Discrete Curvature Computation

work report

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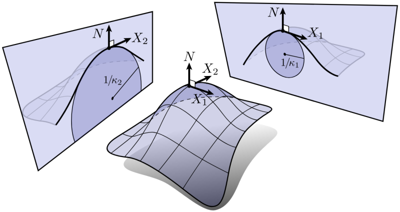
Outline

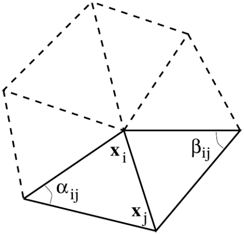
Review

- X is a tangent vector of the surface, B is a 2×2 matrix.
- Normal curvature is a quadratic form of tangent vector $X^T BX$.
- B is called curvature tensor.



Review

- $\kappa_H = \frac{1}{2} tr(B)$ is called mean curvature
- $\kappa_G = \det(B)$ is called Gauss curvature
- Last time, we use *Discrete Geometry Operator*, which computes
  - κ_H by linear combination of coordinate of vertices of 1-ring,
  - κ_G by angular defect, and
  - B by least square.



Targets

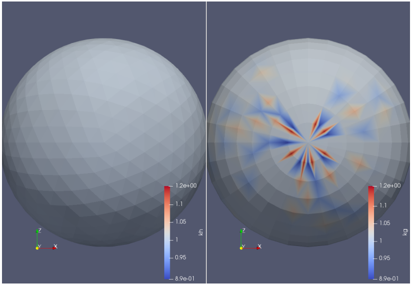
We want the curvature to be

- tessellation independent,
- feature-and-boundary aware, and
- adjustable radius of neighbourhood.

Issues

For example, angular defect method may not converge. Known sufficient condition are:

- vertex valence is 6, or
- the valence is 4 with the one-ring neighbours are aligned with the principal directions.



Issues

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Table 1
Convergence of the angular defect. Surface: $z = (2x^2 + y^2)/2$; Scenario #1, $n = 6$

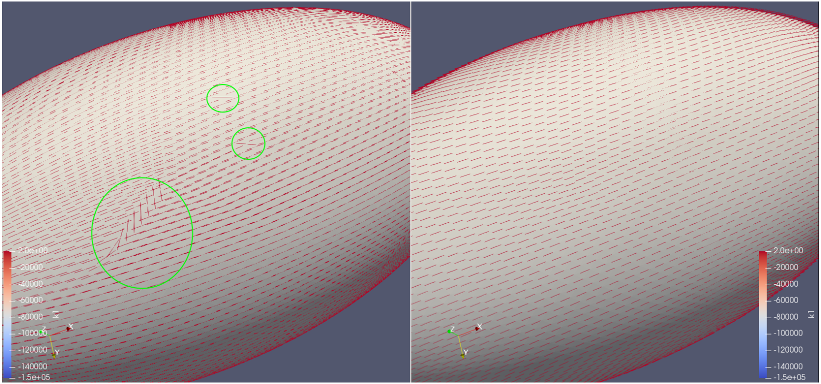
η	δ	δ/η^2	L
1.000	0.97151	0.97151	1.73205
0.500	0.35429	1.41716	1.73205
0.100	0.01716	1.71563	1.73205
0.010	0.00017	1.73188	1.73205
0.001	0.00000	1.73205	1.73205

Table 2
Convergence of the angular defect. Surface: $z = (2x^2 + y^2)/2$; Scenario #1, $n = 8$

η	δ	δ/η^2	L
1.000	0.91462	0.91462	1.61396
0.500	0.33104	1.32416	1.61396
0.100	0.01599	1.59887	1.61396
0.010	0.00016	1.61381	1.61396
0.001	0.00000	1.61396	1.61396

Issues

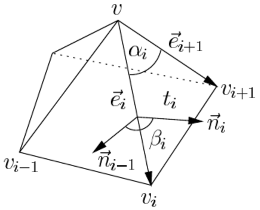
If you are doing least square wrongly...

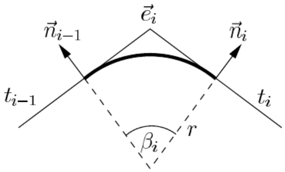


Normal cycle

Normal cycle: a kind of non-linear interpolation.

$$B = \sum_{e \in E} \beta(e) length(e) \vec{e} \otimes \vec{e}$$





Fitting

Polynomial fitting method consists of the following steps:

- Use BFS to gather enough vertexes in the ball of radius r.
- Set a local frame.
- Express gathered data in the local frame and fitting a polynomial.
- Evaluate the curvature tensor.

Reference

name	paper	implementation	curvature species
discrete geometry operator	[MDSB03]	VCG::MeanAndGaussian	mean curvature and gaussian curvature
normal cycle	[CSM03] [DHKL01]	VCG::PrincipalDirectionsNormalCycle and VCG::ComputeSingleVertexCurvature	mean curvature, gaussian curvature and principal curvature and principal direction
polynomial fitting	[CP05] [PPR10]	CGAL::Monge_via_jet_fitting::Monge_form, IGL::principal_curvature	mean curvature, gaussian curvature and principal curvature and principal direction
tensor fitting	[MDSB03] [Tau95]	VCG::PrincipalDirections	principal curvature and principal direction
PCA estimation	[YLH+06]	VCG::PrincipalDirectionsPCA	principal curvature and principal direction

Reference

- Frédéric Cazals and Marc Pouget.
  Estimating differential quantities using polynomial fitting of osculating jets.
  Computer Aided Geometric Design, 22(2):121–146, 2005.

- David Cohen-Steiner and Jean-Marie Morvan.
  Restricted delaunay triangulations and normal cycle.
  In *Proceedings of the nineteenth annual symposium on Computational geometry*, pages 312–321. ACM, 2003.

- Nira Dyn, Kai Hormann, Sun-Jeong Kim, and David Levin.
  Optimizing 3d triangulations using discrete curvature analysis.
  Mathematical methods for curves and surfaces, 1:135–146, 2001.

- Mark Meyer, Mathieu Desbrun, Peter Schröder, and Alan H Barr.
  Discrete differential-geometry operators for triangulated 2-manifolds.