Fast Winding Numbers for Soups and Clouds

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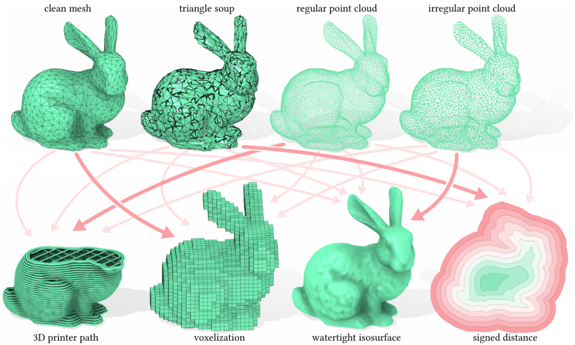
Determining whether a point is inside or outside

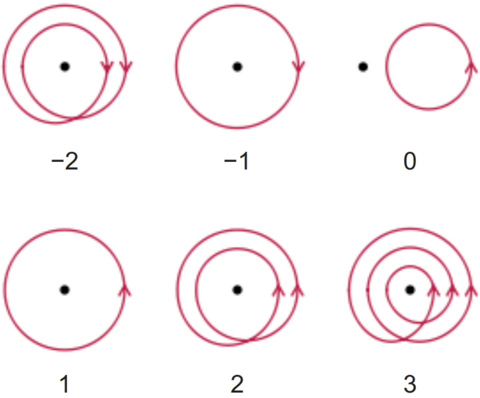


Determining whether a point is inside or outside

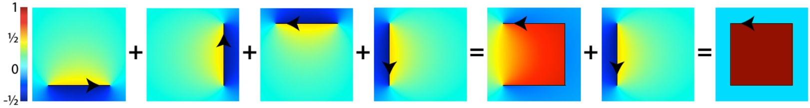
- Challenges
  - Open boundary
  - Degenerate geometry
  - Self-intersections
  - Non-manifold
  - Fast Enough!

What is Winding Number?

- For 2D curve under polar coordinate, $\frac{\theta(1)-\theta(0)}{2\pi}$



What is Winding Number?

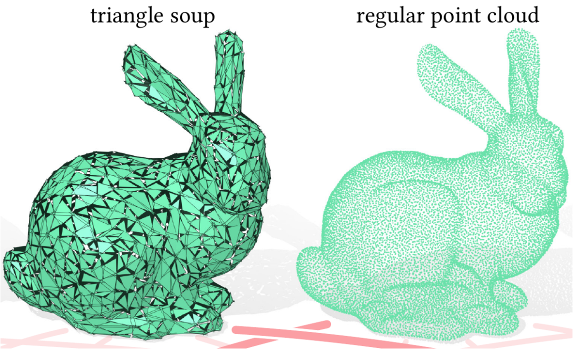


What is Winding Number?

- For suface, $w_S(\mathbf{q}) = \frac{1}{4\pi}\int_S d\Omega(\mathbf{q})$, $d\Omega$ is solid angle.

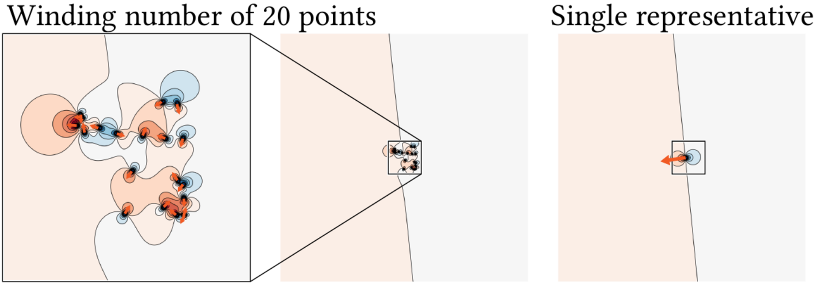
For Point Cloud

- $w_S(\mathbf{q}) \approx \sum_{i=1}^{m} a_i \frac{(\mathbf{p}_i - \mathbf{q}) \cdot \mathbf{n}_i}{4\pi ||\mathbf{p}_i - \mathbf{q}||^3}$
- a_i is the geodesic Voronoi area of the point on the surface.



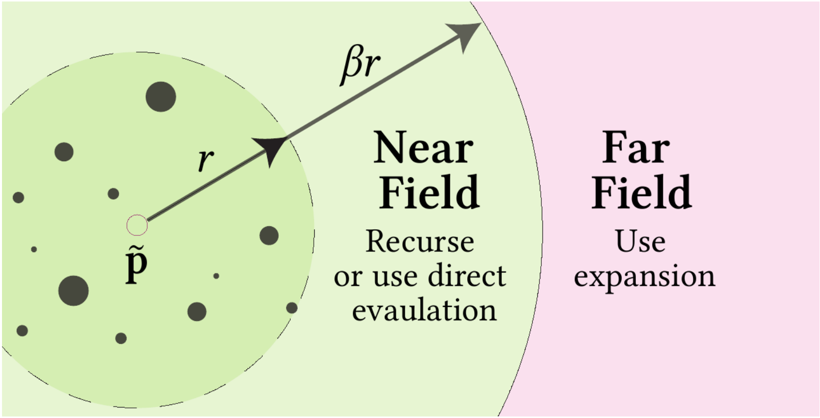
Fast Approximation

- Naive method: n query, m points, complexity: $\mathcal{O}(nm)$



Fast Approximation

- Approximation: use octree or AABB tree for partion the space



Taylor Expansion

- $\nabla G(\mathbf{q}, \tilde{\mathbf{p}}) = \frac{(\tilde{\mathbf{p}} - \mathbf{q})}{4\pi ||\tilde{\mathbf{p}} - \mathbf{q}||^3}$
- $\tilde{\mathbf{p}} = \sum a_i \mathbf{p}_i$
- For point cloud

$$
\begin{aligned}
w(\mathbf{q}) \approx & \left(\sum_{i=1}^{m} a_i \hat{\mathbf{n}}_i \right) \cdot \nabla G(\mathbf{q}, \tilde{\mathbf{p}}) \\
& + \left(\sum_{i=1}^{m} a_i (\mathbf{p}_i - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_i \right) \cdot \nabla^2 G(\mathbf{q}, \tilde{\mathbf{p}}) \\
& + \frac{1}{2} \left(\sum_{i=1}^{m} a_i (\mathbf{p}_i - \tilde{\mathbf{p}}) \otimes (\mathbf{p}_i - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_i \right) \cdot \nabla^3 G(\mathbf{q}, \tilde{\mathbf{p}})
\end{aligned}
$$

Taylor Expansion

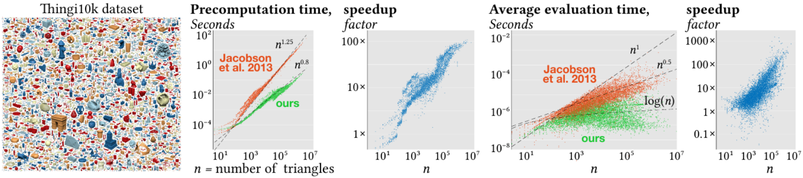
- $\nabla G(\mathbf{q}, \tilde{\mathbf{p}}) = \frac{(\tilde{\mathbf{p}} - \mathbf{q})}{4\pi ||\tilde{\mathbf{p}} - \mathbf{q}||^3}$
- $\tilde{\mathbf{p}} = \sum a_i \mathbf{p}_i$

- For triangle soups

$$
\begin{aligned}
w(\mathbf{q}) \approx & \left(\sum_{t=1}^{m} \int_t \hat{\mathbf{n}}_t \, dA \right) \cdot \nabla G(\mathbf{q}, \tilde{\mathbf{p}}) \\
& + \left(\sum_{t=1}^{m} \int_t (\mathbf{x} - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_t \, dA \right) \cdot \nabla^2 G(\mathbf{q}, \tilde{\mathbf{p}}) \\
& + \frac{1}{2} \left(\sum_{t=1}^{m} \int_t (\mathbf{x} - \tilde{\mathbf{p}}) \otimes ((\mathbf{x} - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_t) \, dA \right) \cdot \nabla^3 G(\mathbf{q}, \tilde{\mathbf{p}})
\end{aligned}
$$

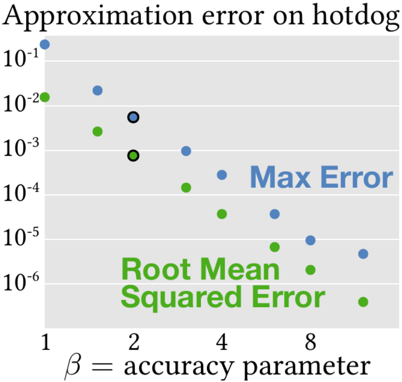
Experiments and Application

- Test on *Thingi10k*



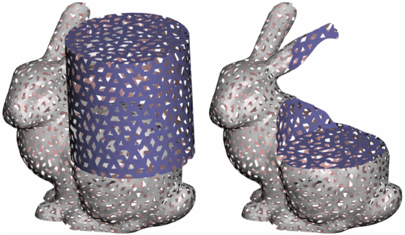
Experiments and Application

- Test on different β.



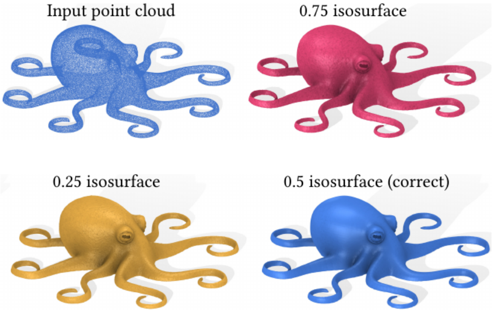
Application

- Boolean operation.



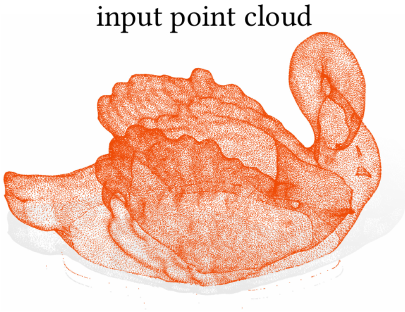
Application

- Extract $\frac{1}{2}$ iso-value surface



Application

- Directly output *toolpath* for 3D print



Application

- Voxelization for skining weight