МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РФ

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Кафедра: 806 «Вычислительная математика и программирование»

Лабораторная работа № 7 по курсу «Численные методы»

Группа: М8О-407Б-21

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Вариант 7:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2u, \tag{1}$$

$$U(0, y) = cos(y), \tag{2}$$

$$U(\frac{\pi}{2}, y) = 0, \tag{3}$$

$$U(x, 0) = cos(x), \tag{4}$$

$$U(x, \frac{\pi}{2}) = 0 \tag{5}$$

Аналитическое решение:

$$U(x, y) = cos(x) \cdot cos(y) \tag{6}$$

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:
```
def U0y(y: float):
    return np.cos(y)

def Ux0(x: float):
    return np.cos(x)

def Uly(y: float):
    return 0

def Uxl(x: float):
    return 0

def f(x:float, y:float):
    return 0

def Uans(x:float, y:float):
    return np.cos(x)*np.cos(y)

c = -2
startx = 0
starty = 0
finishx = np.pi / 2
finishy = np.pi / 2
kx = 20
ky = 20
hx = (finishx - startx) / (kx - 1)
hy = (finishy - starty) / (ky - 1)
xs = np.linspace(startx, finishx, kx)
ys = np.linspace(starty, finishy, ky)
xgrid, ygrid = np.meshgrid(xs, ys)
zans = np.zeros((kx, ky), dtype=np.float64)
for i, x in enumerate(xs):
    for j, y in enumerate(ys):
        zans[i][j] = Uans(x,y)
```

In [3]:
```
answer_lib = np.zeros((kx, ky), dtype=np.float64)
cur_step = np.zeros((kx, ky), dtype=np.float64)

for i, y in enumerate(ys):
    cur_step[0][i] = U0y(y)
    cur_step[-1][i] = Uly(y)
for i, x in enumerate(xs):
    cur_step[i][0] = Ux0(x)
    cur_step[i][-1] = Uxl(x)

for i in range(1, kx - 1):
    coef = (cur_step[i][-1] - cur_step[i][0]) / (ys[-1] - ys[0])
    for j in range(1, ky - 1):
        cur_step[i][j] = cur_step[i][0] + coef*(ys[j] - ys[0])

next_step = np.array(cur_step, copy=True)
```

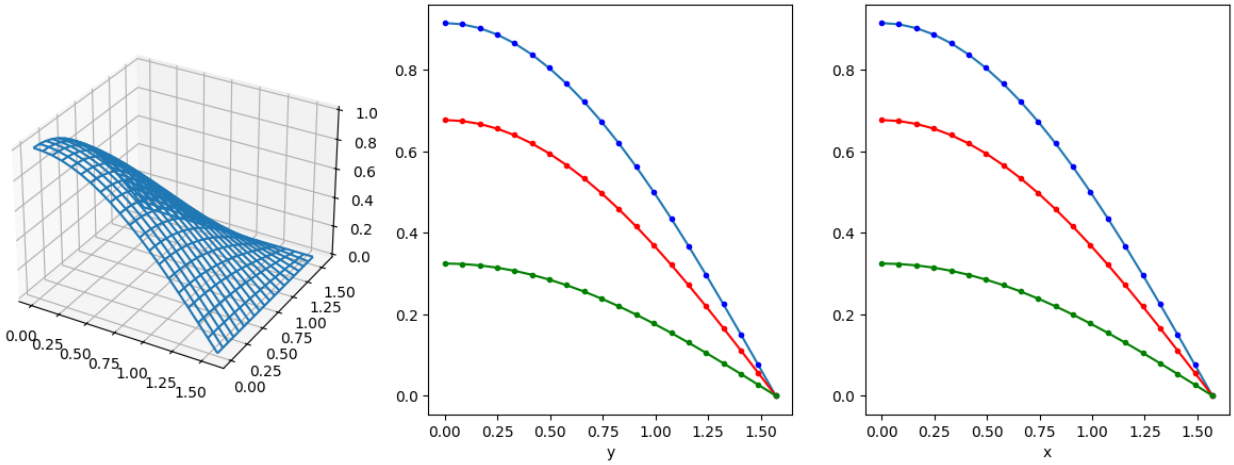
In [4]:
```
def get_error(cur_step, next_step):
    return np.max(np.abs(next_step - cur_step))
```

In [5]:
```
error = 1 * 10**(-5)
count = 0
while True:
    cur_step = np.array(next_step)
    for i in range(1, kx - 1):
        for j in range(1, ky - 1):
            next_step[i][j] = (
                hx**2 * cur_step[i+1][j]
                + hx**2 * cur_step[i-1][j]
                + hy**2 * cur_step[i][j+1]
                + hy**2 * cur_step[i][j-1]
                - f(xs[i], ys[j]) * hx**2 * hy**2
            ) / (2 * (hx**2 + hy**2) + hx**2 * hy**2 * c)
    count += 1
    if get_error(cur_step, next_step) < error:
        break

print("Conut iterations: {iterations}".format(iterations = count))

answer_lib = np.array(next_step)

plt.rcParams['figure.figsize'] = [15, 5]
fig = plt.figure()
ax_3d = fig.add_subplot(1,3,1, projection='3d')
ax_3d.plot_wireframe(xgrid, ygrid, answer_lib.transpose())
axx = fig.add_subplot(1,3,2)
axx.plot(ys, answer_lib[kx // 4])
axx.plot(ys, zans[kx // 4], '.b')
axx.plot(ys, answer_lib[kx // 4 * 2], 'r')
axx.plot(ys, zans[kx // 4 * 2], '.r')
axx.plot(ys, answer_lib[kx // 4 * 3], 'g')
axx.plot(ys, zans[kx // 4 * 3], '.g')
plt.xlabel('y')
axy = fig.add_subplot(1,3,3)
axy.plot(xs, answer_lib[:, ky // 4])
axy.plot(xs, zans[:, ky // 4], '.b')
```

```
axy.plot(xs, answer_lib[:, ky // 4 * 2], 'r')
axy.plot(xs, zans[:, ky // 4 * 2], '.r')
axy.plot(xs, answer_lib[:, ky // 4 * 3], 'g')
axy.plot(xs, zans[:, ky // 4 * 3], '.g')
plt.xlabel('x')
```

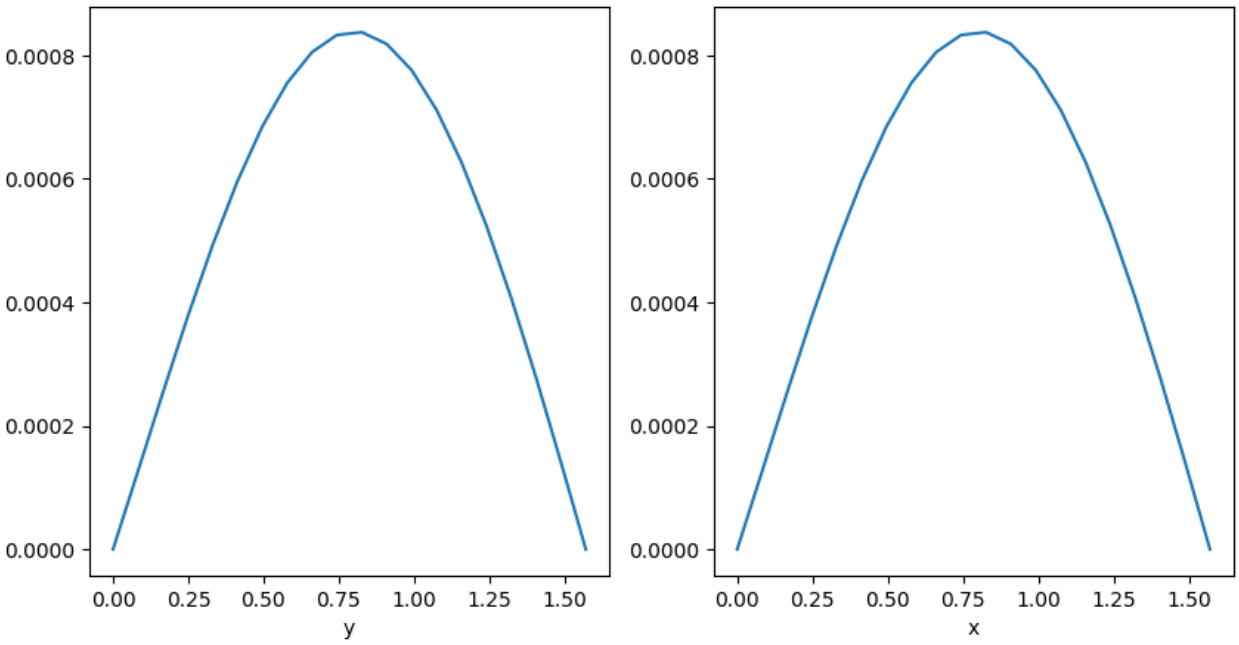
```
Conut iterations: 507
```

Out[5]: Text(0.5, 0, 'x')



In [6]:
```
plt.rcParams['figure.figsize'] = [10, 5]
plt.subplot (1, 2, 1)
plt.plot(ys, np.max(np.abs(answer_lib - zans), axis=0))
plt.xlabel('y')
plt.subplot (1, 2, 2)
plt.plot(xs, np.max(np.abs(answer_lib - zans), axis=1))
plt.xlabel('x')
```

Out[6]: Text(0.5, 0, 'x')



Итерационный метод с релаксацией

In [7]:
```
answer_relax = np.zeros((kx, ky), dtype=np.float64)
relax_coef = 0.5
cur_step = np.zeros((kx, ky), dtype=np.float64)
```

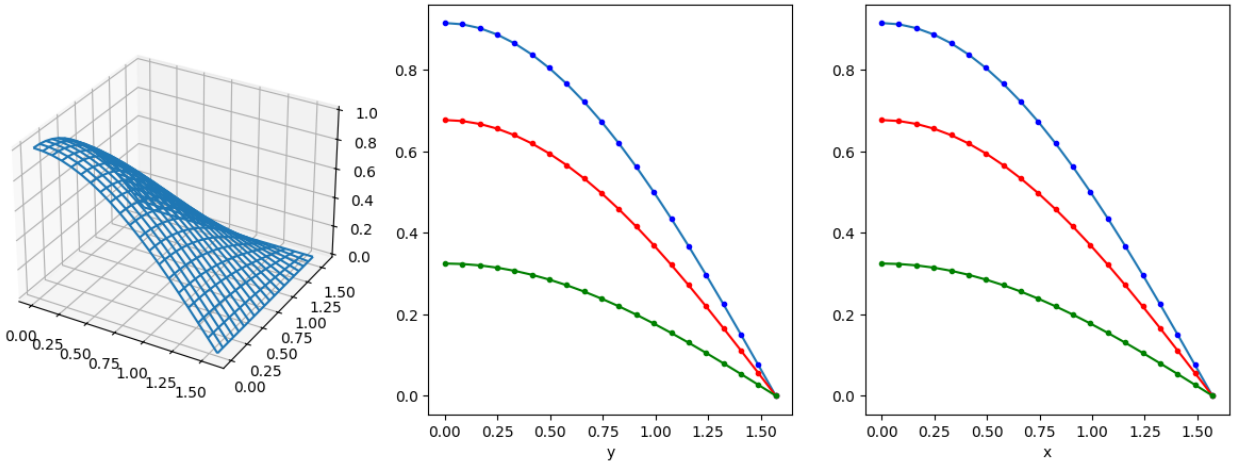
```
for i, y in enumerate(ys):
    cur_step[0][i] = U0y(y)
    cur_step[-1][i] = Uly(y)
for i, x in enumerate(xs):
    cur_step[i][0] = Ux0(x)
    cur_step[i][-1] = Uxl(x)

for i in range(1, kx - 1):
    coef = (cur_step[i][-1] - cur_step[i][0]) / (ys[-1] - ys[0])
    for j in range(1, ky - 1):
        cur_step[i][j] = cur_step[i][0] + coef*(ys[j] - ys[0])

next_step = np.array(cur_step, copy=True)
```

In [8]:

```
error = 1 * 10**(-5)
count = 0
while True:
    cur_step = np.array(next_step)*relax_coef + (1 - relax_coef)*cur_step
    for i in range(1, kx - 1):
        for j in range(1, ky - 1):
            next_step[i][j] = (
                hx**2 * cur_step[i+1][j]
                + hx**2 * cur_step[i-1][j]
                + hy**2 * cur_step[i][j+1]
                + hy**2 * cur_step[i][j-1]
                - f(xs[i], ys[j]) * hx**2 * hy**2
            ) / (2 * (hx**2 + hy**2) + hx**2 * hy**2 * c)
    count += 1
    if get_error(cur_step, next_step) < error:
        break

print("Conut iterations: {iterations}".format(iterations = count))

answer_relax = np.array(next_step)

plt.rcParams['figure.figsize'] = [15, 5]
fig = plt.figure()
ax_3d = fig.add_subplot(1,3,1, projection='3d')
ax_3d.plot_wireframe(xgrid, ygrid, answer_relax.transpose())
axx = fig.add_subplot(1,3,2)
axx.plot(ys, answer_relax[kx // 4])
axx.plot(ys, zans[kx // 4], '.b')
axx.plot(ys, answer_relax[kx // 4 * 2], 'r')
axx.plot(ys, zans[kx // 4 * 2], '.r')
axx.plot(ys, answer_relax[kx // 4 * 3], 'g')
axx.plot(ys, zans[kx // 4 * 3], '.g')
plt.xlabel('y')
axy = fig.add_subplot(1,3,3)
axy.plot(xs, answer_relax[:, ky // 4])
axy.plot(xs, zans[:, ky // 4], '.b')
axy.plot(xs, answer_relax[:, ky // 4 * 2], 'r')
axy.plot(xs, zans[:, ky // 4 * 2], '.r')
axy.plot(xs, answer_relax[:, ky // 4 * 3], 'g')
axy.plot(xs, zans[:, ky // 4 * 3], '.g')
plt.xlabel('x')
```

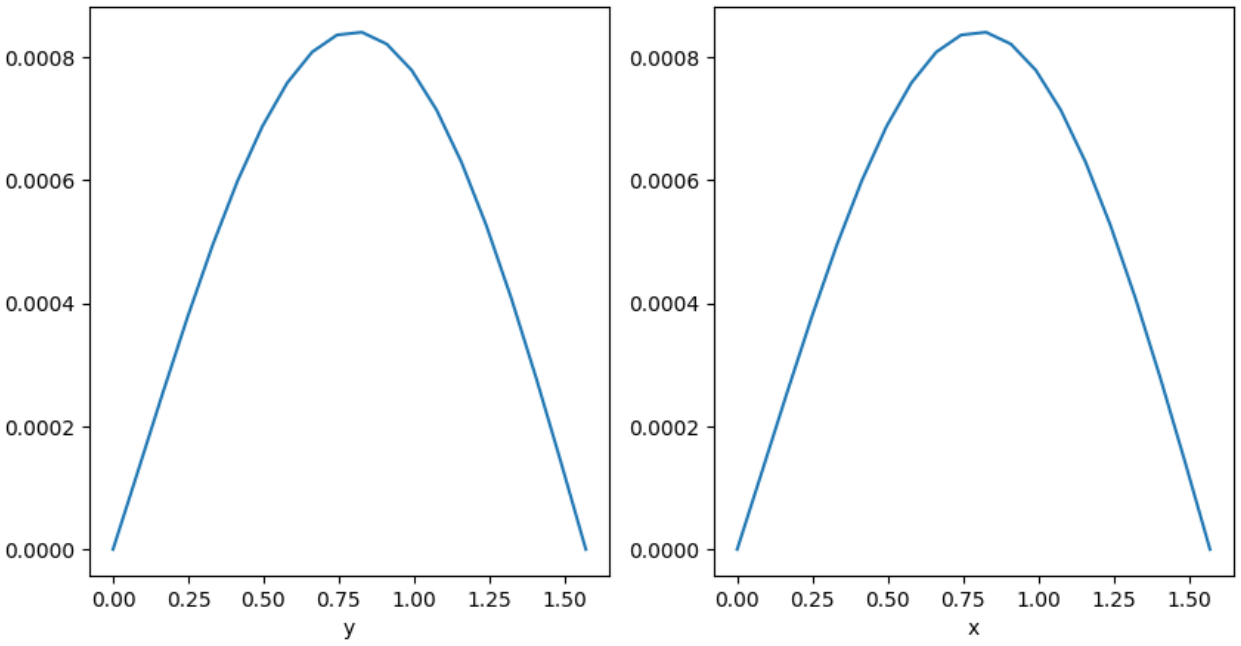
```
Conut iterations: 1015
```

Out[8]: Text(0.5, 0, 'x')



In [9]:
```
plt.rcParams['figure.figsize'] = [10, 5]
plt.subplot (1, 2, 1)
plt.plot(ys, np.max(np.abs(answer_relax - zans), axis=0))
plt.xlabel('y')
plt.subplot (1, 2, 2)
plt.plot(xs, np.max(np.abs(answer_relax - zans), axis=1))
plt.xlabel('x')
```

Out[9]: Text(0.5, 0, 'x')



Метод Зейделя

In [10]:
```
answer_zeydel = np.zeros((kx, ky), dtype=np.float64)
cur_step = np.zeros((kx, ky), dtype=np.float64)

for i, y in enumerate(ys):
    cur_step[0][i] = U0y(y)
    cur_step[-1][i] = Uly(y)
for i, x in enumerate(xs):
    cur_step[i][0] = Ux0(x)
```

```
    cur_step[i][-1] = Uxl(x)

for i in range(1, kx - 1):
    coef = (cur_step[i][-1] - cur_step[i][0]) / (ys[-1] - ys[0])
    for j in range(1, ky - 1):
        cur_step[i][j] = cur_step[i][0] + coef*(ys[j] - ys[0])

next_step = np.array(cur_step, copy=True)
```

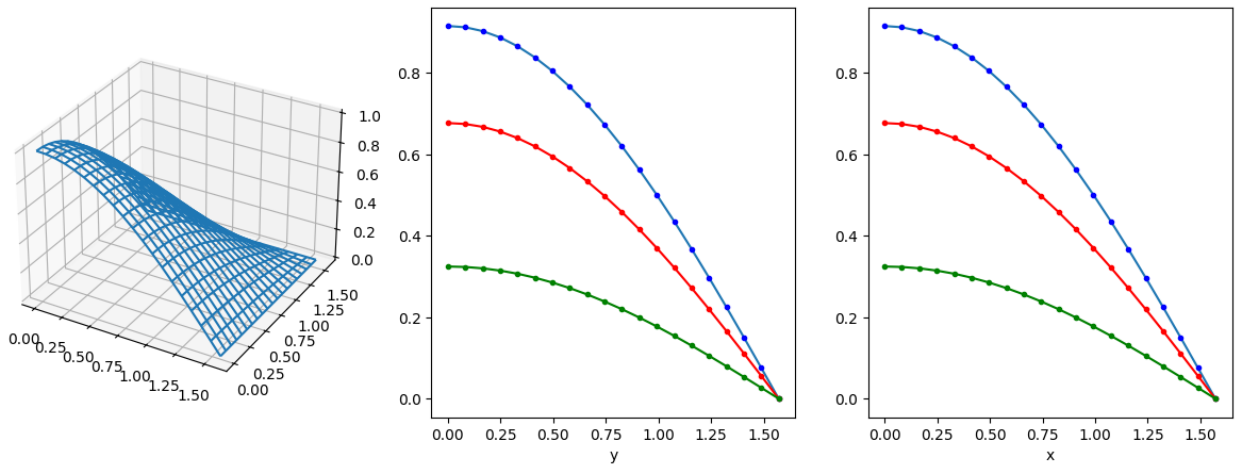
In [11]:

```
error = 1 * 10**(-5)
count = 0
while True:
    cur_step = np.array(next_step)
    for i in range(1, kx - 1):
        for j in range(1, ky - 1):
            next_step[i][j] = (
                hx**2 * cur_step[i+1][j]
                + hx**2 * next_step[i-1][j]
                + hy**2 * cur_step[i][j+1]
                + hy**2 * next_step[i][j-1]
                - f(xs[i], ys[j]) * hx**2 * hy**2
            ) / (2 * (hx**2 + hy**2) + hx**2 * hy**2 * c)
    count += 1
    if get_error(cur_step, next_step) < error:
        break

print("Conut iterations: {iterations}".format(iterations = count))

answer_zeydel = np.array(next_step)

plt.rcParams['figure.figsize'] = [15, 5]
fig = plt.figure()
ax_3d = fig.add_subplot(1,3,1, projection='3d')
ax_3d.plot_wireframe(xgrid, ygrid, answer_zeydel.transpose())
axx = fig.add_subplot(1,3,2)
axx.plot(ys, answer_zeydel[kx // 4])
axx.plot(ys, zans[kx // 4], '.b')
axx.plot(ys, answer_zeydel[kx // 4 * 2], 'r')
axx.plot(ys, zans[kx // 4 * 2], '.r')
axx.plot(ys, answer_zeydel[kx // 4 * 3], 'g')
axx.plot(ys, zans[kx // 4 * 3], '.g')
plt.xlabel('y')
axy = fig.add_subplot(1,3,3)
axy.plot(xs, answer_zeydel[:, ky // 4])
axy.plot(xs, zans[:, ky // 4], '.b')
axy.plot(xs, answer_zeydel[:, ky // 4 * 2], 'r')
axy.plot(xs, zans[:, ky // 4 * 2], '.r')
axy.plot(xs, answer_zeydel[:, ky // 4 * 3], 'g')
axy.plot(xs, zans[:, ky // 4 * 3], '.g')
plt.xlabel('x')
```

```
Conut iterations: 285
```

Out[11]:

```
Text(0.5, 0, 'x')
```



In [12]:
```
plt.rcParams['figure.figsize'] = [10, 5]
plt.subplot (1, 2, 1)
plt.plot(ys, np.max(np.abs(answer_zeydel - zans), axis=0))
plt.xlabel('y')
plt.subplot (1, 2, 2)
plt.plot(xs, np.max(np.abs(answer_zeydel - zans), axis=1))
plt.xlabel('x')
```

Out[12]: Text(0.5, 0, 'x')