, dog: $tType ).
thf(human_decl,type, human: $tType ).
thf(owner_of_decl,type, owner_of: dog > human ).
thf(owns_decl,type, owns: human > dog > $o ).

thf(owns_defn,definition,
   ( owns = ( ^ [H: human,D: dog] : ( H = ( owner_of @ D ) ) ) ) ).

thf(hate_the_multi_biter_dog,axiom,
   ! [Huddle: dog > $o]: ?[Group: human > $o]:
    ![D: dog]: ? [H: human]:
    ( (Huddle @ D) & (Group @ H) & (owns @ H @ D) ) ).
```

Extend languages with new operator:

- New kind of connective:

```
{ connective_name }
```

- `connective_name` is either TPTP-defined: e.g. `{ $necessary }`, `{ $possible }`, `{ $knows }`, ...
- or `connective_name` is system-defined: e.g. `{ $$future }`, `{ $$obligation }`, `{ $$permission }`, ...

Resulting languages:

- Non-classical extended first-order form (NXF)
  - first-order application style:

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{ connective_name }(a,b)
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Non-Classical TPTP

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Resulting languages:

- Non-classical extended first-order form (NXF)
  - first-order-like application style:

```
{ connective_name } @ (a,b)
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- Non-classical higher-order form (NHF)
  - canonical higher-order application style (curried):

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{ connective_name } @ a @ b
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Example in NXF:

```
tff(possible_dog_bit_owner,axiom,
   {$dia} @ (? [D: dog] : bit(D,owner_of(D),1)) ).

tff(jon_says_necessary_truth,axiom,
   ! [S: $o] : ( says(jon,S) => {$box} @ (S) ) ).
```

Example in NHF:

```
thf(possible_jon_owns_biter,axiom,
   ! [D: dog] :
    ( ( bit @ D @ jon @ 1 )
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thf(jon_says_he_must_feed_odie,axiom,
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Parameterized connectives

Optional parameters: Every NCL connective may be parameterized

- For logics with families of operators, e.g. ...
  - multi-modal logics: $\Box_i$
  - term-modal logics: $[t]\varphi$
  - propositional dynamic logic: $[p \cup q]\varphi$
  - epistemic logic: $K_A\varphi$, $C_{\{A,B,C\}}\phi$, ...

Representation: key-value arguments

```
{ connective_name(param1 := value1, param2 := value2, ...) }
```

- ... where the params are functors,
- and the values are arbitrary terms

Allow hashed (#ed) index value as first argument:

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{ connective_name(#index, param1 := value1, param2 := value2, ...) }
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Parameterized exmages


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tff(alice_knows_its_possible_odie_bit_jon,axiom,
   {$knows(#alice)} @ ({$dia} @ (bit(odie,jon,1)) ).
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tff(jon_says_common_knowledge,axiom,
   ! [S: $o] :
     ( says(jon,S) => {$common($agents:=[alice,bob,claire])} @ (S) ) ).
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thf(alice_knows_jon_owns_a_dog,axiom,
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thf(alice_and_bob_know_jon_might_lie,axiom,
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Object-level syntax is not enough

From which logic does the formula $p \vee q$ come from?

Classical logic? Wrong! I meant intuitionistic logic.

From which logic does the formula $\Box\phi \rightarrow \phi$ come from?

Modal logic **K**? Wrong! I meant **S5** ...

- Formula syntax alone not enough to let ATP systems know which logic we're in
- Introduce: Logic specification

```
tff(formula_name, logic, logic_name == [ properties ] ).
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where ...

- *logic_name* is the name of the logic family,
- *properties* is a comma-separated sequence of key-value pairs

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Case study: Modal logics

Example formulas of modal logic

Mono-modal:

- $\Box \text{raining} \rightarrow \Diamond \text{raining}$
- $\forall P\, (\Diamond \text{rich}(P) \lor \Diamond \neg \text{rich}(P))$
- $\neg \Box (\exists X\, \text{rich}(X))$

Multi-modal:

- $\Box_a \text{raining} \rightarrow \Diamond_b \text{raining}$
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Representation in TPTP

Connectives (mono-modal)

- `{ $box }`
- `{ $dia }`

Connectives (multi-modal)

- `{ $box(#i)}`
- `{ $dia(#i)}`

Examples from above:

```
tff(1, axiom, { $box } @ (raining) => { $dia } @ (raining) ).
tff(2, axiom, ![P]: ( { $dia } @ (rich(P)) | ~({ $dia } @ (rich(P))) ).
tff(3, axiom, ~ { $box } @ ( ? [X]: rich(X) ) ).
```

We also offer user-friendly names: `{ $necessary }`, `{ $possible }`, `{ $knows }`, ...

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Examples from above:

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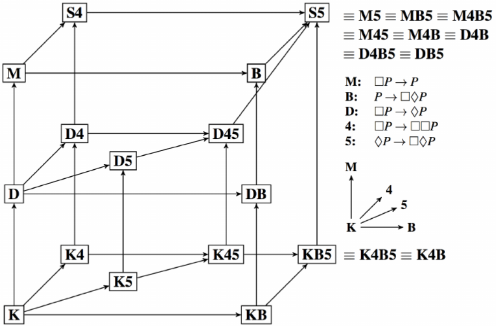
Modal logic: A family of many different logics

- Many parameters exist to create more specific modal logics
- Popular example: Properties of the box operator



Logic dimensions of modal logic

1. **Axiomatization of $\Box_i$**
2. **Quantification**
3. **Rigidity**

Logic dimensions of modal logic

1. **Axiomatization of $\Box_i$**
   - What properties does the box operators have?
   - Depending on the application domain

   Some popular axiom schemes:

Name	**Axiom scheme**	**Condition on R_i**	**Corr. formula**
K	$\Box_i(s \supset t) \supset (\Box_i s \supset \Box_i t)$	—	—
B	$s \supset \Box_i \Diamond_i s$	symmetric	$wR_iv \supset vR_iw$
D	$\Box_i s \supset \Diamond_i s$	serial	$\exists v. wR_iv$
T/M	$\Box_i s \supset s$	reflexive	wR_iw
4	$\Box_i s \supset \Box_i \Box_i s$	transitive	$(wR_iv \land vR_iu) \supset wR_iu$
5	$\Diamond_i s \supset \Box_i \Diamond_i s$	euclidean	$(wR_iv \land wR_iu) \supset vR_iu$
...	...	...	...

2. **Quantification**

3. **Rigidity**

Logic dimensions of modal logic

1. **Axiomatization of $\Box_i$**
   - What properties does the box operators have?
2. **Quantification**
   - What is the meaning of $\forall$?
   - Several popular choices exist
     (1) Varying domains: No restrictions
     (2) Constant domains: $\mathcal{D}_w = \mathcal{D}_v$ for all worlds $w, v \in W$
     (3) Cumulative domains: $\mathcal{D}_w \subseteq \mathcal{D}_v$ whenever $(w, v) \in R^i$
     (4) Decreasing domains: $\mathcal{D}_w \supseteq \mathcal{D}_v$ whenever $(w, v) \in R^i$
3. **Rigidity**

Logic dimensions of modal logic

1. **Axiomatization of $\Box_i$**
   - What properties does the box operators have?
2. **Quantification**
   - What is the meaning of $\forall$?
3. **Rigidity**
   - Do all constants $c \in \Sigma$ denote the same object at every world?
   - Several popular choices exist
     (1) Flexible constants: $\mathcal{I}_w$ may vary for each world w
     (2) Rigid constants: $\mathcal{I}_w(c) = \mathcal{I}_v(c)$
     for all worlds $w, v \in W$ and all $c \in \Sigma$

Logic dimensions of modal logic

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   - What is the meaning of $\forall$?
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   - Do all constants $c \in \Sigma$ denote the same object at every world?

$\longrightarrow$ at least $10 \times 4 \times 2 = 80$ distinct logics

Use logic specification to encode specific logic

`tff(`*formula_name*`, logic, $modal == [ `*properties*` ] ).`

- `$modalities` for the properties of $\Box_i$
- `$domains` for the properties of $\mathcal{D}_w$
- `$designation` for the properties of $\mathcal{I}_w$

Allowed values:

Use logic specification to encode specific logic

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Allowed values:

`$modalities:` `$modal_system_`X or [`$modal_axiom_`Y_1`,...,$modal_axiom_`Y_n]
for ...
$X \in \{K, KB, K4, K5, K45, KB5, \ldots, S4, S5\}$,
$Y_i \in \{K, T, B, D, 4, 5, C\}$.

`$domains:` `$constant`, `$varying`, `$cumulative`, `$decreasing`

`$designation:` `$rigid`, `$flexible`

- May be assigned per modality/type/constant/...

Example

Simple example:

```
tff(simple_spec,logic,
   $modal == [
     $designation == $rigid,
     $domains == [ $constant, some_user_type == $varying ],
     $modalities == $modal_system_S5 ] ).
```

More complex example:

```
tff(complex_spec,logic,
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State of TPTP and tool support

TPTP integration

- Modal logics planned for next TPTP release (v9.X?)
- Tool available for QMLTP -> TPTP translation
- TPTP4X utility to be extended to bridge to other syntax formats
- Syntax prolog parseable
- scala-tptp-parser package available

Automation of modal logics

- Offer semantical embedding to HOL for flexible automation
- Tool available: LET (Logic Embedding Tool)
- Accessible via SystemBeforeTPTP
- Included into Leo-III, accessible via SystemOnTPTP

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Shallow semantical embeddings

- Encode semantics of logic L into HOL
- Translate problem using encoding function $\lceil . \rceil$
- Pass translated problem to HOL systems

$$\begin{tikzcd} & \text{HOL} \arrow[dl] \arrow[dr] & \\ \text{Syntax L} \arrow[rr, dashed] & & \text{Semantics L} \end{tikzcd}$$

Off-the-shelf reasoning for non-classical logics

$$\Psi \models^{L} \varphi \qquad \text{iff} \qquad \{\lceil \psi \rceil \mid \psi \in \Psi\} \cup \mathrm{meta}(L) \models^{HOL} \lceil \varphi \rceil$$

Enabling technology for

- Explorative logical analysis
- Empirical studies
- Automation prototyping

... *fruitful for, e.g., deontic logic context: Logical setting still not settled upon*

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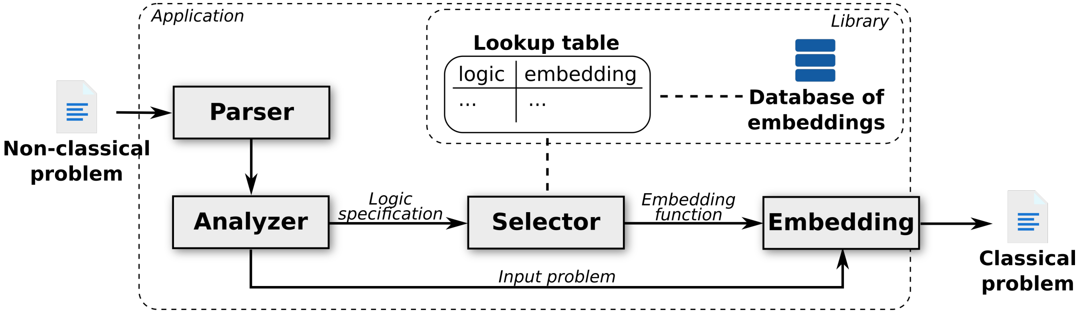
Top-level picture: Logic Embedding Tool (LET)



- Implemented in Scala, available at GitHub `leoprover/logic-embedding`
- Outputs classical TPTP THF (monomorphic or polymorphic)

Possible (exotic?) future

Dynamic epistemic logics (PAL)

- Does $[\varphi!]\varphi$ hold?

```
tff(c1, conjecture, {$box($announce := phi)} @ (phi) ).
```

- Does $[\varphi!]\,(C_{a,b,c}\,\varphi)$ hold?

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tff(c2, conjecture, {$box($announce := phi)} @ ({common($agents := [a,b,c])} @ (phi)) ).
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(Propositional) Dynamic logics (PDL)

- Does $[p \cup (p;q)^*]\varphi$ hold?

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tff(c3, conjecture, {$box($program := $choice(p,$loop($sequence([p,q]))))} @ (phi) ).
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Term modal logics

- Does $\Box_{f(x)}\varphi$ hold?

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Of course, concrete syntax needs to be discussed.

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Limits of the syntax

- There is bound to be a limit on what the extension syntactically express
- ... we did not yet discover it.

Can you provide us with logic representation challenges?

Tools

- *... what tools to we still need?*

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Summary

- Quite generic syntax extension
- Current focus: Modal logic (as proof-of-concept)
- More logics to come (with your help?)

More information at `tptp.org`:

TPTP subprojects:

- **System on TPTP** Services for solving problems and recommending systems.
- **TSTP** The TSTP (Thousands of Solutions from Theorem Provers) Solution Library is a lib automated theorem proving (ATP) systems. In particular, it contains solutions to TPTP pr
- **TMTP** The TMTP (Thousands of Models for Theorem Provers) Model Library is a library of m theorem proving (ATP) systems. In particular, it contains models for TPTP axiomatizatio
- **CASC** The TPTP is used to supply problems for the CADE ATP System Competition.
- **TPTP Proposals** We are working on new features for the TPTP, as explained in the following proposals. appreciated.
  - Extended TFF (TFX), Updated THF
  - Non-classical Logics
  - Answer Extraction
  - The TPTP Process Instruction Language
  - ATP System Building Conventions