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2023-04-09

# EE444 Homework 1

## Question 1)

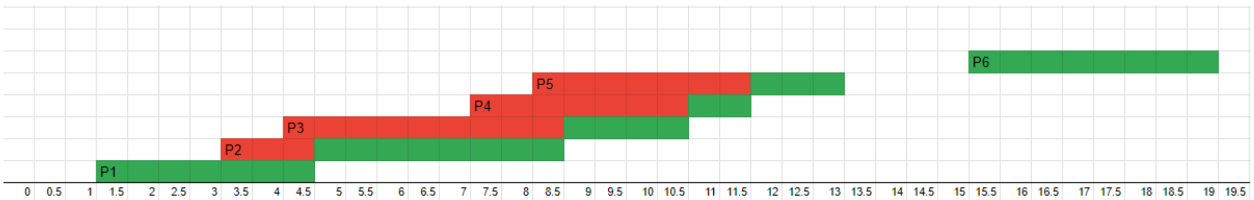


*Figure 1 Packet Visualization in FCFS*

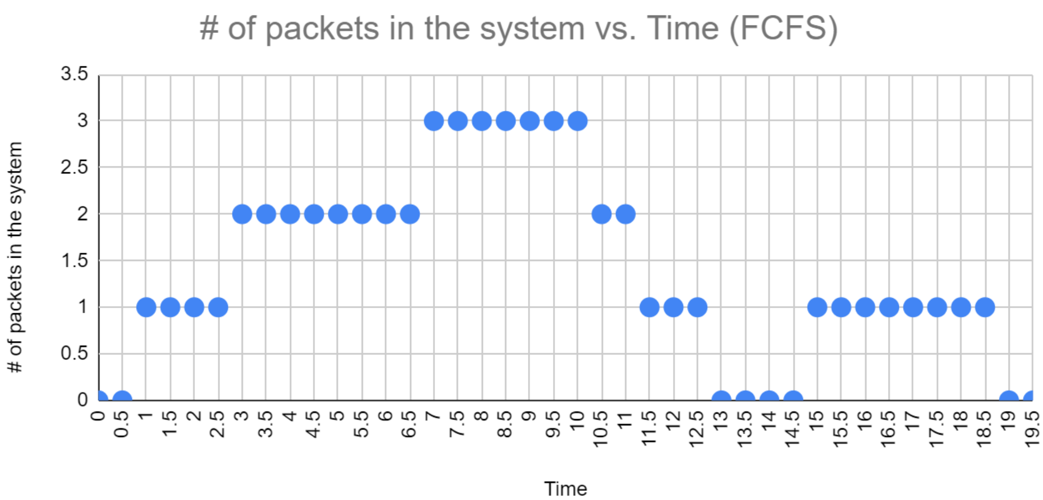


*Figure 2 # of Packets in the System vs Time FCFS*


*Figure 3 Packet Visualization in LCFS*

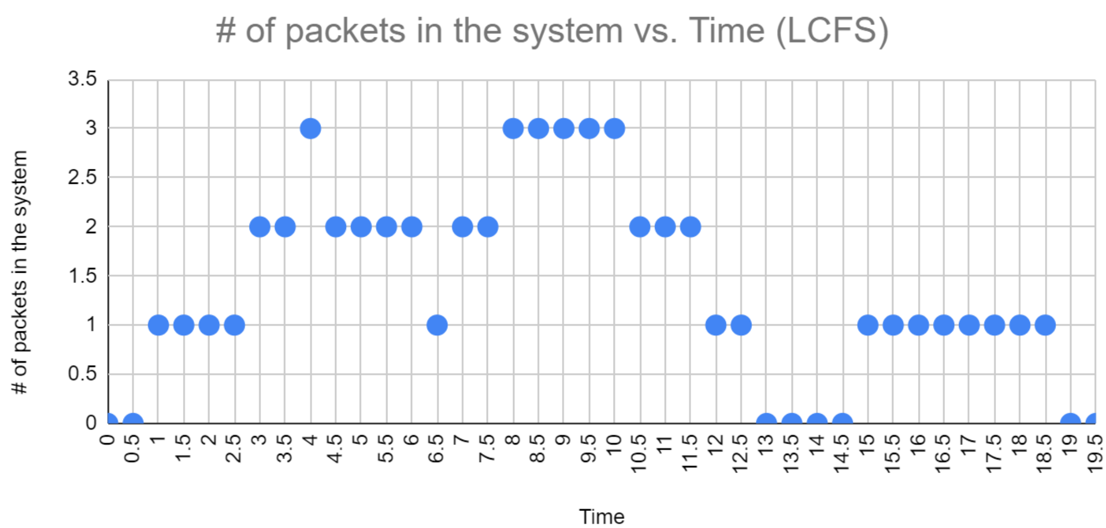


*Figure 4 # of Packets in the System vs Time LCFS*

In time range between t = 0 and t = 19, FCFS has 3 seconds and LCFS has 3 seconds, which are equal. It is because there is no loss of packets. So even though the service time values are different, all the packets are sent in the same time frame.

$$E[IA]_{FCFS} = \frac{(4+2+6+2+14)}{5} * 0.5 = 2.8 = \frac{1}{\lambda_{FCFS}} \rightarrow \lambda_{FCFS} \cong 0.36 \ \frac{packet}{sec}$$

$$E[T_s]_{FCFS} = \frac{(7+11+13+9+10+8)}{6} * 0.5 \cong 4.84 \ \frac{sec}{packet}$$

$$E[N_s]_{FCFS} = \lambda_{FCFS} * E[T_s]_{FCFS} \cong 1.74 \ (From\ Little's\ Law)$$

$$E[IA]_{LCFS} = E[IA]_{FCFS} = 2.8 = \frac{1}{\lambda_{LCFS}} \rightarrow \lambda_{LCFS} \cong 0.36 \ \frac{packet}{sec}$$

$$E[T_s]_{LCFS} = \frac{(7+15+5+12+8+8)}{6} * 0.5 \cong 4.59 \ \frac{sec}{packet}$$

$$E[N_s]_{LCFS} = \lambda_{LCFS} * E[T_s]_{LCFS} \cong 1.65 \ (From\ Little's\ Law)$$

On average, FCFS had fewer packets in the system, and spent more time waiting in the queue before being served. The rate of packet arrivals per second is the same for both methods because same arrival times are used. From Little's Law, while rate of packet arrivals per second is constant, as expected time a packet spends in the system decreases, expected time of packets in the system decreases.

Question 2:

a)


Balance equation

$\pi_0 \lambda = \pi_1 \mu \quad \Rightarrow \pi_1 = \pi_0 \cdot \frac{\lambda}{\mu} = \pi_0 \cdot \rho$

$\pi_1 (\lambda + \mu) = \pi_2 \mu + \pi_0 \cdot \lambda \quad \Rightarrow \pi_2 \mu = \pi_0 \frac{\lambda}{\mu} (\lambda + \mu) - \pi_0 \lambda$

$\Rightarrow \pi_2 \mu = \pi_0 \left[ \frac{\lambda^2}{\mu} + \lambda - \lambda \right]$

$\Rightarrow \pi_2 = \pi_0 \frac{\lambda^2}{\mu^2}$

$\pi_i = \pi_0 \left(\frac{\lambda}{\mu}\right)^i = \pi_0 \cdot \rho^i$

b) Utilization = Prob(System is not empty) $= 1 - \pi_0 = \rho$ (Refer to Queueing Teory document)

c) Prob(loss) $= \pi_N = \pi_0 \cdot \rho^N$

d) $X = \min(\lambda, (N-1)\mu)$

e) $E[N_s] = \lambda (1 - P(N \text{ jobs})) E[T_s]$

f) $E[T_s] = E[T_Q] + E[s]$

$E[T_Q] = \dfrac{(N - E[N_s])}{\mu (N - \lambda)}$ and $E[s] = 1/\mu$

$\therefore \quad E[T_s] = \dfrac{(N - E[N_s])}{\mu (N - \lambda)} + \dfrac{1}{\mu}$

Question 3)

a) $P(\text{packets} \geq n) = 1 - P(\text{packets} < n)$

$$P(\text{packets} < n) = \sum_{k=0}^{n-1} P(N=k) = \sum_{k=0}^{n-1} \left( e^{-\lambda\tau} \cdot \frac{(\lambda\tau)^k}{k!} \right) \quad , \lambda = \rho\mu$$

$$\therefore P(\text{packets} \geq n) = 1 - \sum_{k=0}^{n-1} \left( e^{-\rho\mu\tau} \cdot \frac{(\rho\mu\tau)^k}{k!} \right)$$

b)