

CS159 Lecture 5: LMPC and Model Learning

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Caltech

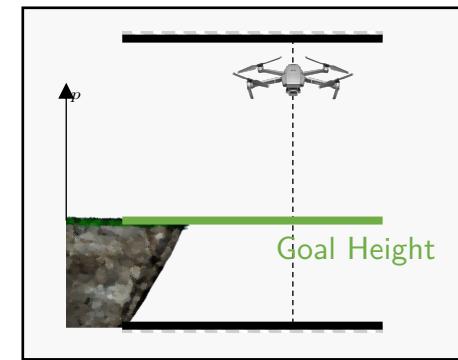
Spring 2021

Today's Class

- ▶ Recap of Lecture #4
- ▶ LMPC implementation
- ▶ Autonomous Racing Experiments
- ▶ Model learning in MPC (a brief summary)
- ▶ Planning under uncertainty

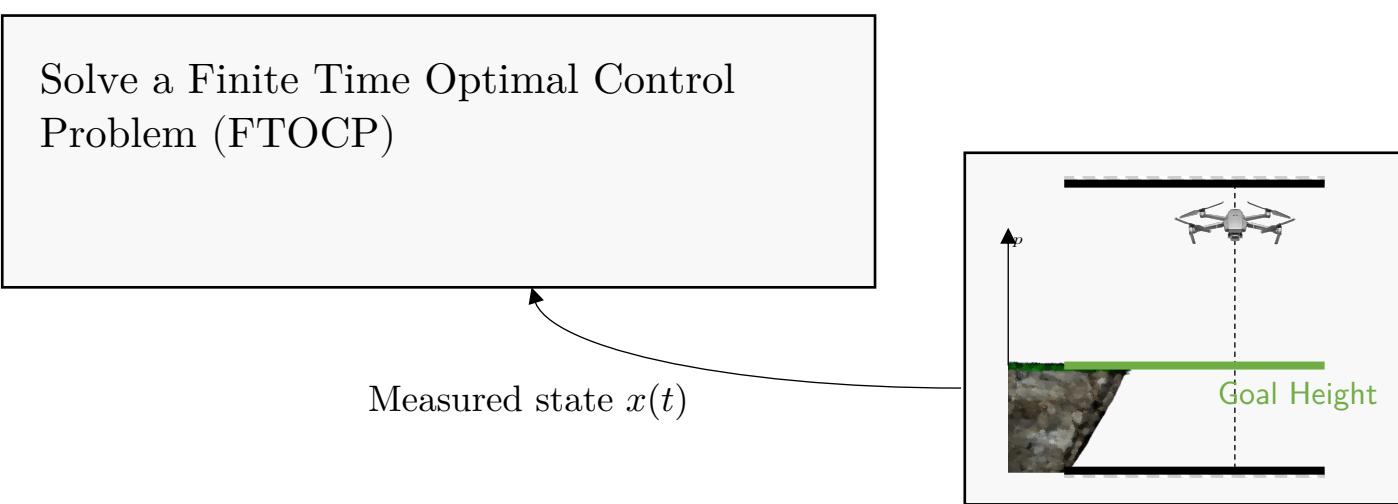
Recap of Lecture #4

Why did we focus on feasibility?



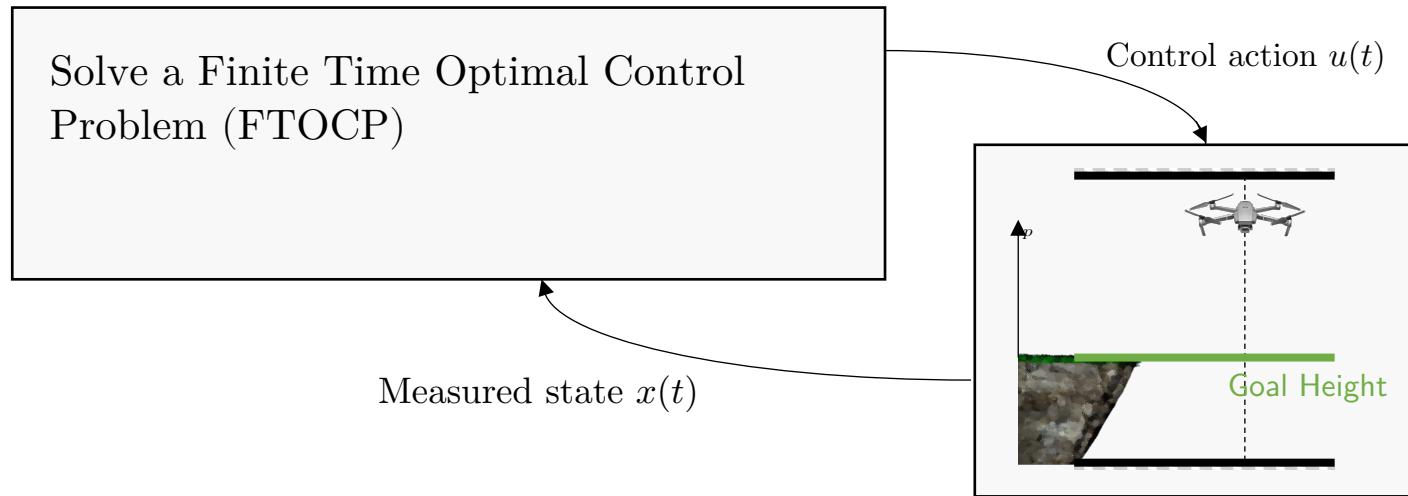
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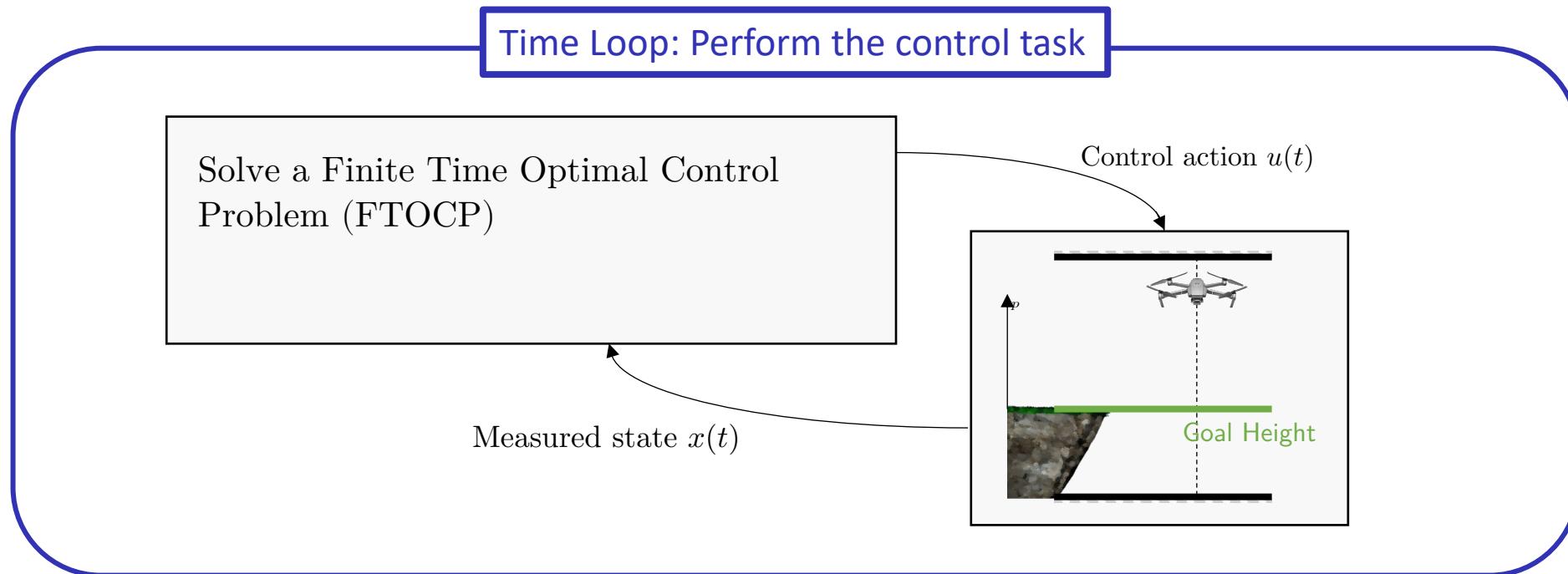
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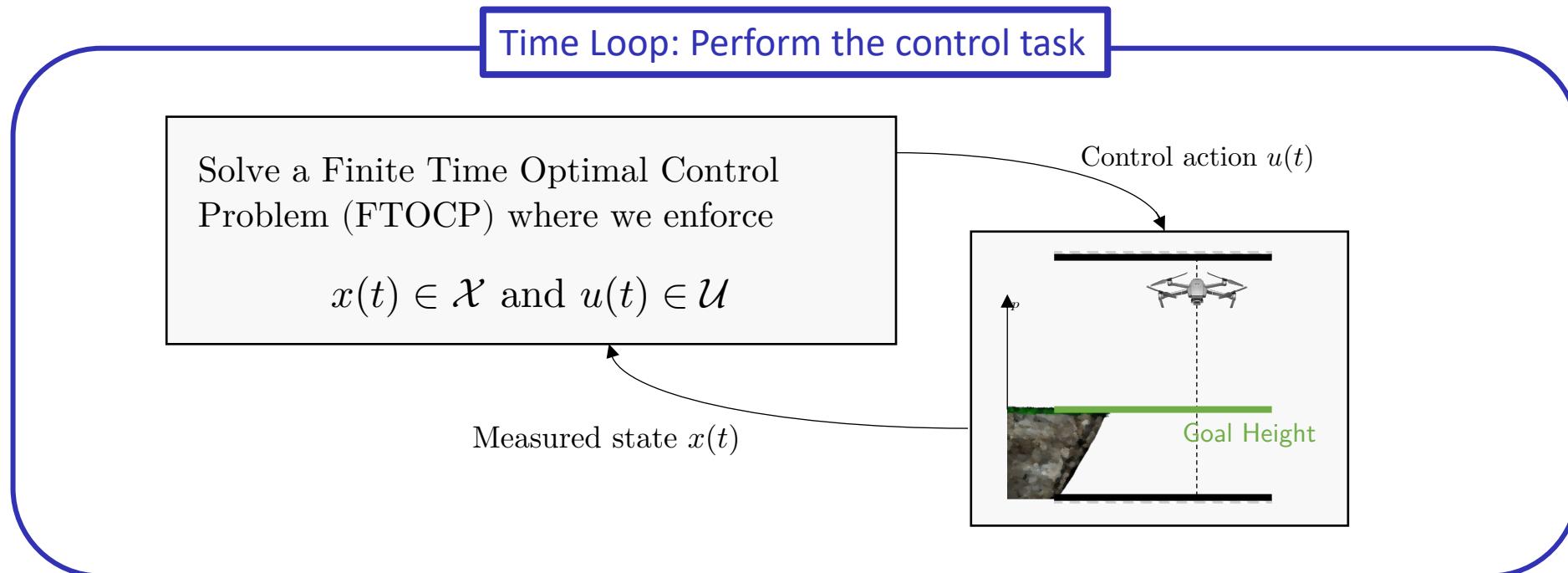
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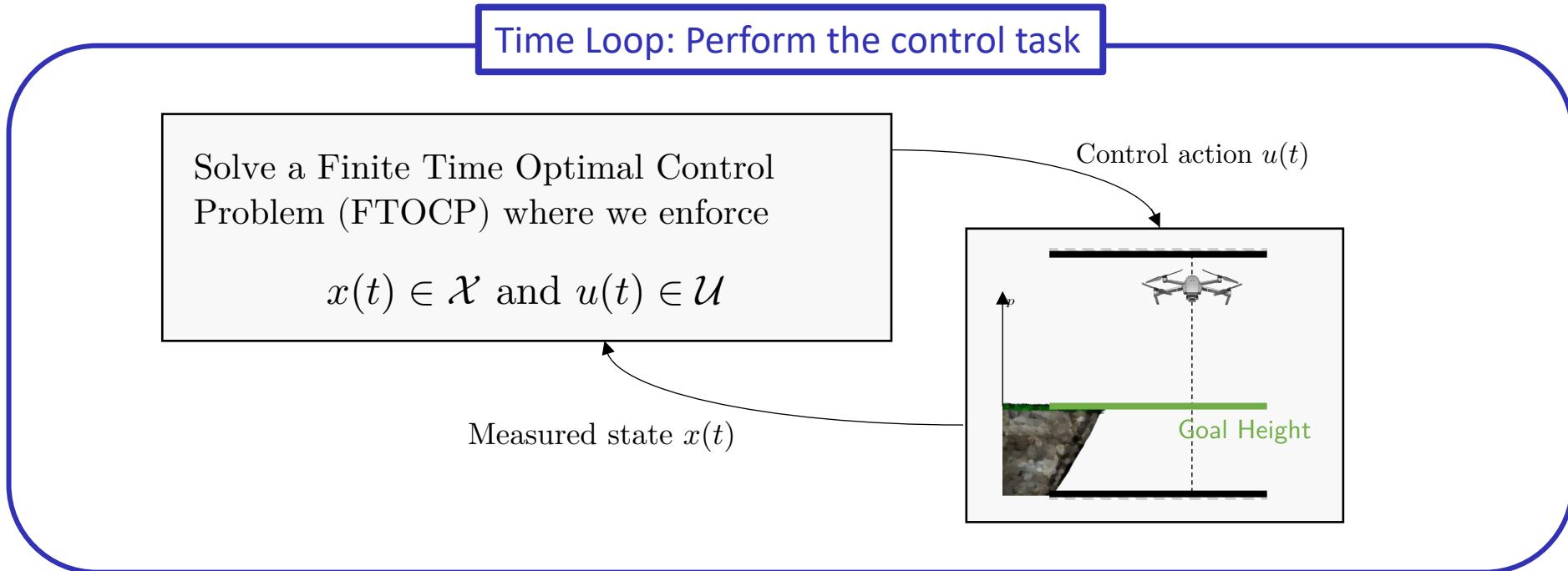
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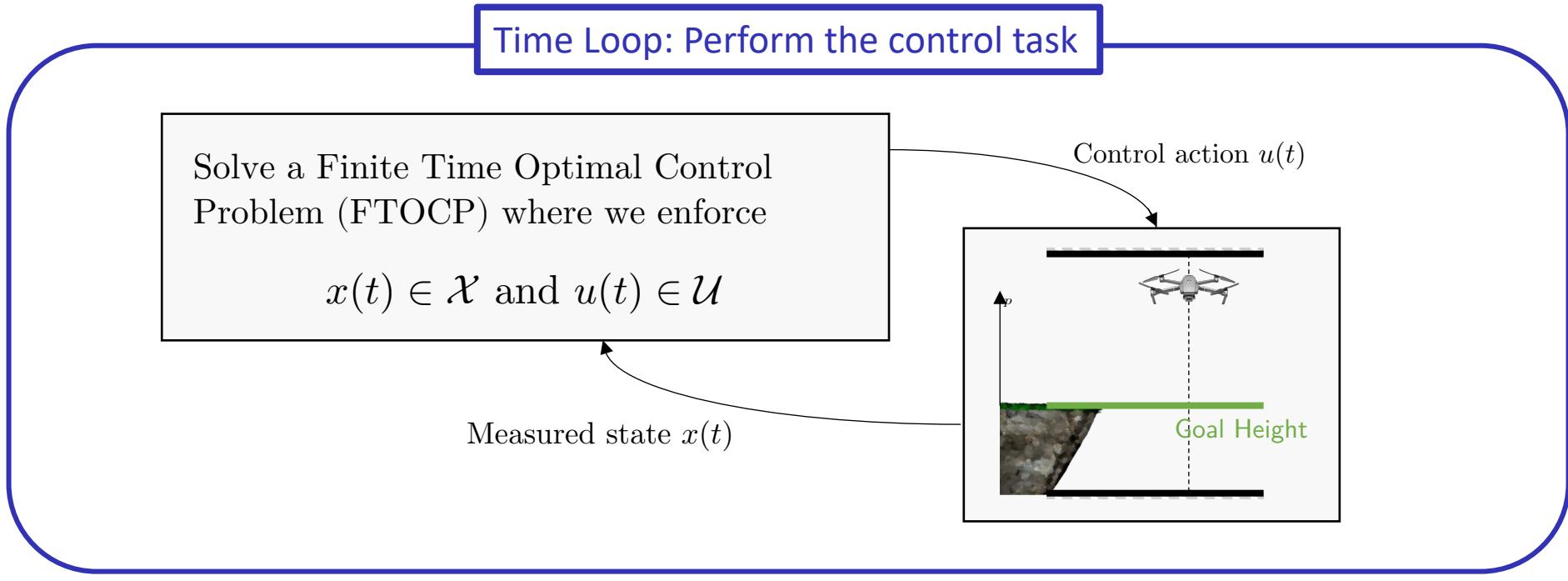
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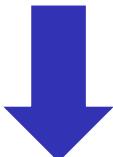


Recap of Lecture #4

Why did we focus on feasibility?



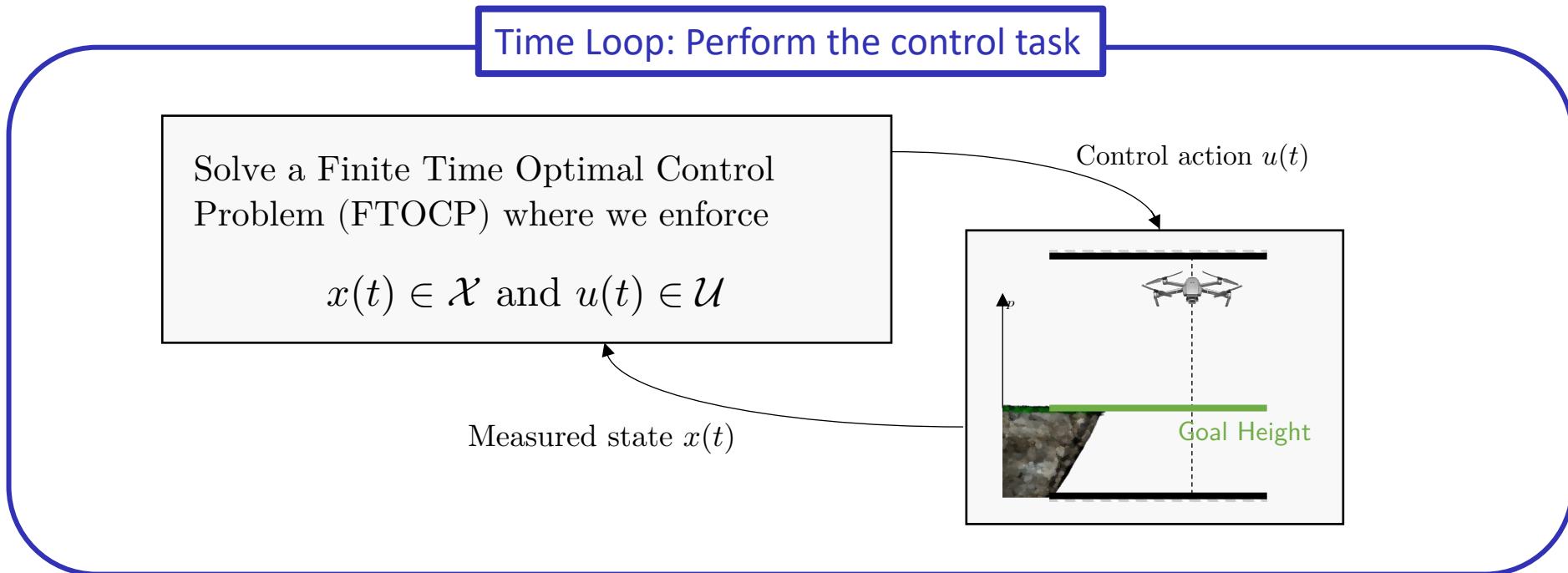
Feasibility of the FTCOP at all times



Safety

Recap of Lecture #4

Why did we focus on feasibility?



Feasibility of the FTCOP at all times



Not true when constraints are relaxed with slack variables!

Safety

Recap of Lecture #4

Stability Proof:

Recap of Lecture #4

Stability Proof:

We have shown that

Recap of Lecture #4

Stability Proof:

We have shown that

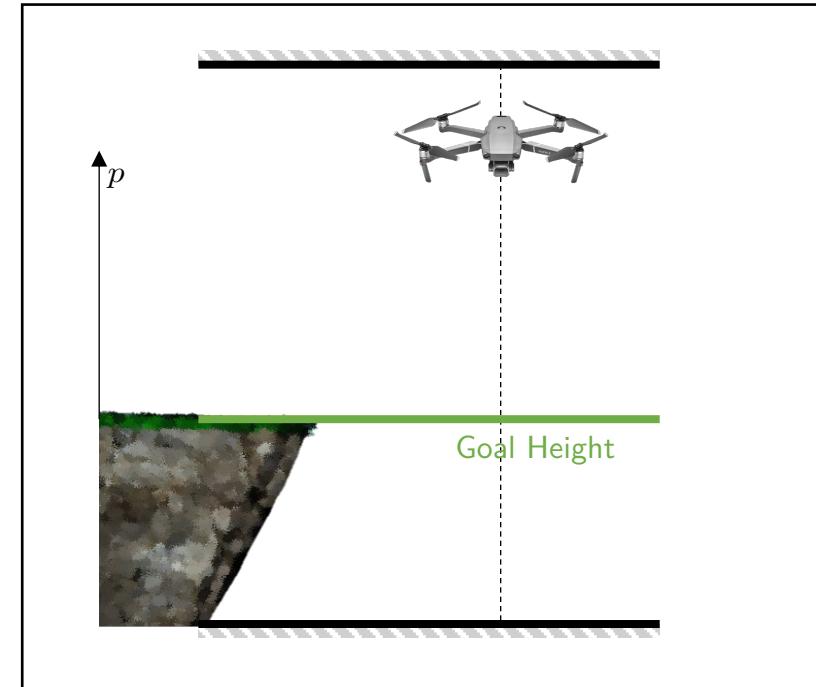
$$J_t^*(x(t)) > J_{t+1}^*(x(t+1)), \forall x(t) \neq 0$$

Recap of Lecture #4

Stability Proof:

We have shown that

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Recap of Lecture #4

Stability Proof:

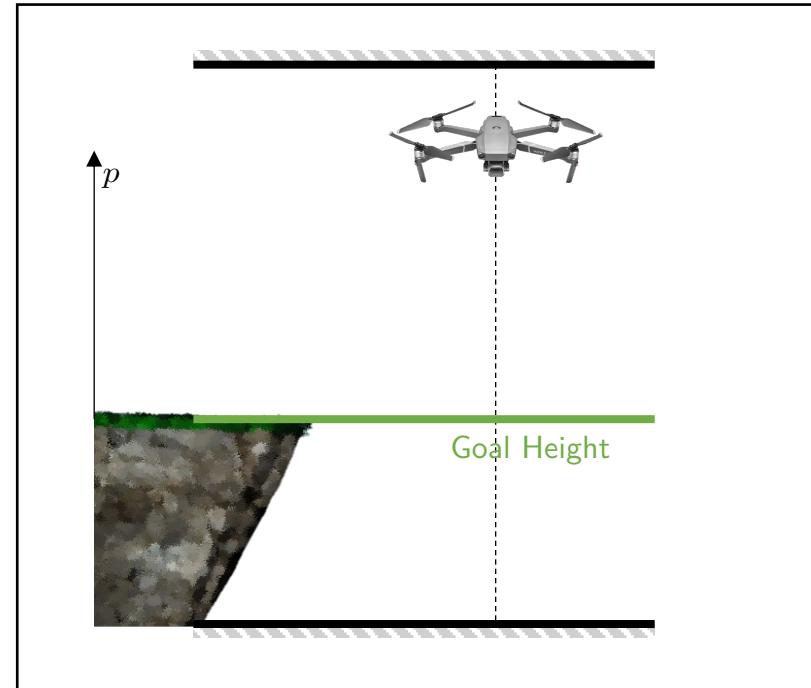
We have shown that

$$J_t^*(x(t)) > J_{t+1}^*(x(t+1)), \forall x(t) \neq 0$$

$$J_t^*(x) > 0, \forall x \neq 0,$$

$$\|x\|_2 \rightarrow \infty \implies J_t^*(x) \rightarrow \infty$$

$$J_t^*(0) = 0$$

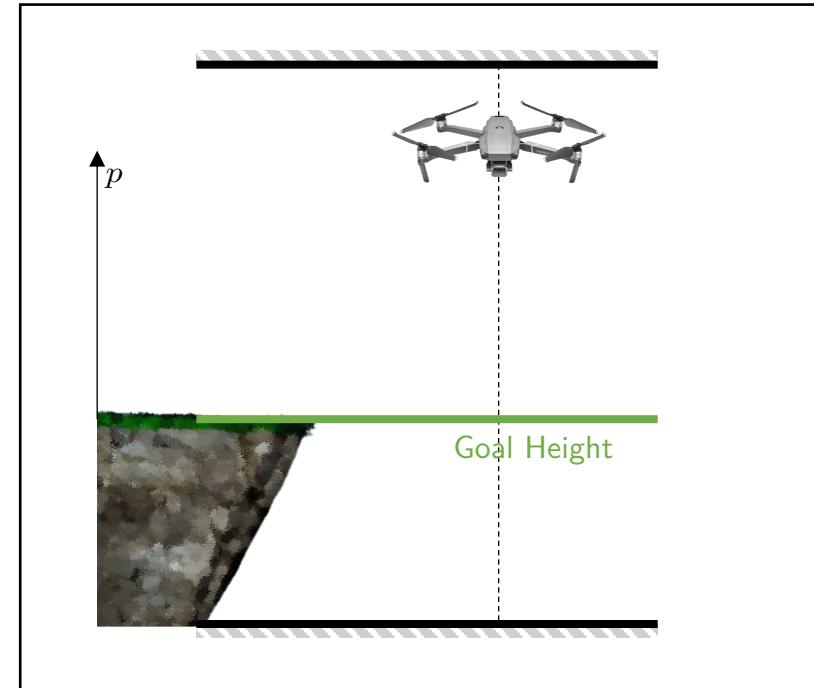


Recap of Lecture #4

Stability Proof:

We have shown that

$$\left. \begin{array}{l} J_t^*(x(t)) > J_{t+1}^*(x(t+1)), \forall x(t) \neq 0 \\ J_t^*(x) > 0, \forall x \neq 0, \\ \|x\|_2 \rightarrow \infty \implies J_t^*(x) \rightarrow \infty \\ J_t^*(0) = 0 \end{array} \right\} \lim_{t \rightarrow \infty} J_t^*(x(t)) = 0$$

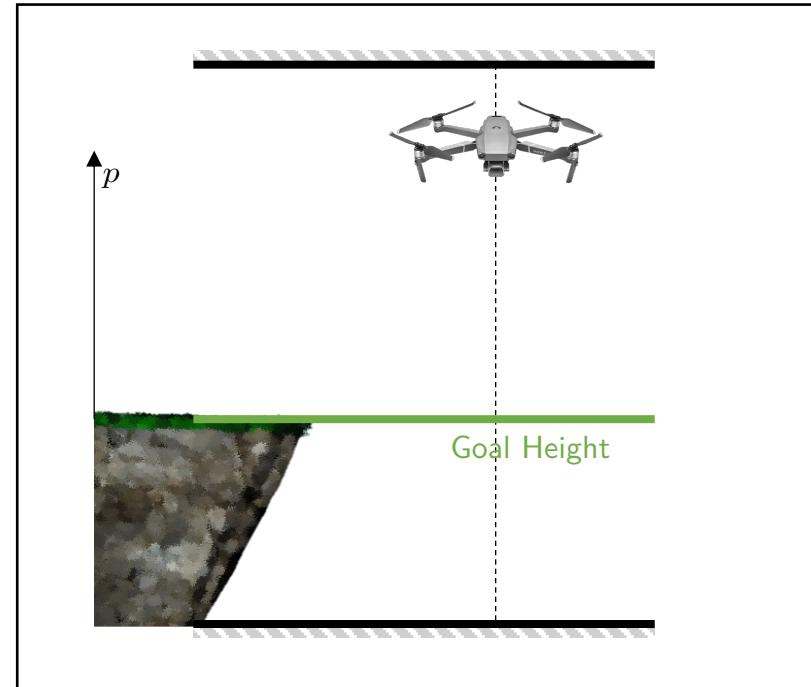


Recap of Lecture #4

Stability Proof:

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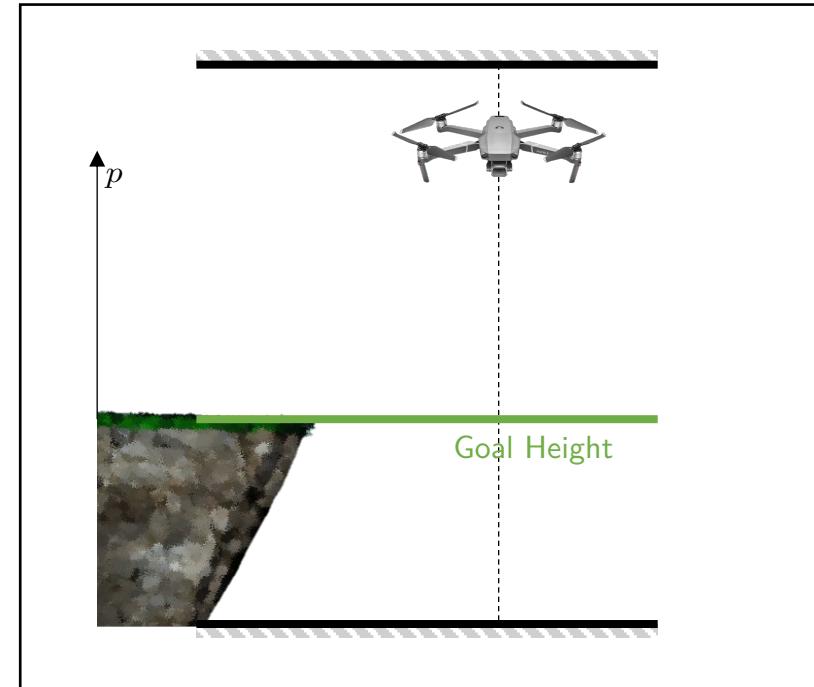
To show stability we need to that $J_t^*(x)$ is Lipchitz continuous and assume that the state and input constraint sets are compact

Recap of Lecture #4

Stability Proof:

We have shown that

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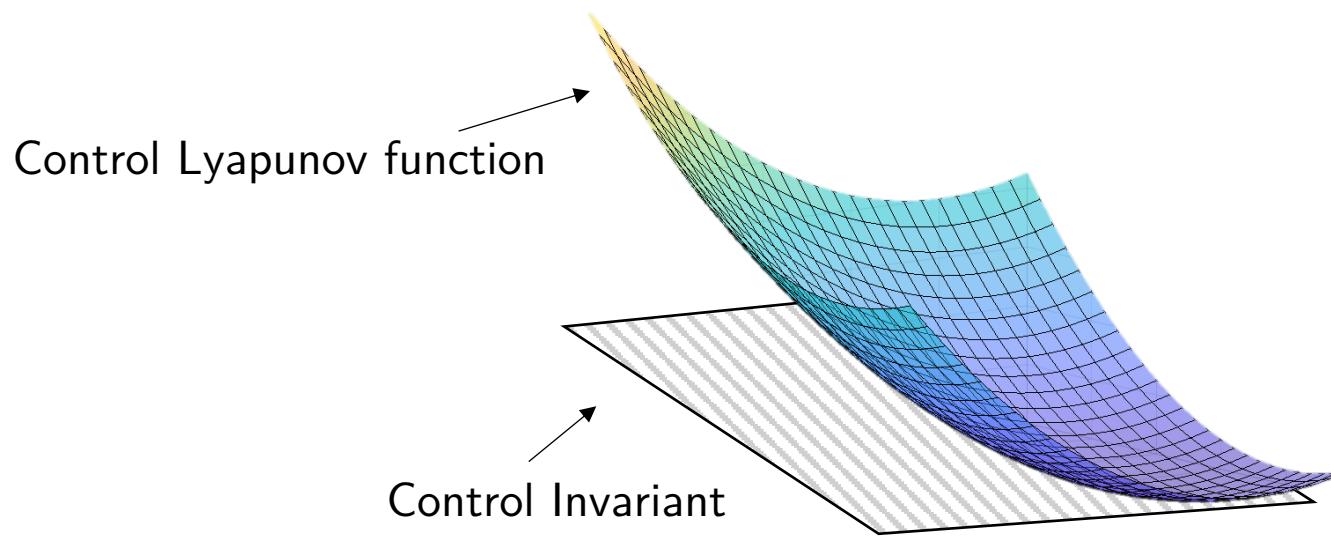
To show stability we need to that $J_t^*(x)$ is Lipchitz continuous and that

$$\alpha_1(x) \leq J_t^*(x) \leq \alpha_2(x)$$

where α_\succcurlyeq and α_\preccurlyeq are class K functions (can be relaxed to continuity around the origin).

Recap of Lecture #4

Design of the terminal MPC components



Recap of Lecture #4

Design of the terminal MPC components

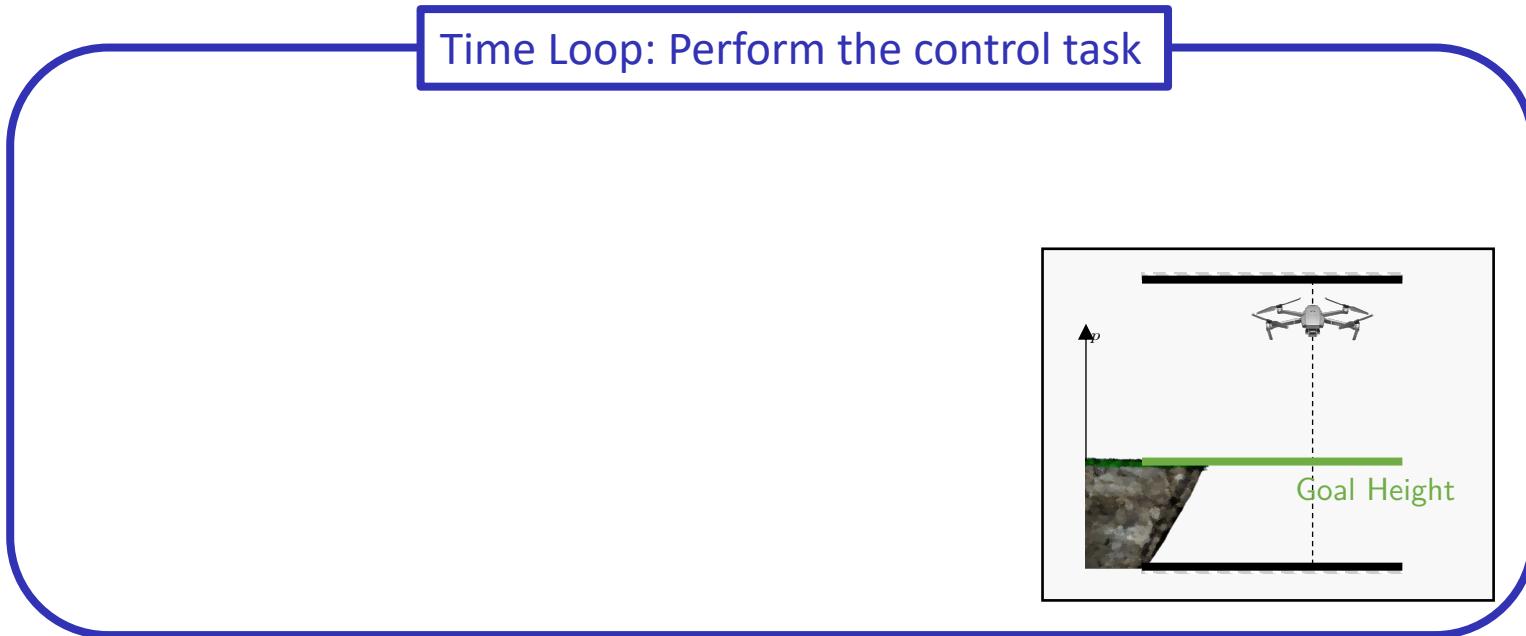
Control Lyapunov function

Control Invariant

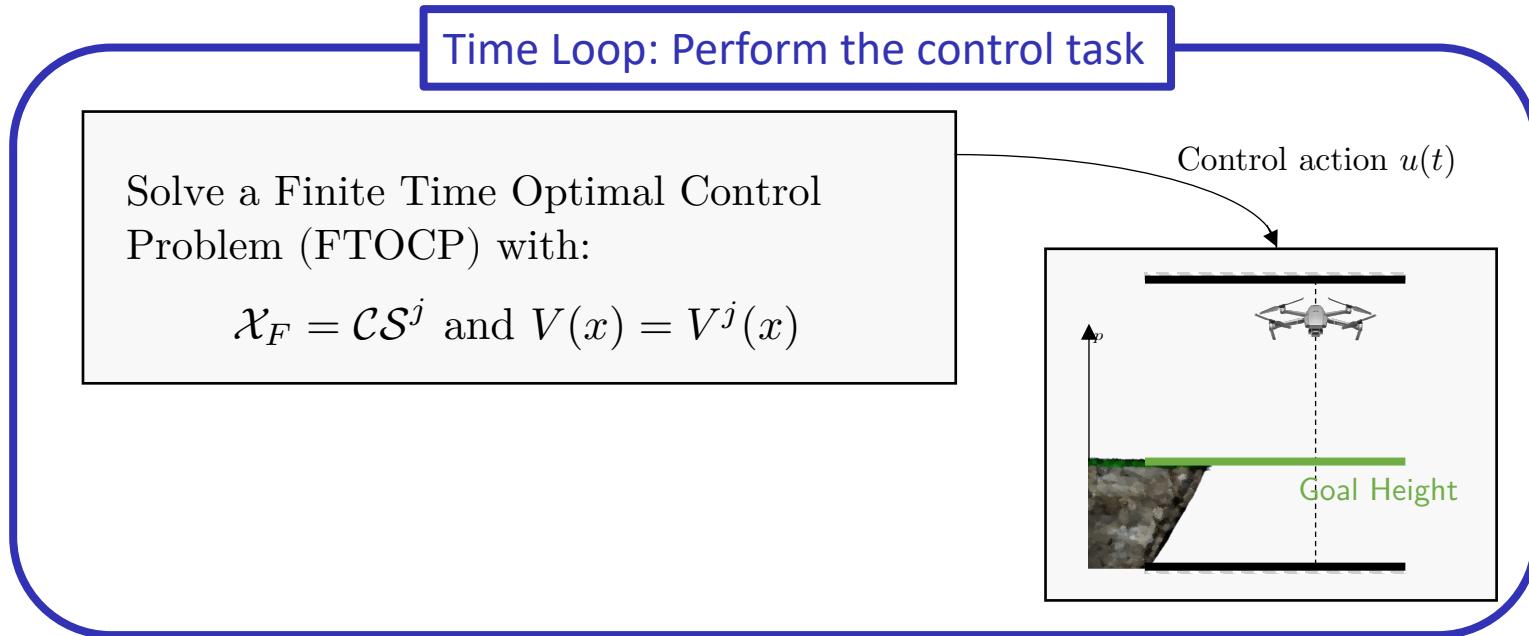


Safety and Stability

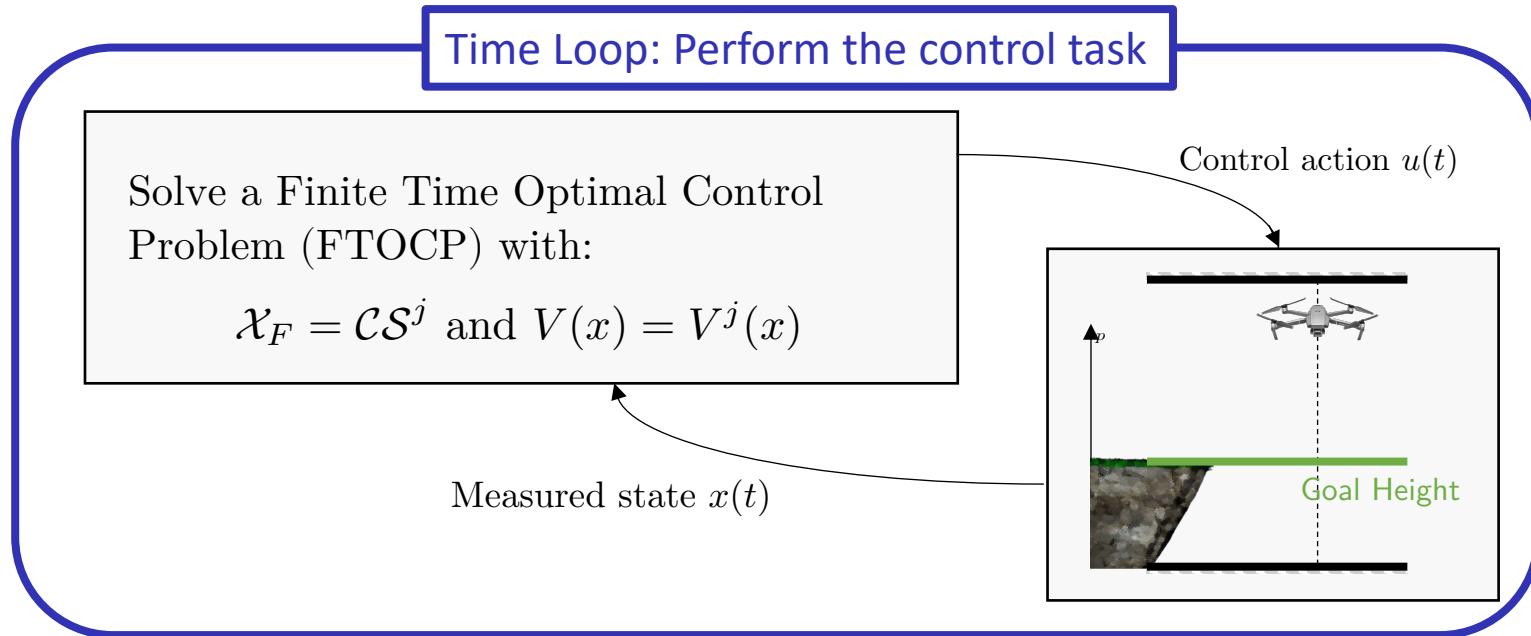
LMPC Diagram



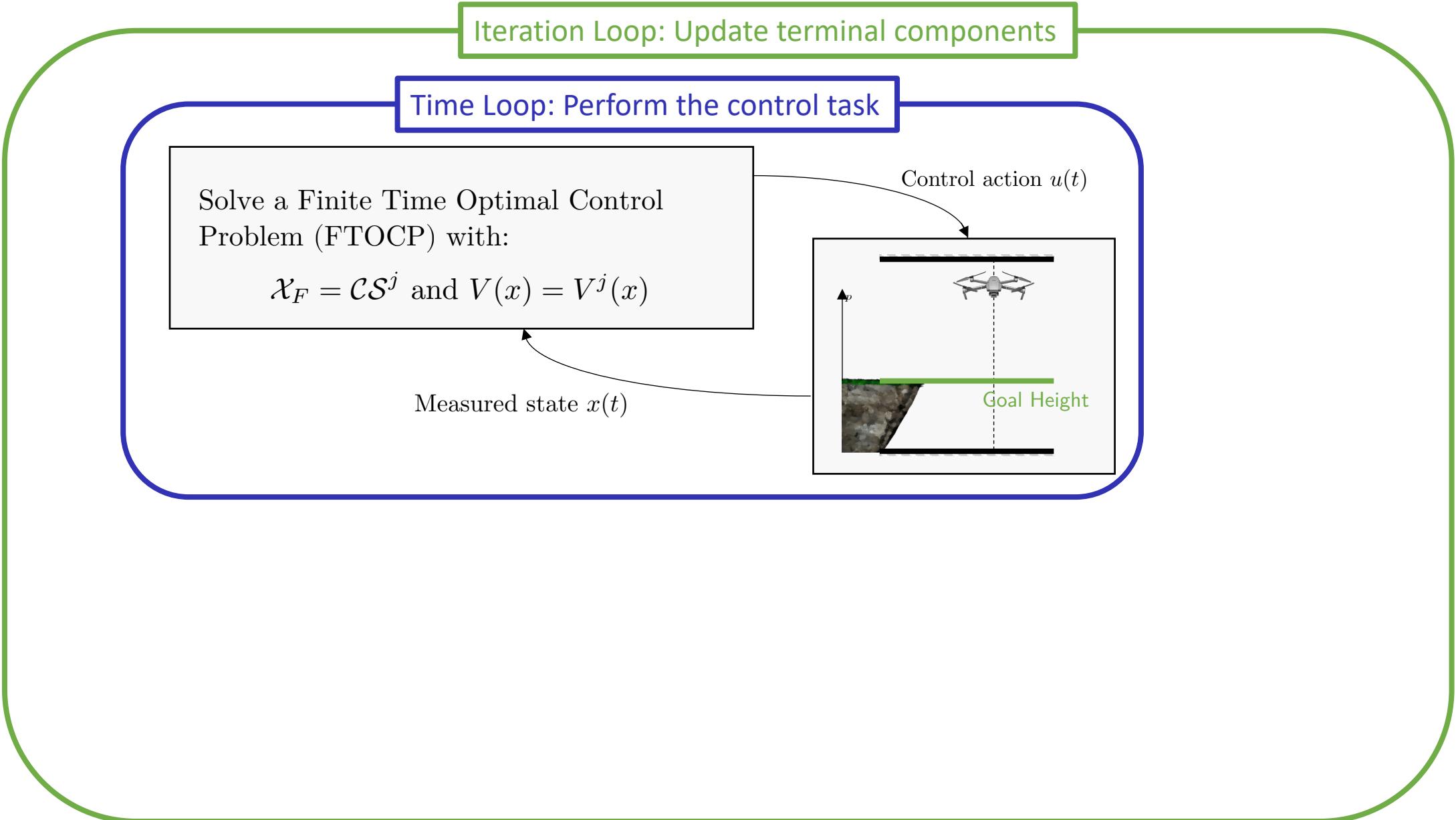
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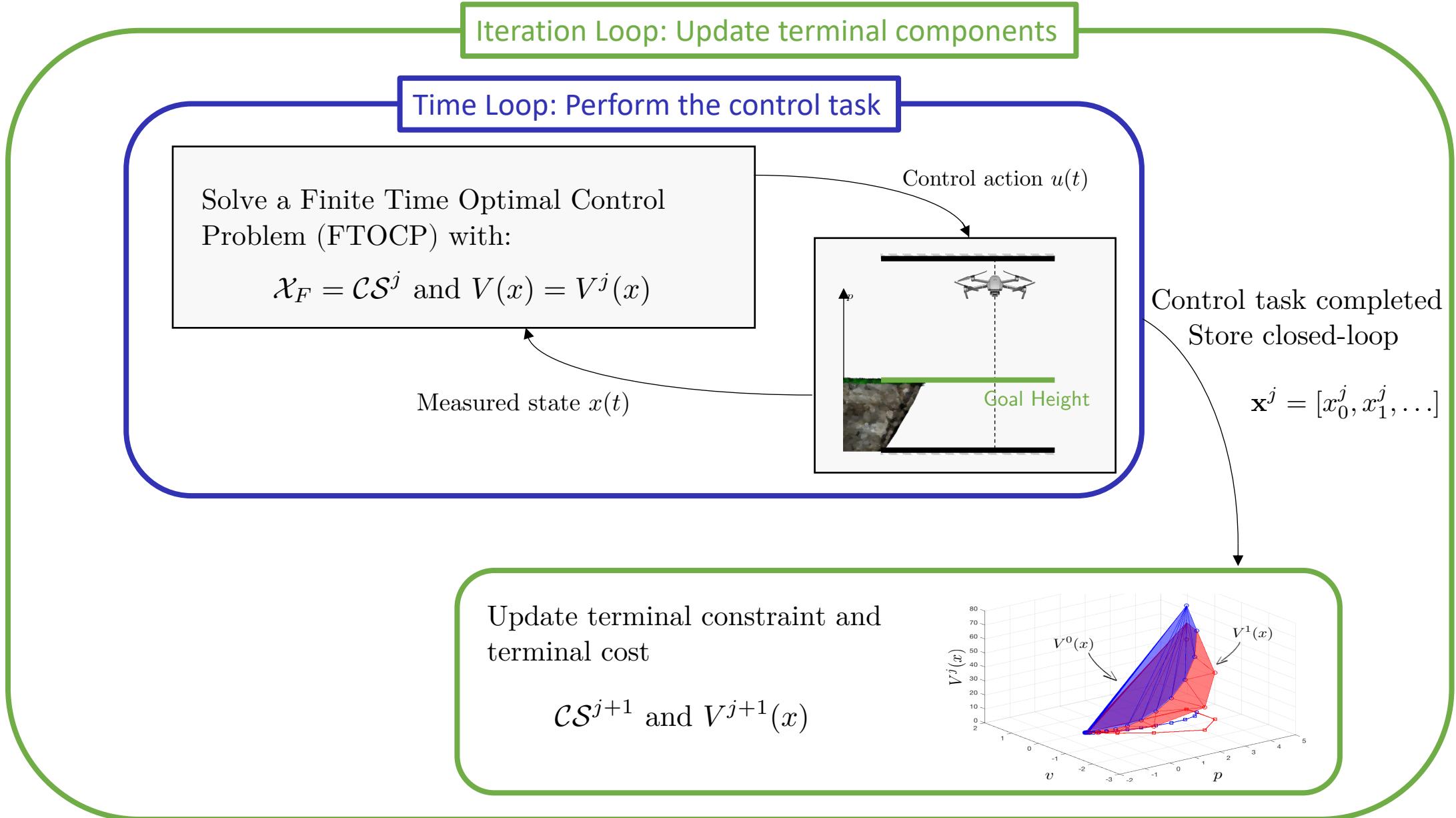
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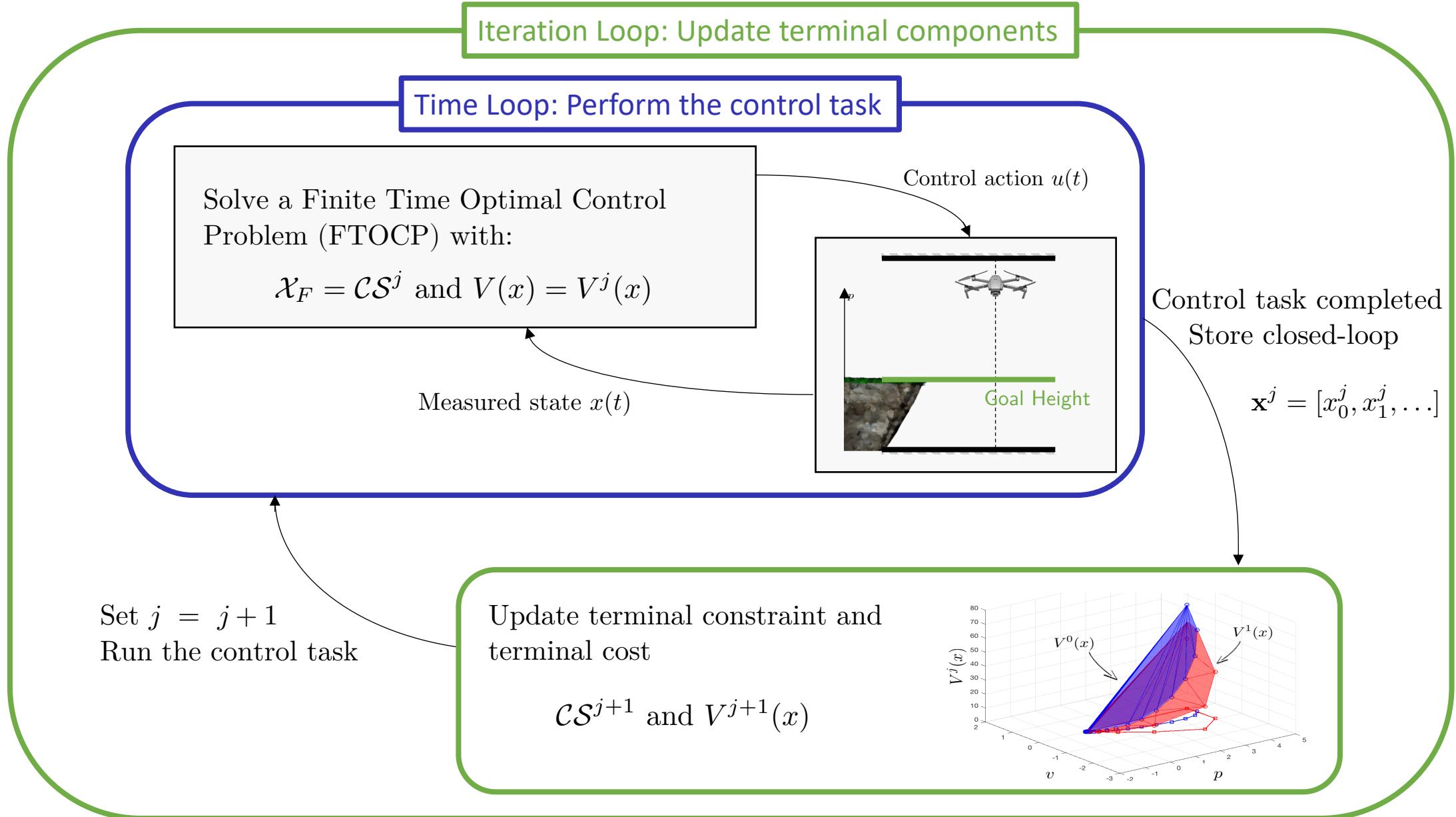
LMPC Diagram



LMPC Diagram



LMPC Diagram



LMPC Recap

Theorem

Let $\mathbf{x}^j = [x_0^j, x_1^j, \dots]$ be the closed-loop trajectory from the starting state x_S at iteration j . Consider sequence $\{\mathbf{x}^j\}$ of closed-loop trajectories and assume that for $c < \infty$ we have that

$$\mathbf{x}^c = \mathbf{x}^{c+1}$$

Then we have that

- ▶ At each iteration state and input constraints are satisfied.
- ▶ The closed-loop cost $J_{0 \rightarrow \infty}^j(x_S)$ is non-increasing, i.e.,

$$J_{0 \rightarrow \infty}^{j+1}(x_S) = \sum_{t=0}^{\infty} h(x_t^{j+1}, u_t^{j+1}) \leq \sum_{t=0}^{\infty} h(x_t^j, u_t^j) = J_{0 \rightarrow \infty}^j(x_S)$$

- ▶ $\mathbf{x}^c = \mathbf{x}^*$, under mild conditions (LICQ holds at each time t)

Non-increasing closed-loop cost: Proof

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Step 1: $J_{0 \rightarrow \infty}^{j-1}(x_S) \geq J_{0 \rightarrow N}^{LMPC,j}(x_S)$

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$$J_{0 \rightarrow \infty}^{j-1}(x_S) = \sum_{k=0}^{\infty} h(x_k^{j-1}, u_k^{j-1}) = \sum_{k=0}^{N-1} h(x_k^{j-1}, u_k^{j-1}) + \sum_{k=N}^{\infty} h(x_k^{j-1}, u_k^{j-1})$$

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Cost associated with a feasible trajectory

Non-increasing closed-loop cost: Proof

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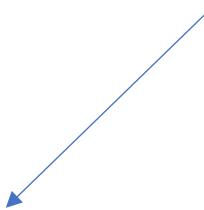
Step 2: $J_{0 \rightarrow N}^{LMPC,j}(x_S) \geq J_{0 \rightarrow \infty}^j(x_S)$

Non-increasing closed-loop cost: Proof

Step 1: $J_{0 \rightarrow \infty}^{j-1}(x_S) \geq J_{0 \rightarrow N}^{LMPC,j}(x_S)$

From standard MPC arguments

Step 2: $J_{0 \rightarrow N}^{LMPC,j}(x_S) \geq J_{0 \rightarrow \infty}^j(x_S)$



$$J_{1 \rightarrow 1+N}^{LMPC,j}(x_1^j) - J_{0 \rightarrow N}^{LMPC,j}(x_0^j) \leq -h(x_0^j, u_0^j)$$

Non-increasing closed-loop cost: Proof

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$$\rightarrow J_{0 \rightarrow N}^{LMPC,j}(x_0^j) \geq \lim_{t \rightarrow \infty} J_{t \rightarrow t+N}^{LMPC,j}(x_t^j) + \sum_{k=0}^{\infty} h(x_k^j, u_k^j)$$

Non-increasing closed-loop cost: Proof

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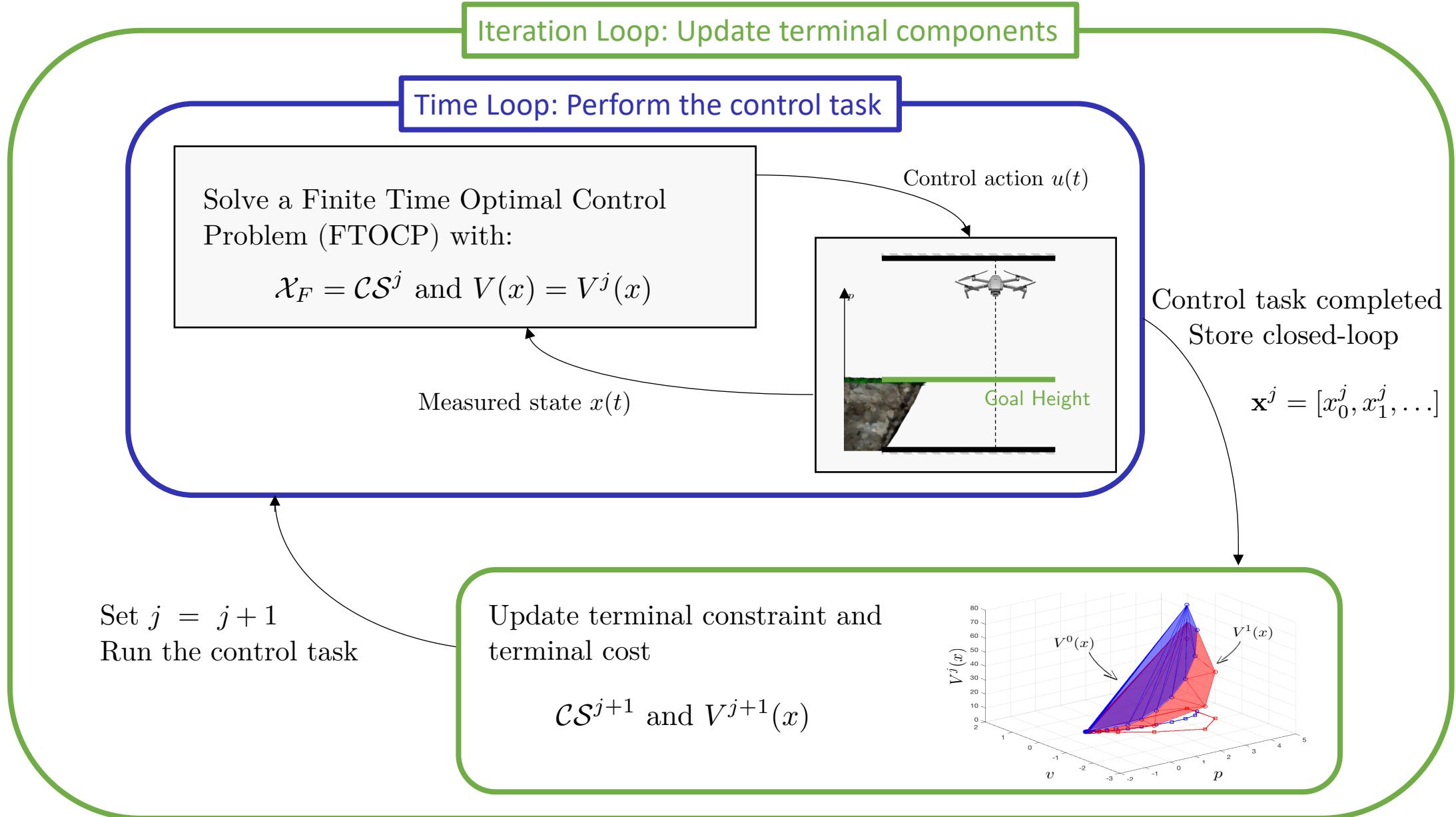
Step 2: $J_{0 \rightarrow N}^{LMPC,j}(x_S) \geq J_{0 \rightarrow \infty}^j(x_S)$

Conclusion

The iteration cost $J_{0 \rightarrow \infty}^j(x_s)$ is non-increasing at each iteration,

$$J_{0 \rightarrow \infty}^{j-1}(x_S) \geq J_{0 \rightarrow N}^{LMPC,j}(x_S) \geq J_{0 \rightarrow \infty}^j(x_S)$$

LMPC Diagram



Linear(ized) LMPC

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (FTOCP)

$$J_{t \rightarrow t+N}^{\text{LMPC},j}(x_t^j) = \min_{u_{t|t}, \dots, u_{t+N-1|t}} \sum_{k=t}^{t+N-1} h(x_{k|t}, u_{k|t}) + V^{j-1}(x_{t+N|t})$$

s.t.

$$x_{k+1|t} = Ax_{k|t} + Bu_{k|t}, \quad \forall k \in [t, \dots, t+N-1]$$

$$x_{t|t} = x_t^j,$$

$$x_{k|t} \in \mathcal{X}, \quad u_{k|t} \in \mathcal{U}, \quad \forall k \in [t, \dots, t+N-1]$$

$$x_{t+N|t} \in \mathcal{CS}^{j-1}$$

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$$V^{j-1}(x_{t+N|t})$$

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$$x_{k|t} \in \mathcal{X}, \quad u_{k|t} \in \mathcal{U}, \quad \forall k \in [t, \dots, t+N-1]$$



$$V^{j-1}(x_{t+N|t})$$

$$x_{t+N|t} = \sum_{i=0}^{j-1} \sum_k x_k^i \lambda_k^i, \quad \sum_{i=0}^{j-1} \sum_k \lambda_k^i = 1, \quad \lambda_k^i \geq 0.$$

$$x_{t+N|t} \in \mathcal{CS}^{j-1}$$

- ▶ Convex optimization problem over inputs and lambdas
- ▶ Safety and performance improvement guarantees still hold (simple proofs as before)
- ▶ Converges to global optimal solution (LICQ required)

Learning MPC for Autonomous Racing

Real-time implementation on the Berkeley Autonomous Race Car (BARC)

Problem Formulation

Minimum Time Control Problem

$$\min_{T, \mathbf{u}} \quad T \quad \text{Control objective}$$

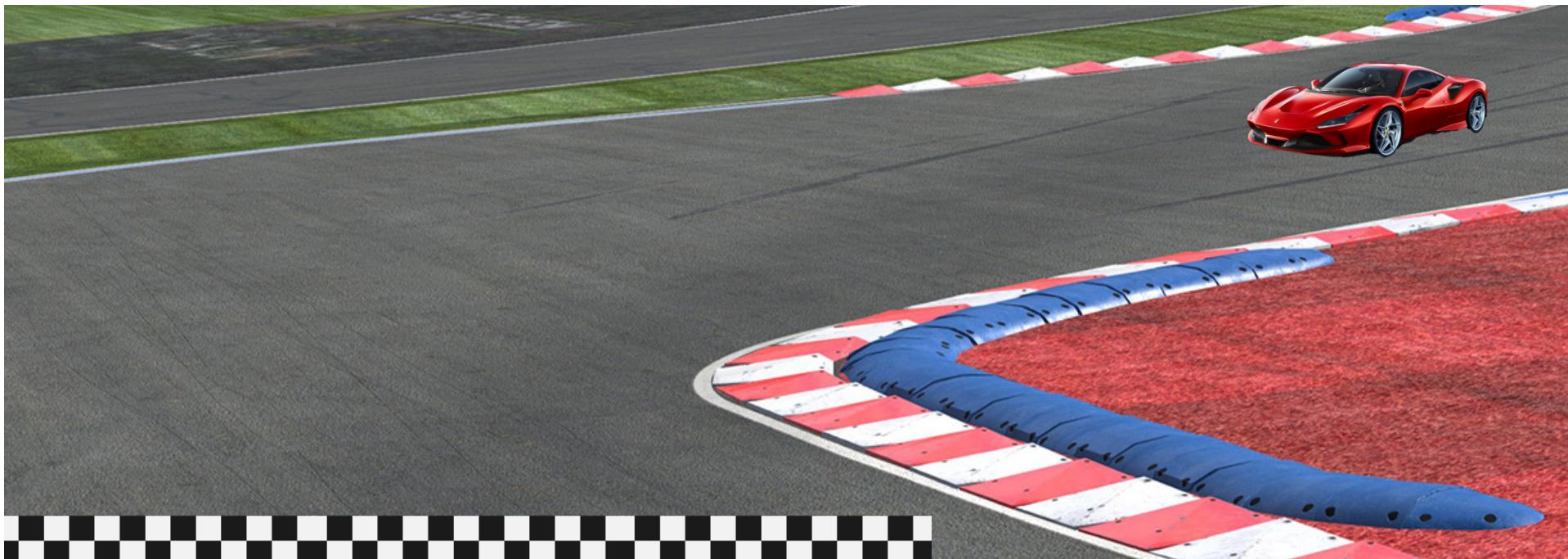
$$x_0 = x_s, \quad x_T = \mathcal{X}_F \quad \text{Start & end position}$$

System dynamics
System constraints

Safety constraints

$$x_{k+1} = f(x_k, u_k), \quad \forall k \in \{0, \dots, T-1\}$$

$$x_k \in \mathcal{X}, \quad u_k \in \mathcal{U}, \quad \forall k \in \{0, \dots, T-1\}$$





Learning Model Predictive Controller full-size vehicle experiments

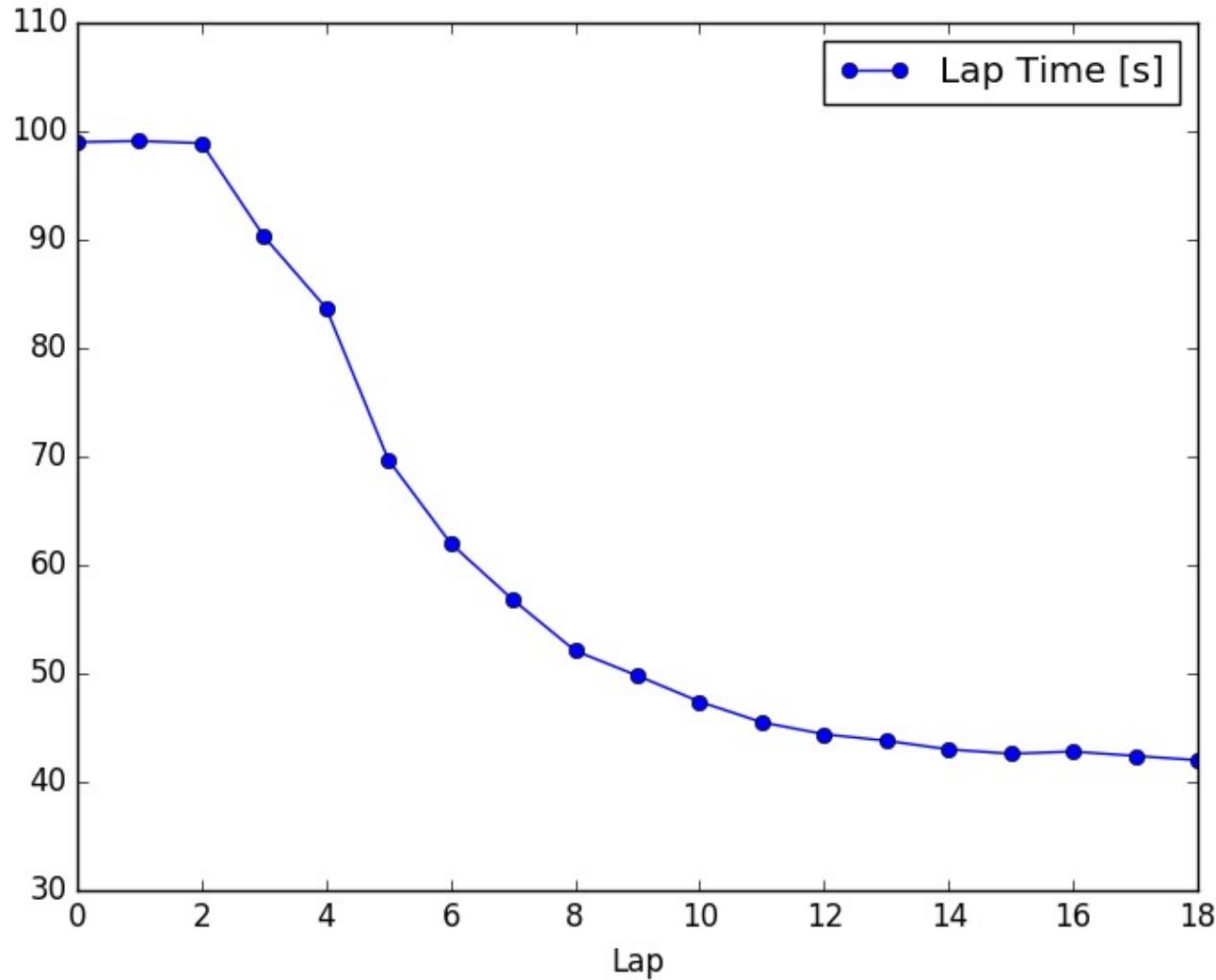
Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia



Learning Model Predictive Controller full-size vehicle experiments

Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia

Lap Time

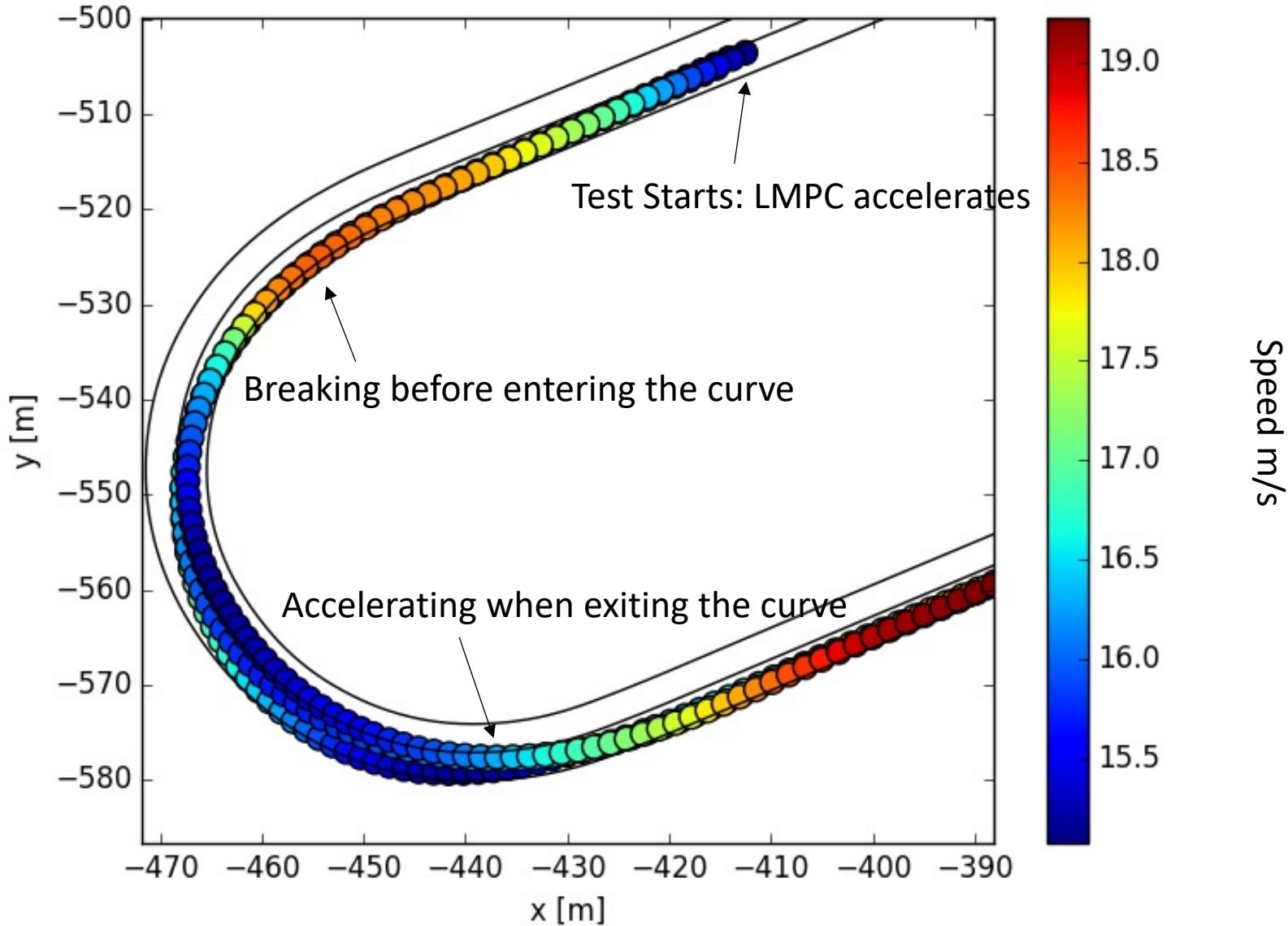




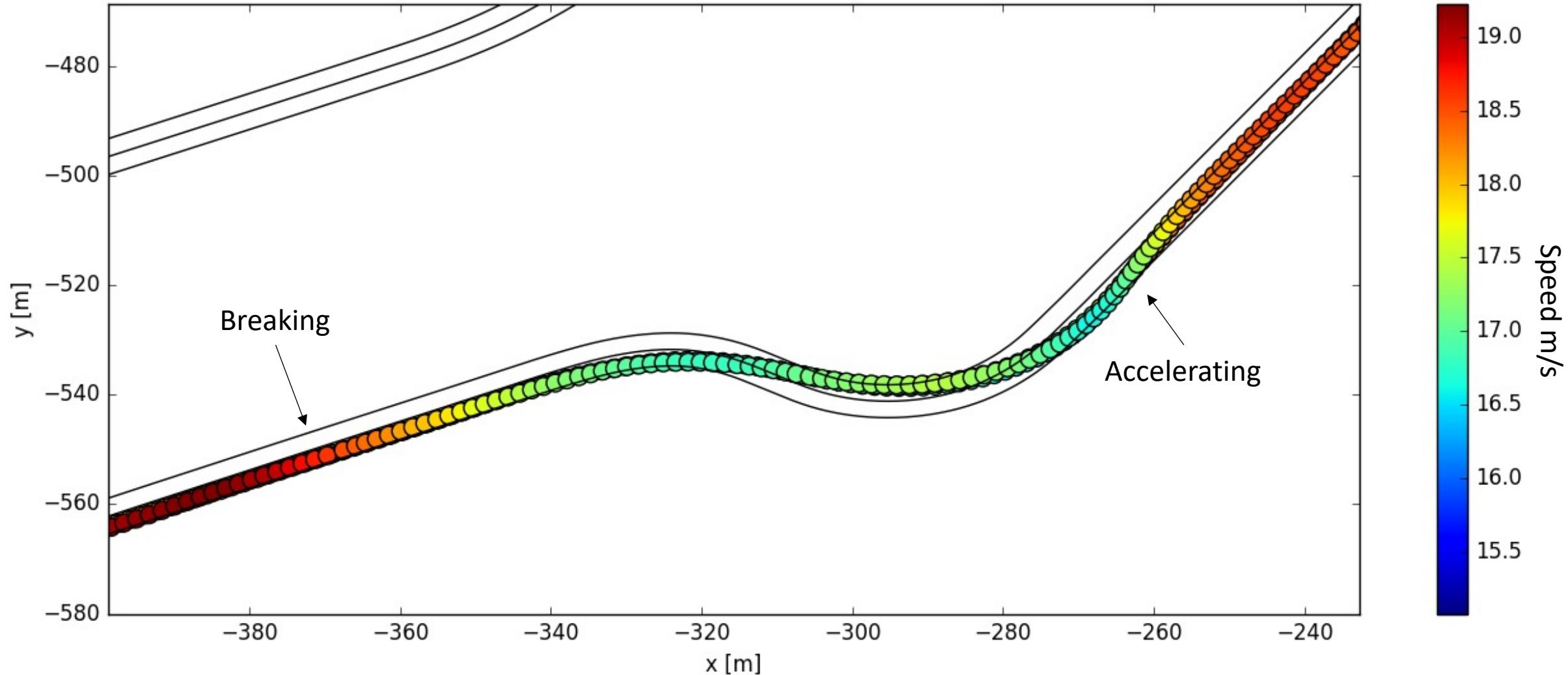
Learning Model Predictive Controller full-size vehicle experiments

Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia

Velocity Profile at Convergence (Curve 1)



Velocity Profile at Convergence (Chicane)



Problem Formulation

Minimum Time Control Problem

$$\min_{T, \mathbf{u}} \quad T \quad \text{Control objective}$$

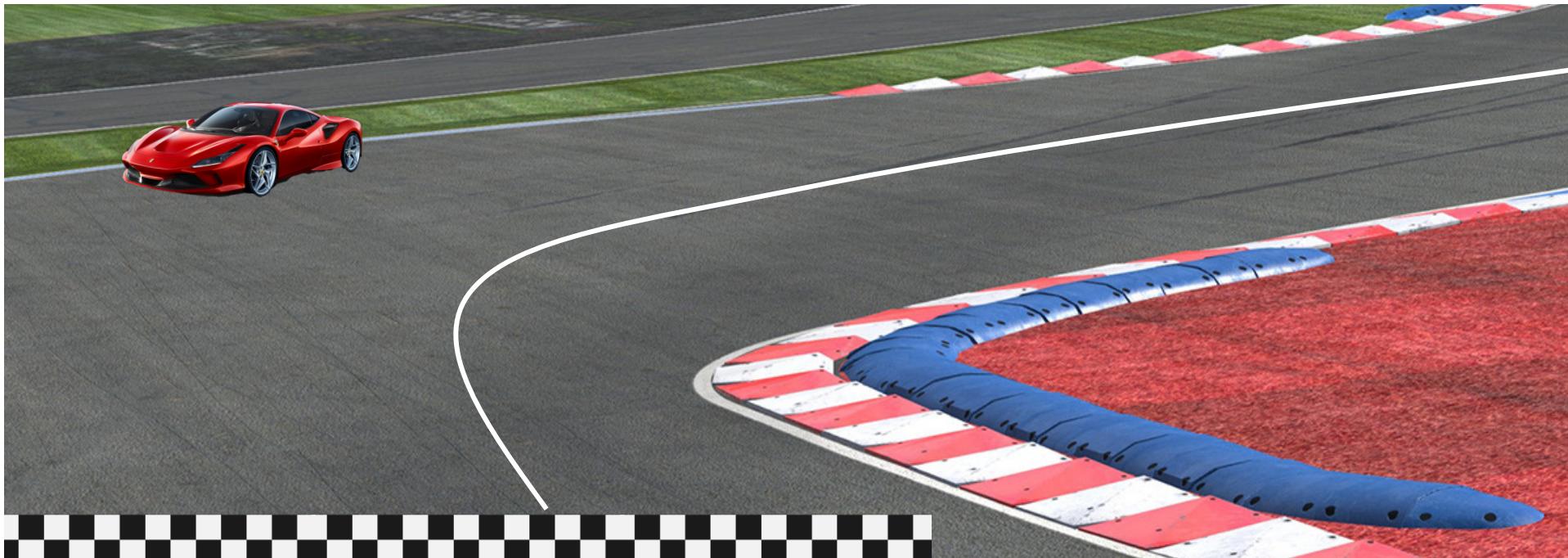
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System dynamics
System constraints

Safety constraints

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$$x_k \in \mathcal{X}, \quad u_k \in \mathcal{U}, \quad \forall k \in \{0, \dots, T-1\}$$



Problem Formulation

Minimum Time Control Problem

$$\min_{T, \mathbf{u}} \quad T \quad \text{Control objective}$$

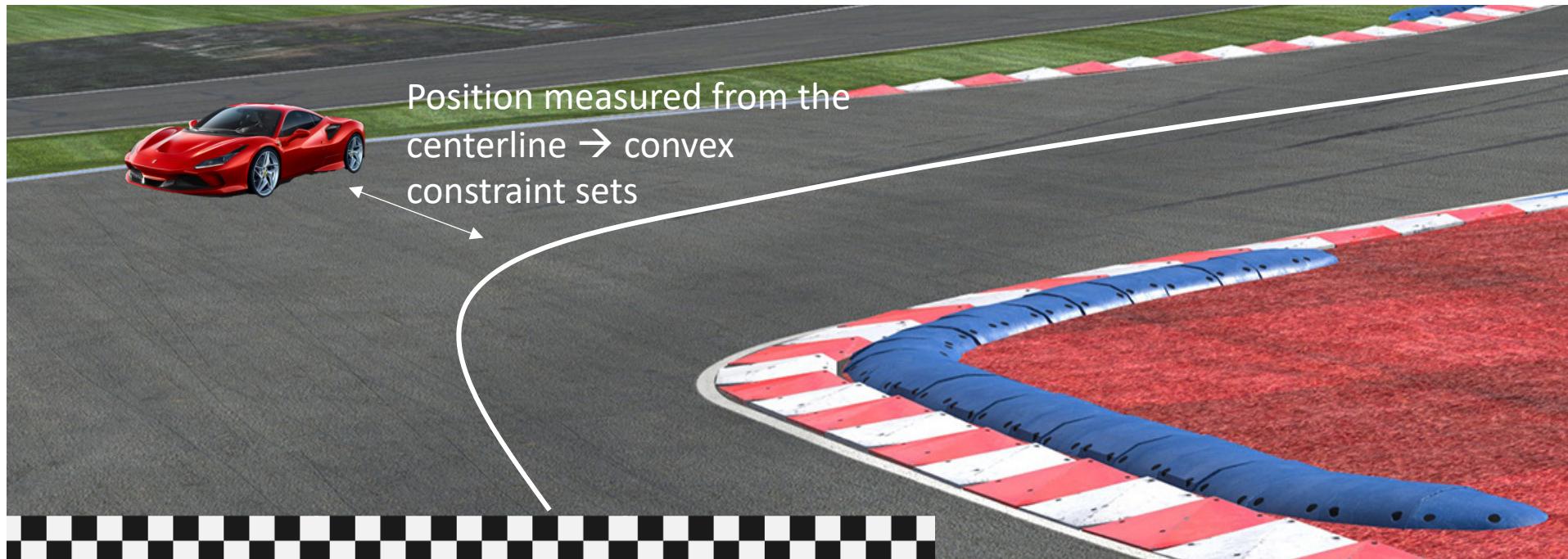
$$x_0 = x_s, \quad x_T = \mathcal{X}_F \quad \text{Start & end position}$$

System dynamics
System constraints

Safety constraints

$$x_{k+1} = f(x_k, u_k), \quad \forall k \in \{0, \dots, T-1\}$$

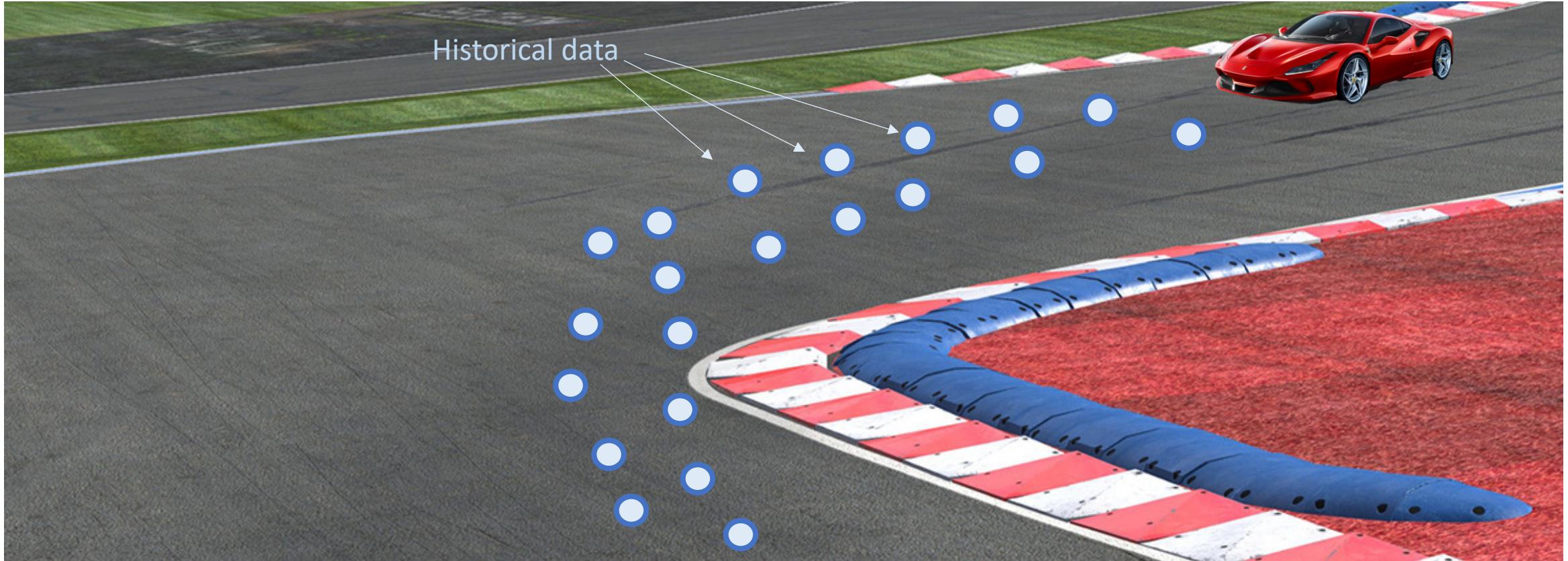
$$x_k \in \mathcal{X}, \quad u_k \in \mathcal{U}, \quad \forall k \in \{0, \dots, T-1\}$$



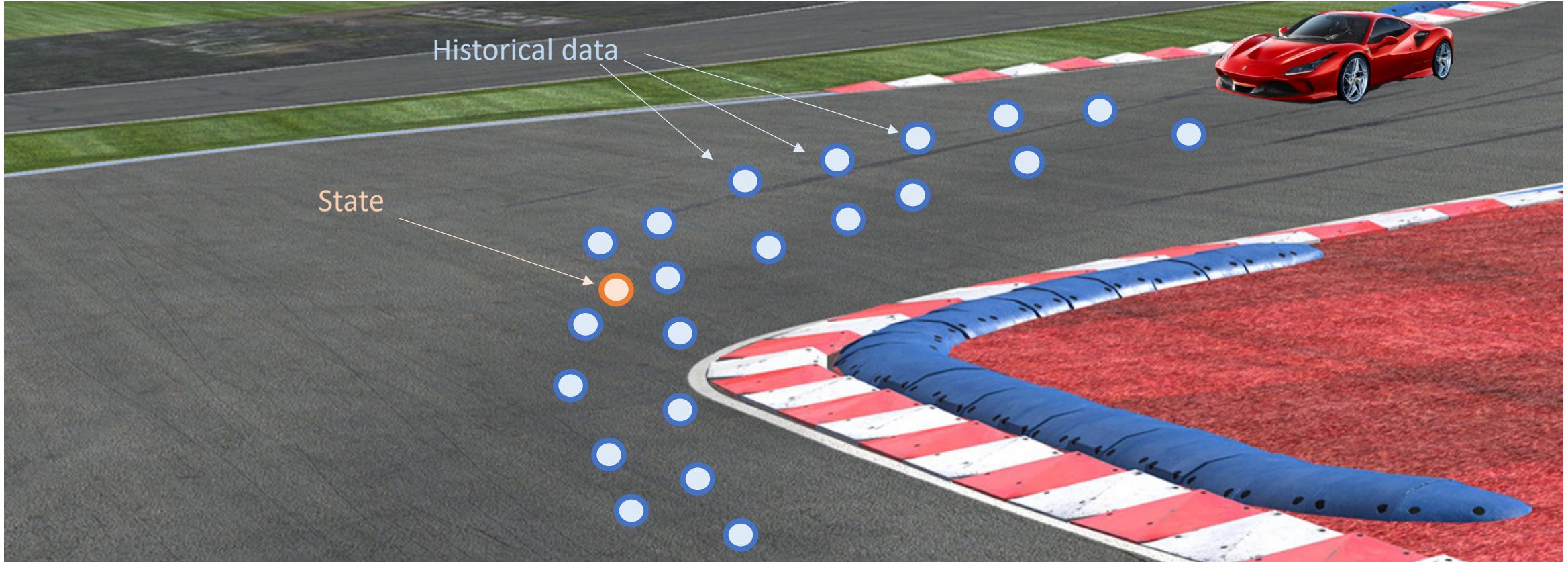
Local Approximations



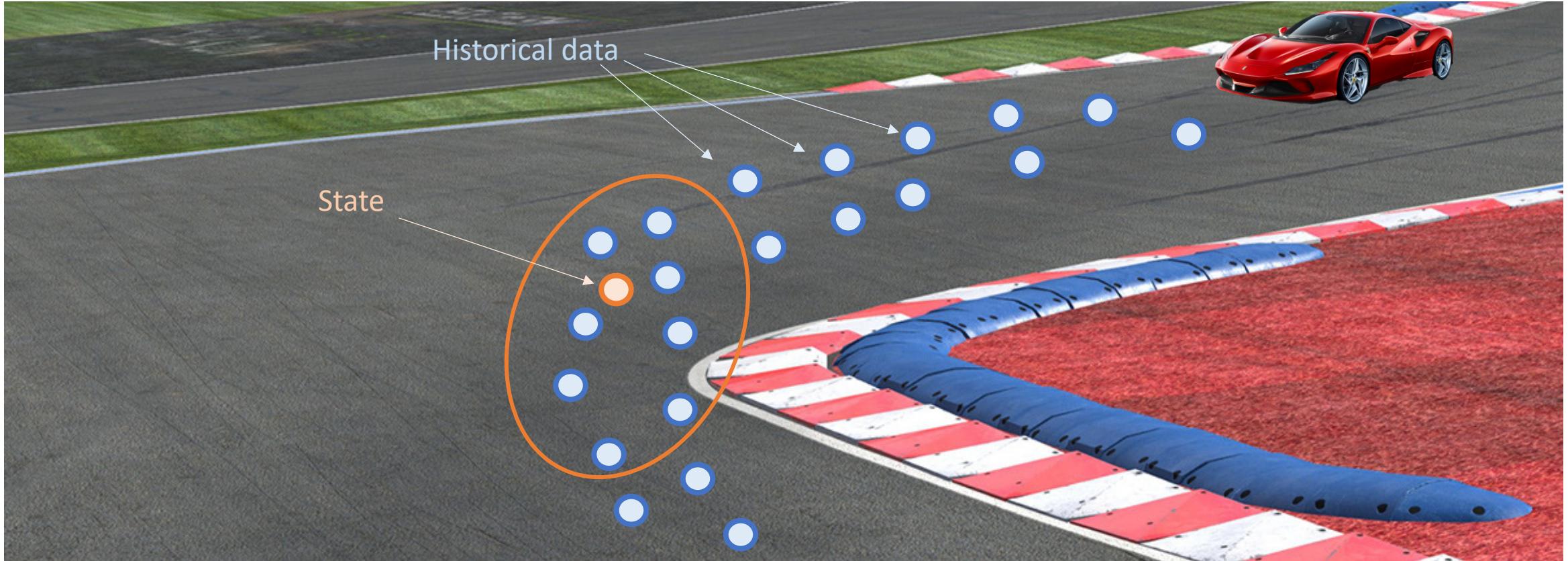
Local Approximations



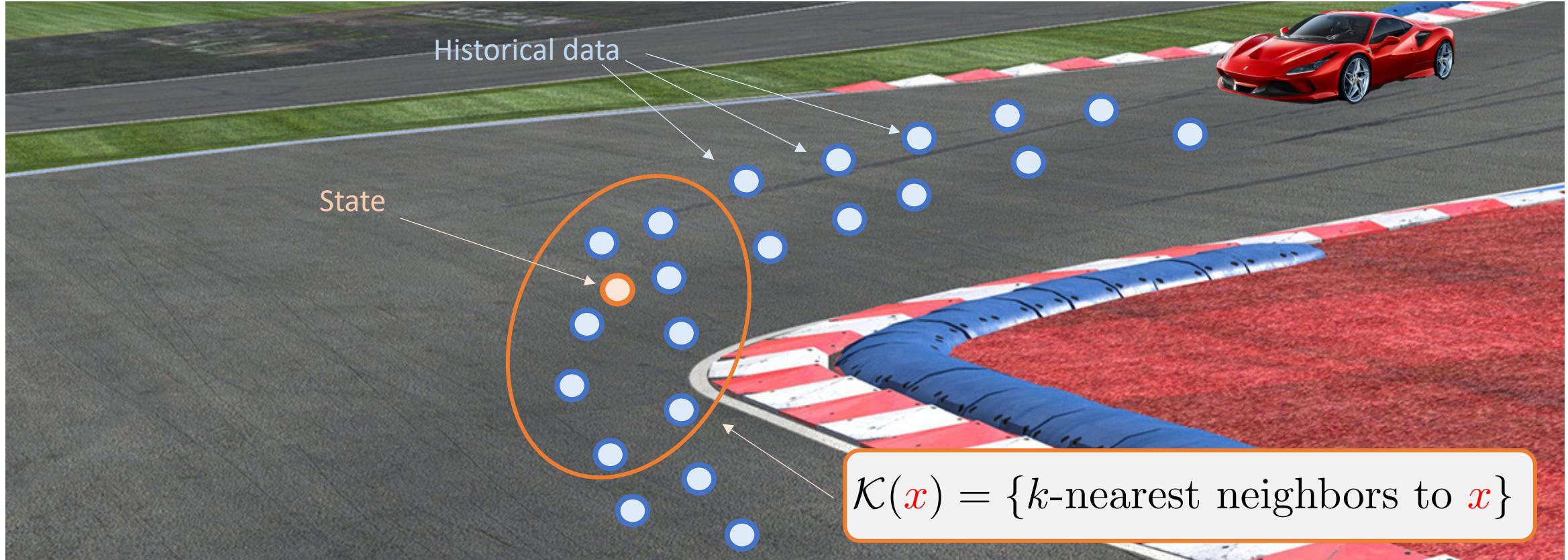
Local Approximations



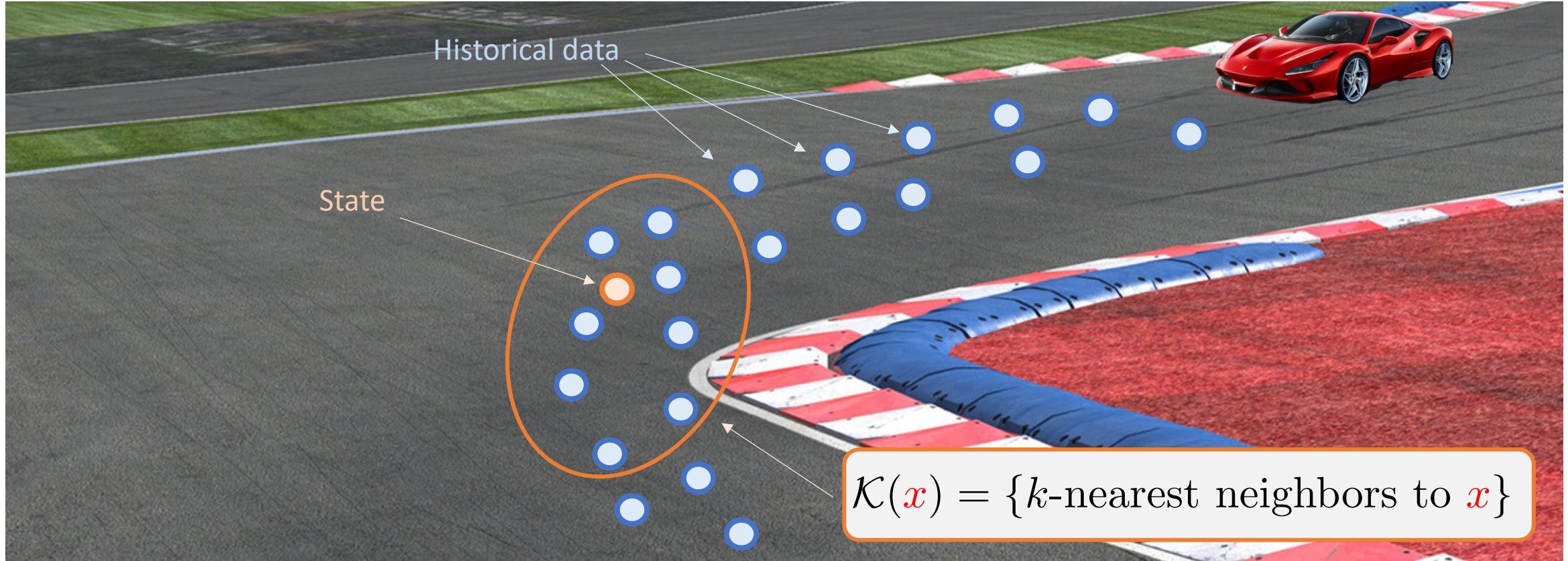
Local Approximations



Local Approximations



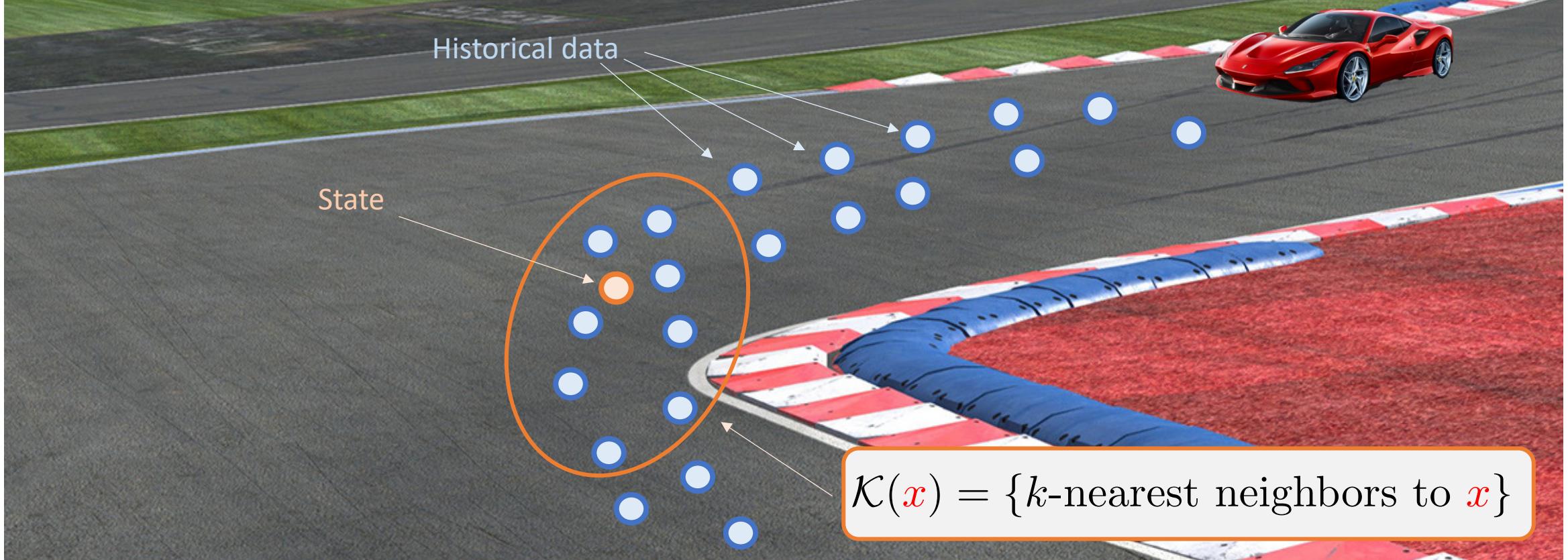
Local Approximations



Local convex safe set approximation:

$$\mathcal{CS}^j(\textcolor{red}{x}) = \text{conv} \left(\cup_{x_t^j \in \mathcal{K}(\textcolor{red}{x})} x_t^j \right)$$

Local Approximations



Local value function approximation:

$$V^j(\bar{x}, \textcolor{red}{x}) = \min_{\lambda_t^i \geq 0} \sum_{x_t^i \in \mathcal{K}^j(\textcolor{red}{x})} J_t^i(x_t^i) \lambda_t^i$$

subject to $\sum_{x_t^i \in \mathcal{K}^j(\textcolor{red}{x})} x_t^i \lambda_t^i = \bar{x}, \sum_i \sum_t \lambda_t^i = 1$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

$$J_{t \rightarrow t+N}^{\text{LMPC},j}(x_t^j) = \min_{u_{t|t}^j, \dots, u_{t+N-1|t}^j} \sum_{k=t}^{t+N-1} h(x_{k|t}^j, u_{k|t}^j) + V^{j-1}(x_{t+N|t}^j, x_{t+N}^+)$$

s.t.

$$x_{k+1|t}^j = A_{k|t}^j x_{k|t}^j + B_{k|t}^j u_{k|t}^j + C_{k|t}^j$$

$$x_{t|t}^j = x_t^j,$$

$$x_{k|t}^j \in \mathcal{X}, \quad u_{k|t}^j \in \mathcal{U}, \quad \forall k \in [t, \dots, t+N-1]$$

$$x_{t+N|t}^j \in \mathcal{CS}^{j-1}(x_{t+N}^+),$$

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$$x_{t+N|t}^j \in \mathcal{CS}^{j-1}(x_{t+N}^+),$$

Computed online
using LMPC strategy

where $x_{t+N}^+ = x_{t+N-1|t-1}^*$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

$$J_{t \rightarrow t+N}^{\text{LMPC},j}(x_t^j) = \min_{u_{t|t}^j, \dots, u_{t+N-1|t}^j} \sum_{k=t}^{t+N-1} h(x_{k|t}^j, u_{k|t}^j) + V^{j-1}(x_{t+N|t}^j, x_{t+N}^+)$$

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$$x_{t+N|t}^j \in \mathcal{CS}^{j-1}(x_{t+N}^+),$$

Computed online
using LMPC strategy

where $x_{t+N}^+ = x_{t+N-1|t-1}^*$ or $x_{t+N}^+ = \hat{f}(x_{t+N-1|t-1}^*, u_f)$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

$$J_{t \rightarrow t+N}^{\text{LMPC},j}(x_t^j) = \min_{u_{t|t}^j, \dots, u_{t+N-1|t}^j} \sum_{k=t}^{t+N-1} h(x_{k|t}^j, u_{k|t}^j) + V^{j-1}(x_{t+N|t}^j, x_{t+N}^+)$$

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online

Computed online
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Learning Model Predictive Controller

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$$J_{t \rightarrow t+N}^{\text{LMPC},j}(x_t^j) = \min_{u_{t|t}^j, \dots, u_{t+N-1|t}^j} \sum_{k=t}^{t+N-1} h(x_{k|t}^j, u_{k|t}^j) + V^{j-1}(x_{t+N|t}^j, x_{t+N}^+)$$

s.t.

$$x_{k+1|t}^j = A_{k|t}^j x_{k|t}^j + B_{k|t}^j u_{k|t}^j + C_{k|t}^j$$

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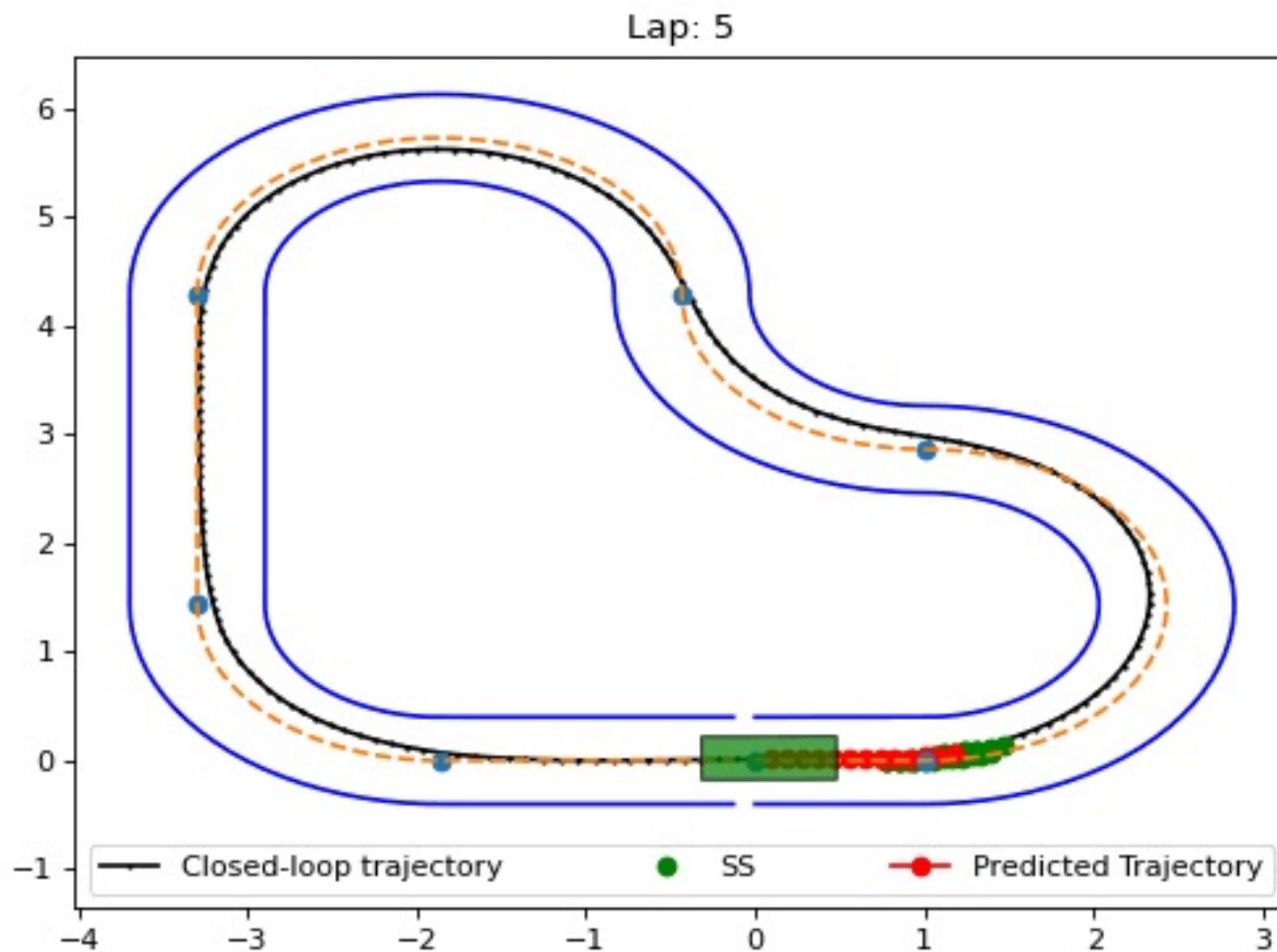
$$x_{t+N|t}^j \in \mathcal{CS}^{j-1}(x_{t+N}^+),$$

Computed
online

Computed online
using LMPC strategy

Then apply to the system the control input $u_t^j = u_{t|t}^{*,j}$

Safe Set and Value Function Approximations



System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$\begin{aligned}\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m} \sum_i F_{x_i} \\ \ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m} \sum_i F_{y_i} \\ \ddot{\psi} &= \frac{1}{I_z} (a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}})) \\ \dot{X} &= \dot{x} \cos \psi - \dot{y} \sin \psi, \quad \dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi\end{aligned}$$

System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$\ddot{x} = \dot{y}\dot{\psi} + \frac{1}{m} \sum_i F_{x_i}$$

$$\ddot{y} = -\dot{x}\dot{\psi} + \frac{1}{m} \sum_i F_{y_i}$$

$$\ddot{\psi} = \frac{1}{I_z} (a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}}))$$

$$\dot{X} = \dot{x} \cos \psi - \dot{y} \sin \psi, \quad \dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi$$

Kinematic Equations

System ID in Autonomous Racing

- ▶ Nonlinear Dynamical System,

$$\begin{aligned}\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m} \sum_i F_{x_i} \\ \ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m} \sum_i F_{y_i} \\ \ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}}) \\ \dot{X} &= \dot{x} \cos \psi - \dot{y} \sin \psi, \quad \dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi\end{aligned}$$

Kinematic Equations

- ▶ Identifying the Dynamical System

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} x_{k|t}^j + \begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

System ID in Autonomous Racing

- ▶ Nonlinear Dynamical System,

$$\begin{aligned}\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m} \sum_i F_{x_i} \\ \ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m} \sum_i F_{y_i} \\ \ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}})) \\ \dot{X} &= \dot{x} \cos \psi - \dot{y} \sin \psi, \quad \dot{Y} = \dot{x} \sin \psi + \dot{y} \cos \psi\end{aligned}$$

Dynamic Equations
Kinematic Equations

- ▶ Identifying the Dynamical System

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} x_{k|t}^j + \begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

System ID in Autonomous Racing

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Dynamic Equations
Kinematic Equations

- ▶ Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \boxed{\arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||}, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \\ \boxed{\text{Linearized Kinematics}} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- ▶ Compute trajectory to linearize around from previous optimal inputs and stored data

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- ▶ Compute trajectory to linearize around from previous optimal inputs and stored data
- ▶ Enforce model-based sparsity in local linear regression

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- ▶ Compute trajectory to linearize around from previous optimal inputs and stored data
- ▶ Enforce model-based sparsity in local linear regression
- ▶ Perform local linear regression at the linearization points using a subset of the stored data

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x_{k+1|t}^j = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \arg \min \sum_{i,s} K(x_{k|t}^j - x_s^i) || \Lambda_y \begin{bmatrix} x_{k|t}^j \\ u_{k|t}^j \\ 1 \end{bmatrix} - y_{s+1}^i ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\} \\ \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] x_{k|t}^j + \left[\begin{array}{c} \hline \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{array} \right] \begin{bmatrix} u_{k|t}^j \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

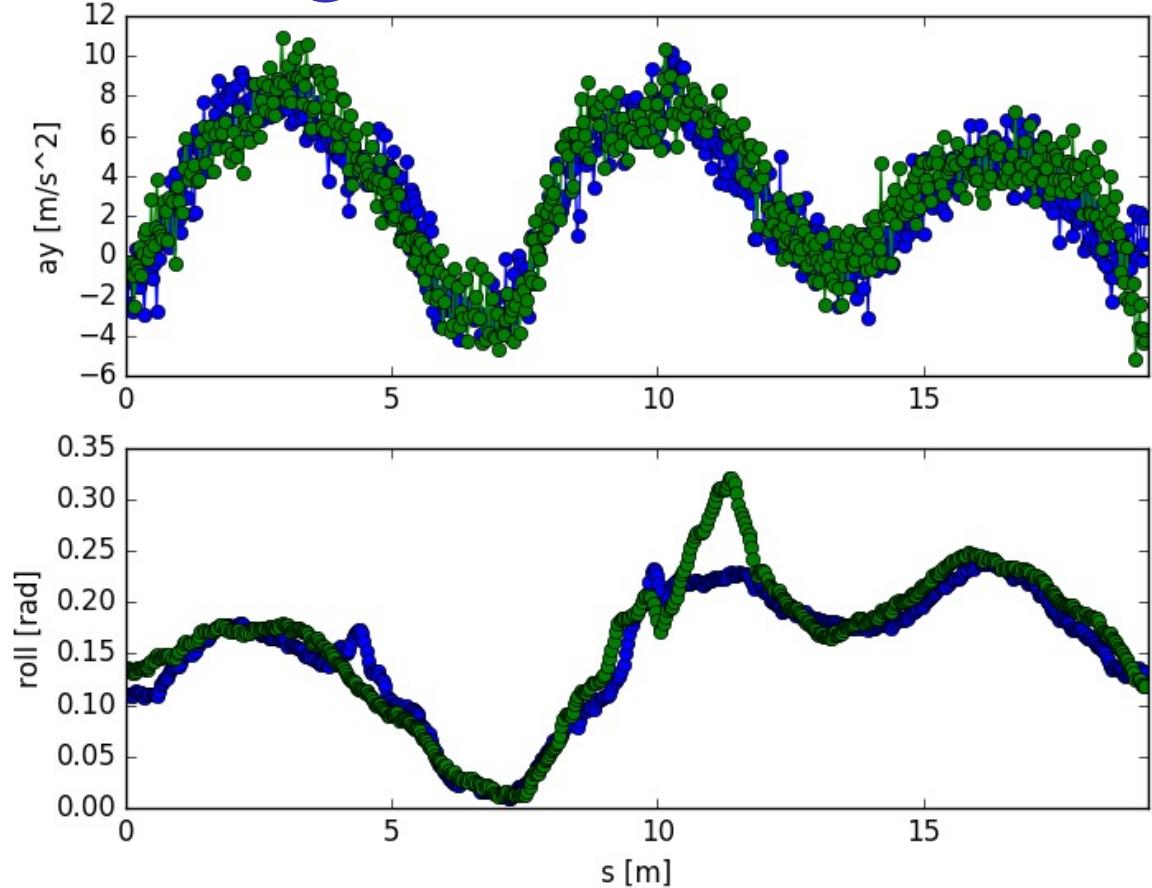
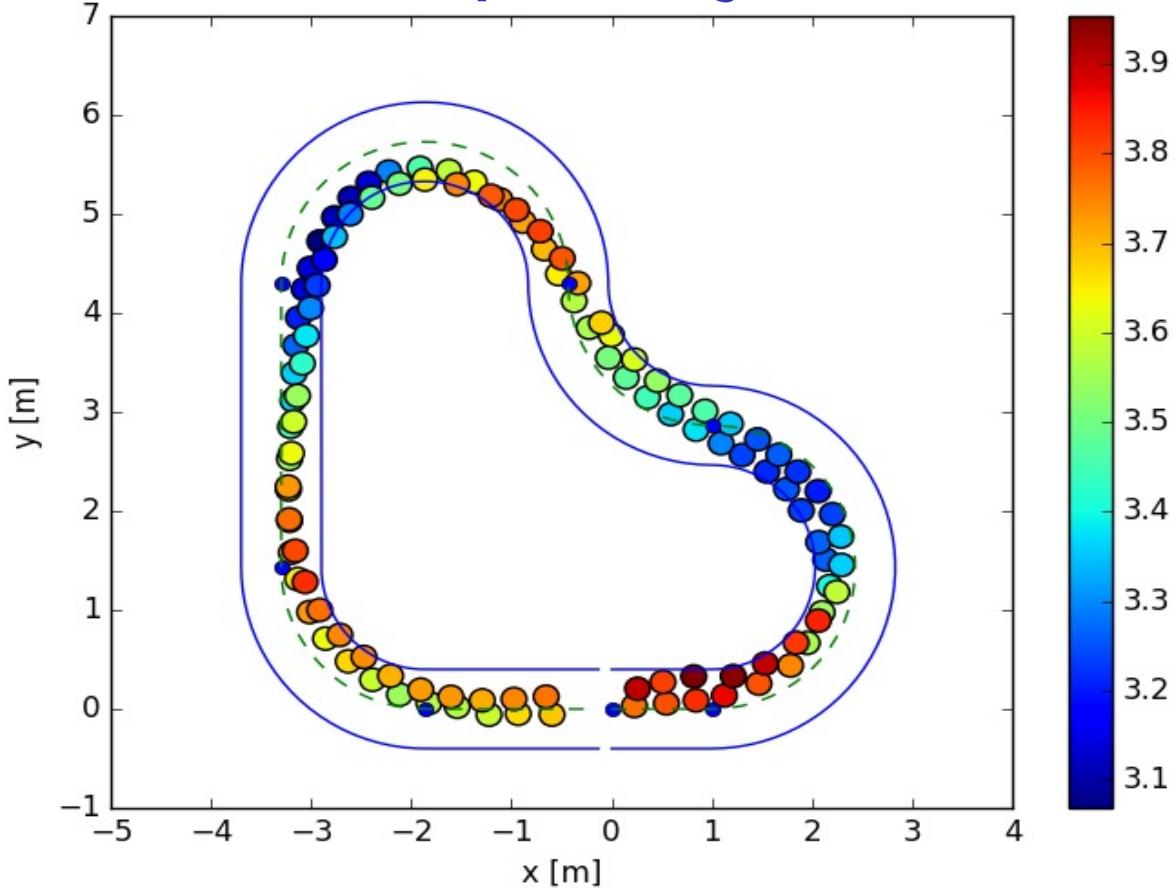
Implementation Details

- ▶ Compute trajectory to linearize around from previous optimal inputs and stored data
- ▶ Enforce model-based sparsity in local linear regression
- ▶ Perform local linear regression at the linearization points using a subset of the stored data
- ▶ Use kernel $K()$ to weight differently data as a function of distance to the linearization trajectory



Learning Model Predictive Control for Autonomous Racing

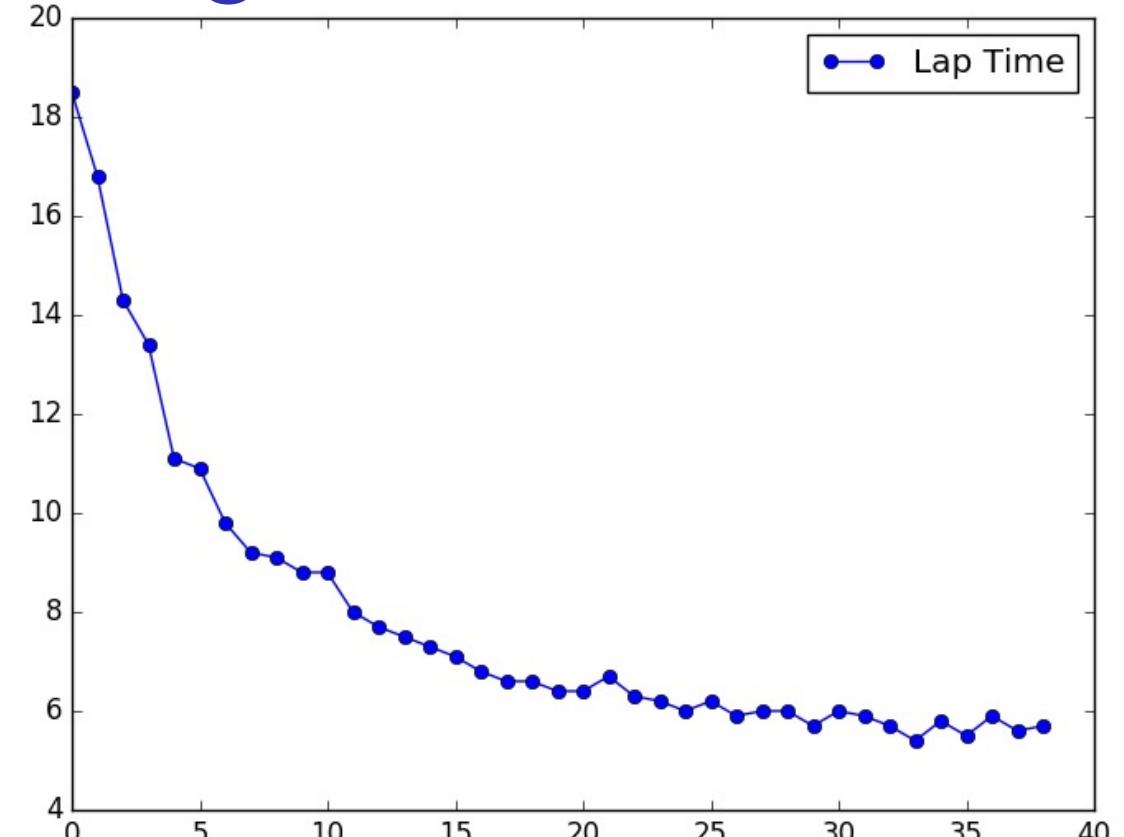
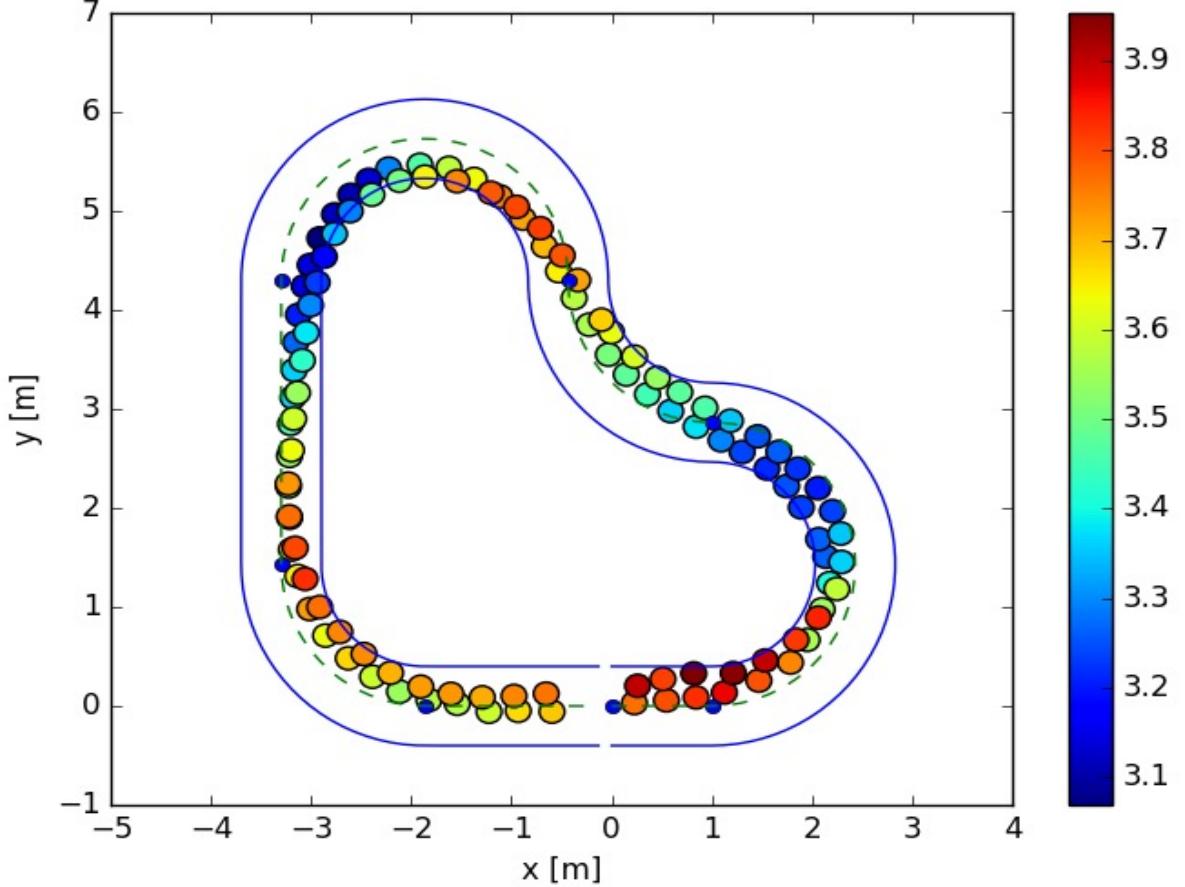
Closed-loop Trajectories at Convergence



Remarks

- ▶ A **reference trajectory is not needed** for the controller implementation
- ▶ The controller converges after few laps: the learning process is data efficient
- ▶ The **controller safely explores the state space** iteratively improving the lap time

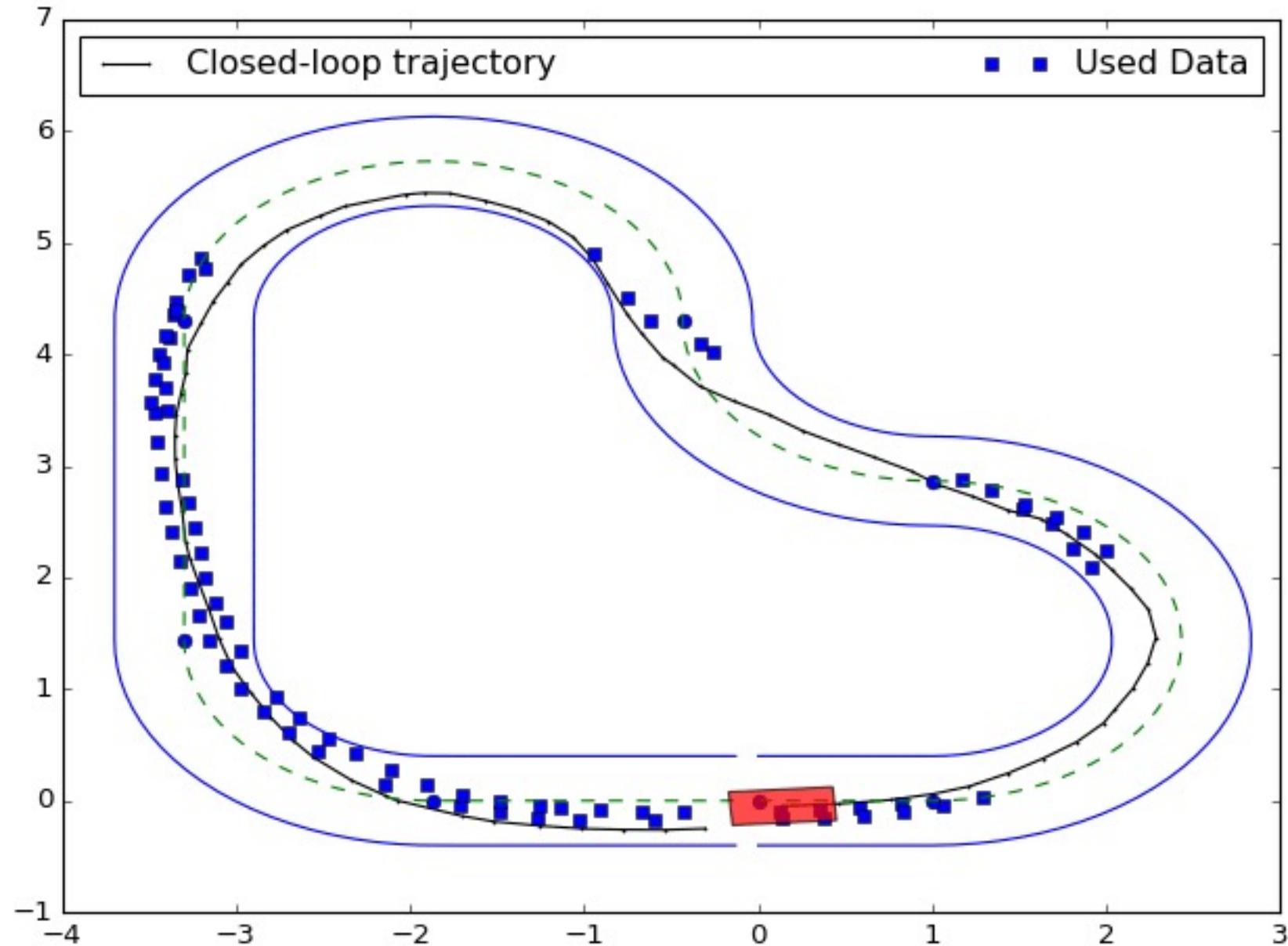
Closed-loop Trajectories at Convergence



Remarks

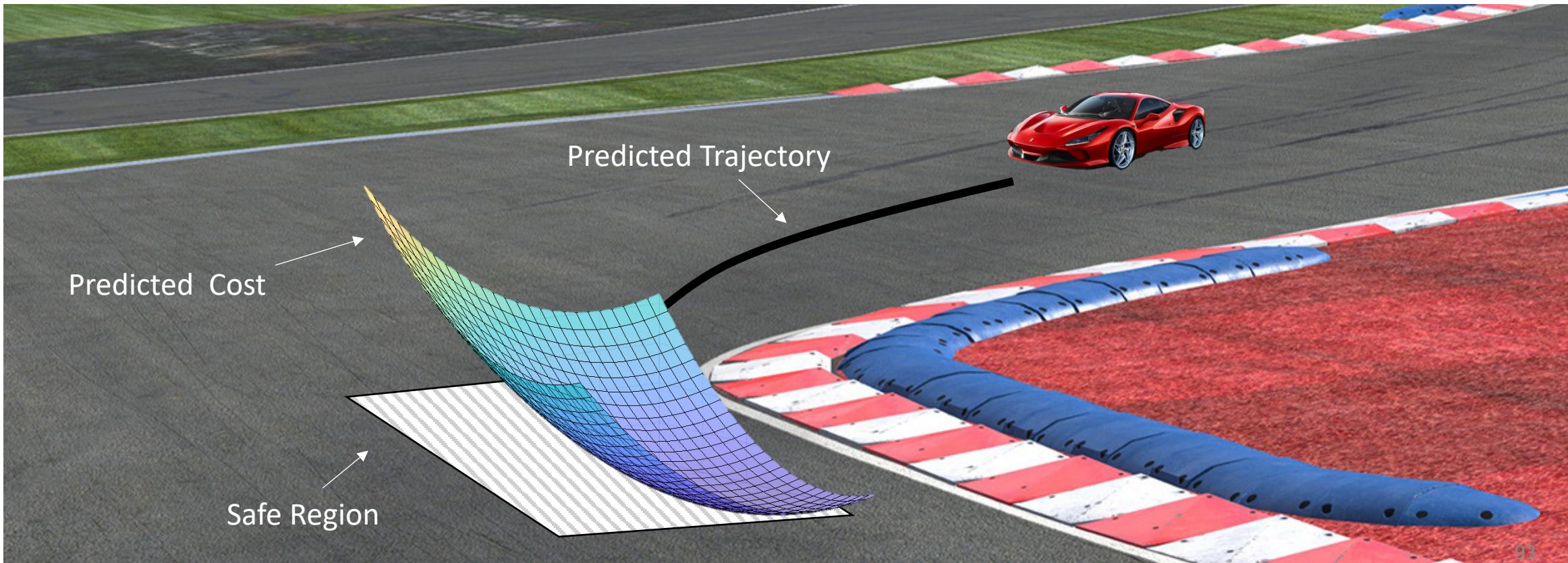
- ▶ A **reference trajectory is not needed** for the controller implementation
- ▶ The controller converges after few laps: the learning process is data efficient
- ▶ The **controller safely explores the state space** iteratively improving the lap time

Data Point Selection



The key components

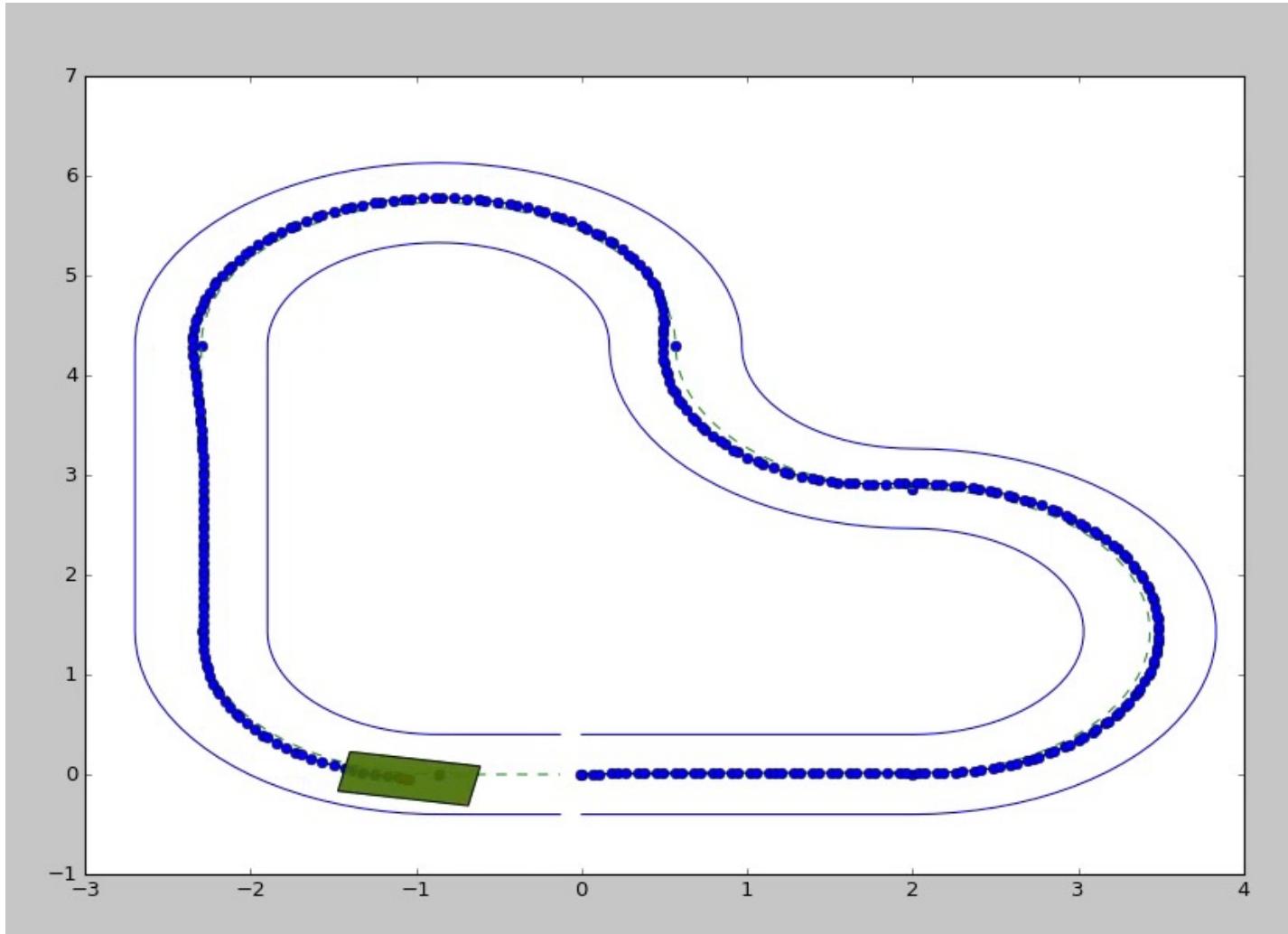
- ▶ Predicted trajectory given by prediction model
- ▶ Predicted cost estimated by value function approximation
- ▶ Safe region estimated by the safe set



Do you need the safe set? – Yes

LMPC without Invariant Set

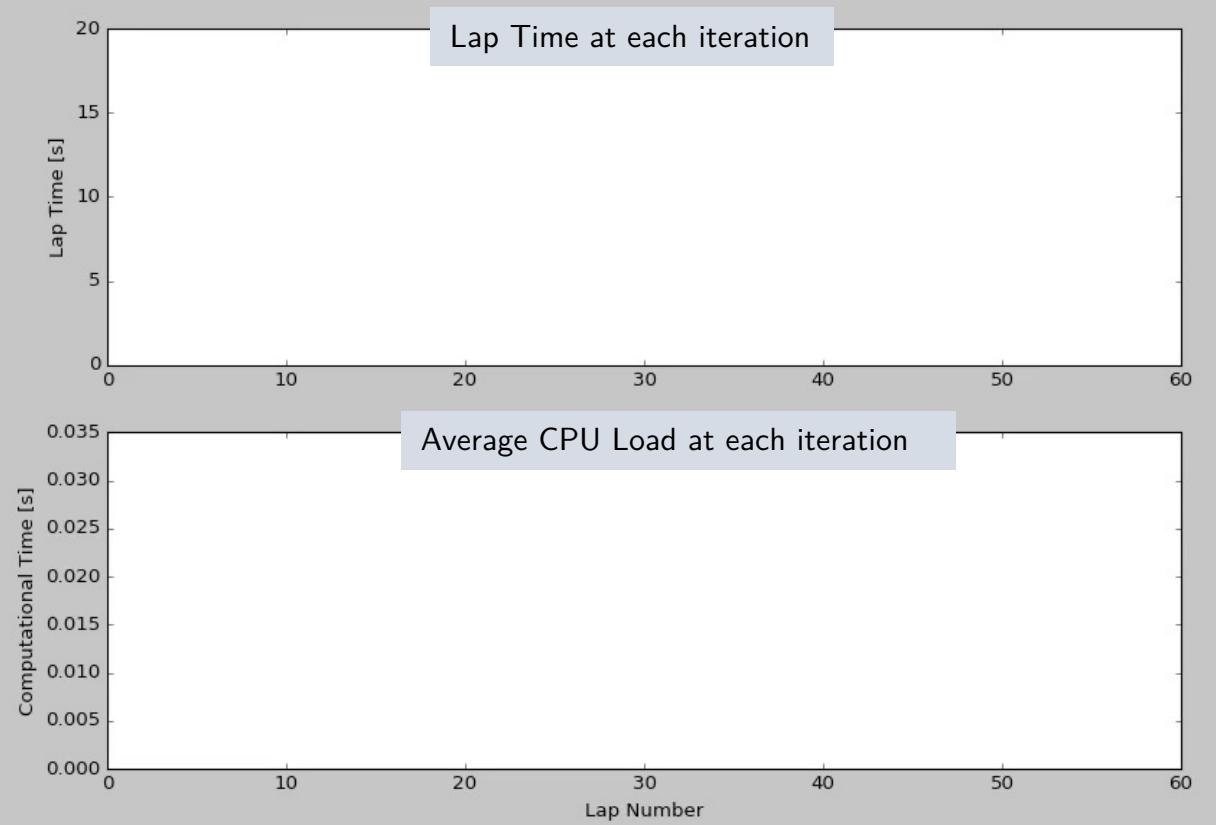
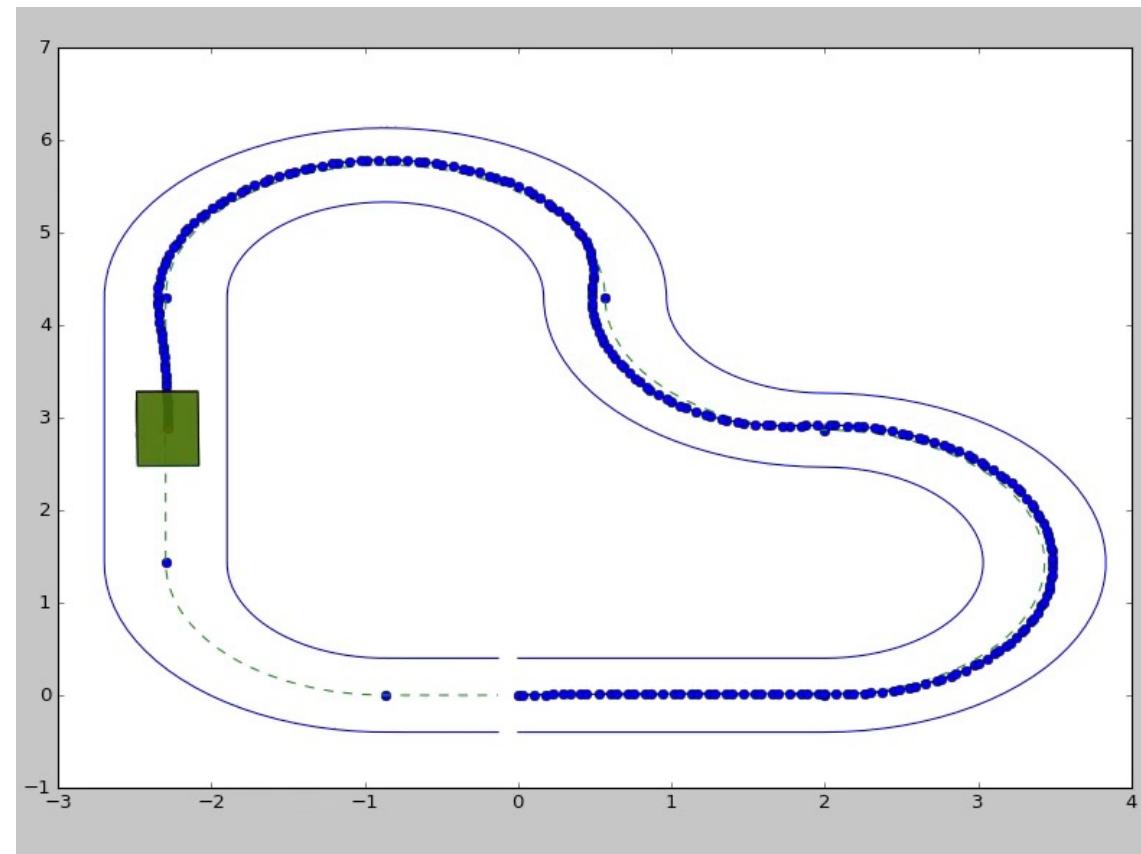
The controller extrapolates the value function on the Vx dimension



Do you need to Predict to Learn? Yes

When the LMPC horizon is $N = 1$ the controller

- ▶ solves the Bellman equation using the value function approximation
- ▶ does not explore the state space as it cannot plan outside the safe set



Comparison with Approximate DP (aka RL)

- ▶ Some references:
 - ❖ Bertsekas paper connecting MPC and ADP [1], books on RL and OC [2,3]
 - ❖ Lewis and Vrabie survey [4]
 - ❖ Recht survey [5]

- ▶ LMPC highlights
 - ❖ Continuous state and action formulation
 - ❖ Constraints satisfaction and Sampled Safe Sets
 - ❖ Terminal constructed locally based on cost/model driven exploration
 - ❖ Terminal at stored state is “exact” and upperbounds at intermediate points

[1] D. Bertsekas, “Dynamic programming and suboptimal control: A survey from ADP to MPC.” European Journal of Control 11.4-5 (2005)

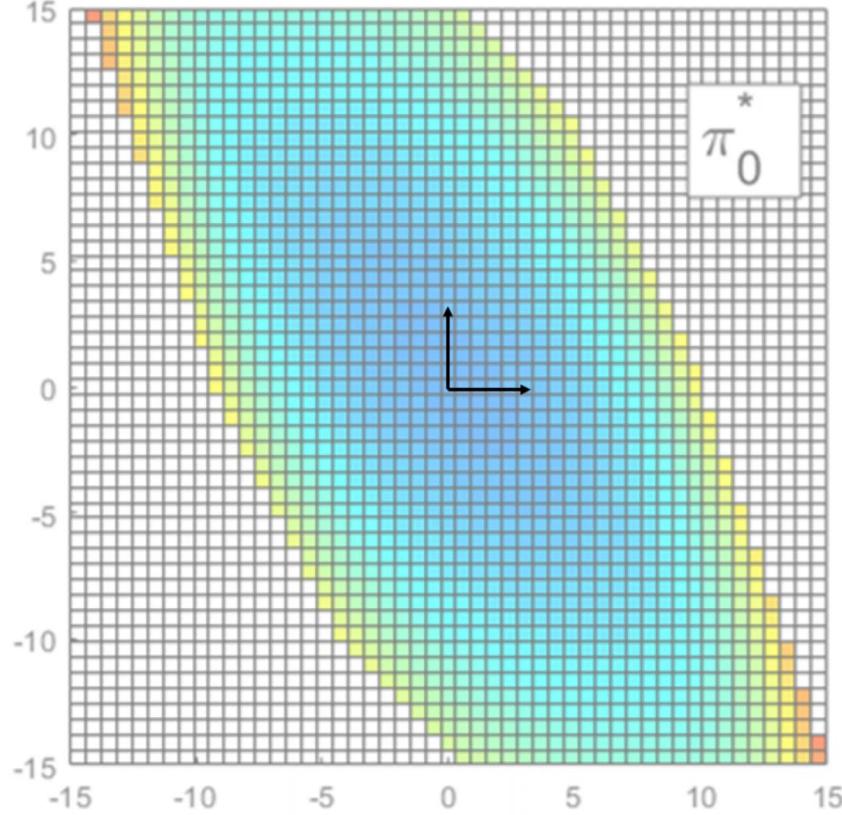
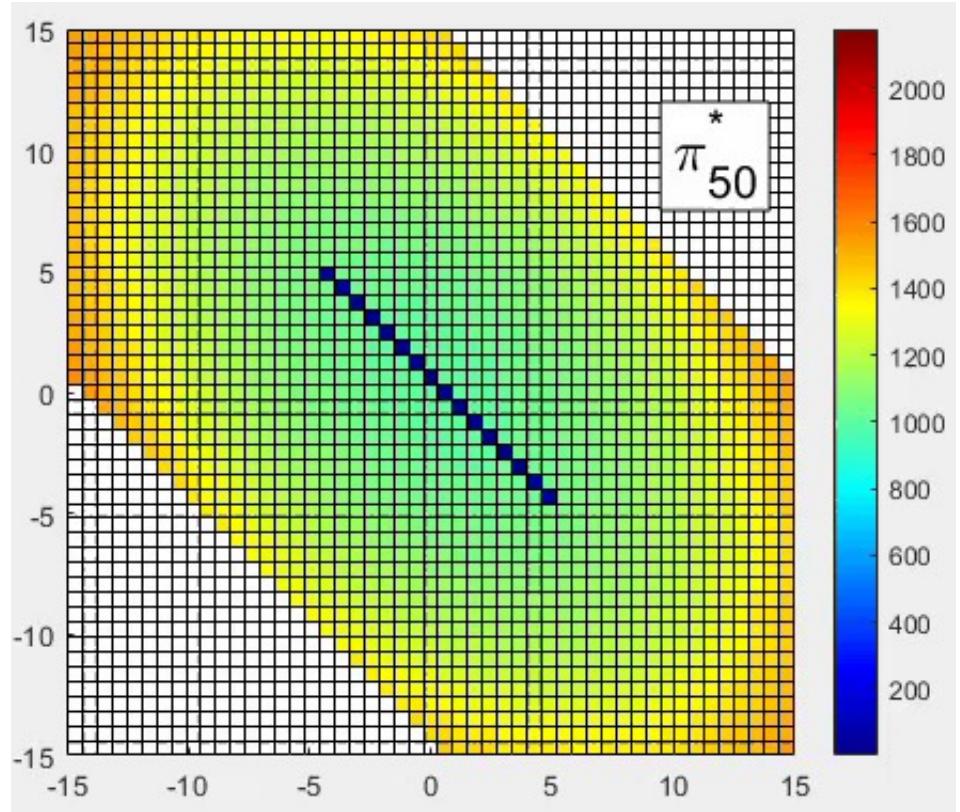
[2] D. Bertsekas, “Reinforcement learning and optimal control.” Athena Scientific, 2019.

[3] D. Bertsekas, “Distributed Reinforcement Learning” http://web.mit.edu/dimitrib/www/RL_2_Rollout_&_PI.pdf

[4] F. Lewis, Frank, and D. Vrabie. "Reinforcement learning and adaptive dynamic programming for feedback control." IEEE circuits and systems magazine 9.3 (2009)

[5] R. Benjamin. "A tour of reinforcement learning: The view from continuous control." Annual Review of Control, Robotics, and Autonomous Systems 2 (2019)

Forward Value Iteration



Dynamic Programming:

- ▶ Gridding, global properties
- ▶ Backward, one-step iteration

LMPC:

- ▶ No Gridding, local properties
- ▶ Forward, multi-step prediction
- ▶ LICQ required for optimality

Model Learning in MPC

A short and non-comprehensive summary

The complexity of the prediction model

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Linear

$$x_{k+1} = Ax_k + Bu_k + w_k$$

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A screenshot of a Google Scholar search results page. The search query 'learning LQR' is entered in the search bar. The results section shows two articles:

- Trajectory tracking using online learning LQR with adaptive learning control of a leg-exoskeleton for disorder gait rehabilitation**
N Ajanaromvat, M Pamichkun - Mechatronics, 2018 - Elsevier
Precise trajectory tracking of gait pattern under varied load condition is necessary for rehabilitation using leg exoskeleton. In this paper, online iterative learning linear quadratic regulator (OILLR) with adaptive iterative learning control is proposed to control trajectory ...
☆ 99 Cited by 17 Related articles All 2 versions
- Learning robust control for LQR systems with multiplicative noise via policy gradient**
B Gravell, PM Esfahani, T Summers - arXiv preprint arXiv:1905.13547, 2019 - arxiv.org
The linear quadratic regulator (LQR) problem has reemerged as an important theoretical benchmark for reinforcement learning-based control of complex dynamical systems with continuous state and action spaces. In contrast with nearly all recent work in this area, we ...
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Filtering options on the left include: Any time, Since 2021, Since 2020, Since 2017, Custom range..., Date range (2018), Sort by relevance, Sort by date, and include patents.

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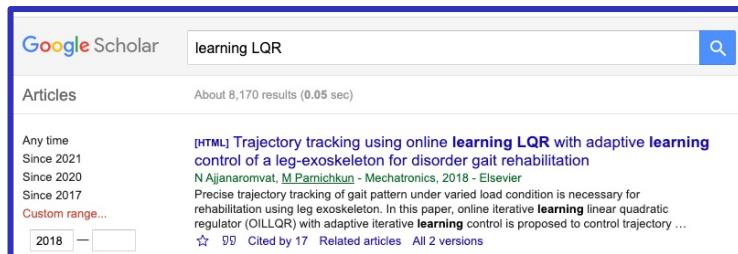
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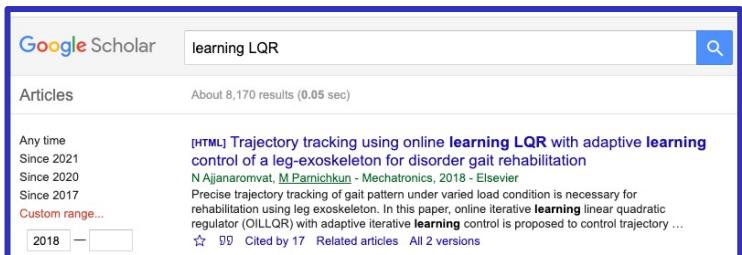
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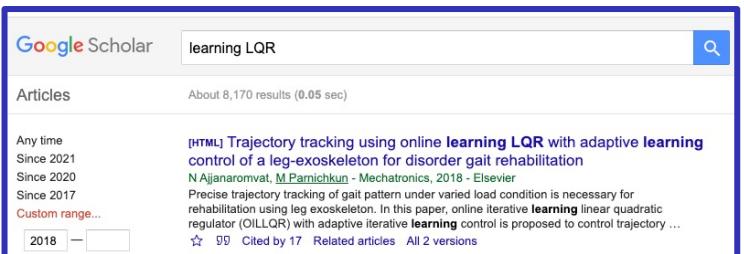
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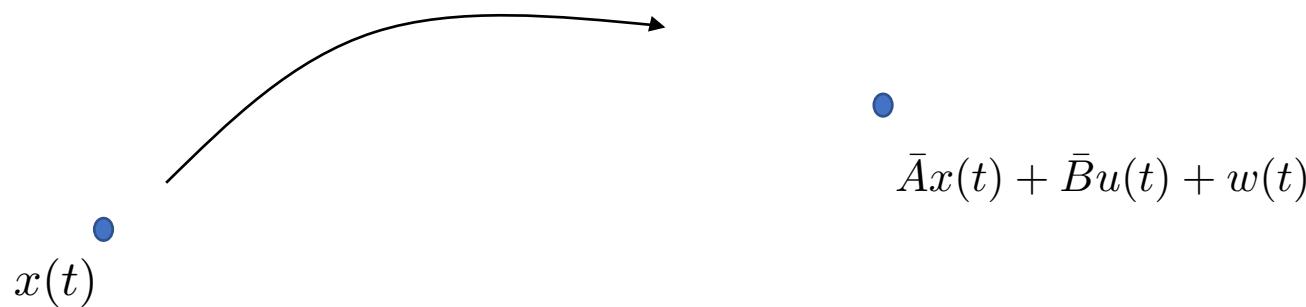
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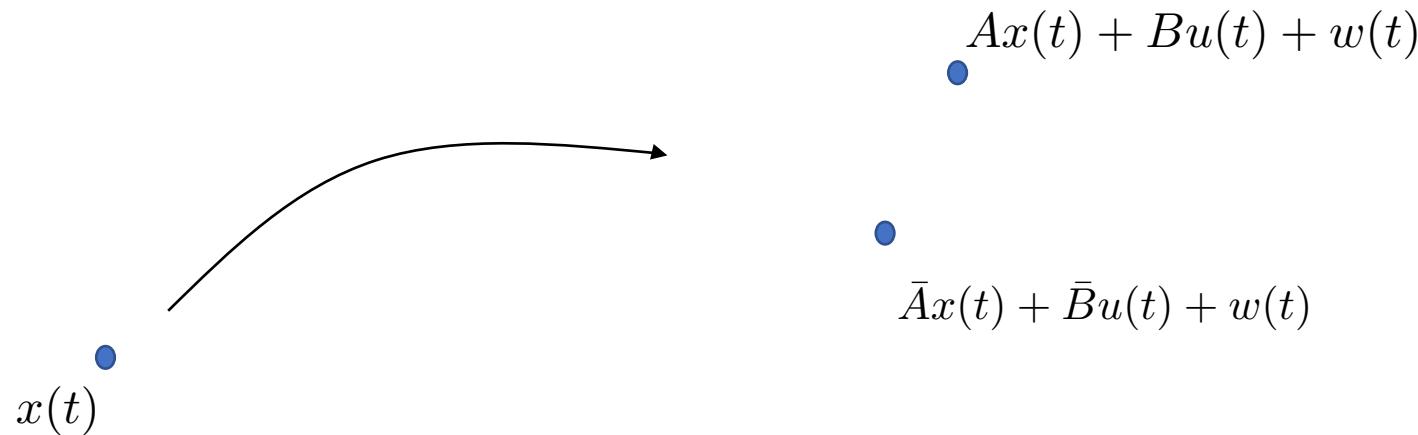


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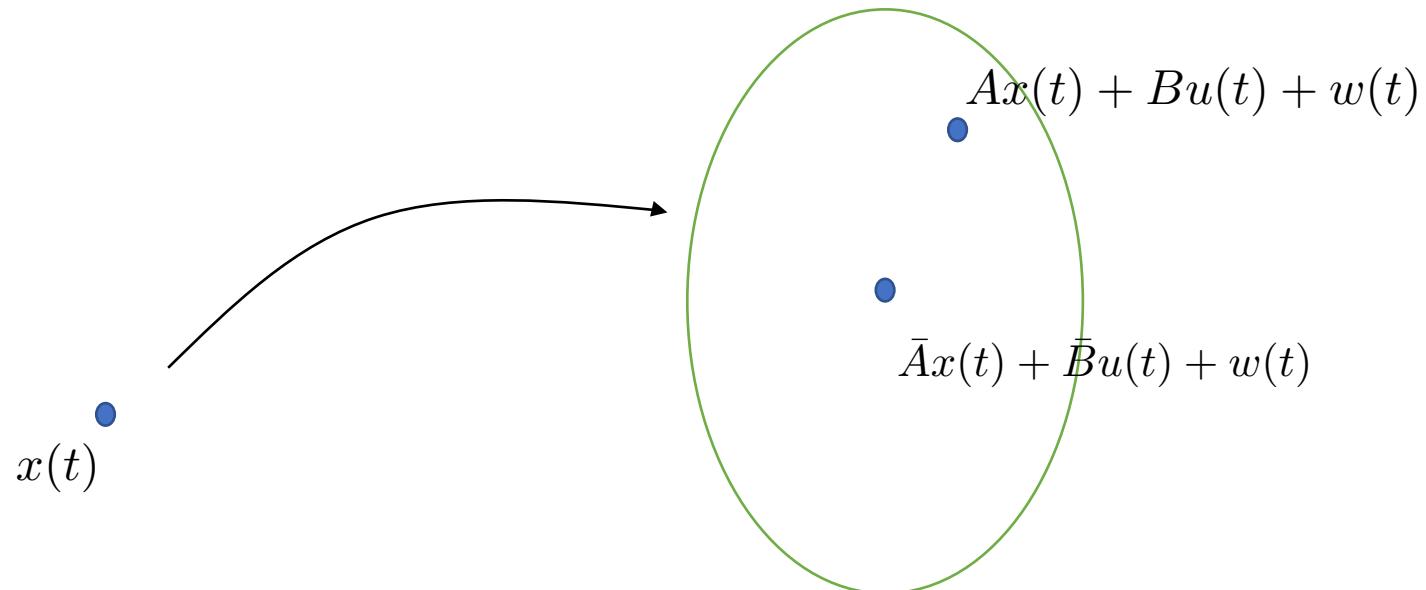


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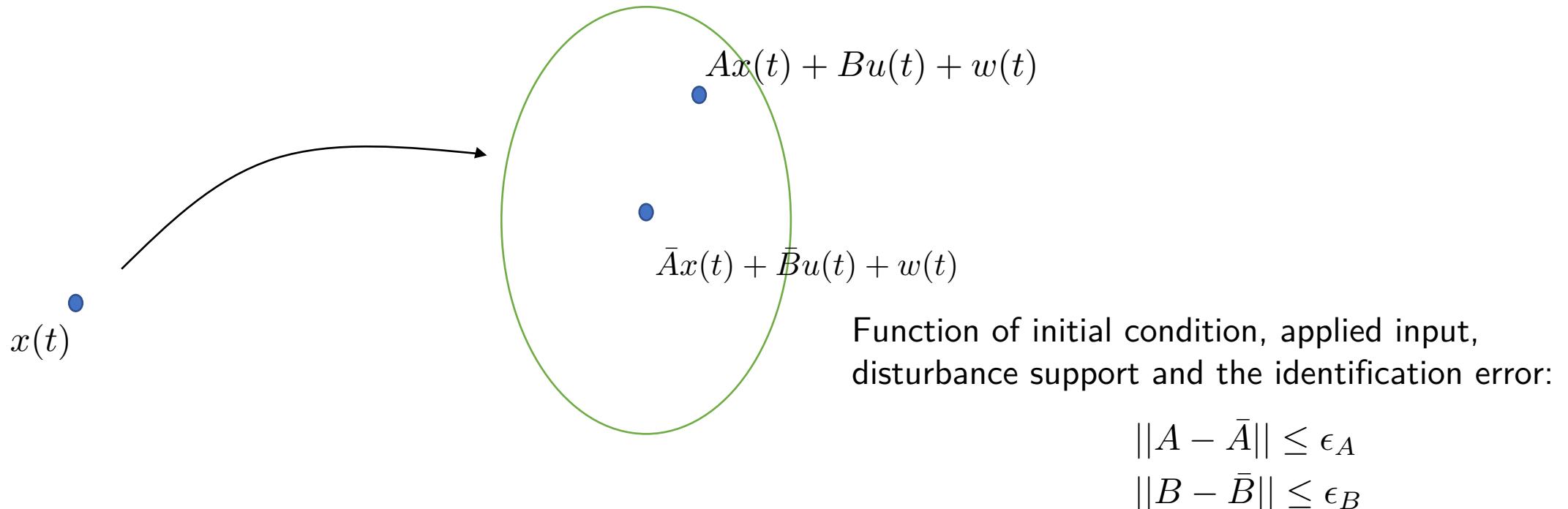


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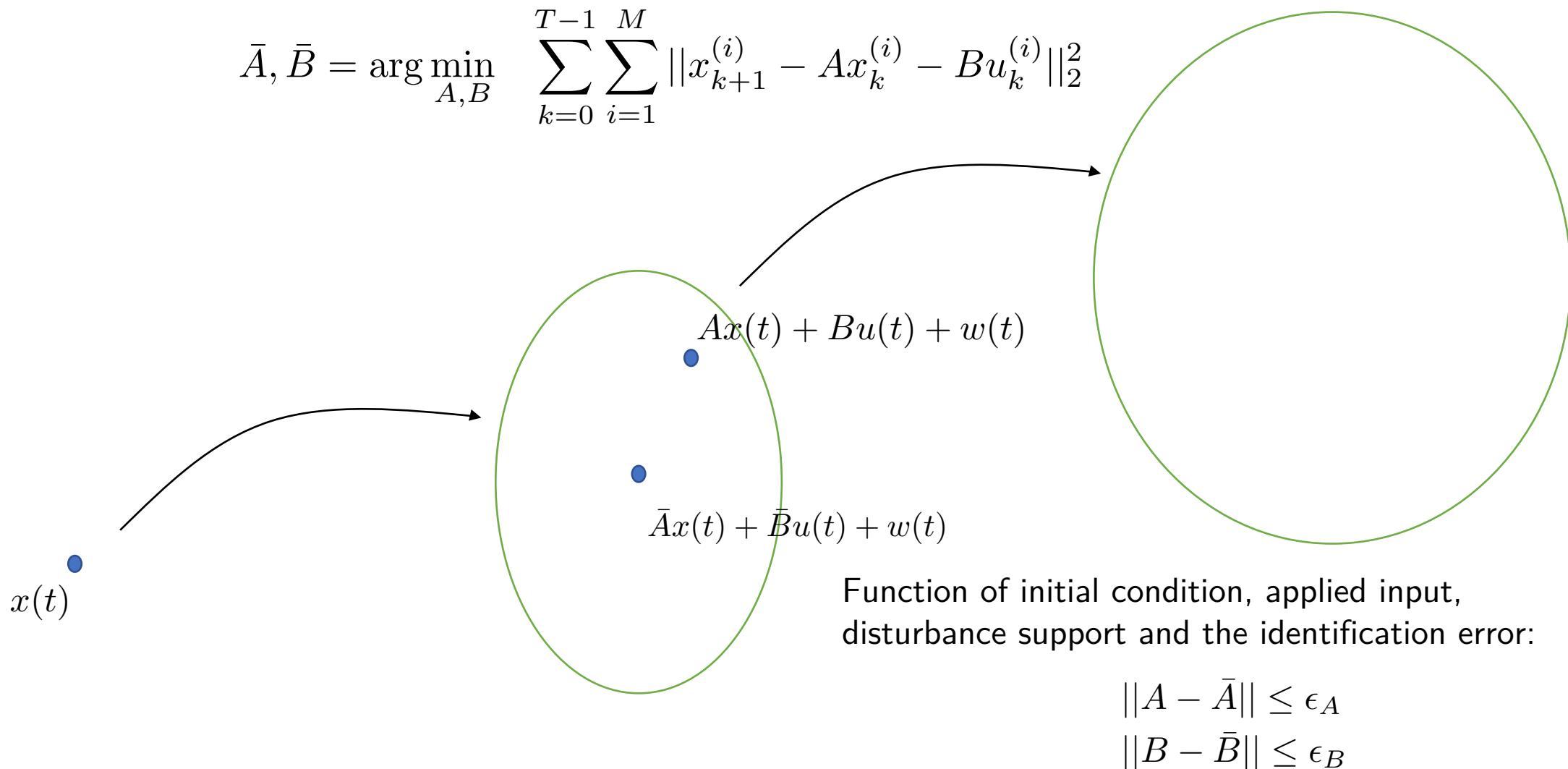
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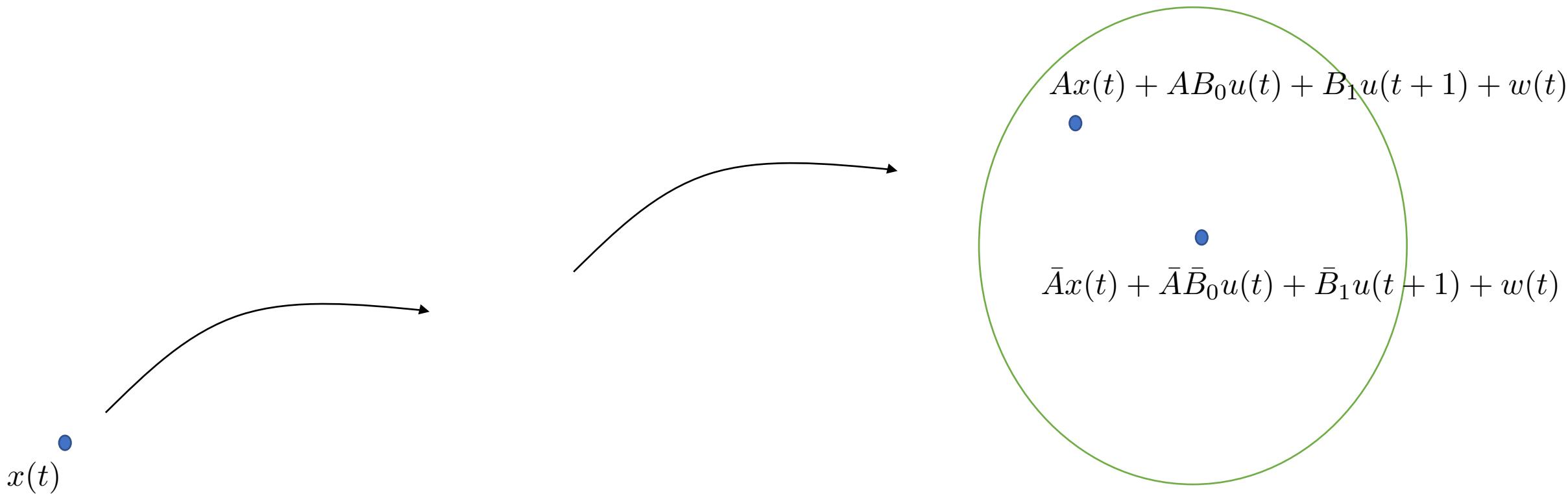
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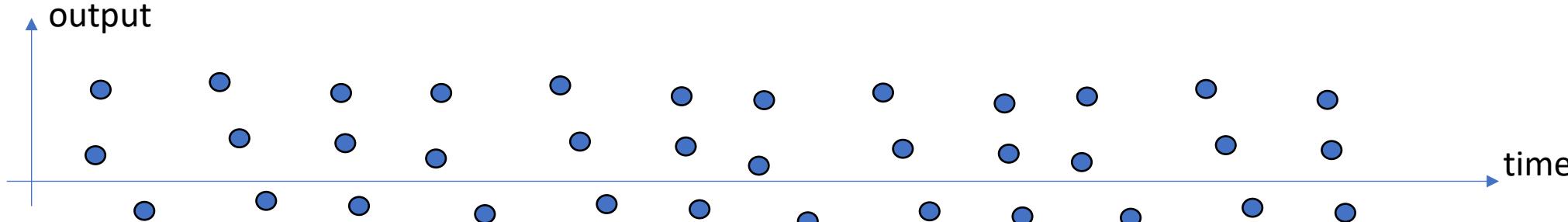
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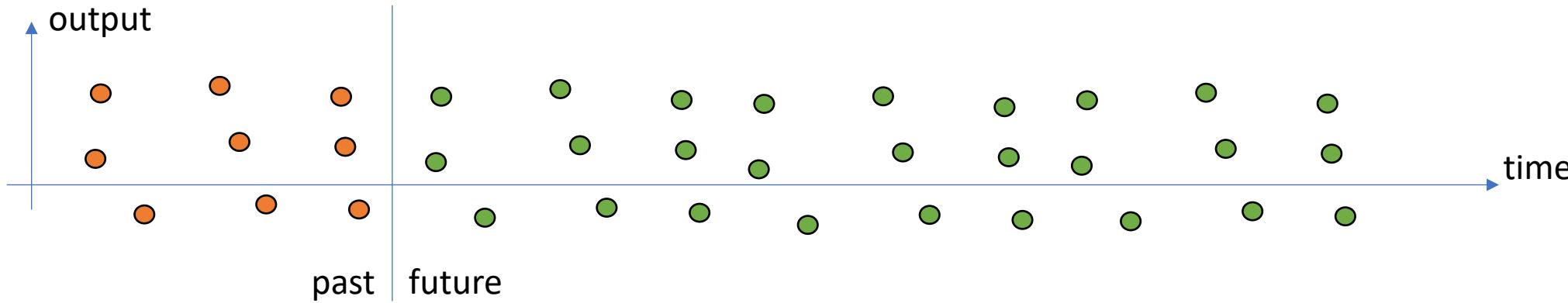


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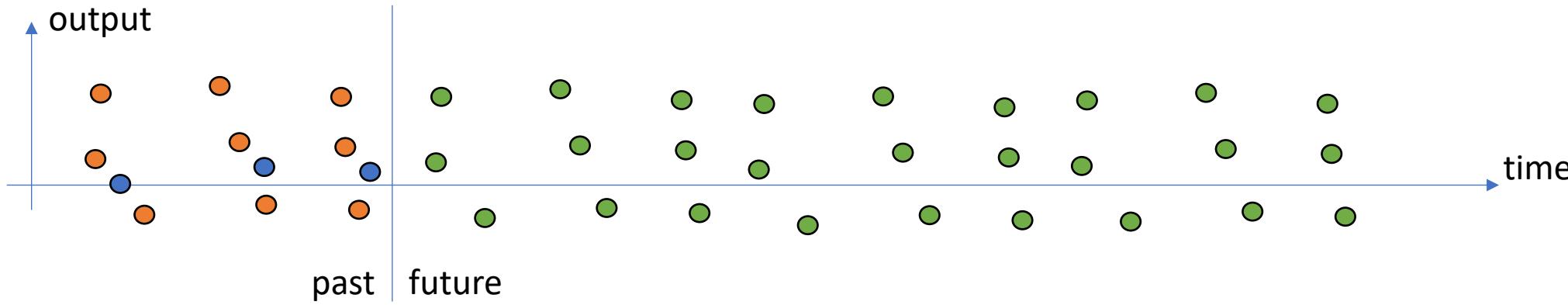
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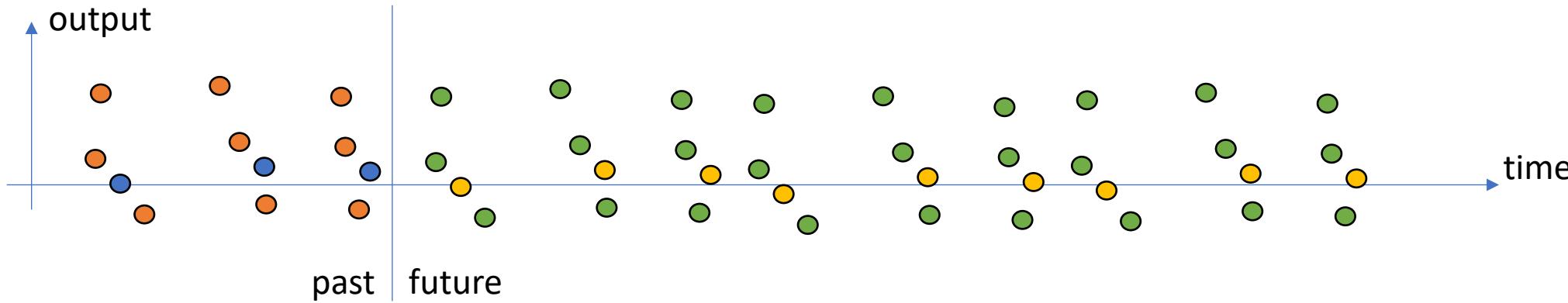
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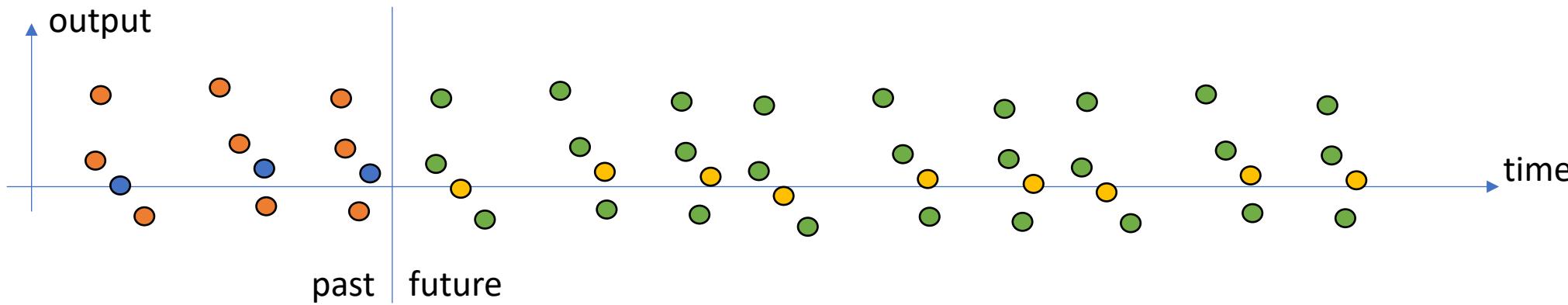
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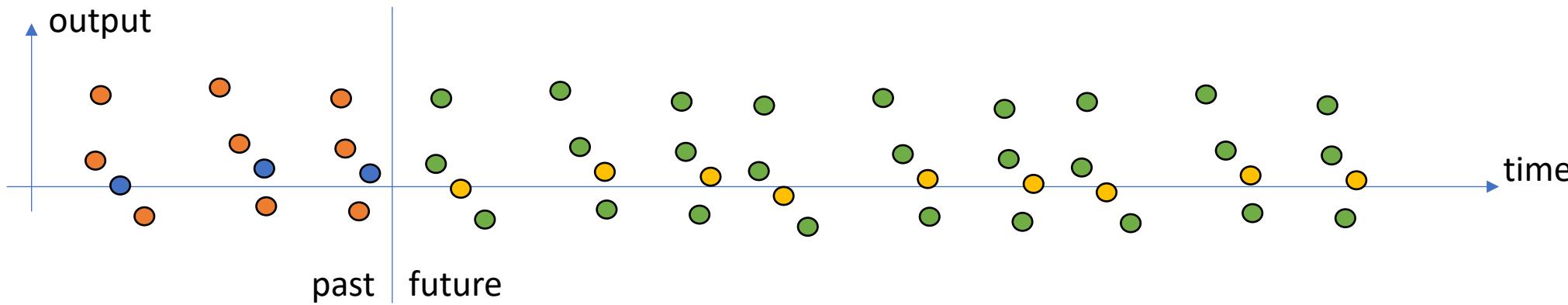


Construct the Hankel Matrix using Data

$$\mathcal{H}_L(u) := \begin{pmatrix} u_1 & \cdots & u_{T-L+1} \\ \vdots & \ddots & \vdots \\ u_L & \cdots & u_T \end{pmatrix}$$

$$\begin{pmatrix} U_p \\ U_f \end{pmatrix} := \mathcal{H}_{T_{\text{ini}}+N}(u^d), \quad \begin{pmatrix} Y_p \\ Y_f \end{pmatrix} := \mathcal{H}_{T_{\text{ini}}+N}(y^d),$$

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Replace the model

$$\underset{g, u, y}{\text{minimize}} \quad \sum_{k=0}^{N-1} \left(\|y_k - r_{t+k}\|_Q^2 + \|u_k\|_R^2 \right)$$

$$\text{subject to} \quad \begin{pmatrix} U_p \\ Y_p \\ U_f \\ Y_f \end{pmatrix} g = \begin{pmatrix} u_{\text{ini}} \\ y_{\text{ini}} \\ u \\ y \end{pmatrix},$$

$$u_k \in \mathcal{U}, \forall k \in \{0, \dots, N-1\},$$

$$y_k \in \mathcal{Y}, \forall k \in \{0, \dots, N-1\}.$$

The complexity of the prediction model

Linear

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Marco C., Campi, and Erik Weyer. "Finite sample properties of system identification methods." *IEEE Transactions on Automatic Control* 47.8 (2002): 1329-1334.

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Google Scholar search results for "learning LQR":

Articles: About 8,170 results (0.05 sec)

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Results:

- HTML Trajectory tracking using online learning LQR with adaptive learning control of a leg-exoskeleton for disorder gait rehabilitation N Aljanaromat, M Pamichuk - Mechatronics, 2018 - Elsevier
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Enrico Terzi, Lorenzo Fagiano, Marcello Farina, Riccardo Scattolini. Learning multi-step prediction models for receding horizon control. In *2018 European Control Conference (ECC)* 2018 Jun 12 (pp. 1335-1340). IEEE.

known Gray Box unknown

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Monimoy Bujarbaruah, Xiaojing Zhang, Ugo Rosolia, and Francesco Borrelli. "Adaptive MPC for iterative tasks." In *2018 IEEE Conference on Decision and Control (CDC)*, pp. 6322-6327. IEEE, 2018.

Black Box

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Kendall Lowrey, Aravind Rajeswaran, Sham Kakade, Emanuel Todorov, and Igor Mordatch. "Plan online, learn offline: Efficient learning and exploration via model-based control." In *Proc. Int. Conf. Mach. Learn.*, 2019.

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Gray Box Model

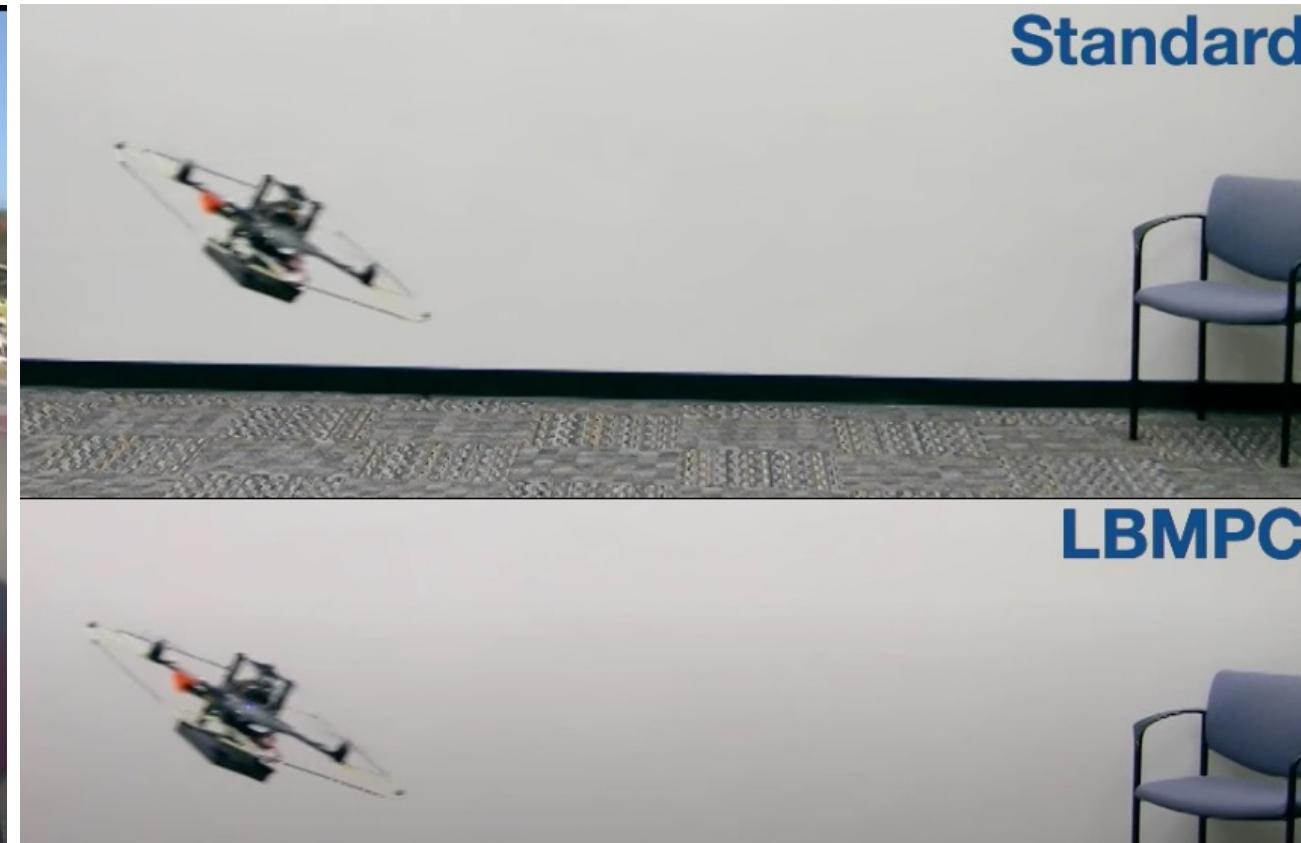
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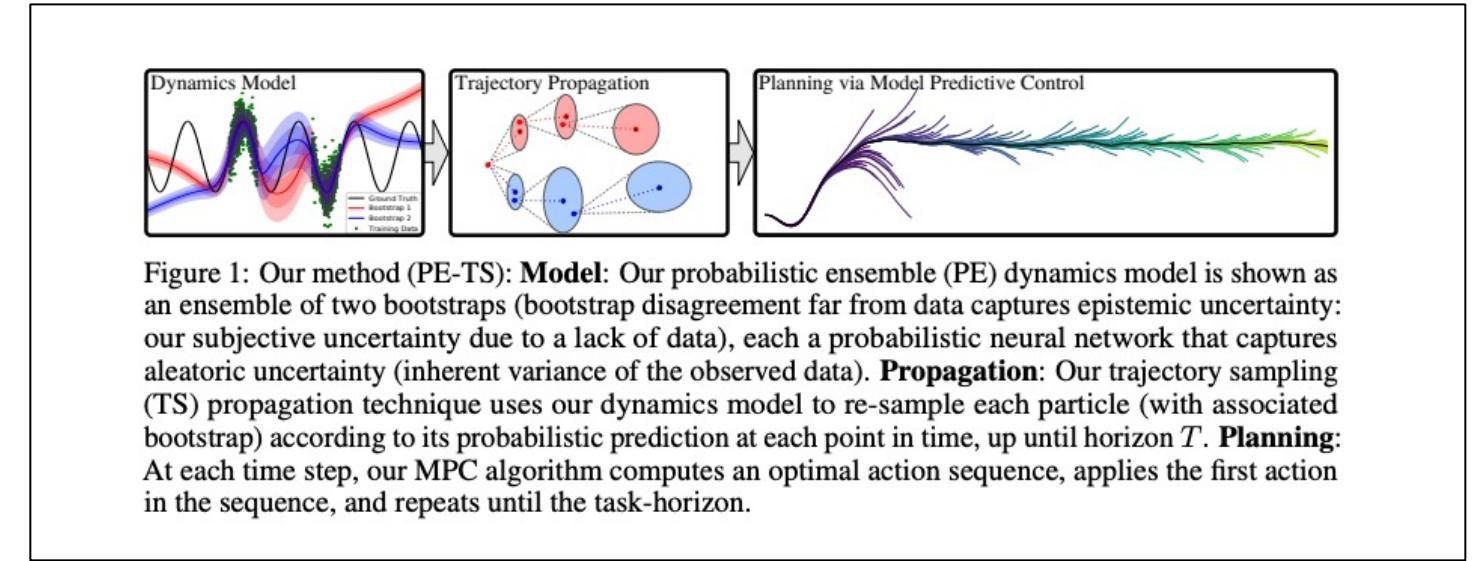
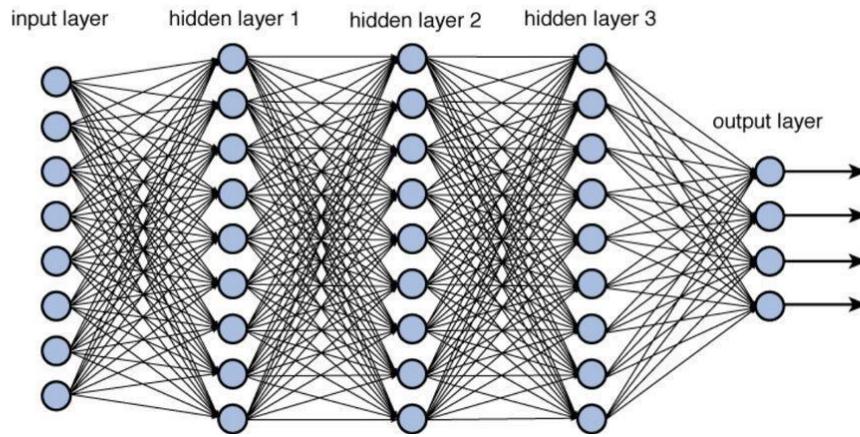
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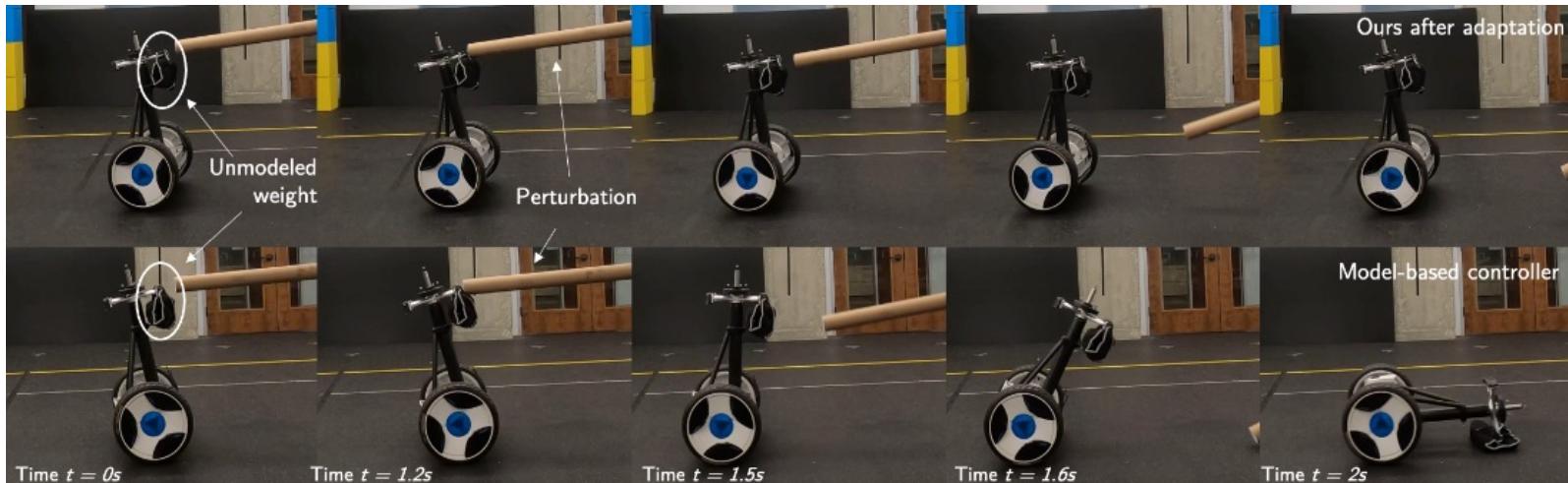
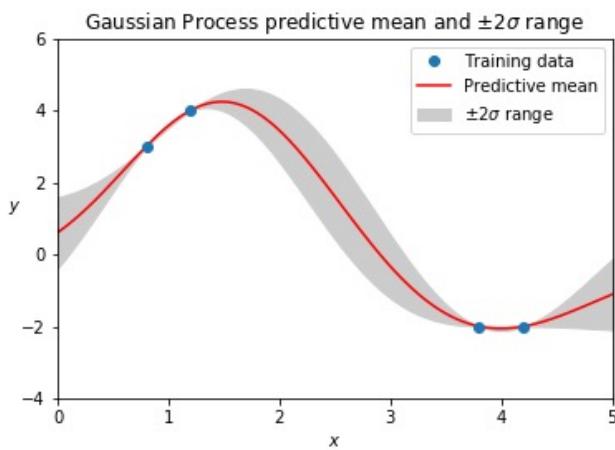
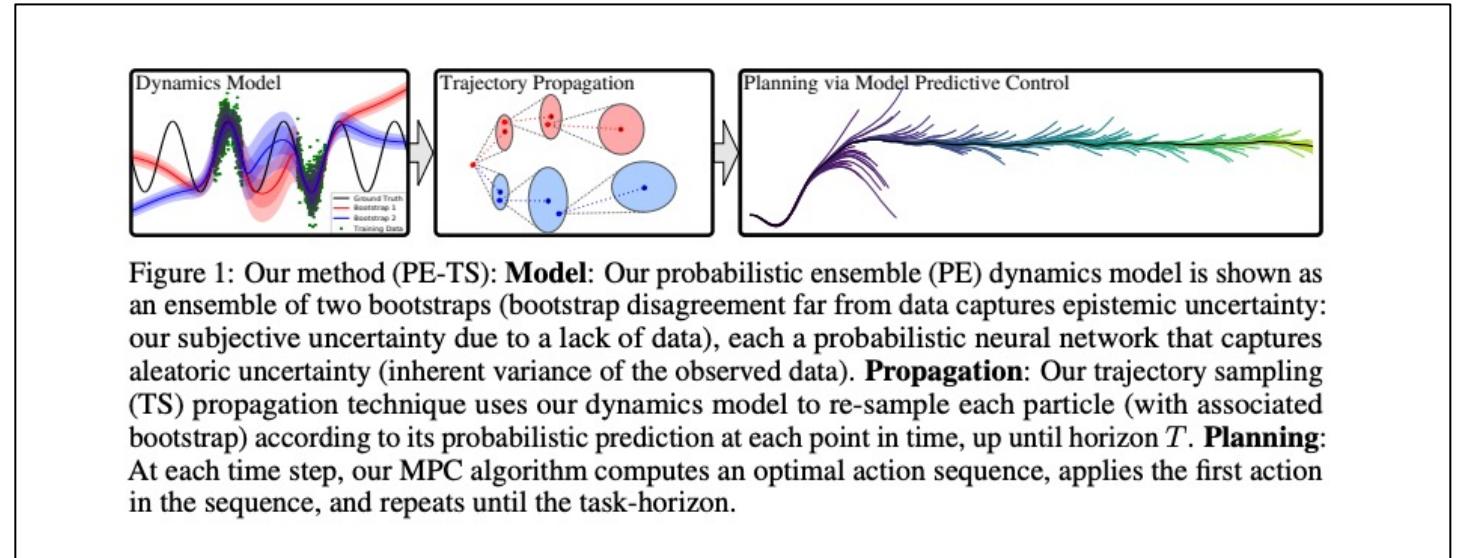
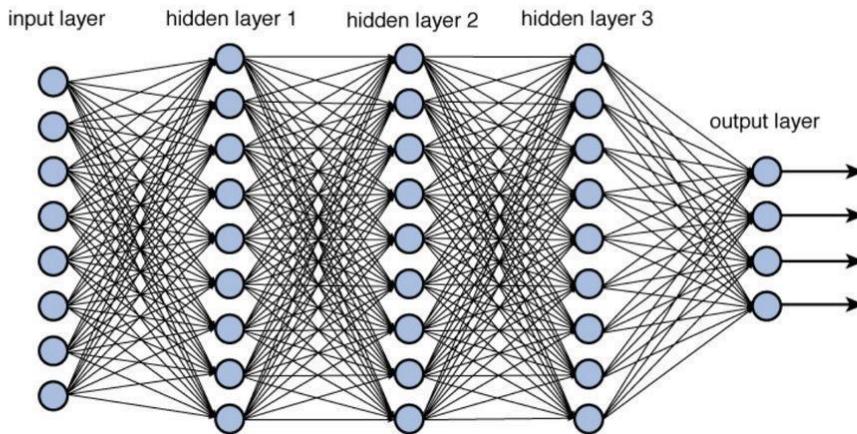
Deisenroth, Marc, and Carl E. Rasmussen. "PILCO: A model-based and data-efficient approach to policy search." In *Proceedings of the 28th International Conference on machine learning (ICML-11)*, pp. 465-472. 2011.

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Black Box Model



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Ivan D. Jimenez, Rodriguez, Ugo Rosolia, Aaron D. Ames, and Yisong Yue. "Learning Unstable Dynamics with One Minute of Data: A Differentiation-based Gaussian Process Approach." *arXiv preprint arXiv:2103.04548* (2021).

Planning under uncertainty

Optimizing over control policies

Why is planning in uncertain environments harder?

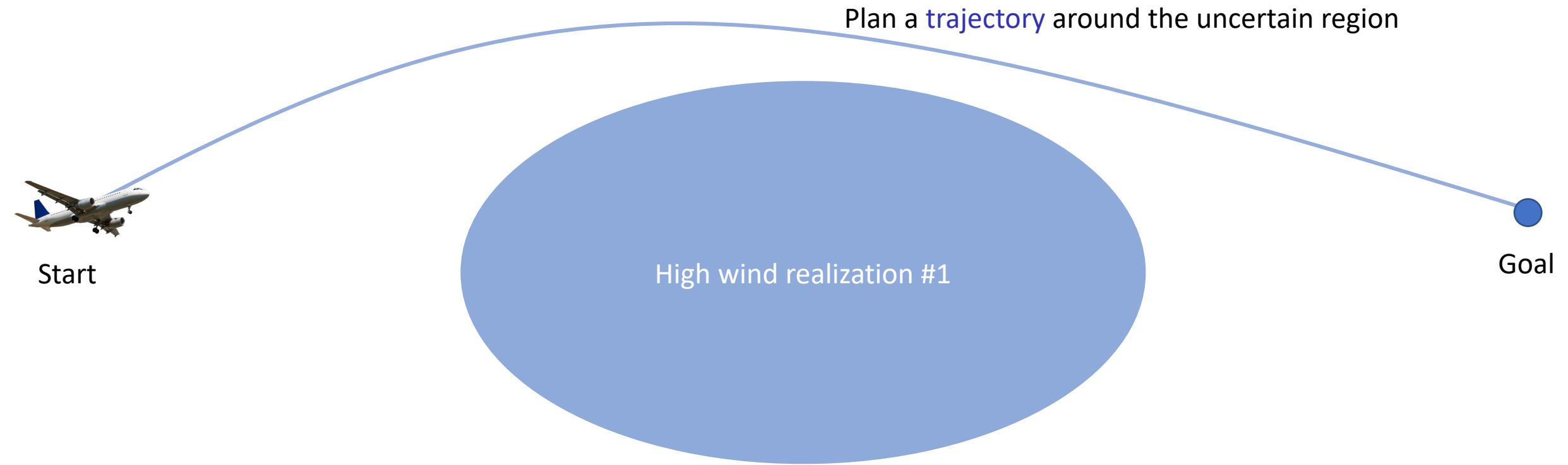


Start



Goal

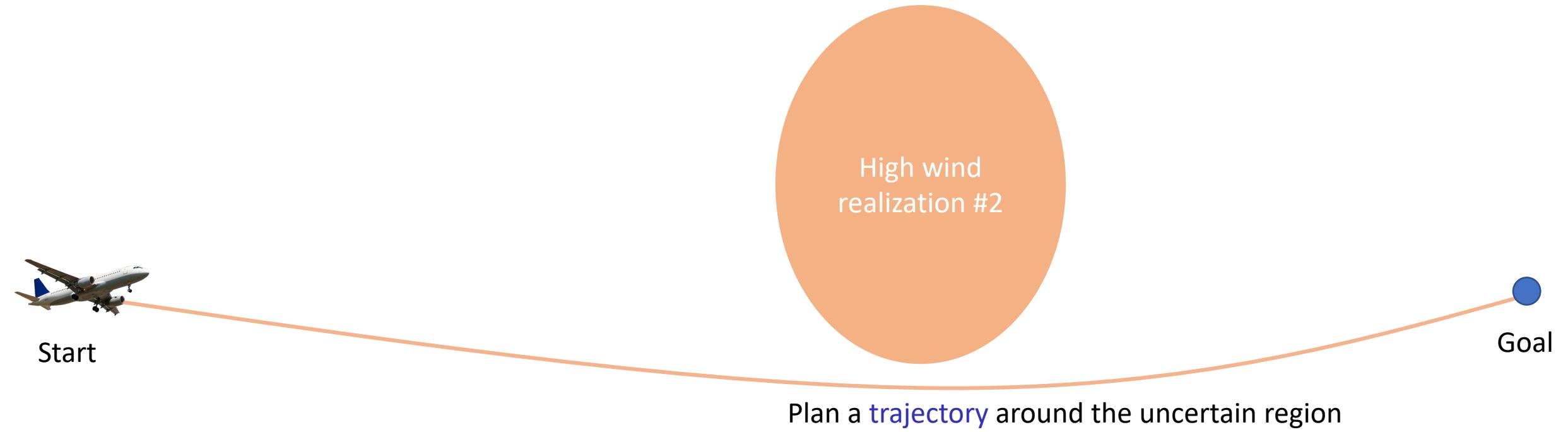
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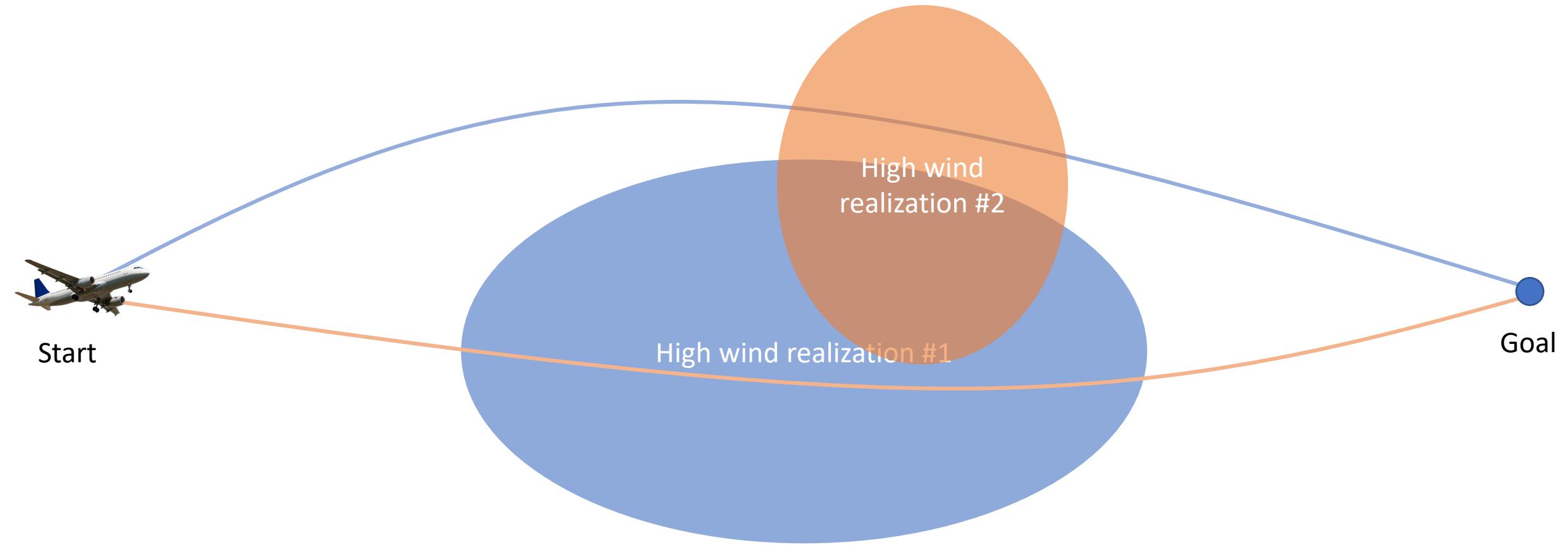
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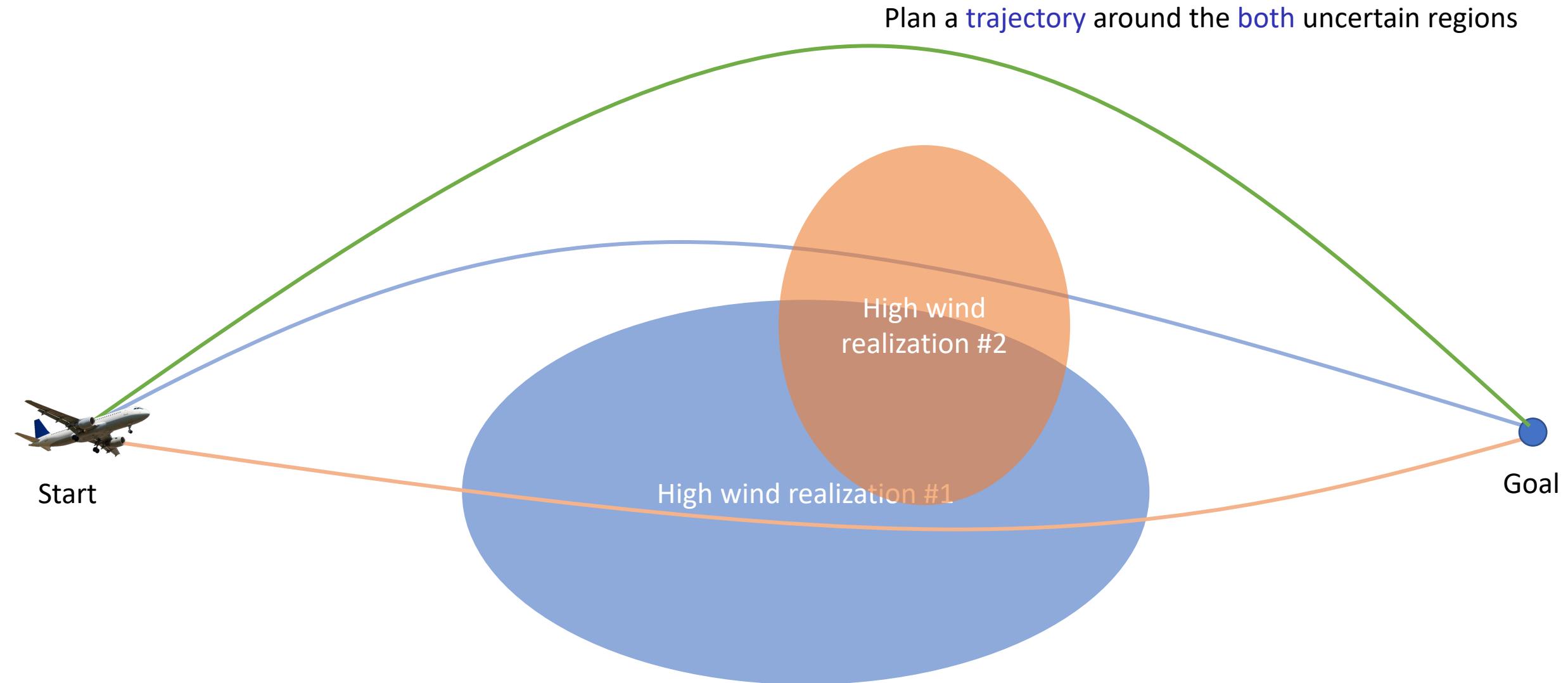
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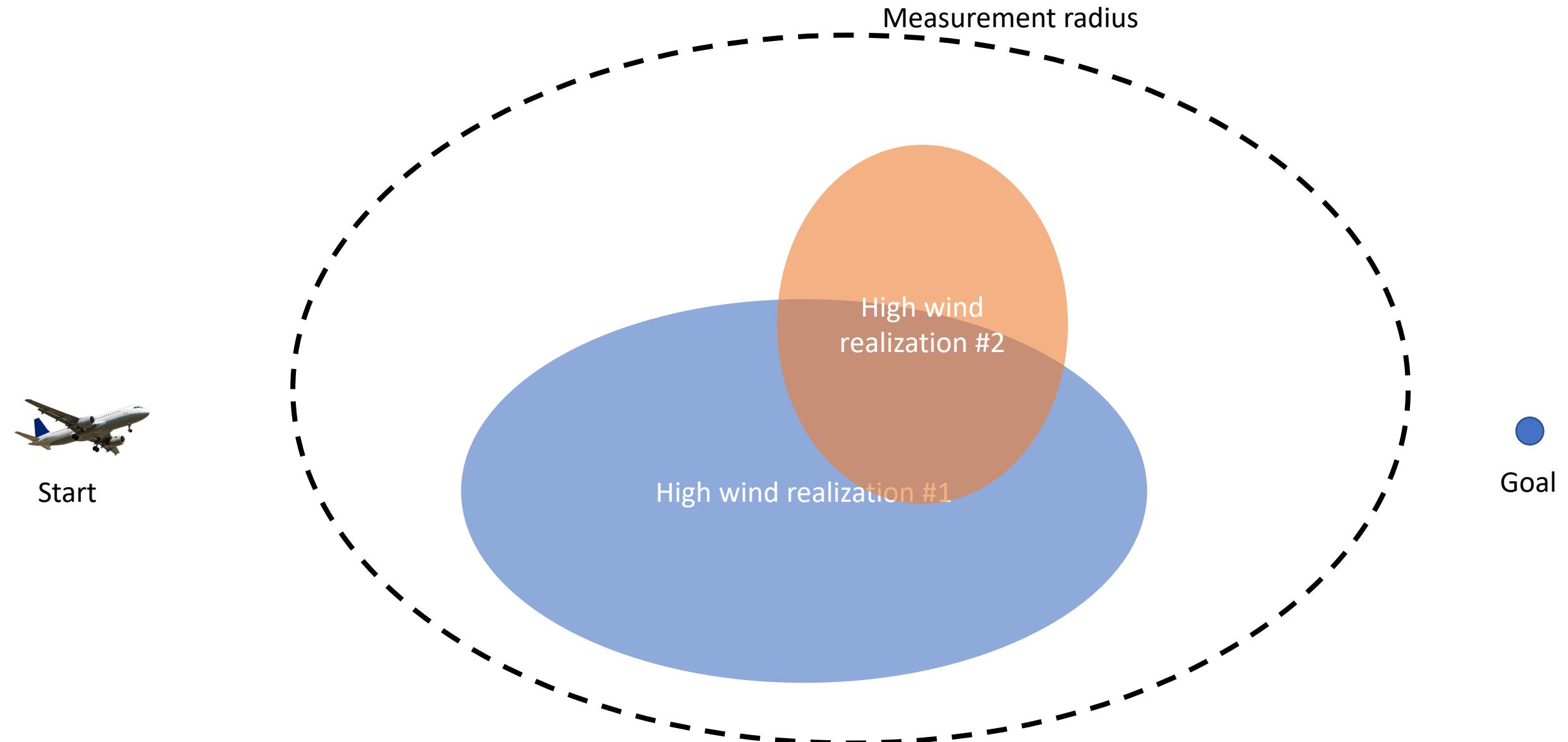
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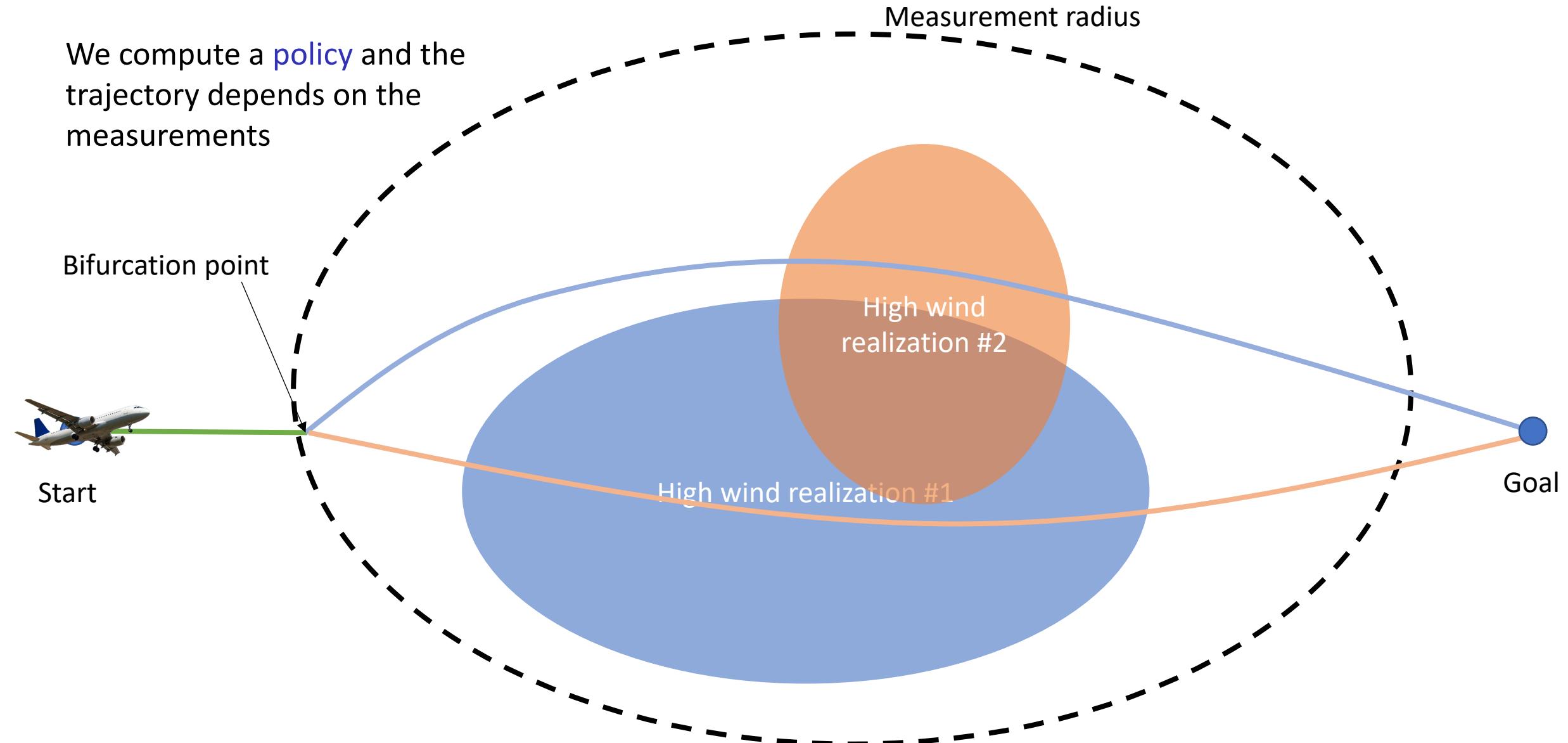


Why is planning in uncertain environments harder?



Why is planning in uncertain environments harder?

We compute a **policy** and the trajectory depends on the measurements



Problem Formulation

True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$

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Robust constraints:

Given a control policy $\pi : \mathbb{R}^n \rightarrow \mathbb{R}^d$ designed using the nominal model, we want to guarantee that the closed-loop system

$$x_{k+1} = Ax_k + B\pi(x_k) + w_k$$

satisfies $x_k \in \mathcal{X}, \pi(x_k) \in \mathcal{U}$ for all $w_k \in \mathcal{W}$.

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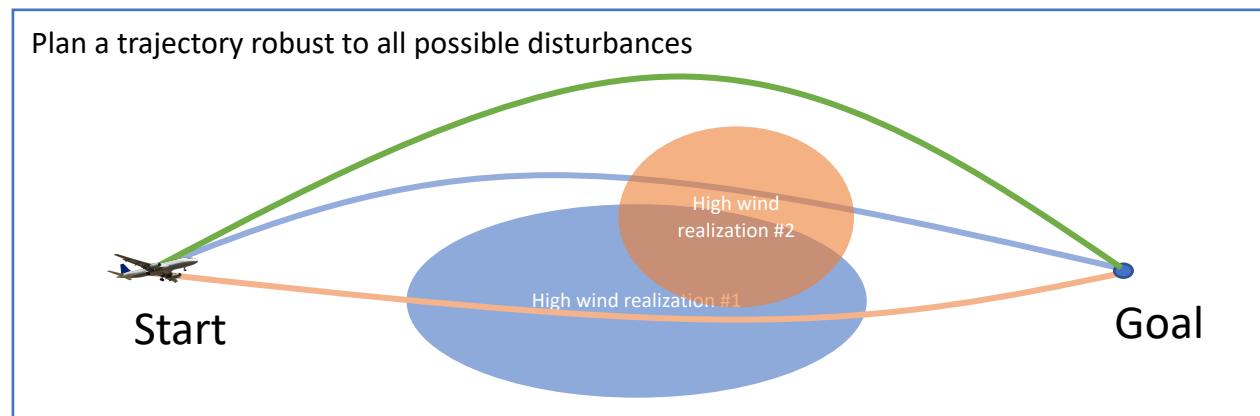
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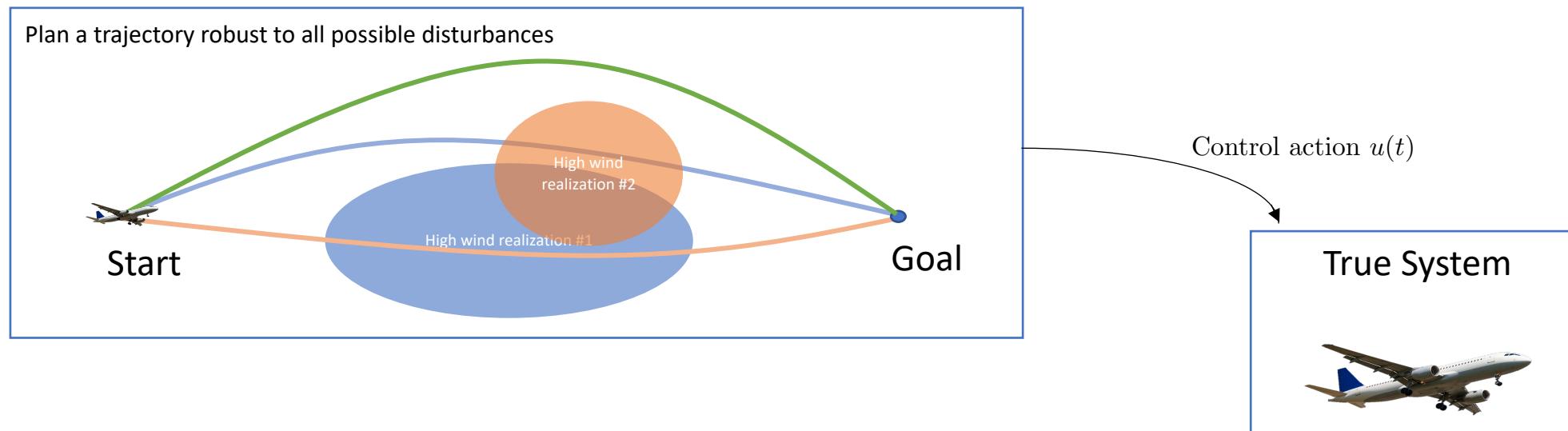
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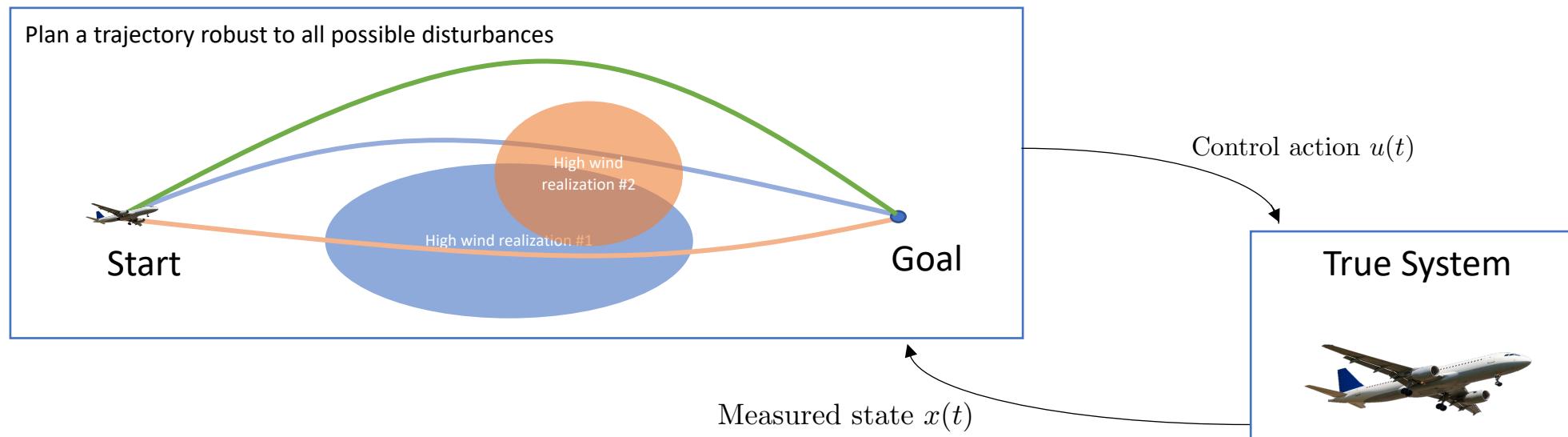
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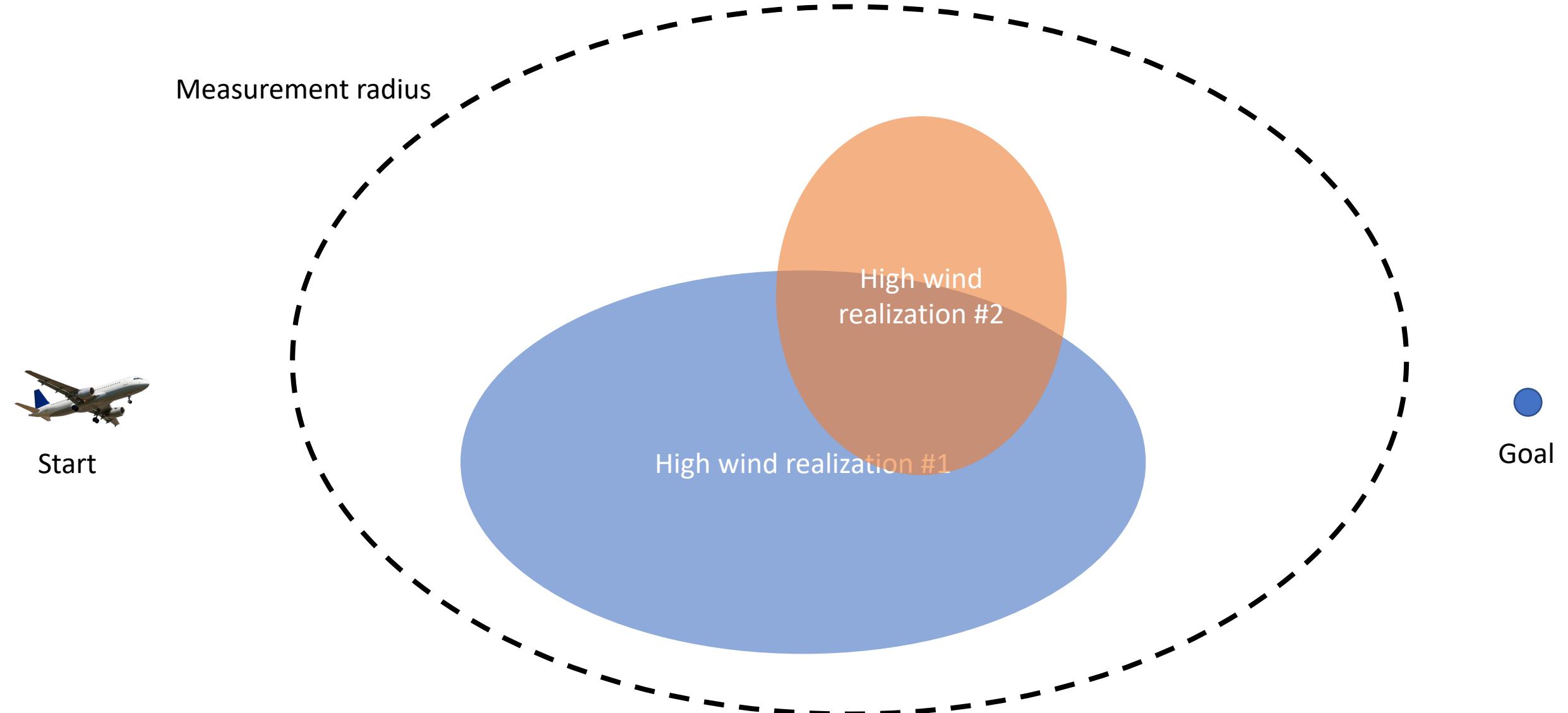
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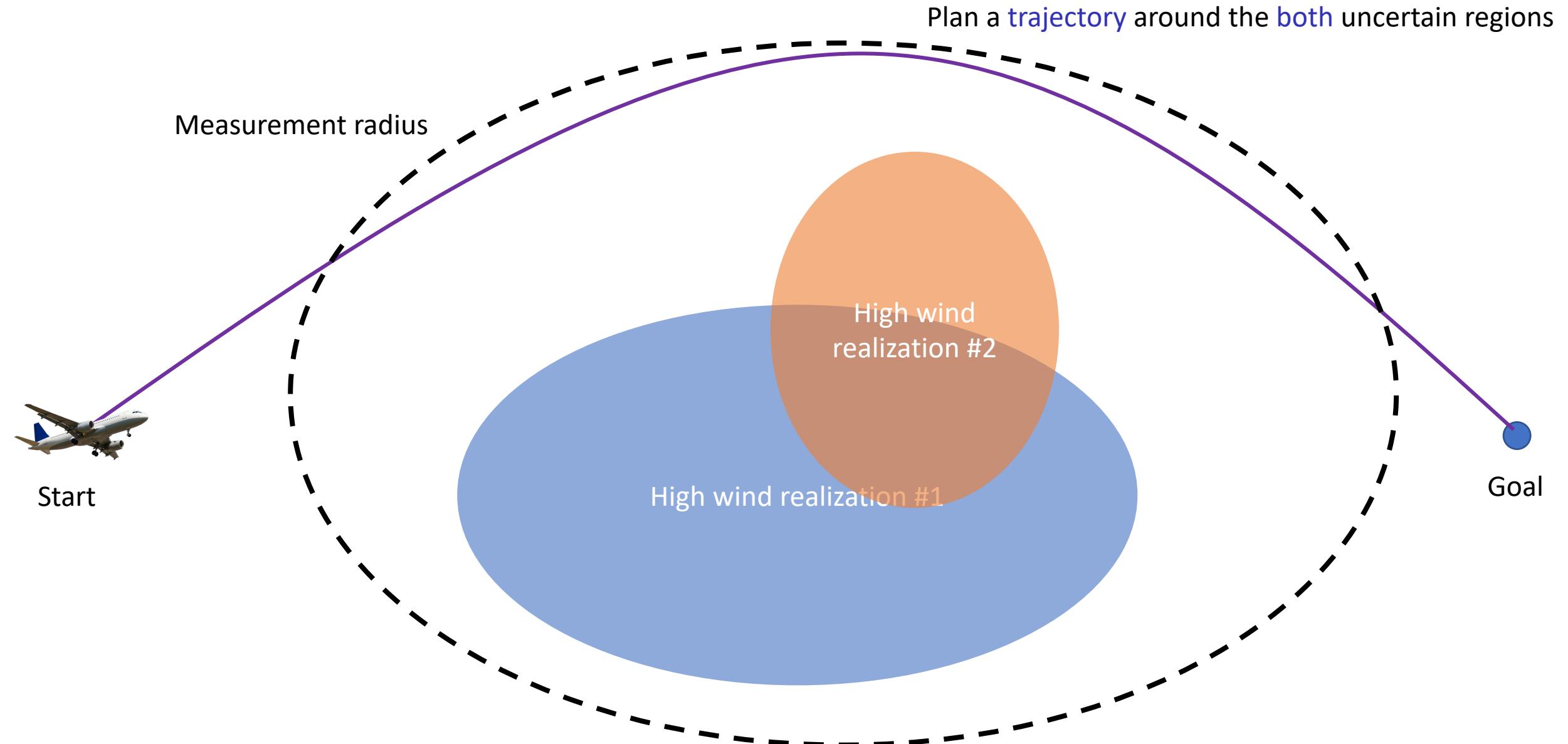
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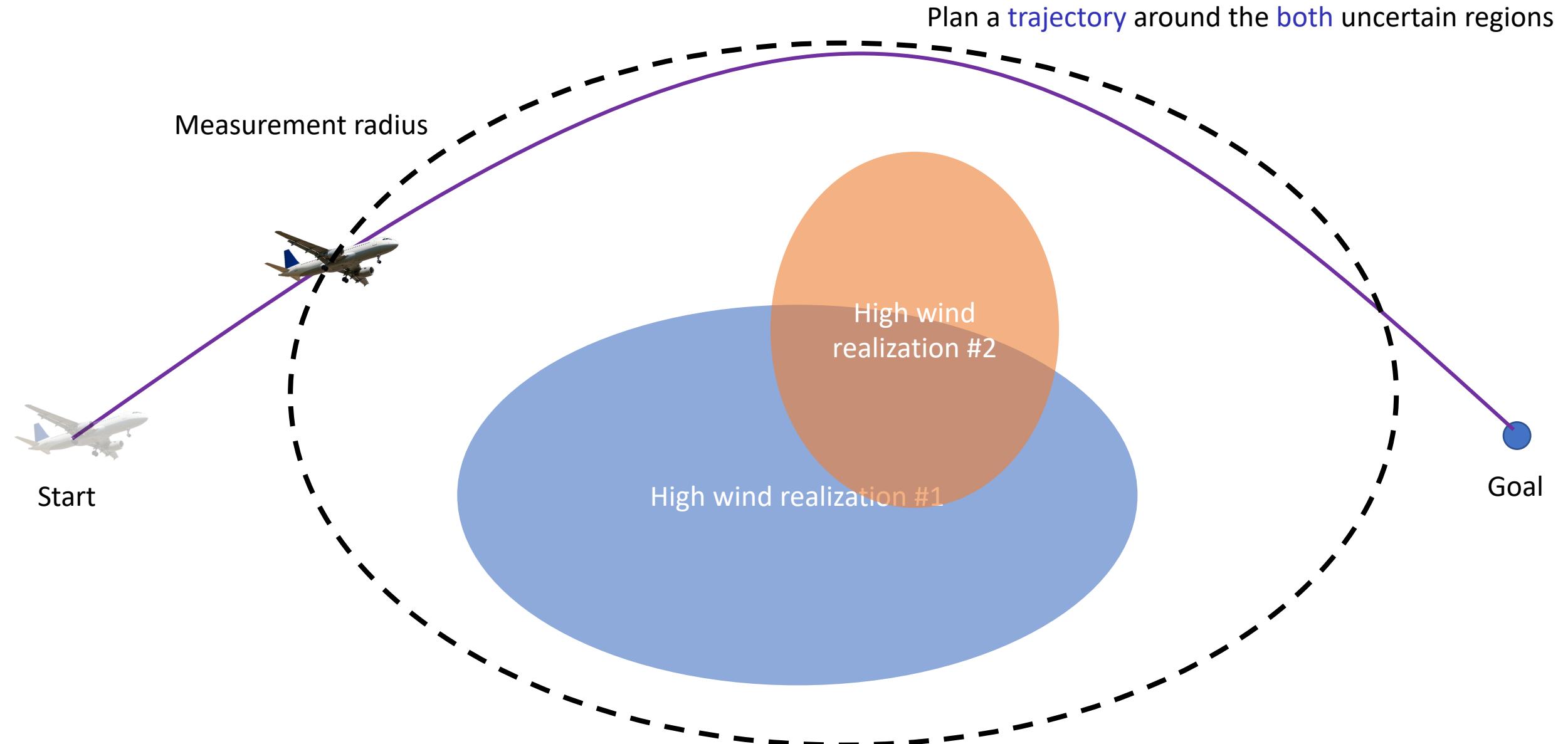
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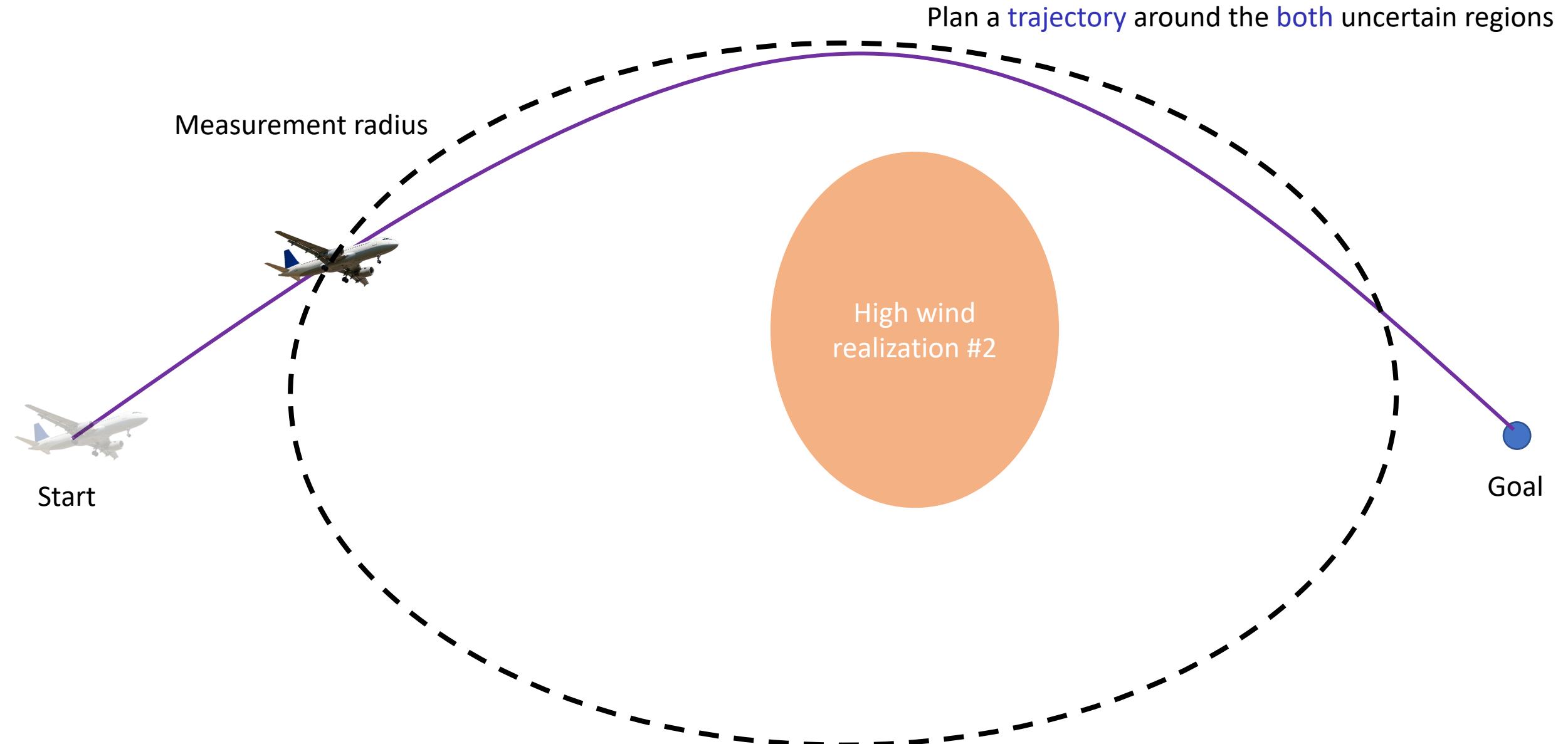
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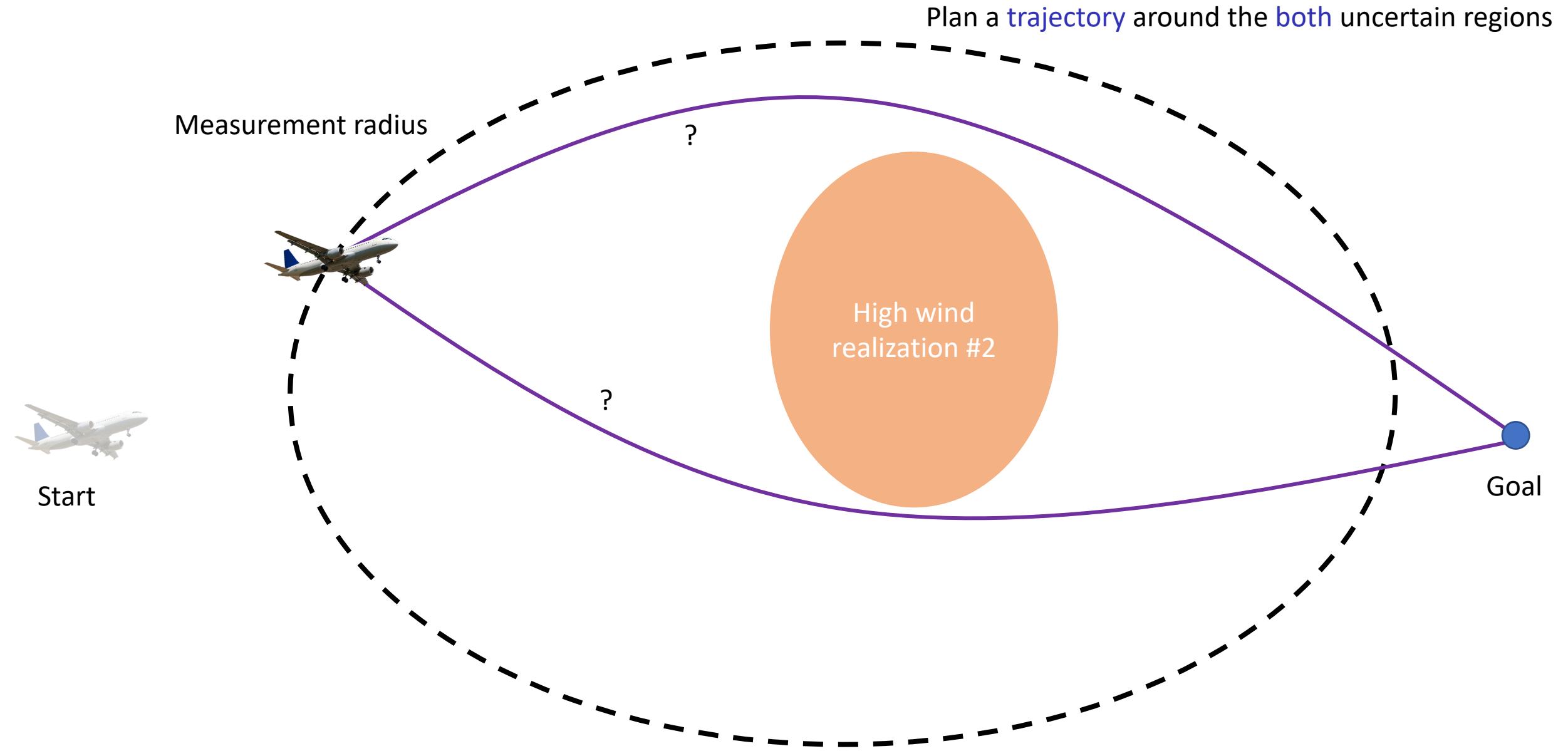
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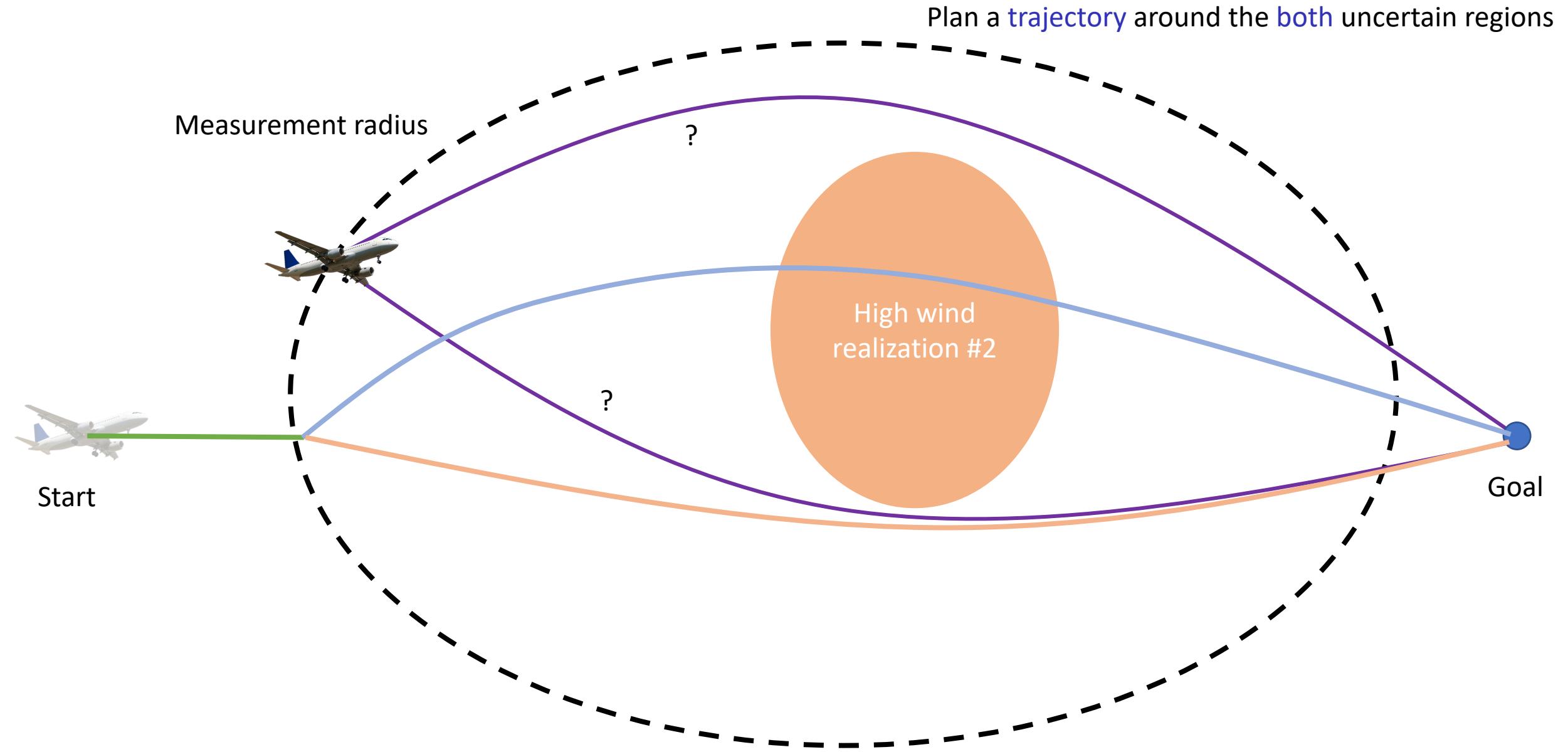
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Why should I care?

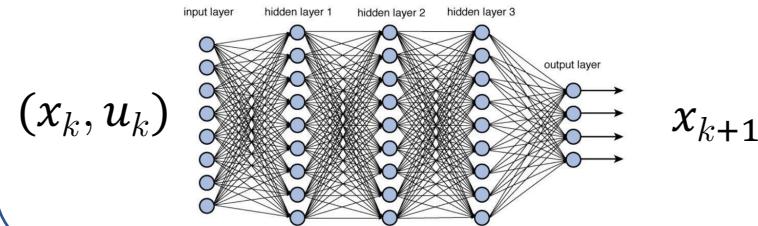
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Model-based RL

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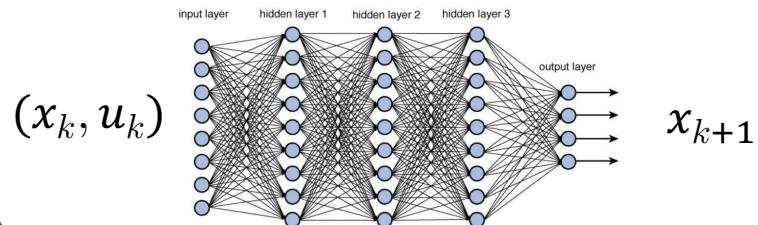
Step 1: Learn a Model



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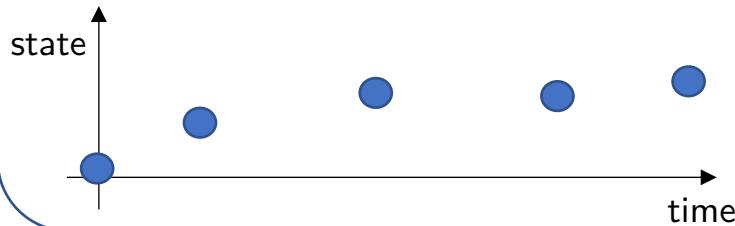
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Step 1: Learn a Model



Step 2: Receding horizon planning

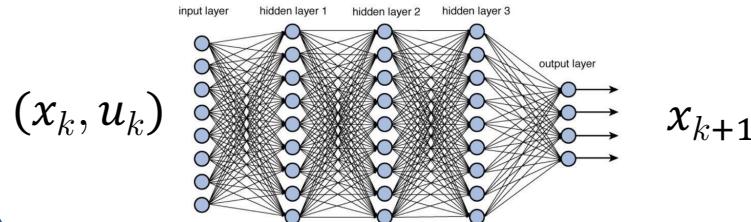
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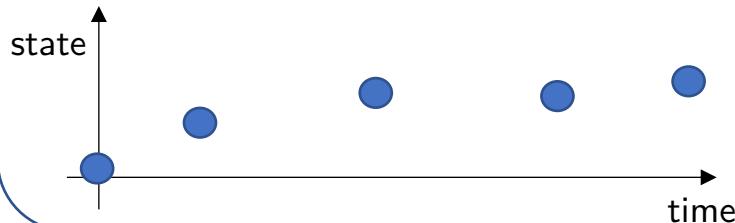
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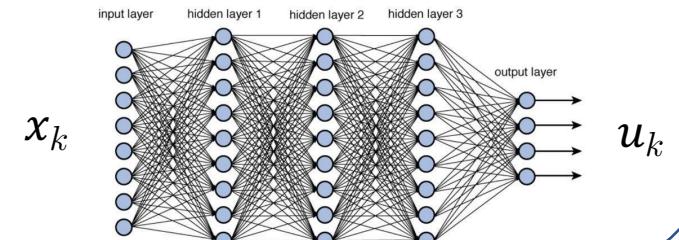
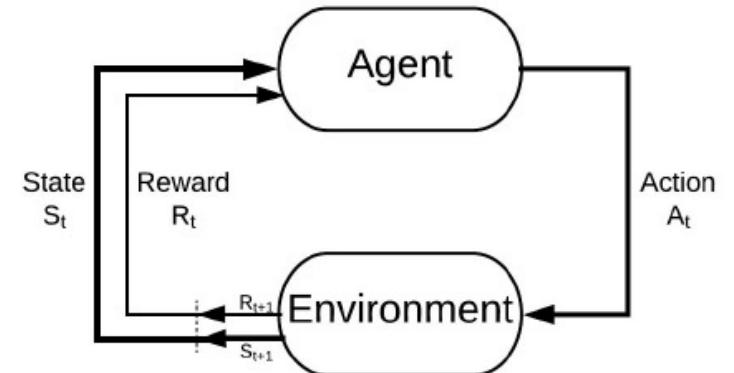
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Model-free RL

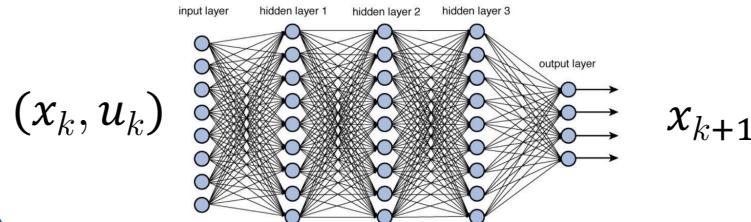
Step 1: Learn a policy



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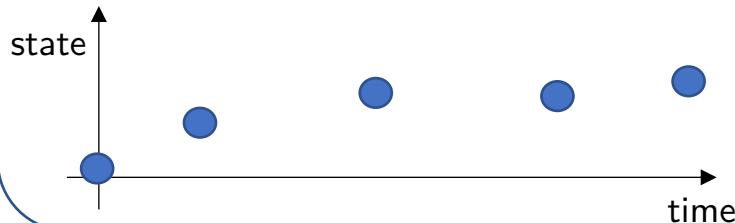
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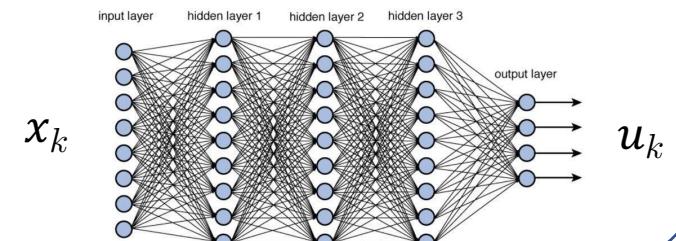
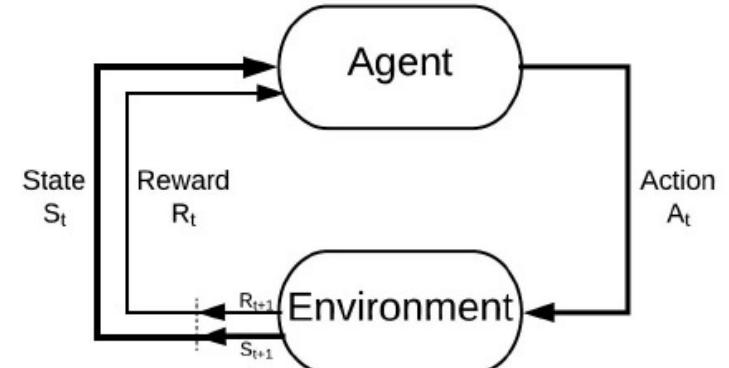
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Model-free RL

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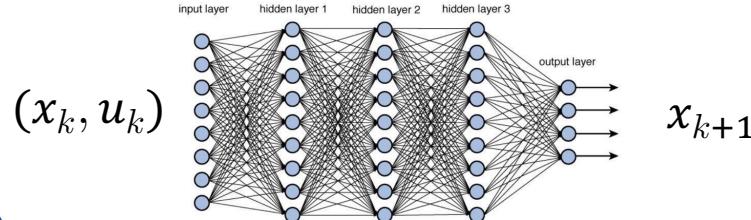


Which one has better asymptotic performance?

Why should I care?

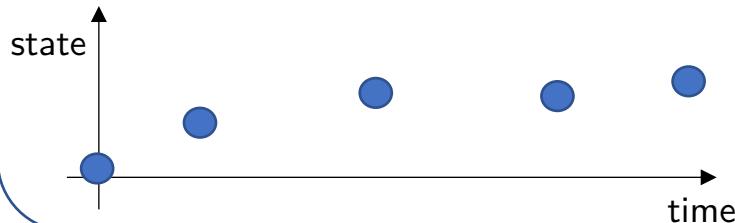
Model-based RL

Step 1: Learn a Model



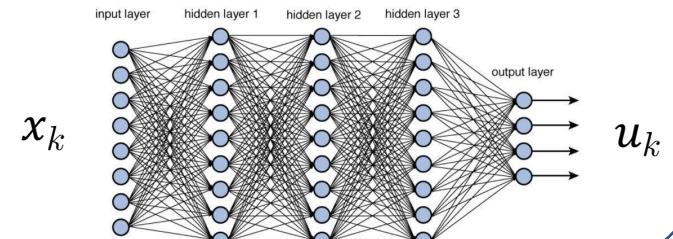
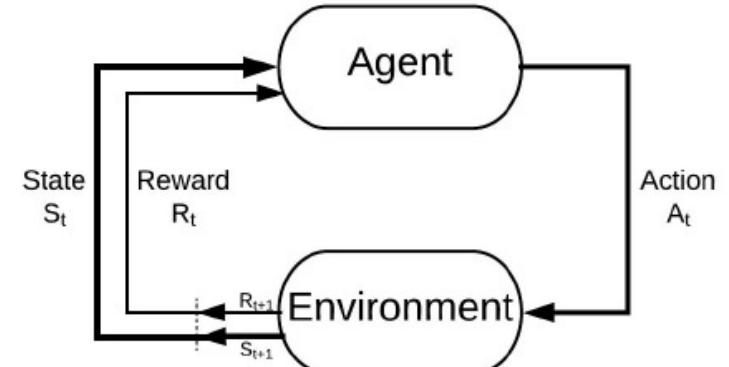
Step 2: Receding horizon planning

$$\min \mathbb{E}_{\substack{a_t \sim \pi \\ s_t \sim p}} [\sum_{t=0}^H \gamma^t c(a_t, s_t)]$$



Model-free RL

Step 1: Learn a policy

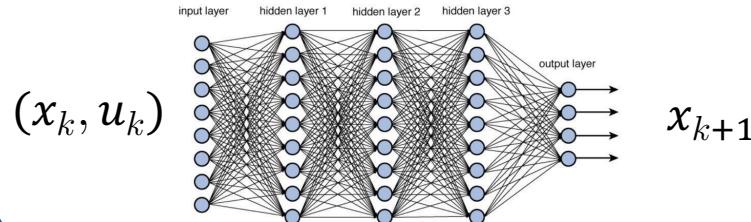


What can go wrong with model-based RL?

Why should I care?

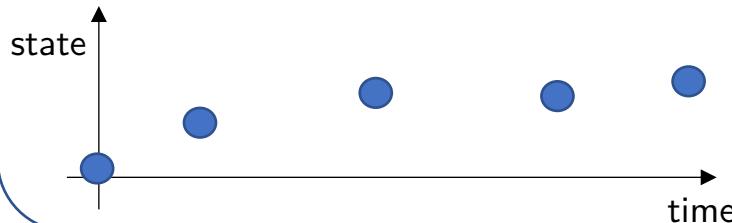
Model-based RL

Step 1: Learn a Model



Step 2: Receding horizon planning

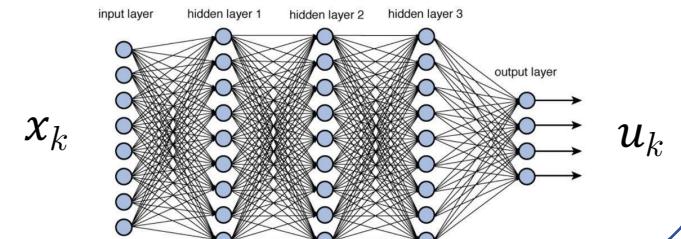
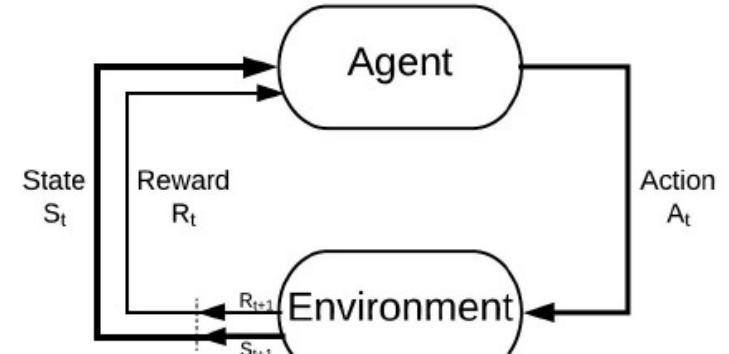
$$\min \mathbb{E}_{\substack{a_t \sim \pi \\ s_t \sim p}} [\sum_{t=0}^H \gamma^t c(a_t, s_t)]$$



Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



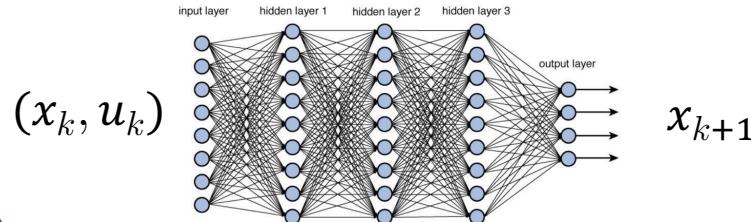
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- ▶ The prediction horizon is short → need a value function and safe set

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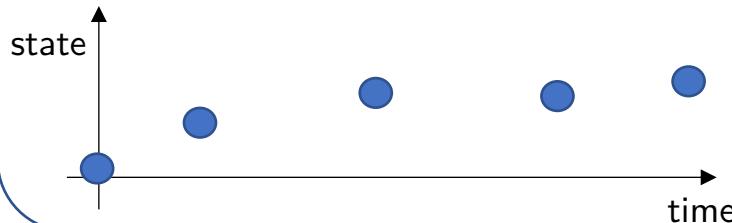
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Step 2: Receding horizon planning

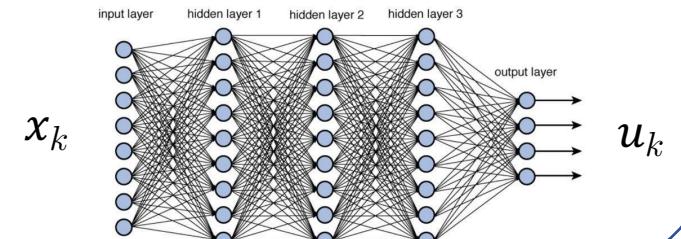
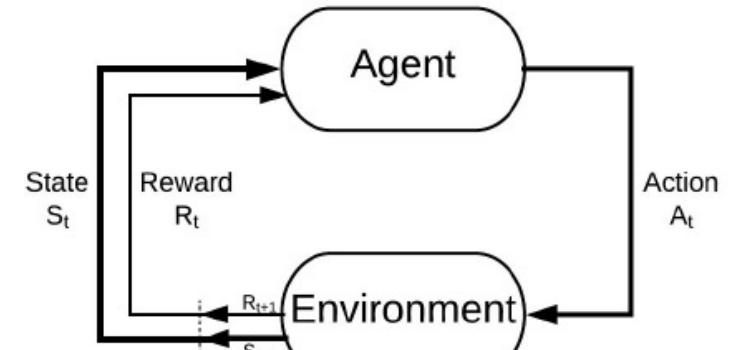
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Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



What can go wrong with model-based RL?

- ▶ The prediction horizon is short → need a value function and safe set
- ▶ Planning trajectories is suboptimal! We should plan over policies

Why should I care?

Model-based RL

Benchmarking Model-Based Reinforcement Learning

Tingwu Wang^{1,2}, Xuchan Bao^{1,2}, Ignasi Clavera³, Jerrick Hoang^{1,2}, Yeming Wen^{1,2}, Eric Langlois^{1,2}, Shunshi Zhang^{1,2}, Guodong Zhang^{1,2}, Pieter Abbeel³ & Jimmy Ba^{1,2}

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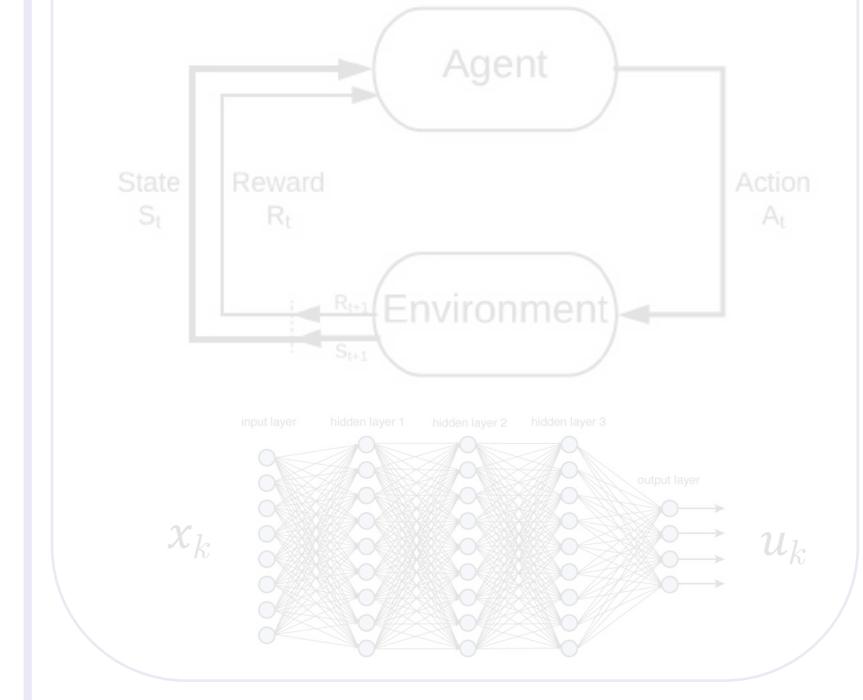
Abstract

Model-based reinforcement learning (MBRL) is widely seen as having the potential to be significantly more sample efficient than model-free RL. However, research in model-based RL has not been very standardized. It is fairly common for authors to experiment with self-designed environments, and there are several separate lines of research, which are sometimes closed-sourced or not reproducible. Accordingly, it is an open question how these various existing MBRL algorithms perform relative to each other. To facilitate research in MBRL, in this paper we gather a wide collection of MBRL algorithms and propose over 18 benchmarking environments specially designed for MBRL. We benchmark these algorithms with unified problem settings, including noisy environments. Beyond cataloguing performance, we explore and unify the underlying algorithmic differences across MBRL algorithms. We characterize three key research challenges for future MBRL research: the dynamics bottleneck, the planning horizon dilemma, and the early-termination dilemma. Finally, to maximally facilitate future research on MBRL, we open-source our benchmark in <http://www.cs.toronto.edu/~tingwuwang/mbrl.html>.

has better performance?

Model-free RL

Step 1: Learn a policy



value function and safe set
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Why should I care?

Model-based RL

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Model-free RL

Model-Based Reinforcement Learning via Meta-Policy Optimization

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Abstract: Model-based reinforcement learning approaches carry the promise of being data efficient. However, due to challenges in learning dynamics models that sufficiently match the real-world dynamics, they struggle to achieve the same asymptotic performance as model-free methods. We propose Model-Based Meta-Policy-Optimization (MB-MPO), an approach that foregoes the strong reliance on accurate learned dynamics models. Using an ensemble of learned dynamic models, MB-MPO meta-learns a policy that can quickly adapt to any model in the ensemble with one policy gradient step. This steers the meta-policy towards internalizing consistent dynamics predictions among the ensemble while shifting the burden of behaving optimally w.r.t. the model discrepancies towards the adaptation step. Our experiments show that MB-MPO is more robust to model imperfections than previous model-based approaches. Finally, we demonstrate that our approach is able to match the asymptotic performance of model-free methods while requiring significantly less experience.

Keywords: Reinforcement Learning, Meta-Learning, Model-Based, Model-Free

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How do we fix this problem?

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1136

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 43, NO. 8, AUGUST 1998

Statement 2: Under the assumptions of this theorem for any $0 \leq t_0 \leq t$

$$\begin{aligned}\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}^*}(t_0) &\geq \Phi_{\tilde{u}^*}(t) - \Phi_{\tilde{u}^*}(t_0) \\ &= -\frac{1}{h} \Delta \vartheta^T(t) L^{-1} \Delta \vartheta(t) + o_\omega(h)\end{aligned}$$

where

$$\Phi_{\tilde{u}}(t) := \int_0^t (2\tilde{u}^T(\tau)d\vartheta(\tau) + \tilde{u}^T(\tau)L\tilde{u}(\tau)d\tau) \quad (20)$$

$$\begin{aligned}\Delta \vartheta(t) &:= B_{ab}^T P[B_{ab}(u_{comp}(t) - x^*(t))h \\ &+ (K + A_0 C_0^+) \Delta y(t)], \quad \Delta y(t) = y(t) - y(t-h)\end{aligned}$$

and this minimum is reachable for

$$\begin{aligned}\dot{\tilde{u}}(t) dt &= \tilde{u}^*(t) dt := -L^{-1} B_{ab}^T P[B_{ab}(u_{comp}(t) + x^*(t))dt \\ &+ (K + A_0 C_0^+) dy(t)].\end{aligned}$$

As a result, we have: $d\Phi_{\tilde{u}^*}(t) \leq 0$ (in symbolic form).

Proof of Statement 2: Using the Euler–Maruyama's formula [3], [9] we obtain the following relation: $(h := t - t_0 \rightarrow 0), \Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}^*}(t_0) = 2\tilde{u}^T(t) \Delta \vartheta(t) + \tilde{u}^T(t) L\tilde{u}(t)h + o_\omega(h)$.

Minimizing then the right side for each fixed t , we derive

$$\begin{aligned}\min_{\tilde{u}(t)} [\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}^*}(t_0)] &= \Phi_{\tilde{u}^*}(t) - \Phi_{\tilde{u}^*}(t_0) \\ &= -\frac{1}{h} \Delta \vartheta^T(t) L^{-1} \Delta \vartheta(t) + o_\omega(h) \leq o_\omega(h).\end{aligned}\quad (21)$$

Hence, taking into account the definition (20), we have $d\Phi_{\tilde{u}^*}(t) \leq 0$. \square

Rewriting (18) in differential form and taking into account that P is the solution of the Riccati equation (see Assumption 4 of this theorem) we finally derive

$$dV(e(t)) \stackrel{a.s.}{\leq} [\varphi(t) + I(t)] dt + S^T(t) dw(t) - e^T(t) Q e(t) dt.$$

According to the Kronecker lemma, the Large Number law for martingale, and Skorohod lemma (see [2] and [4]), we obtain the result of the theorem. Details of this proof can be found in [13].

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Automatica 41 (2005) 219–224

Brief paper

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Optimization over state feedback policies for robust control with constraints

Paul J. Goulart^{a,*}, Eric C. Kerrigan^b, Jan M. Maciejowski^a^aDepartment of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1FW, UK^bDepartment of Aeronautics and Department of Electrical and Electronic Engineering, Imperial College London, Exhibition Road, London SW7 2BY, UK

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Abstract

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1. Introduction

This paper is concerned with the control of constrained discrete-time linear systems that are subject to additive, but bounded disturbances on the state. The main aim is to provide results that allow for the efficient computation of an optimal and stabilizing state feedback control policy that ensures a given set of state and input constraints are satisfied for all time, despite the presence of the disturbances. This is a problem that has been studied for some time now in the optimal control literature (Bertsekas & Rhodes, 1973) and a number of different

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It is generally accepted that if disturbances accounted for in the formulation of a constraint control problem, then the optimization has to be admissible state feedback policies, rather than open sequences, otherwise infeasibility and instability may occur (Mayne et al., 2000). However, optimization (nonlinear) feedback policies is particularly constraints have to be satisfied. Current proposals for this using finite dimensional optimization, such

Abstract

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Keywords: Robust model predictive control; Robustness; Bounded disturbances

1. Introduction

Model predictive control is widely employed for the control of constrained systems and an extensive literature on the subject exists some of which is reviewed in Bemporad and Morari (1999); Allgöwer, Badgwell, Qin, Rawlings, and Wright (1999); Mayne, Rawlings, Rao, and Scokaert (2000). Several methods for achieving robustness have been considered. The simplest is to ignore the disturbance and rely on the inherent robustness of deterministic model predictive control applied to the nominal system (Scokaert & Rawlings, 1995; Marruedo, Álamo, & Camacho, 2002). *Open-loop* model predictive control that determines the

$\{u_0, u_1, \dots, u_{N-1}\}$ of control actions was proposed in Zheng and Morari (1993); this method cannot contain the 'spread' of predicted trajectories resulting from disturbances. Hence *feedback* model predictive control in which the decision variable is a *policy* π , which is a sequence $\{\mu_0(\cdot), \mu_1(\cdot), \dots, \mu_{N-1}(\cdot)\}$ of control laws, was advocated in, for example, Mayne (1995); Kothare, Balakrishnan, and Morari (1996); Lee and Yu (1997); Scokaert and Mayne (1998); De Nicalo, Magni, and Scattolini (2000); Magni, Nijmeijer, and van der Schaft (2001); Magni, De Nicolao, Scattolini, and Allgöwer (2003). Determination of a control policy is usually prohibitively difficult so research has concentrated on various simplifications and relaxations (Mayne

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The problem is NP-hard!

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Brief paper

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Paul J. Goulart^{a,*}, Eric C. Kerrigan^b, Jan M. Maciejowski^a^aDepartment of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1FW, UK^bDepartment of Aeronautics and Department of Electrical and Electronic Engineering, Imperial College London, Exhibition Road, London SW7 2BY, UK

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Statement 2: Under the assumptions of this theorem for any $0 \leq t_0 \leq t$

$$\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}^*}(t_0) \geq \Phi_{\tilde{u}^*}(t) - \Phi_{\tilde{u}^*}(t_0) \\ = -\frac{1}{h} \Delta \vartheta^T(t) L^{-1} \Delta \vartheta(t) + o_\omega(h)$$

where

$$\Phi_{\tilde{u}}(t) := \int_0^t (2\tilde{u}^T(\tau)d\vartheta(\tau) + \tilde{u}^T(\tau)L\tilde{u}(\tau)d\tau) \quad (20)$$

$$\Delta \vartheta(t) := B_{ab}^T P[B_{ab}(u_{comp}(t) - x^*(t))h \\ + (K + A_0 C_0^+) \Delta y(t)], \quad \Delta y(t) = y(t) - y(t-h)$$

and this minimum is reachable for

$$\dot{\tilde{u}}(t) dt = \tilde{u}^*(t) dt := -L^{-1} B_{ab}^T P[B_{ab}(u_{comp}(t) + x^*(t)) dt \\ + (K + A_0 C_0^+) dy(t)].$$

As a result, we have: $d\Phi_{\tilde{u}^*}(t) \leq 0$ (in symbolic form).

Proof of Statement 2: Using the Euler–Maruyama's formula [3], [9] we obtain the following relation: $(h := t - t_0 \rightarrow 0), \Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}^*}(t_0) = 2\tilde{u}^T(t)\Delta \vartheta(t) + \tilde{u}^T(t)L\tilde{u}(t)h + o_\omega(h)$.

Minimizing then the right side for each fixed t , we derive

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Hence, taking into account the definition (20), we have $d\Phi_{\tilde{u}^*}(t) \leq 0$. \square

Rewriting (18) in differential form and taking into account that P is the solution of the Riccati equation (see Assumption 4 of this theorem) we finally derive

$$dV(e(t)) \stackrel{a.s.}{\leq} [\varphi(t) + I(t)] dt + S^T(t) dw(t) - e^T(t)Q_e e(t) dt.$$

According to the Kronecker lemma, the Large Number law for martingale, and Skorohod lemma (see [2] and [4]), we obtain the result of the theorem. Details of this proof can be found in [13].

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Brief paper

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Optimization over state feedback policies for robust control with constraints

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Abstract

This paper is concerned with the optimal control of linear discrete-time systems subject to unknown but bounded state dist mixed polytopic constraints on the state and input. It is shown that the class of admissible affine state feedback control policies with prior states is equivalent to the class of admissible feedback policies that are affine functions of the past disturbance sequence. that a broad class of constrained finite horizon robust and optimal control problems, where the optimization is over affine state policies, can be solved in a computationally efficient fashion using convex optimization methods. This equivalence result is used robust receding horizon control (RHC) state feedback policy such that the closed-loop system is input-to-state stable (ISS) and the are satisfied for all time and all allowable disturbance sequences. The cost to be minimized in the associated finite horizon op problem is quadratic in the disturbance-free state and input sequences. The value of the receding horizon control law can be calc sample instant using a single, tractable and convex quadratic program (QP) if the disturbance set is polytopic, or a tractable secon program (SOCP) if the disturbance set is given by a 2-norm bound. © 2006 Elsevier Ltd. All rights reserved.

Keywords: Robust control; Constraint satisfaction; Robust optimization; Predictive control; Optimal control

1. Introduction

This paper is concerned with the control of constrained discrete-time linear systems that are subject to additive, but bounded disturbances on the state. The main aim is to provide results that allow for the efficient computation of an optimal and stabilizing state feedback control policy that ensures a given set of state and input constraints are satisfied for all time, despite the presence of the disturbances. This is a problem that has been studied for some time now in the optimal control literature (Bertsekas & Rhodes, 1973) and a number of different

& Diaz-Bobillo, 1995) or predictive control (Mayne et al., 2000; Mayne, Rawlings, Rao, & Scokaert, 2000).

It is generally accepted that if disturbances accounted for in the formulation of a constraint control problem, then the optimization has to be admissible state feedback policies, rather than open sequences, otherwise infeasibility and instability may occur (Mayne et al., 2000). However, optimization trary (nonlinear) feedback policies is particularly constraints have to be satisfied. Current proposals for this using finite dimensional optimization, such as

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This paper provides a novel solution to the problem of robust model predictive control of constrained, linear, discrete-time systems in the presence of bounded disturbances. The optimal control problem that is solved online includes, uniquely, the initial state of the model employed in the problem as a decision variable. The associated value function is zero in a disturbance invariant set that serves as the 'origin' when bounded disturbances are present, and permits a strong stability result, namely robust exponential stability of the disturbance invariant set for the controlled system with bounded disturbances, to be obtained. The resultant online algorithm is a quadratic program of similar complexity to that required in conventional model predictive control. © 2004 Elsevier Ltd. All rights reserved.

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