Gilbert-Johnson-Keerthi Algorithm

Minliang LIN

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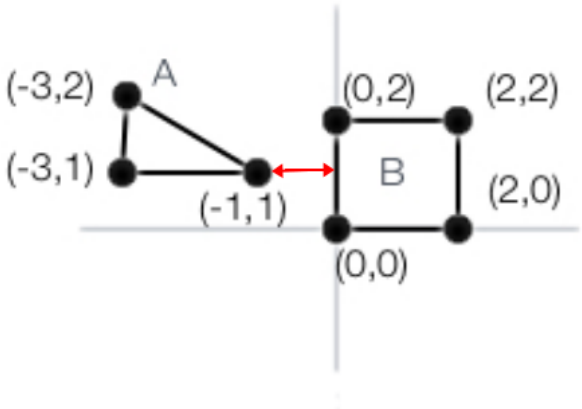
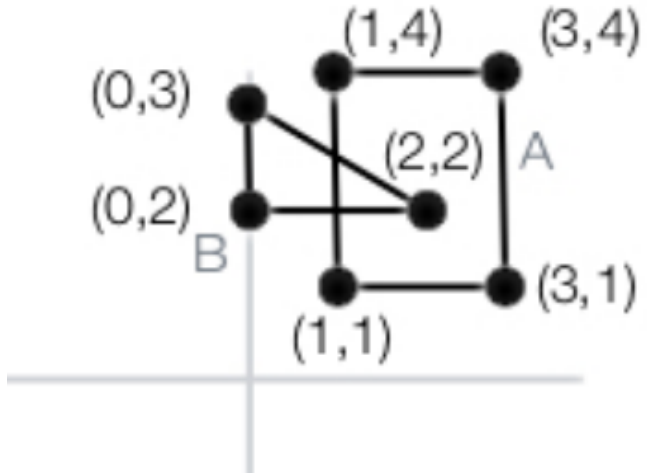
Outline

Minimum distance of two convex sets

- $minDistance(A, B) = min\{\|a - b\| \mid a \in A, b \in B\}$



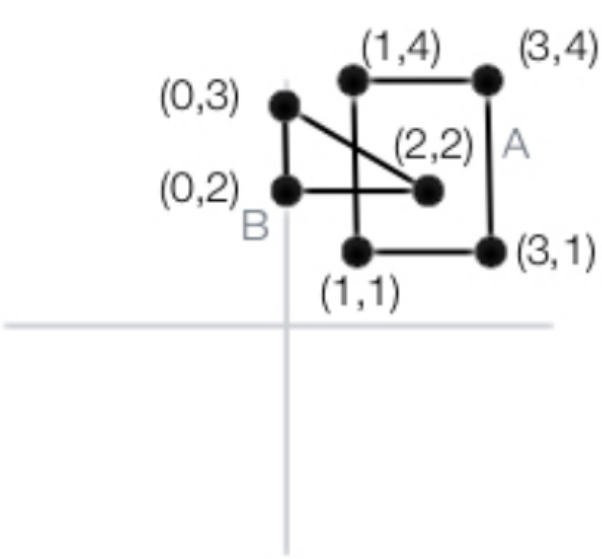
min distance $= 1$

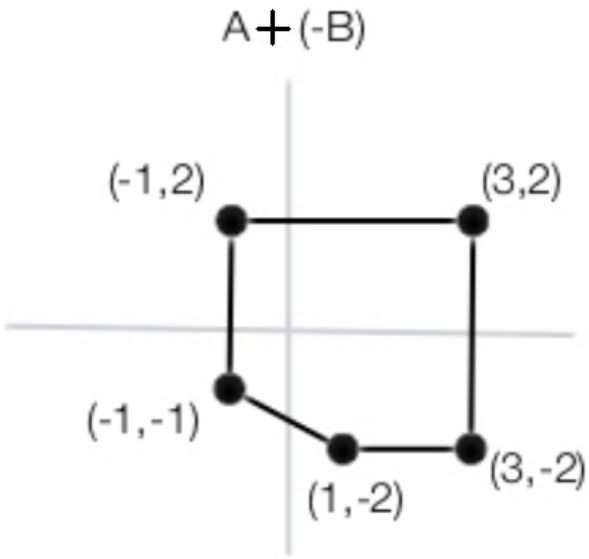


min distance $= 0$

Minkowski Sum

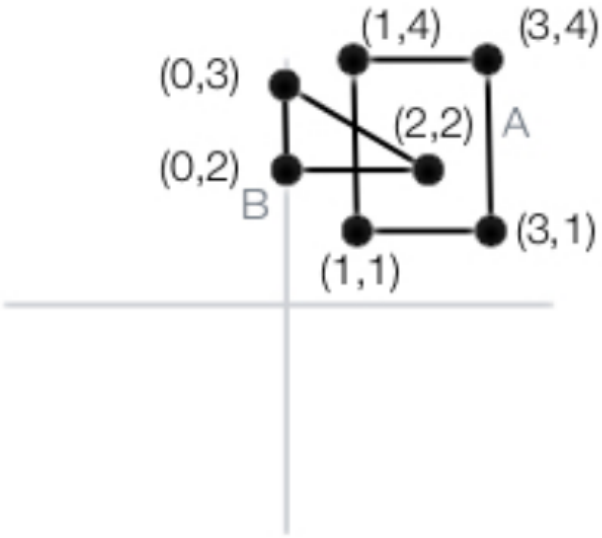
- $C = A + (-B) = \{a - b | a \in A, b \in B\}$

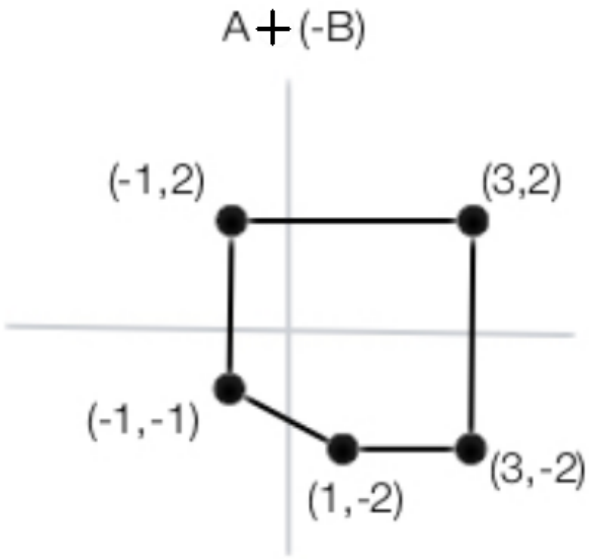




Minkowski Sum

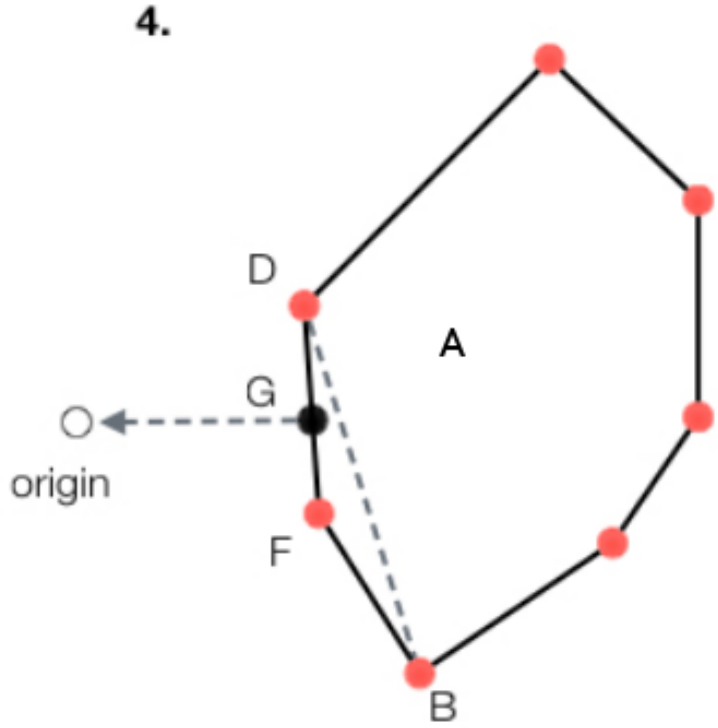
- $C = A + (-B) = \{a - b | a \in A, b \in B\}$
- $minDistance(A, B) = min\{\|c\| \mid c \in C\}$





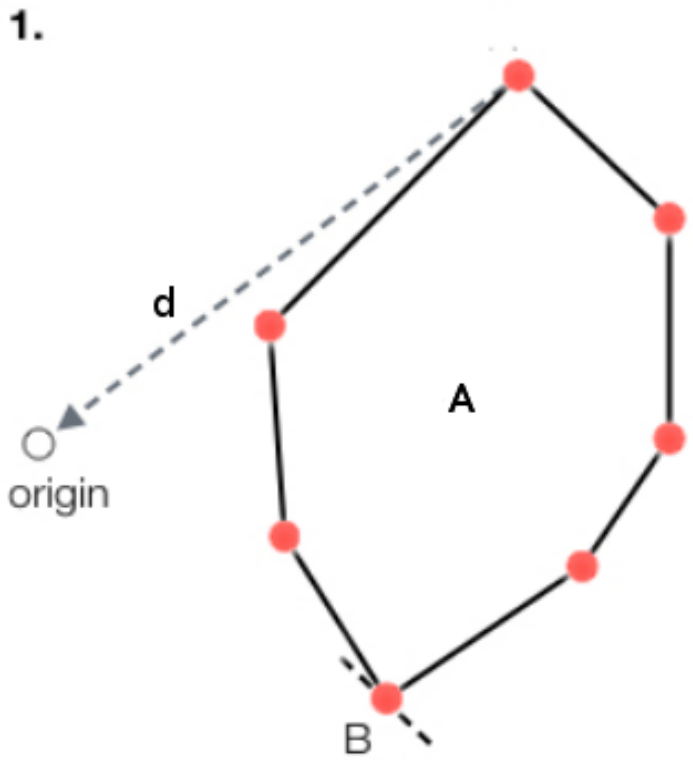
The Main Idea of GJK for **Collision Detection**

- Try to enclose the origin with a triangle/tetrahedron/line/point(i.e. simplex) $\tau_k \subseteq C$ iteratively.
- $min \left\| t \right\| \approx min \left\| c \right\|, t \in \tau_k, c \in C$.



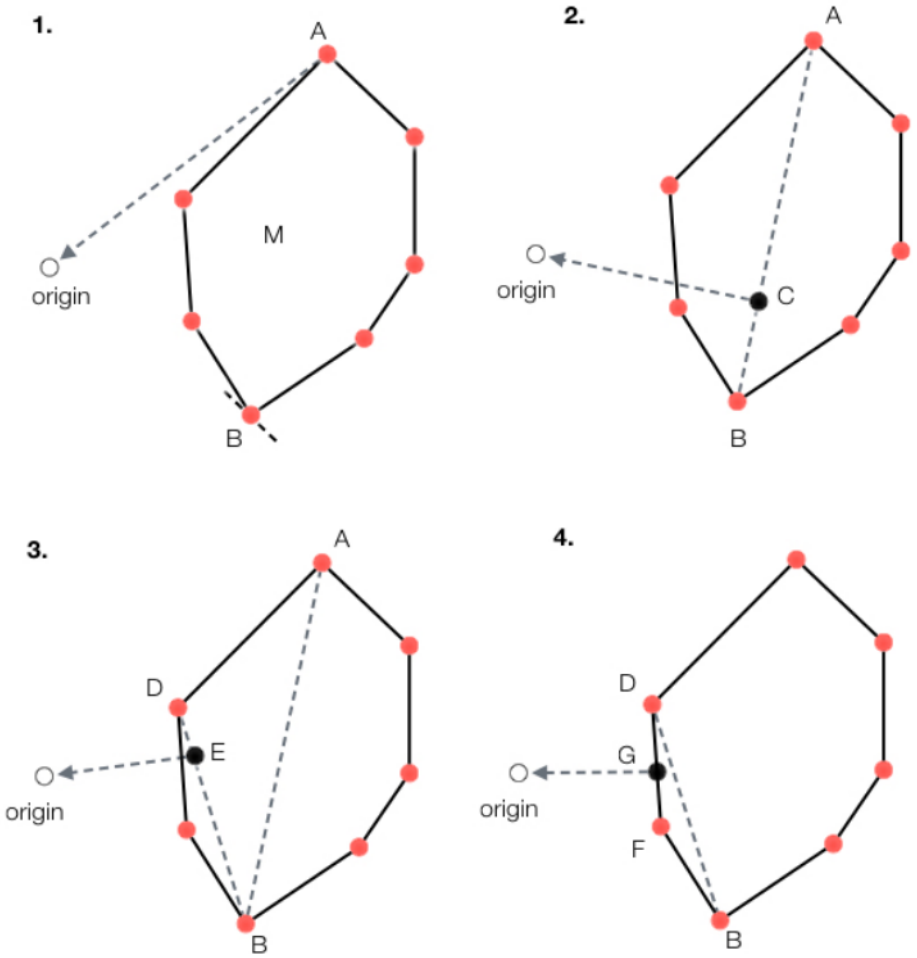
The Main Idea of GJK for **Collision Detection**

- Given set A and direction d,
  $Support(A, d) = \{s | s \in A, s \cdot d = max\{w \cdot d \mid w \in A\}\}$



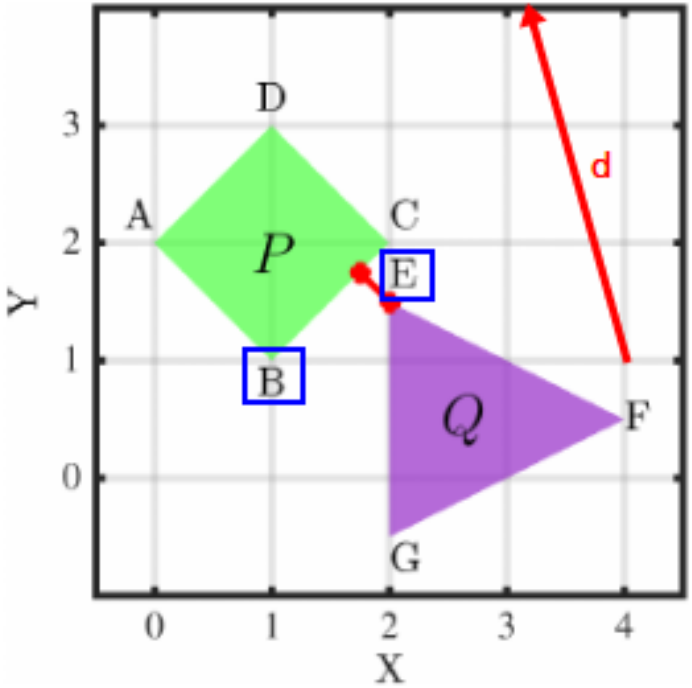
The Main Idea of GJK for **Collision Detection**

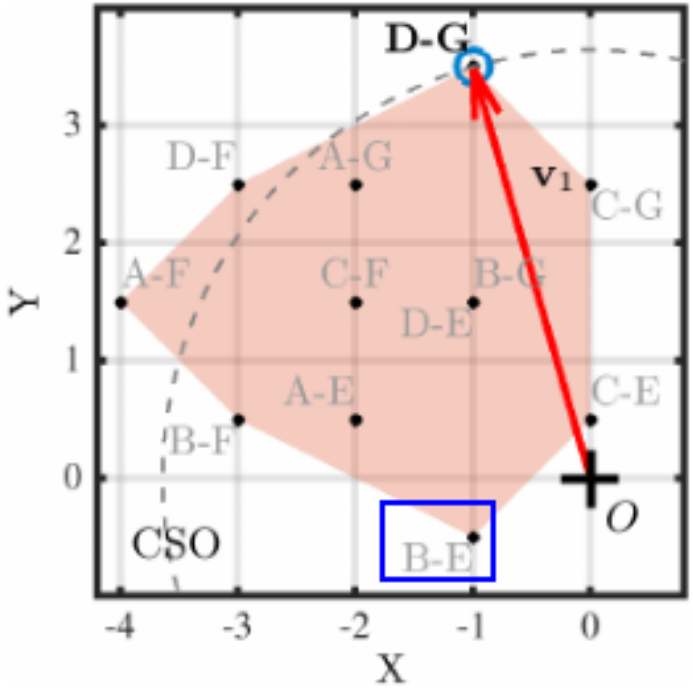
```
S = Support(C, random_direction)
[] = S
D = -S
Loop:
  S = Support(C, D)
  If dot(S, D) < 0:
    NO INTERSECTION, BREAK
  [] += S
  [], D, contains_origin = NearestSimplex([])
  If contains_origin
    INTERSECTION, BREAK
```



Finding Support

Simple for polytope:
$Support(A + (-B), d) = Support(A, d) - Support(B, -d)$
If A is m-polytope, B is n-polytope. Finding support is $O(m + n)$.





Update Simplex and Direction

Given simplex τ and direction d.

- Enumerate all Voronoi region to locate the origin

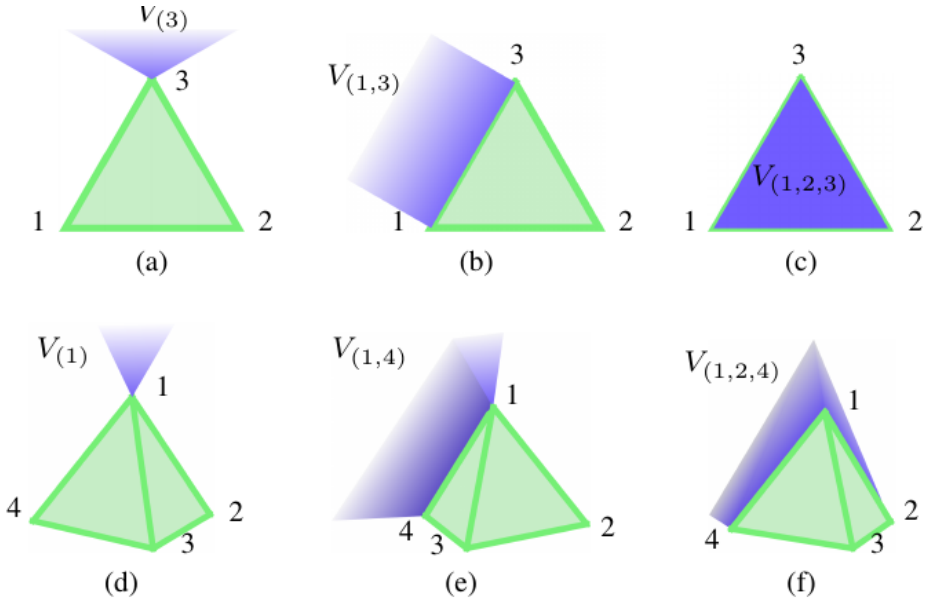
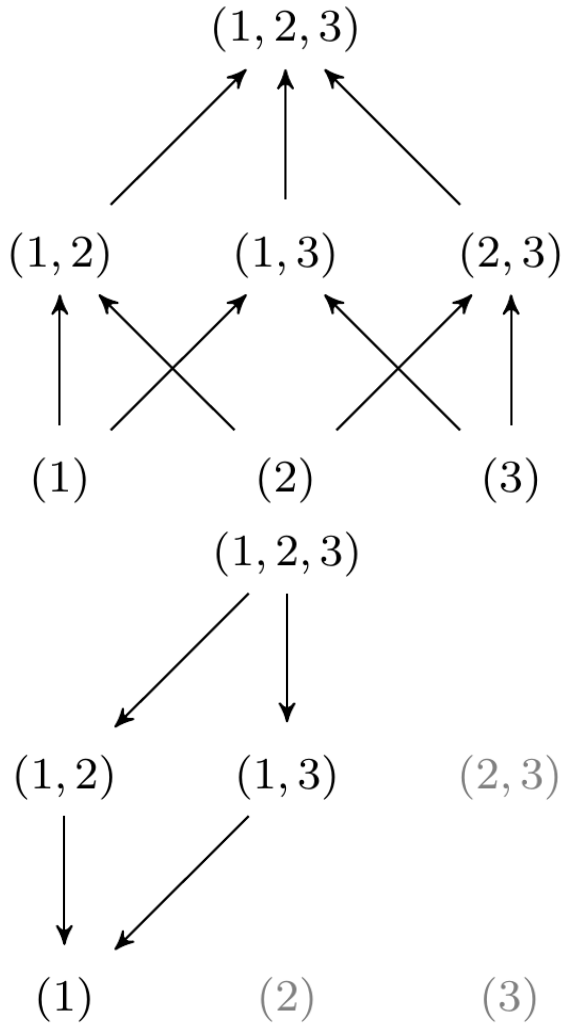


Fig. 4: Voronoi regions of a 2-simplex (a)-(c) and 3-simplex (d)-(f).

Update Simplex and Direction

Given simplex τ and direction d.

- Different enumerative method



Update Simplex and Direction

Given simplex τ and direction d.

- My understanding: minimum square or projection.
  - If $O \in \tau$, return INTERSECTION
  - Else update the direction