CS159 Lecture 6: Planning under uncertainty and uncertain LMPC

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Caltech

Spring 2021

Today's Class

- Planning under uncertainty
- Uncertain LMPC
- Discussion
- Key takeaways

Planning under uncertainty

Optimizing over control policies

Why is planning in uncertain environments harder?

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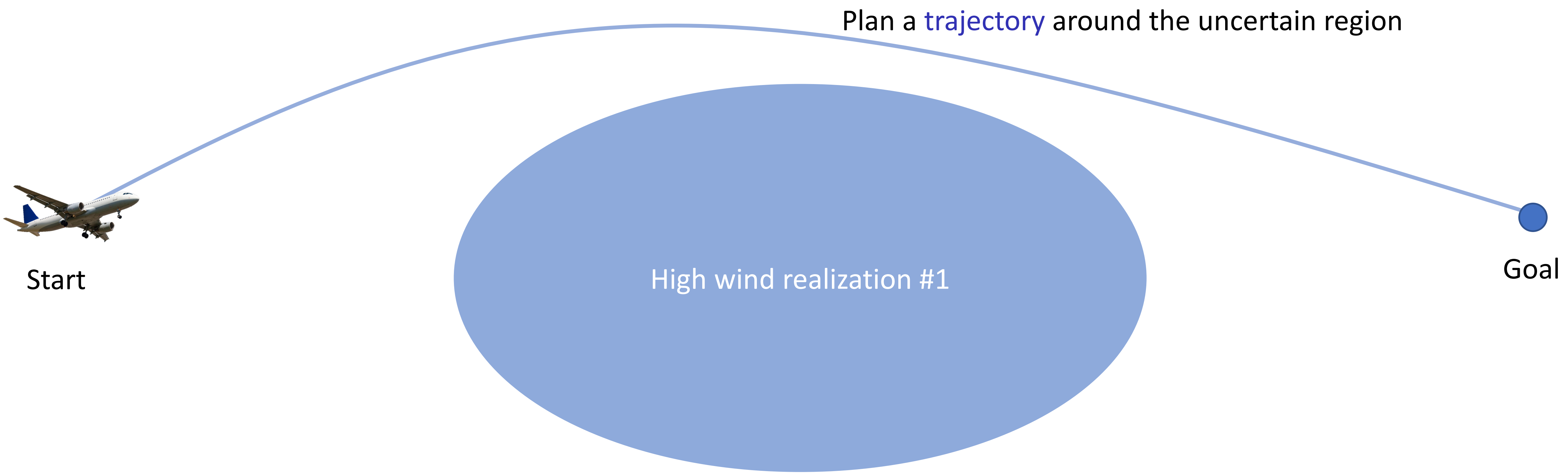

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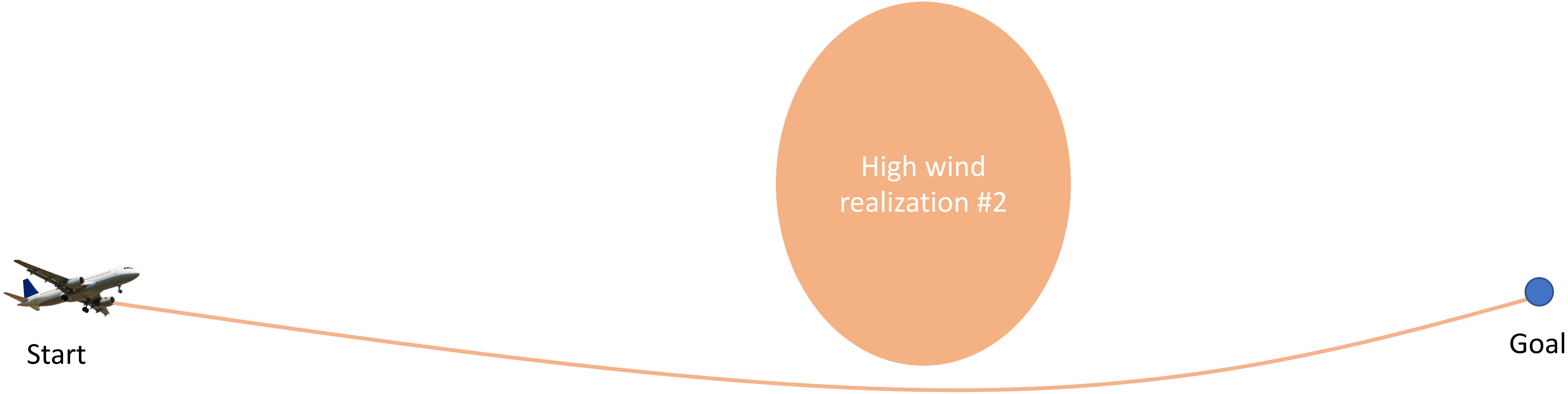

Plan a trajectory around the uncertain region

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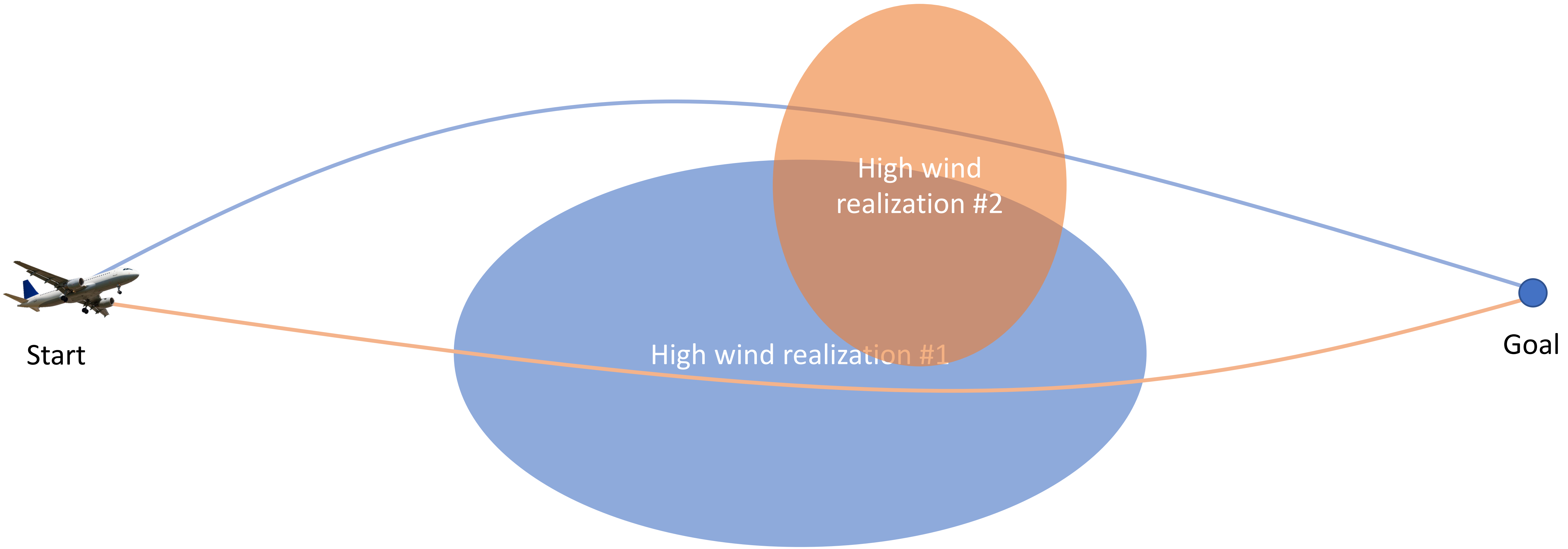

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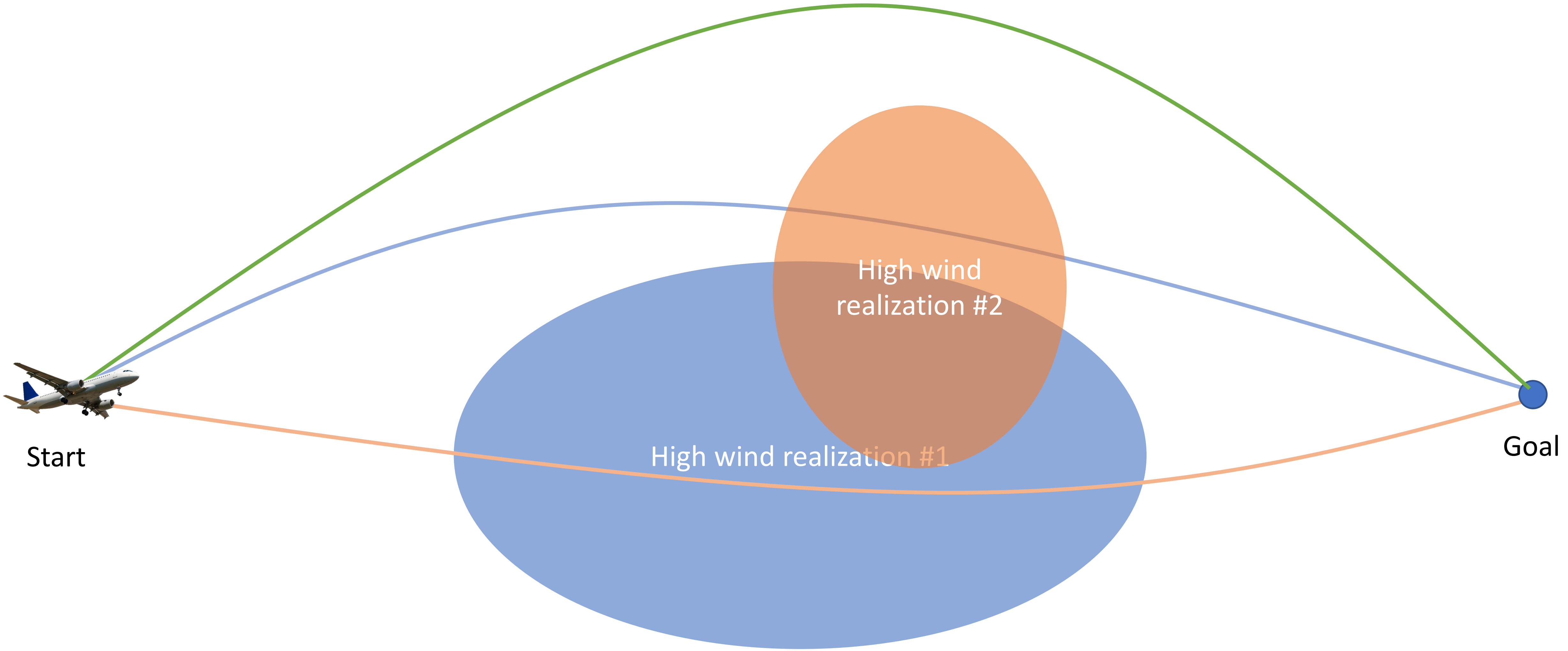
Plan a trajectory around both uncertain regions



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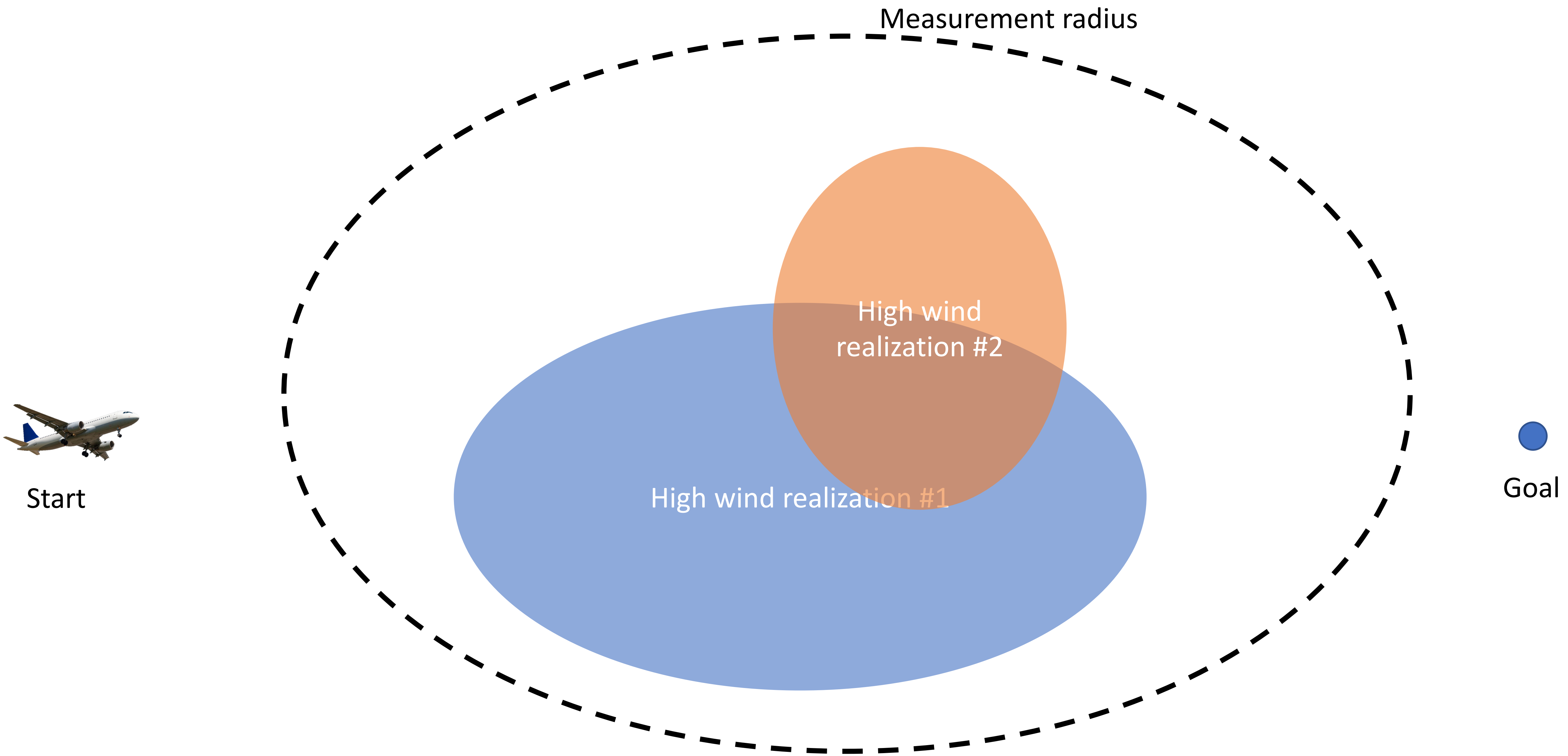
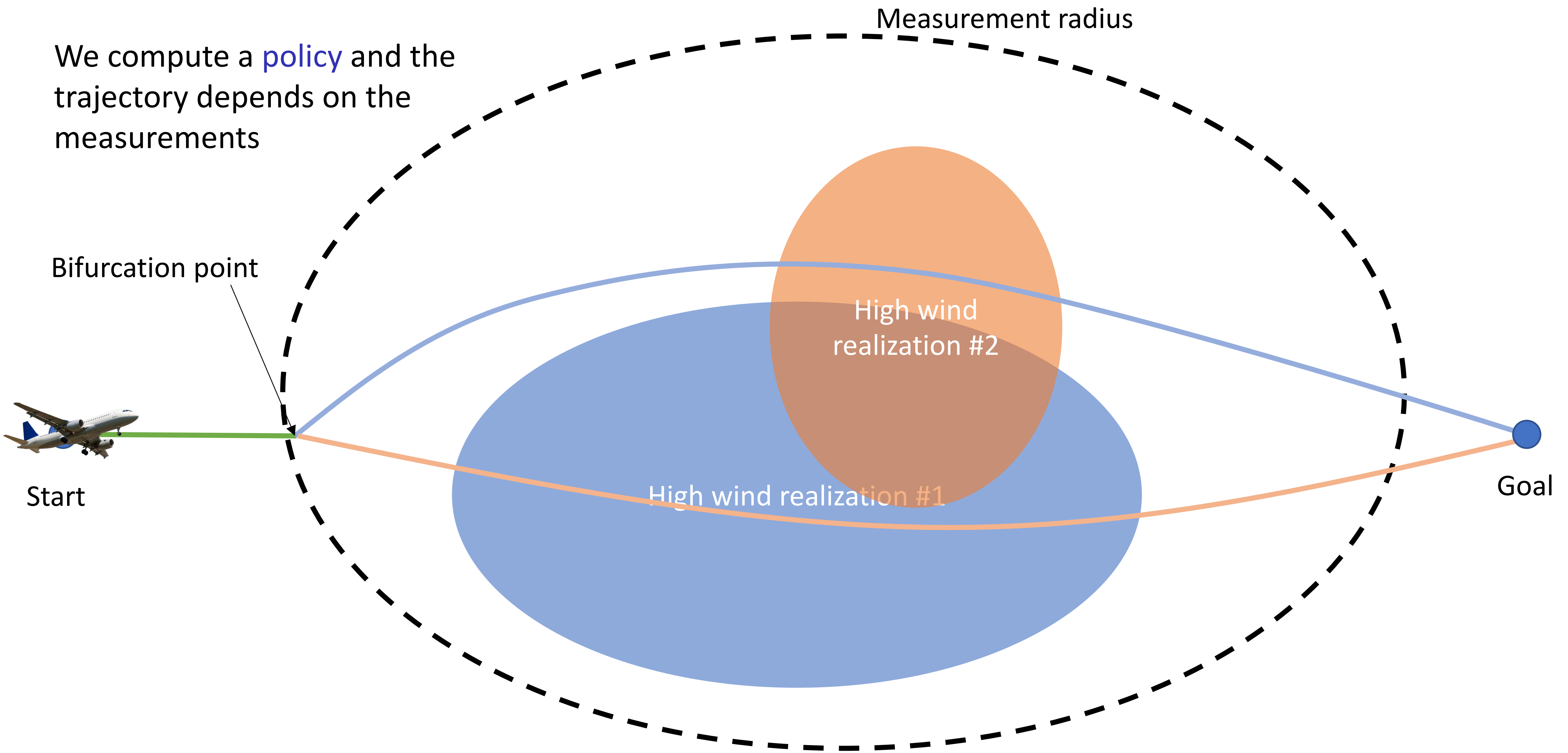


Why is planning in uncertain environments harder?

We compute a policy and the trajectory depends on the measurements



Problem Formulation

True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$

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Nominal model: $\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}\bar{u}_k + w_k$ where $||A - \bar{A}|| \leq \epsilon_A, ||B - \bar{B}|| \leq \epsilon_B$

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Robust constraints:

Given a control policy $\pi : \mathbb{R}^n \to \mathbb{R}^d$ designed using the nominal model, we want to guarantee that the closed-loop system

$$x_{k+1} = Ax_k + B\pi(x_k) + w_k$$

satisfies $x_k \in \mathcal{X}, \pi(x_k) \in \mathcal{U} \text{ for all } w_k \in \mathcal{W}.$

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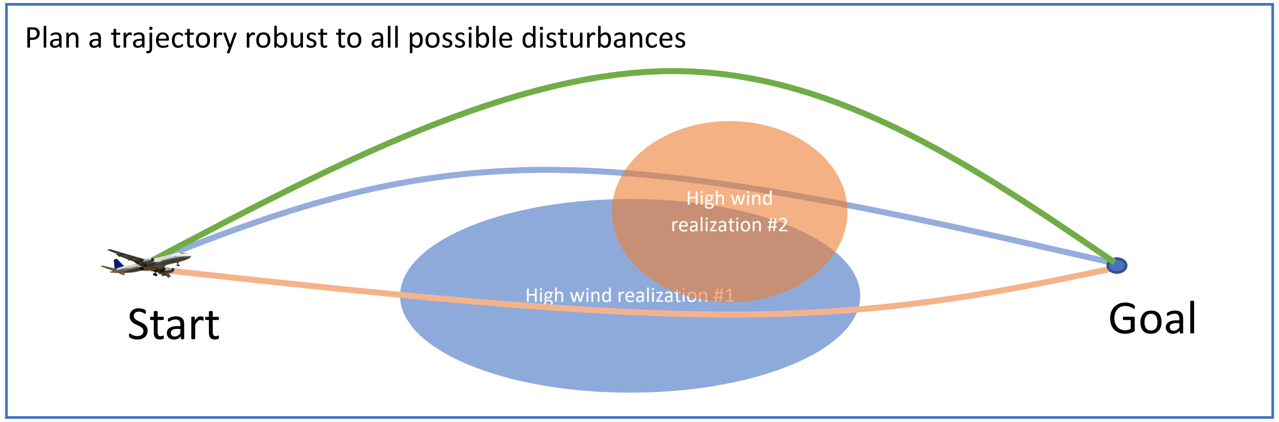

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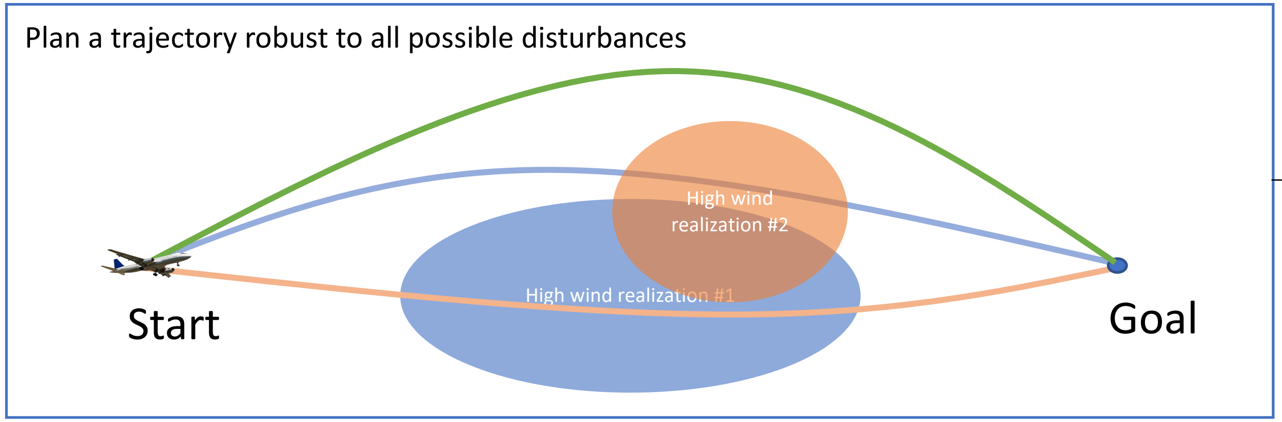

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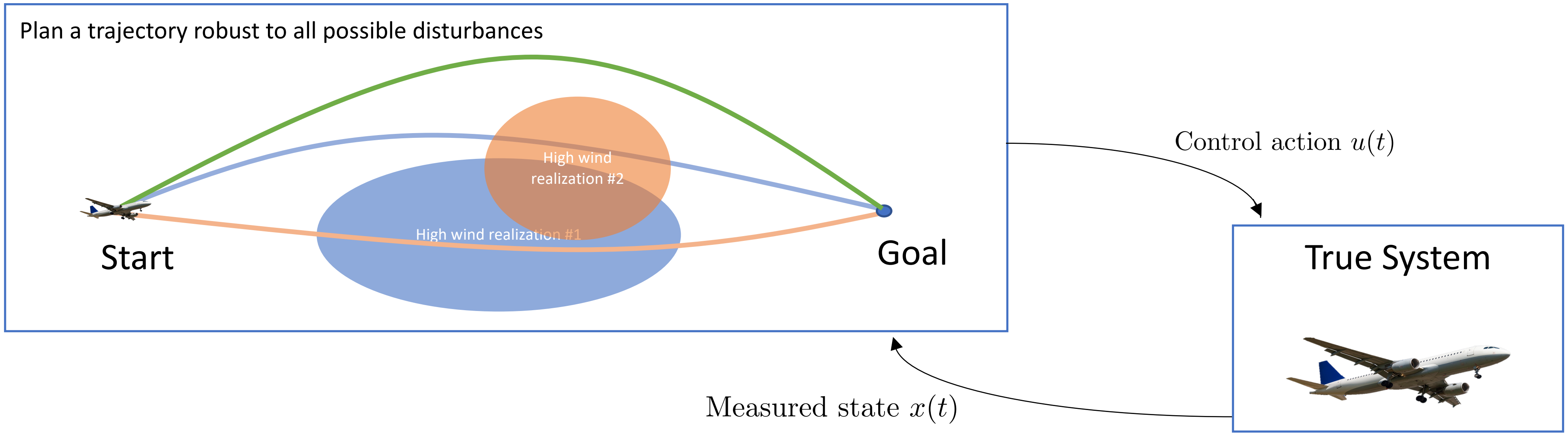

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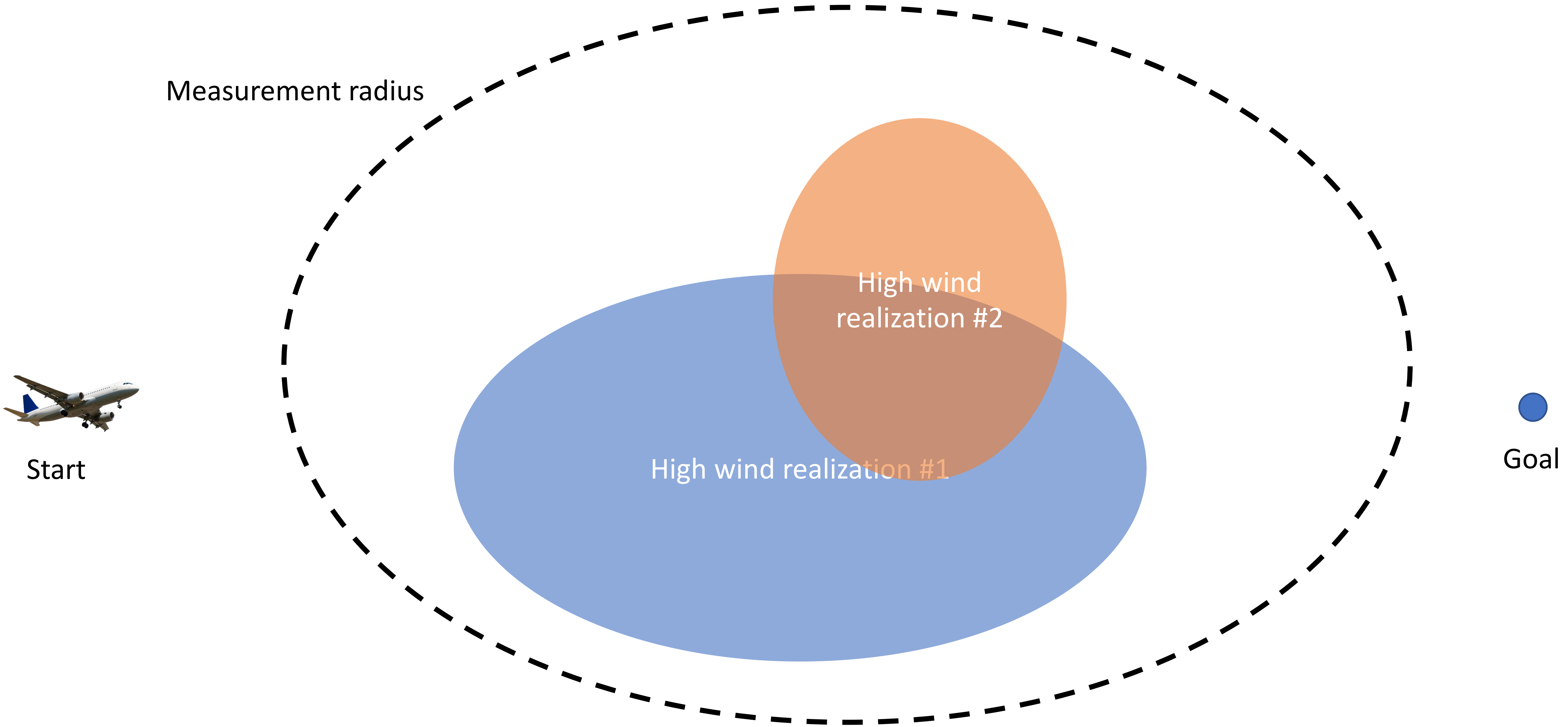

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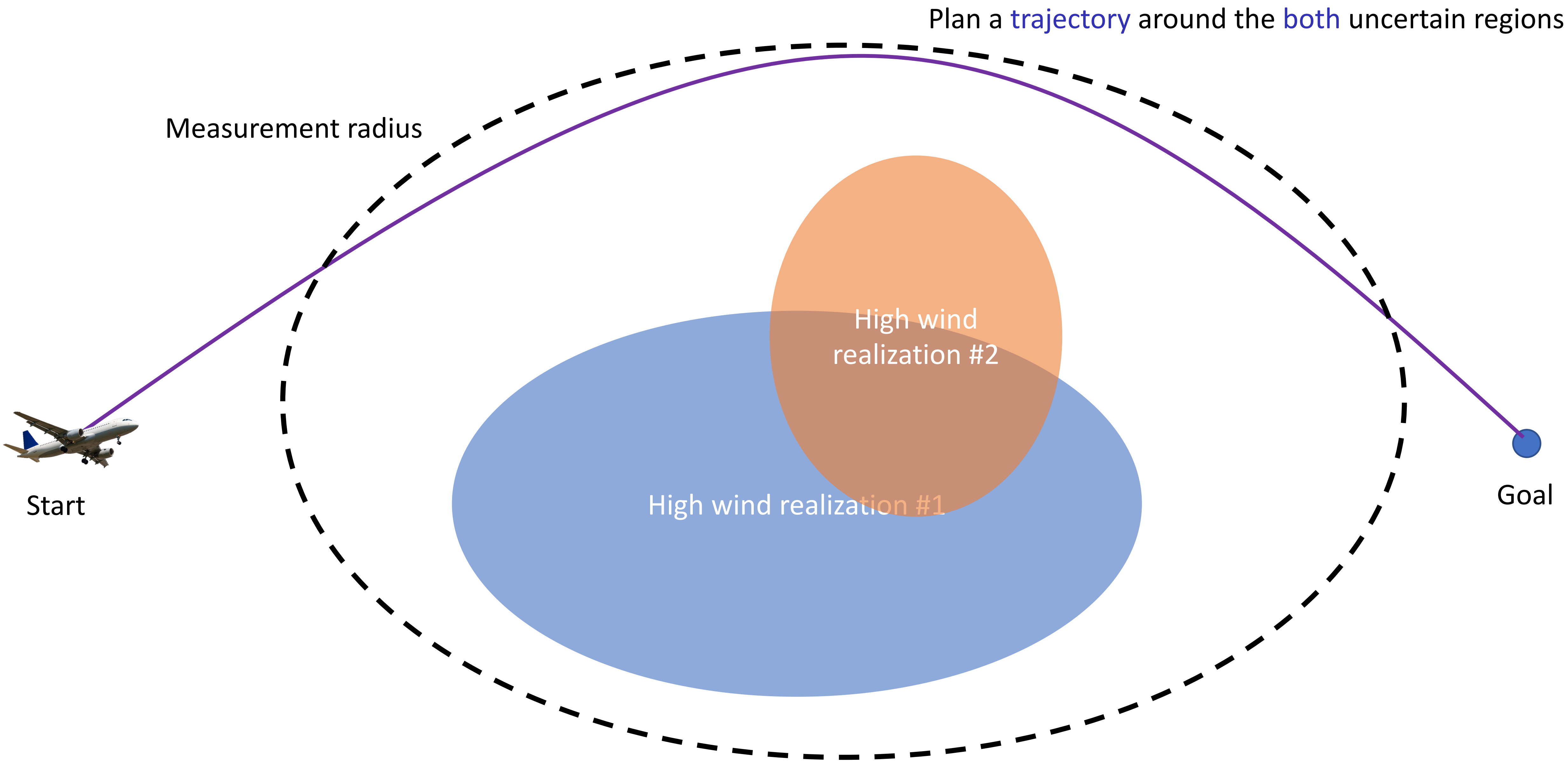

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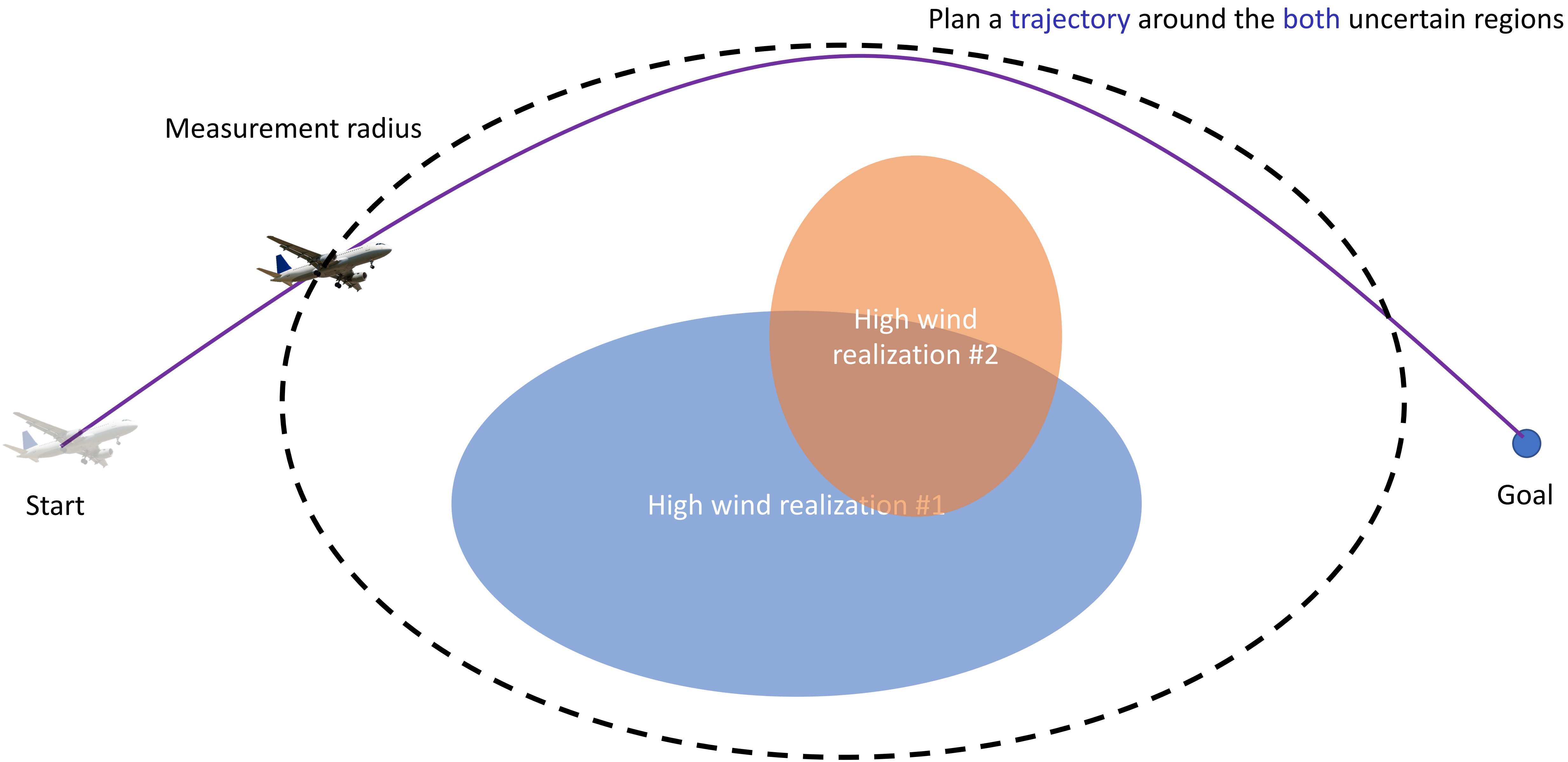

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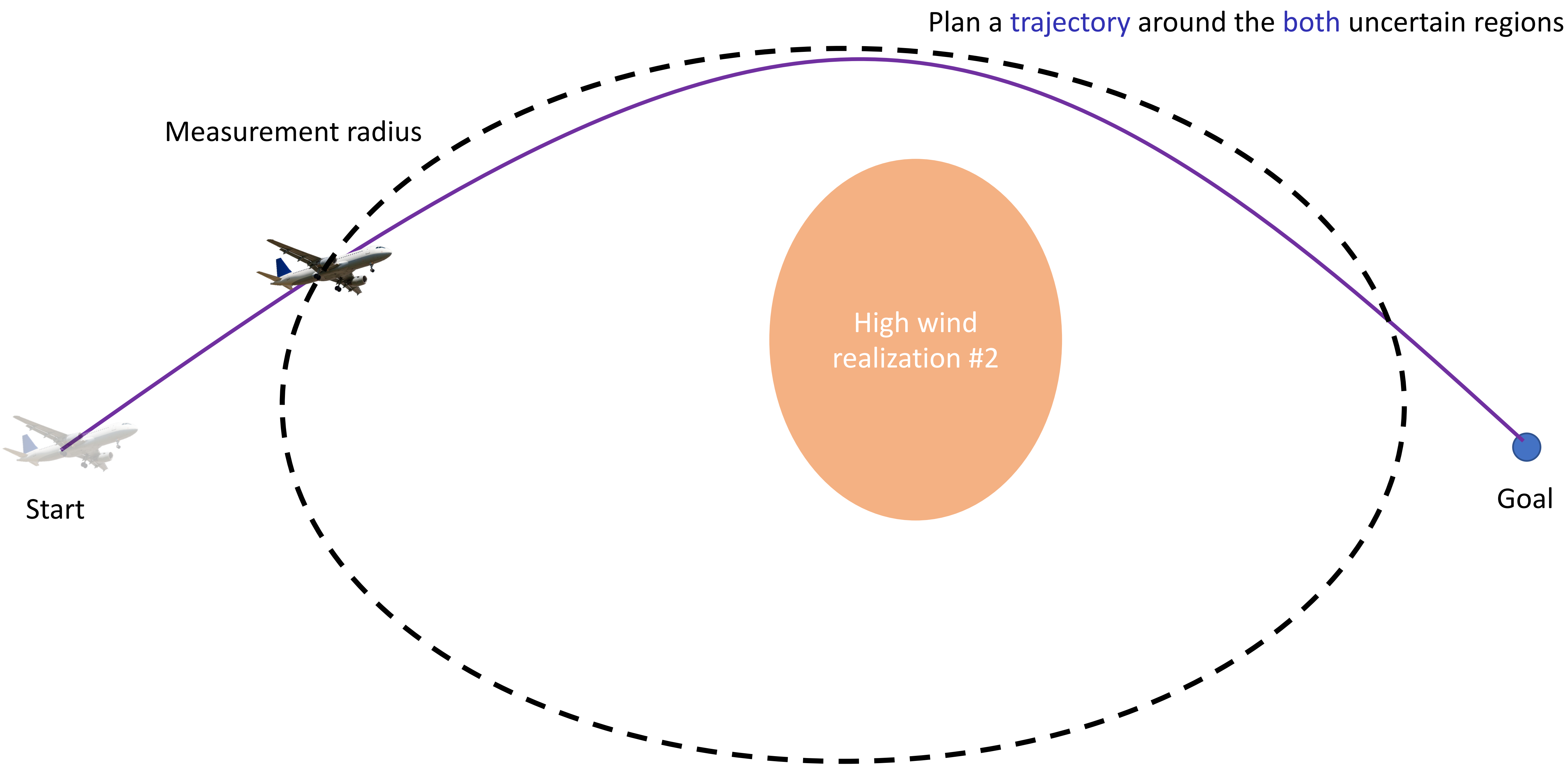

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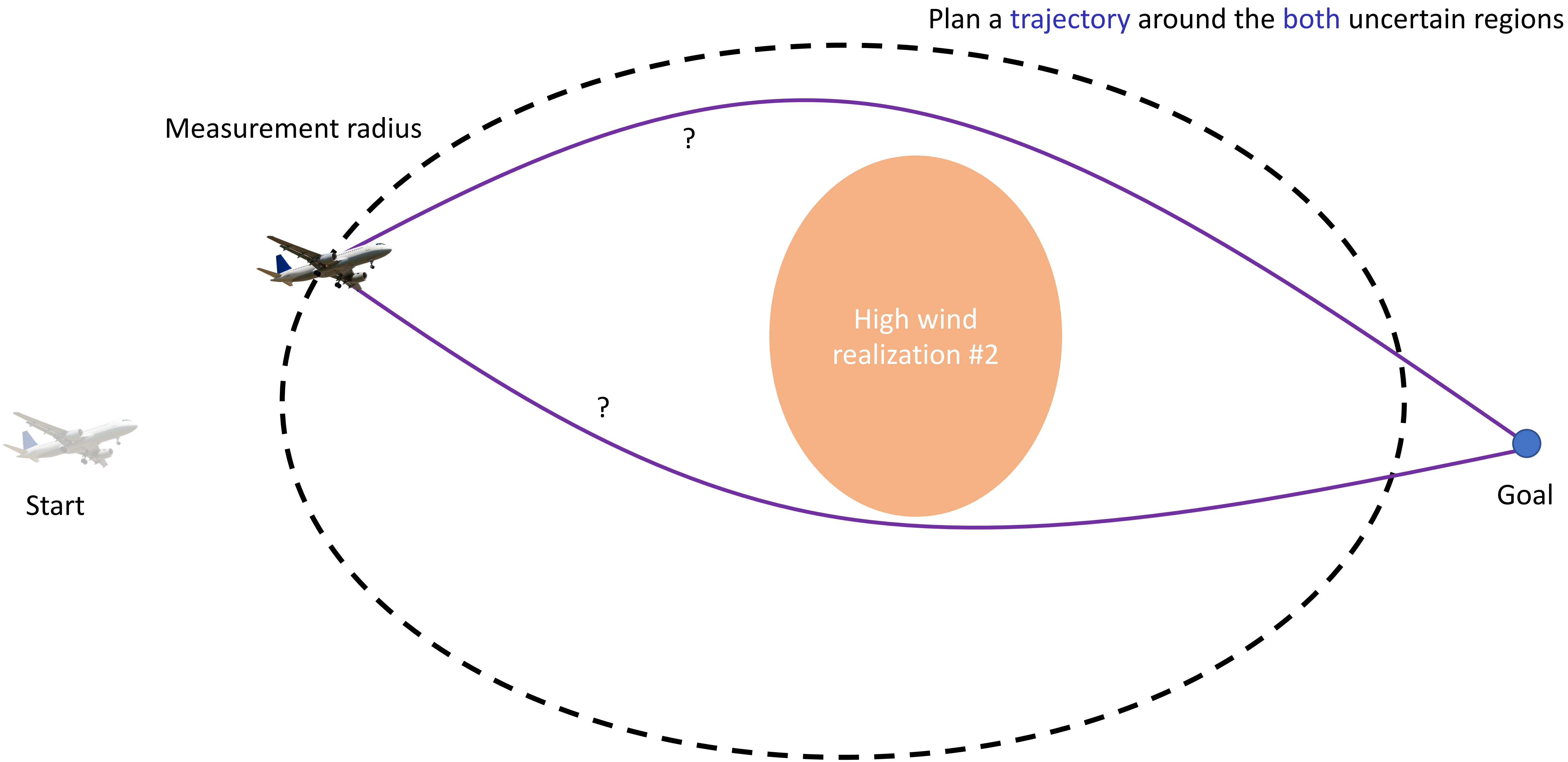

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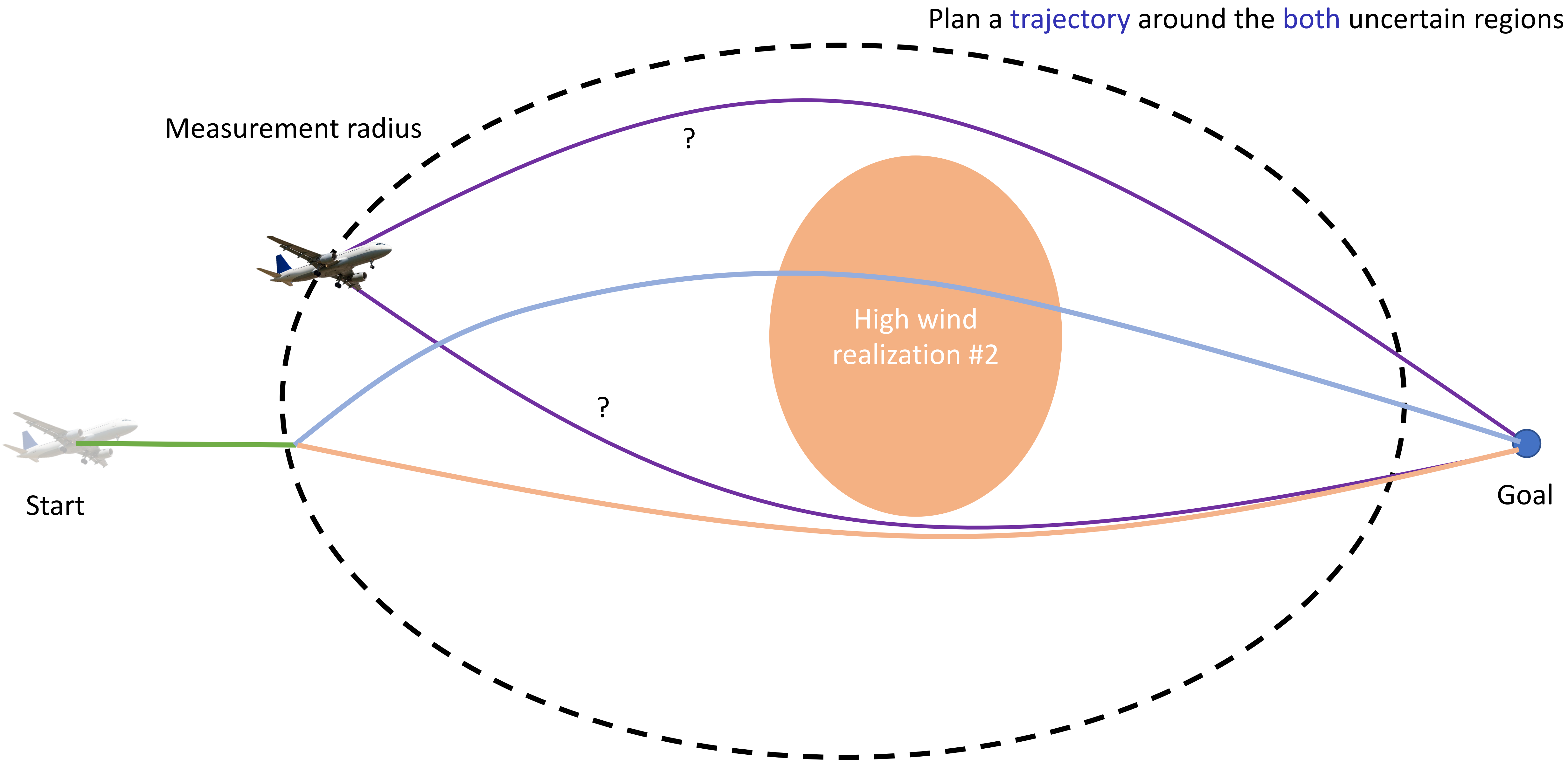

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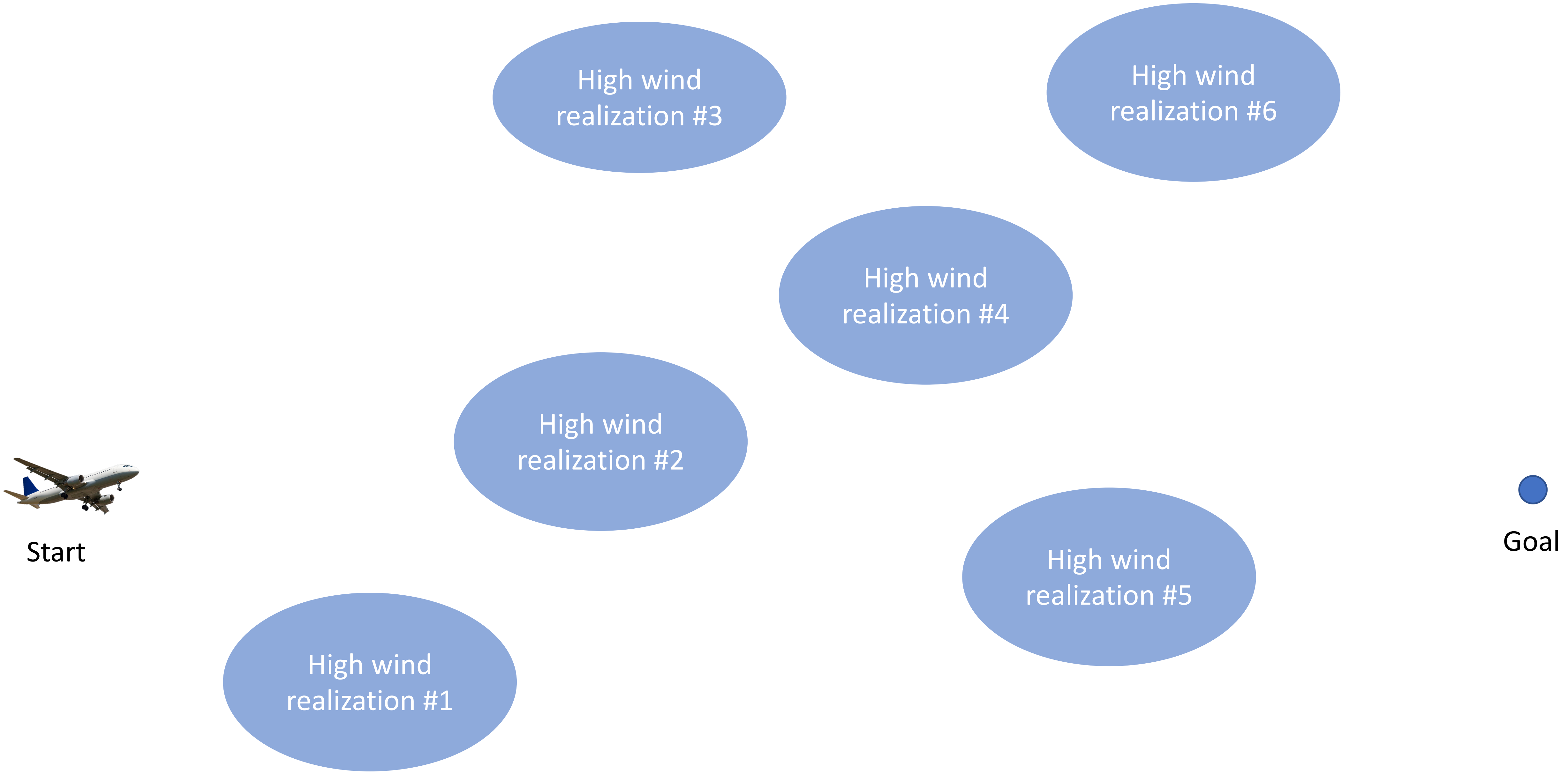

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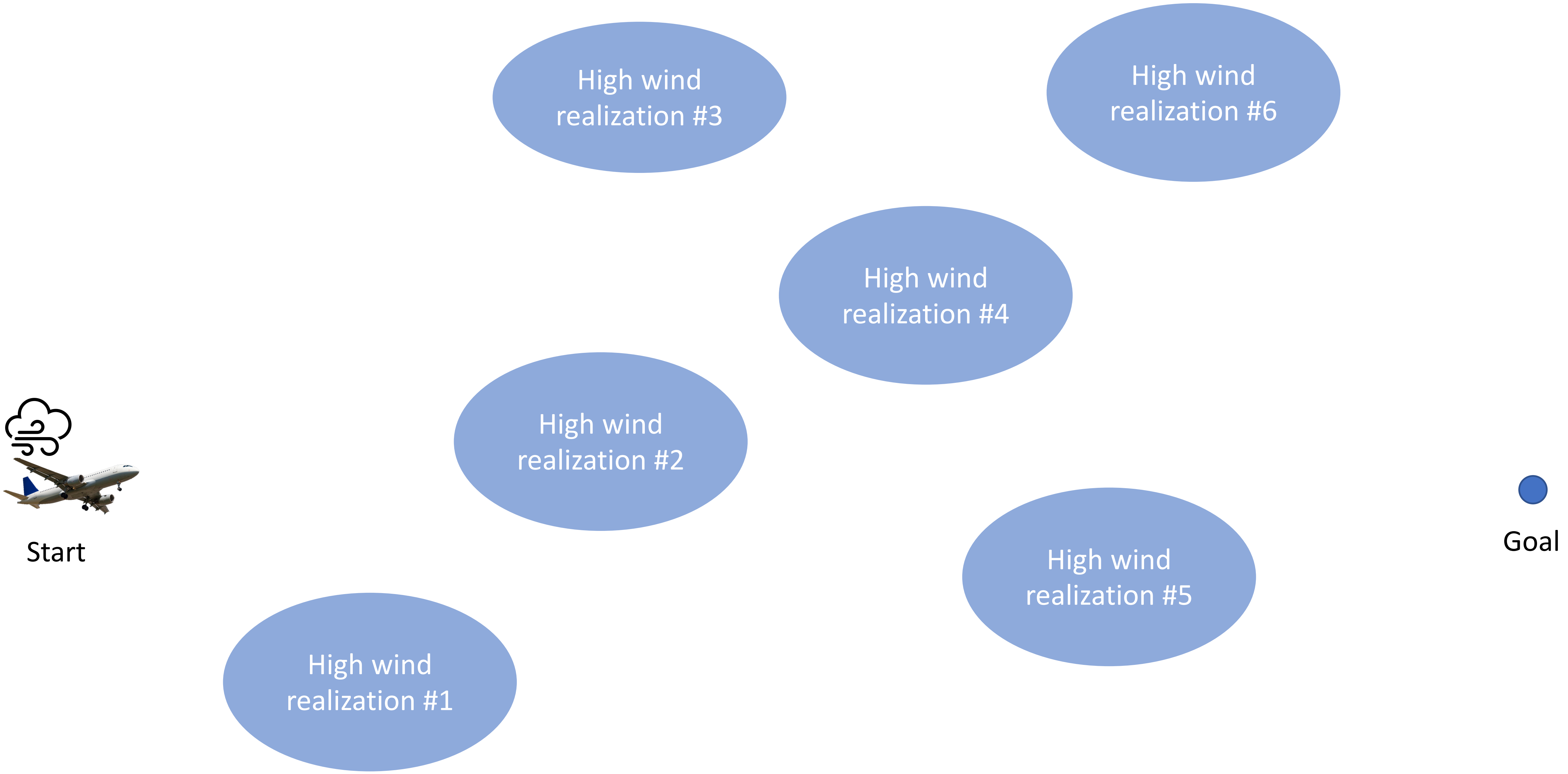

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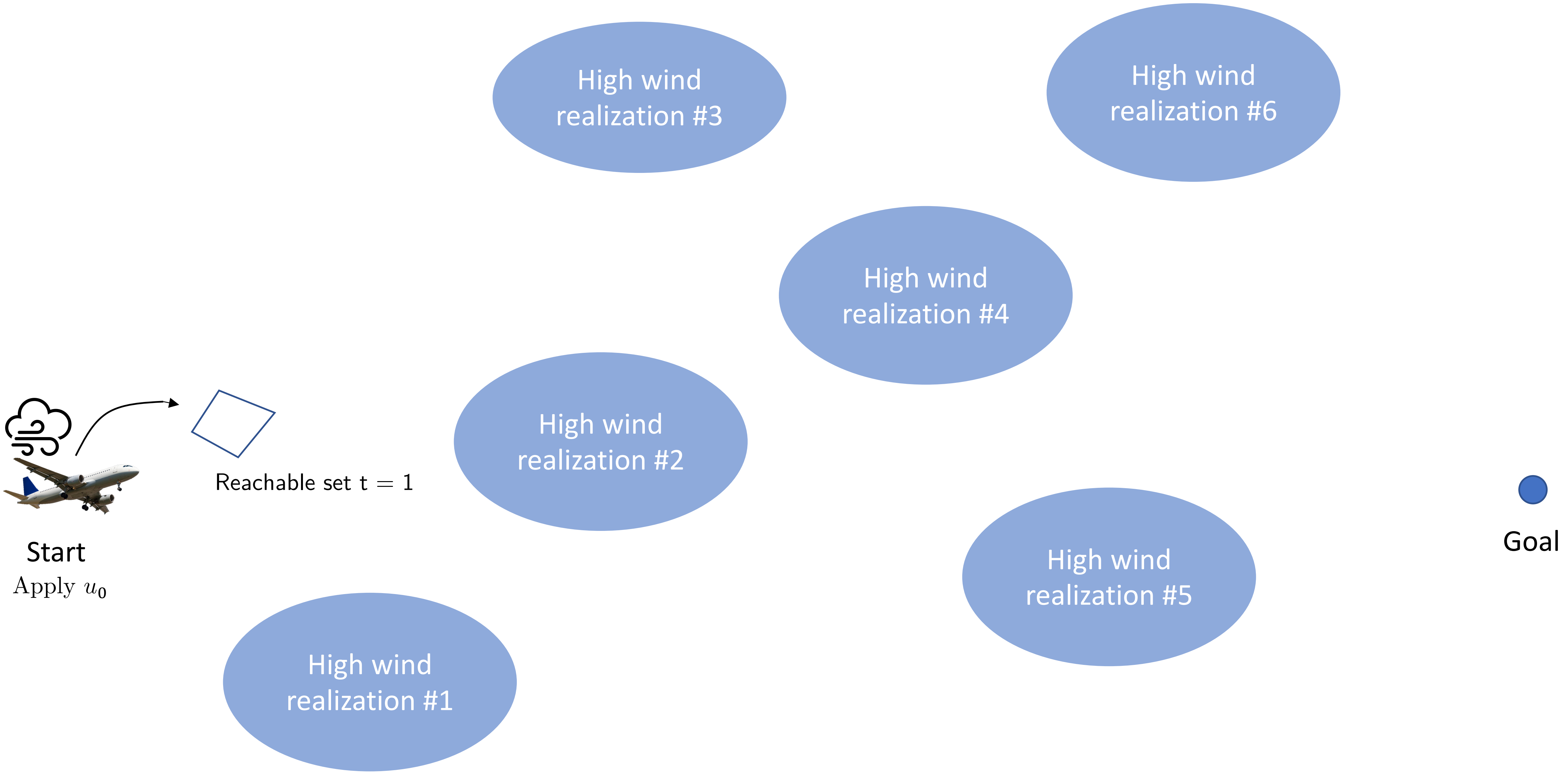

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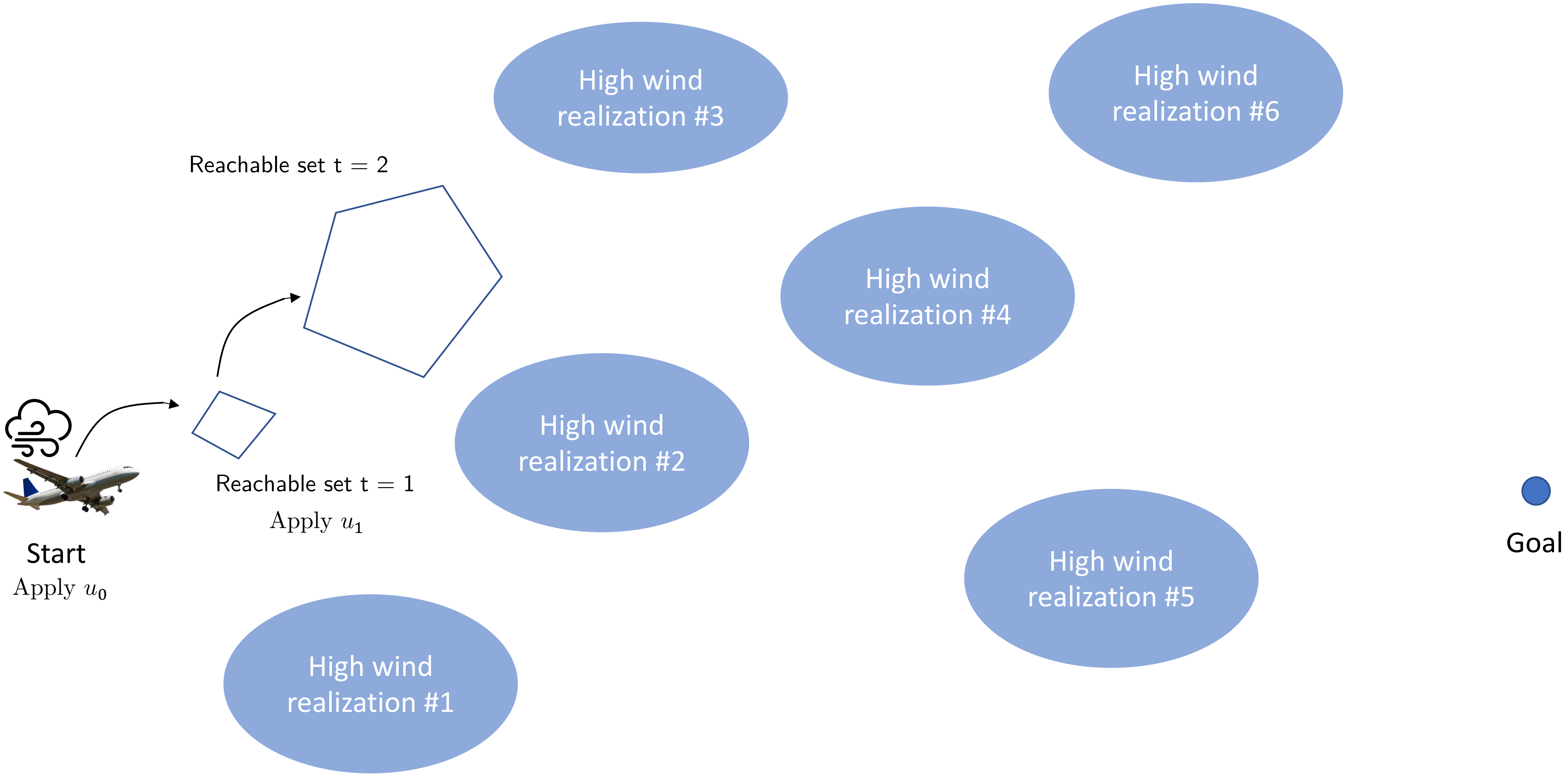

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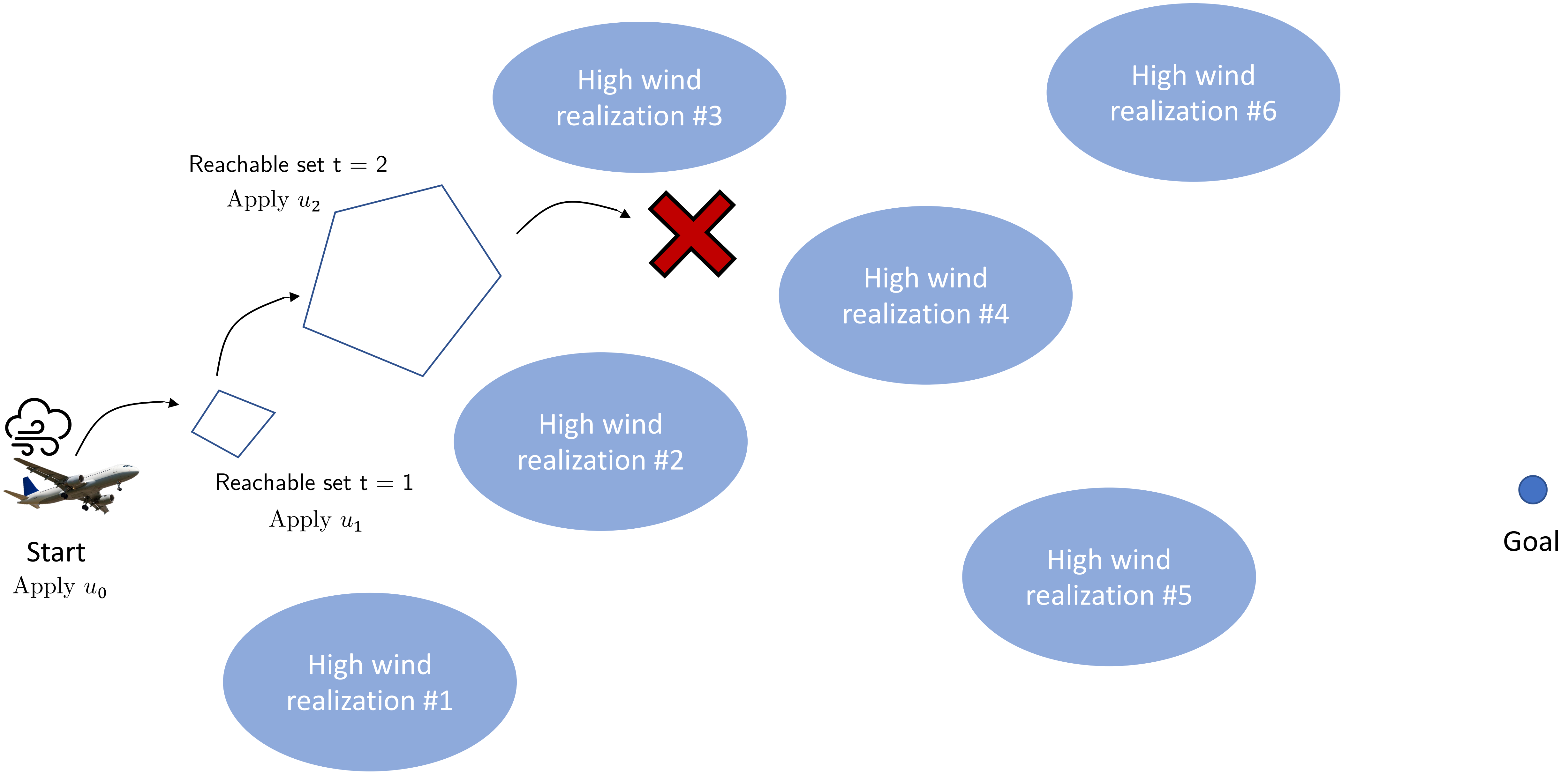
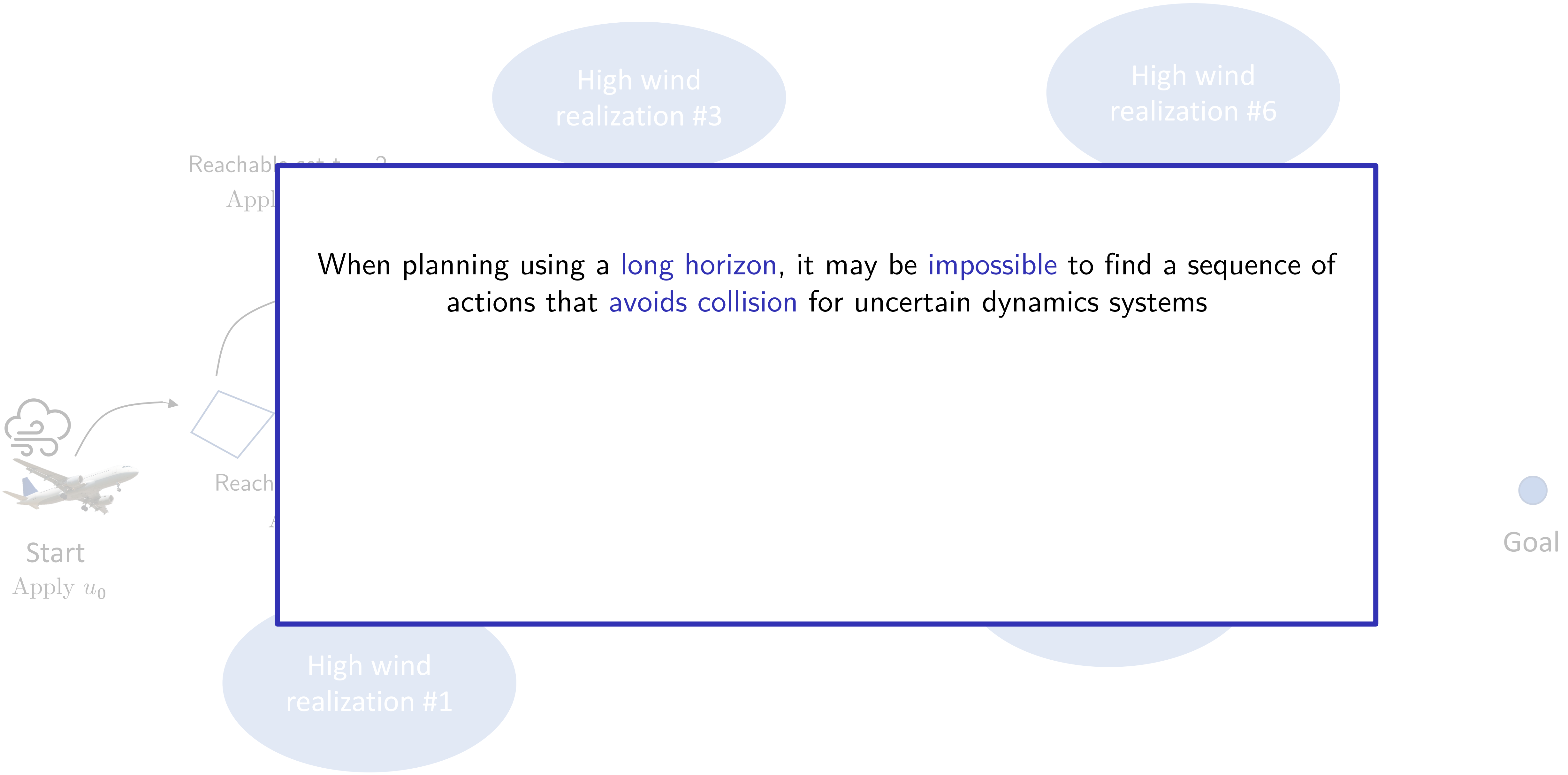


The horizon dilemma in MBRL



When planning using a long horizon, it may be impossible to find a sequence of actions that avoids collision for uncertain dynamics systems

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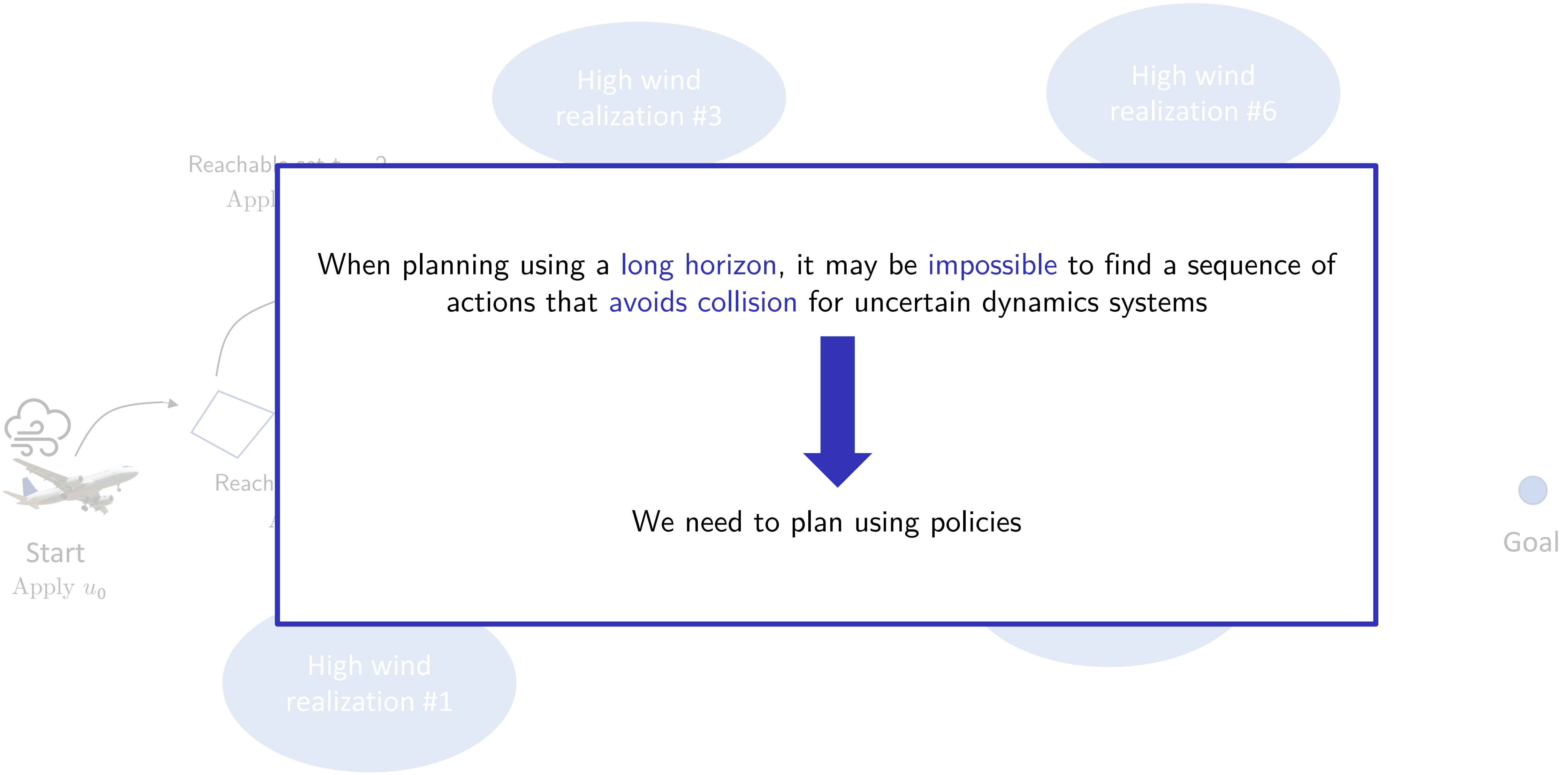

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We need to plan using policies

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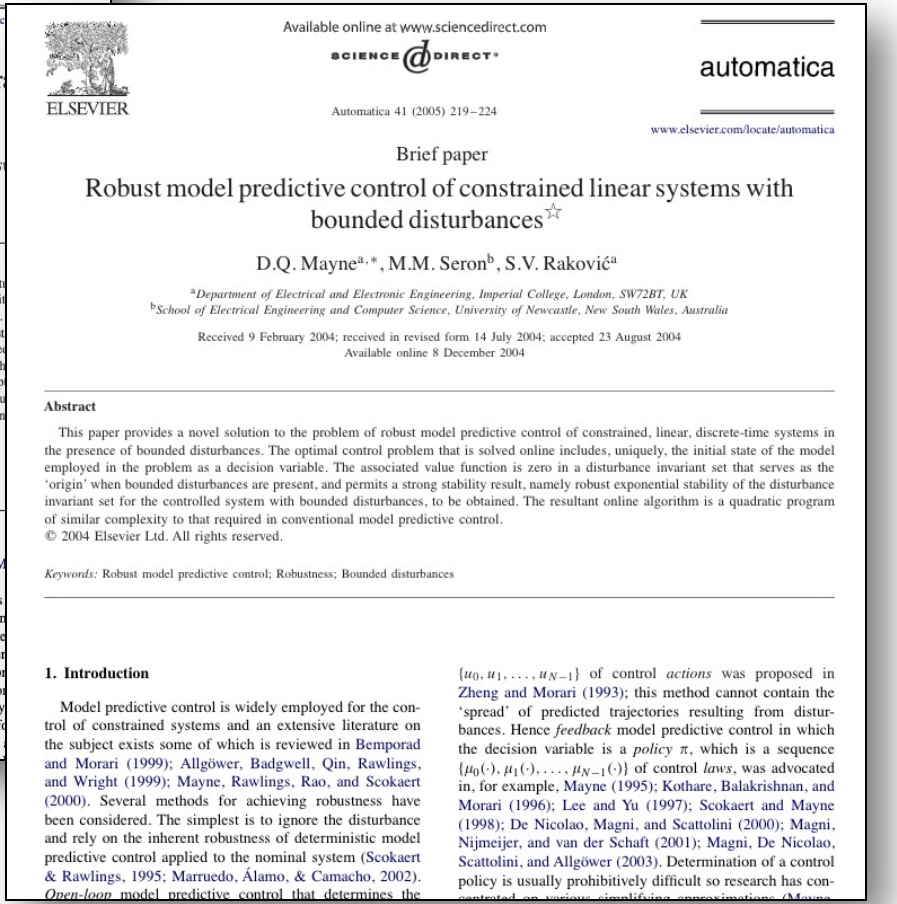

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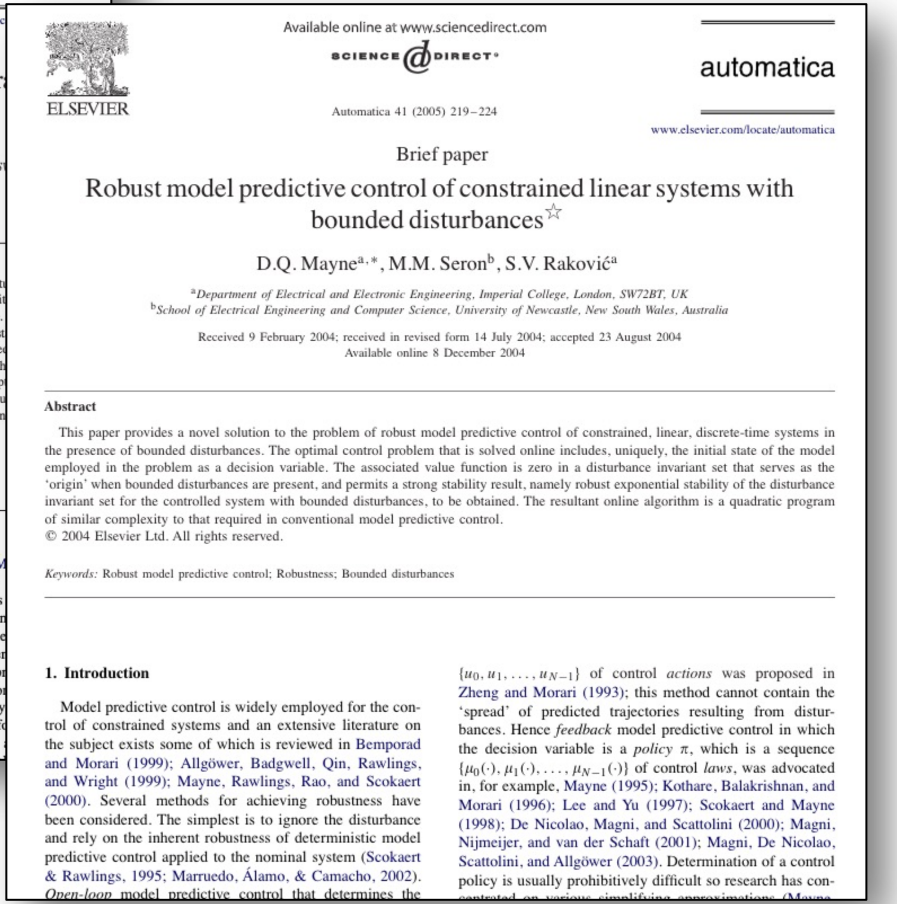

An elegant approximation out of many

How do we fix this problem?

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 43, NO. 8, AUGUST 1998

Min–Max Feedback Model Predictive Control for Constrained Linear Systems

P. O. M. Scokaert and D. Q. Mayne

The problem is NP-hard!

Automatica 42 (2006) 523–533

Optimization over state feedback policies for robust control with constra…

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Brief paper

Robust model predictive control of constrained linear systems with bounded disturbances

D.Q. Mayne[a,*], M.M. Seron[b], S.V. Raković[a]

What people do in practice

How do we fix this problem?

Statement 2: Under the assumptions of this theorem for any $0 \le t_0 \le t$

$$\Phi_{\bar u}(t) - \Phi_{\bar u}(t_0) \ge \Phi_{\bar u^*}(t) - \Phi_{\bar u^*}(t_0) = -\frac{1}{h}\Delta\vartheta^T(t)L^{-1}\Delta\vartheta(t) + o_\omega(h)$$

where

$$\Phi_{\bar u}(t) := \int_0^t (2\bar u^T(\tau)d\vartheta(\tau) + \bar u^T(\tau)L\bar u(\tau)d\tau) \tag{20}$$

$$\Delta\vartheta(t) := B_{u0}^T P[B_{u0}(u_{comp}(t) - x^*(t))h + (K + A_0C_0^+)\Delta y(t)], \quad \Delta y(t) = y(t) - y(t-h)$$

and this minimum is reachable for

$$\bar u(t)\,dt = \bar u^*(t)\,dt := -L^{-1}B_{u0}^T P[B_{u0}(u_{comp}(t) + x^*(t))\,dt + (K + A_0C_0^+)\,dy(t)].$$

As a result, we have: $d\Phi_{\bar u^*}(t) \le 0$ (in symbolic form).

Proof of Statement 2: Using the Euler–Maruyama's formula [3], [9] we obtain the following relation: $(h := t - t_0 \to 0)$: $\Phi_{\bar u}(t) - \Phi_{\bar u}(t_0) = 2\bar u^T(t)\Delta\vartheta(t) + \bar u^T(t)L\bar u(t)h + o_\omega(h)$.

Minimizing then the right side for each fixed t, we derive

$$\min_{\bar u(t)}[\Phi_{\bar u}(t) - \Phi_{\bar u}(t_0)] = \Phi_{\bar u^*}(t) - \Phi_{\bar u^*}(t_0) = -\frac{1}{h}\Delta\vartheta^T(t)L^{-1}\Delta\vartheta(t) + o_\omega(h) \le o_\omega(h). \tag{21}$$

Hence, taking into account the definition (20), we have $d\Phi_{\bar u^*}(t) \le 0$. □

Rewriting (18) in differential form and taking into account that P is the solution of the Riccati equation (see Assumption 4 of this theorem) we finally derive

$$dV(e(t)) \overset{a.s.}{\le} [\varphi(t) + I(t)]\,dt + S^T(t)\,dw(t) - e^T(t)Q_e e(t)\,dt.$$

According to the Kronecker lemma, the Large Number law for martingale, and Skorohod lemma (see [2] and [4]), we obtain the result of the theorem. Details of this proof can be found in [13].

References

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Min–Max Feedback Model Predictive Control for Constrained Linear Systems

P. O. M. Scokaert and D. Q. Mayne

***Abstract*—Min–max feedback formulations of model predictive control are discussed, both in the fixed and variable horizon contexts. The control schemes the authors discuss introduce, in the control optimization, the notion that feedback is present in the receding-horizon implementation of the control. This leads to improved performance, compared to standard model predictive control, and resolves the feasibility difficulties that arise with the min–max techniques that are documented in the literature. The stabilizing properties of the methods are discussed as well as some practical implementation details.**

***Index Terms*—Feedback, min–max optimization, model predictive control.**

I. Introduction

Model predictive control (MPC) is a control methodology that is becoming mature. The basic aspects of the method are now well understood, and stabilizing formulations of the control law are documented in the literature, both for linear and nonlinear processes [1], [2].

The MPC strategy optimizes an open-loop control sequence, at each sample, to minimize a nominal cost function, subject to some state and input constraints. The optimization is usually based on the assumption that the process model is exact and that future disturbances are constant. Because the control law ignores the effects of possible future changes in disturbance and model mismatch, closed-loop performance can be poor [3], with likely violations of the constraints, when disturbances or model mismatch are present.

Some formulations of MPC have been proposed that address these issues [4], [5]. These methods rely on a min–max optimization of predicted performance. However, these formulations optimize a single control profile over all possible disturbance (or model mismatch)

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Brief paper

Robust model predictive control of constrained linear systems with bounded disturbances

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[a] Department of Electrical and Electronic Engineering, Imperial College, London, SW72BT, UK

[b] School of Electrical Engineering and Computer Science, University of Newcastle, New South Wales, Australia

Received 9 February 2004; received in revised form 14 July 2004; accepted 23 August 2004

Available online 8 December 2004

Abstract

This paper provides a novel solution to the problem of robust model predictive control of constrained, linear, discrete-time systems in the presence of bounded disturbances. The optimal control problem that is solved online includes, uniquely, the initial state of the model employed in the problem as a decision variable. The associated value function is zero in a disturbance invariant set that serves as the 'origin' when bounded disturbances are present, and permits a strong stability result, namely robust exponential stability of the disturbance invariant set for the controlled system with bounded disturbances, to be obtained. The resultant online algorithm is a quadratic program of similar complexity to that required in conventional model predictive control.

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Keywords: Robust model predictive control; Robustness; Bounded disturbances

What people do in practice

Optimizing over policies

True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$

Pierre OM Scokaert, and David Q. Mayne. "Min-max feedback model predictive control for constrained linear systems." IEEE Transactions on Automatic control 43, no. 8 (1998): 1136-1142.

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Assumption: the disturbance support $\mathcal{W} = \text{conv}(w^{(0)}, \ldots, w^{(n_d)})$.

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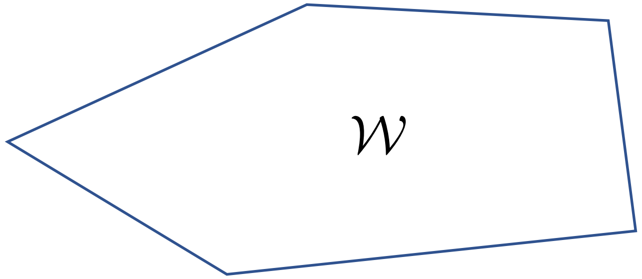

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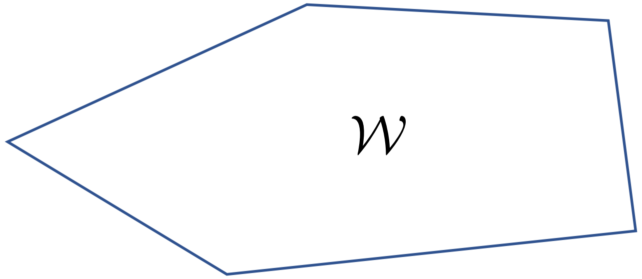

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The problem that we want to solve

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\text{subject to} \quad & x_{k+1|t} = Ax_{k|t} + Bu_{k|t} + w_{k|t} \\
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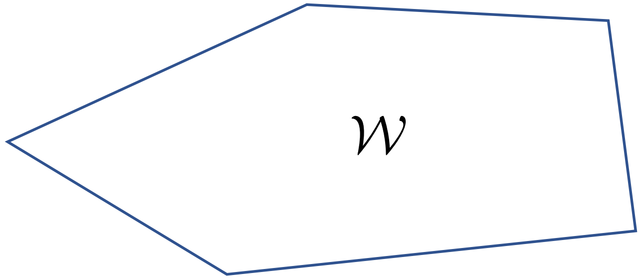

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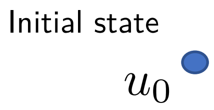
Optimization over policies

Finite dimensional reformulation

Optimize over a tree



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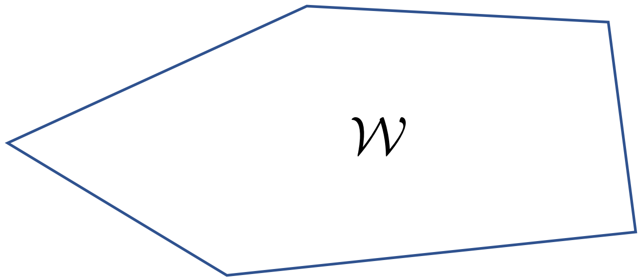

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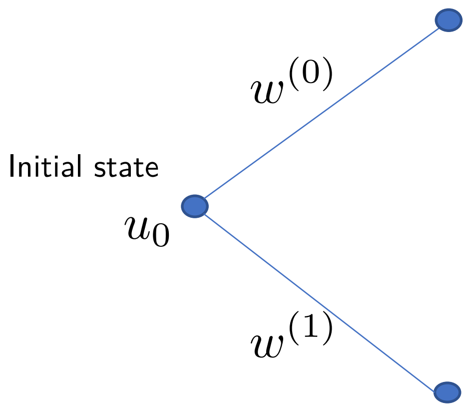

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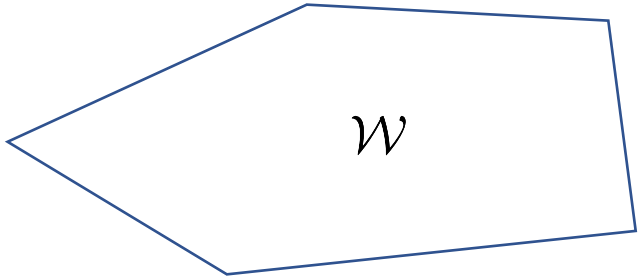
True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$

Assumption: the disturbance support $\mathcal{W} = \mathrm{conv}(w^{(0)}, \ldots, w^{(n_d)})$.



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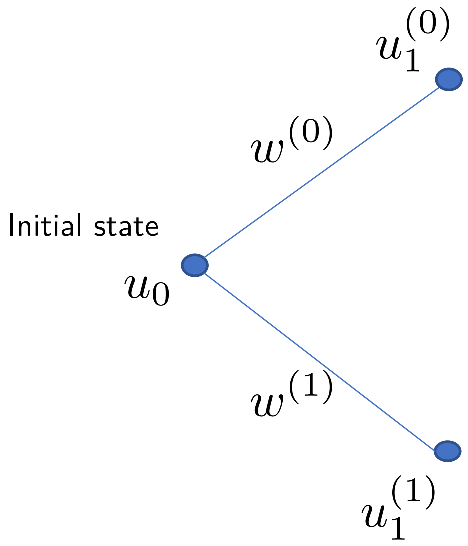
Optimization over policies

Finite dimensional reformulation

Optimize over a tree



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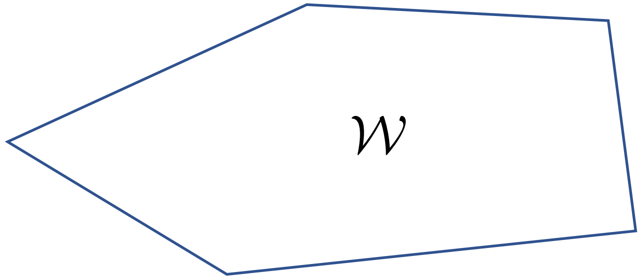

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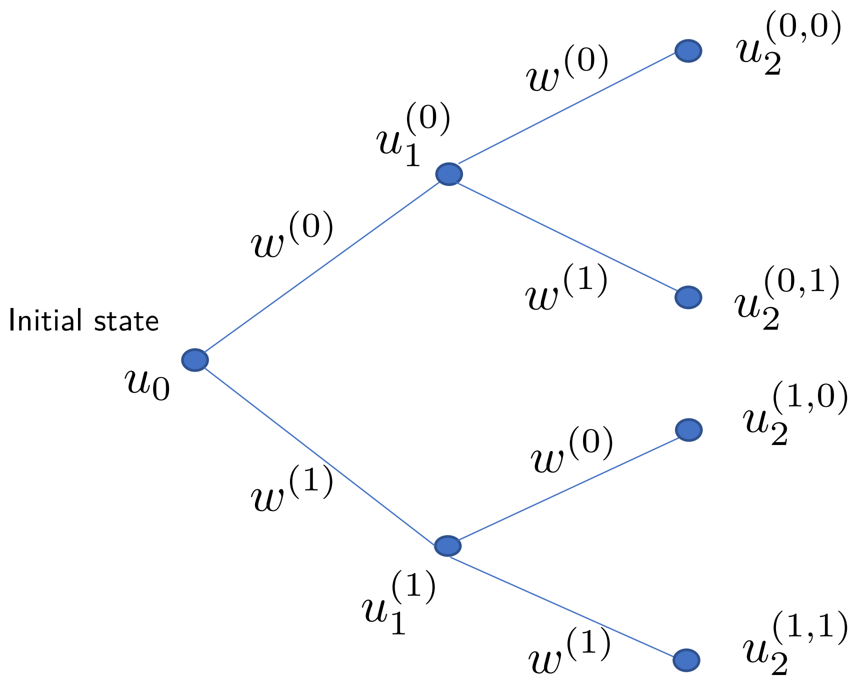

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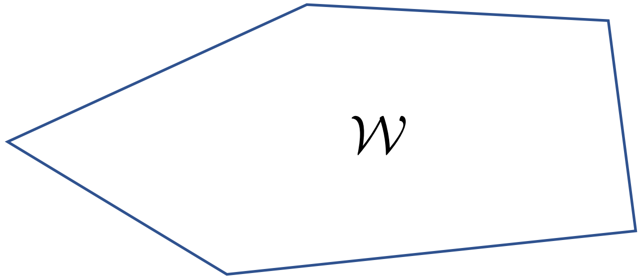

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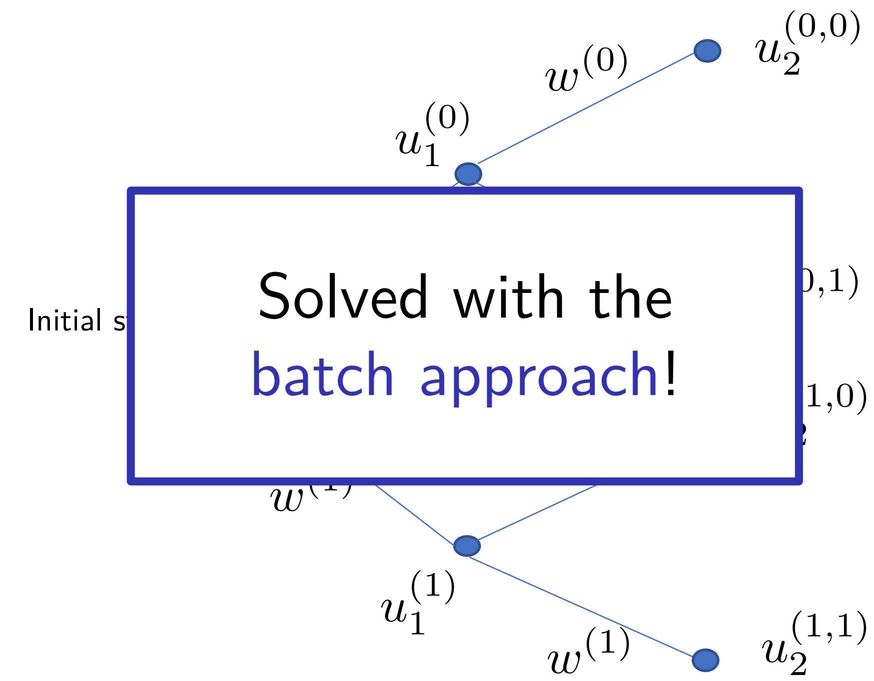

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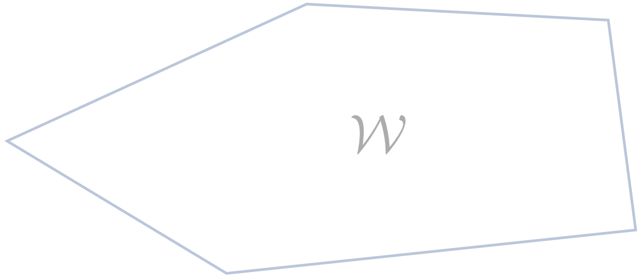


The problem that we want to solve

Optimization over policies

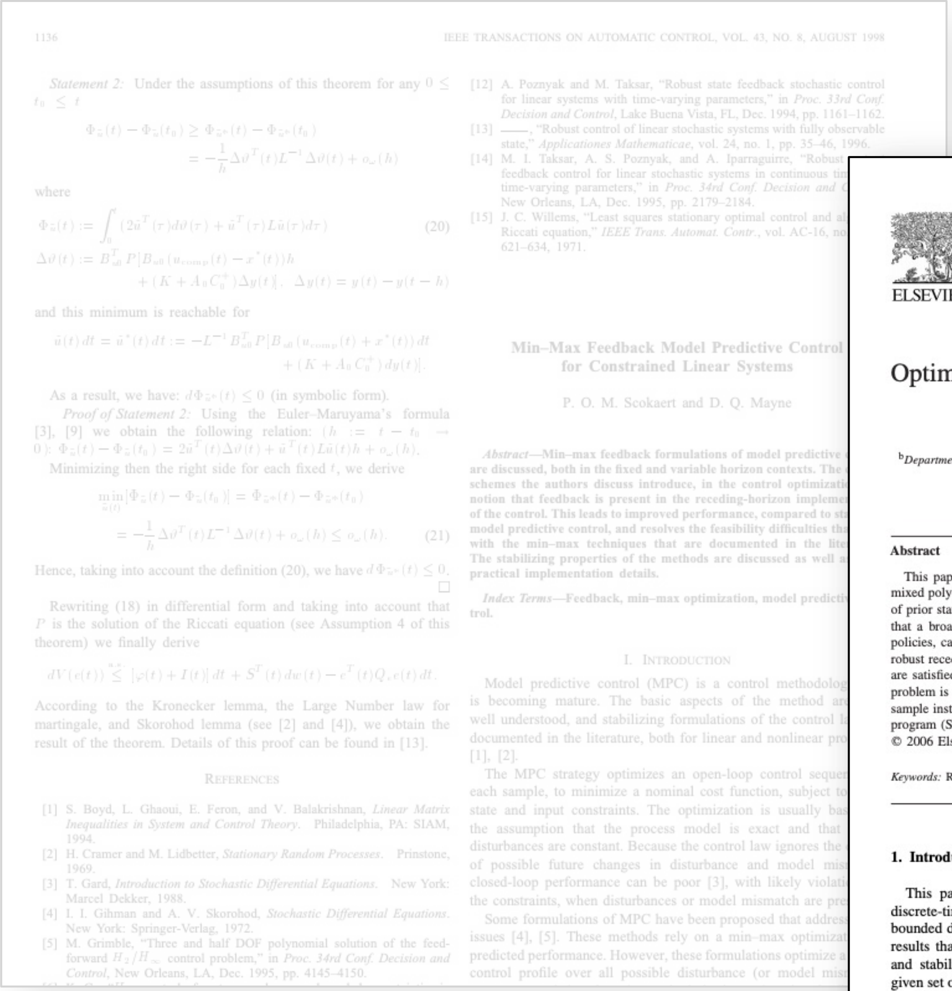
Optimize over a tree

This approach can be used also when A and B are uncertain.

For further details check:
Francesco Borrelli, Alberto Bemporad, and Manfred Morari. *Predictive control for linear and hybrid systems*. Cambridge University Press, 2017.

Pierre OM Scokaert, and David Q. Mayne. "Min-max feedback model predictive control for constrained linear systems." IEEE Transactions on Automatic control 43, no. 8 (1998): 1136-1142.

How do we fix this problem?



IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 43, NO. 8, AUGUST 1998

Min–Max Feedback Model Predictive Control for Constrained Linear Systems

P. O. M. Scokaert and D. Q. Mayne

The problem is NP-hard!

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automatica

Automatica 42 (2006) 523–533

Optimization over state feedback policies for robust control with constr

Paul J. Goulart[a,*], Eric C. Kerrigan[b], Jan M. Maciejowski[a]

[a]Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK

[b]Department of Aeronautics and Department of Electrical and Electronic Engineering, Imperial College London, Exhibition Road, London S

Received 15 March 2005; received in revised form 8 June 2005; accepted 25 August 2005

Abstract

This paper is concerned with the optimal control of linear discrete-time systems subject to unknown but bounded state dist mixed polytopic constraints on the state and input. It is shown that the class of admissible affine state feedback control policies wi of prior states is equivalent to the class of admissible feedback policies that are affine functions of the past disturbance sequence. that a broad class of constrained finite horizon robust and optimal control problems, where the optimization is over affine st policies, can be solved in a computationally efficient fashion using convex optimization methods. This equivalence result is use robust receding horizon control (RHC) state feedback policy such that the closed-loop system is input-to-state stable (ISS) and th are satisfied for all time and all allowable disturbance sequences. The cost to be minimized in the associated finite horizon op problem is quadratic in the disturbance-free state and input sequences. The value of the receding horizon control law can be calcu sample instant using a single, tractable and convex quadratic program (QP) if the disturbance set is polytopic, or a tractable secor program (SOCP) if the disturbance set is given by a 2-norm bound.

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Keywords: Robust control; Constraint satisfaction; Robust optimization; Predictive control; Optimal control

1. Introduction

This paper is concerned with the control of constrained discrete-time linear systems that are subject to additive, but bounded disturbances on the state. The main aim is to provide results that allow for the efficient computation of an optimal and stabilizing state feedback control policy that ensures a given set of state and input constraints are satisfied for all time, despite the presence of the disturbances. This is a problem that has been studied for some time now in the optimal control literature (Bertsekas & Rhodes, 1973) and a number of different

An elegant approximation out of many

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Automatica 41 (2005) 219–224

Brief paper

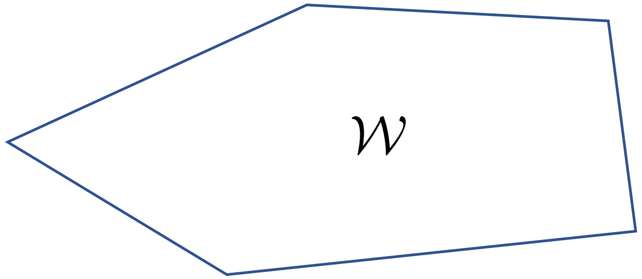
Robust model predictive control of constrained linear systems with bounded disturbances

D.Q. Mayne[a,*], M.M. Seron[b], S.V. Raković[a]

What people do in practice

Optimizing over disturbance feedback policies

True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$



Assumption: the disturbance support $\mathcal{W} = \mathrm{conv}(w^{(0)}, \ldots, w^{(n_d)})$.

The problem that we want to solve

$$\begin{aligned}
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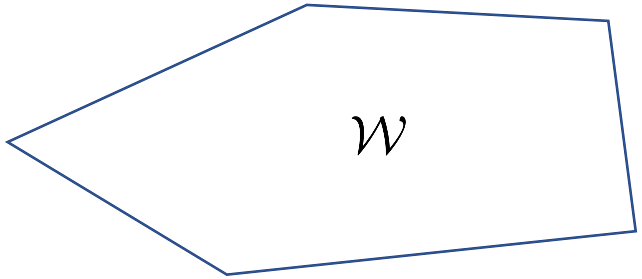

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Optimization over policies

Tractable approximation

Optimize over disturbance feedback policies

$$u_{k|t} = \pi(x(t), w_{t|t}, \ldots, w_{k-1|t})$$

Function of disturbances!

Paul J. Goulart, Eric C. Kerrigan, and Jan M. Maciejowski. "Optimization over state feedback policies for robust control with constraints." Automatica 42, no. 4 (2006): 523-533.

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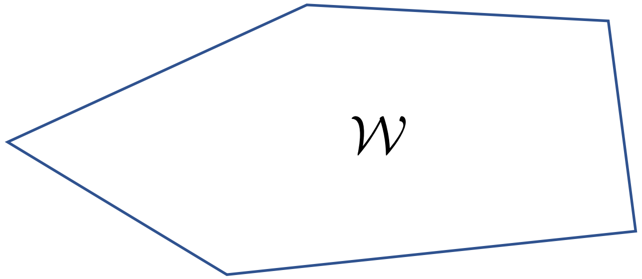

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Optimization over policies

Tractable approximation

Optimize over disturbance feedback policies

$$u_{k|t} = \pi(x(t), w_{t|t}, \ldots, w_{k-1|t})$$

Function of disturbances!

The problem is tractable when we pick:

$$u_{k|t} = M_{k|t} \begin{bmatrix} x(t) \\ w_{t|t} \\ \vdots \\ w_{k-1|t} \end{bmatrix}$$

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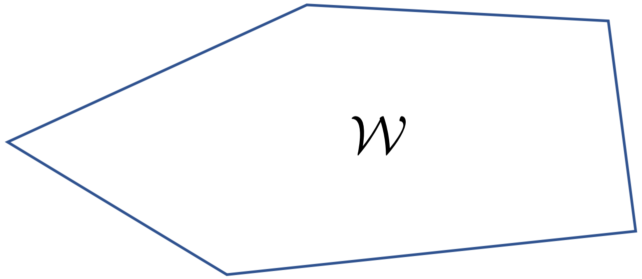

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$$u_{k|t} = M_{k|t} \begin{bmatrix} x(t) \\ w_{t|t} \\ \vdots \\ w_{k-1|t} \end{bmatrix} = K_{k|t} \begin{bmatrix} x(t) \\ x_{t+1|t} \\ \vdots \\ x_{k|t} \end{bmatrix}$$

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The pro

$\min_{\pi}$

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Optimizatio

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This approach is not convex when A and B are uncertain → we need to overapproximate the uncertainty to convexify the problem!

For more details check:

Aharon Ben-Tal, Alexander Goryashko, Elana Guslitzer, and Arkadi Nemirovski. "Adjustable robust solutions of uncertain linear programs." *Mathematical programming* 99, no. 2 (2004): 351-376.

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Strategies to handle uncertainty in A and B (conservative, but tractable):

- Sarah Dean, Horia Mania, Nikolai Matni, Benjamin Recht, and Stephen Tu. "On the sample complexity of the linear quadratic regulator." Foundations of Computational Mathematics (2019): 1-47.
- Sarah Dean, Stephen Tu, Nikolai Matni, and Benjamin Recht. "Safely learning to control the constrained linear quadratic regulator." In *2019 American Control Conference (ACC)*, pp. 5582-5588. IEEE, 2019.
- Wilbur Langson, Ioannis Chryssochoos, S. V. Raković, and David Q. Mayne. "Robust model predictive control using tubes." *Automatica* 40, no. 1 (2004): 125-133.
- Monimoy Bujarbaruah, Ugo Rosolia, Yvonne R. Stürz, Xiaojing Zhang, and Francesco Borrelli. "Robust MPC for LTI Systems with Parametric and Additive Uncertainty: A Novel Constraint Tightening Approach." *To appear in American Control Conference (ACC)* (2020).

Paul J. Goulart, Eric C. Kerrigan, and Jan M. Maciejowski. "Optimization over state feedback policies for robust control with constraints." Automatica 42, no. 4 (2006): 523-533.

How do we fix this problem?

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 43, NO. 8, AUGUST 1998

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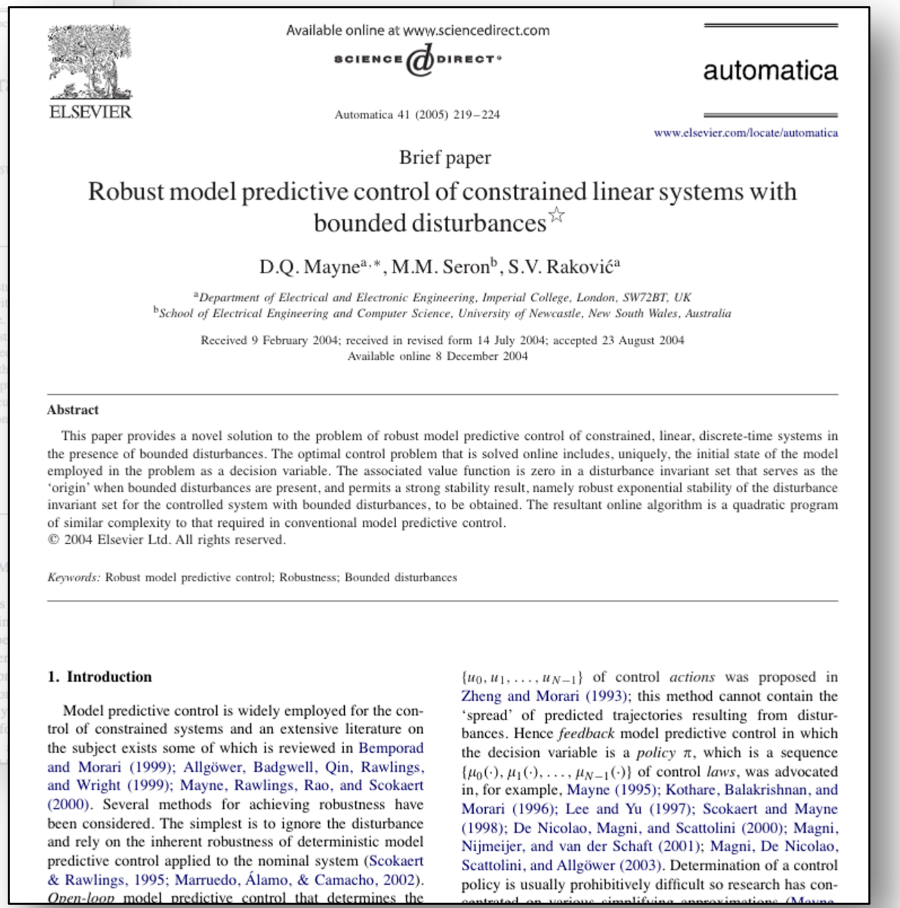

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Automatica 41 (2005) 219–224

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Brief paper

Robust model predictive control of constrained linear systems with bounded disturbances

D.Q. Mayne[a,*], M.M. Seron[b], S.V. Raković[a]

[a]Department of Electrical and Electronic Engineering, Imperial College, London, SW72BT, UK

[b]School of Electrical Engineering and Computer Science, University of Newcastle, New South Wales, Australia

Received 9 February 2004; received in revised form 14 July 2004; accepted 23 August 2004
Available online 8 December 2004

Abstract

This paper provides a novel solution to the problem of robust model predictive control of constrained, linear, discrete-time systems in the presence of bounded disturbances. The optimal control problem that is solved online includes, uniquely, the initial state of the model employed in the problem as a decision variable. The associated value function is zero in a disturbance invariant set that serves as the 'origin' when bounded disturbances are present, and permits a strong stability result, namely robust exponential stability of the disturbance invariant set for the controlled system with bounded disturbances, to be obtained. The resultant online algorithm is a quadratic program of similar complexity to that required in conventional model predictive control.

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Keywords: Robust model predictive control; Robustness; Bounded disturbances

What people do in practice

Fixed tube robust MPC

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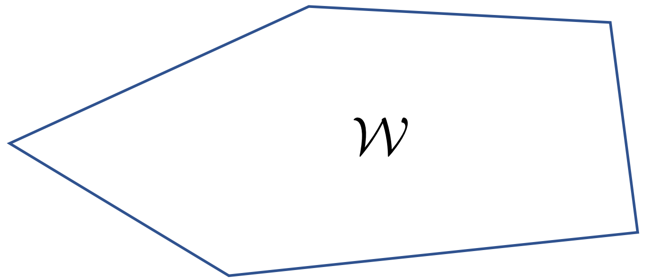

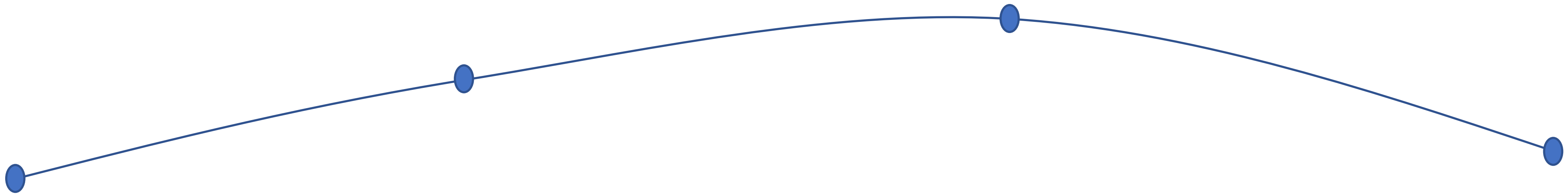

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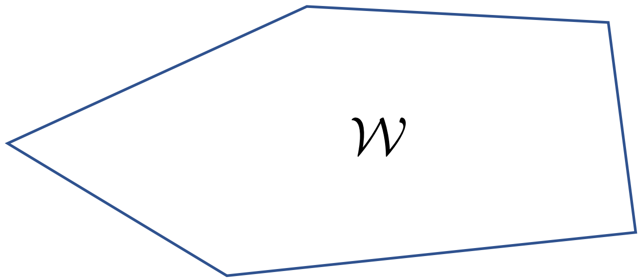

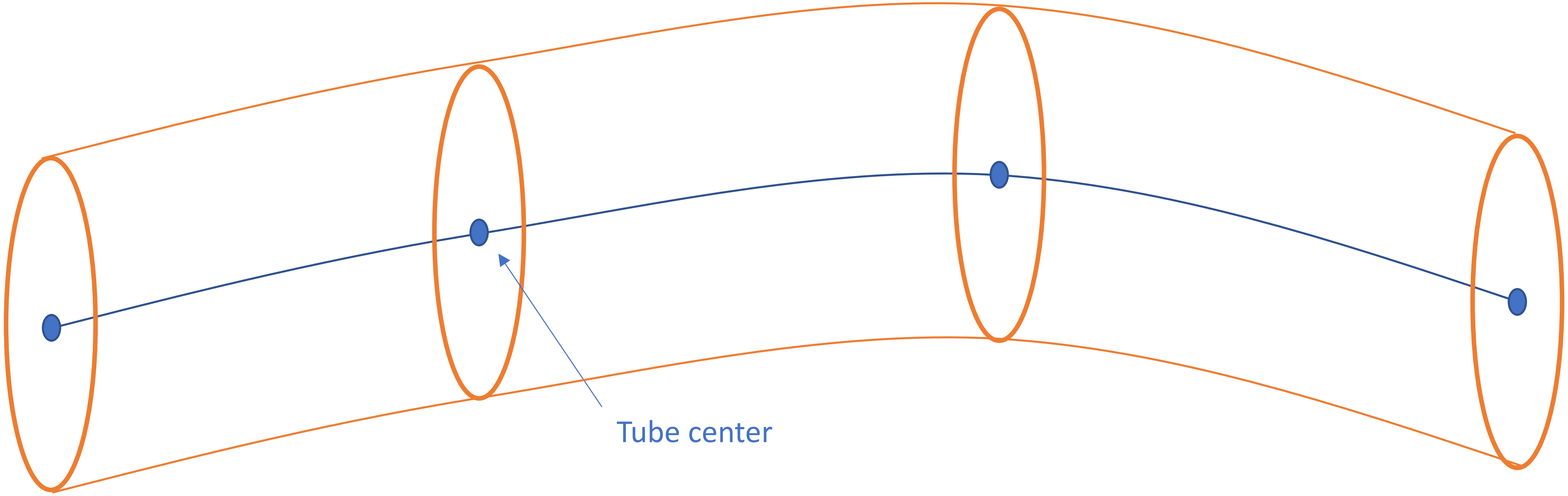

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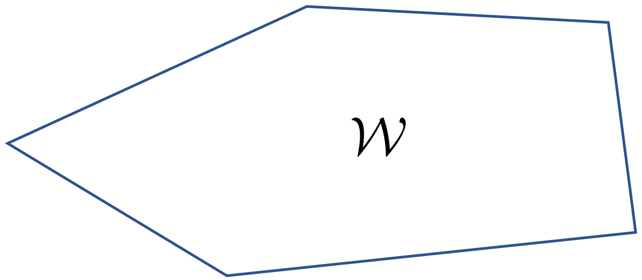

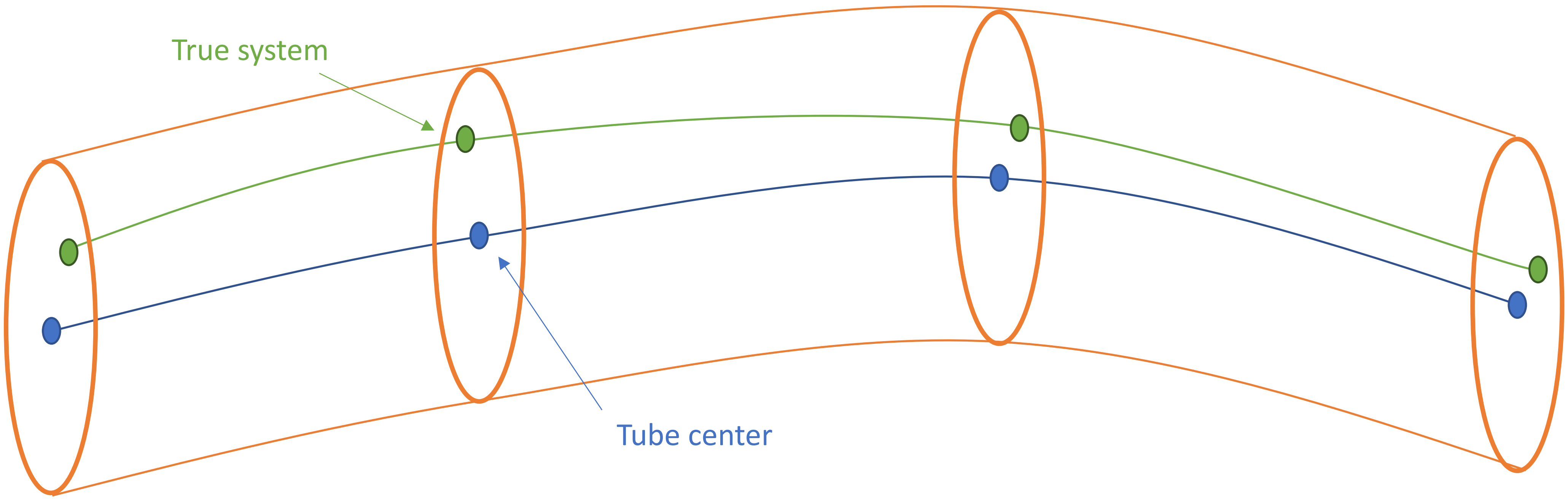

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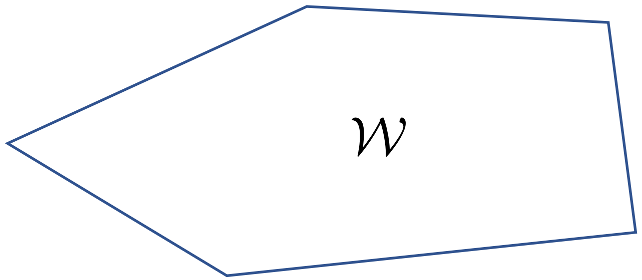

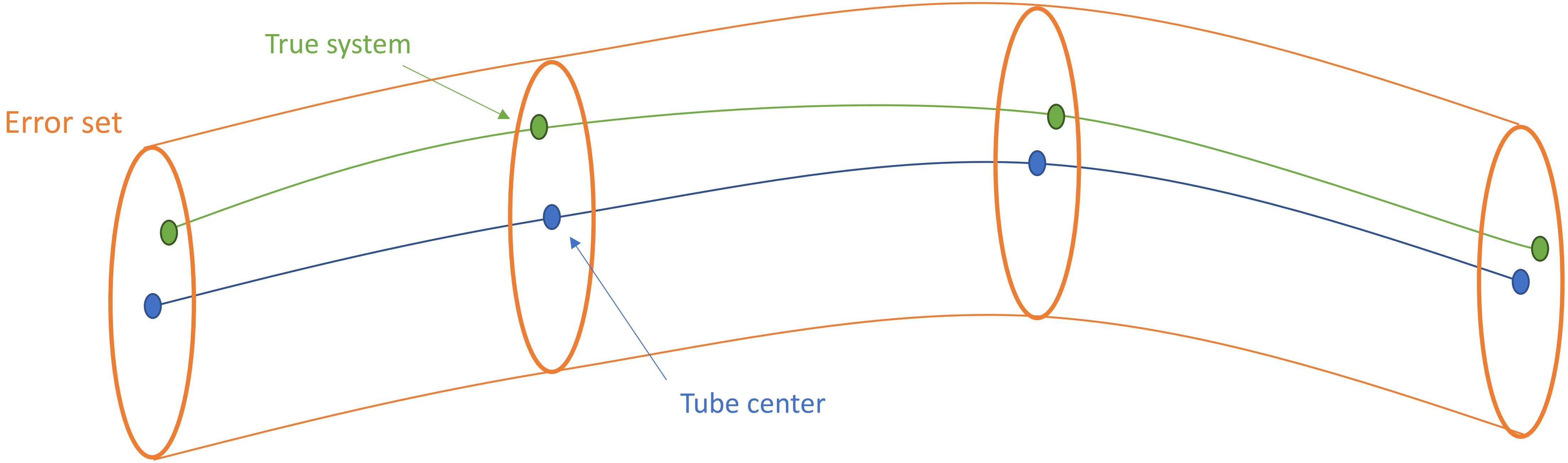

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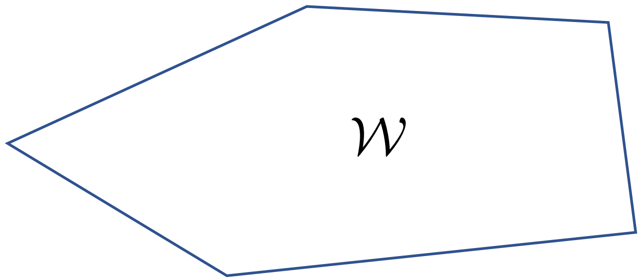

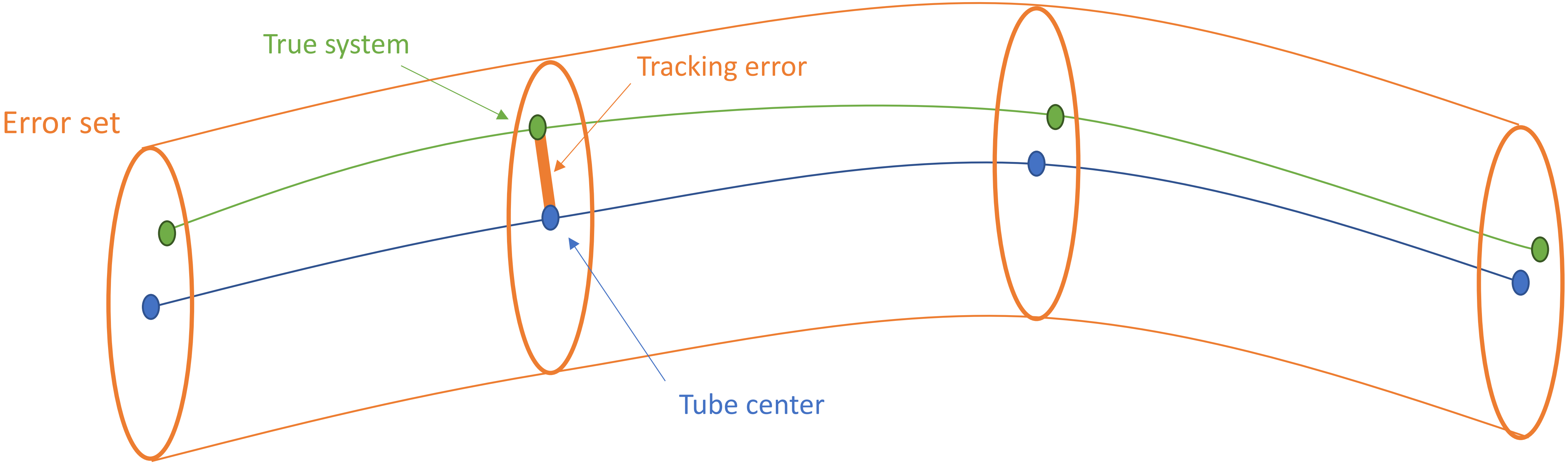

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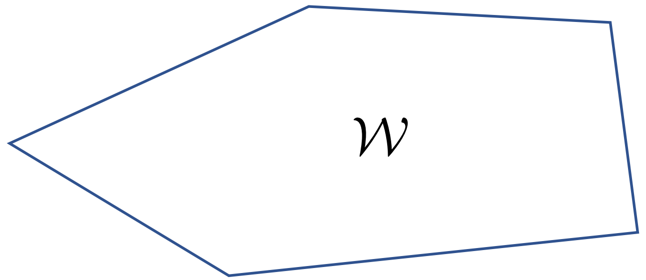

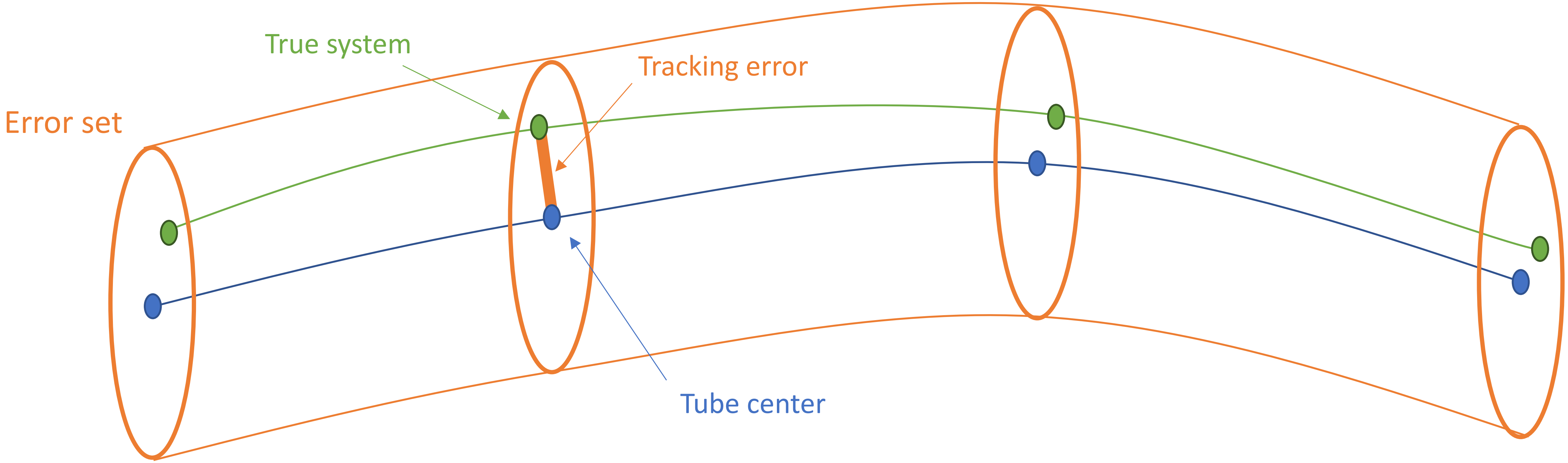

Key Ideas:

- Tube center planned via MPC
- Feedback to reduce tracking error

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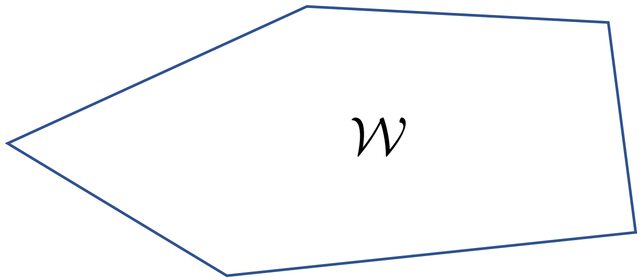

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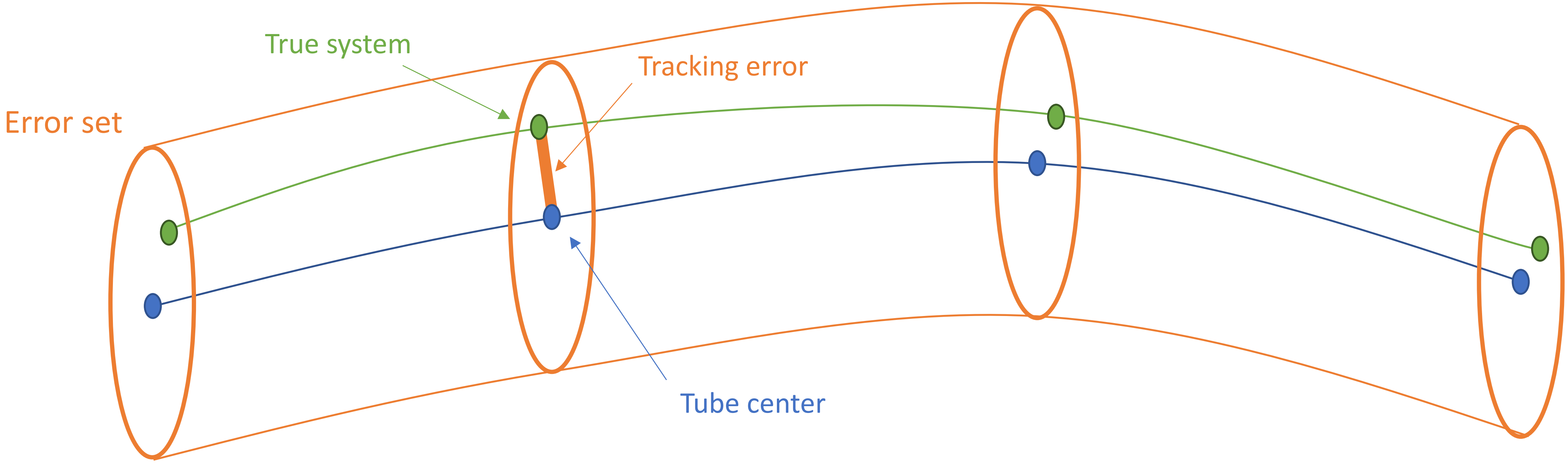

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$$\pi(x) = \pi^{\mathrm{MPC}}(x) + \pi^{\mathrm{tracking}}(x, \text{tube center})$$

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LMPC for uncertain systems

A sample-based approach

LMPC for uncertain systems – Problem Formulation

LMPC for uncertain systems – Problem Formulation

- Model Choice: Linear system subject to bounded additive uncertainty,

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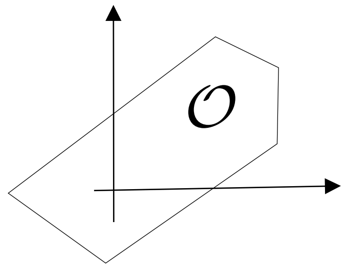
- Cost: Worst Case.
- Convergence: Cannot regulate to a point but to a robust positive invariant $\mathcal{O}$.



$$\forall x \in \mathcal{O} \to (A - KB)x + w \in \mathcal{O}, \; \forall w \in \mathcal{W}$$

LMPC for uncertain systems – Differences w/ Nominal

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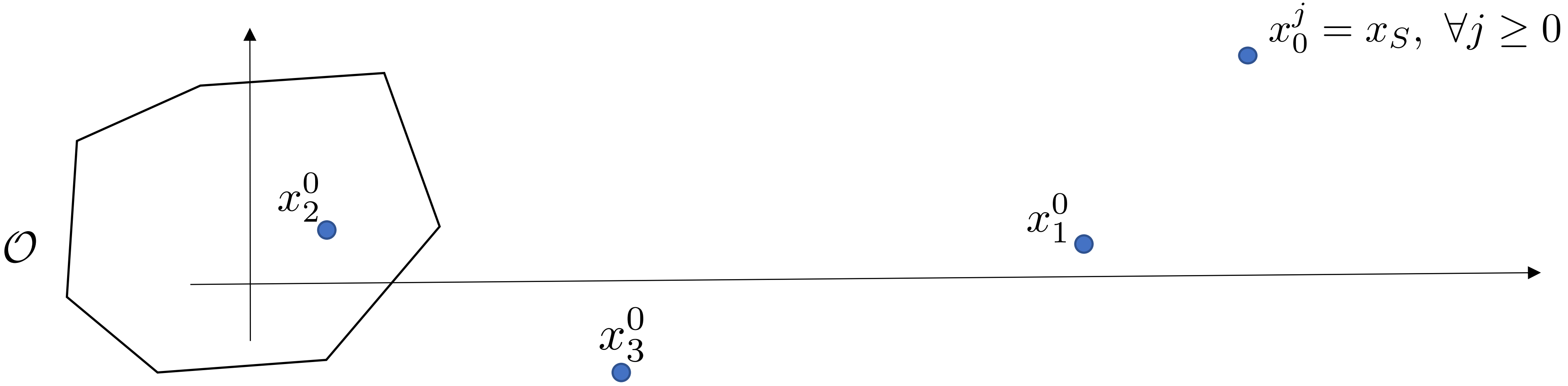
Given a feasible trajectory which drives the system to the terminal robust invariant



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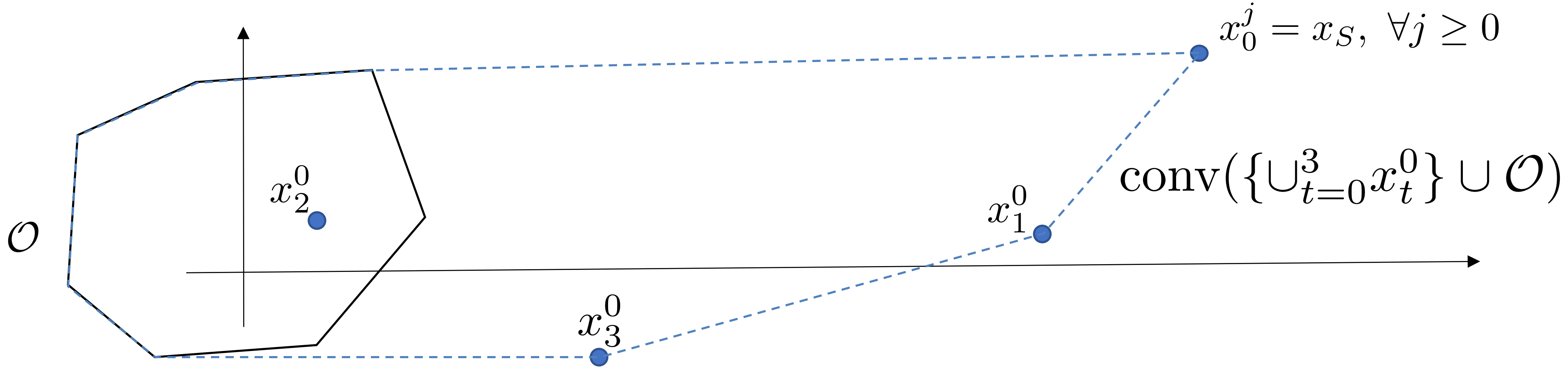

The convex hull of the stored data is not a robust control invariant.

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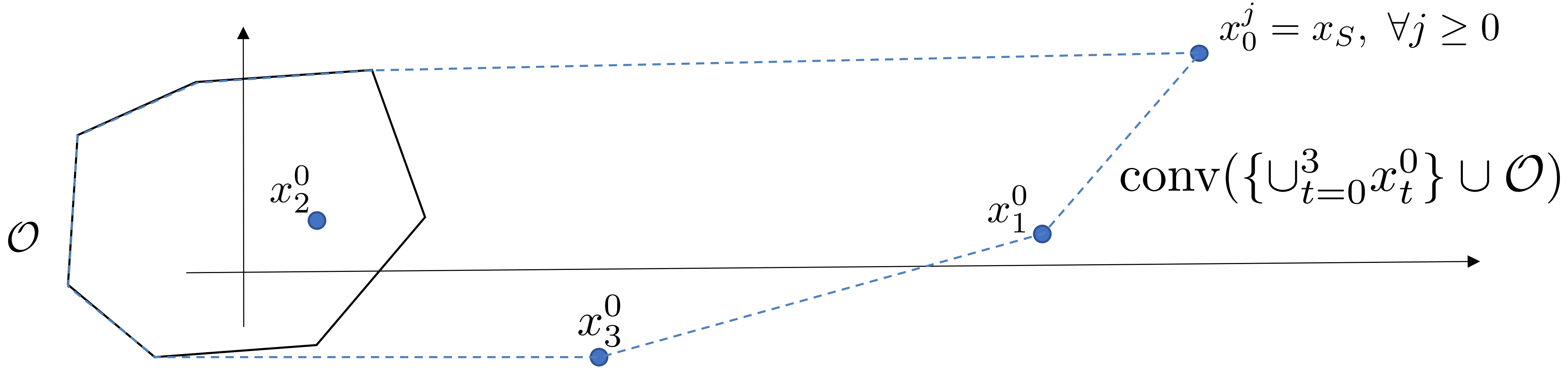

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For this reason,

A. the safe set

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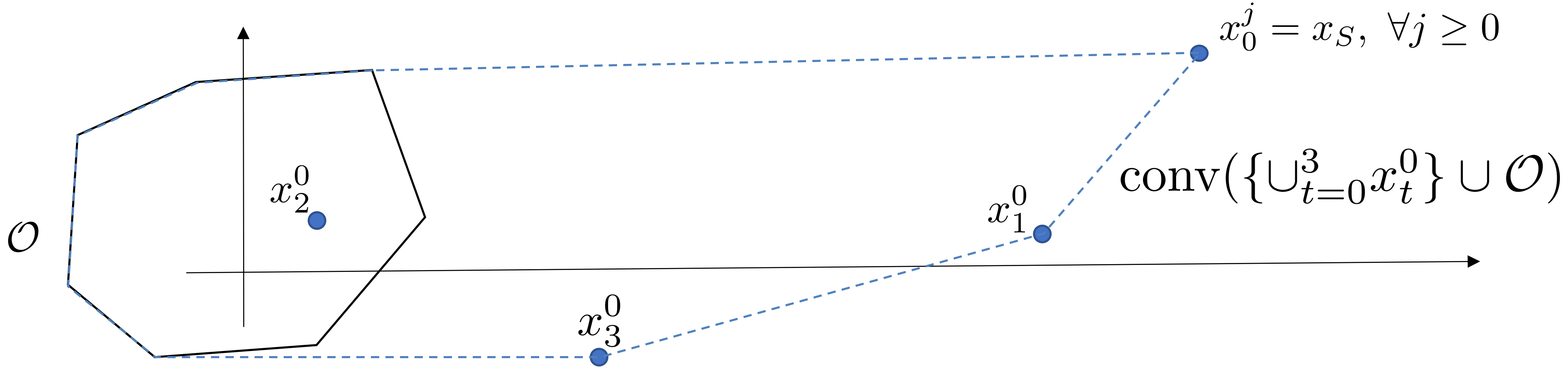

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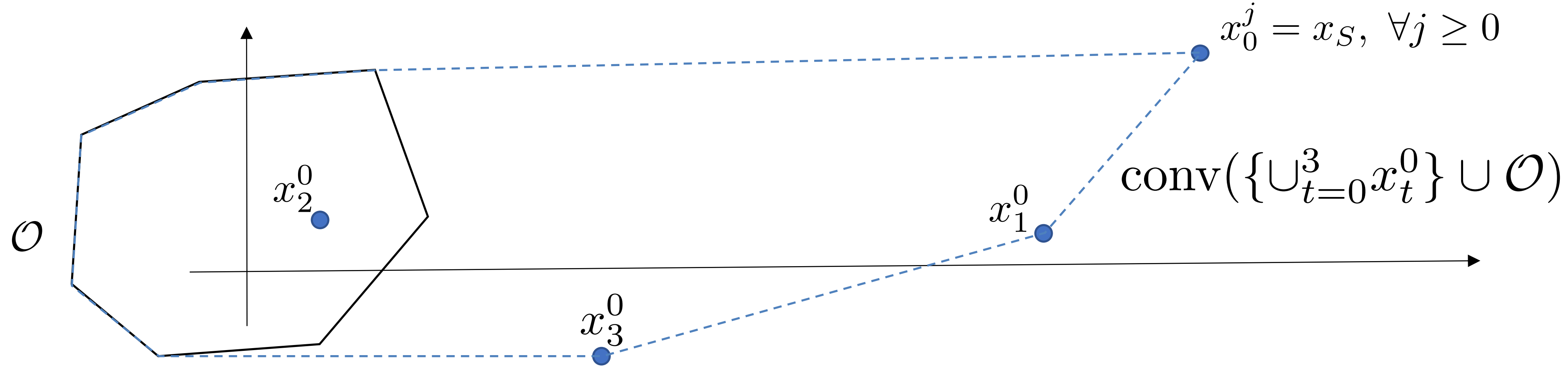


The convex hull of the stored data is not a robust control invariant.

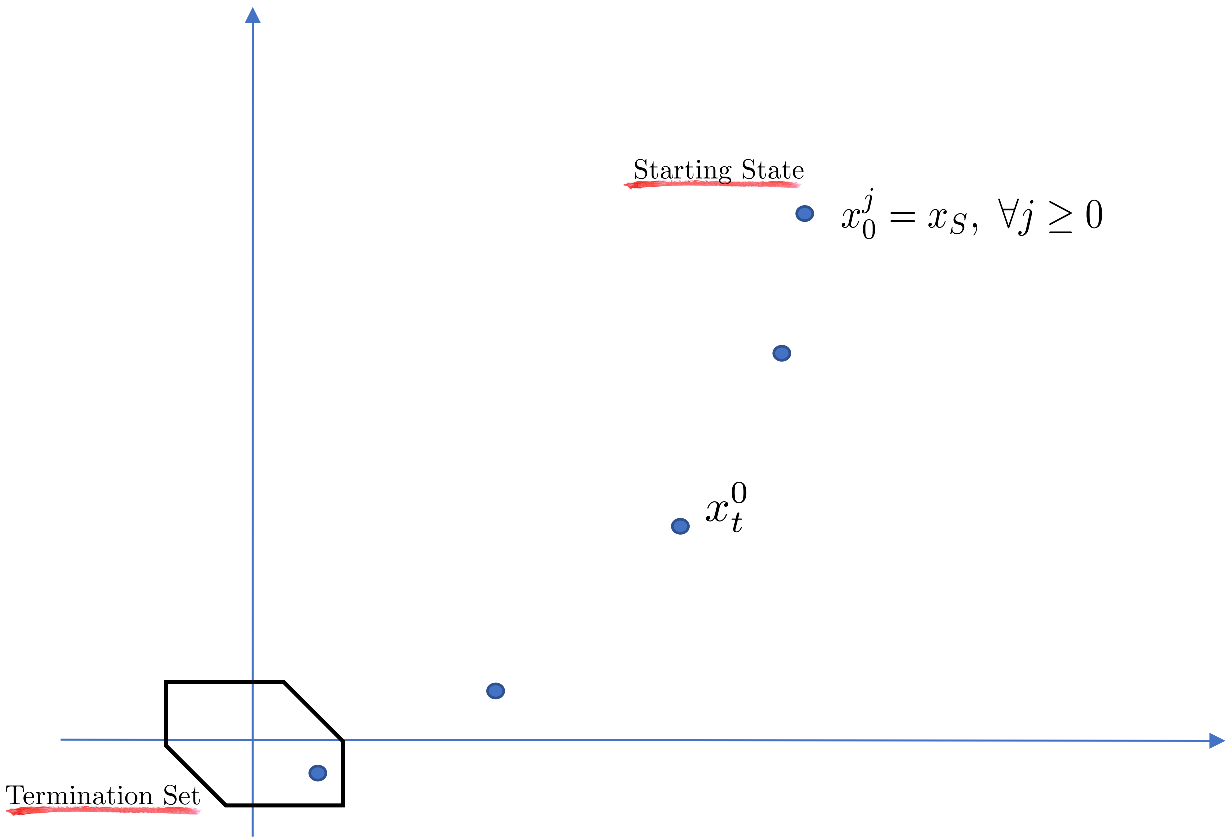
For this reason,

A. the safe set

B. the terminal cost function

need to be designed differently in Robust LMPC.

Example: Constrained LQR Uncertain System



Control Problem

Consider the uncertain system

$$x_{k+1} = Ax_k + Bu_k + w_k, \text{ with } w_k \in \mathcal{W} \ \forall k \geq 0$$

Our goal is to design a safe policy which steers the system to a terminal set

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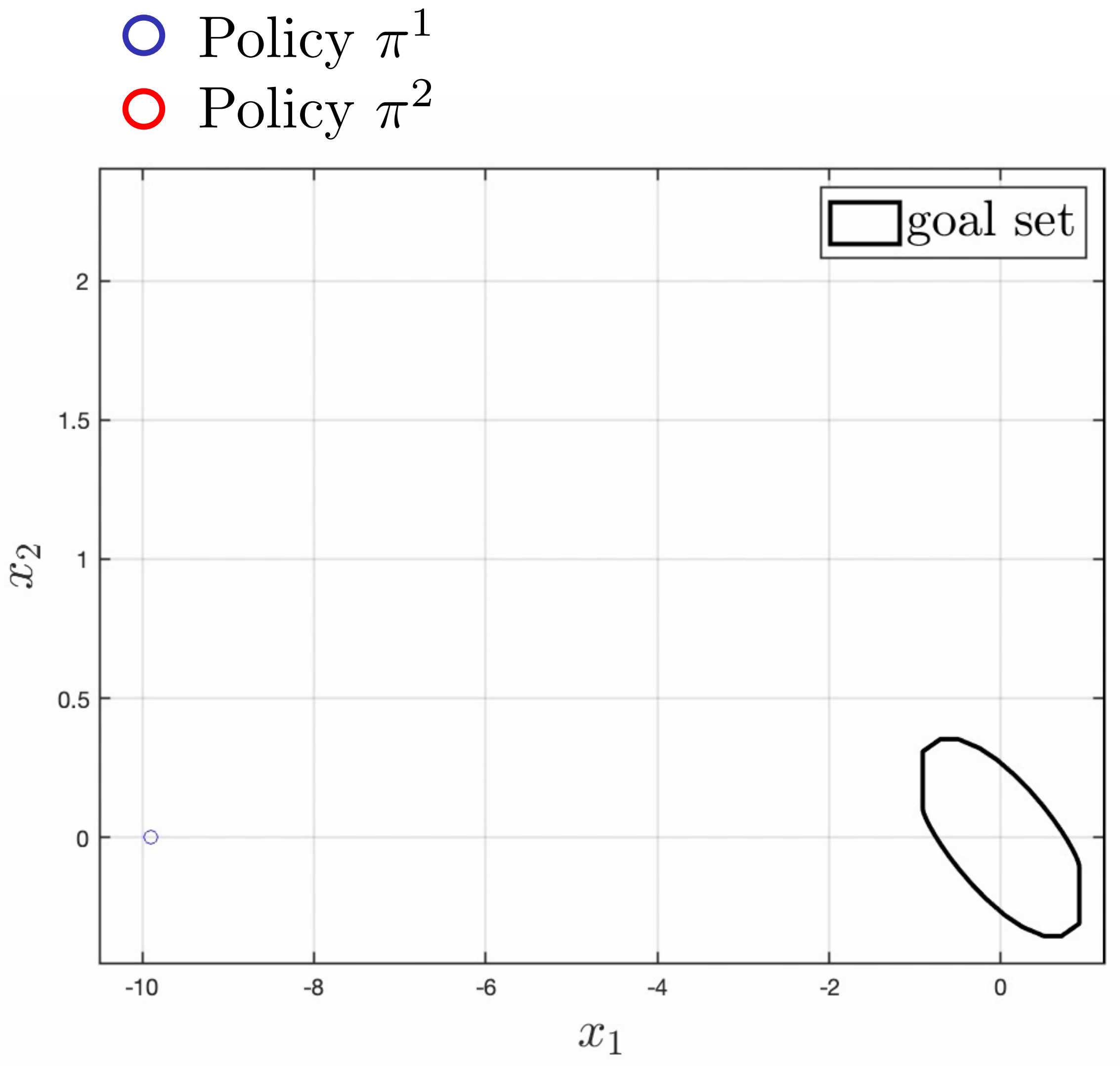

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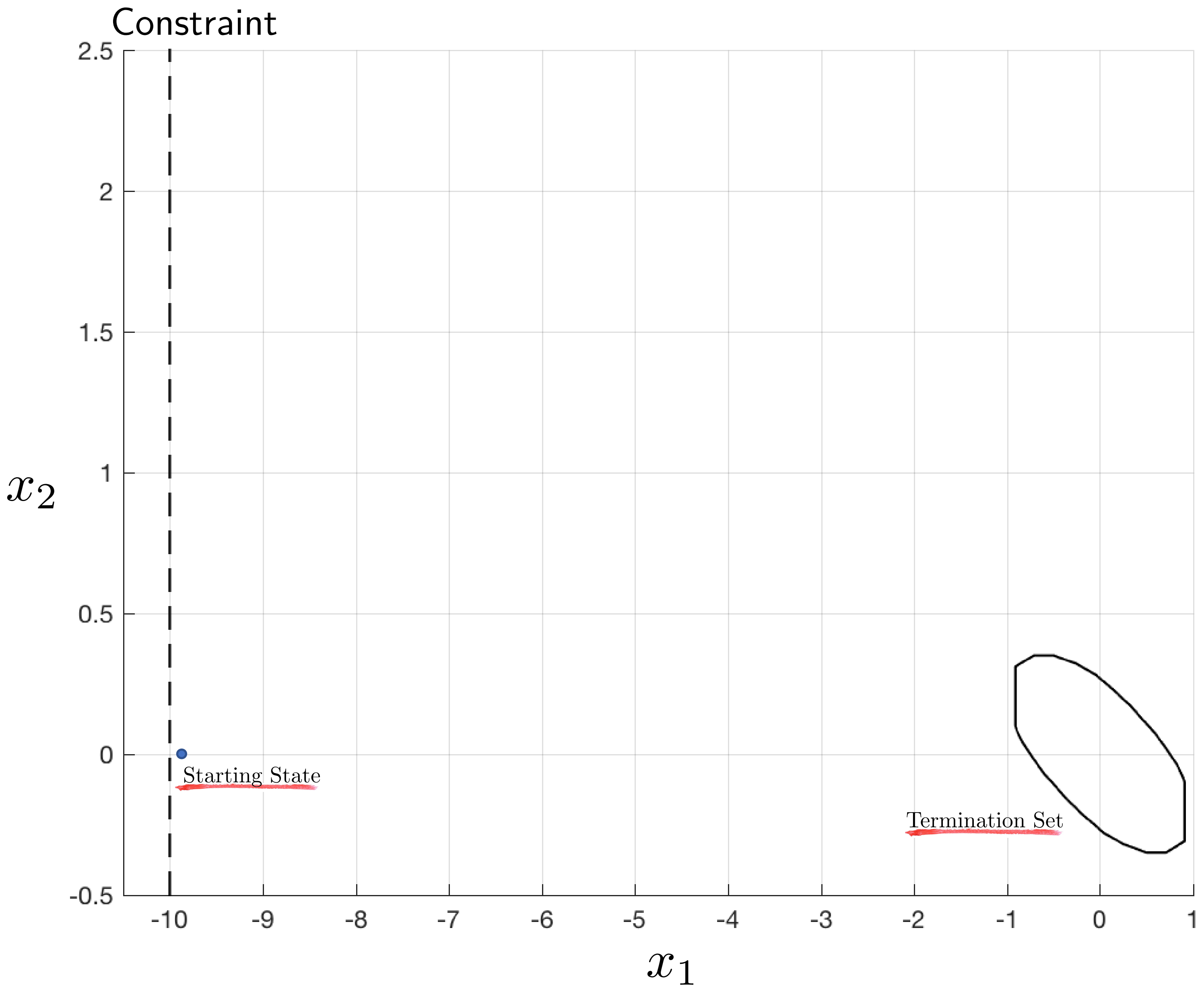

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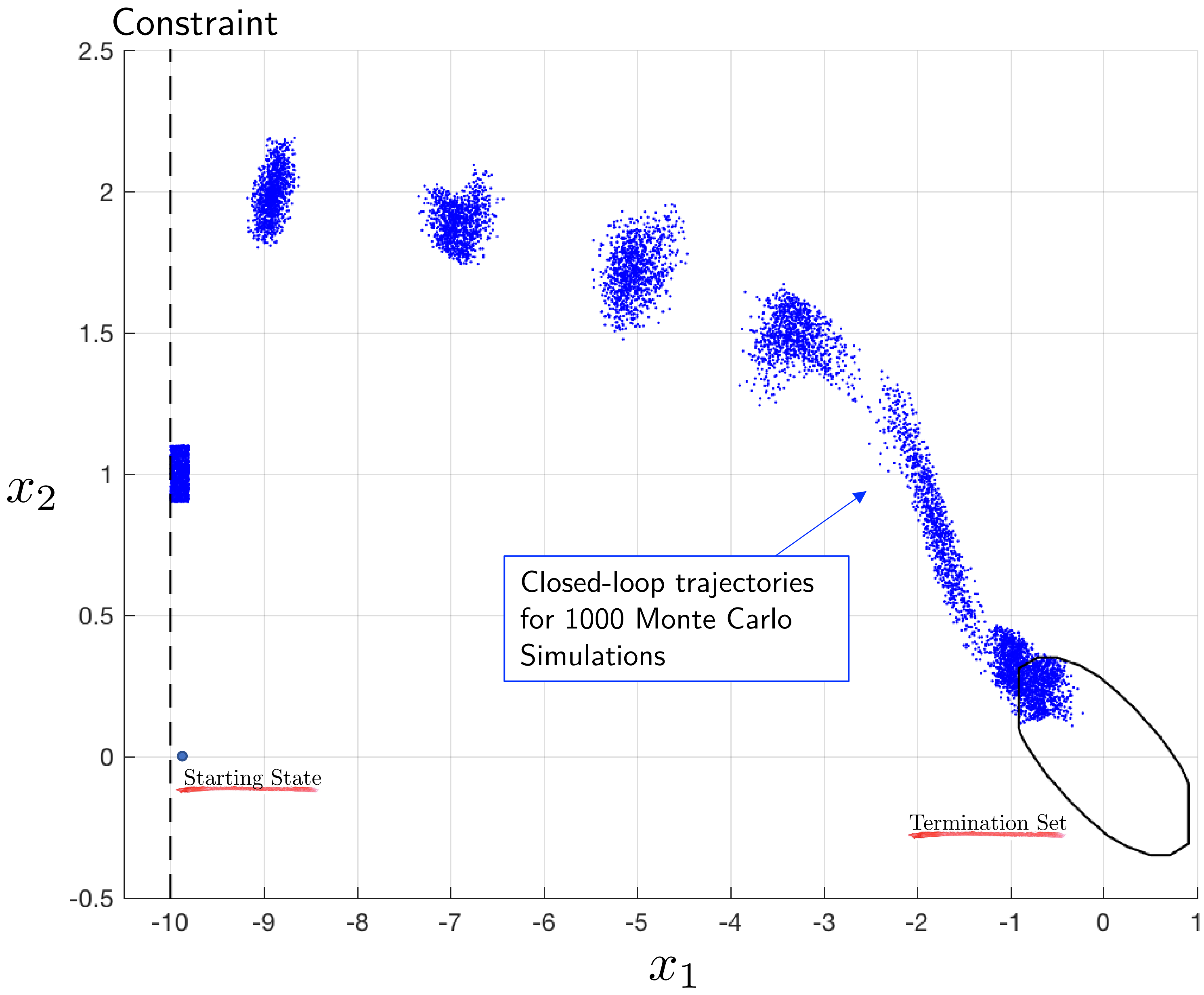

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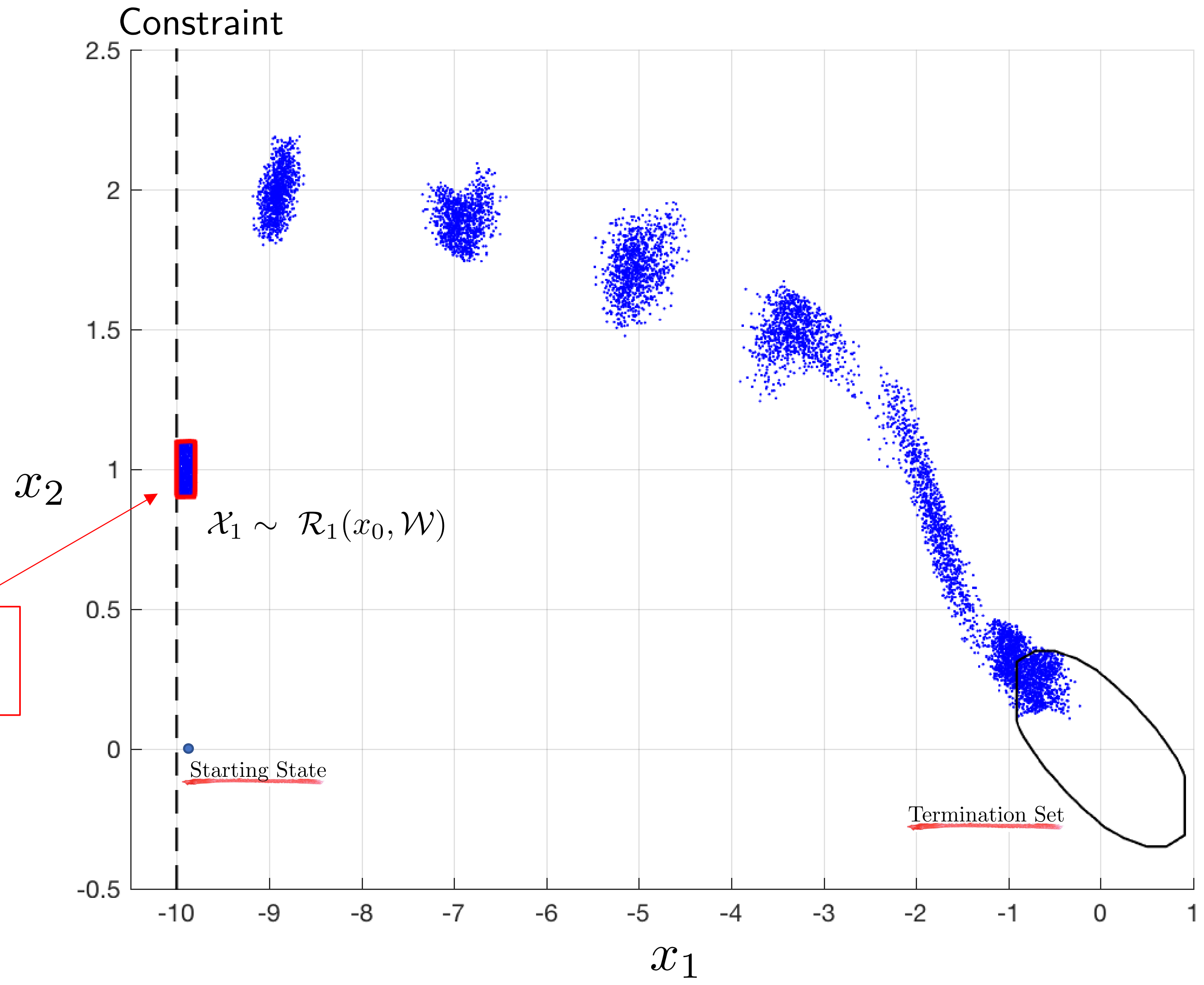
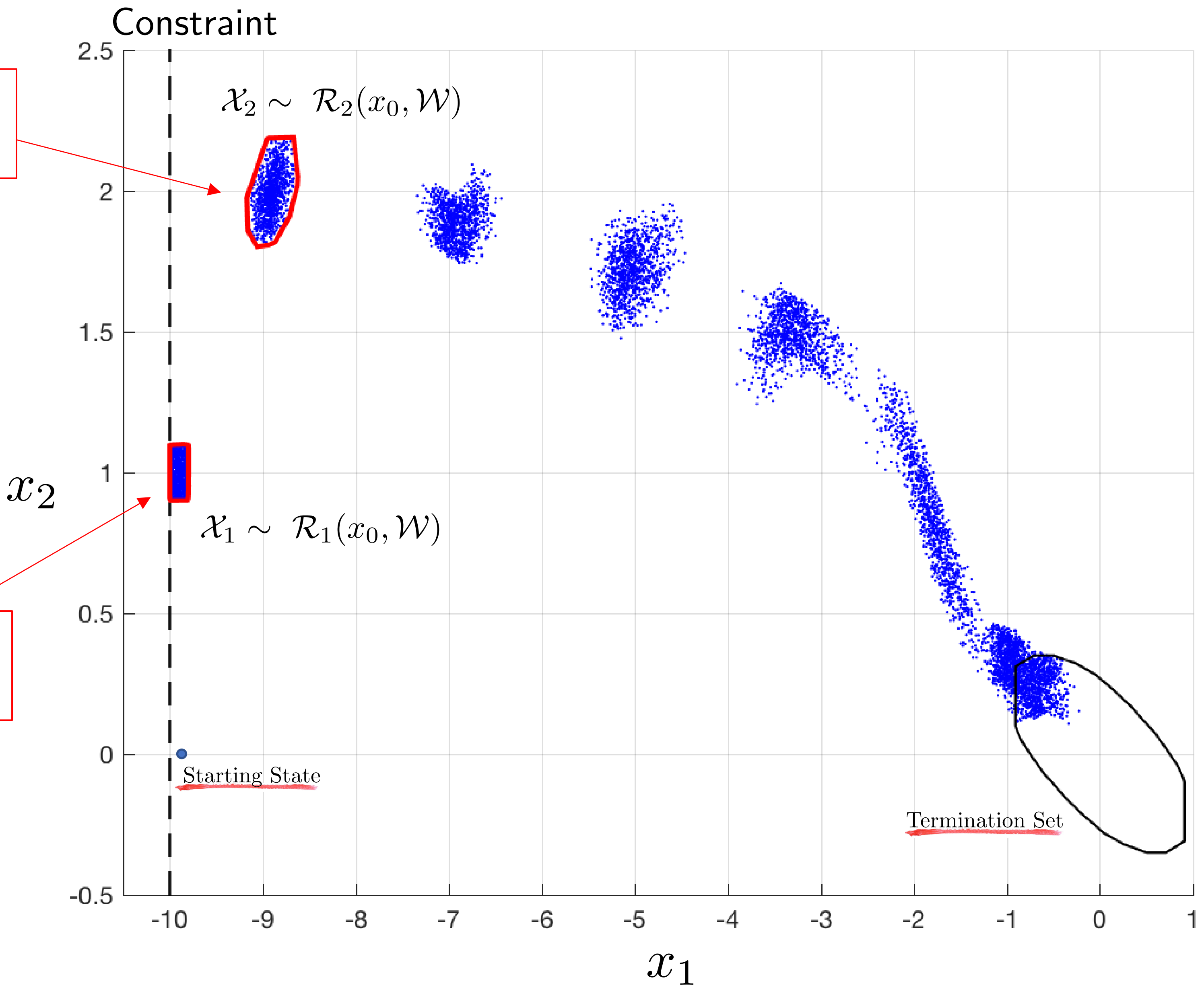


Convex hull realized state at time *t=1*

Example: Constrained LQR Uncertain System

Convex hull realized state at time *t=2*

Convex hull realized state at time *t=1*



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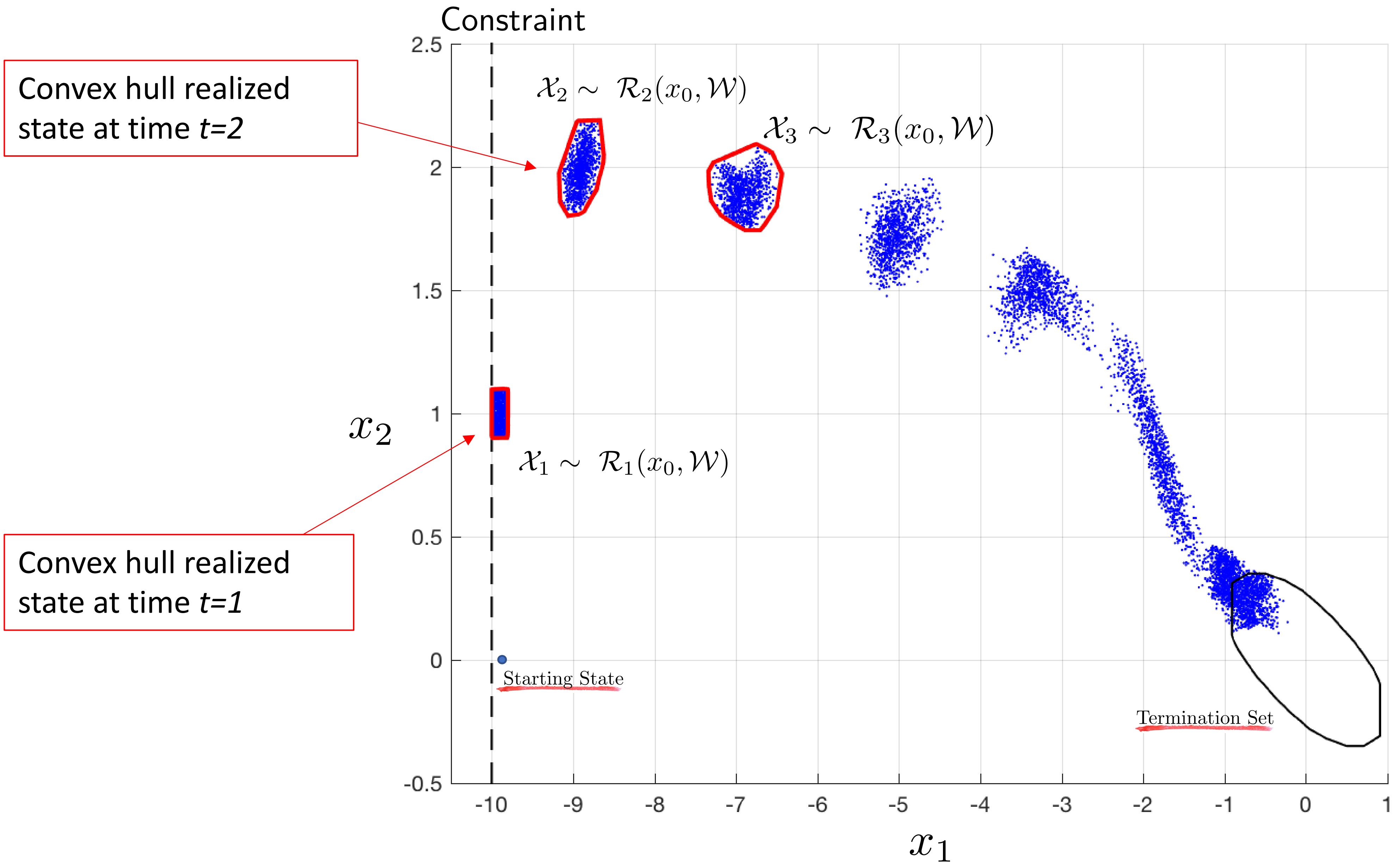

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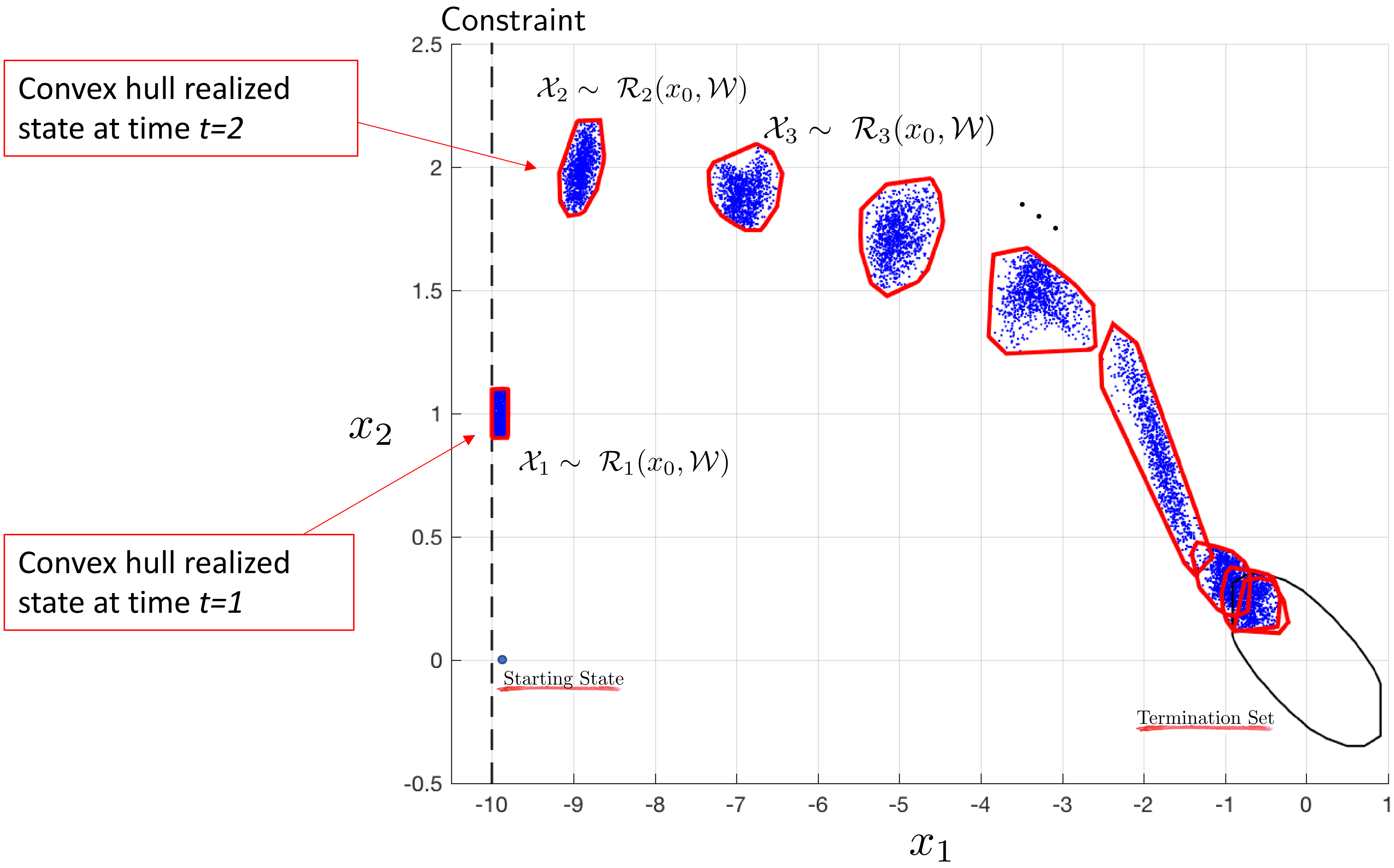

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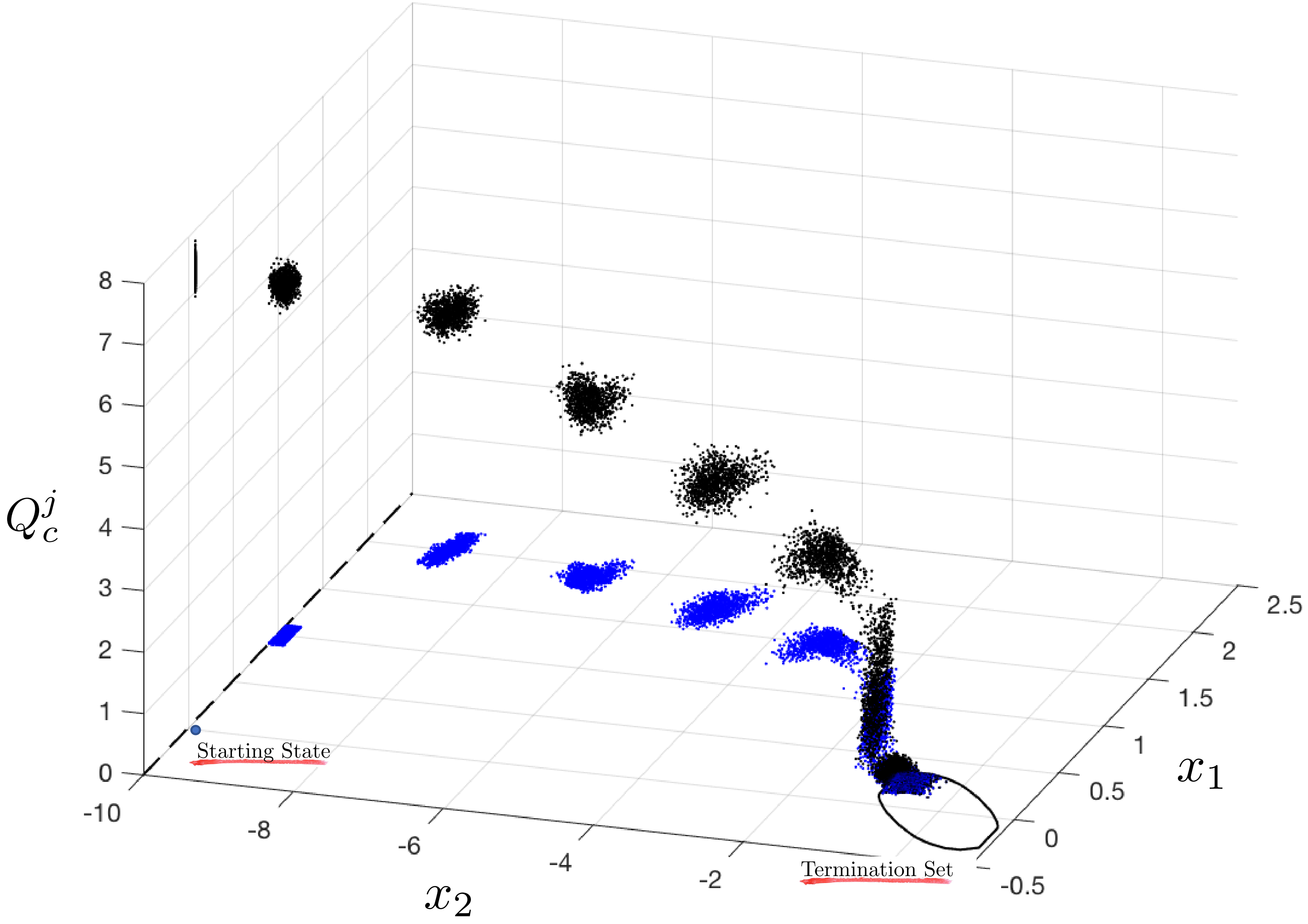

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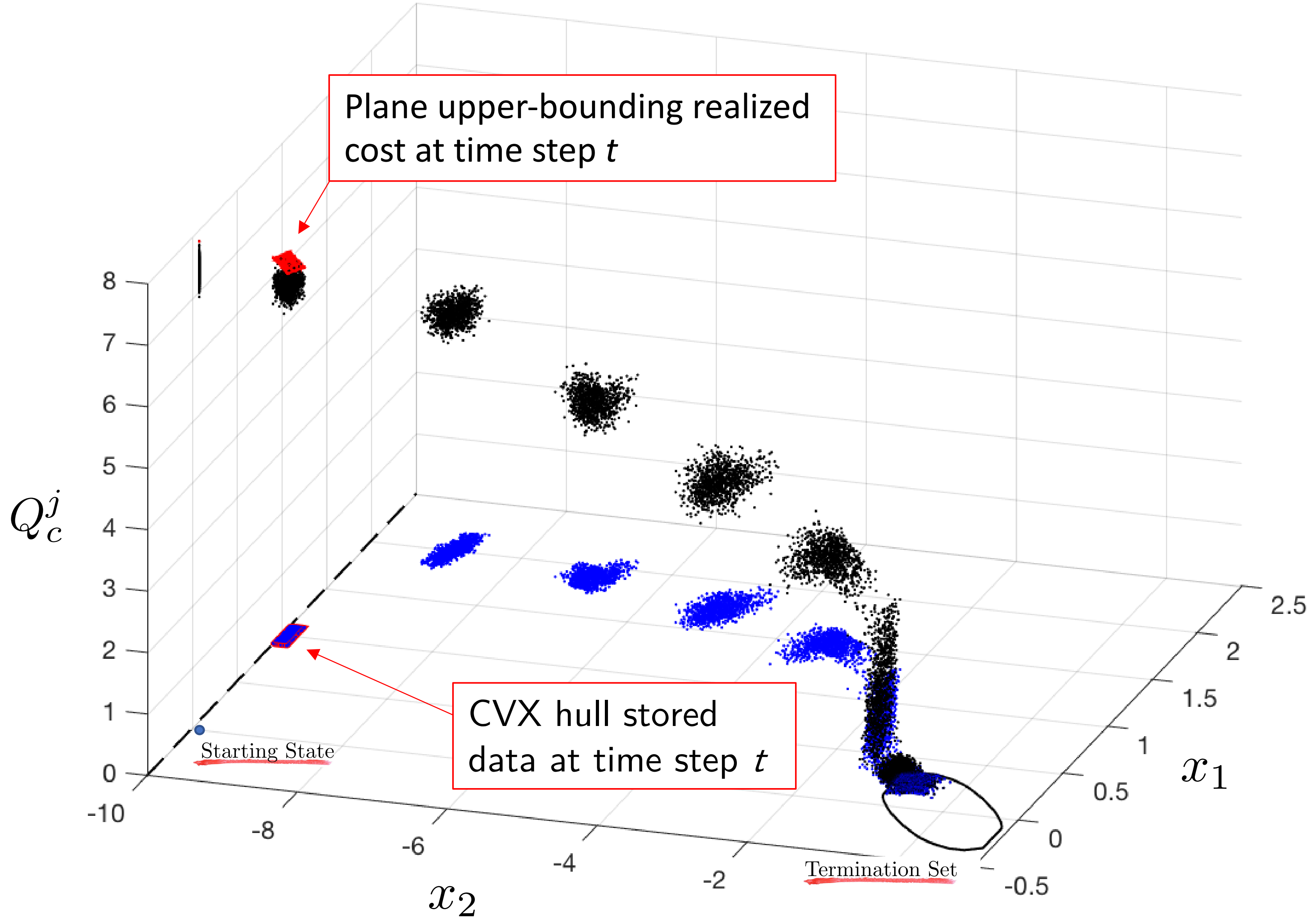

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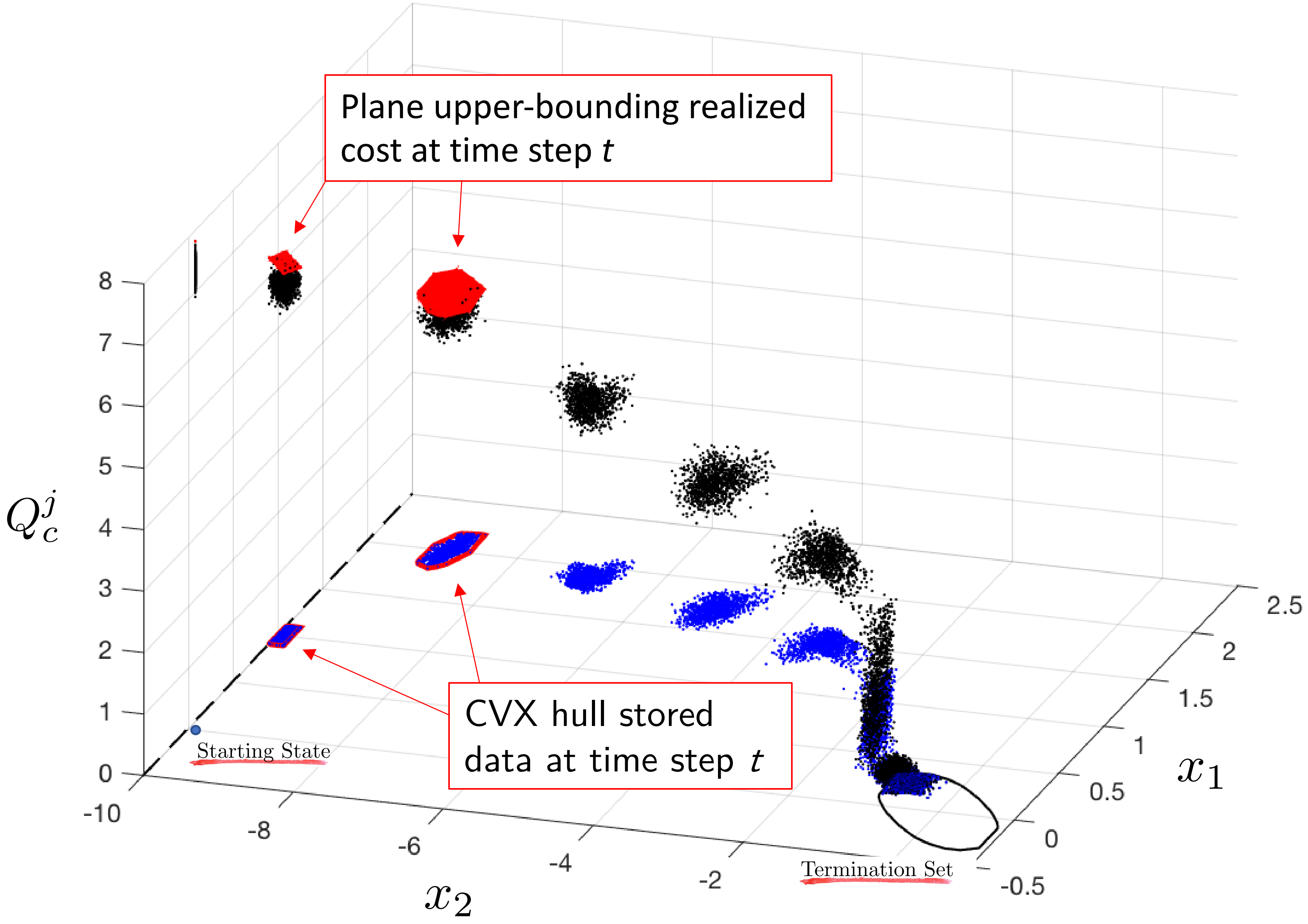

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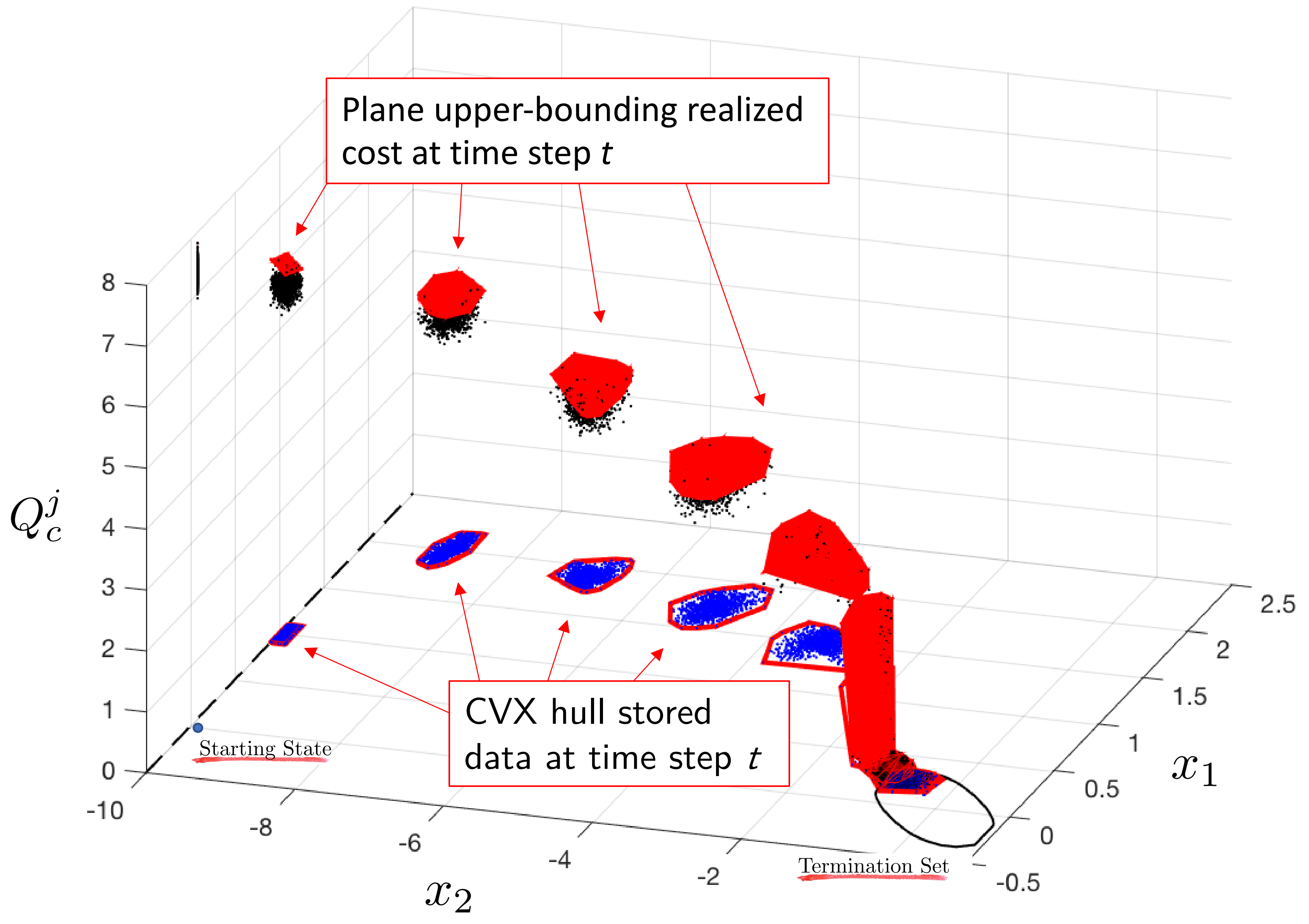

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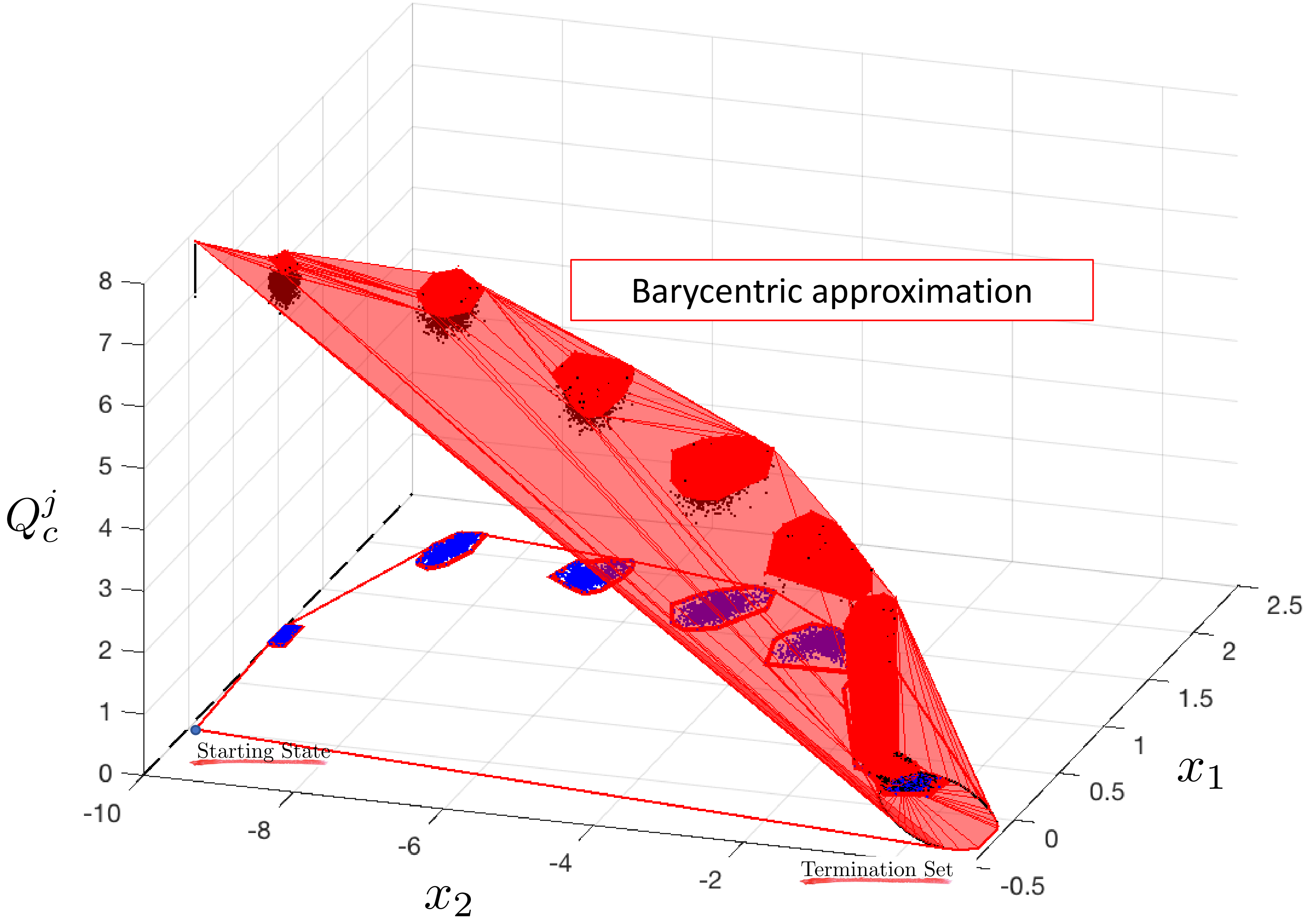

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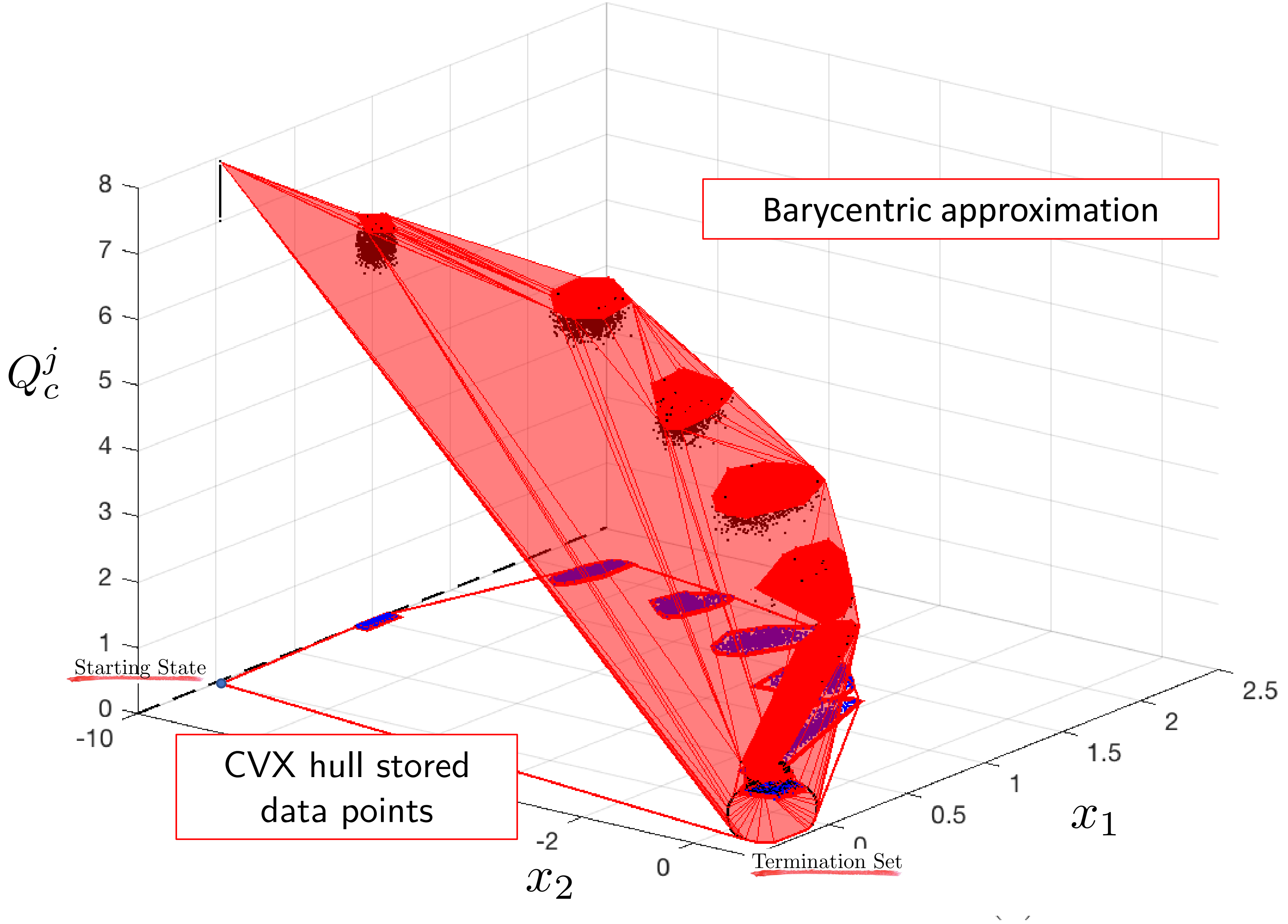

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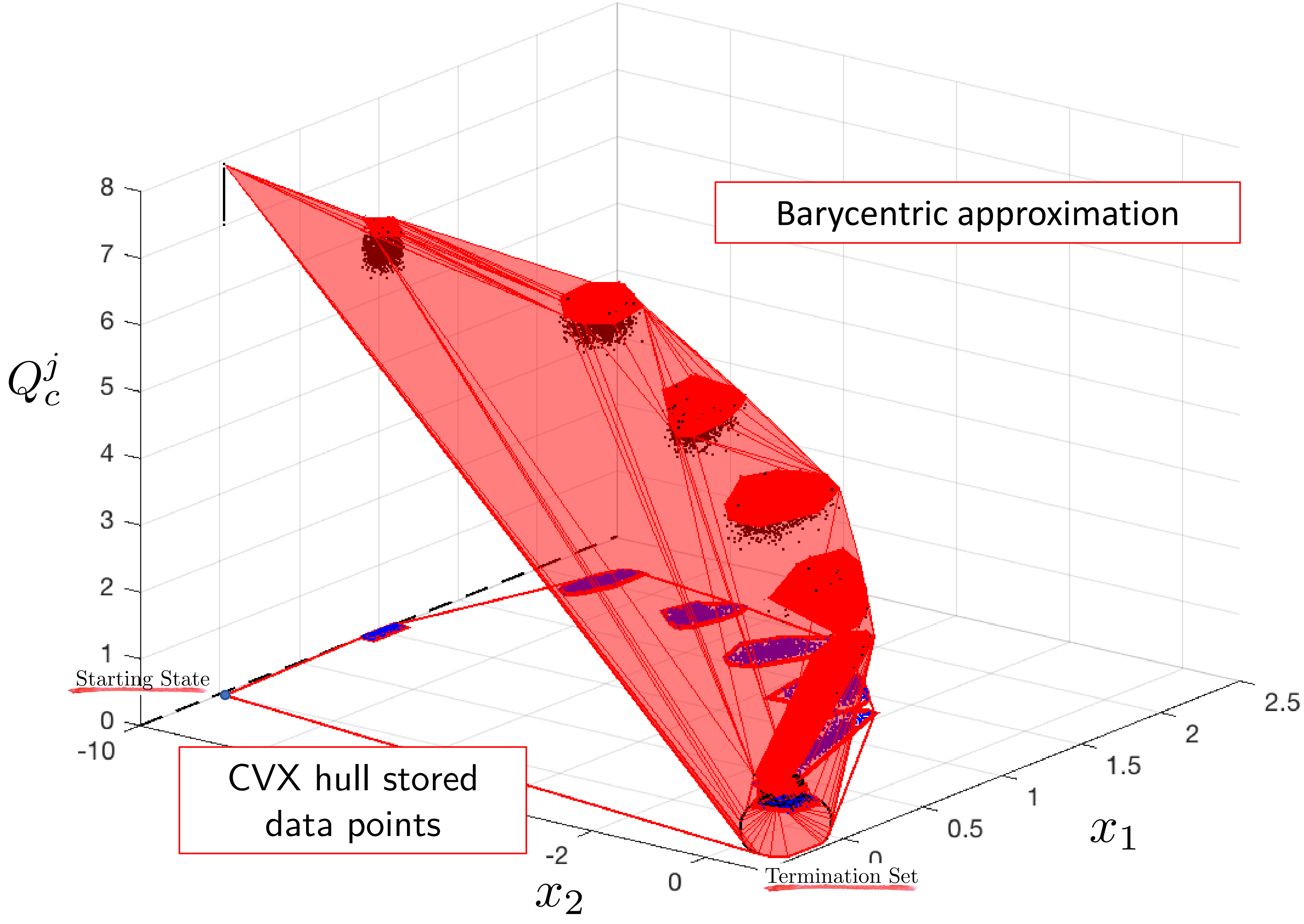

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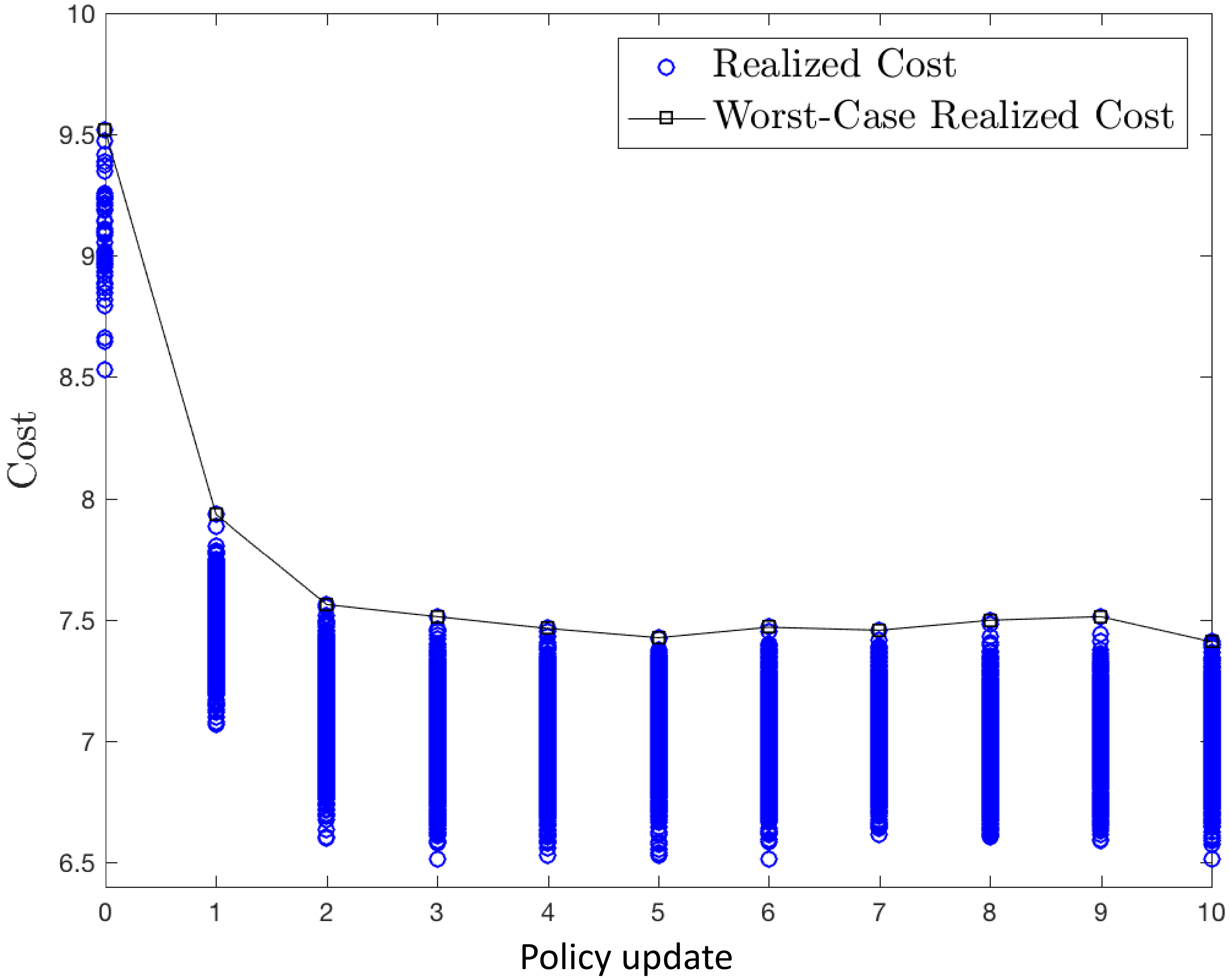

Summary Deterministic Vs Robust LMPC

LMPC for deterministic systems

LMPC for uncertain systems

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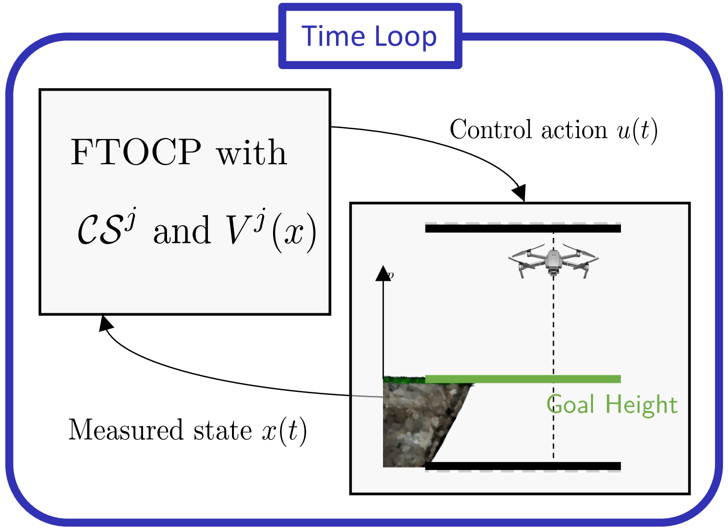

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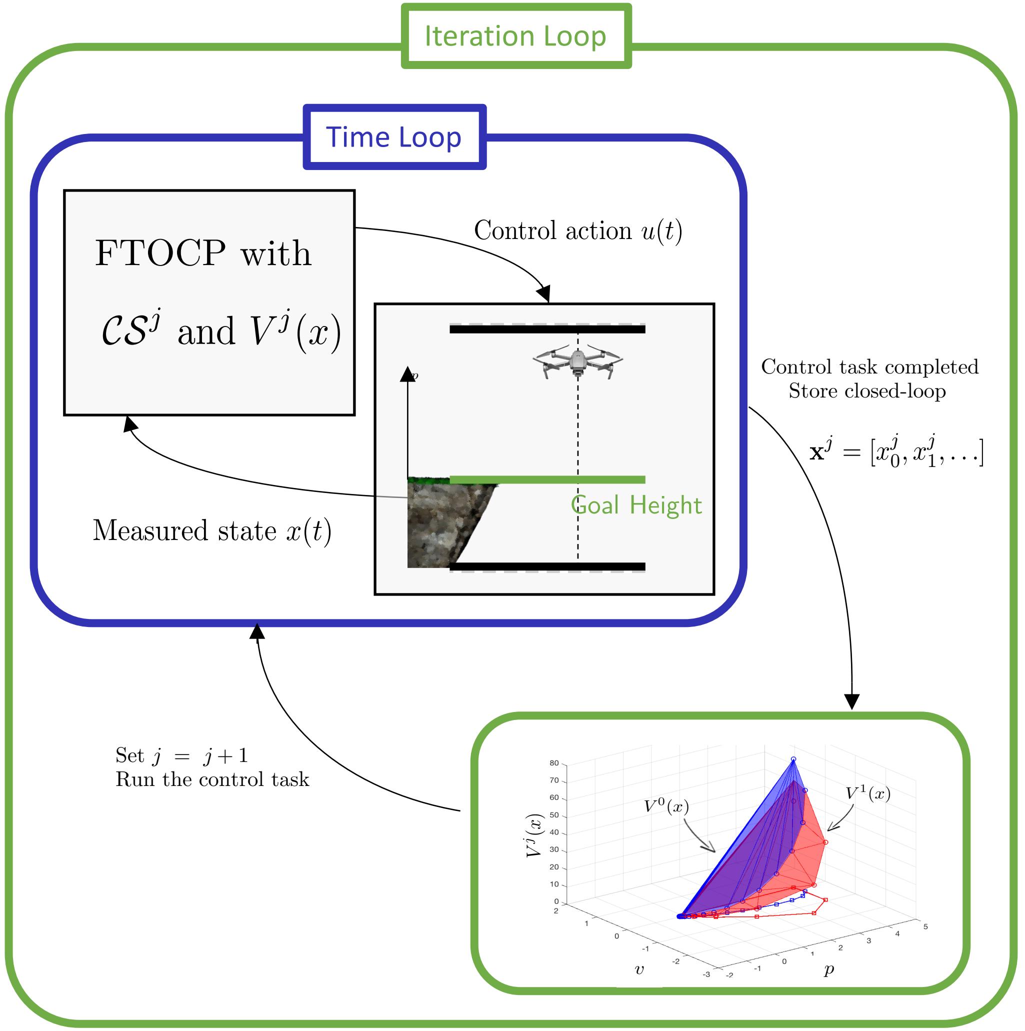

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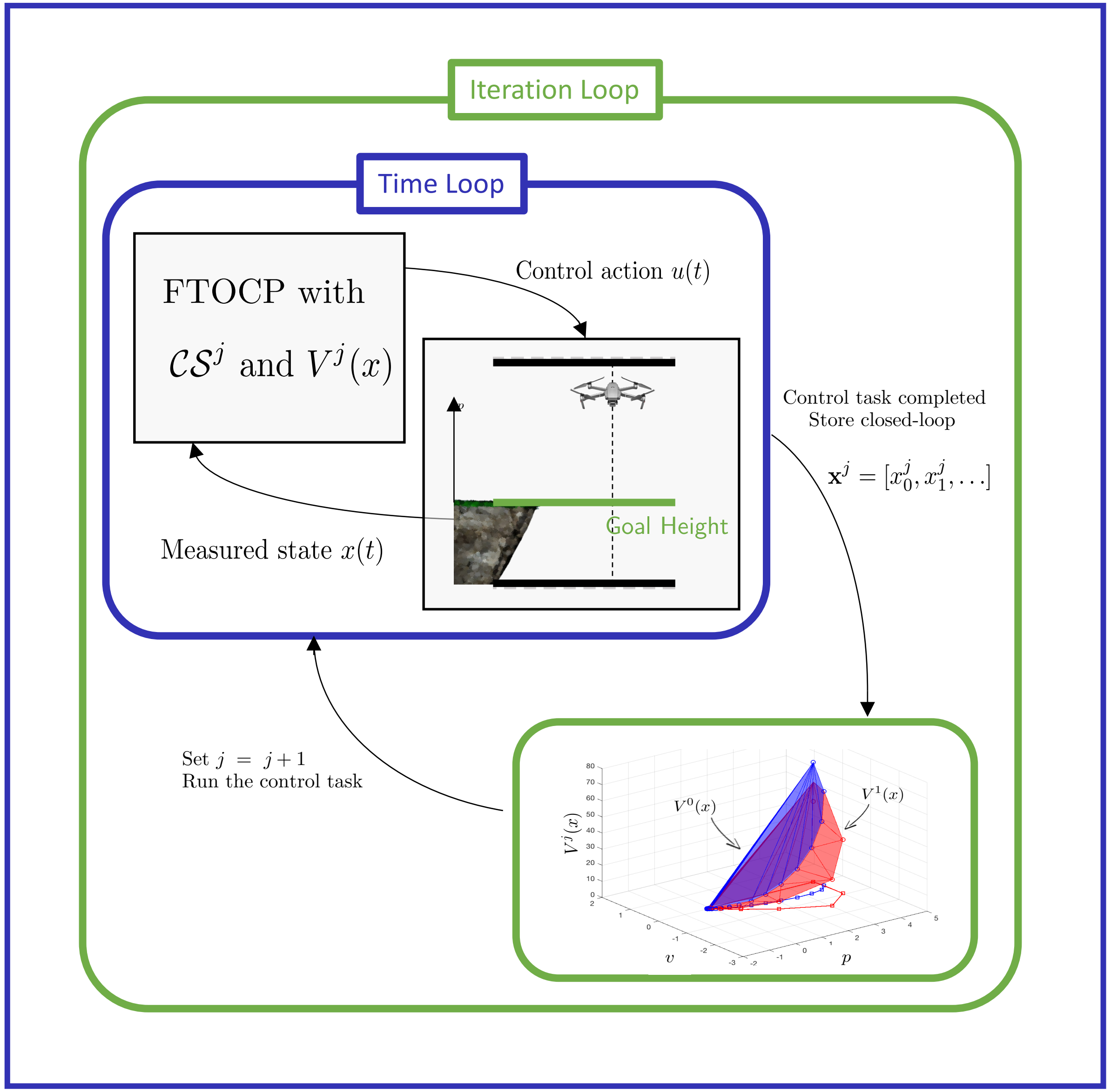

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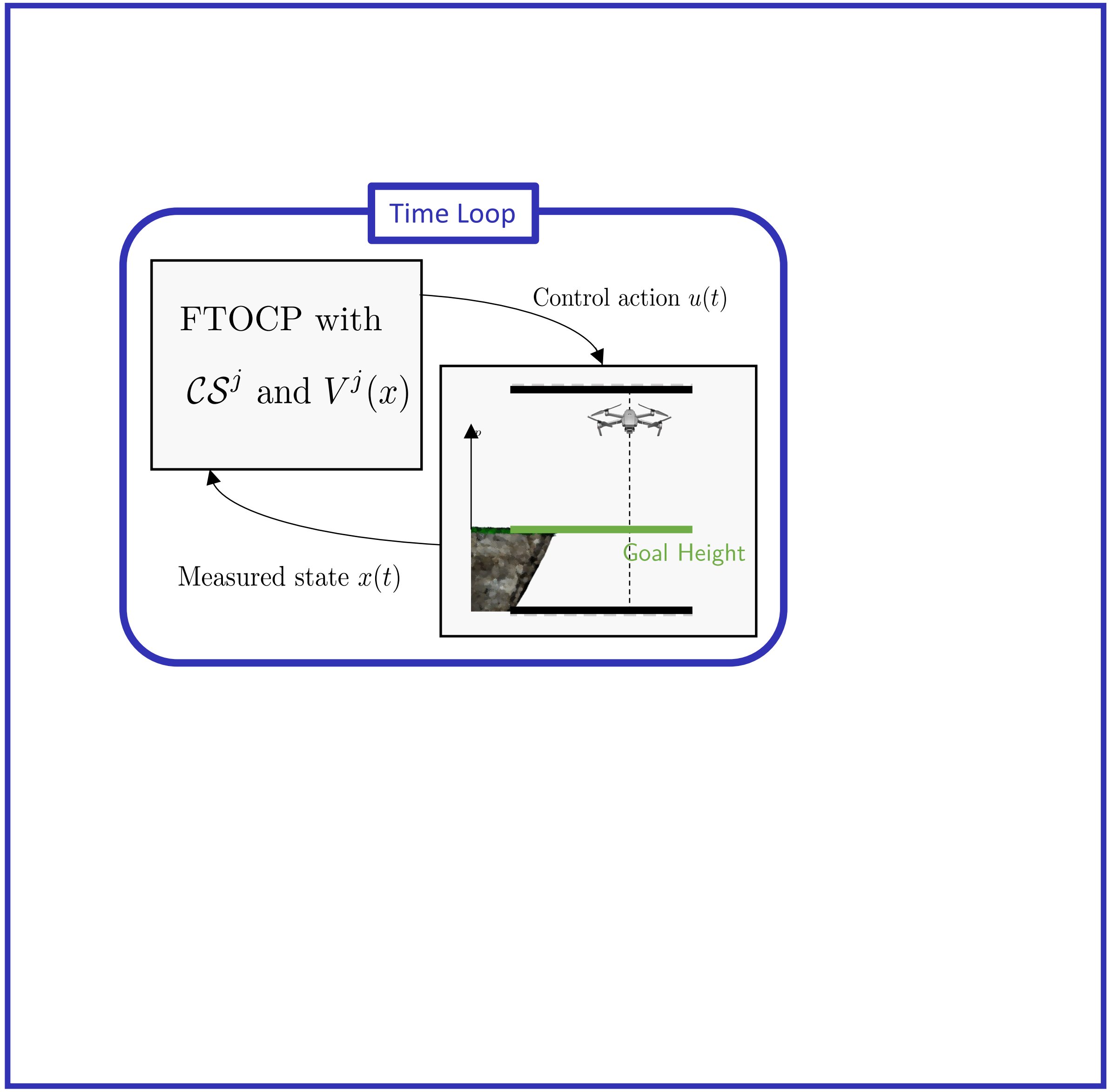

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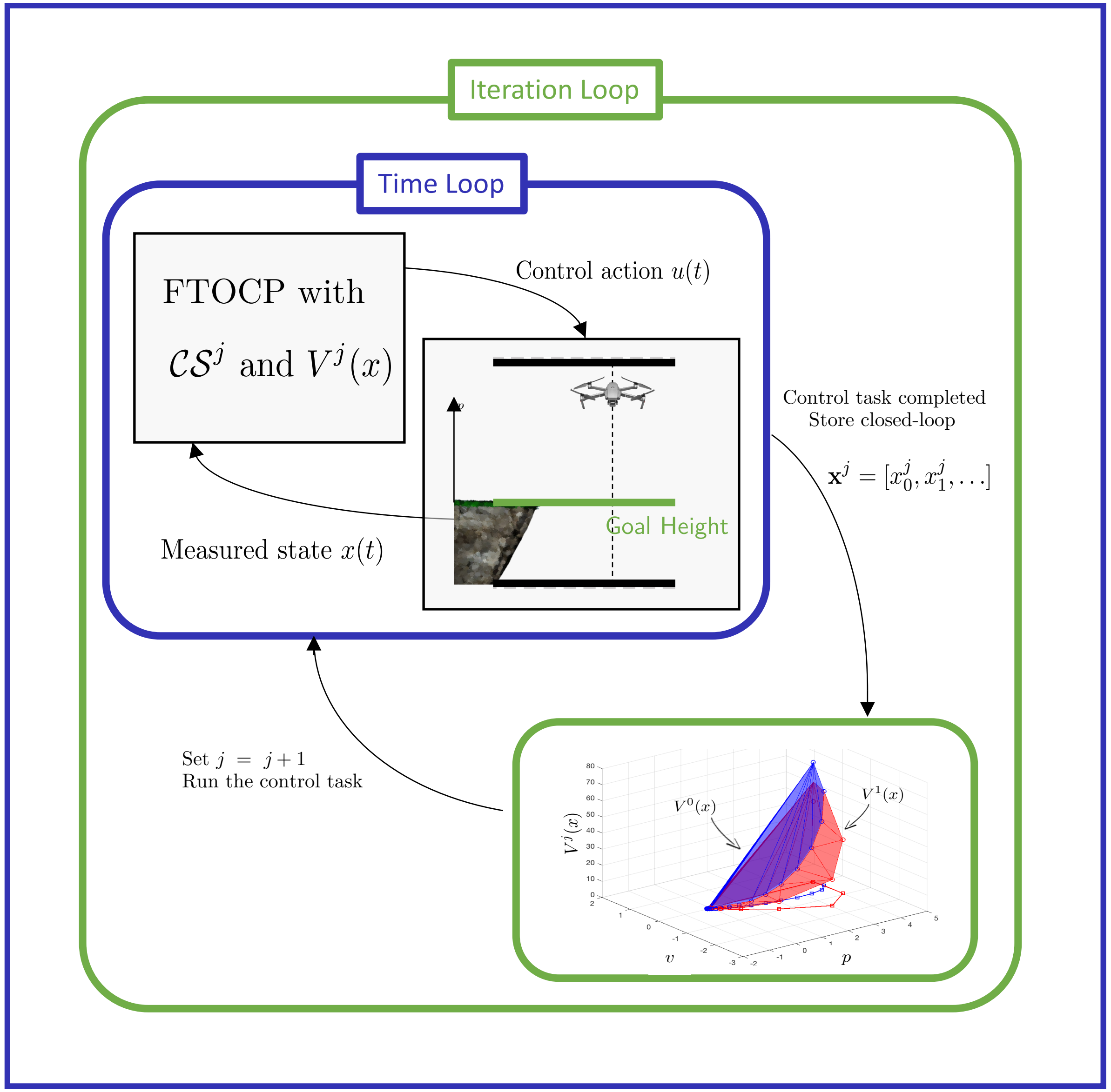

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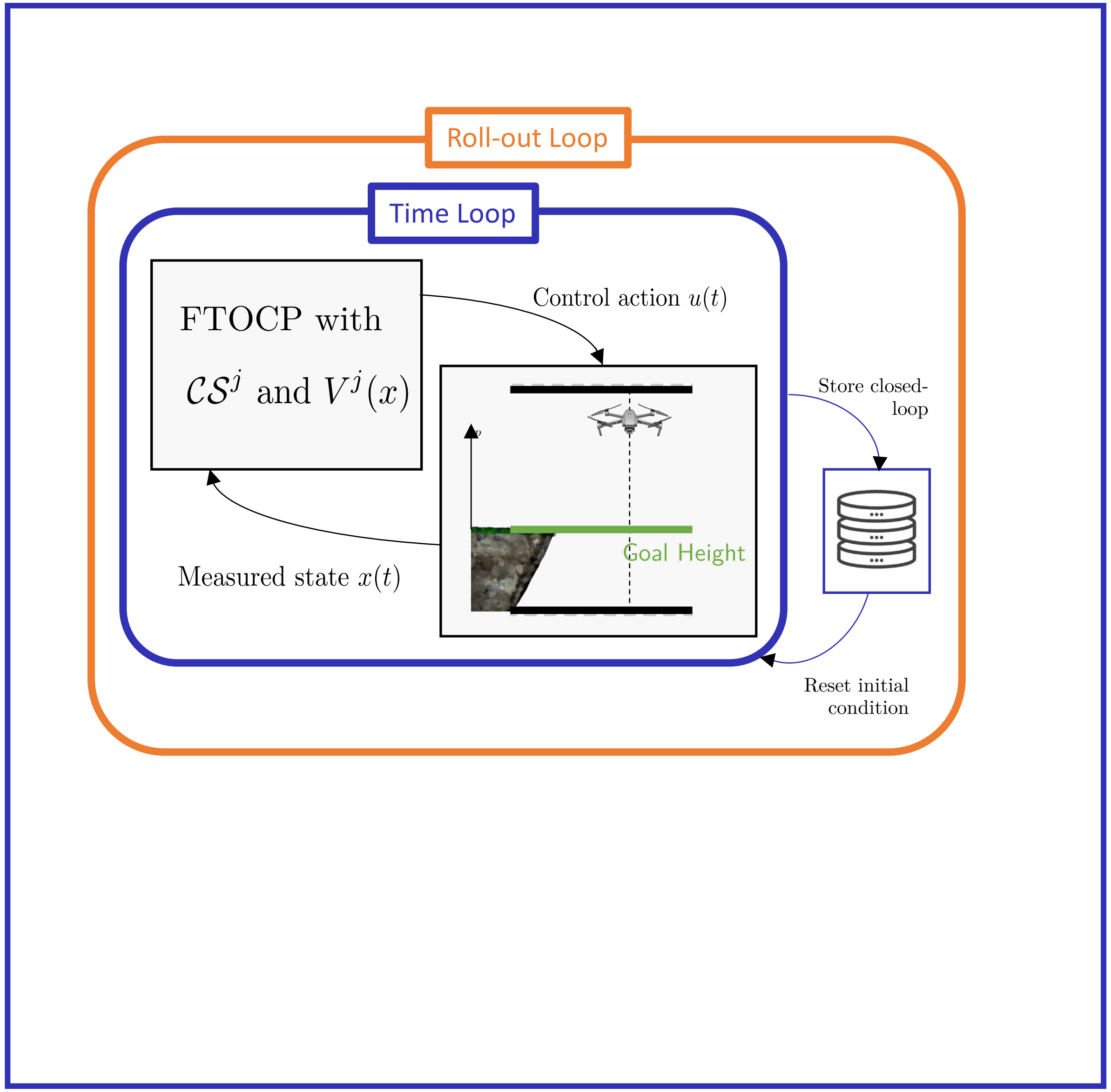

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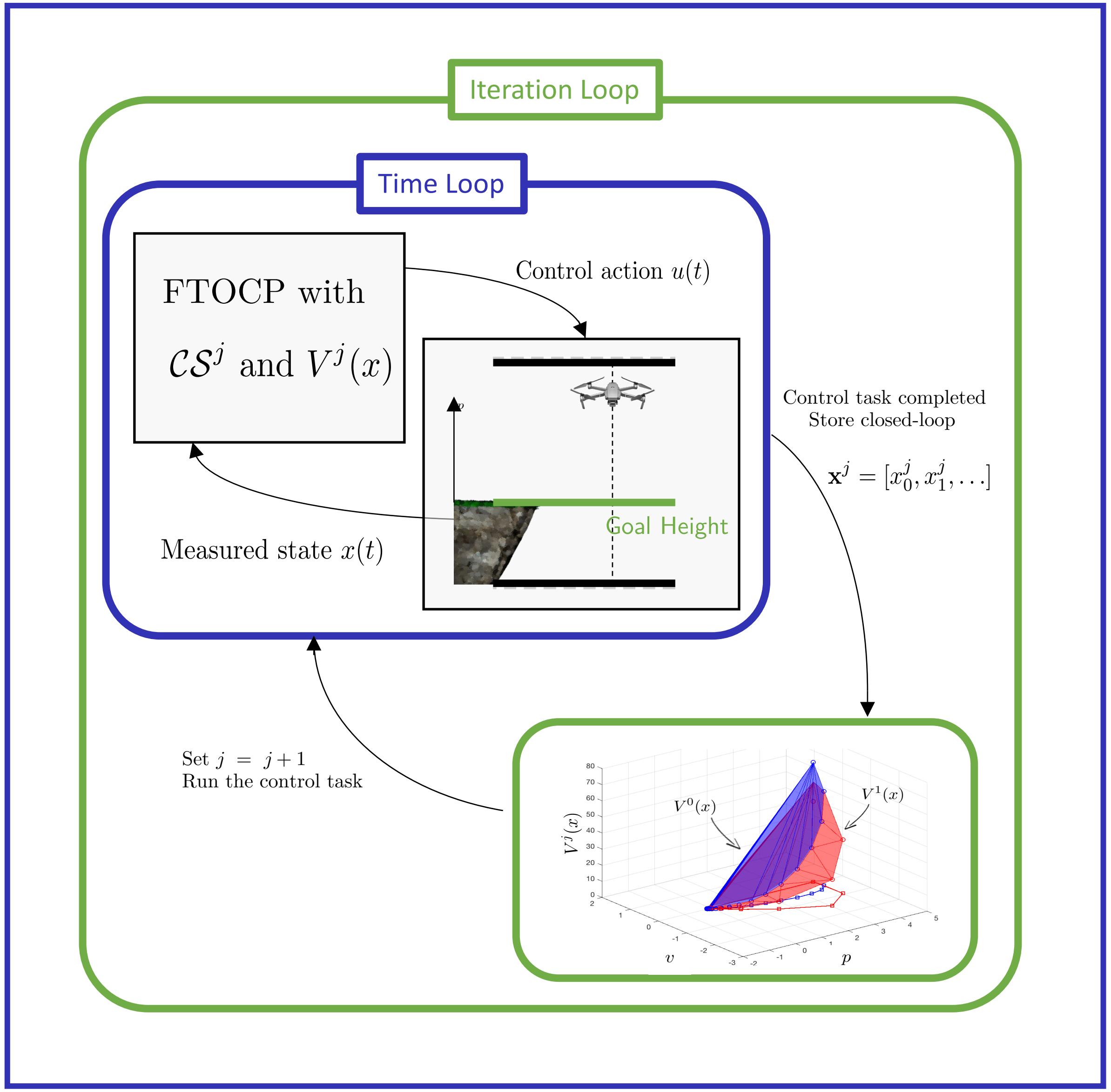

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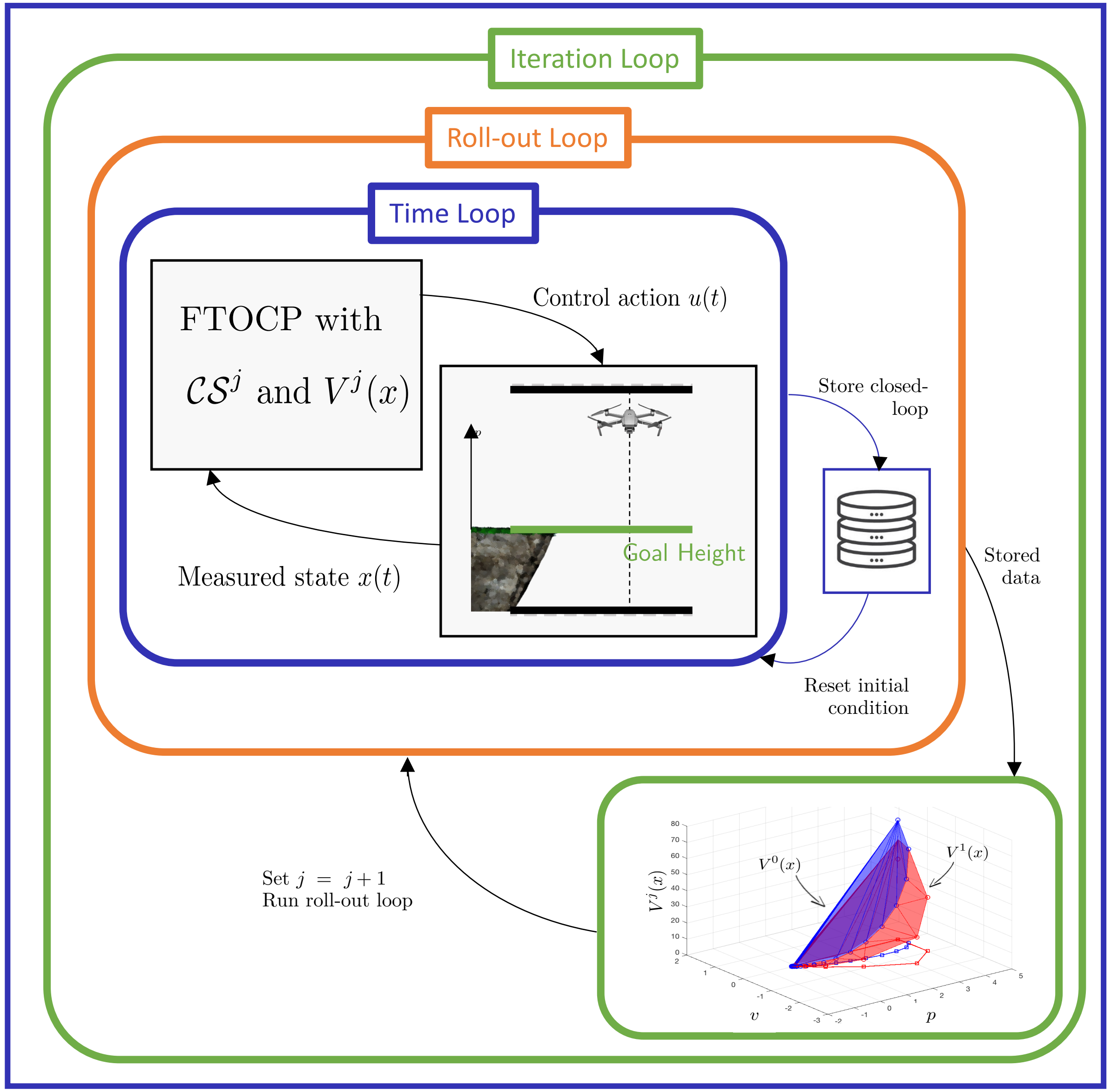


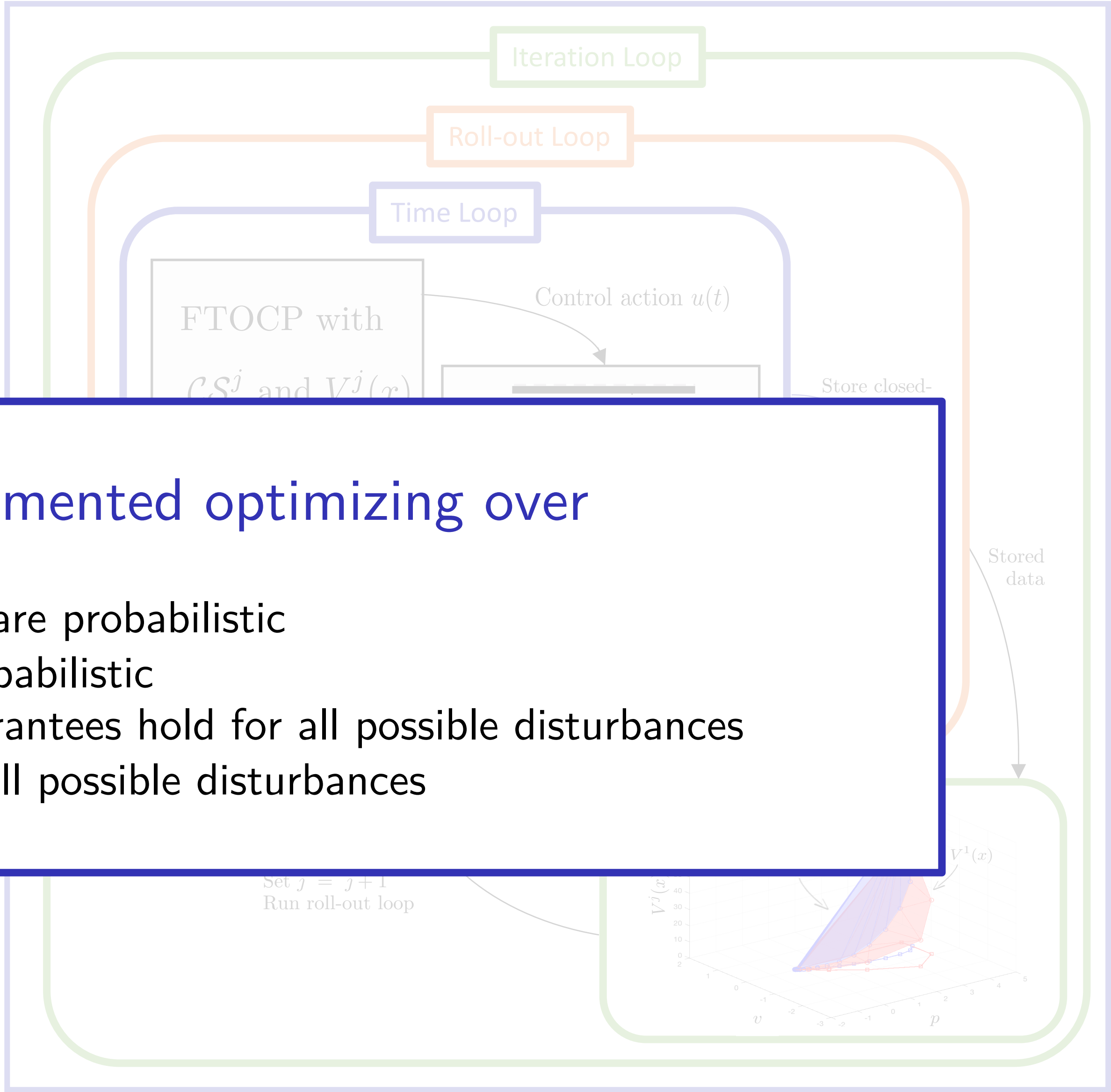
LMPC for uncertain systems

Summary Deterministic Vs Robust LMPC

The strategy can be implemented optimizing over

1. Feedback policies (NP-hard) → guarantees are probabilistic
2. Disturbance feedback → guarantees are probabilistic
3. Disturbance feedback + reachability → guarantees hold for all possible disturbances
4. Fixed tube strategy → guarantees hold for all possible disturbances

LMPC for deterministic systems



LMPC for uncertain systems

Discussion

Some high-level ideas and on-going research

Three types of learning

Learning classification borrowed from Prof. Borrelli

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Skill acquisition

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Performance Improvement

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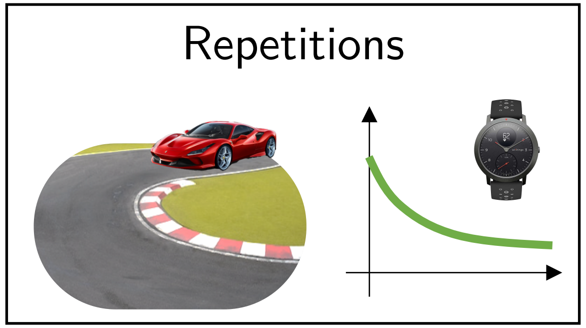
Skill acquisition



Performance Improvement

Three types of learning

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Repetitions

Analyze

Watch replay

Skill acquisition

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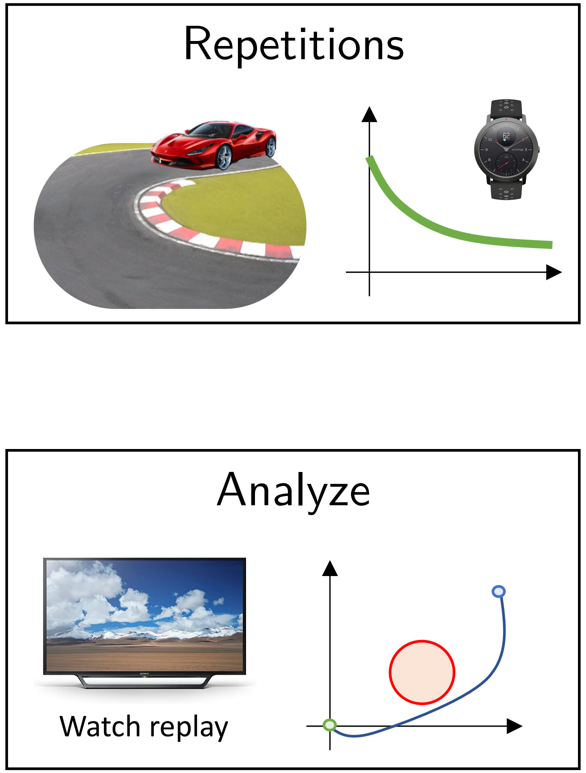

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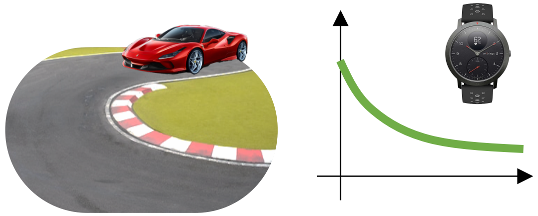

Analyze

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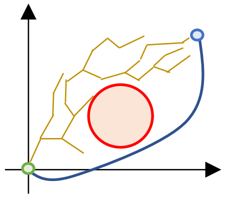

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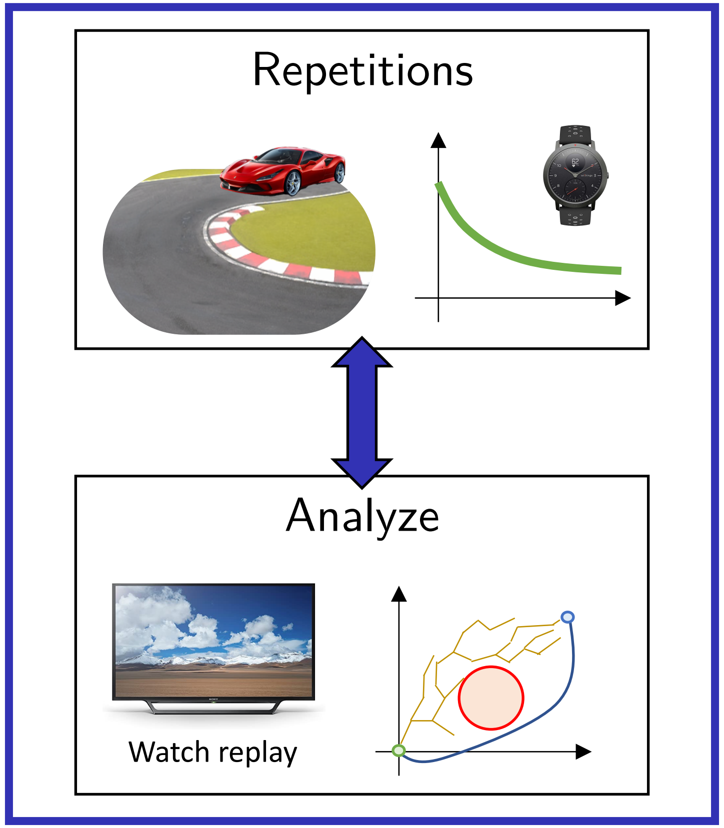

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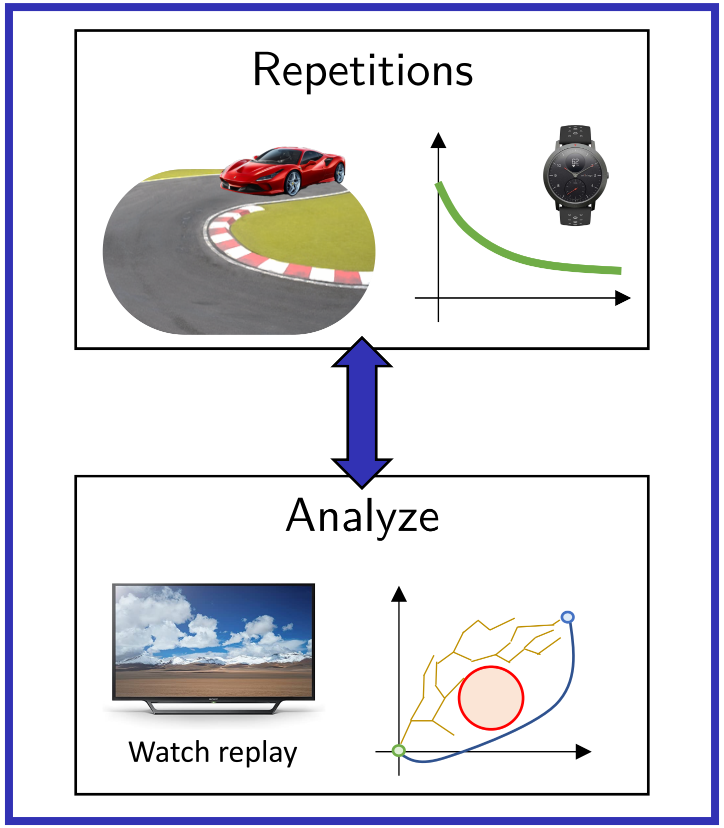

Performance Improvement

Building muscle memory

Three types of learning

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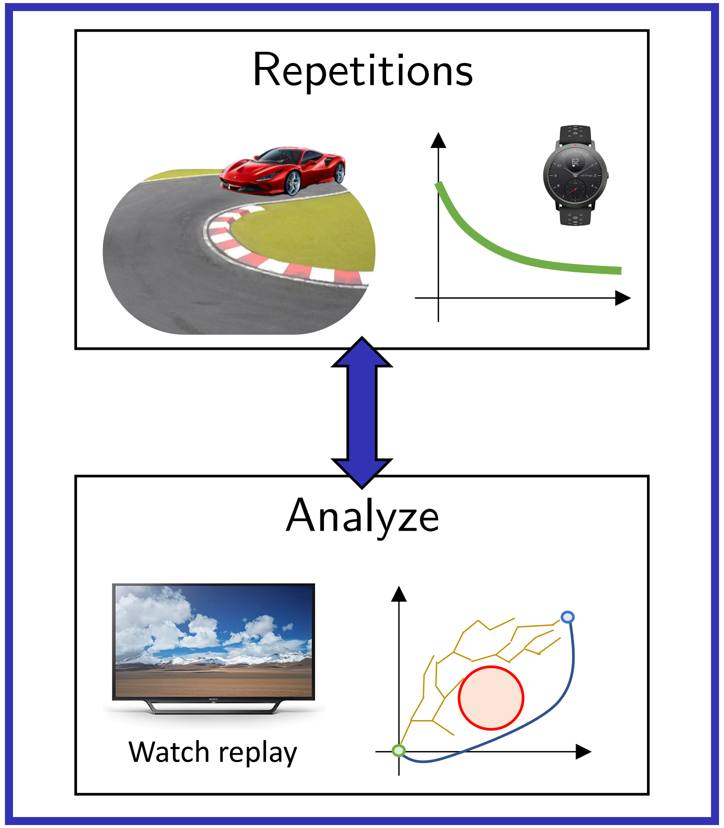
Skill acquisition



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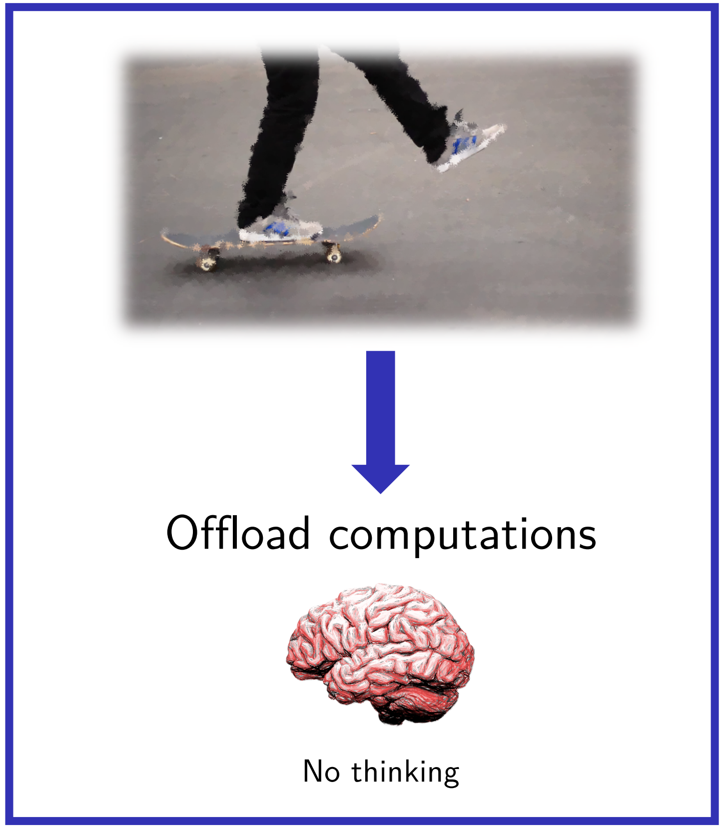

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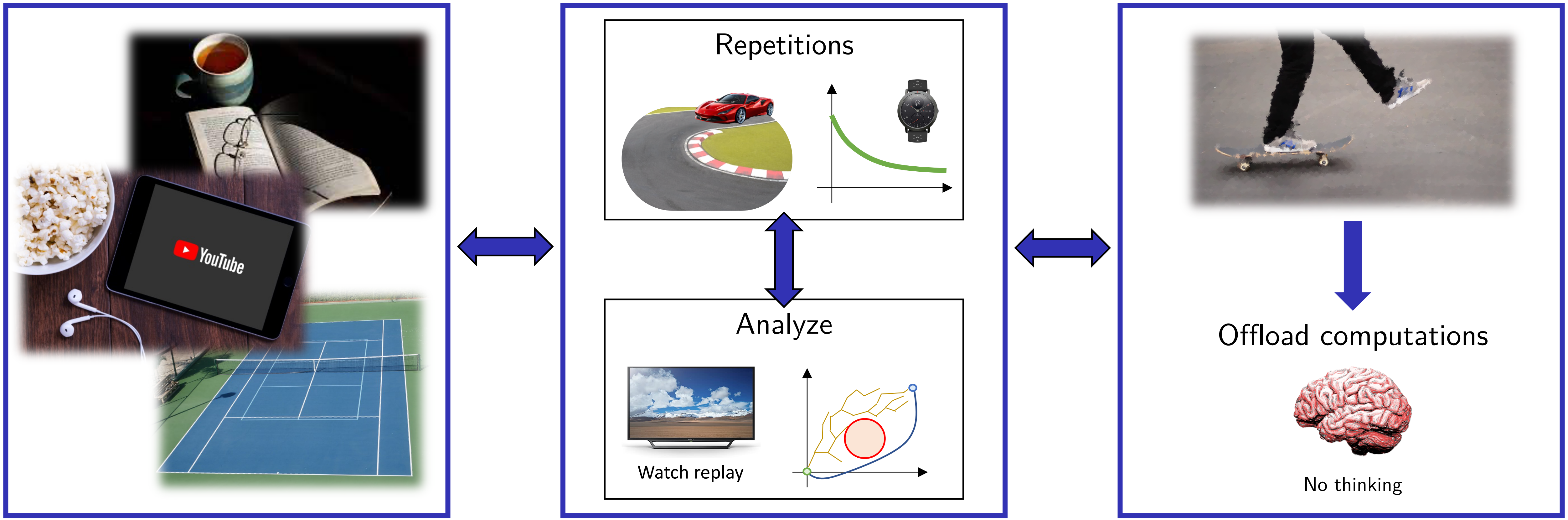

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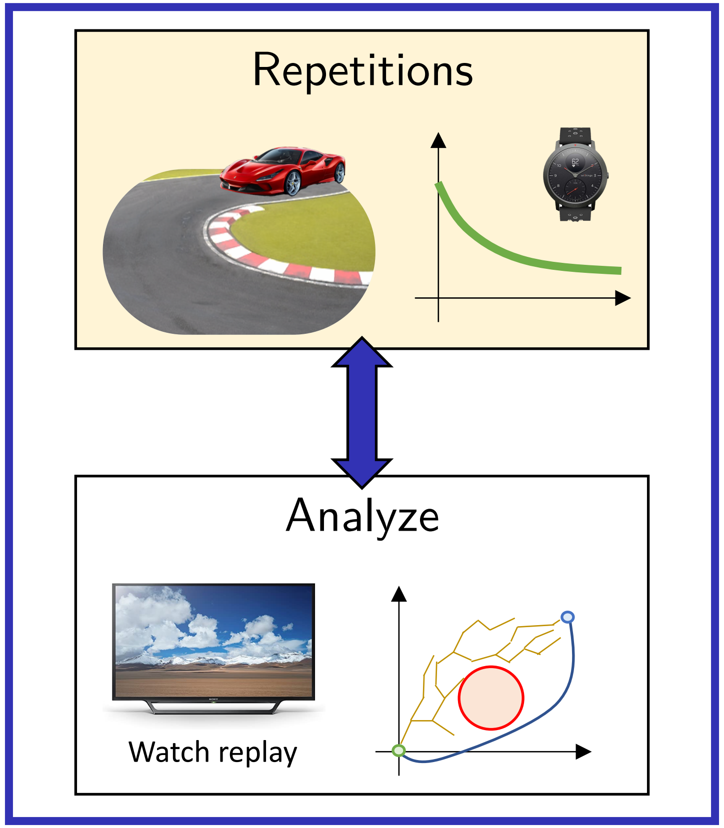

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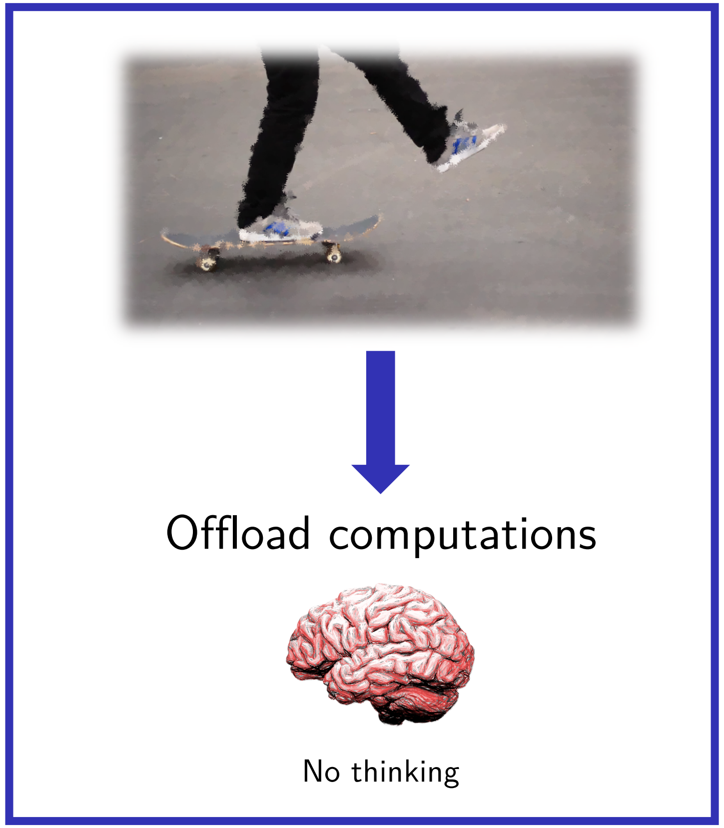

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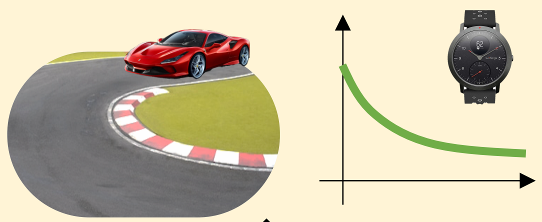

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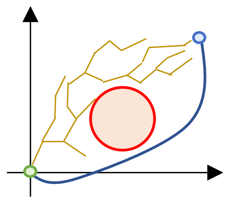
Watch replay



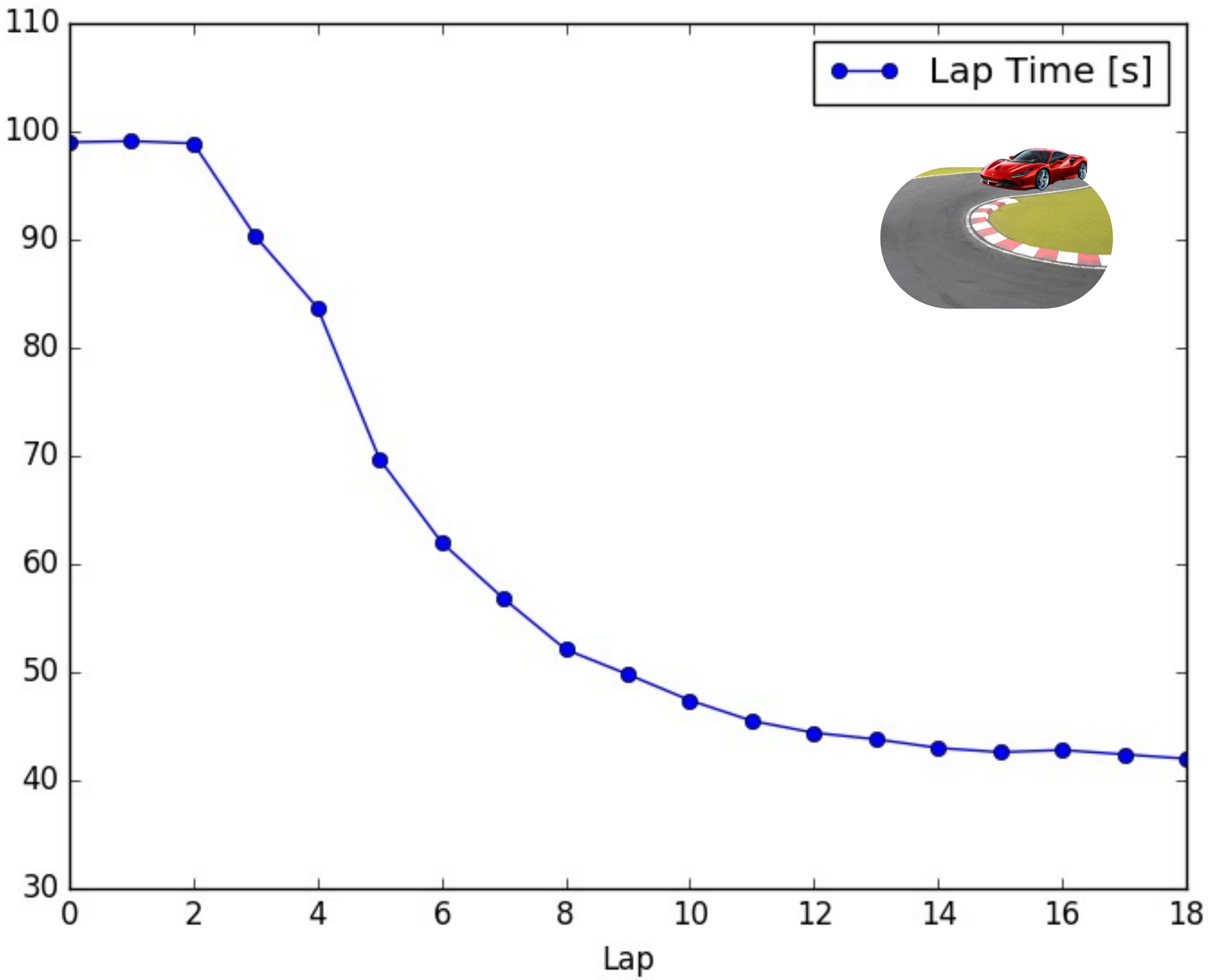
Performance Improvement

Offload computations

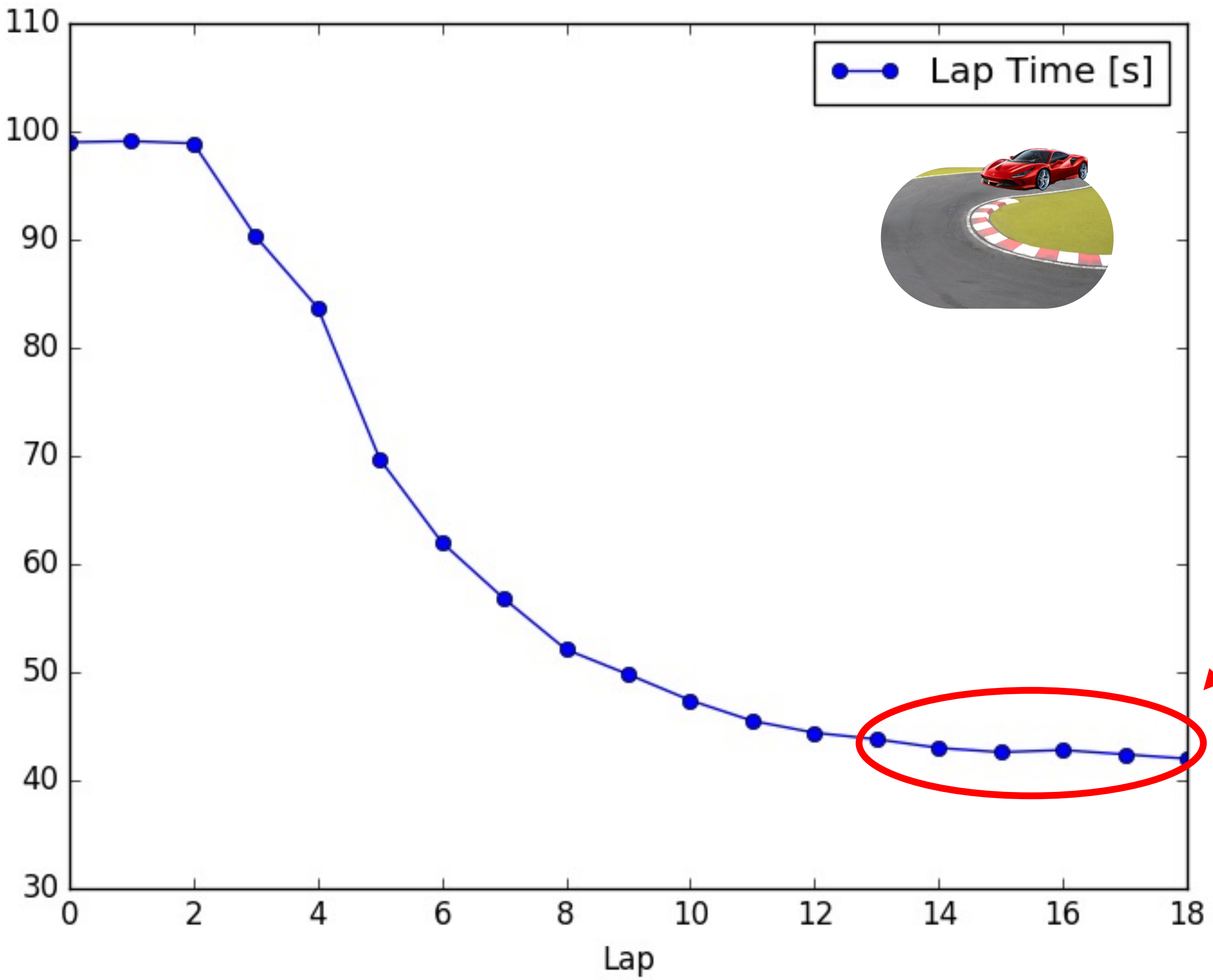
No thinking

Building muscle memory

Motivation: Lap Time Improvement

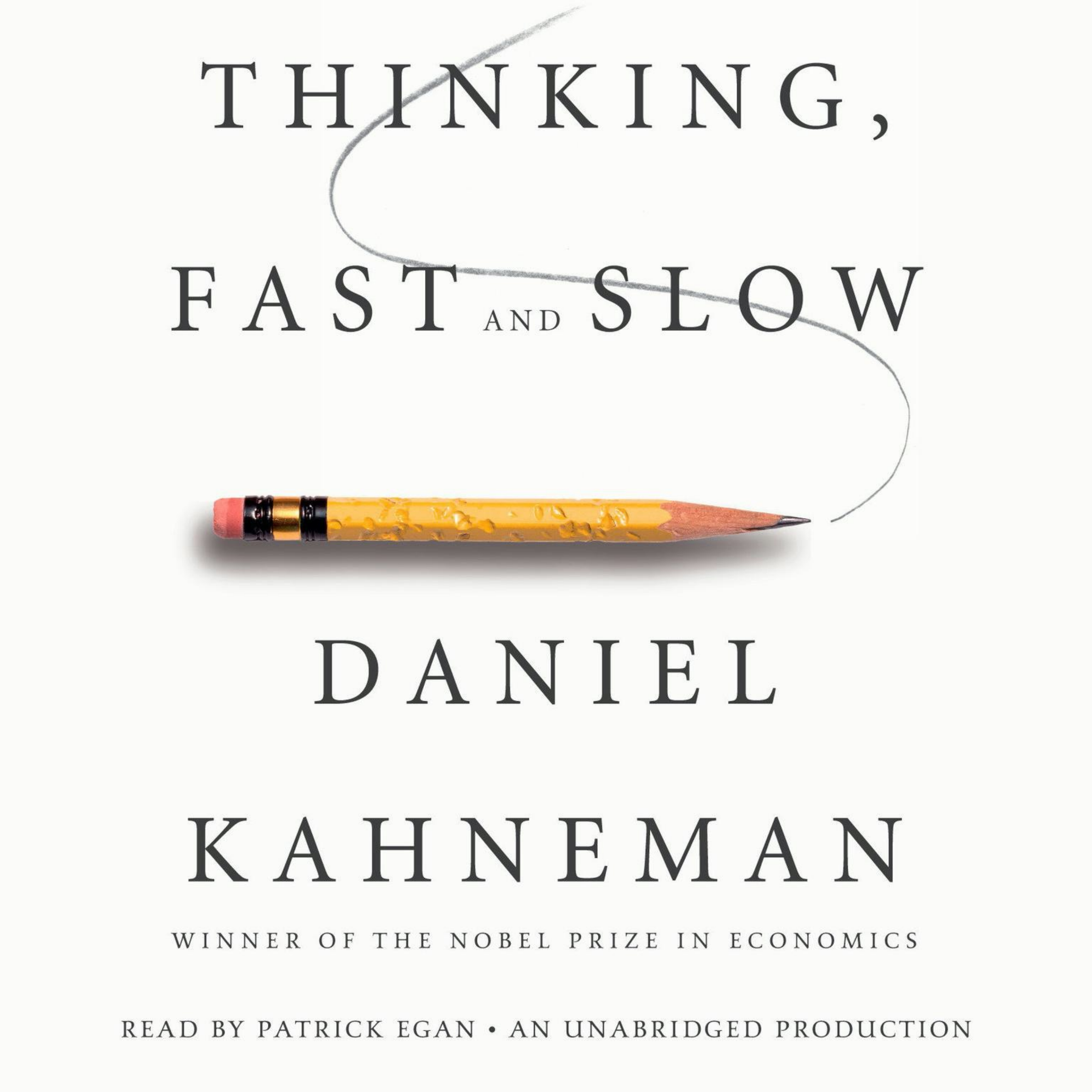


Goal: Reduce Computational Complexity



At convergence the **control policy** does not change

Learning as Computational Load Reduction



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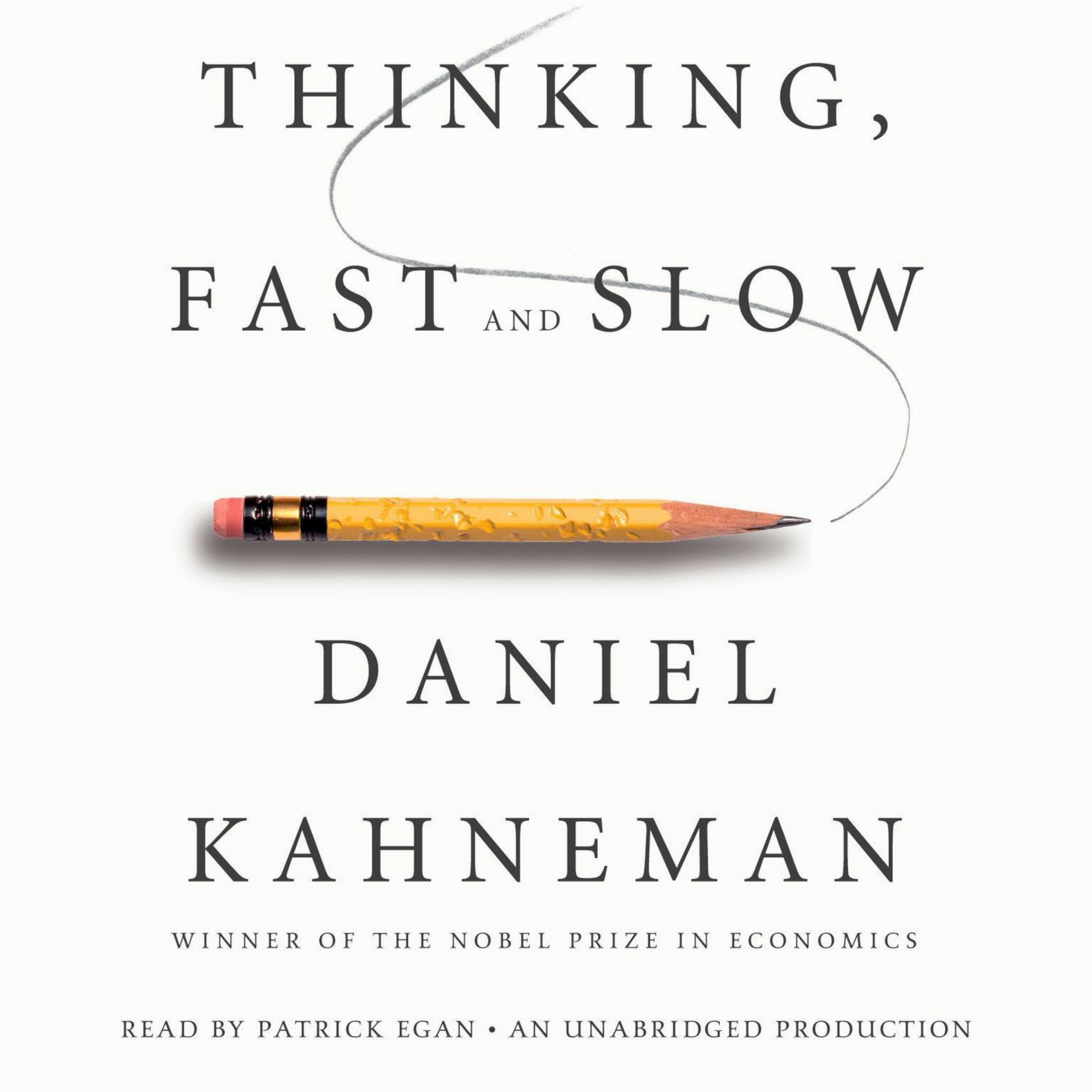
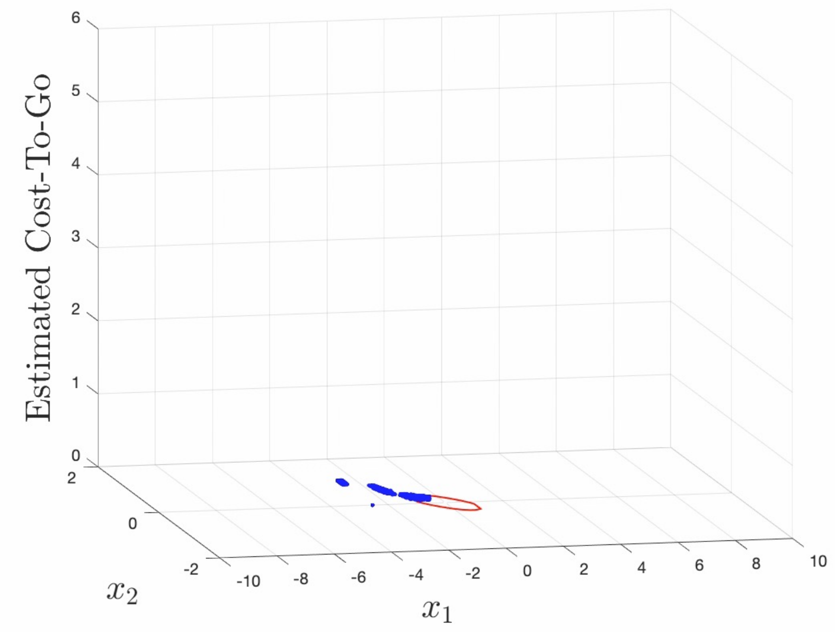


At convergence: the value function approximation does not change as we collect more data



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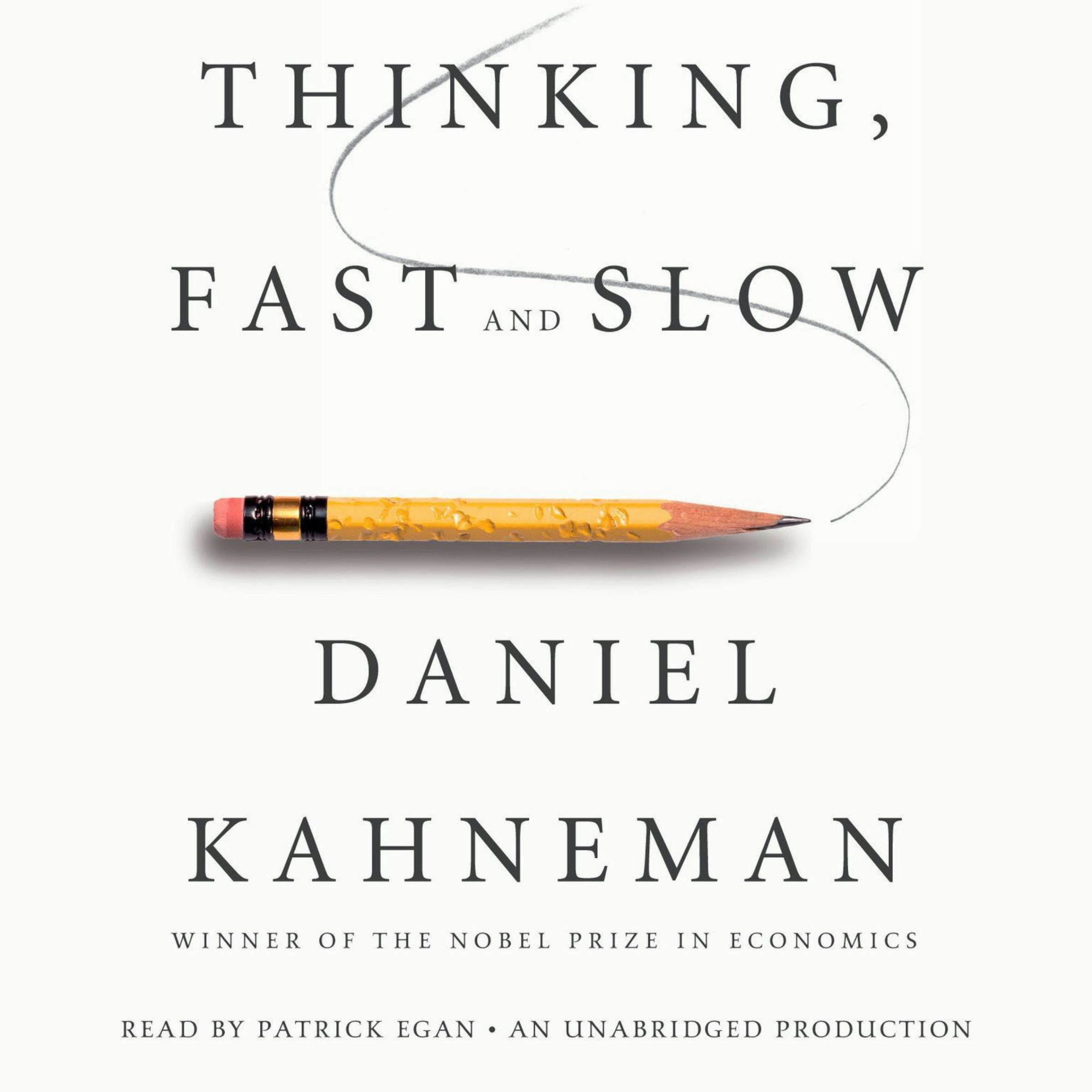

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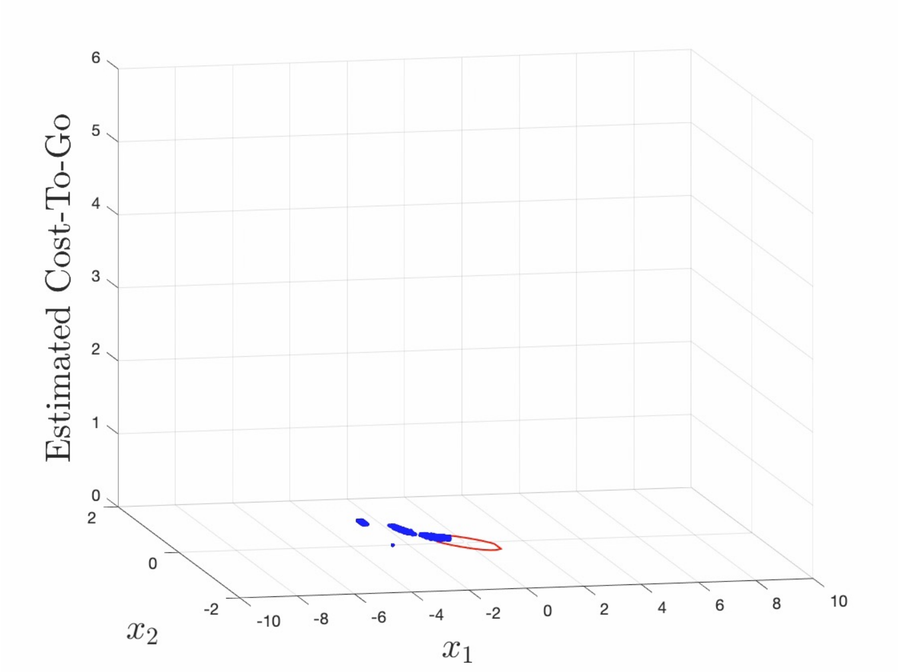
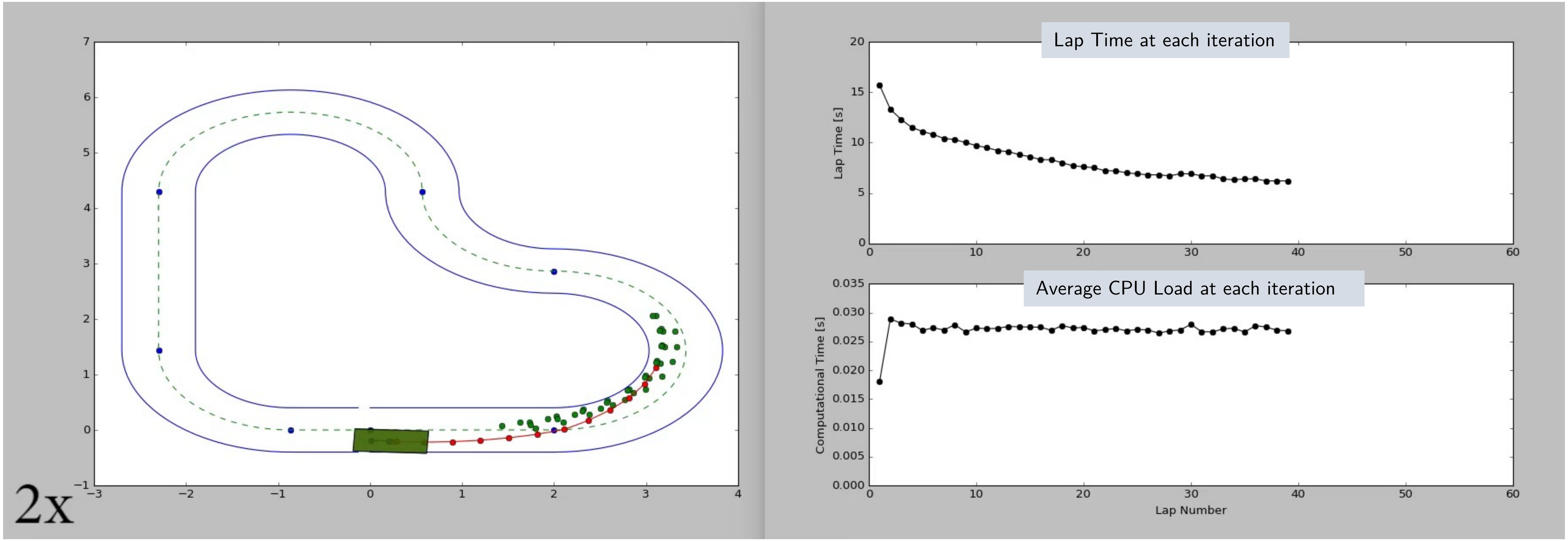


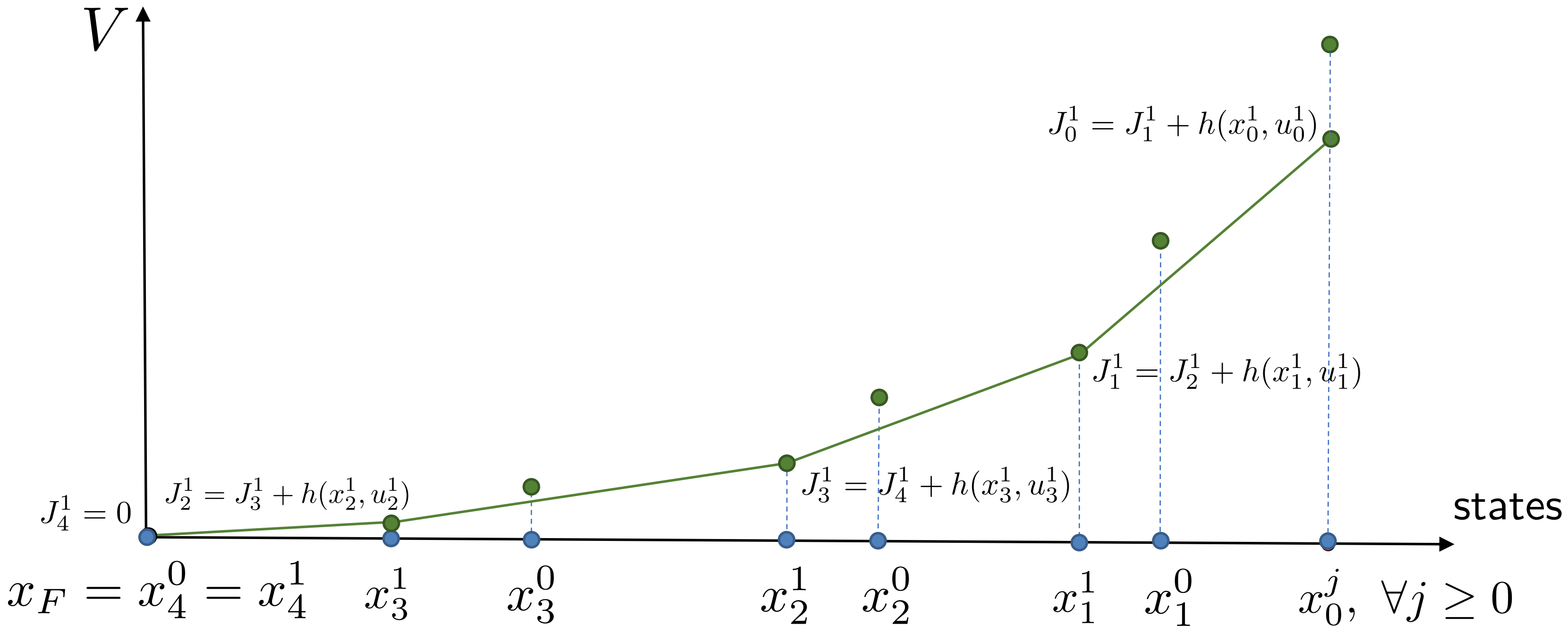
Can we design a policy which used data and **"plays back"** the inputs that we computed with the expensive policy

Do you need to Predict at Convergence?

Do you need to Predict at Convergence? No



Data-Based Policy



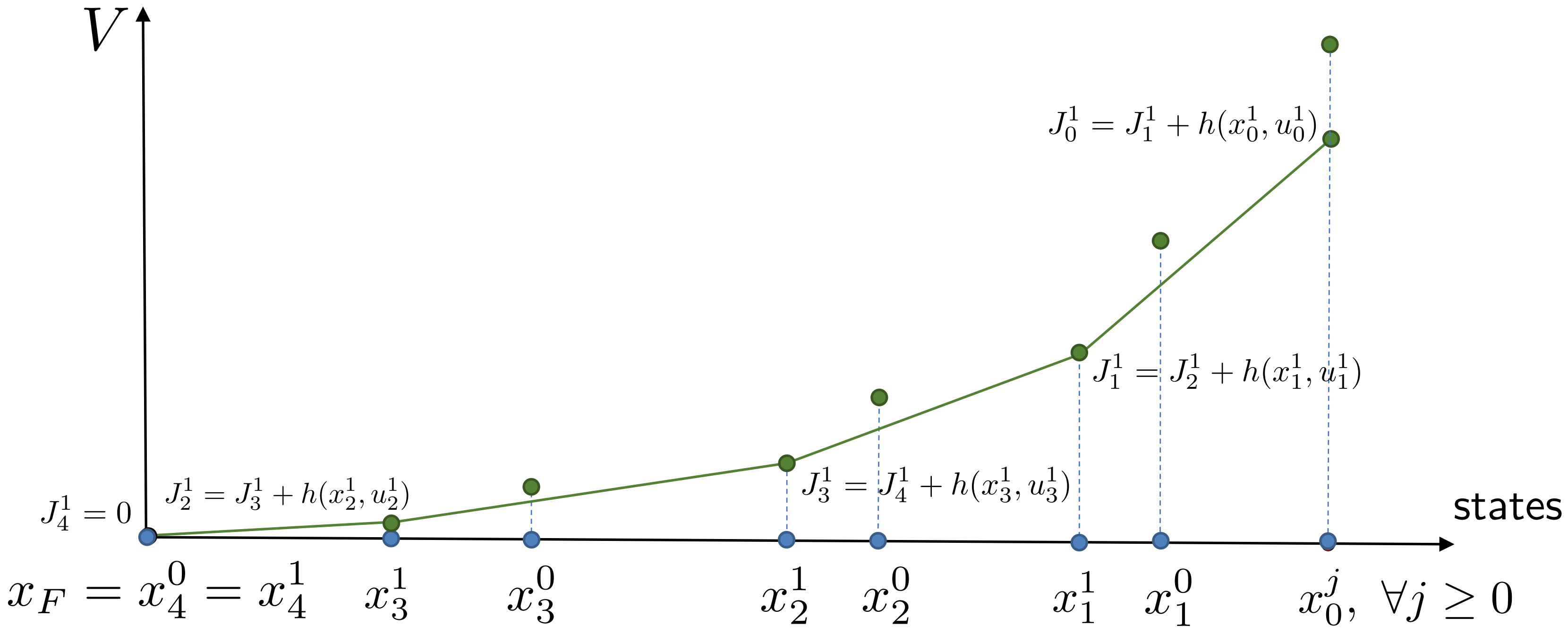
Multipliers Defining the Value Function Approximation

$$[\lambda_0^{0,*}, \ldots, \lambda_{T^j}^{T^j,*}] = \arg \min_{\lambda_i^j \in [0,1]} \sum_i \sum_j J_i^j \lambda_i^j$$

$$\text{s.t} \qquad \sum_i \sum_j x_i^j \lambda_i^j = x,$$

$$\sum_i \sum_j \lambda_i^j = 1$$

Data-Based Policy



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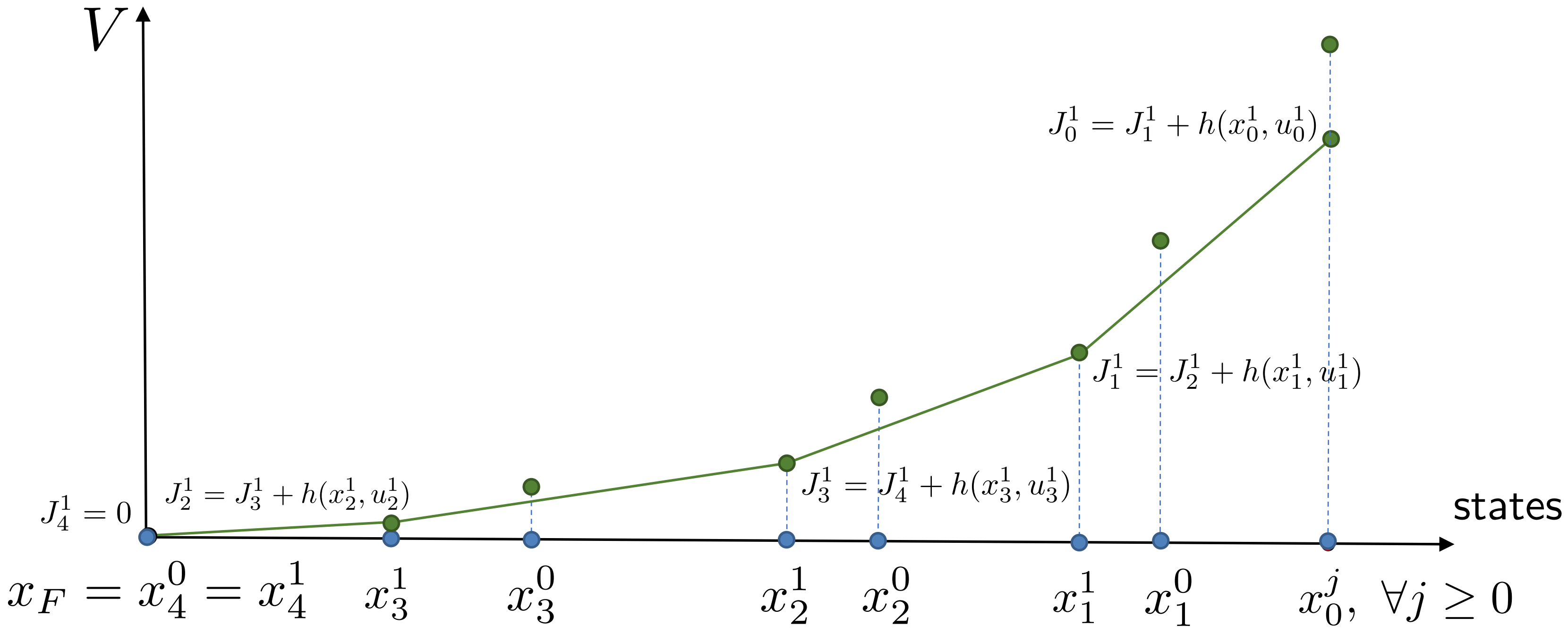
$$\sum_i \sum_j \lambda_i^j = 1$$

Data-Based Policy

$$\kappa^j(x) = \sum_i \sum_j u_i^j \lambda_i^{j,*}$$

Data-Based Policy



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$$\text{s.t} \quad \sum_i \sum_j x_i^j \lambda_i^j = x,$$

$$\sum_i \sum_j \lambda_i^j = 1$$

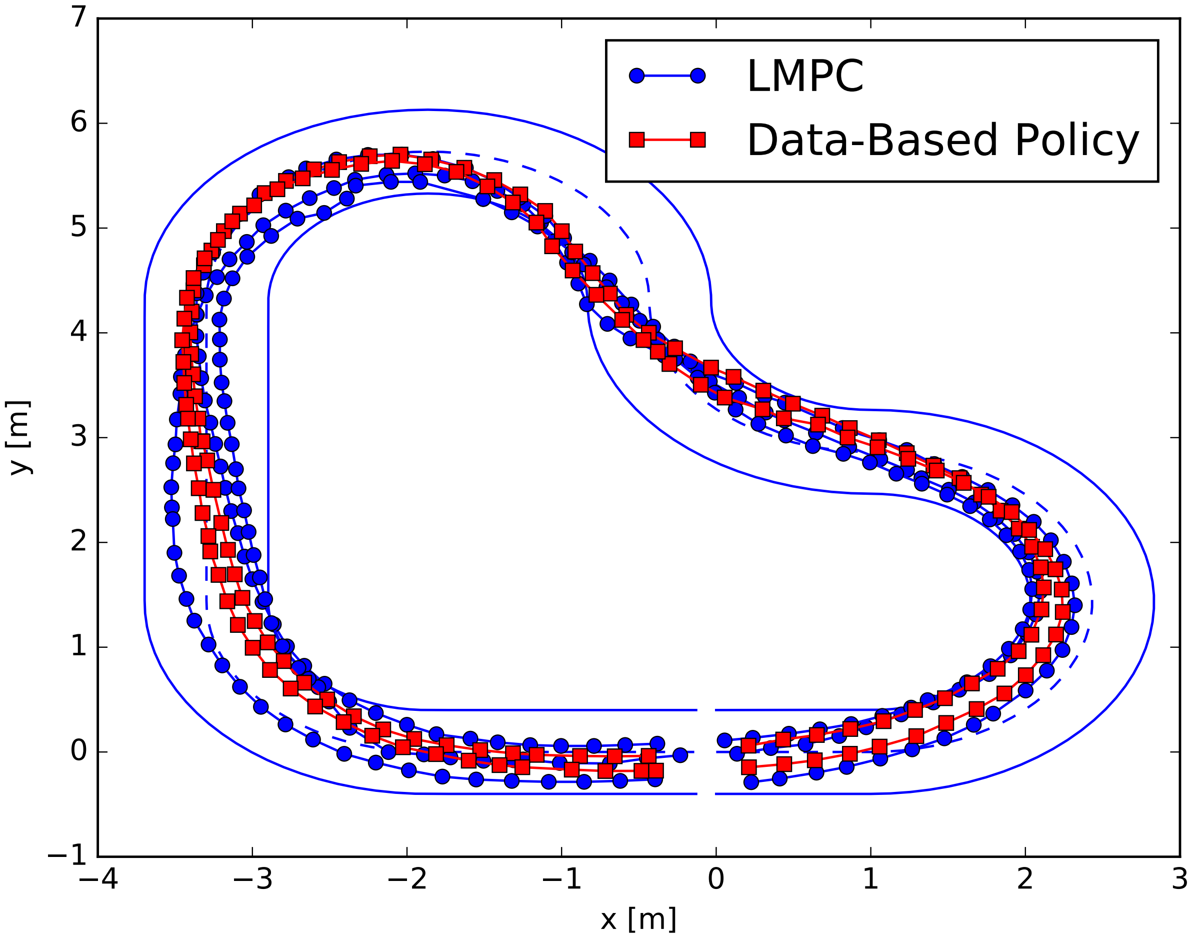
Data-Based Policy

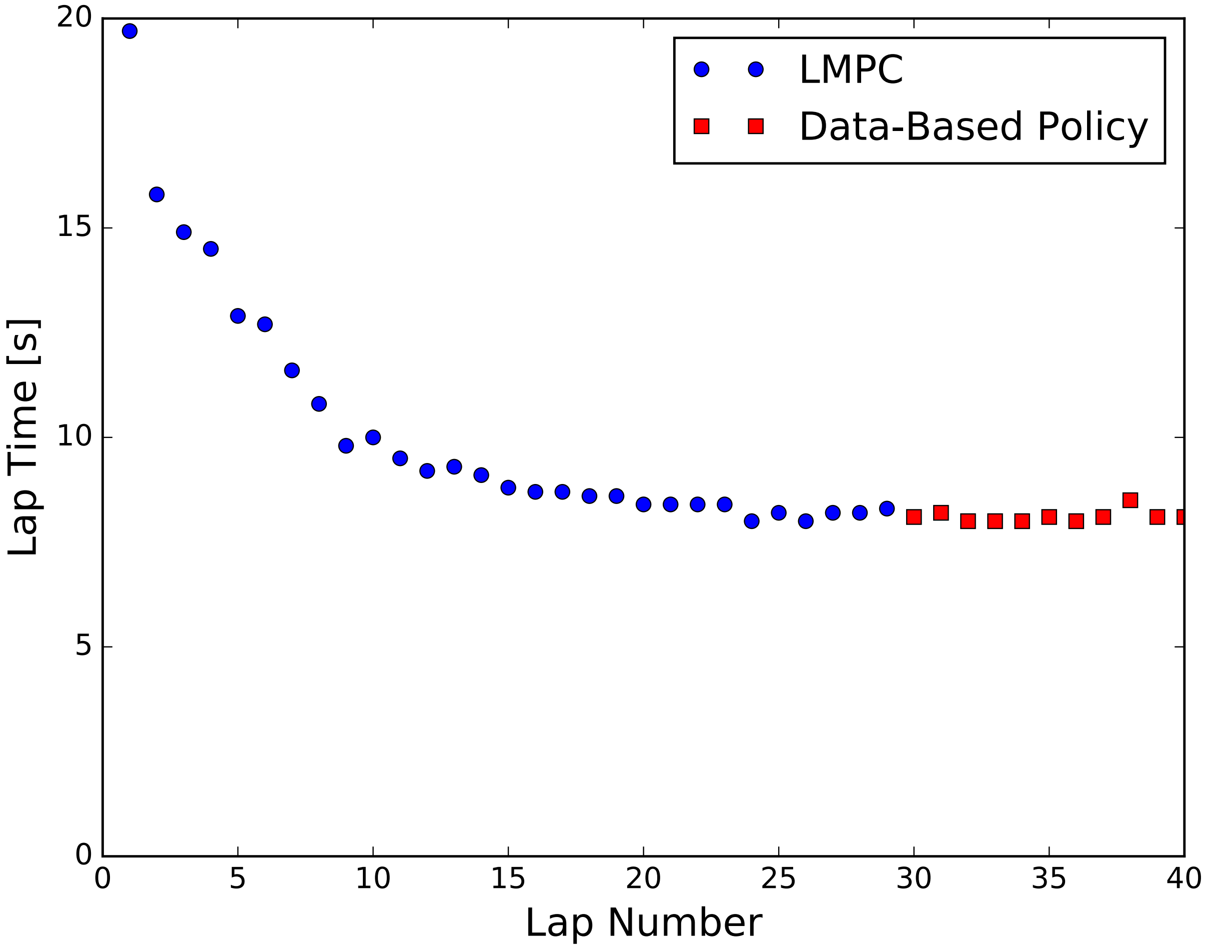
$$\kappa^j(x) = \sum_i \sum_j u_i^j \lambda_i^{j,*}$$

Historical Data

Berkeley Autonomous Race Car Implementation

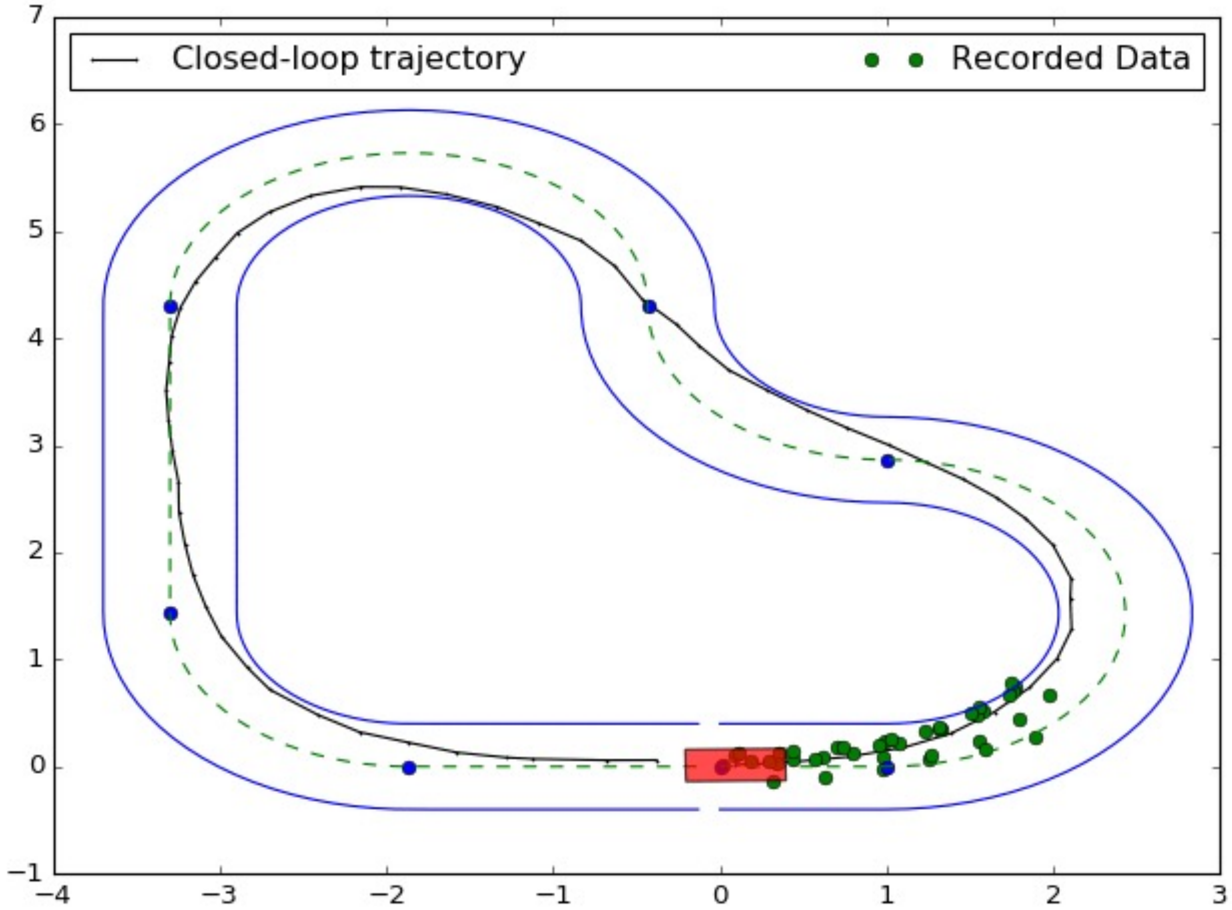
Berkeley Autonomous Race Car Implementation





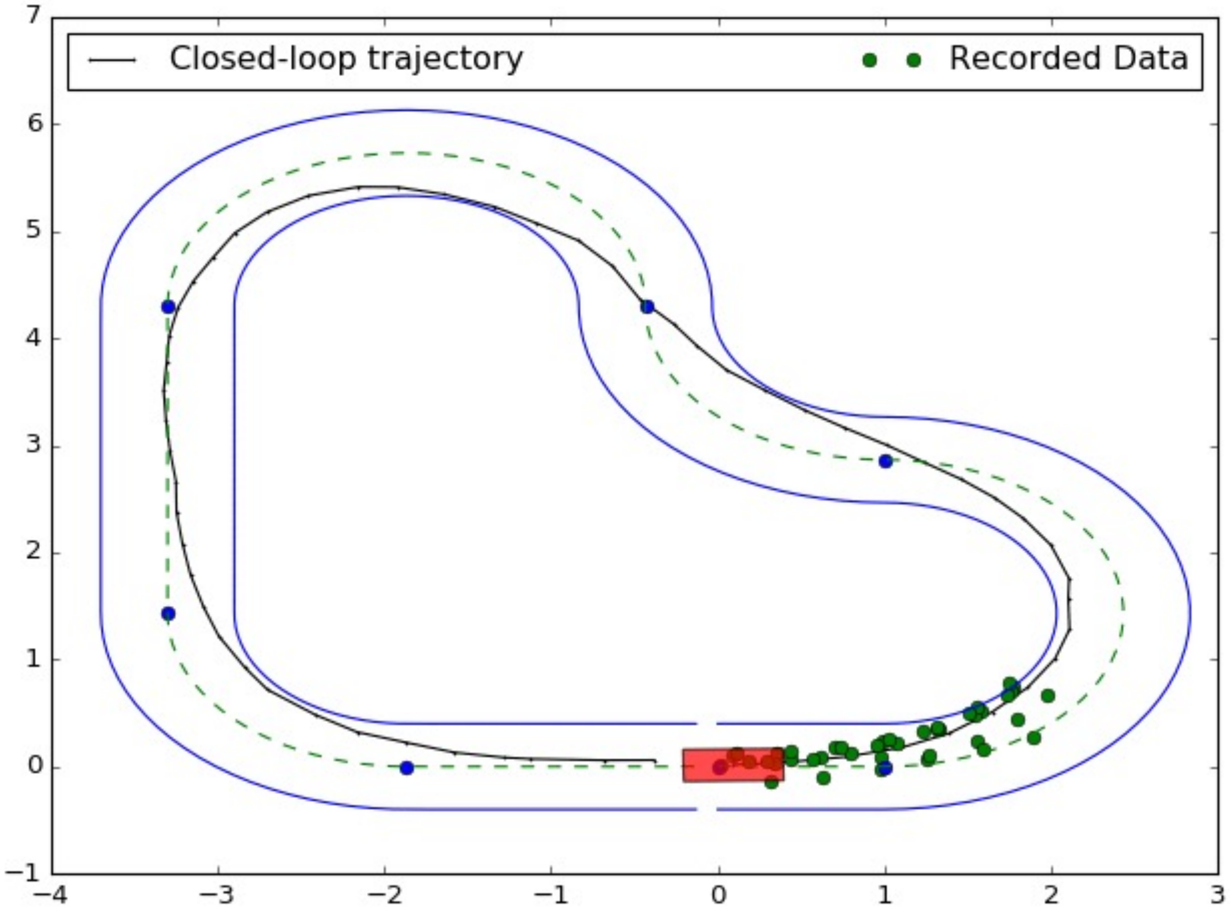
- A **reference trajectory** is **not needed** for the controller implementation
- The controller converges after few laps: the learning process is **data efficient**
- The controller **safely explores** the state space **iteratively improving** the **lap time**

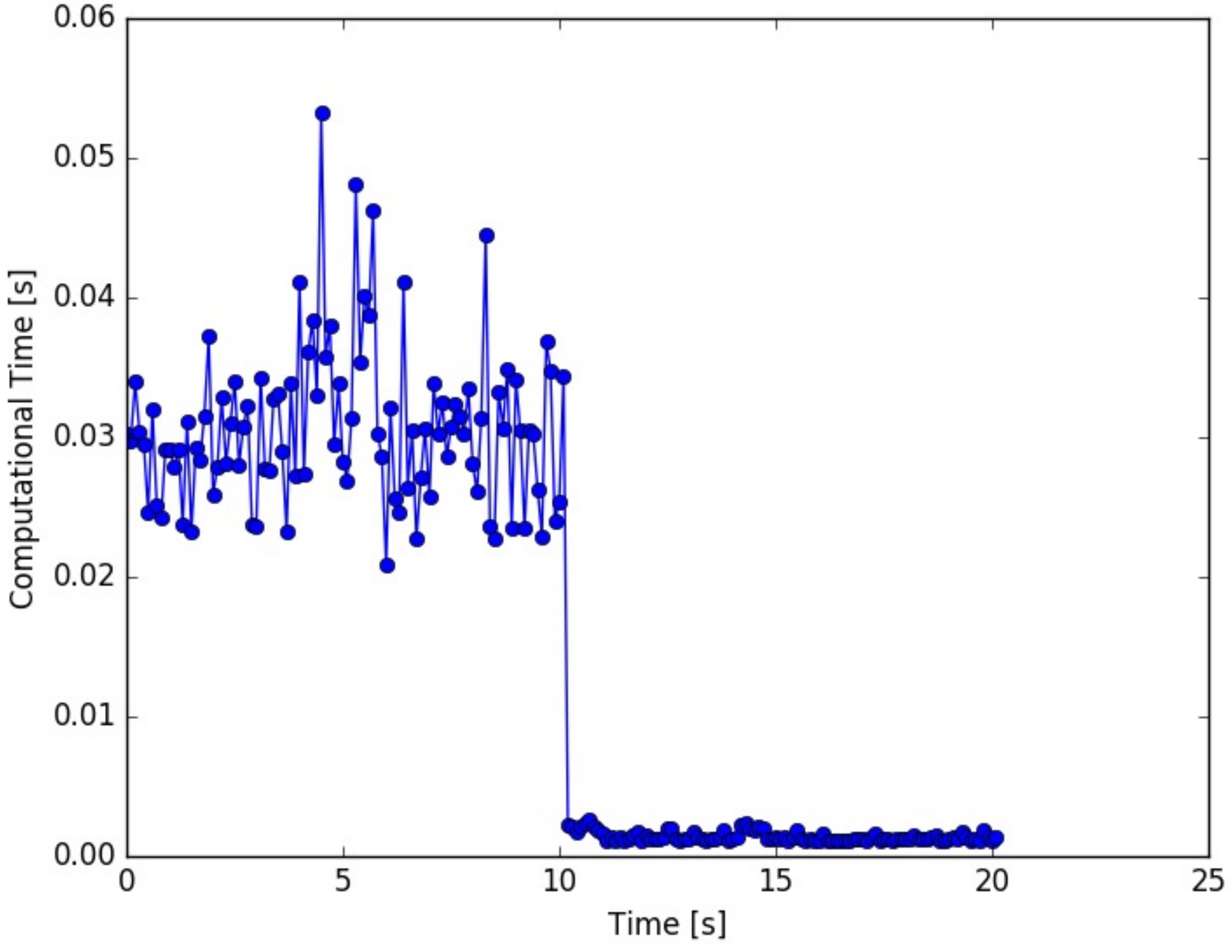
Berkeley Autonomous Race Car Implementation



At each time t the data-based policy is computed with the K nearest neighbors (in green) to the current state. This allows us to account for the nonlinearity of the vehicle

Berkeley Autonomous Race Car Implementation





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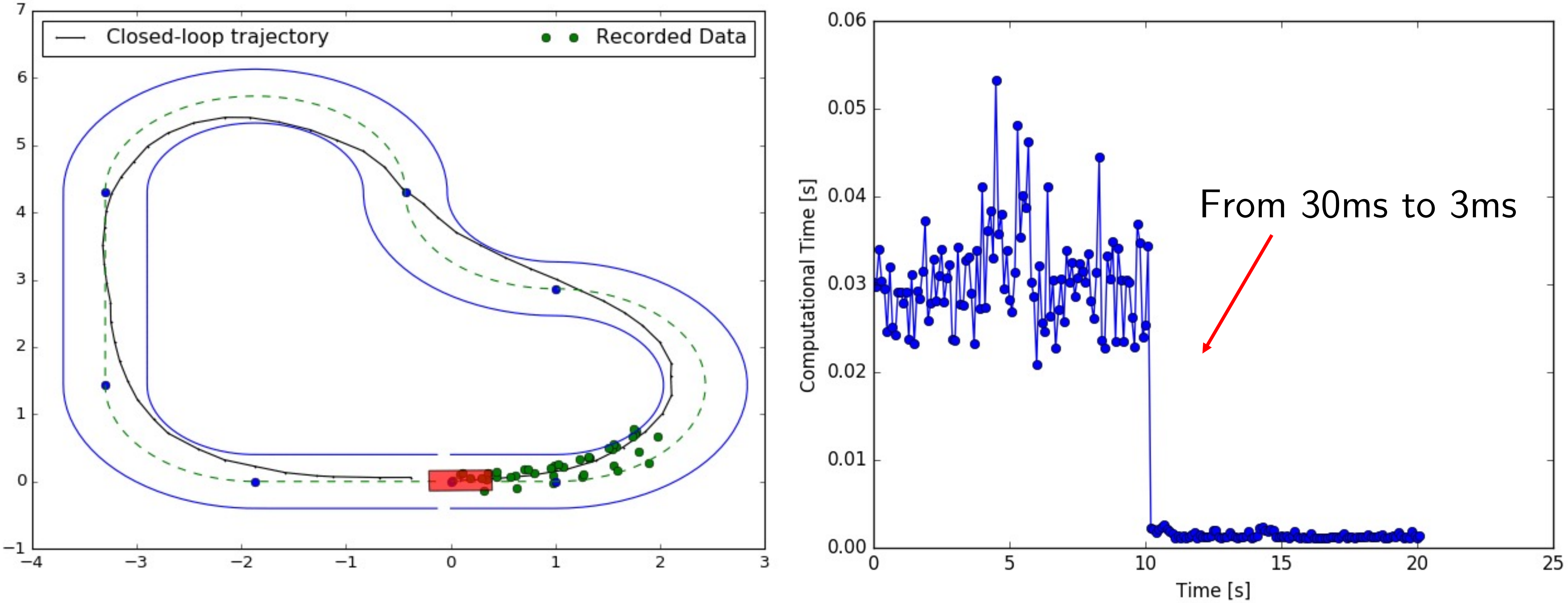
Berkeley Autonomous Race Car Implementation



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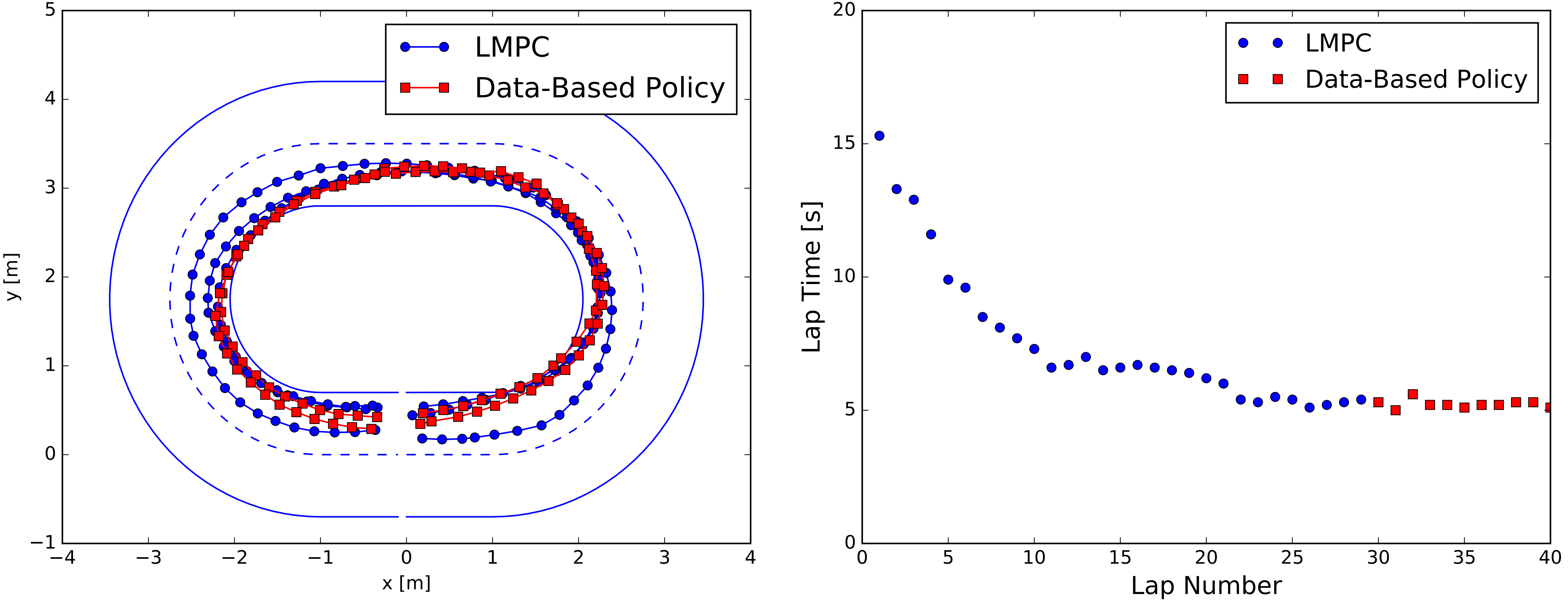
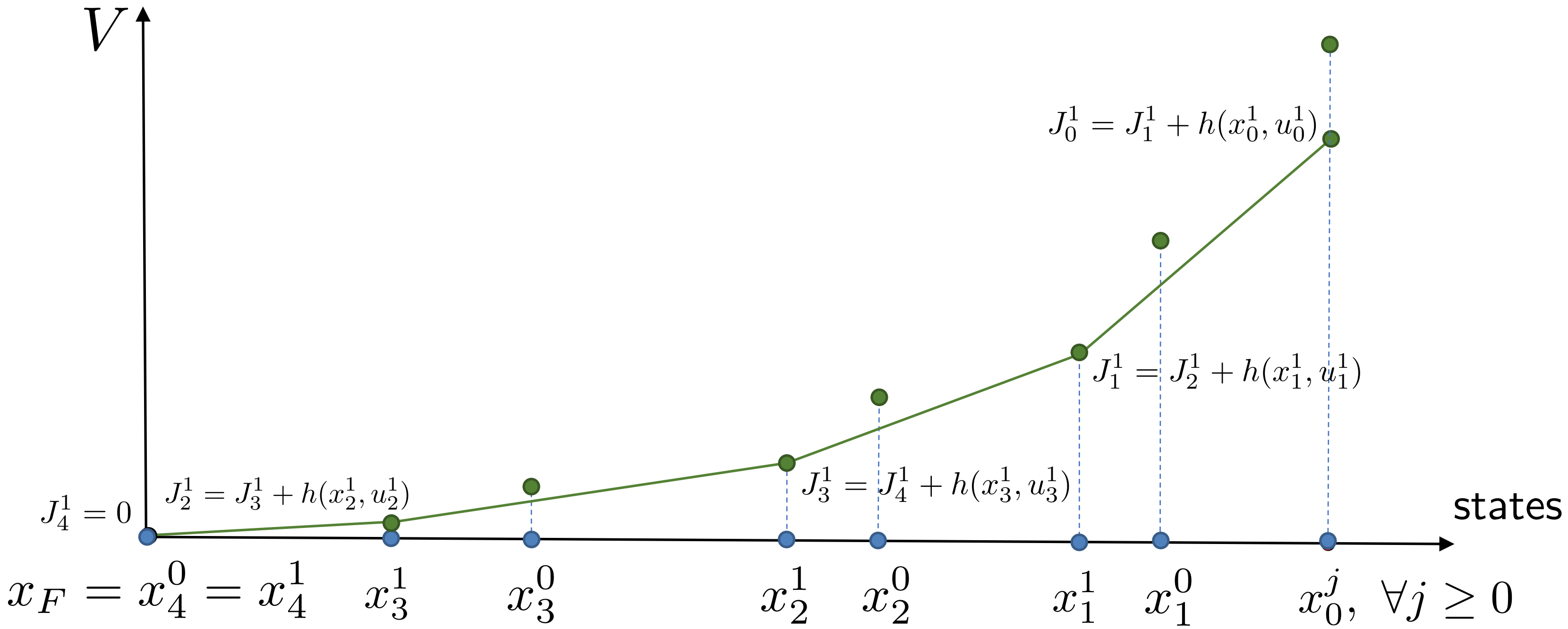
Berkeley Autonomous Race Car Implementation



Data-Based Policy



Multipliers Defining the Value Function Approximation

$$[\lambda_0^{0,*}, \ldots, \lambda_{T^j}^{T^j,*}] = \arg \min_{\lambda_i^j \in [0,1]} \sum_i \sum_j J_i^j \lambda_i^j$$

$$\text{s.t} \quad \sum_i \sum_j x_i^j \lambda_i^j = x,$$

$$\sum_i \sum_j \lambda_i^j = 1$$

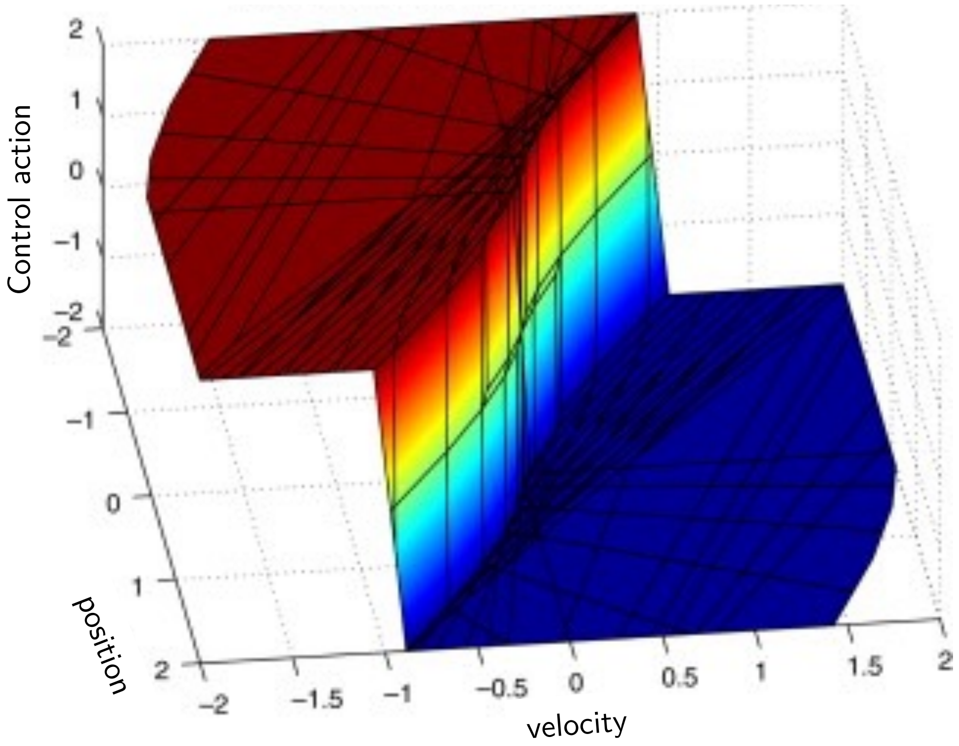
Data-Based Policy

$$\kappa^j(x) = \sum_i \sum_j u_i^j \lambda_i^{j,*}$$

Historical Data

Speeding-up the solver

Leverage the explicit solution



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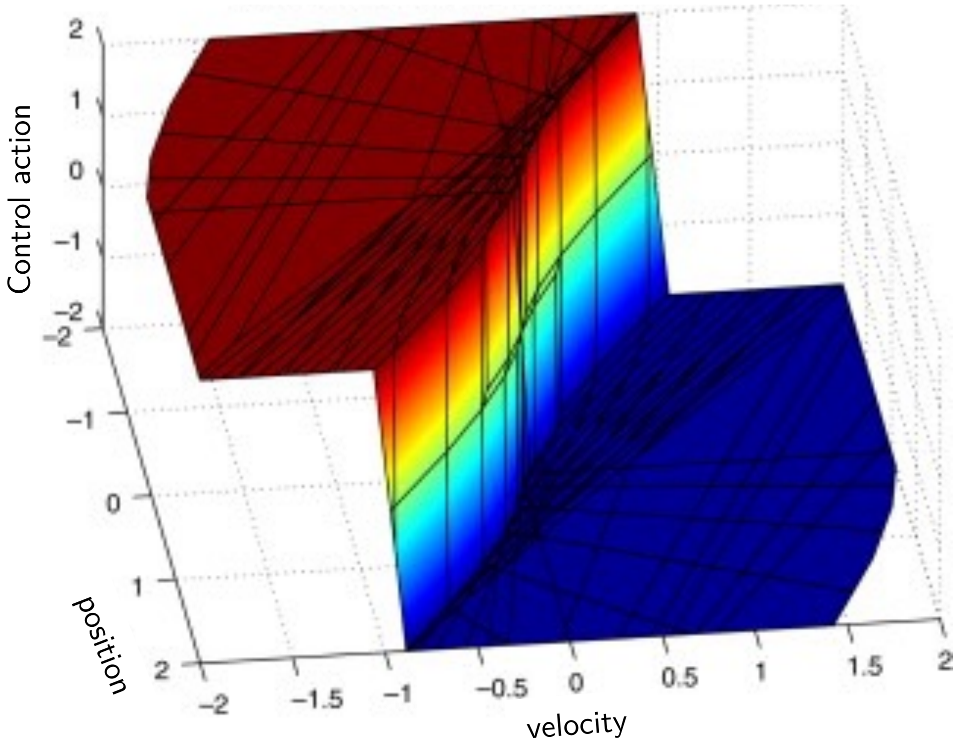
Speeding-up the solver

Leverage the explicit solution



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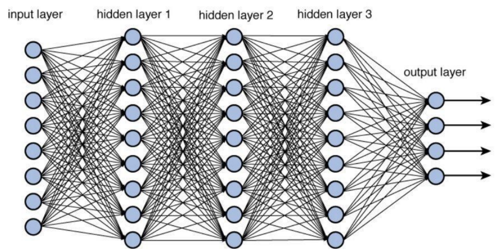
Offline

Primal Neural-Network



Learns input policy $\pi^p(x)$

Steven Chen, Kelsey Saulnier, Nikolay Atanasov, Daniel D. Lee, Vijay Kumar, George J. Pappas, and Manfred Morari. "Approximating explicit model predictive control using constrained neural networks." In *2018 Annual American control conference (ACC)*

Speeding-up the solver

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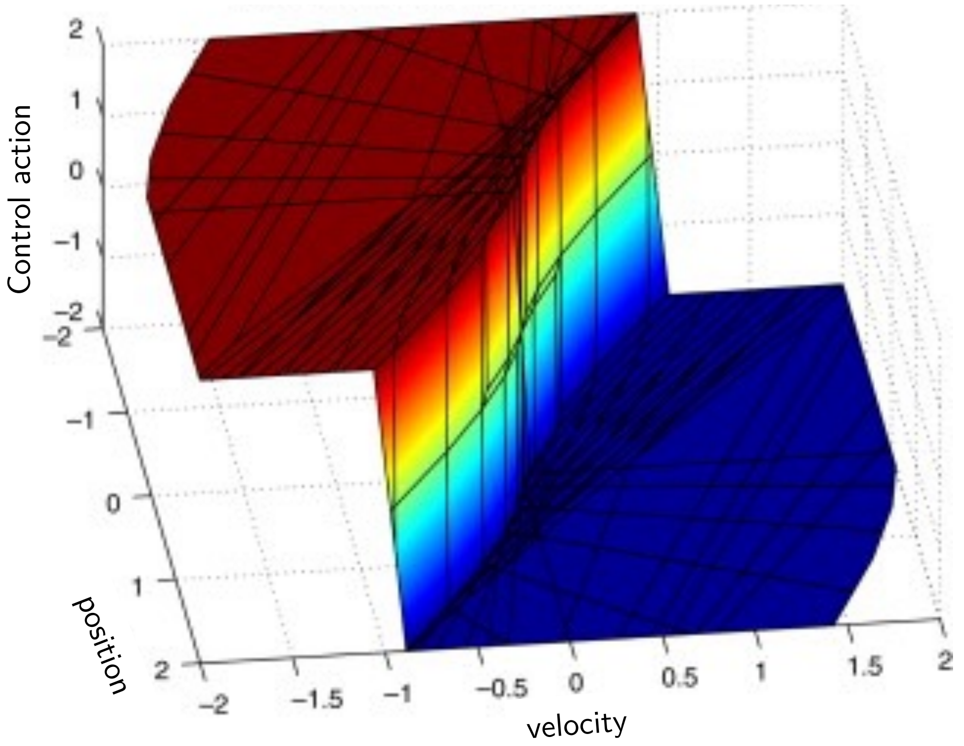

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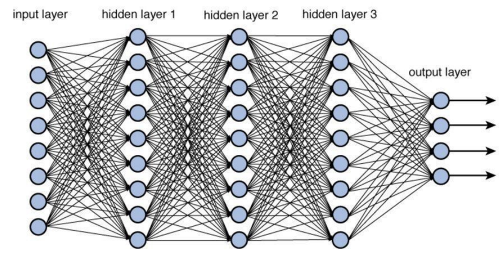

Learns input policy $\pi^p(x)$

Online

Apply the control policy $\pi^p(x)$

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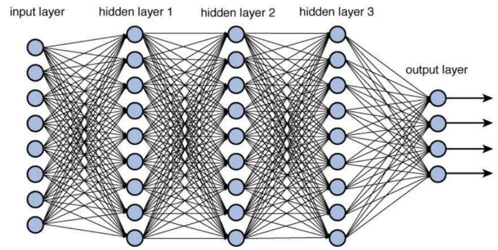
Primal-Dual Neural Network

Primal-Dual Neural Network

Offline

Primal Neural-Network



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Primal-Dual Neural Network

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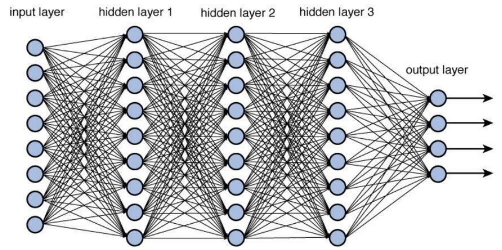
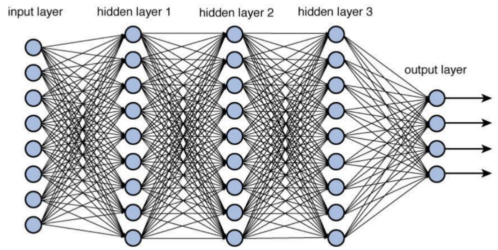


Learns input policy $\pi^p(x)$

Dual Neural-Network



Learns optimality *certificate* $\pi^d(x)$

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Primal-Dual Neural Network

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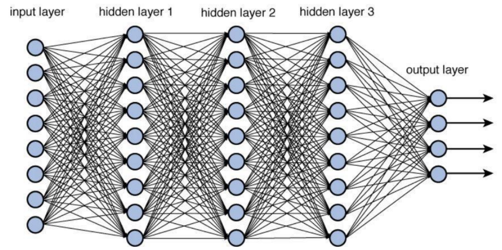
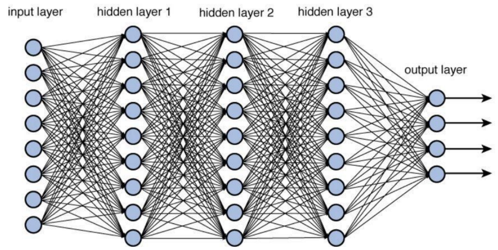


Learns input policy $\pi^p(x)$

+

Dual Neural-Network



Learns optimality *certificate* $\pi^d(x)$

Online

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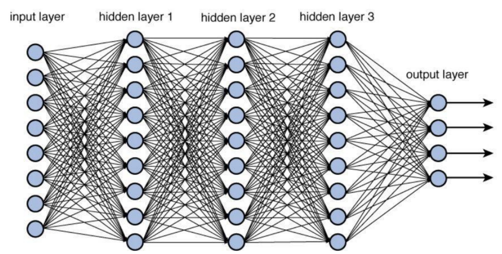

Learns input policy $\pi^p(x)$

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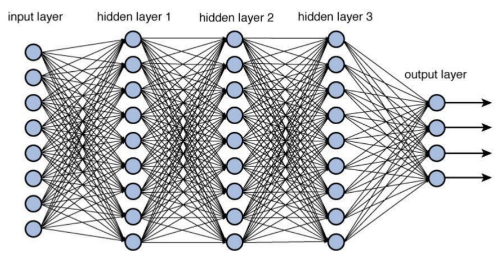
Dual Neural-Network



Learns optimality *certificate* $\pi^d(x)$

Online

Compute primal cost $J_N^p(x(t), \pi^p)$

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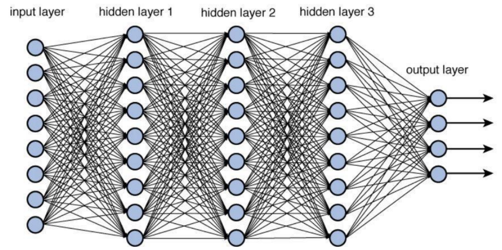
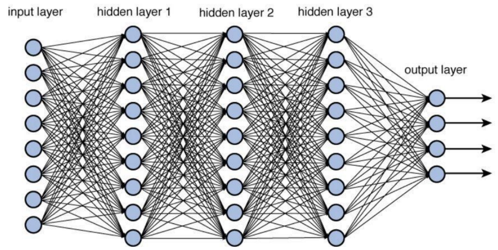
Primal Neural-Network



Learns input policy $\pi^p(x)$

Dual Neural-Network



Learns optimality *certificate* $\pi^d(x)$

Online

Compute primal cost $J_N^p(x(t), \pi^p)$

Compute dual cost $J_N^d(x(t), \pi^d)$

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Primal-Dual Neural Network

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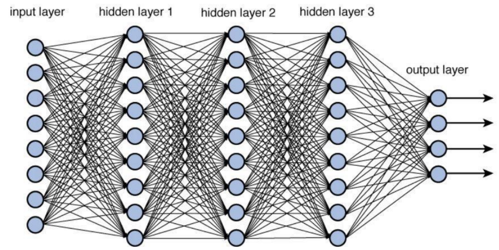
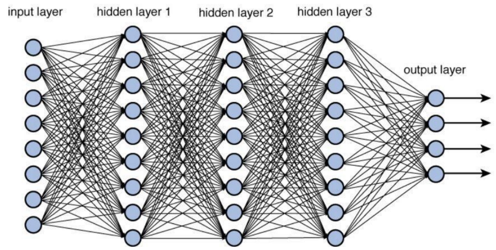
Primal Neural-Network



Learns input policy $\pi^p(x)$

Dual Neural-Network



Learns optimality *certificate* $\pi^d(x)$

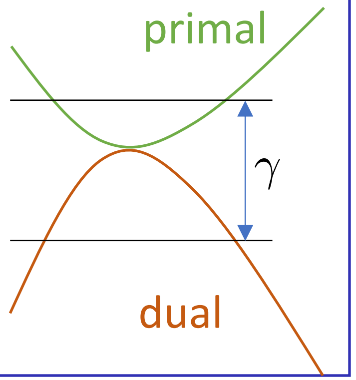
Online

Compute primal cost $J_N^p(x(t), \pi^p)$

Compute dual cost $J_N^d(x(t), \pi^d)$

If (primal $=$ feasible) and (dual $=$ feasible) and $|J_N^d(x(t), \pi^d) - J_N^d(x(t), \pi^d)| \leq \gamma$ then

Apply $u_t = \pi^p(x)$



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Primal-Dual Neural Network

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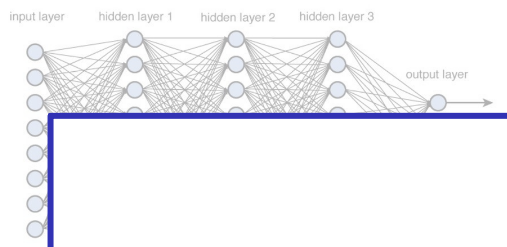
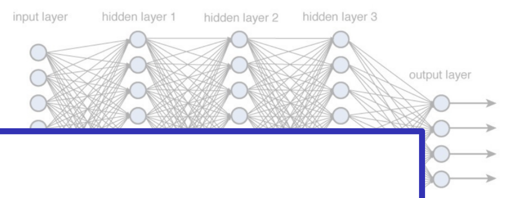
Primal Neural-Network



Dual Neural-Network



Lea…

…ate $\pi^d(x)$

This strategy guarantees probabilistic bounds on the performance (assumptions listed in the paper)

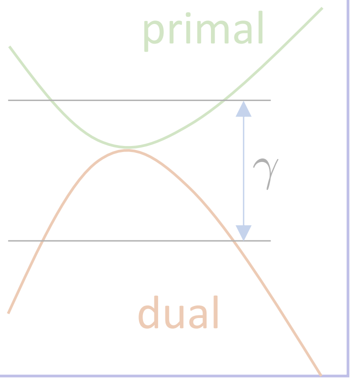
Online

Compute primal cost $J_N^p(x(t), \pi^p)$

Compute dual cost $J_N^d(x(t), \pi^d)$

If (primal = feasible) and (dual = feasible) and $|J_N^d(x(t), \pi^d) - J_N^d(x(t), \pi^d)| \leq \gamma$ then

Apply $u_t = \pi^p(x)$



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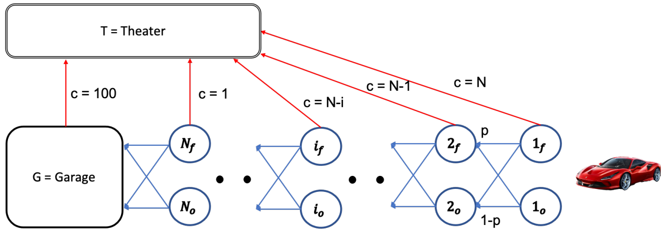
Key takeaways

Two slides summary

Lecture #1



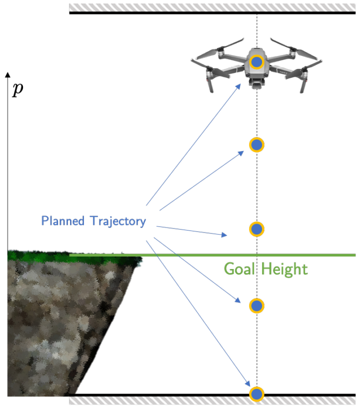
- Control problem with discrete state and action spaces can be solved using value iteration and policy iteration
- For large problem we can use Approximate Dynamic Programming (ADP). The engine behind alphaGo (more details: http://web.mit.edu/dimitrib/www/Slides_Lecture13_RLOC.pdf)

Lecture #2

- A control policy for the Linear Quadratic Regulator (LQR) problem can be computed exactly using dynamic programming
- For constrained systems is hard to compute a policy, but it is "easy" to compute a trajectory using the batch approach

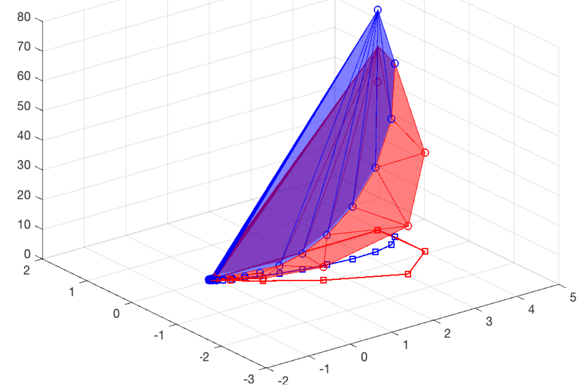
$$\begin{aligned} \min_{z} \quad & z^\top Q z \\ \text{s.t.} \quad & Fz = x(t) \\ & Gz \leq Ex(t) + b \end{aligned}$$

Lecture #3



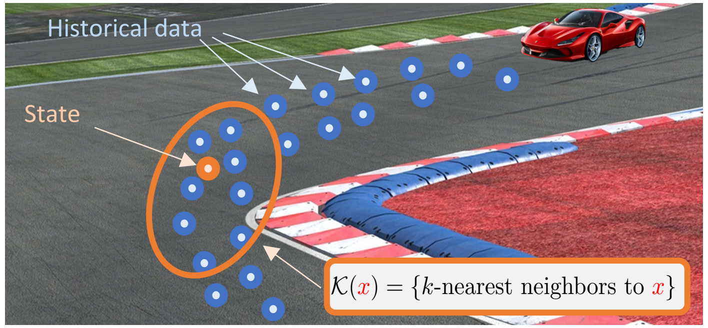
- A control policy for constrained problems can be defined iteratively replanning a trajectory over a time horizon N
- Receding horizon strategies may lead to safety constraint violation and instabilities when the terminal components are not designed correctly

Lecture #4



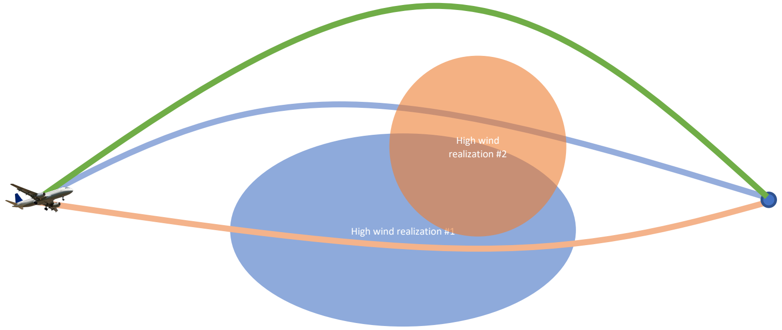
- When terminal cost is a control Lyapunov function and the terminal set is an invariant set, the MPC policy guarantees safety and stability
- We can compute the terminal set and a terminal cost iteratively from historical data. This iterative strategy guarantees safety, stability and performance improvement

Lecture #5



- A control strategy tailored to uncertain Linear Time-Varying (LTV) systems could work surprisingly well for smooth systems
- There are several strategies for model learning! Pick the one that best fit your application keeping in mind the accuracy-computation trade off

Lecture #6



- When disturbances are acting on the system, we should plan of policies to reduce conservatism and the uncertainty along the planned trajectory
- For stochastic system, we should collect several closed-loop trajectories associated with a control policy to estimate the value function and safe set