$\boxed{1.}$

3.3 / 43.

There exists a real number $\varepsilon > 0$ such that for every real number $\delta > 0$, there exists a real number x such that $a - c < x < a + c$ and $x \neq a$, and $f(x)$ does not satisfy $L - \varepsilon < f(x) < L + \varepsilon$

3.3/44

a. The statement is true. The unique number is 1. Because $1 \cdot y = y$ for all real numbers y. If y is any real number such that for instance, $x \cdot 2 = 2$, then deviding both sides by 2 gives $x = \frac{2}{2} = 1$.

b. The statement is false. The number with the given property can be 1 and -1. Because $\frac{1}{1} = 1$, $\frac{1}{-1} = -1$, 1 and -1 are both integers, and therefore, the statement is false.

c. The statement is true. The unique number y can be $-x$. Because $x + y = 0$ shows $x = -y$, so $y = -x$. If x is any real number such that for instance, $2 + y = 0$, then substracting both sides by 2 gives $y=-2$. So y is unique and it can only be $-x$.

45.

There exists a unique x belongs to D such that $P(x)$.

$\boxed{2}$

3.3 15b $\exists x \in D, (P(x) \wedge Q(x))$ and $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$

They have the same truth value.
If $\exists x \in D, (P(x) \wedge Q(x))$ is true, then $P(x) \wedge Q(x)$ is true for one x. $\therefore$ $P(x)$ is true and $Q(x)$ is true
therefore, the second statement is true

3.3 57 $\forall x \in D (P(x) \vee Q(x))$, and $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$
True. $P(x) \vee Q(x)$ is true, at least one is true.
Second statement is true if $P(x)$ or $Q(x)$ is true.
$\therefore$ They have same true value

3. 3.4 #30

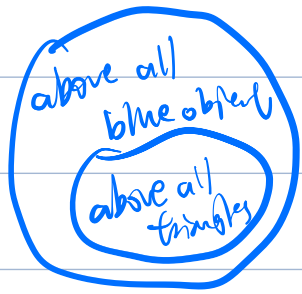
a) 1. ∀ object x, if x is above all the triangles, then x is above all the blue objects.

2. ∀ object x, if x is a square, then x is above all the gray objects.

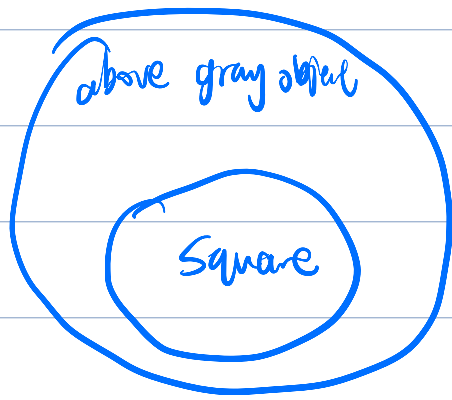
3. ∀ object x, if x is black, then x is a square.

4. ∀ object x, if x is above all the gray objects, then x is above all the triangles.

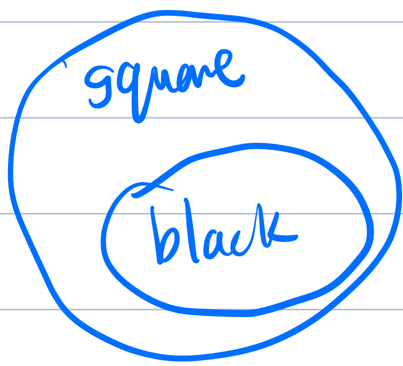
b) 1.



2.



3.



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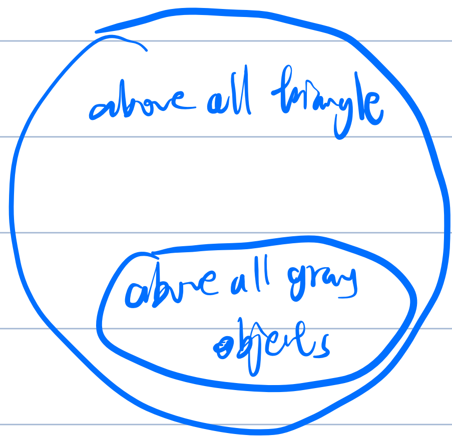


c) 3. ∀ object, if x black then x is a square

2. ∀ object x, if x is a square, then x is above all the gray objects

4. ∀ object x, if x is above all the gray objects, then x is above all the triangles.

1. ∀ object x, if x is above all the triangles, then x is above all the blue objects.

$\boxed{4}$ 3.4/32

a) 1. ∀ logic example x, if I work x without grumbling, it is the one I understood.

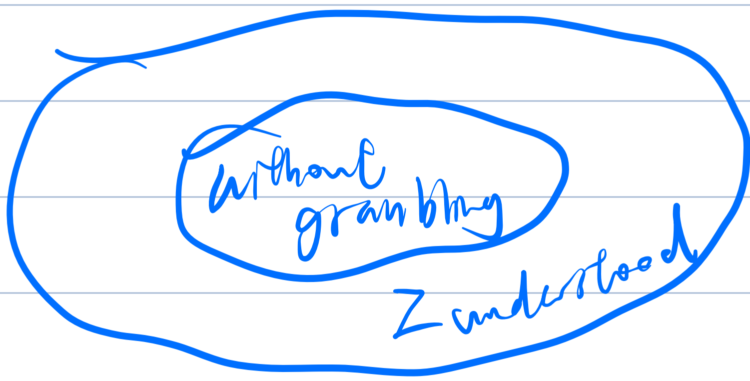
2. ∀ logic example x, if arguments in x is not arranged in regular order, then I'm not used to x.

3. ∀ logic example x, if my head ache happens, x is not easy.

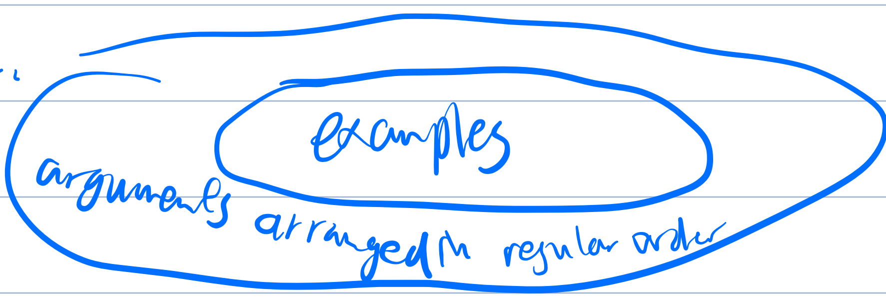
4. ∀ logic example x, if arguments are not arranged in regular order like the ones I'm used to, then I cannot understood.

5. ∀ logic example, if I grumble at x, then x gives me a headache.

b) 1.



2.



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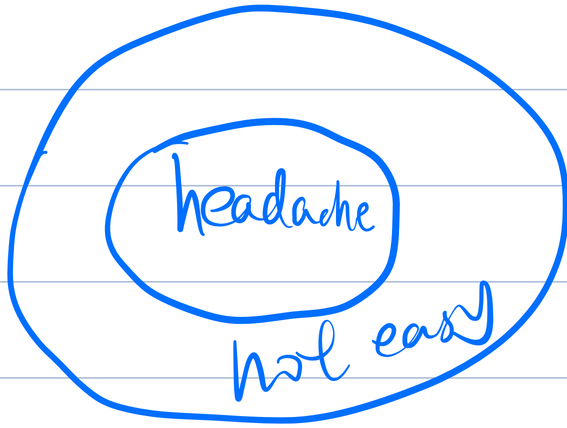
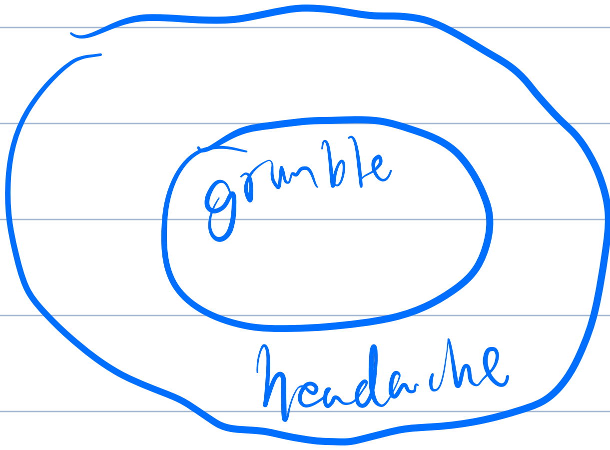


4.

5



c) 2. $\forall$ logic example x, if arguments in x is not arranged in regular order, then I'm not used to x

4. $\forall$ logic example x, if arguments are not arranged in regular order like the ones I'm used to, then I cannot understood

1. $\forall$ logic example x, if I work x without grumbling, it is the one I understood

5. $\forall$ logic example, if I grumble at x, then x gives me a headache.

3. $\forall$ logic example x, if my headache happens, x is not easy.

3.4 / #34

a) 1. ∀ writer a, if a understand human nature, then a is clever

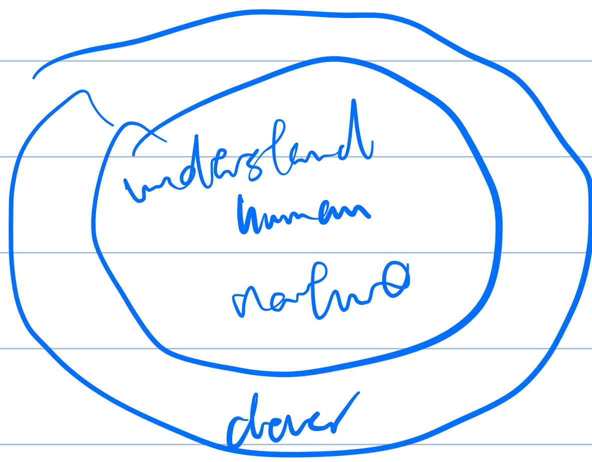
2. ∀ person a, if a is a true poet, then he can stir the human heart.

3. ∀ writer a, if a is Shakespeare, then a write Hamlet.

4. ∀ writer a, if a can stir the human heart, then he understand human nature.

5. ∀ poet b, if b write Hamlet, b is a true poet.

b) 1.



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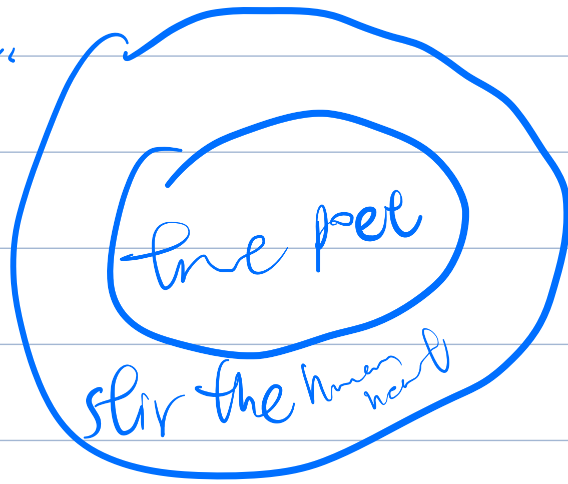

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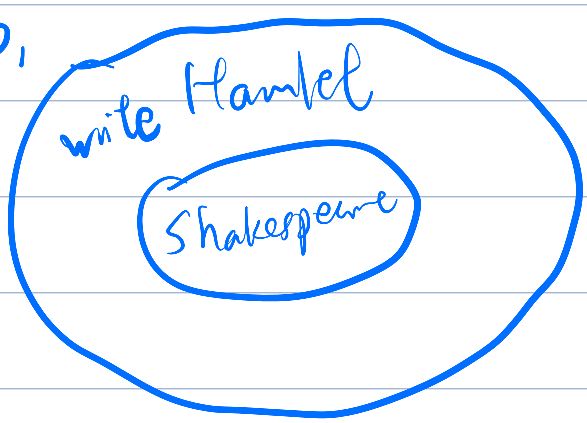

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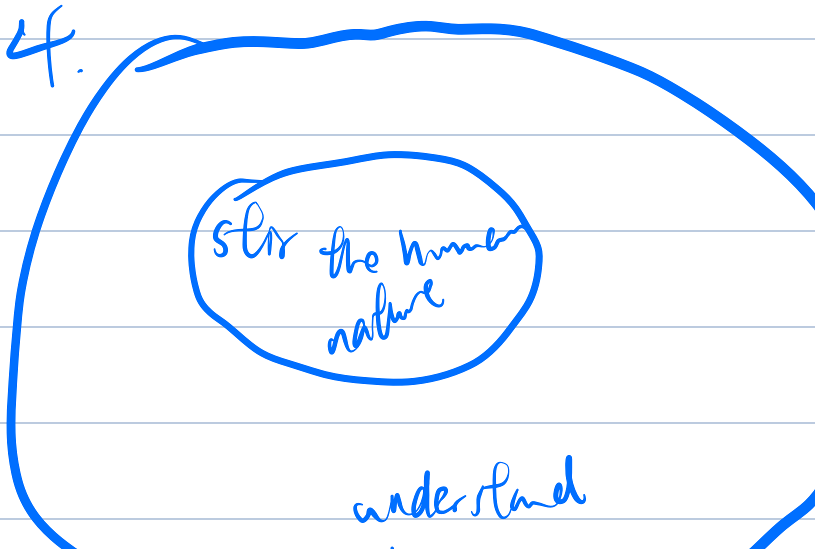
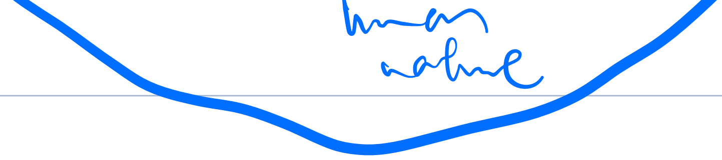
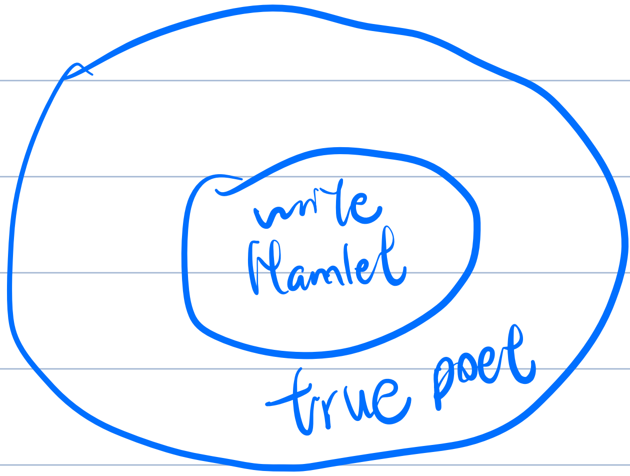




5.



3 ∀ writer a, if a is Shakespeare then a write Hamlet

5. ∀ poet b, if b write Hamlet, b is a true poet.

2. ∀ person a, if a is a true poet, then he can stir the human heart.

4. ∀ writer a, if a can stir the human heart, then he understand human nature.

1. ∀ writer a, if a understand human nature, then a is clever

$\boxed{5}$ 4.1/#7

For example, let $a=0$, $b=4$,

Then $a, b \in \mathbb{R}$ and

$$\sqrt{a+b} = \sqrt{0+4} = 2$$

$$\sqrt{a}+\sqrt{b} = 0+\sqrt{4} = 2$$

$$\therefore \sqrt{a+b} = \sqrt{a}+\sqrt{b}$$

#11

For example, for $25=5^2$, $9=3^2$, $16=4^2$,

25 is a perfect square that is the sum of 16 and 9, while 16 and 9 are both perfect square.

#16 For example, $n=8$. $n \in \mathbb{Z}$, $n=2k$, $k \in \mathbb{Z}$

$n^2+1 = 8^2+1 = 65$ $65 = 5 \times 13$

65 is not a prime number.

$\therefore$ The statement is wrong

$\boxed{6}$ 4.1 #22

$n=1$, $n^2-n+11 = 1-1+11 = 11$

$n=2$, $n^2-n+11 = 4-2+11 = 13$

$n=3$, $n^2-n+11 = 9-3+11 = 17$

$n=4$, $n^2-n+11 = 16-4+11 = 23$

$n=5$, $n^2-n+11 = 25-5+11 = 31$

$n=6$, $n^2-n+11 = 36-6+11 = 41$

$n=7$, $n^2-n+11 = 49-7+11 = 53$

$n=8$, $n^2-n+11 = 64-8+11 = 67$

$n=9$, $n^2-n+11 = 81-9+11 = 83$

$n=10$, $n^2-n+11 = 100-10+11 = 101$

#29 (a) $\forall$ integer a, if a is even then $-a$ is even.

$\forall$ even integer a, $-a$ is even.

If a is any even integer, then $-a$ is even.

(b) (a) definition of even

(b) multiply both side with (-1).

(c) any product of integers is an integer

(d) any product of integers is an integer.

$\boxed{7}$

4.2 #8

Suppose $m \in 2k$ $k \in \mathbb{Z}$, $n \in 2k_2+1$, $k_2 \in \mathbb{Z}+1$

$5m+3n = 5\cdot 2k + 3\cdot(2k_2+1)$ by substitution

$= 10k + 6k_2 + 3$ by algebra

$= 2(5k+3k_2+1)+1$ by algebra

$5k+3k_2+1 \in \mathbb{Z}$ by sum of integer is integer.

let $r \in 5k+3k_2+1$ $\therefore r \in \mathbb{Z}$

$\therefore 5m+3n = 2r+1$ by substitution

$2r+1 \in$ odd by definition of odd

$\therefore 5m+3n \in$ odd number

4.2 #9

Suppose $n > 4$, $n = k^2$ $k \in \mathbb{Z}$

$k^2 > 4$ substitution

$k > 2$ square root

let $k = 2 + k_1$ $k_1 \in \mathbb{Z}^+$

$n-1 = k^2 - 1$

$= (2+k_1)^2 - 1$ substitution

$= (2+k_1+1)(2+k_1-1)$ Algebra

$= (k_1+3)(k_1+1)$ Algebra

$r_1 = k_1+3 \in \mathbb{Z}$, $r_2 = k_1+1 \in \mathbb{Z}$

$\therefore n-1 = r_1 \cdot r_2$, $r_1 > 1$, $r_2 > 1$ substitution

$\therefore n-1$ is not prime definition of prime

4.2 #14

Suppose $k \in \mathbb{Z}$, $k \geq 4$

$2k^2 - 5k + 2 = (2k-1)(k-2)$

$\because k \geq 4$

$\therefore 2k-1 > 1$, $k-2 > 1$

$\therefore 2k^2-5k+2$ is not prime definition of prime

$\boxed{Q\ 8}$ #18

"$m \cdot n$ is even" can not be assumed

#19 "$n = 2k$," "$m = 2k$," cannot use same constant k,

should be k_1 and k_2

$\boxed{\#9}$ False

#2b Suppose $a \in \mathbb{Z}$,

$b = a+1, \in \mathbb{Z}$

$c = b+1 = a+2, \in \mathbb{Z}$

$a \cdot b \cdot c = a+(a+1)+(a+2)$

Counter example $a, b, c = 2, 3, 4$

$a \cdot b \cdot c = 9 \in$ even

$\therefore$ false statement

30. Suppose $m \in \mathbb{Z}$,

$m^2 - 4 = (m+2)(m-2)$

$\because m > 2$,

$\therefore m+2 > 4$,

$m-2 > 0$

Counter example: $m = 3$, $\therefore$ statement is false.

#31 Suppose $n \in \mathbb{Z}$

$n^2 - n + 11 = n(n-1) + 11$

let $n = 11k$, $k \in \mathbb{Z}$

$\therefore 11k(11k-1) + 11 = (11k)^2$

counterexample: $n=11$

statement is false

Q10 #14

a) $\forall x \in \mathbb{Q},\ x^3 \in \mathbb{Q}$

b) Suppose $x \in \mathbb{R}$

let $x = \frac{a}{b}$, $a \in \mathbb{Z}$, $b \in \mathbb{Z} - 0$

$x^3 = \frac{a^3}{b^3}$ $\quad a^3 \in \mathbb{Z}$

$b^3 \in \mathbb{Z} - 0$

$\therefore x^3 \in \mathbb{Q}$

#18 Suppose $r, s \in \mathbb{R}$

$r = \frac{a}{b}$ $\quad s = \frac{c}{d}$, $\quad a, c \in \mathbb{Z}$

$b, d \in \mathbb{Z} - 0$

$\frac{r+s}{2} = \frac{ad+bc}{2bd}$ $\quad ad+bc \in \mathbb{Z}$

$2bd \in \mathbb{Z} - 0$

$\therefore \frac{r+s}{2} \in \mathbb{Q}$ definition of rational number