МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ РФ

Федеральное государственное бюджетное образовательное учреждение высшего образования «Московский Авиационный Институт» (Национальный Исследовательский Университет)

Институт: №8 «Информационные технологии и прикладная математика»

Кафедра: 806 «Вычислительная математика и программирование»

Лабораторная работа № 6
по курсу «Численные методы»

Группа: М8О-407Б-21

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Оценка:

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In [7]:
```python
from main import *
import matplotlib.pyplot as plt
from matplotlib import cm
from mpl_toolkits.mplot3d import Axes3D
```

Вариант 7:

$$\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial u}{\partial x} - 3u, \tag{1}$$
$$U(0,t) = exp(-t) \cdot cos(2t), \tag{2}$$
$$U(\frac{\pi}{2}, t) = 0, \tag{3}$$
$$U(x,0) = exp(-x) \cdot cos(x), \tag{4}$$
$$U_t(x,0) = -exp(-x) \cdot cos(x) \tag{5}$$

Аналитическое решение:

$$U(x,t) = exp(-t-x) \cdot cos(x) \cdot cos(2t) \tag{6}$$

В данной лабораторной работе используется 3 вида аппроксимации граничных условий:

1. двухточечная аппроксимация с первым порядком
2. трехтоточная аппроксимация со вторым порядком
3. двухточечная аппроксимация со вторым порядком

In [8]:
```python
def __init__(self, params, equation_type):
    self.data = Data(params)
    self.h = 0
    self.tau = 0
    self.sigma = 0
    try:
        self.solve_func = getattr(self, f'{equation_type}_solver')
    except:
        raise Exception("This type does not exist")

def solve(self, N, K, T):
    self.h = self.data.l / N
    self.tau = T / K
    self.sigma = (self.tau ** 2) / (self.h ** 2)
    return self.solve_func(N, K, T)

def analyticSolve(self, N, K, T):
    self.h = self.data.l / N
    self.tau = T / K
    self.sigma = (self.tau ** 2) / (self.h ** 2)
    u = np.zeros((K, N))
    for k in range(K):
        for j in range(N):
            u[k][j] = self.data.solution(j * self.h, k * self.tau)
    return u
```

```
def calculate(self, N, K):
    u = np.zeros((K, N))

    for j in range(0, N - 1):
        x = j * self.h
        u[0][j] = self.data.psi1(x)

        if self.data.approximation == 'p1':
            u[1][j] = self.data.psi1(x) + self.data.psi2(x) * self.tau +
                        (self.tau ** 2 / 2)
        elif self.data.approximation == 'p2':
            u[1][j] = self.data.psi1(x) + self.data.psi2(x) * self.tau +
                        (self.data.psi1_dir2(x) + self.data.b * self.data
                        self.data.c * self.data.psi1(x) + self.data.f())

    return u

def implicit_solver(self, N, K, T):
    u = self.calculate(N, K)

    a = np.zeros(N)
    b = np.zeros(N)
    c = np.zeros(N)
    d = np.zeros(N)

    for k in range(2, K):
        for j in range(1, N):
            a[j] = self.sigma
            b[j] = -(1 + 2 * self.sigma)
            c[j] = self.sigma
            d[j] = -2 * u[k - 1][j] + u[k - 2][j]

        if self.data.bound_type == 'a1p2':
            b[0] = self.data.alpha / self.h / (self.data.beta - self.data
            c[0] = 1
            d[0] = 1 / (self.data.beta - self.data.alpha / self.h) * self
            a[-1] = -self.data.gamma / self.h / (self.data.delta + self.d
            d[-1] = 1 / (self.data.delta + self.data.gamma / self.h) * se

        elif self.data.bound_type == 'a2p3':
            k1 = 2 * self.h * self.data.beta - 3 * self.data.alpha
            omega = self.tau ** 2 * self.data.b / (2 * self.h)
            xi = self.data.d * self.tau / 2

            b[0] = 4 * self.data.alpha - self.data.alpha / (self.sigma +
                    (1 + xi + 2 * self.sigma - self.data.c * self.tau **
            c[0] = k1 - self.data.alpha * (omega - self.sigma) / (omega +
            d[0] = 2 * self.h * self.data.phi0(k * self.tau) + self.data.
            a[-1] = -self.data.gamma / (omega - self.sigma) * \
                    (1 + xi + 2 * self.sigma - self.data.c * self.tau **
            d[-1] = 2 * self.h * self.data.phi1(k * self.tau) - self.data

        elif self.data.bound_type == 'a2p2':
            b[0] = 2 * self.data.a / self.h
            c[0] = -2 * self.data.a / self.h + self.h / self.tau ** 2 - s
                    -self.data.d * self.h / (2 * self.tau) + \
```

```python
                    self.data.beta / self.data.alpha * (2 * self.data.a +
            d[0] = self.h / self.tau ** 2 * (u[k - 2][0] - 2 * u[k - 1][0
                    -self.data.d * self.h / (2 * self.tau) * u[k - 2][0]
                    (2 * self.data.a - self.data.b * self.h) / self.data.
            a[-1] = -b[0]
            d[-1] = self.h / self.tau ** 2 * (-u[k - 2][0] + 2 * u[k - 1]
                    self.data.d * self.h / (2 * self.tau) * u[k - 2][0] +
                    (2 * self.data.a + self.data.b * self.h) / self.data.

        u[k] = tma(a, b, c, d)

    return u

def _left_bound_a1p2(self, u, k, t):
    coeff = self.data.alpha / self.h
    return (-coeff * u[k - 1][1] + self.data.phi0(t)) / (self.data.beta -

def _right_bound_a1p2(self, u, k, t):
    coeff = self.data.gamma / self.h
    return (coeff * u[k - 1][-2] + self.data.phi1(t)) / (self.data.delta

def _left_bound_a2p2(self, u, k, t):
    n = self.data.c * self.h - 2 * self.data.a / self.h - self.h / self.t
        (2 * self.tau) + self.data.beta / self.data.alpha * (2 * self.dat
    return 1 / n * (- 2 * self.data.a / self.h * u[k][1] +
                    self.h / self.tau ** 2 * (u[k - 2][0] - 2 * u[k - 1][
                    -self.data.d * self.h / (2 * self.tau) * u[k - 2][0]
                    (2 * self.data.a - self.data.b * self.h) / self.data.

def _right_bound_a2p2(self, u, k, t):
    n = -self.data.c * self.h + 2 * self.data.a / self.h + self.h / self.
        (2 * self.tau) + self.data.delta / self.data.gamma * (2 * self.da
    return 1 / n * (2 * self.data.a / self.h * u[k][-2] +
                    self.h / self.tau ** 2 * (2 * u[k - 1][-1] - u[k - 2]
                    self.data.d * self.h / (2 * self.tau) * u[k - 2][-1]
                    (2 * self.data.a + self.data.b * self.h) / self.data.

def _left_bound_a2p3(self, u, k, t):
    denom = 2 * self.h * self.data.beta - 3 * self.data.alpha
    return self.data.alpha / denom * u[k - 1][2] - 4 * self.data.alpha /
            2 * self.h / denom * self.data.phi0(t)

def _right_bound_a2p3(self, u, k, t):
    denom = 2 * self.h * self.data.delta + 3 * self.data.gamma
    return 4 * self.data.gamma / denom * u[k - 1][-2] - self.data.gamma /
            2 * self.h / denom * self.data.phi1(t)

def explicit_solver(self, N, K, T):
    global left_bound, right_bound
    u = self.calculate(N, K)

    # for j in range(1, N - 1):
    #     u[1][j] = self.data.ps1()
    if self.data.bound_type == 'a1p2':
        left_bound = self._left_bound_a1p2
        right_bound = self._right_bound_a1p2
```

```
    elif self.data.bound_type == 'a2p2':
        left_bound = self._left_bound_a2p2
        right_bound = self._right_bound_a2p2

    elif self.data.bound_type == 'a2p3':
        left_bound = self._left_bound_a2p3
        right_bound = self._right_bound_a2p3

    for k in range(2, K):
        t = k * self.tau
        for j in range(1, N - 1):
            # u[k][j] = self.sigma * u[k - 1][j + 1] + (2 - 2 * self.sigm
            #           self.sigma * u[k - 1][j - 1] - u[k - 2][j]
            quadr = self.tau ** 2
            tmp1 = self.sigma + self.data.b * quadr / (2 * self.h)
            tmp2 = self.sigma - self.data.b * quadr / (2 * self.h)
            u[k][j] = u[k - 1][j + 1] * tmp1 + \
                u[k - 1][j] * (-2 * self.sigma + 2 + self.data.c * quadr)
                u[k - 1][j - 1] * tmp2 - u[k - 2][j] + quadr * self.data.

        u[k][0] = left_bound(u, k, t)
        u[k][-1] = right_bound(u, k, t)

    return u
```

Input equation type (example: explicit)

In [9]:
```
equation_type = str(input())
```

In [10]:
```
N = 70
K = 764
T = 1
params = {
        'a': 1,
        'b': 2,
        'c': -3,
        'd': 2,
        'l': np.pi / 2,
        'f': lambda: 0,
        'alpha': 1,
        'beta': 0,
        'gamma': 1,
        'delta': 0,
        'psi1': lambda x: np.exp(-x) * np.cos(x),
        'psi2': lambda x: -np.exp(-x) * np.cos(x),
        'psi1_dir1': lambda x: -np.exp(-x) * np.sin(x) - np.exp(-x) * np.
        'psi1_dir2': lambda x: 2 * np.exp(-x) * np.sin(x),
        'phi0': lambda t: np.exp(-t) * np.cos(2 * t),
        'phi1': lambda t: 0,
        'bound_type': 'a1p2',
        'approximation': 'p1',
        'solution': lambda x, t: np.exp(-t - x) * np.cos(x) * np.cos(2 *
    }
```

In [11]:
```
params['bound_type'] = 'a1p2'
```

In [12]:
```python
solver = HyperbolicSolver(params, equation_type)
```

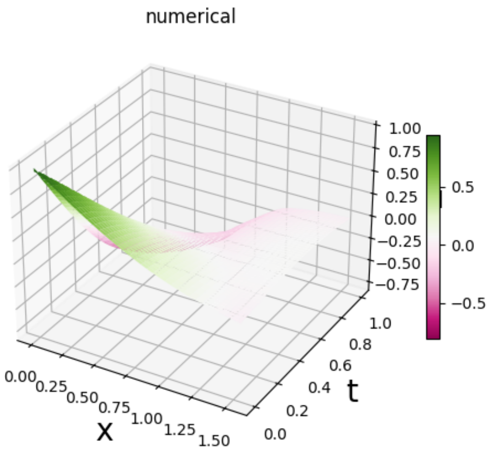
In [13]:
```python
dict_ans = {
        'numerical': solver.solve(N, K, T).tolist(),
        'analytic': solver.analyticSolve(N, K, T).tolist()
    }
```

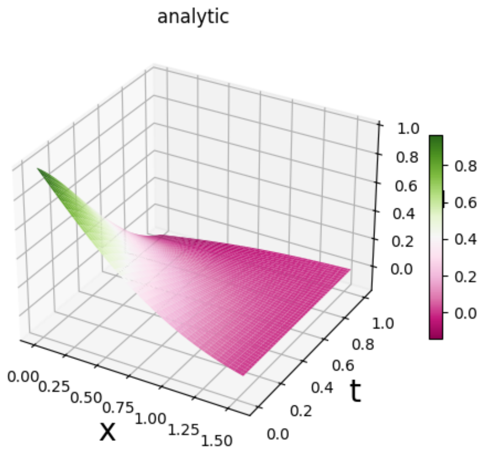
In [14]:
```python
print("Sigma:", solver.sigma)
```

```
Sigma: 0.003402276526462098
```

In [15]:
```python
def draw(dict_, N, K, T, save_file="plot.png"):
    fig = plt.figure(figsize=plt.figaspect(0.3))
    # Make data
    x = np.arange(0, np.pi / 2, np.pi / 2 / N)
    t = np.arange(0, T, T / K)
    x, t = np.meshgrid(x, t)
    z1 = np.array(dict_['numerical'])
    z2 = np.array(dict_['analytic'])

    # Plot the surface.
    ax = fig.add_subplot(1, 2, 1, projection='3d')
    plt.title('numerical')
    ax.set_xlabel('x', fontsize=20)
    ax.set_ylabel('t', fontsize=20)
    ax.set_zlabel('u', fontsize=20)
    surf = ax.plot_surface(x, t, z1, cmap=cm.PiYG,
                           linewidth=0, antialiased=True)
    fig.colorbar(surf, shrink=0.5, aspect=15)

    ax = fig.add_subplot(1, 2, 2, projection='3d')
    ax.set_xlabel('x', fontsize=20)
    ax.set_ylabel('t', fontsize=20)
    ax.set_zlabel('u', fontsize=20)
    plt.title('analytic')
    surf = ax.plot_surface(x, t, z2, cmap=cm.PiYG,
                                  linewidth=0, antialiased=True)
    # # Customize the z axis
    # ax.set_zlim(-1.01, 1.01)

    # # Add a color bar which maps values to colors.
    fig.colorbar(surf, shrink=0.5, aspect=15)

    plt.savefig(save_file)
    plt.show()
```

In [16]:
```python
draw(dict_ans, N, K, T)
```





In [17]:

```
def draw_u_x(dict_, N, K, T, time, save_file="plot_u_x.png"):
    fig = plt.figure()
    x = np.arange(0, np.pi / 2, np.pi / 2 / N)
    t = np.arange(0, T, T / K)
    z1 = np.array(dict_['numerical'])
    z2 = np.array(dict_['analytic'])


    plt.title('U from x')
    plt.plot(x, z1[time], color='r', label='numerical')
    plt.plot(x, z2[time], color='b', label='analytic')
    plt.legend(loc='best')
    plt.ylabel('U')
    plt.xlabel('x')
    plt.savefig(save_file)
    plt.show()

    err = []
    error = compare_error(dict_ans)
    for i in range(len(error)):
        tmp = 0
        for j in error[i]:
            tmp += j
        err.append(tmp/len(error[i])/100)
    plt.title('Error from t')
    plt.plot(t, err, color='b', label='err')
    plt.legend(loc='best')
    plt.ylabel('Err')
    plt.xlabel('t')
    plt.savefig('err.png')
    plt.show()
```

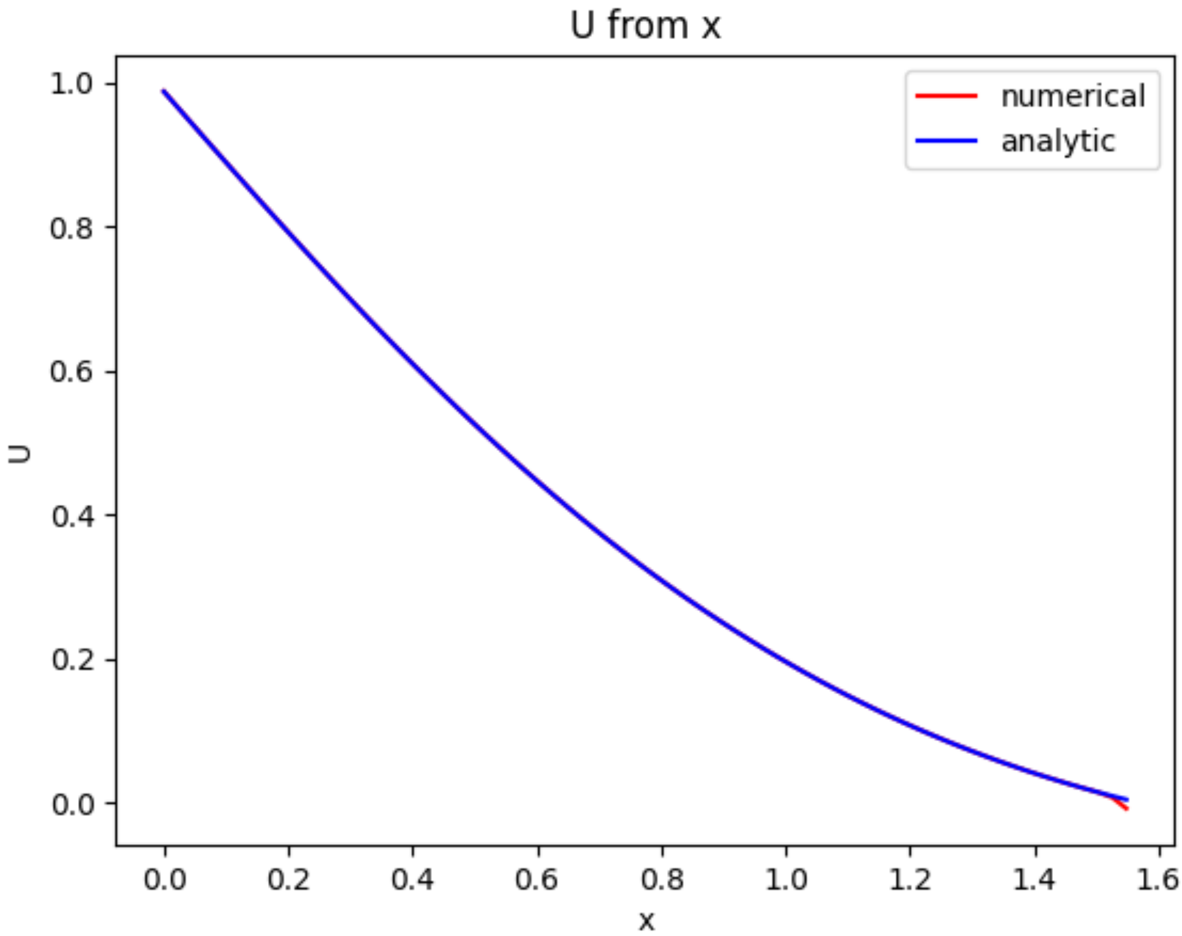
Time check

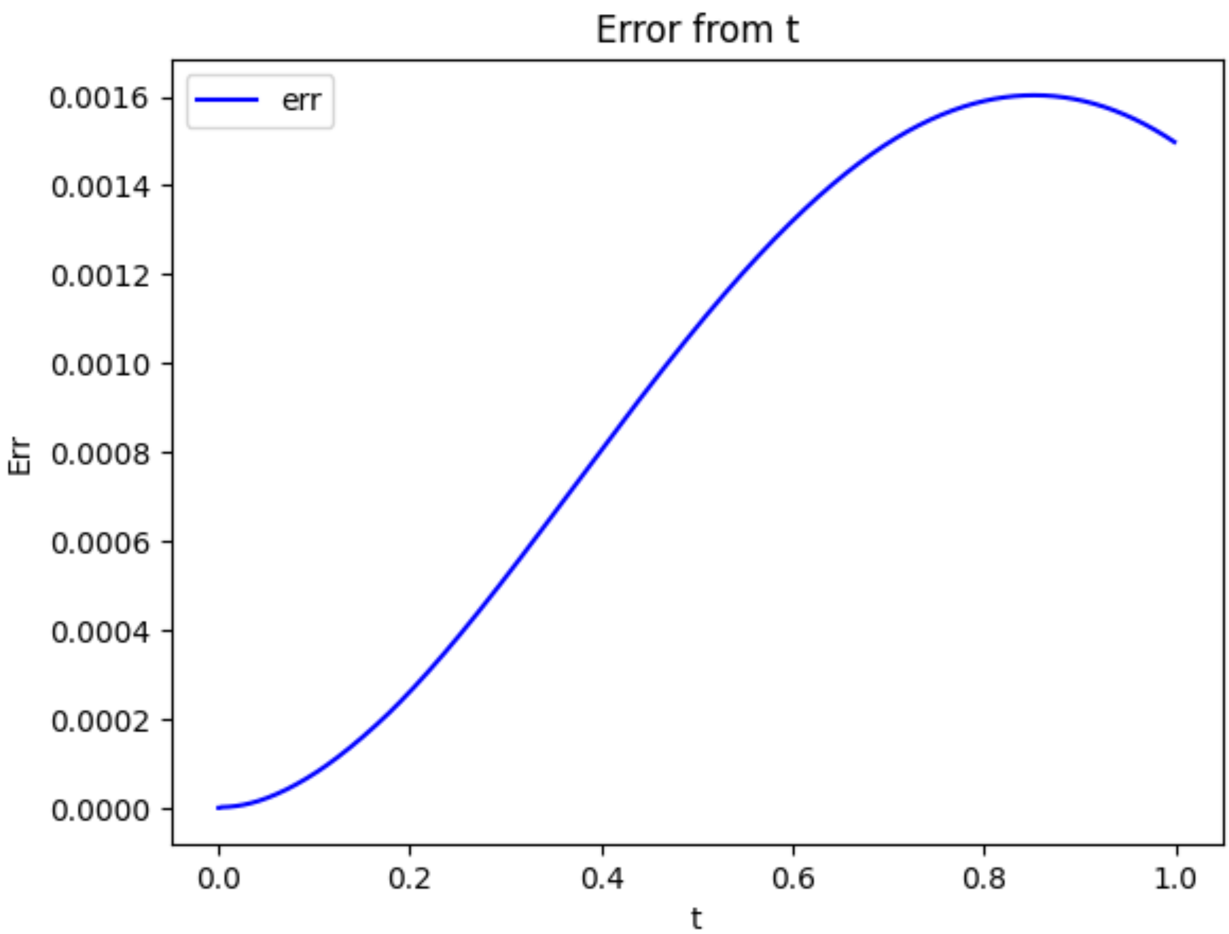
In [18]:

```
curr_time = int(input())
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
Cell In[18], line 1
----> 1 curr_time = int(input())

ValueError: invalid literal for int() with base 10: 'explicit'
```

In [15]:
```
draw_u_x(dict_ans, N, K, T, curr_time)
```





In [16]:
```
error = compare_error(dict_ans)
avg_err = 0.0
for i in error:
    for j in i:
        avg_err += j
    avg_err /= N
```

First elements in error array:

In [17]:
```
print(error[0])
```

```
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.
0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0047702677544667815]
```

Middle elements in error array:

In [18]:
```
print(error[int(K/2)])
```

```
[0.16354750614019037, 0.16354628559001544, 0.16355867753100806, 0.16356645
39265091, 0.1635523446425644, 0.16350014538150087, 0.1633947730110783, 0.1
6322230494805362, 0.16297019089644943, 0.16262705015932483, 0.162183114280
75283, 0.16162999537410358, 0.1609607146250759, 0.16017029778696418, 0.159
25514439559274, 0.15821313370460616, 0.1570446454836768, 0.155751892769870
06, 0.15433792163702695, 0.15280776347503822, 0.15117010095729774, 0.14943
672820537054, 0.147620103292764 1, 0.14573113329407864, 0.1437783911492544
2, 0.14176846924933145, 0.13970668850929702, 0.1375976726163916, 0.1354456
7650913228, 0.1332547360403458, 0.13102872559986495, 0.12877137808360742,
0.12648629204240 5, 0.12417693510503532, 0.12184664634946447, 0.11949863771
893605, 0.11713599166525665, 0.11476164220882938, 0.1123782892764207, 0.10
99880713756671, 0.10759146239171855, 0.10518401191239947, 0.10274810508743
978, 0.10023597605428283, 0.09754422640541437, 0.09449739498981158, 0.0908
9024701623838, 0.08665011537289066, 0.08208367710504162, 0.077934163927827
63, 0.07487657458589562, 0.07265577669918591, 0.07005549309351339, 0.06637
784667067592, 0.06246928517184719, 0.05910229627827187, 0.0554895020054585
46, 0.05124486929305845, 0.04724014037277467, 0.043180351823768656, 0.0386
3392455846405, 0.03431045703182581, 0.02972847569171562, 0.02497180051053
16, 0.020323167513119036, 0.015306295174725305, 0.010476948343357 62, 0.005
304478345612902, 0.0002869041970881962, 0.005023299525969112]
```

Last elements in error array:

In [19]:
```python
print(error[-1])
```

```
[0.16258372637029164, 0.1625842940375307, 0.1628798504468028, 0.1634529029
0877865, 0.16428548185619288, 0.1653591885804307, 0.16665524382621644, 0.1
6815453842836012, 0.16983768746198852, 0.17168508070990143, 0.173676944562
32002, 0.17579338651147292, 0.17801445672981525, 0.18032016213625748, 0.18
269044569902765, 0.18510499648345297, 0.18754269257698275, 0.1899803174679
6123, 0.19238983963965614, 0.1947336534145205, 0.19695757610785308, 0.1989
8346884324747, 0.20070744338046823, 0.20201393077939767, 0.202816240450714
34, 0.20312151190616246, 0.2030879670269747, 0.2030116808242782, 0.2031901
5479867789, 0.20369980820553296, 0.20425817944266553, 0.20436593177691653,
0.20369847749880232, 0.20238316394812628, 0.20080144077120018, 0.199096946
87200805, 0.19703435083431808, 0.19441788193475626, 0.19139882203163064,
0.18816333351731923, 0.1846081379167883, 0.18062095914833678, 0.1763380862
4472008, 0.17183787833246103, 0.1670183498921263, 0.16191495553250695, 0.1
5662924019141874, 0.1511005399442012, 0.1453363219317275, 0.13941632432974
97, 0.13329268408348632, 0.12698751722142584, 0.12055054485656182, 0.11393
965335092811, 0.1072041136772855, 0.1003492481610346, 0.09336387920080794,
0.08629721606423978, 0.07912182090416837, 0.07187735016298923, 0.064561126
66208401, 0.05718274120672338 4, 0.04976610571952224, 0.04230153772337431,
0.03482510041624329, 0.02731911623111283 8, 0.019823382464950134, 0.0123201
13626406866, 0.004847371071450732, 0.002610851813121852]
```

In [20]:
```python
print(f'Average error in each N: {avg_err}')
```

```
Average error in each N: 0.15194907342786956
```

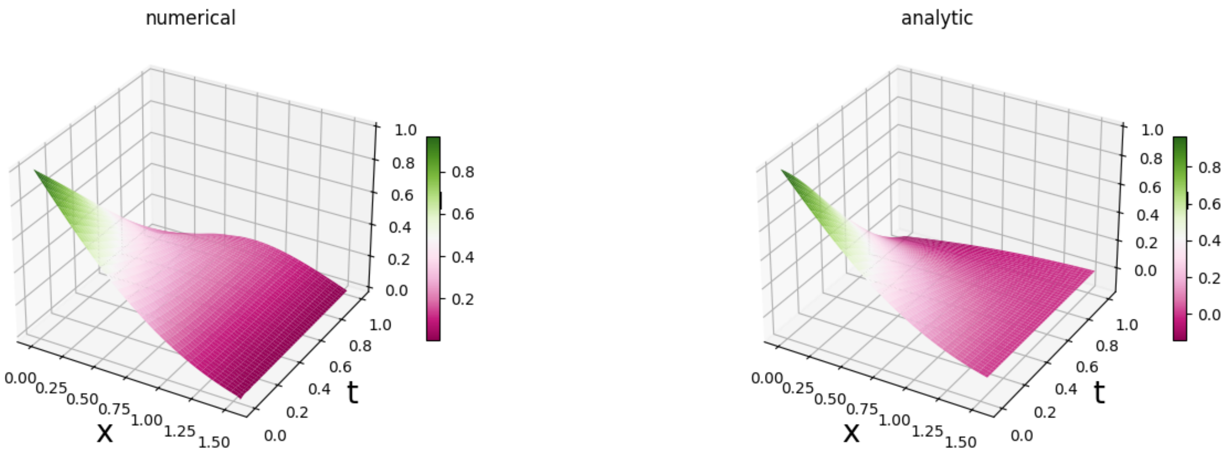
In [21]:
```python
print(f'Average error\t\t: {avg_err / K}')
```

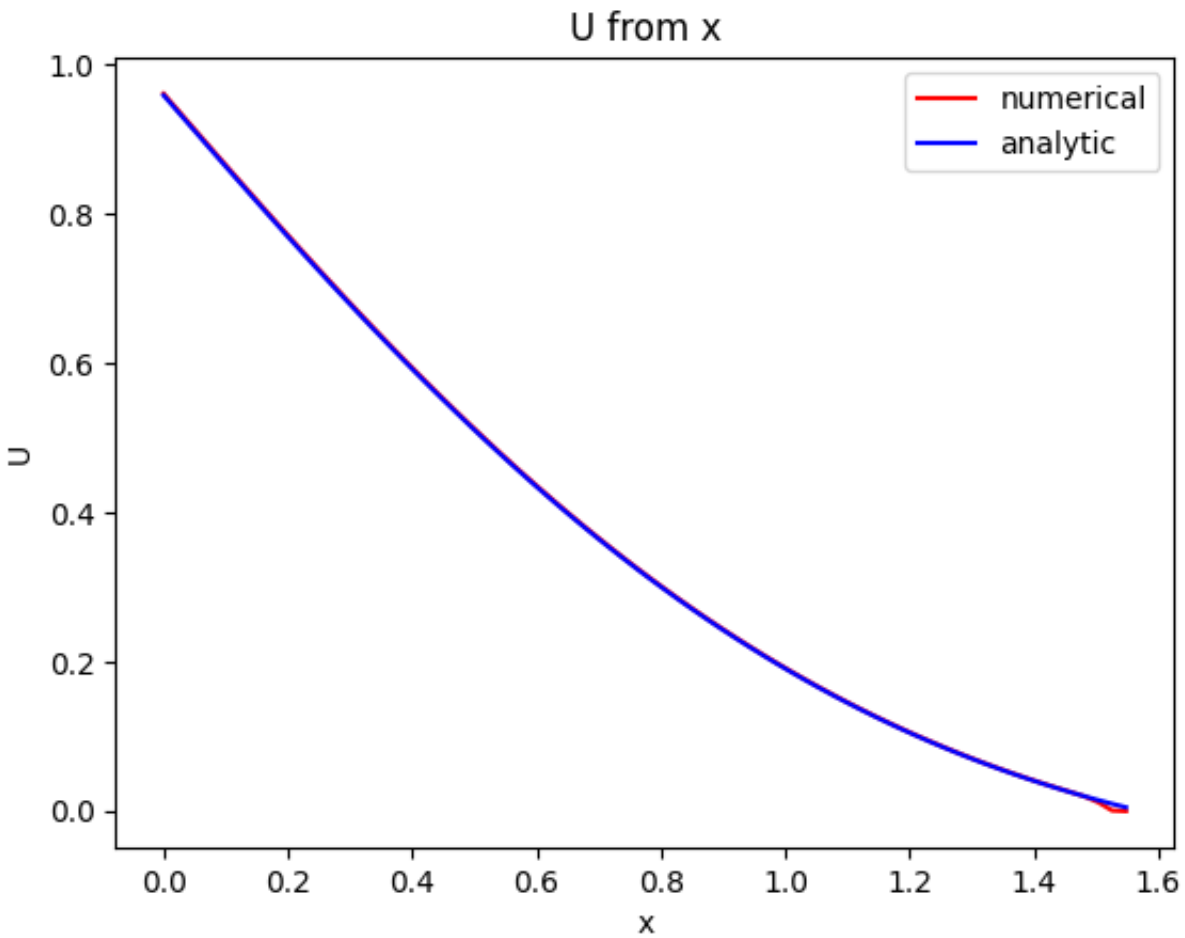
```
Average error		: 0.0001988862217642272
```

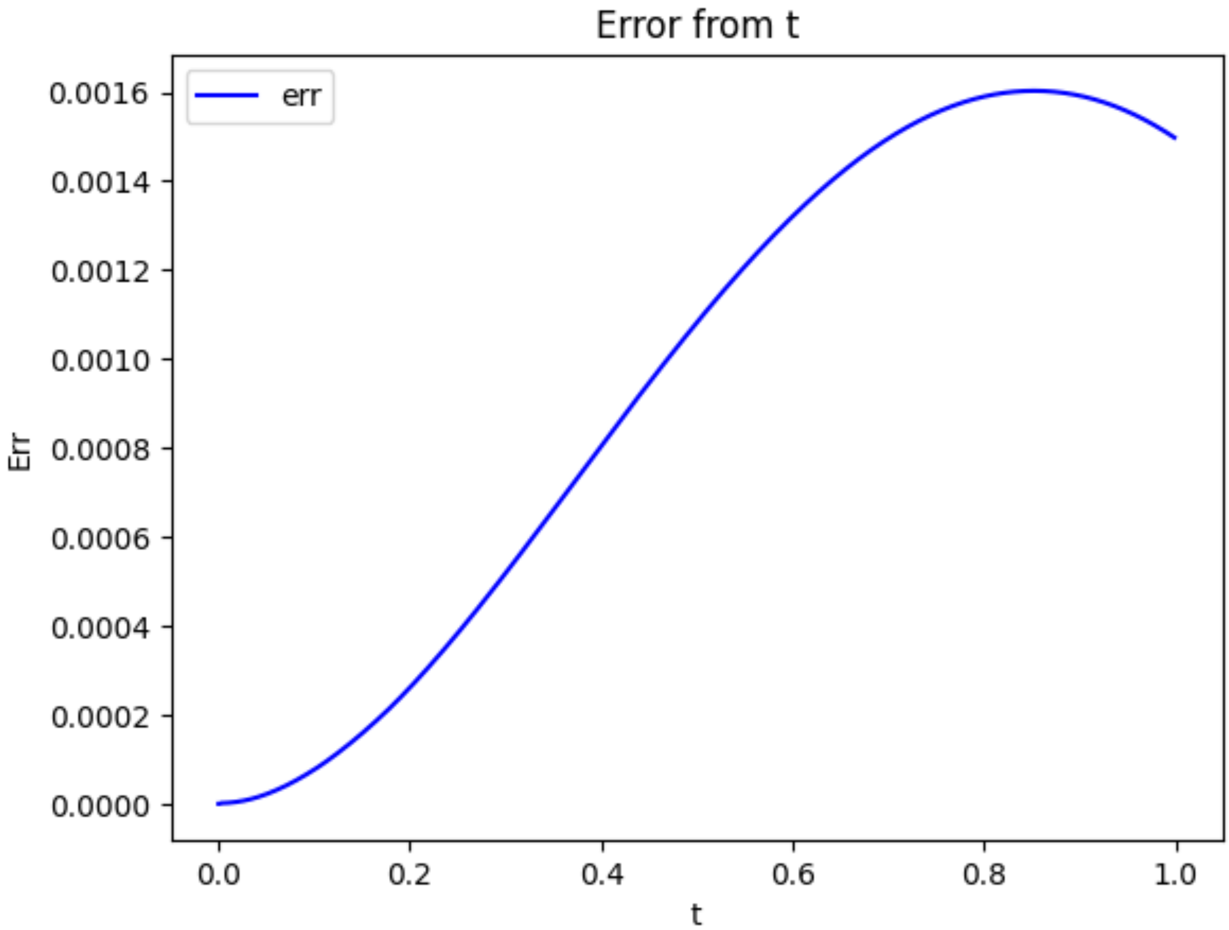
In [22]:
```python
equation_type = str(input())
```

In [23]:
```python
N = 70
K = 764
```

```python
T = 1
curr_time = 30
params['bound_type'] = 'a1p2'
solver = HyperbolicSolver(params, equation_type)
dict_ans = {
        'numerical': solver.solve(N, K, T).tolist(),
        'analytic': solver.analyticSolve(N, K, T).tolist()
    }
draw(dict_ans, N, K, T)
draw_u_x(dict_ans, N, K, T, curr_time)
```






Error from t
err
Err
0.0016
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
0.0
0.2
0.4
0.6
0.8
1.0
t