1.

a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Because in the column $\neg(p \vee q)$ and $\neg p \wedge \neg q$, each line is same. Therefore, $\neg(p \vee q) \equiv \neg p \wedge \neg q$

b)

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Because in the column $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$, each line is same. Therefore $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

2. $(p \vee q) \to r \equiv \neg(p \vee q) \vee r$ by Theorem *

$\equiv (\neg p \wedge \neg q) \vee r$ by DeMorgan's law

$\equiv r \vee (\neg p \wedge \neg q)$ by the commutative law

$\equiv (r \vee \neg p) \wedge (r \vee \neg q)$ by the distributive law

$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$ by the commutative law

$\equiv (p \to r) \wedge (q \to r)$ by Theorem *

3.

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

- when p is false and q is true, and p is true and q is false, $p \to q$ is not equal to $q \to p$.
- When p and q are both true and both false, $p \to q$ is equal to $q \to p$.

4. $[(p \to q) \wedge (p \to \neg q)] \to \neg p$

$\equiv \neg[(\neg p \vee q) \wedge (\neg p \vee \neg q)] \vee \neg p$ by Theorem *

$\equiv \neg[\neg p \vee (q \wedge \neg q)] \vee \neg p$ by the distribution law

$\equiv \neg[\neg p \vee \bot] \vee \neg p$ by the negation law

$\equiv \neg(\neg p) \vee \neg p$ by the identity law

$\equiv p \vee \neg p$ by double negative law

$\equiv T$ by the negation law

$\boxed{5}$ (31)

a) 01, 02, 11, 12

b) 21, 22

c) 11, 10, 21, 20

$\boxed{6.}$ (46.a)

$p \oplus p \equiv (p \vee p) \wedge \neg(p \wedge p)$

$\equiv p \wedge \neg p$ by the idempotent laws

$\equiv \bot$ by the negation law

$(p \oplus p) \oplus p \equiv \bot \oplus p$

$\equiv (\bot \vee p) \wedge \neg(\bot \wedge p)$

$\equiv p \wedge \neg \bot$ by the identity and universal bound

$\equiv p \wedge T$ by tautology & contradiction negation

$\equiv p$ by identity laws

$\boxed{7.}$ 22, 23 / b, c, e, g

Contrapositive :

b: If tomorrow is not January, then today is not New Year's Eve.

c: If r is not rational, then the decimal expansion of r is not terminating.

e: If x is not positive and x is not 0, then x is negative.

g: If n is not divisble by 2 or n is not divisible by 3, then n is not divisible by 6.

Converse:

b: If tomorrow is January, then today is New Year's Eve.

c: If r is rational, then the decimal expansion of r is terminating.

e: If x is positive or x is 0, then x is nonnegative.
g: If n is divisible by 2 and n is divisible by 3, then n is divisible by 6.

Inverse:
b: If today is not New Year's Eve, then tomorrow is not January.

c: If the decimal expansion of r is not terminating, then r is not rational.
e: If x is negative, then x is not positive and x is not 0.
g: If n is not divisible by 6, then n is not divisible by 2 or n is not divisible by 3.

$\boxed{8}$ 2.2
14/a $p \to q \vee r$, $p \wedge \neg q \to r$, $p \wedge \neg r \to q$

$p \to q \vee r \equiv \neg p \vee (q \vee r)$ by theorem *
$\equiv \neg p \vee q \vee r$ by associativity law

$p \wedge \neg q \to r \equiv \neg (p \wedge \neg q) \vee r$ by theorem *
$\equiv \neg p \vee q \vee r$ by De Morgan's law
$\equiv p \to q \vee r$

$$p \wedge \neg r \to q \equiv \neg(p \wedge \neg r) \vee q \quad \text{by theorem *}$$
$$\equiv \neg p \vee r \vee q \quad \text{by De Morgan's law}$$
$$\equiv \neg p \vee q \vee r \quad \text{by commutativity law}$$
$$\equiv p \to q \vee r$$

$$\therefore\ p \to q \vee r \equiv p \wedge \neg q \to r \equiv p \wedge \neg r \to q$$

14/b'

If n is prime and n is not odd, then n is 2.
If n is prime and n is not 2, then n is odd.

38. If it doesn't rain, then Ann will go.

43. If Jim doesn't do homework regularly, then he would not pass the course.

If Jim does homework regularly, then he would pass the course.

$\boxed{9}$

2.3/9.

conclusion: $\neg r$; premises: $p \wedge q \to \neg r$, $p \vee \neg q$, $\neg q \to p$

p	q	r	$\neg r$	$\neg q$	$p \wedge q$	$p \wedge q \to \neg r$	$p \vee \neg q$	$\neg q \to p$
T	T	T	F	F	T	F	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	F	T	F	T
F	T	F	T	F	F	T	F	T
F	F	T	F	T	F	T	T	F
F	F	F	T	T	F	T	T	F

This row shows that it is possible for an argument of this form to have true premises and a false conclusion.

Thus this argument form is invalid

12 (b):

premises: $\neg p$, $p \to q$; conclusion: $\neg q$

p	q	$\neg p$	$p \to q$	$\neg q$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

This row shows that it is possible for an argument of this form to have true premises and a false conclusion. Thus this argument form is invalid.

23. let p: "Oleg is a math major"
q: "Oleg is an economics major"
r: "Oleg is required to take Math 362"

Then argument:

$$p \vee q$$
$$p \to r$$
$$\therefore q \vee \neg r$$

p	q	r	$\neg r$	$p \vee q$ (premise)	$p \to r$ (premise)	$q \vee \neg r$ (conclusion)
T	T	T	F	T	T	T
T	T	F	T	T	F	T
T	F	T	F	T	T	F
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	T

This raw shows that it is possible for an argument of this form to hae true premises and a false conclusion. Thus this argument form is invalid.

$\boxed{10}$.

#29 $p \to q$

$\neg p$

$\therefore \neg q$

invalid: inverse error

38(d)

W and Y are knights,

U, V, X and Z are knaves

$\boxed{11}$ #42

1) $q \to r$ by (b)

$\sim r$ by (d)

$\therefore \sim q$ by Modus Tollens

2) $\sim q \to u \wedge s$ by (e)

$\sim q$ by (1)

$\therefore u \wedge s$ by Modus Ponens

3) $u \wedge s$ by (2)

$\therefore s$ by specialization

4) $\sim q$ by (1)

$p \vee q$ by (a)

$\therefore p$ by elimination

5) p by (4)

s by (3)

$\therefore p \wedge s$ by conjunction

6) $p \wedge s \to t$ by (c)

$p \wedge s$ by (5)

$\therefore t$ by Modus Ponens

#44 1) $\sim s$ by (e)

$\sim s \to \sim t$ by (c)

$\therefore \sim t$ by modus ponens

2) $\sim t$ by (1)

$w \vee t$ by (g)

$\therefore w$ by elimination

3) $\sim s$ by (e)

$r \vee s$ by (b)

$\therefore r$ by elimination

4) $\sim s$ by (e)

$\sim q \vee s$ by (d)

$\therefore \sim q$ by elimination

5) $p \to q$ by (a)

$\sim q$ by (4)

$\therefore \sim p$ by modus tollens

6) $\sim p$ by (5)

r by (3)

$\therefore \sim p \wedge r$ by conjunction

7) $\sim p \wedge r$ by (6)

$\sim p \wedge r \to u$ by (f)

$\therefore u$ by modus ponens

8) u by (7)

w by (2)

$u \wedge w$ by conjunction

$\boxed{12}$

3.1/4. a) $Q(2,1)$: "If $-2<1$ then $(-2)^2<1^2$"

hypothesis: $x=-2, y=1$, $x<y$, true

conclusion: $x^2=4$, $y^2=1$, $4>1$, $x^2>y^2$, false

$\therefore Q(2,1)$ is a conditional statement with a true hypothesis and a false conclusion. So $Q(-2,1)$ is false.

b) $Q(-5, 2)$

c) $Q(3,8)$: "If $3<8$, then $3^2<8^2$"

hypothesis: $x=3$, $y=8$, $x<y$, true

conclusion: $x^2=9$, $y^2=64$, $x^2<y^2$;

$\therefore Q(3,8)$ is a conditional statement with a true hypothesis and true conclusion. So $Q(3,8)$ is true.

d) $Q(4, 9)$

/20. The square root of a real positive number is positive.

The square root of a number must be positive if the number is a positive real number.

/32 b) true

For any real number that is larger than 2, its square is larger than 4.

d) true

for any real number, if its square is greater than 4, its absolute value must greater than 2; also if a real number's absolute value is larger than 2, its square must larger than 4.

$\boxed{13}$

3.2/ 15(b) true

15(d) true

15(e) false counterexample: 36

$\boxed{14}$

3.2/12. Not correct.

negation: There is an irrational number and a rational number and their product is rational.

40. If a number is divisible by 8, then it is divisible by 4.

46. There is a person who has a large income and the person is unhappy.