Discrete Curvature Computation

Work Report

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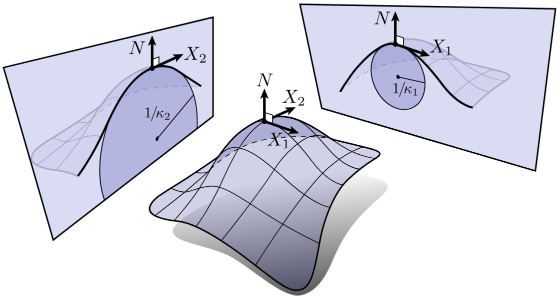
November 9, 2018

Outline

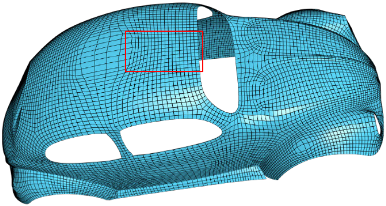
What is curvature?

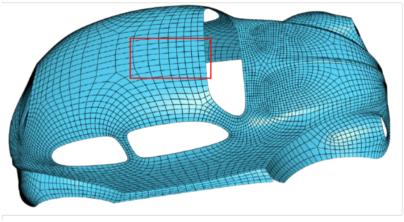
- A number indicating how blend a curve is, which lays on surface
- How fast does a curve leave the tagent plane?



Motivation

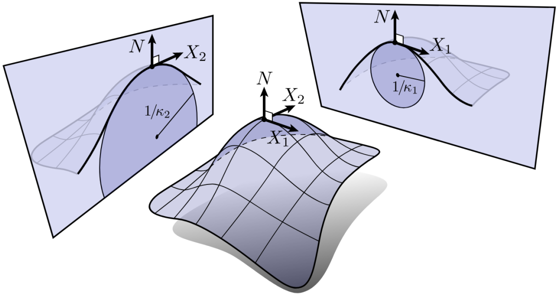
controlling the direction and size of quadrangulation





What is curvature

- A quadratic form of tagent vector $X^T B X$



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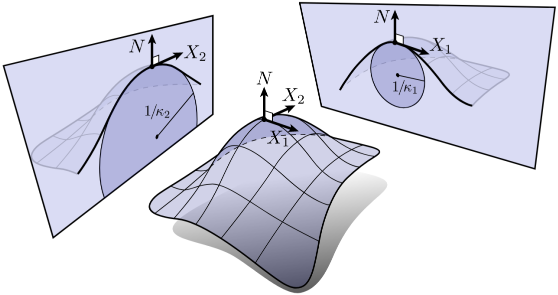

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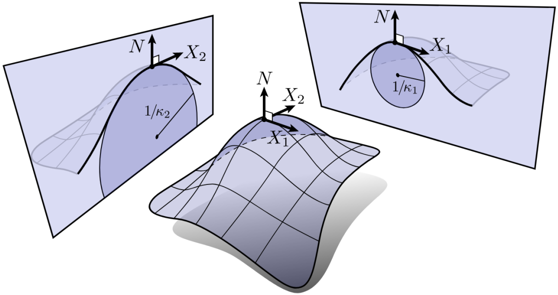
- B is a **linear map / tensor** from dX to dN
- mean curvature $\kappa_H = tr(B)/2$



How to compute **mean curvature** on vertex?

- multiply by normal vector $\mathbf{K}(\mathbf{x}) = \kappa_H \mathbf{n}$
- $\int \int_{\mathcal{A}_M} \mathbf{K}(\mathbf{x}) dA = \frac{1}{2} \sum_{j \in N_1(i)} (cot\alpha_{ij} + cot\beta_{ij})(\mathbf{x}_i - \mathbf{x}_j)$

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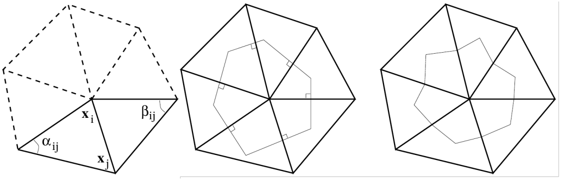

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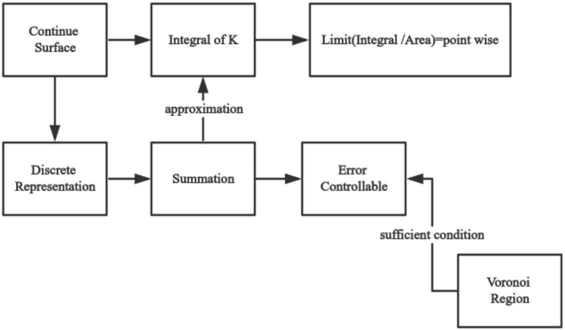

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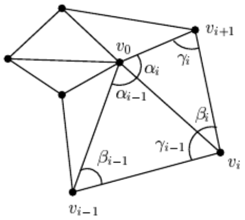

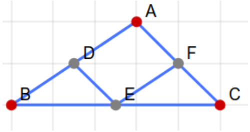

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- $E(a,b,c) = \sum_j w_j(\mathbf{d}_{i,j}^T B \mathbf{d}_{i,j} - \kappa_{i,j}^N)^2 \ s.t. \ a + c = 2\kappa_H, \cancel{ac - b^2 = \kappa_G}$

Problems

- Some pontential trouble makers
  1. Hard constrain of κ_H is replaced by soft constrain.
  2. w_js are not used
  3. Area of computating κ_H is just barycenter area, and cotangents of obtuse angles are not handled, which may cause some negative weight of summation of κ_H.
  4. [MDSB03] states that "the curvature values computed from the lesast squares are often less accurate in practice".
  5. We compute curvature of facet as the average of vertex with uniform weight

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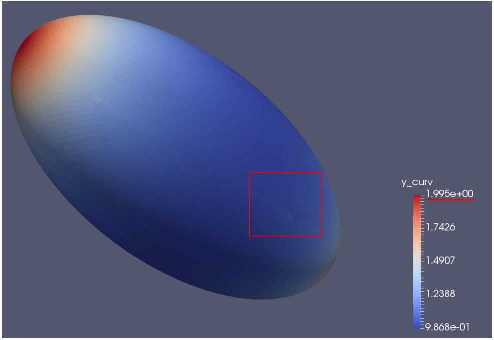

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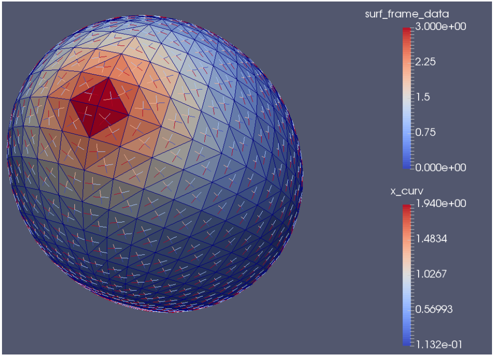
- reflection



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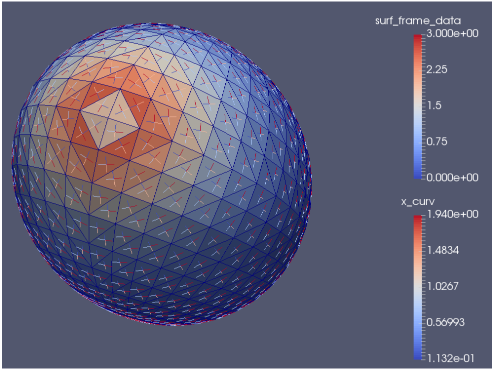

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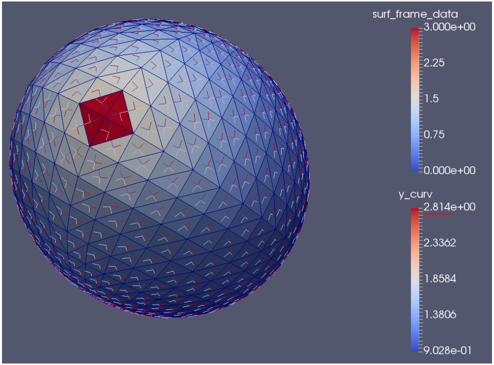

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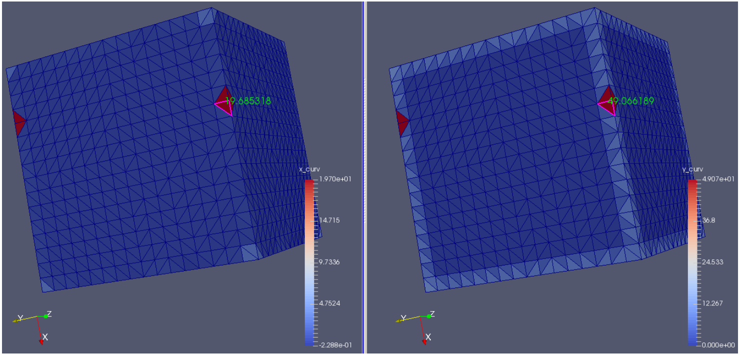
- burst



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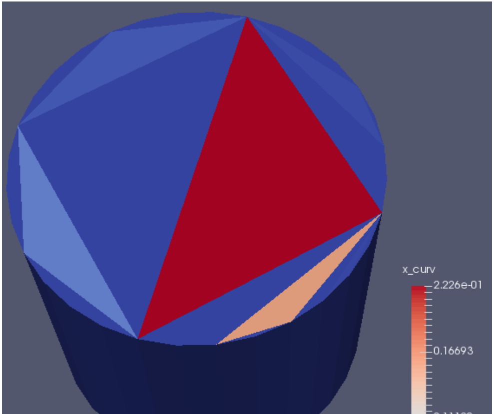
- spreading



Ask more questions

- Pipeline:
  $weight1 * principal\_curvature + weight2 * feature\_line \rightarrow constrain \rightarrow metric \rightarrow frame\_field \rightarrow quad$
- How to reduce weight of bad *principal_curvature*?
  1. conflict with feature
  2. "flat/sphere like" region
     current solution: $weight = min(|e^{|pc_1 - pc_2|} - 1|, 100)$, still fail on the previous big triangle.
  3. other solution: subdivide or other way to change the bad tessellation.

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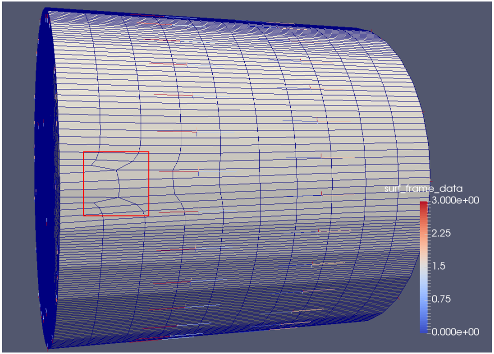
- what is the difference between *metric* and *frame_field*?

Other burst



Reference

Mark Meyer, Mathieu Desbrun, Peter Schröder, and Alan H Barr, *Discrete differential-geometry operators for triangulated 2-manifolds*, Visualization and mathematics III, Springer, 2003, pp. 35–57.