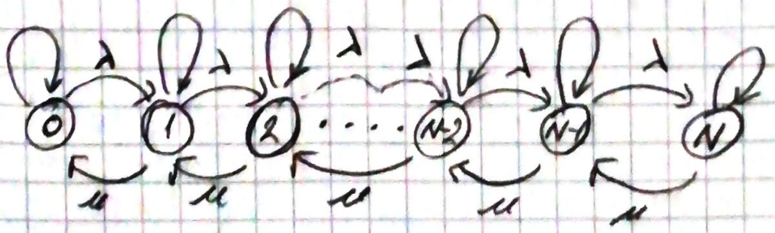
Question 2:

a)



Balance equation

$\pi_0 \lambda = \pi_1 \mu \Rightarrow \pi_1 = \pi_0 \cdot \frac{\lambda}{\mu} = \pi_0 \cdot \rho$

$\pi_1(\lambda+\mu) = \pi_2 \mu + \pi_0 \cdot \lambda \Rightarrow \pi_2 \mu = \pi_0 \frac{\lambda}{\mu}(\lambda+\mu) - \pi_0 \lambda$

$\Rightarrow \pi_2 \mu = \pi_0 \left[\frac{\lambda^2}{\mu} + \lambda - \lambda\right]$

$\Rightarrow \pi_2 = \pi_0 \frac{\lambda^2}{\mu^2}$

$\pi_i = \pi_0 \left(\frac{\lambda}{\mu}\right)^i = \pi_0 \cdot \rho^i$

b) Utilization = Prob(System is not empty) $= 1 - \pi_0 = \rho$ (Refer to Queueing Teory document)

c) $\text{Prob(loss)} = \pi_N = \pi_0 \cdot \rho^N$

d) $X = \min(\lambda, (N-1)\mu)$

e) $E[N_s] = \lambda(1 - P(N \text{ jobs}))\, E[T_s]$

f) $E[T_s] = E[T_Q] + E[s]$

$E[T_Q] = \frac{(N - E[N_s])}{\mu(N-\lambda)}$ and $E[s] = 1/\mu$

$\therefore E[T_s] = \frac{(N - E[N_s])}{\mu(N-\lambda)} + \frac{1}{\mu}$

Question 3)

a) $P(\text{packets} \geq n) = 1 - P(\text{packets} < n)$

$P(\text{packets} < n) = \sum_{k=0}^{n-1} P(N=k) = \sum_{k=0}^{n-1} \left(e^{-\lambda \tau} \cdot \frac{(\lambda \tau)^k}{k!} \right)$, $\lambda = \rho \mu$

$\therefore P(\text{packets} \geq n) = 1 - \sum_{k=0}^{n-1} \left(e^{-\rho \mu \tau} \cdot \frac{(\rho \mu \tau)^k}{k!} \right)$

b)