**EE 444 Introduction to Computer Networks**

**Queuing Homework Assignment. Weight: 10%**
**Due date: April 9 2023 23:59**
**CLEARLY STATE YOUR ASSUMPTIONS, SHOW ALL YOUR WORK**
**Refer to the supplementary Queuing Theory Document. Please use a decent scanner for handwritten solutions.**

**Question 1)** A queuing system starts empty at time t=0. We look at 6 packet arrivals with their service times as in the table below. Assume no other packet arrivals.

| Arrival time | Service time: Length/bitrate |
|---|---|
| 1 | 3.5 |
| 3 | 4 |
| 4 | 2 |
| 7 | 1 |
| 8 | 1.5 |
| 15 | 4 |

Draw a graph that shows the number of packets in the system as a function of time $N(t)$ if
a) the server is FCFS (First come first served)
b) the server is LCFS (Last come first served)

- Compare the times that did the system become empty in both parts a and b? Why do you have this result?
- What are the average values of N, $\lambda$ (packets/sec), Time in the system in parts a and b. Comment on your result.

**Question 2)** Refer to the finite buffer case in the Queuing Theory Document
A single server system has a buffer of $N-1$ packets. Hence, including the packet that is getting service, the maximum number of packets in the system is $N$ at any time. If a packet arrives when there are already $N$ packets in the system the arriving packet is dropped. Arrivals are Poisson with rate $\lambda$ packets/sec and the packet sizes are exponentially distributed with an average service time of $\frac{1}{\mu}$ sec/packet.
a) Draw the Continuous Time Markov Chain for this system and derive an expression for $\pi_i$
b) What is the utilization of this system (Hint: the server is utilized when there is some packet in the system)
c) What is the probability of packet loss?
d) What is the throughput?
e) What is the expected number of packets in this system?
f) What is the expected time in the system for packets that are not dropped?

**Question 3)**
a) Derive the probability that the system has **equal to or more than** $n$ packets in an M/M/1 system as a function of $\rho$
b) We want to limit the probability of packet loss by a probability $P$.
Given a router with a 10 Gbits/sec outgoing line. The packet sizes are exponentially distributed with an average of 1000 Bytes.
Plot the required buffer size in Bytes as a function of $\rho$ for $P=10^{-6}$ and $P=10^{-12}$
$\rho$=0.1, 0.2, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9, 0.92, 0.94, 0.96, 0.98, 0.99
Comment on the results.

**Question 4)** Use a simulator of your choice to simulate the following queuing systems. You can check https://www.mathworks.com/help/simevents/queue-service-and-route-modeling.html
a) M/M/1 with $\lambda = 4$ and $\mu = 5$
b) M/D/1 with $\lambda = 4$ and $\mu = 5$ (μ is constant)
c) M/M/1/5 with $\lambda = 4$ and $\mu = 5$ (Total capacity including the server is 5)
Simulate for 10000 arrivals.

- For parts a, b and c, plot the server utilization and waiting time.
- Compare your results with analytical models for parts a and c.
- Compare the results you achieve in part b to those of part a and comment (We do not cover the expressions for M/D/1 in this course)
- Collect the number of packets in the system at regular time intervals and relate your findings to your Markov Chain Model $\pi_i$ that you learn in the lectures and you derive in Question 2.

**IMPORTANT:**
If you use another simulator please provide the reference. You can write your own event-triggered queue simulator which requires maintaining a priority queue and generating exponentially distributed events. If you are interested in doing that please contact me as soon as possible.