CS159 Lecture 5: LMPC and Model Learning

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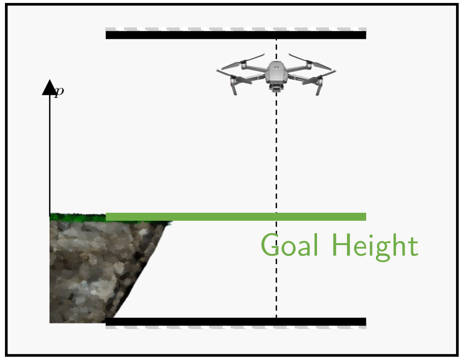
Spring 2021

Today's Class

- Recap of Lecture #4
- LMPC implementation
- Autonomous Racing Experiments
- Model learning in MPC (a brief summary)
- Planning under uncertainty

Recap of Lecture #4

Why did we focus on feasibility?



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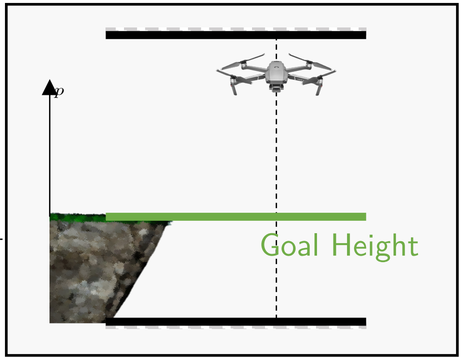
Why did we focus on feasibility?

Solve a Finite Time Optimal Control Problem (FTOCP)

Measured state $x(t)$



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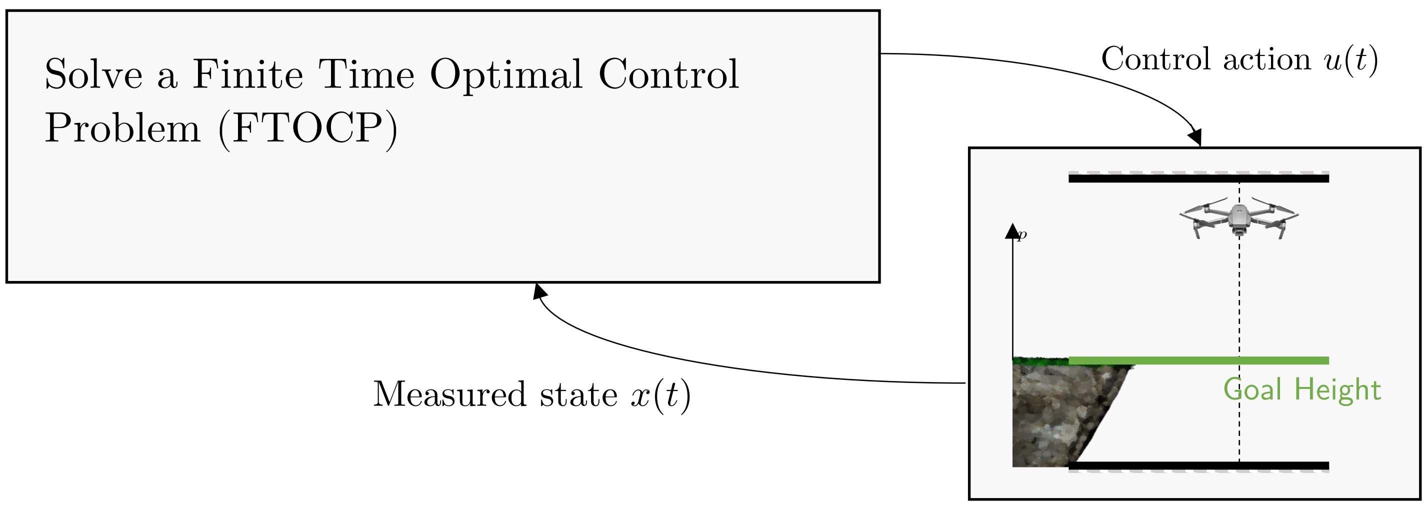

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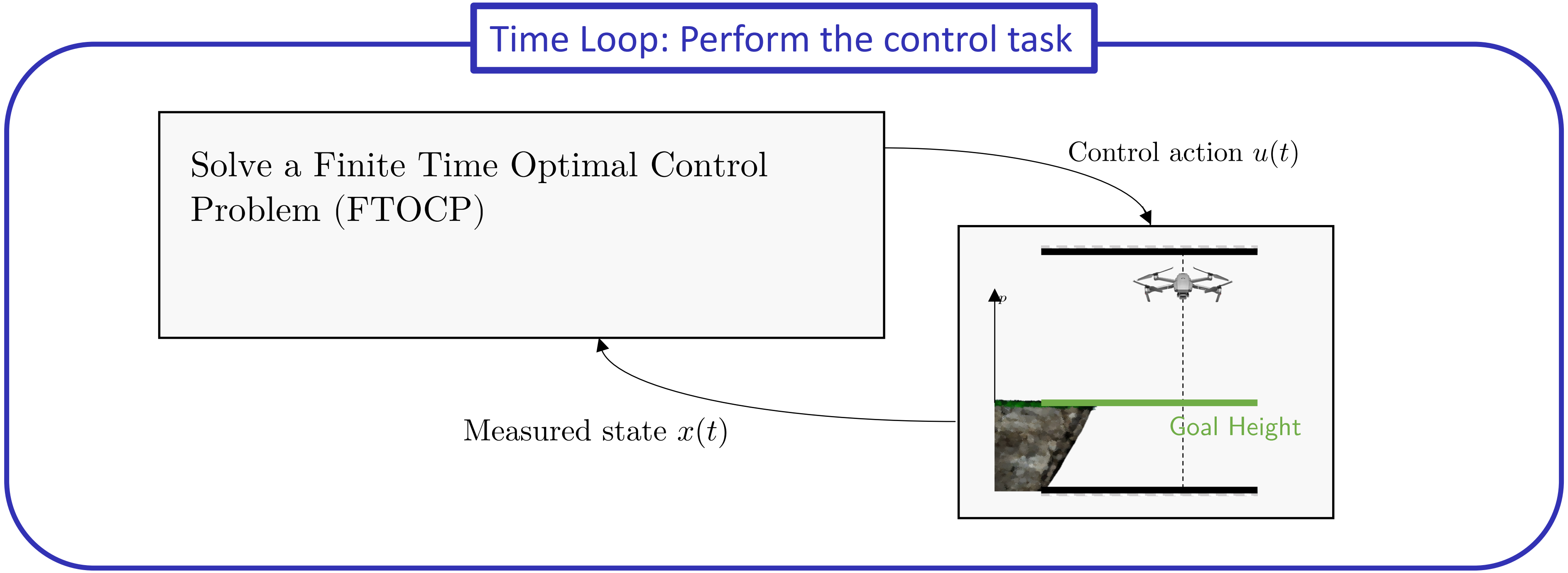

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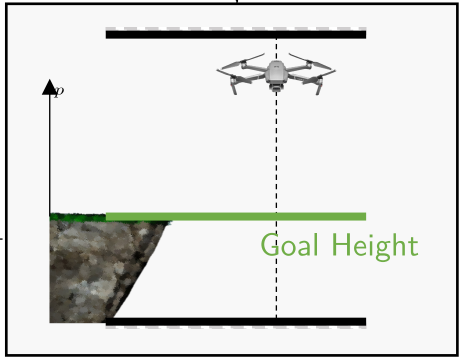
Why did we focus on feasibility?

Time Loop: Perform the control task

Solve a Finite Time Optimal Control Problem (FTOCP) where we enforce

$$x(t) \in \mathcal{X} \text{ and } u(t) \in \mathcal{U}$$

Control action $u(t)$



Measured state $x(t)$

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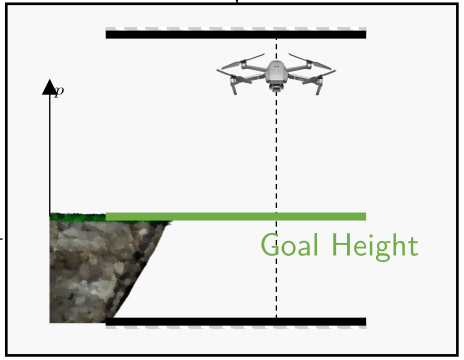

Measured state $x(t)$

Feasibility of the FTCOP at all times

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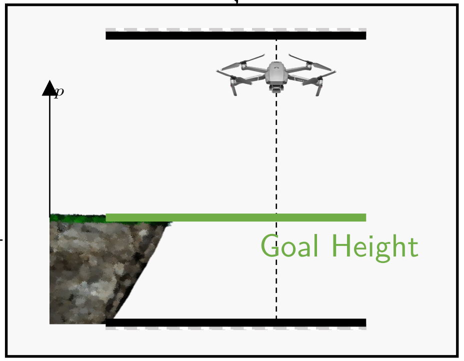

Measured state $x(t)$

Feasibility of the FTCOP at all times

Safety

Recap of Lecture #4

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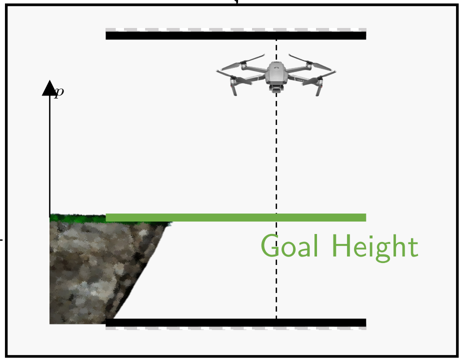

Measured state $x(t)$

Feasibility of the FTCOP at all times

Not true when constraints are relaxed with slack variables!

Safety

Recap of Lecture #4

Stability Proof:

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We have shown that

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$$J_t^*(x(t)) > J_{t+1}^*(x(t+1)),\ \forall x(t) \neq 0$$

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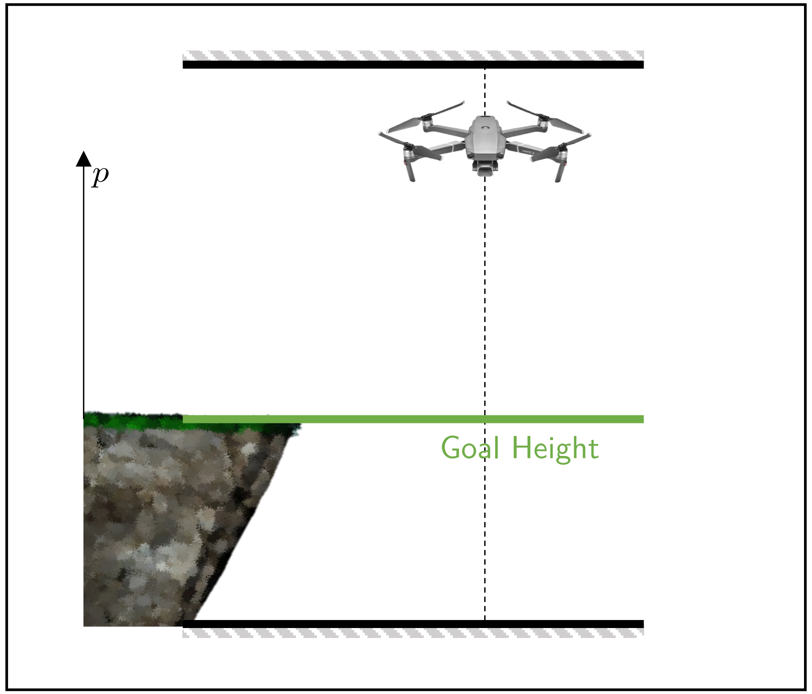


Recap of Lecture #4

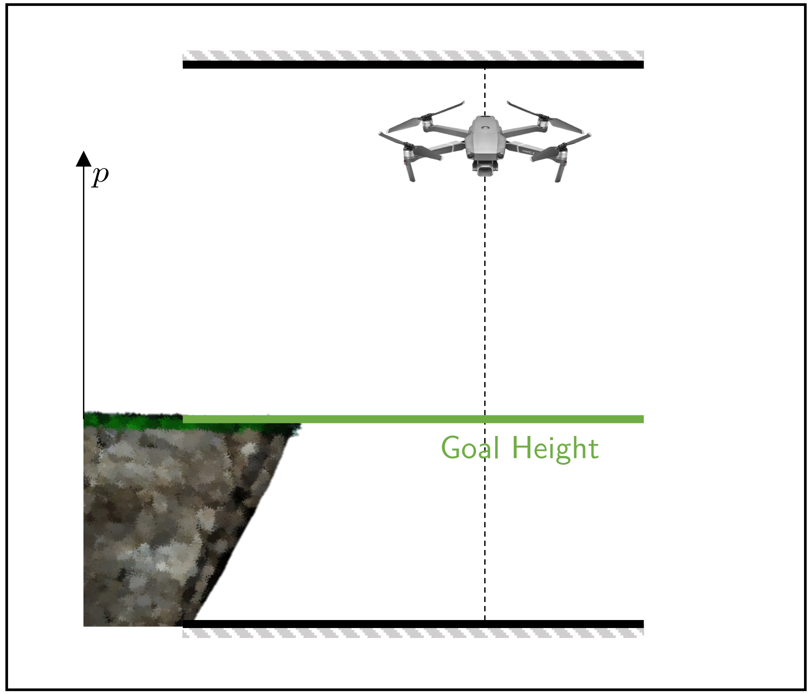
Stability Proof:

We have shown that

$$J_t^*(x(t)) > J_{t+1}^*(x(t+1)),\ \forall x(t) \neq 0$$

$$J_t^*(x) > 0, \forall x \neq 0$$

$$J_t^*(0) = 0$$



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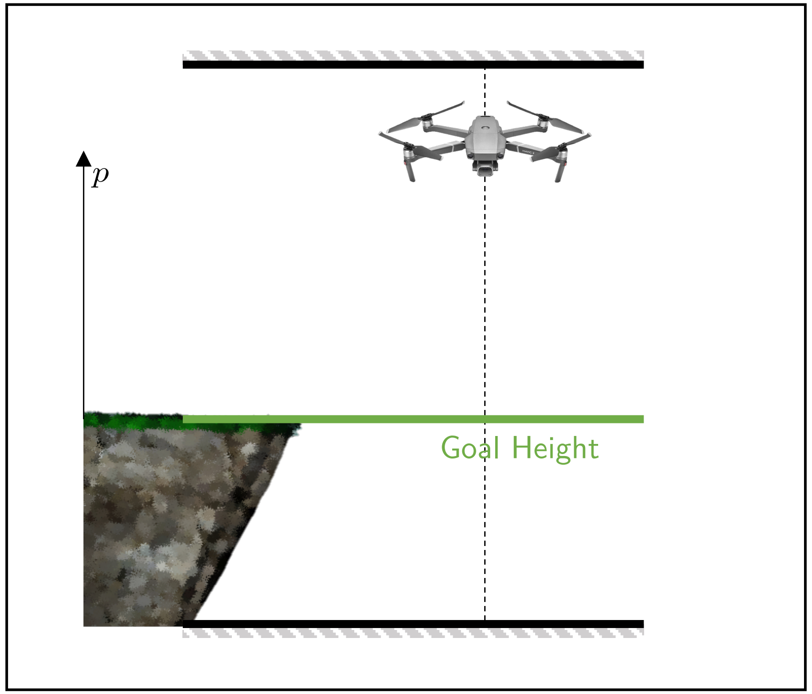

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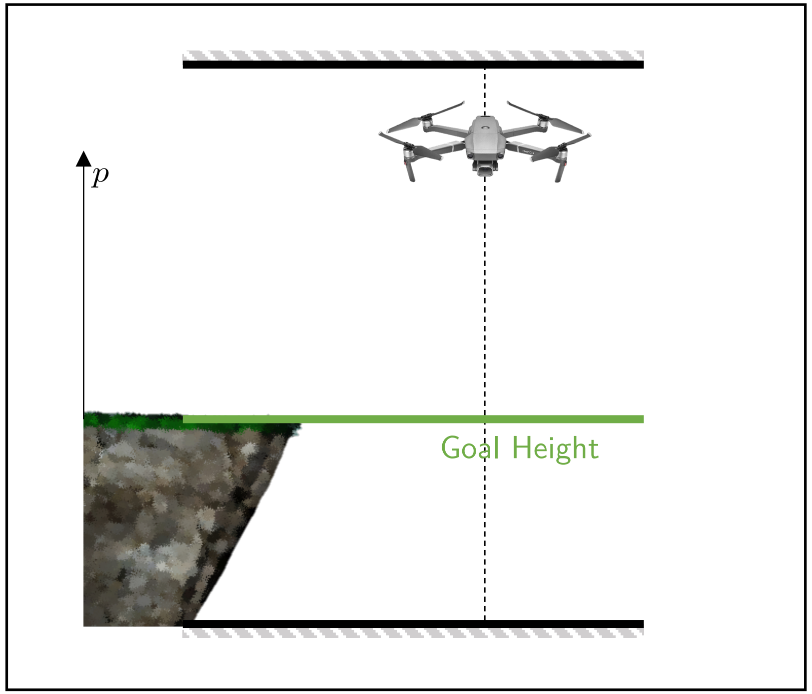

To show stability we need to that $J_t^*(x)$ is Lipchitz continuous and assume that the state and input constraint sets are compact

Francesco Borrelli, Alberto Bemporad, and Manfred Morari. *Predictive control for linear and hybrid systems*. Cambridge University Press, 2017.

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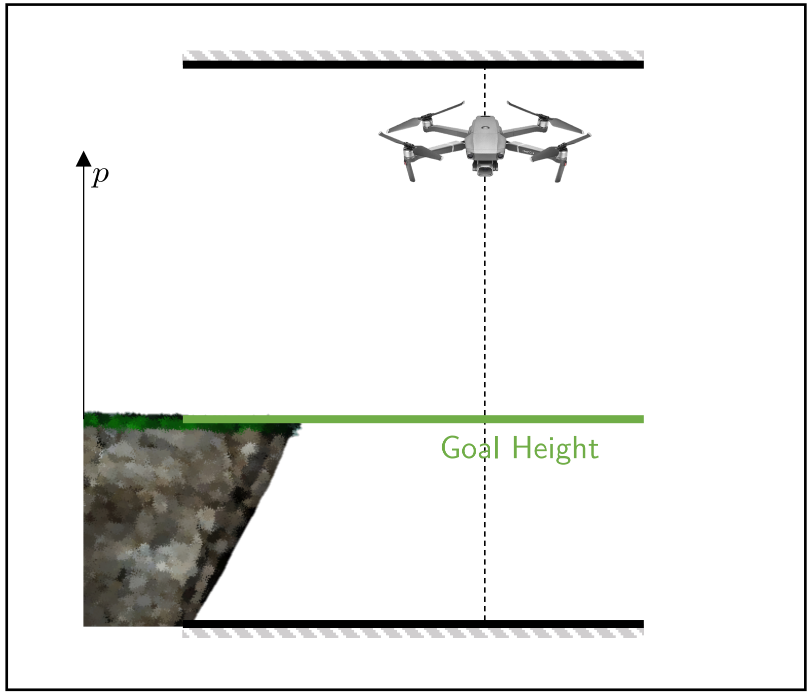
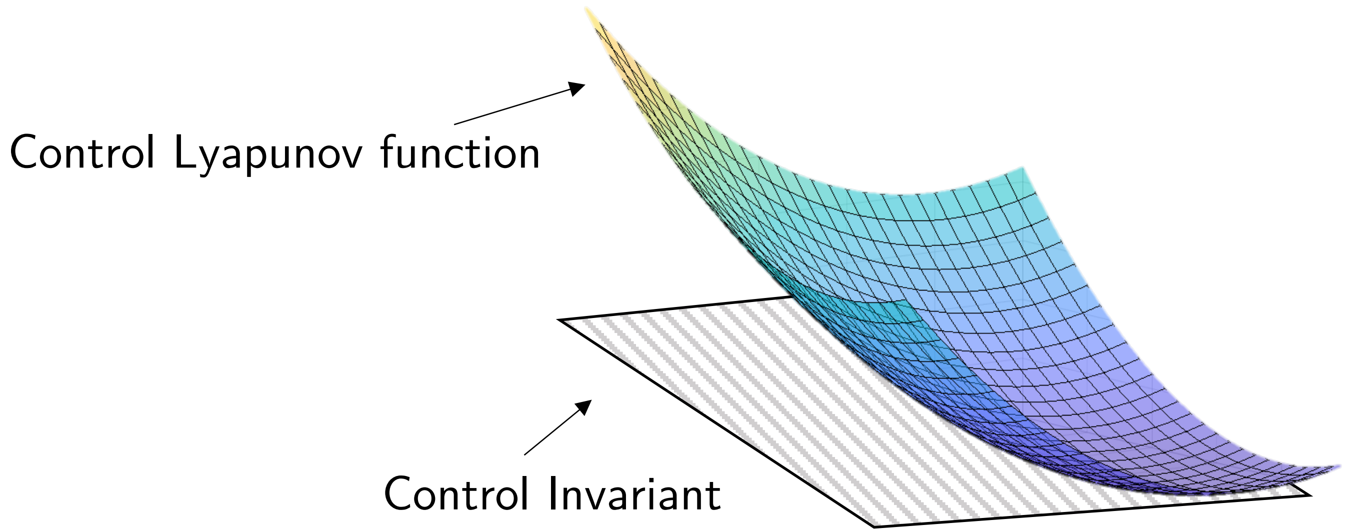


To show stability we need to that $J_t^*(x)$ is Lipchitz continuous and assume that the state and input constraint sets are compact (can be relaxed to continuity around the origin).

Francesco Borrelli, Alberto Bemporad, and Manfred Morari. *Predictive control for linear and hybrid systems*. Cambridge University Press, 2017.
Lars Grüne, and Christopher M. Kellett. "ISS-Lyapunov functions for discontinuous discrete-time systems." IEEE Transactions on Automatic Control 59, no. 11 (2014): 3098-3103.

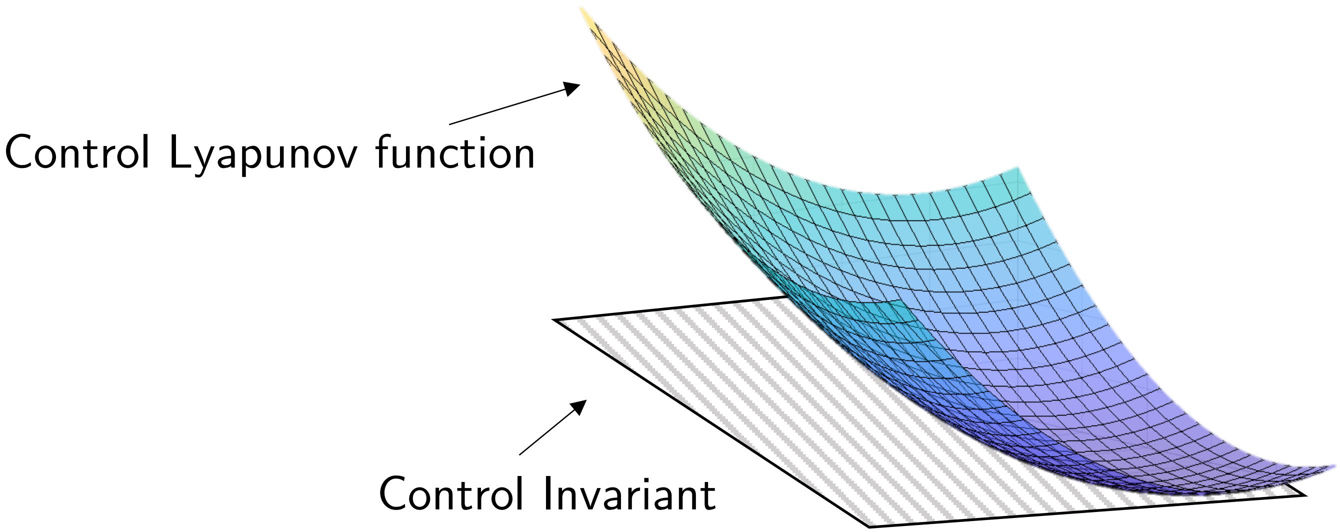
Recap of Lecture #4

Design of the terminal MPC components



Recap of Lecture #4

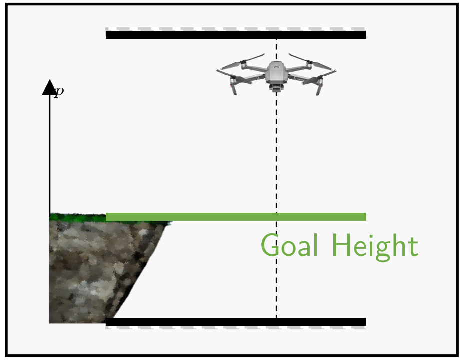
Design of the terminal MPC components



Safety and Stability

LMPC Diagram

Time Loop: Perform the control task



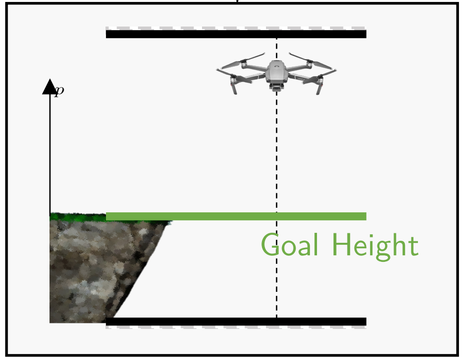
LMPC Diagram

Time Loop: Perform the control task

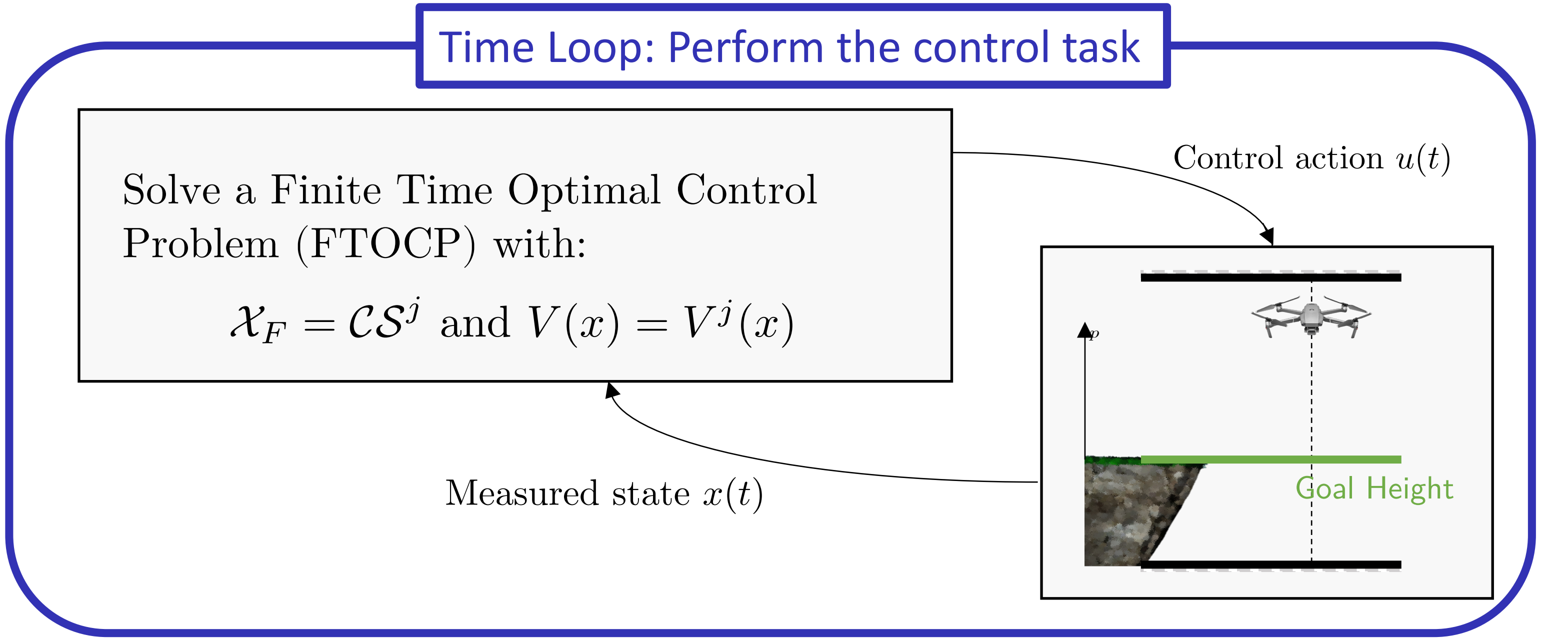
Solve a Finite Time Optimal Control Problem (FTOCP) with:

$$\mathcal{X}_F = \mathcal{CS}^j \text{ and } V(x) = V^j(x)$$

Control action $u(t)$



LMPC Diagram



LMPC Diagram

Iteration Loop: Update terminal components

Time Loop: Perform the control task

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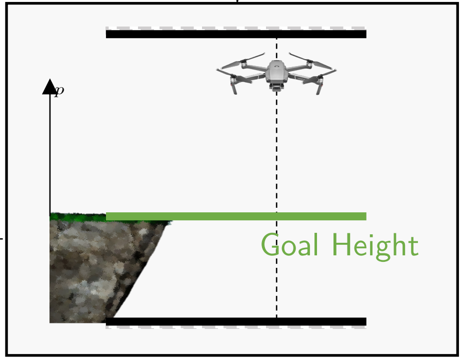
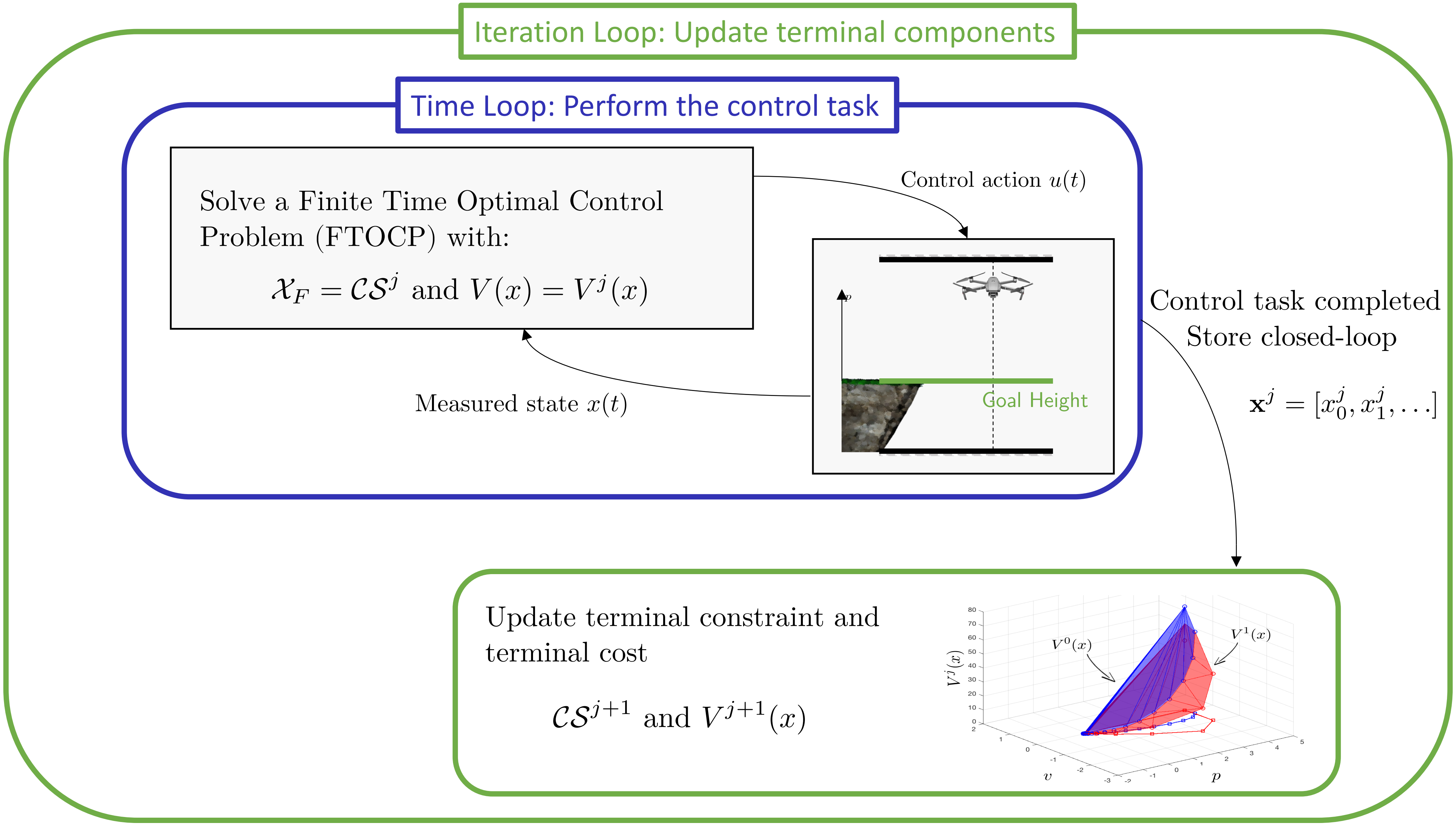
Control action $u(t)$



Measured state $x(t)$

LMPC Diagram



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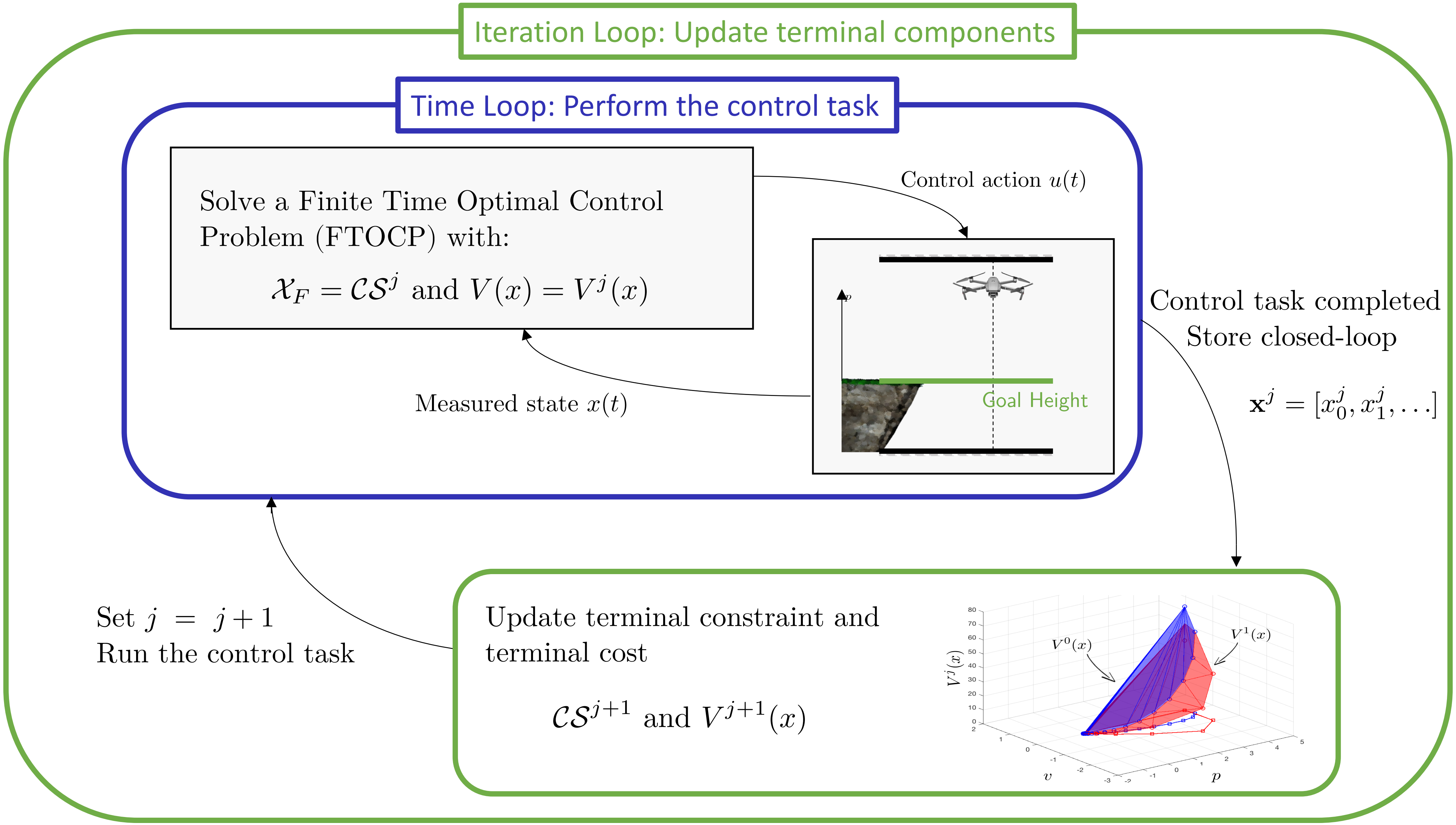

LMPC Recap

Theorem

Let $\mathbf{x}^j = [x_0^j, x_1^j, \ldots]$ be the closed-loop trajectory from the starting state x_S at iteration j. Consider sequence $\{\mathbf{x}^j\}$ of closed-loop trajectories and assume that for $c < \infty$ we have that

$$\mathbf{x}^c = \mathbf{x}^{c+1}$$

Then we have that

- At each iteration state and input constraints are satisfied.
- The closed-loop cost $J_{0\to\infty}^j(x_S)$ is non-increasing, i.e.,

$$J_{0\to\infty}^{j+1}(x_S) = \sum_{t=0}^{\infty} h(x_t^{j+1}, u_t^{j+1}) \leq \sum_{t=0}^{\infty} h(x_t^j, u_t^j) = J_{0\to\infty}^j(x_S)$$

- $\mathbf{x}^c = \mathbf{x}^*$, under mild conditions (LICQ holds at each time t)

Non-increasing closed-loop cost: Proof

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Step 1: $J_{0\rightarrow\infty}^{j-1}(x_S) \geq J_{0\rightarrow N}^{LMPC,j}(x_S)$

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$$J^{j-1}_{0\rightarrow\infty}(x_S) = \sum_{k=0}^{\infty} h(x_k^{j-1}, u_k^{j-1}) = \sum_{k=0}^{N-1} h(x_k^{j-1}, u_k^{j-1}) + \sum_{k=N}^{\infty} h(x_k^{j-1}, u_k^{j-1})$$

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$$\rightarrow \quad J^{j-1}_{0\to\infty}(x_S) \geq \sum_{k=0}^{N-1} h(x^{j-1}_k, u^{j-1}_k) + Q^{j-1}(x^{j-1}_N)$$

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$$\rightarrow \; J^{j-1}_{0\rightarrow\infty}(x_S) \geq \underbrace{\sum_{k=0}^{N-1} h(x_k^{j-1}, u_k^{j-1}) + Q^{j-1}(x_N^{j-1})}_{\text{Cost associated with a feasible trajectory}}$$

Non-increasing closed-loop cost: Proof

Step 1: $J_{0\rightarrow\infty}^{j-1}(x_S) \geq J_{0\rightarrow N}^{LMPC,j}(x_S)$

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Non-increasing closed-loop cost: Proof

Step 1: $J_{0\to\infty}^{j-1}(x_S) \geq J_{0\to N}^{LMPC,j}(x_S)$

From standard MPC arguments

Step 2: $J_{0\to N}^{LMPC,j}(x_S) \geq J_{0\to\infty}^{j}(x_S)$

$$J_{1\to 1+N}^{LMPC,j}(x_1^j) - J_{0\to N}^{LMPC,j}(x_0^j) \leq -h(x_0^j, u_0^j)$$

Non-increasing closed-loop cost: Proof

Step 1: $J^{j-1}_{0\rightarrow\infty}(x_S) \geq J^{LMPC,j}_{0\rightarrow N}(x_S)$

Step 2: $J^{LMPC,j}_{0\rightarrow N}(x_S) \geq J^{j}_{0\rightarrow\infty}(x_S)$

$$J^{LMPC,j}_{1\rightarrow 1+N}(x^j_1) - J^{LMPC,j}_{0\rightarrow N}(x^j_0) \leq -h(x^j_0, u^j_0)$$

$$\rightarrow \; J^{LMPC,j}_{0\rightarrow N}(x^j_0) \geq J^{LMPC,j}_{1\rightarrow 1+N}(x^j_1) + h(x^j_0, u^j_0) \geq J^{LMPC,j}_{2\rightarrow 2+N}(x^j_2) + h(x^j_0, u^j_0) + h(x^j_1, u^j_1)$$

Non-increasing closed-loop cost: Proof

Step 1: $J_{0\to\infty}^{j-1}(x_S) \geq J_{0\to N}^{LMPC,j}(x_S)$

Step 2: $J_{0\to N}^{LMPC,j}(x_S) \geq J_{0\to\infty}^{j}(x_S)$

$$J_{1\to 1+N}^{LMPC,j}(x_1^j) - J_{0\to N}^{LMPC,j}(x_0^j) \leq -h(x_0^j, u_0^j)$$

$$\to \; J_{0\to N}^{LMPC,j}(x_0^j) \geq J_{1\to 1+N}^{LMPC,j}(x_1^j) + h(x_0^j, u_0^j) \geq J_{2\to 2+N}^{LMPC,j}(x_2^j) + h(x_0^j, u_0^j) + h(x_1^j, u_1^j)$$

$$\to \; J_{0\to N}^{LMPC,j}(x_0^j) \geq \lim_{t\to\infty} J_{t\to t+N}^{LMPC,j}(x_t^j) + \sum_{k=0}^{\infty} h(x_k^j, u_k^j)$$

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$$\rightarrow \; J_{0\rightarrow N}^{LMPC,j}(x_0^j) \geq \underset{0}{\cancel{\lim_{t\rightarrow\infty} J_{t\rightarrow t+N}^{LMPC,j}(x_t^j)}} + \sum_{k=0}^{\infty} h(x_k^j, u_k^j)$$

Non-increasing closed-loop cost: Proof

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$$\rightarrow \; J_{0\rightarrow N}^{LMPC,j}(x_0^j) \geq \underbrace{\cancel{\lim_{t\rightarrow\infty} J_{t\rightarrow t+N}^{LMPC,j}(x_t^j)}}_{0} + \sum_{k=0}^{\infty} h(x_k^j, u_k^j)$$

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Non-increasing closed-loop cost: Proof

Step 1: $J^{j-1}_{0\rightarrow\infty}(x_S) \geq J^{LMPC,j}_{0\rightarrow N}(x_S)$

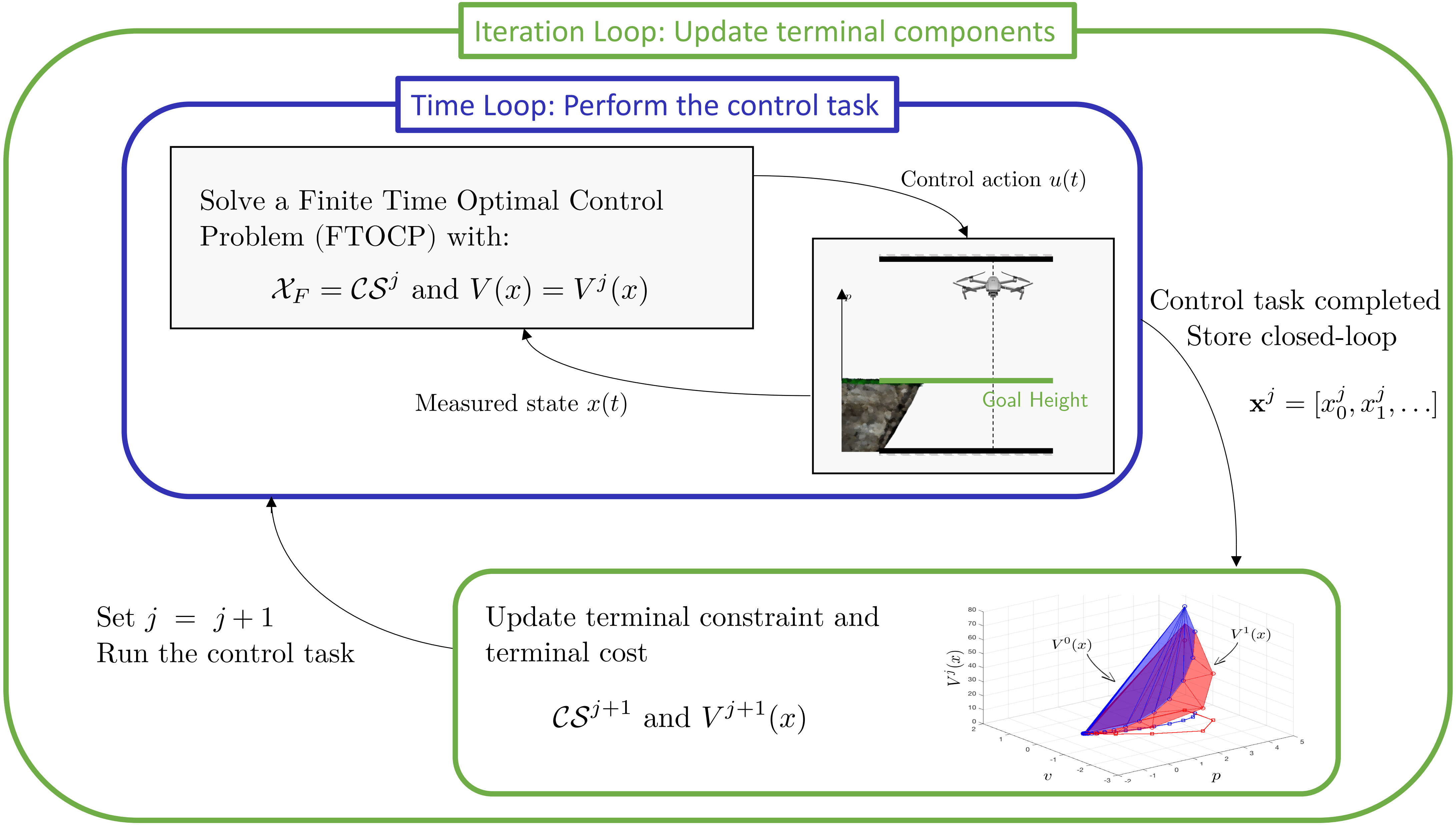
Step 2: $J^{LMPC,j}_{0\rightarrow N}(x_S) \geq J^{j}_{0\rightarrow\infty}(x_S)$

Conclusion

The iteration cost $J^{j}_{0\rightarrow\infty}(x_s)$ is non-increasing at each iteration,

$$J^{j-1}_{0\rightarrow\infty}(x_S) \geq J^{LMPC,j}_{0\rightarrow N}(x_S) \geq J^{j}_{0\rightarrow\infty}(x_S)$$

LMPC Diagram



Linear(ized) LMPC

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (FTOCP)

$$J^{\mathrm{LMPC},j}_{t\to t+N}(x^j_t) = \min_{u_{t|t},\ldots,u_{t+N-1|t}} \sum_{k=t}^{t+N-1} h(x_{k|t}, u_{k|t}) + V^{j-1}(x_{t+N|t})$$

s.t.

$$x_{k+1|t} = Ax_{k|t} + Bu_{k|t}, \ \forall k \in [t, \cdots, t+N-1]$$

$$x_{t|t} = x^j_t,$$

$$x_{k|t} \in \mathcal{X}, \ u_{k|t} \in \mathcal{U}, \ \forall k \in [t, \cdots, t+N-1]$$

$$x_{t+N|t} \in \mathcal{CS}^{j-1}$$

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s.t.

$$x_{k+1|t} = Ax_{k|t} + Bu_{k|t},\ \forall k \in [t, \cdots, t+N-1]$$

$$x_{t|t} = x^j_t,$$

$$x_{k|t} \in \mathcal{X},\ u_{k|t} \in \mathcal{U},\ \forall k \in [t, \cdots, t+N-1]$$

$$\boxed{x_{t+N|t} = \sum_{i=0}^{j-1}\sum_k x^i_k\lambda^i_k,\ \sum_{i=0}^{j-1}\sum_k \lambda^i_k = 1,\ \lambda^i_k \geq 0.} \Longleftrightarrow \boxed{x_{t+N|t} \in \mathcal{CS}^{j-1}}$$

Linear(ized) LMPC

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (FTOCP)

$$J^{\mathrm{LMPC},j}_{t\to t+N}(x^j_t) = \min_{\substack{u_{t|t},\dots,u_{t+N-1|t}\\ \boxed{\lambda^0_0,\dots,\lambda^{j-1}_{T_f}}}} \sum_{k=t}^{t+N-1} h(x_{k|t},u_{k|t}) + \boxed{\sum_{i=0}^{j-1}\sum_k J^i_k(x^i_k)\lambda^i_k}$$

$$\Updownarrow$$

$$\boxed{V^{j-1}(x_{t+N|t})}$$

s.t.

$$x_{k+1|t} = Ax_{k|t} + Bu_{k|t},\ \forall k\in[t,\cdots,t+N-1]$$

$$x_{t|t} = x^j_t,$$

$$x_{k|t}\in\mathcal{X},\ u_{k|t}\in\mathcal{U},\ \forall k\in[t,\cdots,t+N-1]$$

$$\boxed{x_{t+N|t} = \sum_{i=0}^{j-1}\sum_k x^i_k\lambda^i_k,\quad \sum_{i=0}^{j-1}\sum_k \lambda^i_k = 1,\ \lambda^i_k\geq 0.} \Longleftrightarrow \boxed{x_{t+N|t}\in\mathcal{CS}^{j-1}}$$

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$$\boxed{x_{t+N|t} = \sum_{i=0}^{j-1}\sum_k x^i_k\lambda^i_k,\ \ \sum_{i=0}^{j-1}\sum_k \lambda^i_k = 1,\ \lambda^i_k \geq 0.} \Longleftrightarrow \boxed{x_{t+N|t} \in \mathcal{CS}^{j-1}}$$

- Convex optimization problem over inputs and lambdas
- Safety and performance improvement guarantees still hold (simple proofs as before)
- Converges to global optimal solution (LICQ required)

Learning MPC for Autonomous Racing

Real-time implementation on the Berkeley Autonomous Race Car (BARC)

Problem Formulation

Minimum Time Control Problem

$$\min_{T,\mathbf{u}} \boxed{T} \quad \text{Control objective}$$

$$\boxed{x_0 = x_s,\ x_T = \mathcal{X}_F} \quad \text{Start \& end position}$$

System dynamics
System constraints

$$\boxed{x_{k+1} = f(x_k, u_k), \quad \forall k \in \{0, \dots, T-1\}}$$

Safety constraints

$$\boxed{x_k \in \mathcal{X},\ u_k \in \mathcal{U}, \quad \forall k \in \{0, \dots, T-1\}}$$

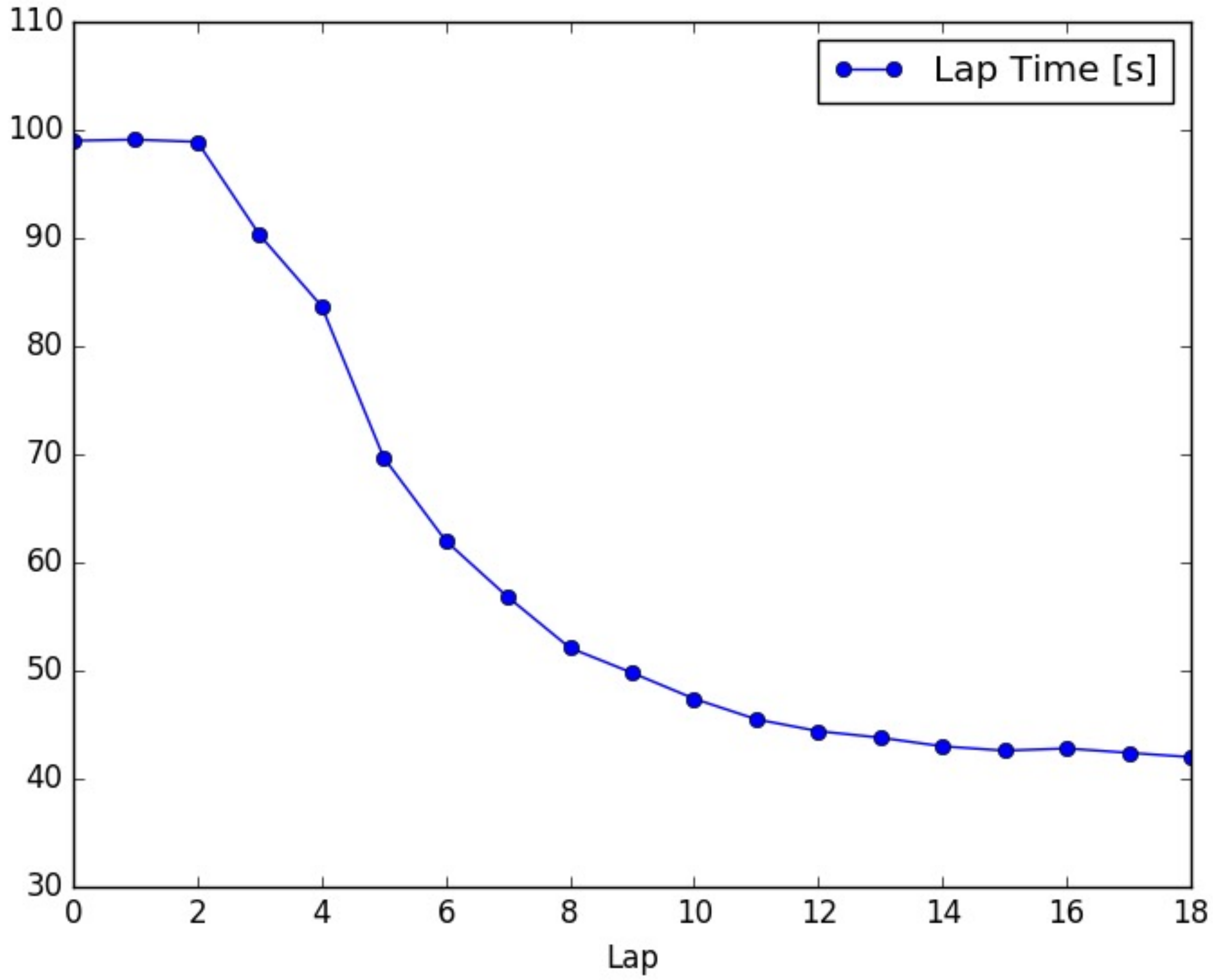
Learning Model Predictive Controller full-size vehicle experiments

Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia

Learning Model Predictive Controller full-size vehicle experiments

Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia

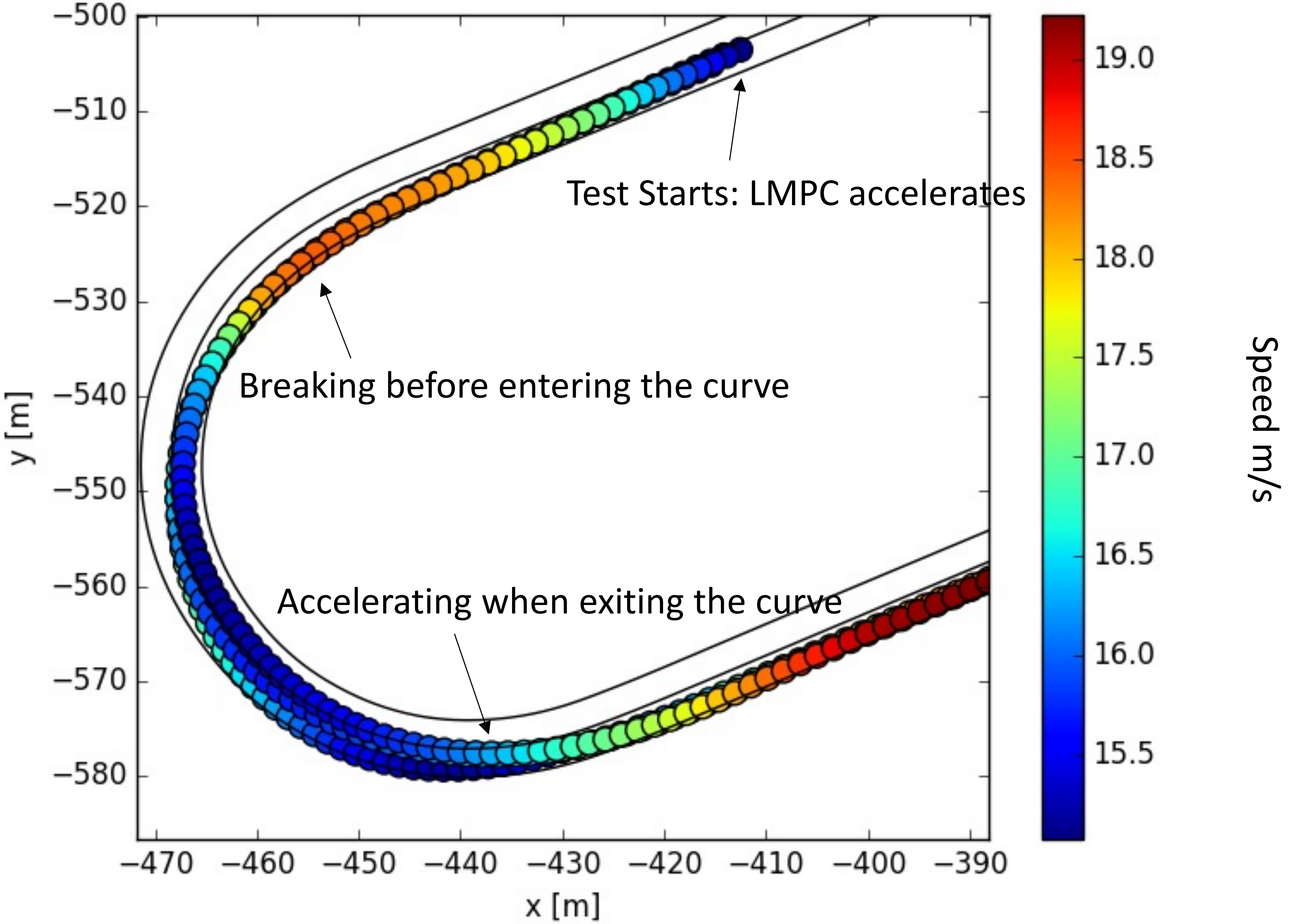
Lap Time



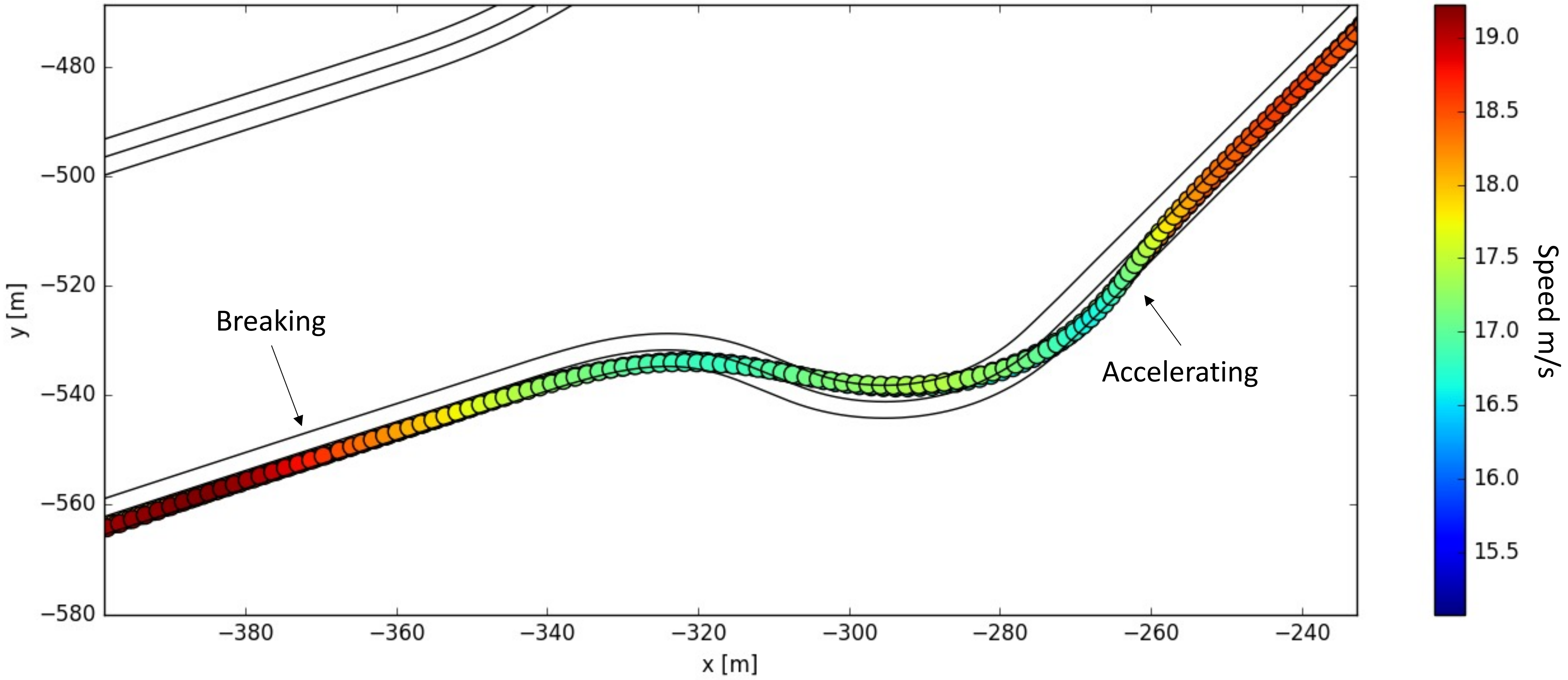
Learning Model Predictive Controller full-size vehicle experiments

Credits: Siddharth Nair, Nitin Kapania and Ugo Rosolia

Velocity Profile at Convergence (Curve 1)



Velocity Profile at Convergence (Chicane)



Problem Formulation

Minimum Time Control Problem

$$\min_{T,\mathbf{u}} \quad \boxed{T}$$ Control objective

$$\boxed{x_0 = x_s, \; x_T = \mathcal{X}_F}$$ Start & end position

System dynamics
System constraints
$$\boxed{x_{k+1} = f(x_k, u_k), \quad \forall k \in \{0, \ldots, T-1\}}$$

Safety constraints
$$\boxed{x_k \in \mathcal{X}, \; u_k \in \mathcal{U}, \quad \forall k \in \{0, \ldots, T-1\}}$$

Problem Formulation

Minimum Time Control Problem

$$\min_{T,\mathbf{u}} \boxed{T}$$ Control objective

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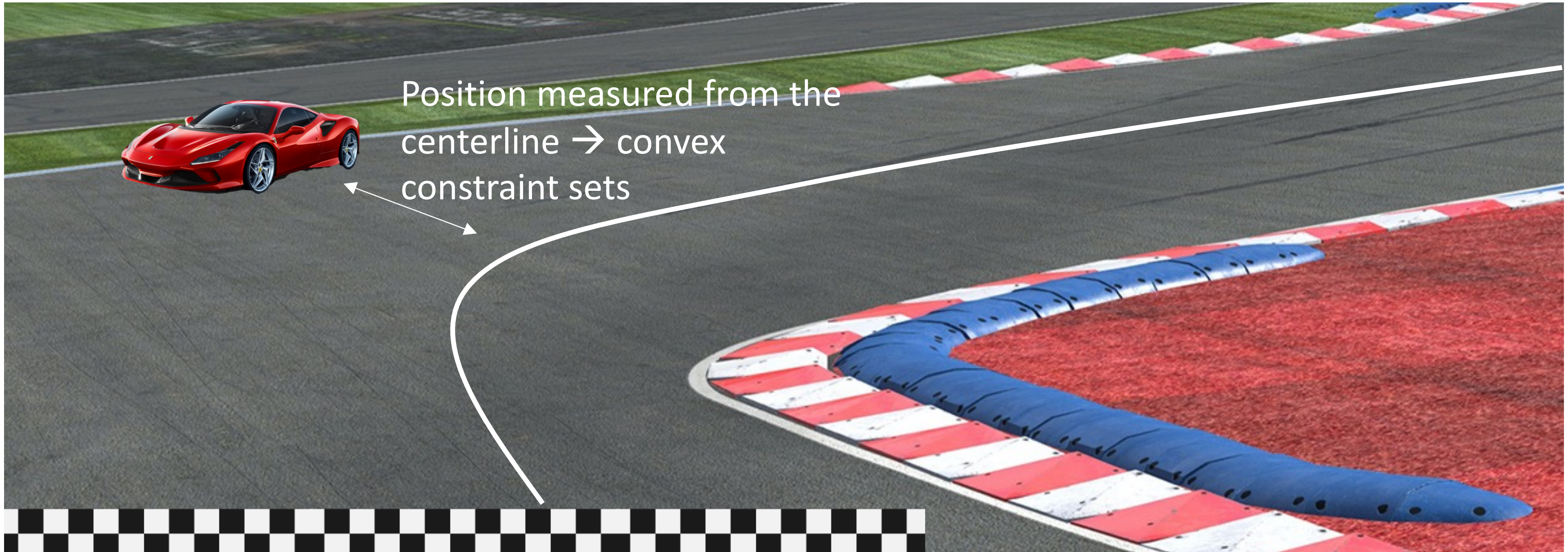
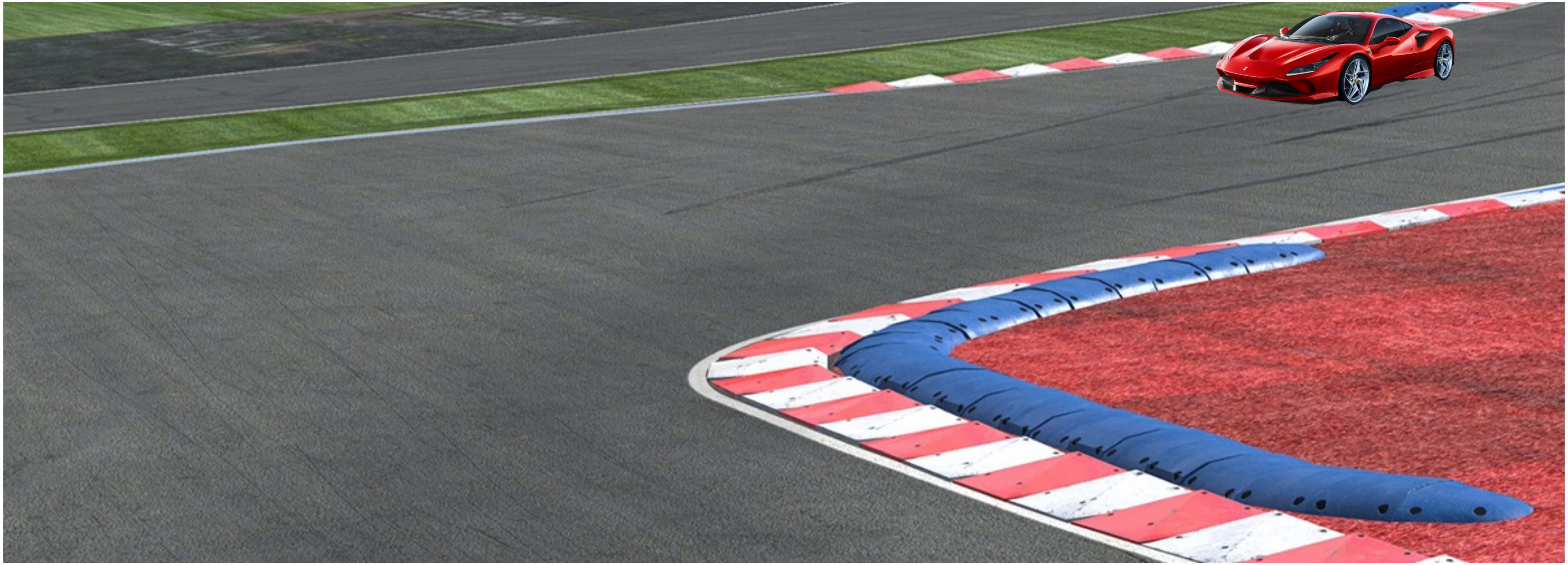
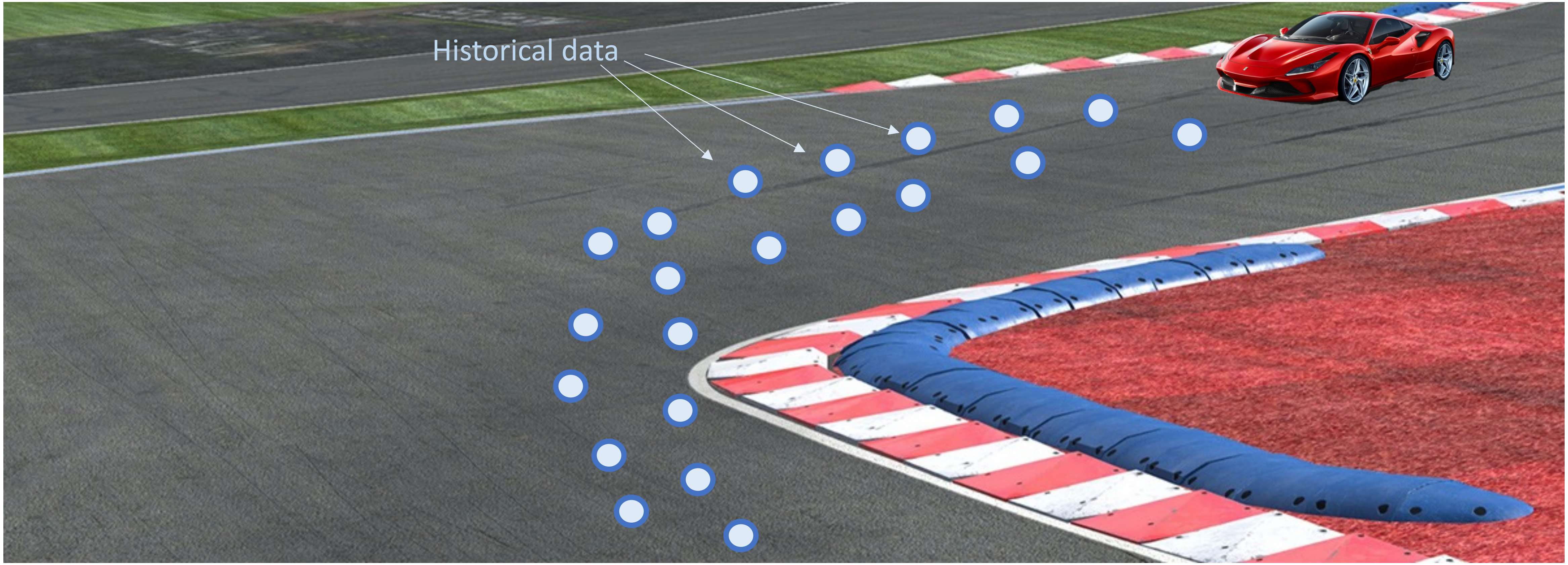


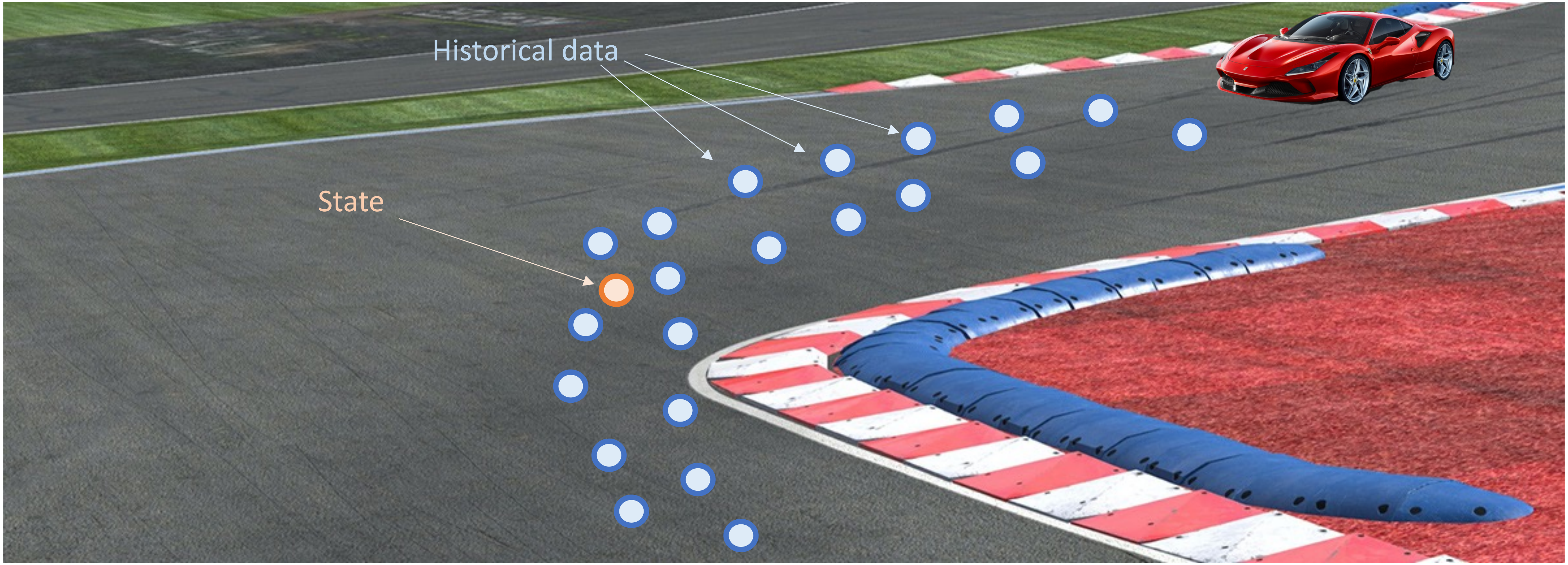
Local Approximations



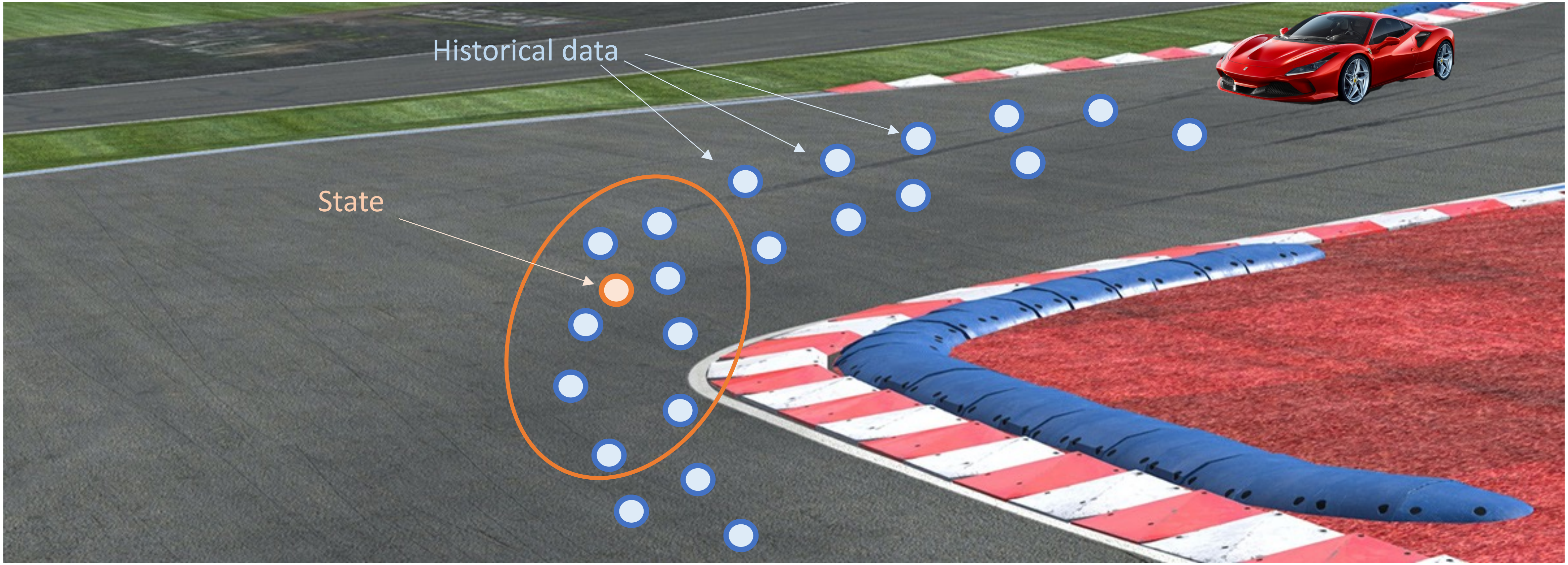
Local Approximations



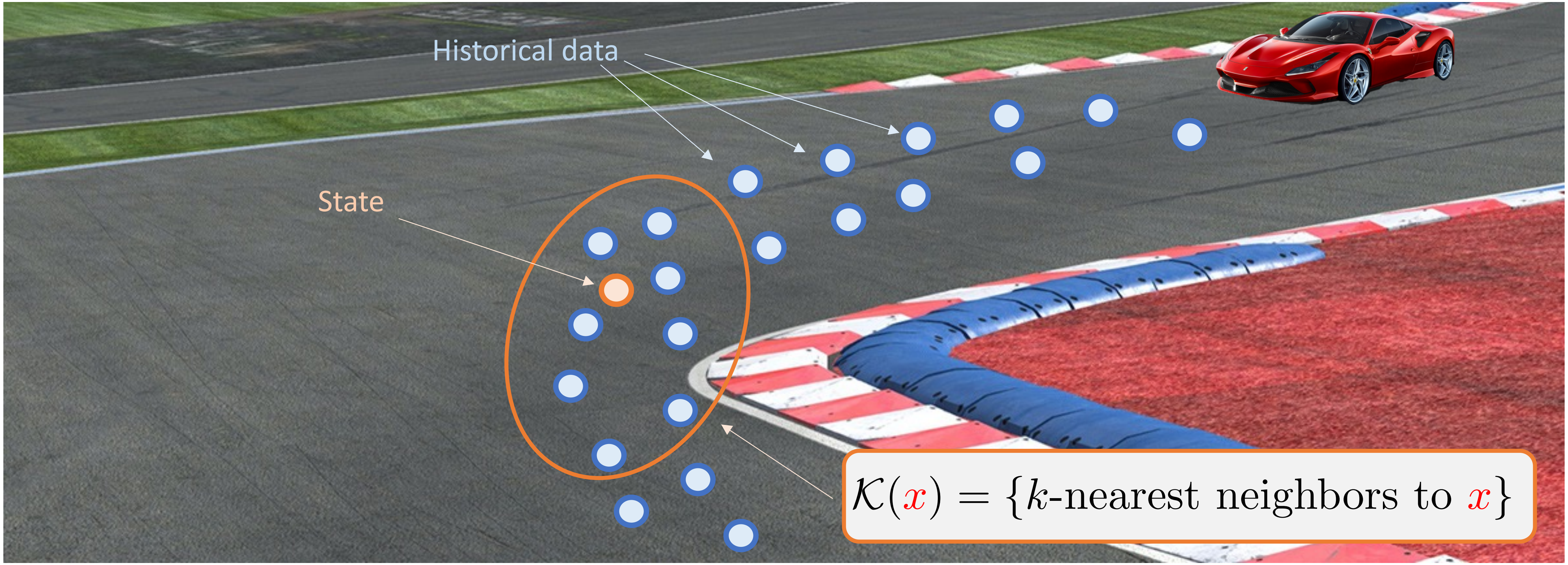
Local Approximations



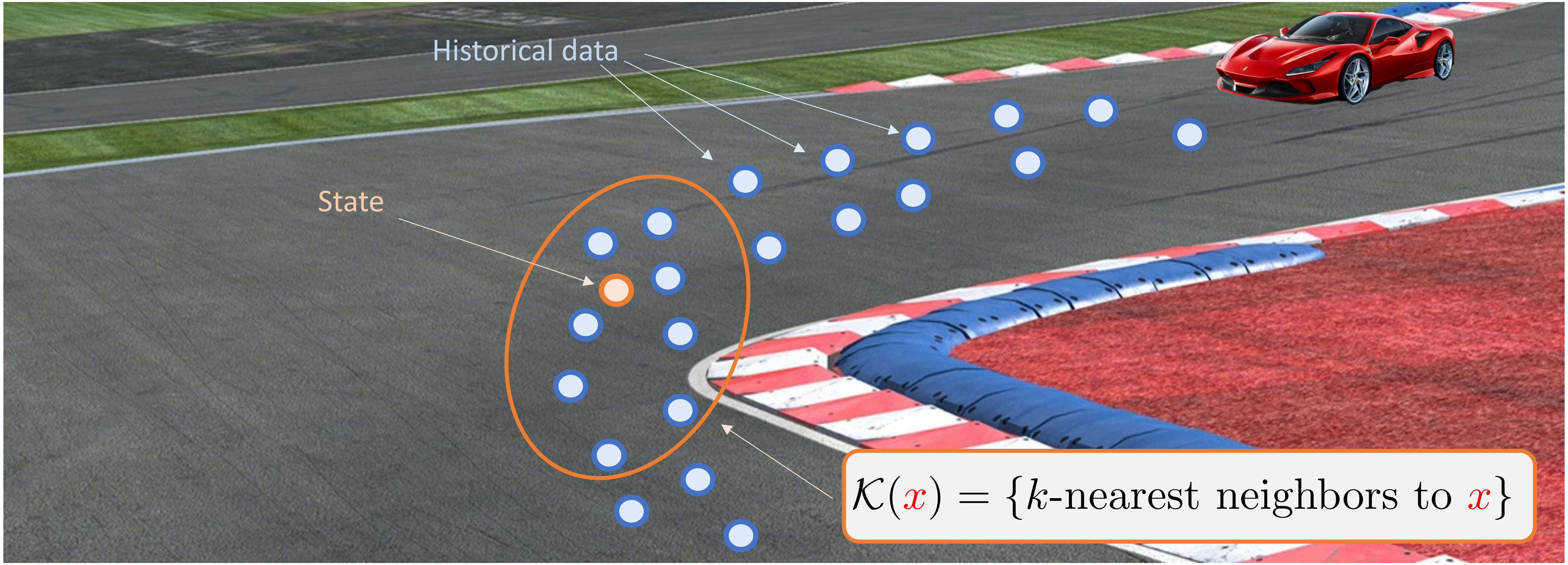
Local Approximations



Local Approximations



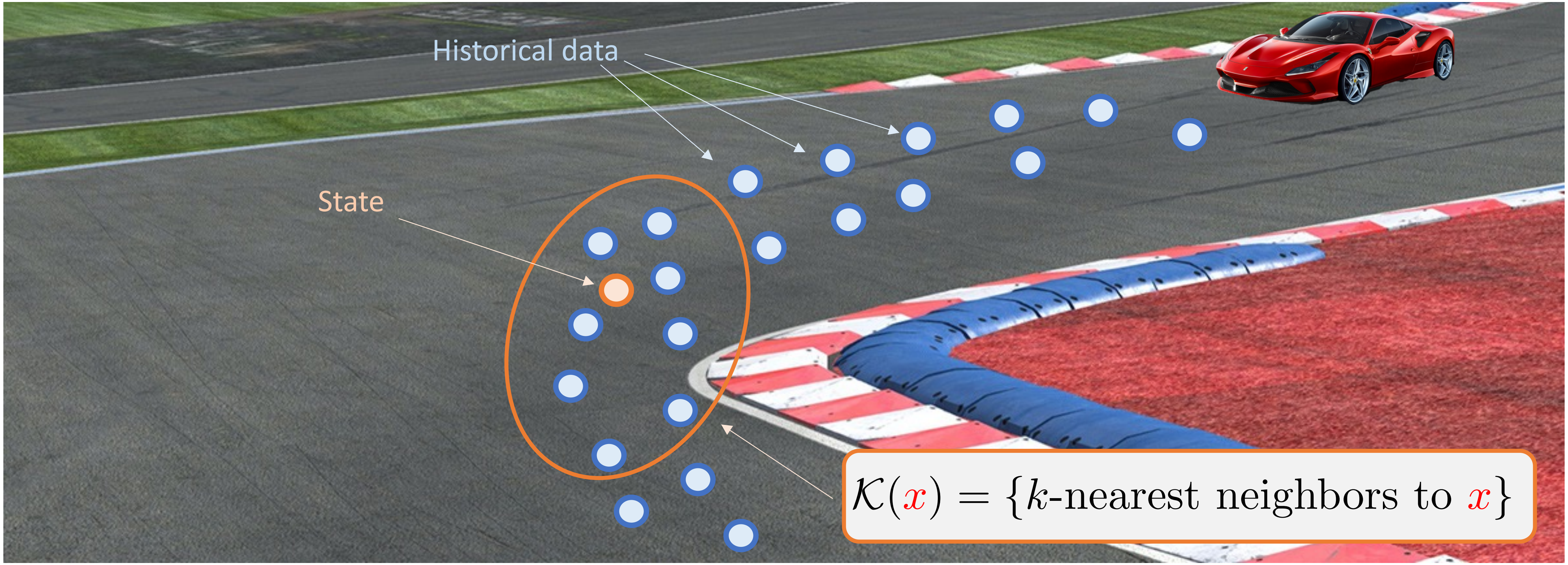
Local Approximations



Local convex safe set approximation:

$$\mathcal{CS}^j(x) = \text{conv}\left(\cup_{x_t^j \in \mathcal{K}(x)} x_t^j \right)$$

Local Approximations



Local value function approximation:

$$V^j(\bar{x}, x) = \min_{\lambda_t^i \geq 0} \quad \sum_{x_t^i \in \mathcal{K}^j(x)} J_t^i(x_t^i)\lambda_t^i$$

$$\text{subject to} \quad \sum_{x_t^i \in \mathcal{K}^j(x)} x_t^i \lambda_t^i = \bar{x}, \sum_i \sum_t \lambda_t^i = 1$$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

$$J^{\mathrm{LMPC},j}_{t\rightarrow t+N}(x^j_t) = \min_{u^j_{t|t},\ldots,u^j_{t+N-1|t}} \sum_{k=t}^{t+N-1} h(x^j_{k|t}, u^j_{k|t}) + V^{j-1}(x^j_{t+N|t}, x^+_{t+N})$$

s.t.

$$x^j_{k+1|t} = A^j_{k|t} x^j_{k|t} + B^j_{k|t} u^j_{k|t} + C^j_{k|t}$$

$$x^j_{t|t} = x^j_t,$$

$$x^j_{k|t} \in \mathcal{X},\ u^j_{k|t} \in \mathcal{U},\ \forall k \in [t, \cdots, t+N-1]$$

$$x^j_{t+N|t} \in \ \mathcal{CS}^{j-1}(x^+_{t+N}),$$

Learning Model Predictive Controller

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$$x^j_{t+N|t} \in \boxed{\mathcal{CS}^{j-1}(x^+_{t+N})},$$

Computed online
using LMPC strategy

where $x^+_{t+N} = x^*_{t+N-1|t-1}$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

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Computed online using LMPC strategy

where $x^+_{t+N} = x^*_{t+N-1|t-1}$ or $x^+_{t+N} = \hat{f}(x^*_{t+N-1|t-1}, u_f)$

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

$$J^{\mathrm{LMPC},j}_{t\rightarrow t+N}(x^j_t) = \min_{u^j_{t|t},\ldots,u^j_{t+N-1|t}} \sum_{k=t}^{t+N-1} h(x^j_{k|t}, u^j_{k|t}) + \boxed{V^{j-1}(x^j_{t+N|t}, x^+_{t+N})}$$

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Computed online

Computed online using LMPC strategy

Learning Model Predictive Controller

At time t of iteration j solve the following Constrained Finite Time Optimal Control Problem (CFTOCP)

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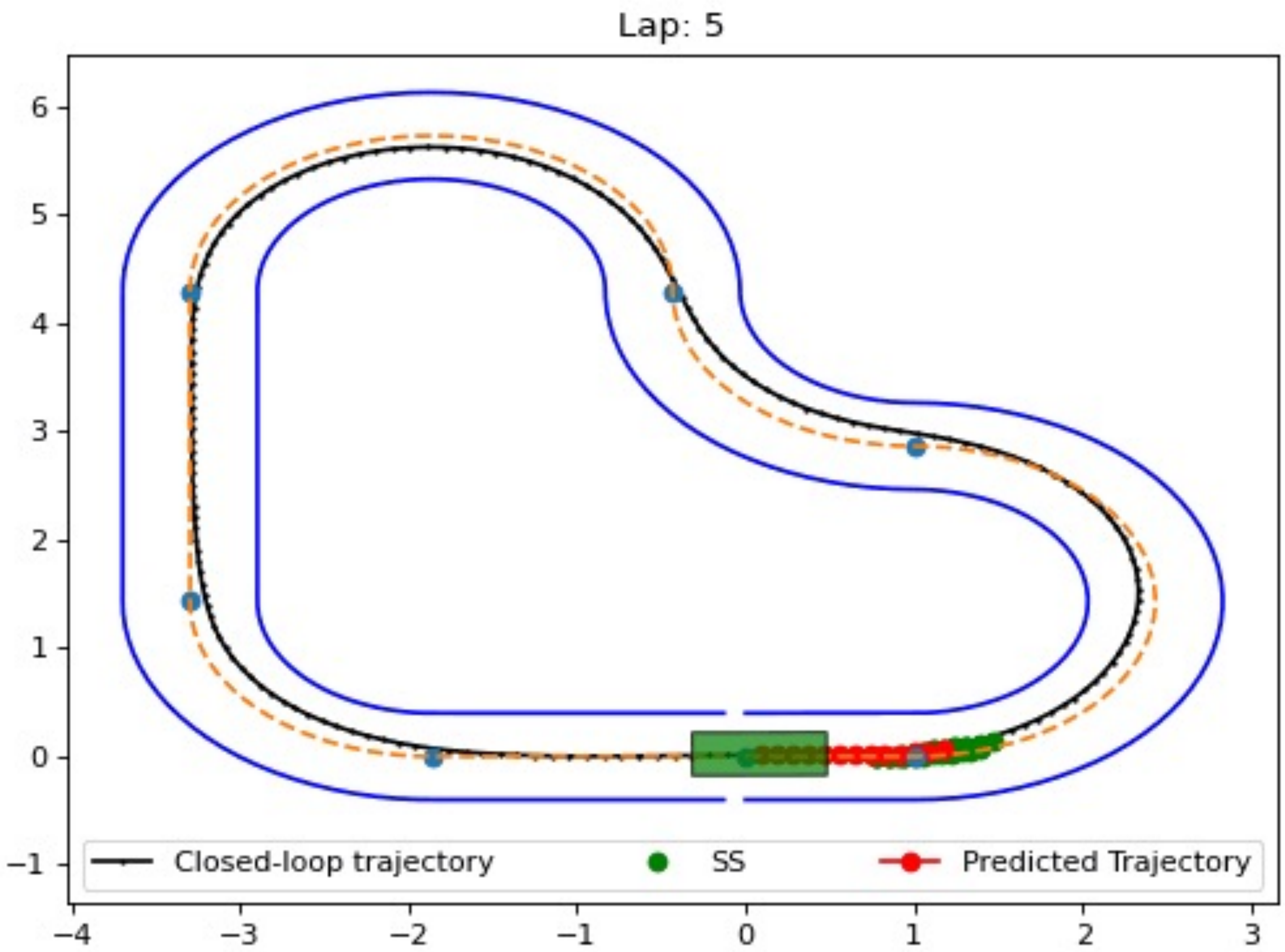
$$x^j_{t+N|t} \in \mathcal{CS}^{j-1}(x^+_{t+N}),$$

Computed online

Computed online using LMPC strategy

Then apply to the system the control input $u^j_t = u^{*,j}_{t|t}$

Safe Set and Value Function Approximations



System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$
\begin{aligned}
\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m}\sum_i F_{x_i} \\
\ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m}\sum_i F_{y_i} \\
\ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}}) \\
\dot{X} &= \dot{x}\cos\psi - \dot{y}\sin\psi, \quad \dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi
\end{aligned}
$$

System ID in Autonomous Racing

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$$
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&\boxed{\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi, \quad \dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi}
\end{aligned}
$$

Kinematic Equations

System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$
\begin{aligned}
\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m}\textstyle\sum_i F_{x_i} \\
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\ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}}) \\
\dot{X} &= \dot{x}\cos\psi - \dot{y}\sin\psi, \quad \dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi
\end{aligned}
$$

Kinematic Equations

- Identifying the Dynamical System

$$
x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} x^j_{k|t} + \begin{bmatrix} \\ \\ \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}
$$

Linearization around predicted trajectory

System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$\boxed{\begin{aligned}\ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m}\sum_i F_{x_i}\\ \ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m}\sum_i F_{y_i}\\ \ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}})\end{aligned}}$$

Dynamic Equations

$$\boxed{\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi, \quad \dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi}$$

Kinematic Equations

- Identifying the Dynamical System

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} x^j_{k|t} + \begin{bmatrix} \\ \\ \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

System ID in Autonomous Racing

- Nonlinear Dynamical System,

$$\begin{aligned} \ddot{x} &= \dot{y}\dot{\psi} + \frac{1}{m}\sum_i F_{x_i} \\ \ddot{y} &= -\dot{x}\dot{\psi} + \frac{1}{m}\sum_i F_{y_i} \\ \ddot{\psi} &= \frac{1}{I_z}(a(F_{y_{1,2}}) - b(F_{y_{2,3}}) + c(-F_{x_{1,3}} + F_{x_{2,4}}) \end{aligned}$$

Dynamic Equations

$$\dot{X} = \dot{x}\cos\psi - \dot{y}\sin\psi, \quad \dot{Y} = \dot{x}\sin\psi + \dot{y}\cos\psi$$

Kinematic Equations

- Identifying the Dynamical System

Local Linear Regression

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{matrix} \multicolumn{3}{c}{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s)\|\Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1}\|, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}} \\ \begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix} & x^j_{k|t} + & \begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix} \end{matrix} \right] \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \boxed{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s) || \Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1} ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}} \\ \begin{bmatrix} \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \end{bmatrix} x^j_{k|t} + \begin{bmatrix} \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \end{bmatrix} \end{array} \right] \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \left[\begin{array}{c} \boxed{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s)||\Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1}||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}} \\ \boxed{\begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix} x^j_{k|t} + \begin{bmatrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{bmatrix}} \end{array} \right] \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- Compute trajectory to linearize around from previous optimal inputs and stored data

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \multicolumn{3}{c}{\boxed{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s)||\Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1}||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}}} \\ \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} & x^j_{k|t} + & \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \end{bmatrix} \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- Compute trajectory to linearize around from previous optimal inputs and stored data
- Enforce model-based sparsity in local linear regression

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \multicolumn{3}{c}{\boxed{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s)||\Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1}||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}}} \\ \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} & x^j_{k|t} + & \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \end{bmatrix} \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}$$

Linearization around predicted trajectory

Implementation Details

- Compute trajectory to linearize around from previous optimal inputs and stored data
- Enforce model-based sparsity in local linear regression
- Perform local linear regression at the linearization points using a subset of the stored data

System Identification – Design Steps

Identifying the Dynamical System

Local Linear Regression

$$
x^j_{k+1|t} = \begin{bmatrix} \dot{x}_{k+1|t} \\ \dot{y}_{k+1|t} \\ \ddot{\psi} \\ \psi_{k+1|t} \\ X_{k+1|t} \\ Y_{k+1|t} \end{bmatrix} = \begin{bmatrix} \boxed{\arg\min \sum_{i,s} K(x^j_{k|t} - x^i_s) || \Lambda_y \begin{bmatrix} x^j_{k|t} \\ u^j_{k|t} \\ 1 \end{bmatrix} - y^i_{s+1} ||, \forall y \in \{\dot{x}, \dot{y}, \ddot{\psi}\}} \\ \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \; x^j_{k|t} + \boxed{\begin{matrix} \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \\ \text{Linearized Kinematics} \end{matrix}} \end{bmatrix} \begin{bmatrix} u^j_{k|t} \\ 1 \end{bmatrix}
$$

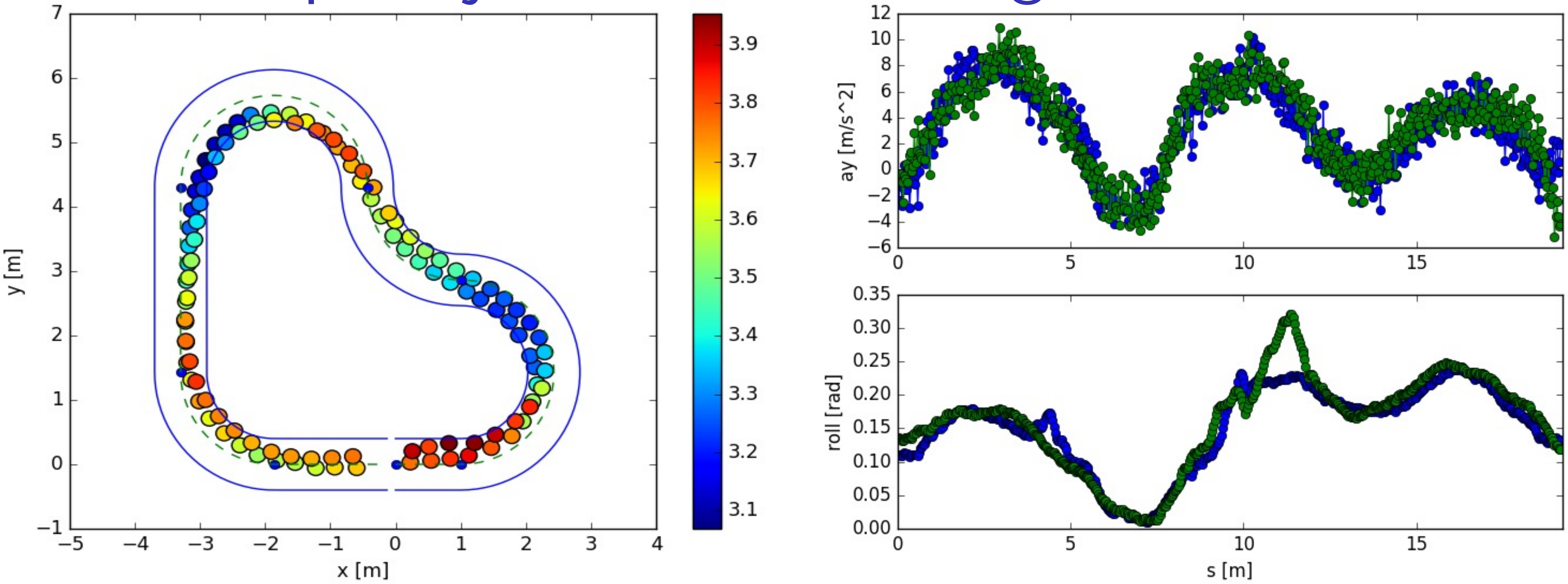
Linearization around predicted trajectory

Implementation Details

- Compute trajectory to linearize around from previous optimal inputs and stored data
- Enforce model-based sparsity in local linear regression
- Perform local linear regression at the linearization points using a subset of the stored data
- Use kernel *K()* to weight differently data as a function of distance to the linearization trajectory

Learning Model Predictictive Control
for Autonomous Racing

Closed-loop Trajectories at Convergence



Remarks

- A **reference trajectory is not needed** for the controller implementation
- The controller converges after few laps: the learning process is data efficient
- The **controller safely explores the state space** iteratively improving the lap time

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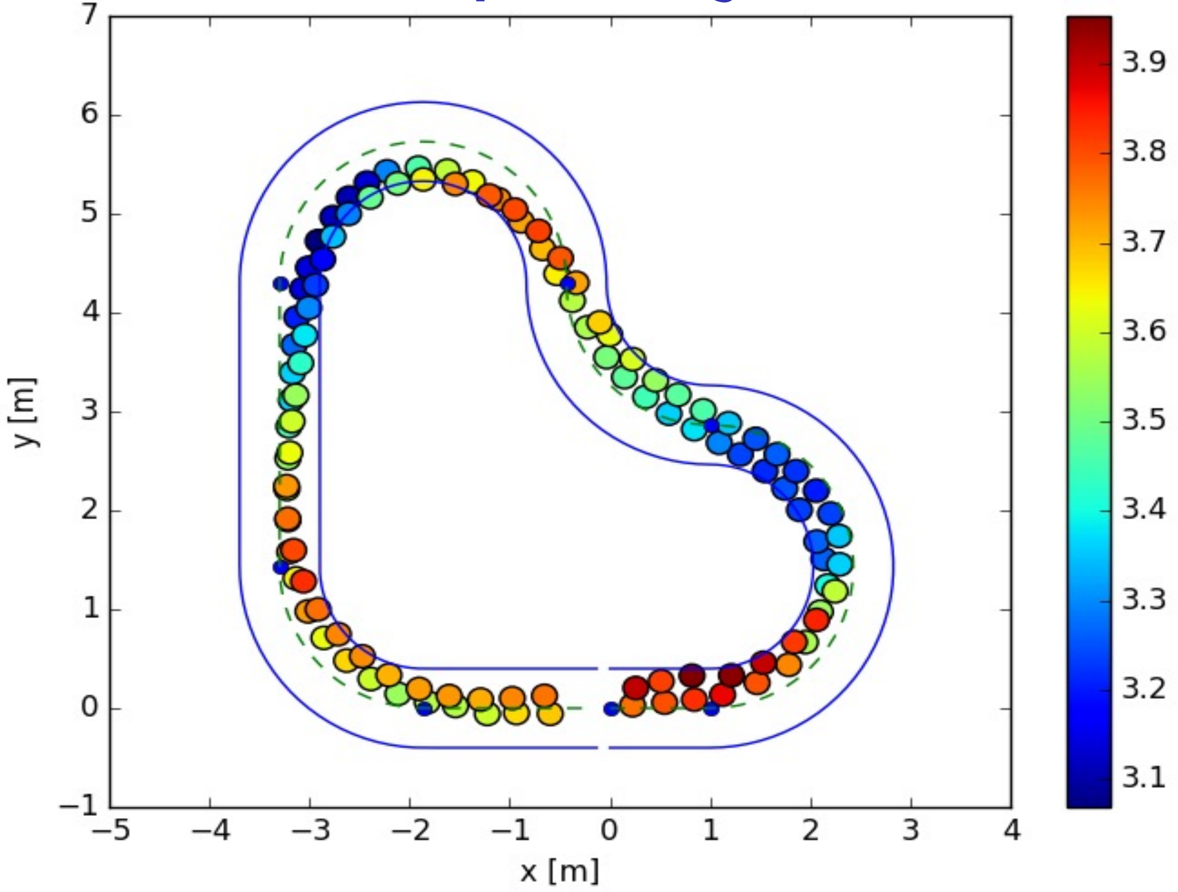

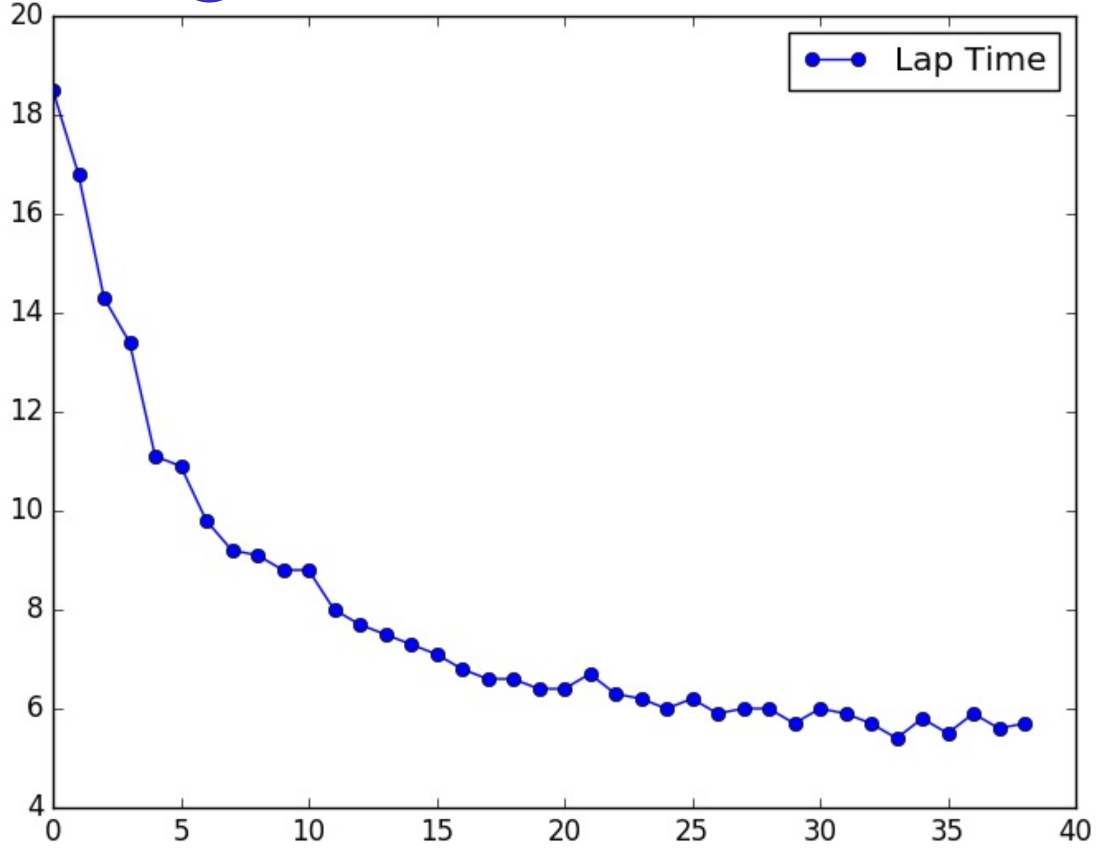

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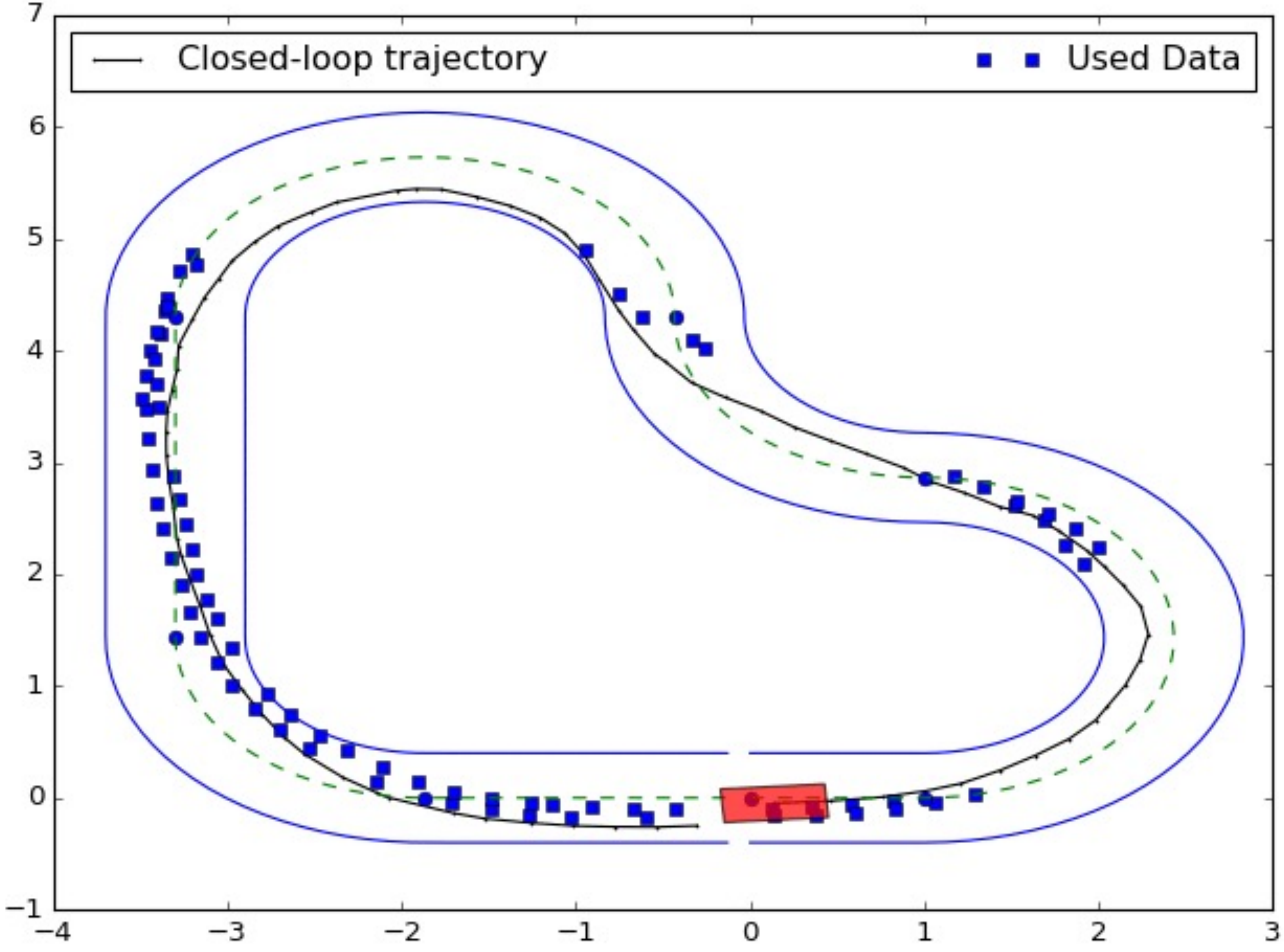
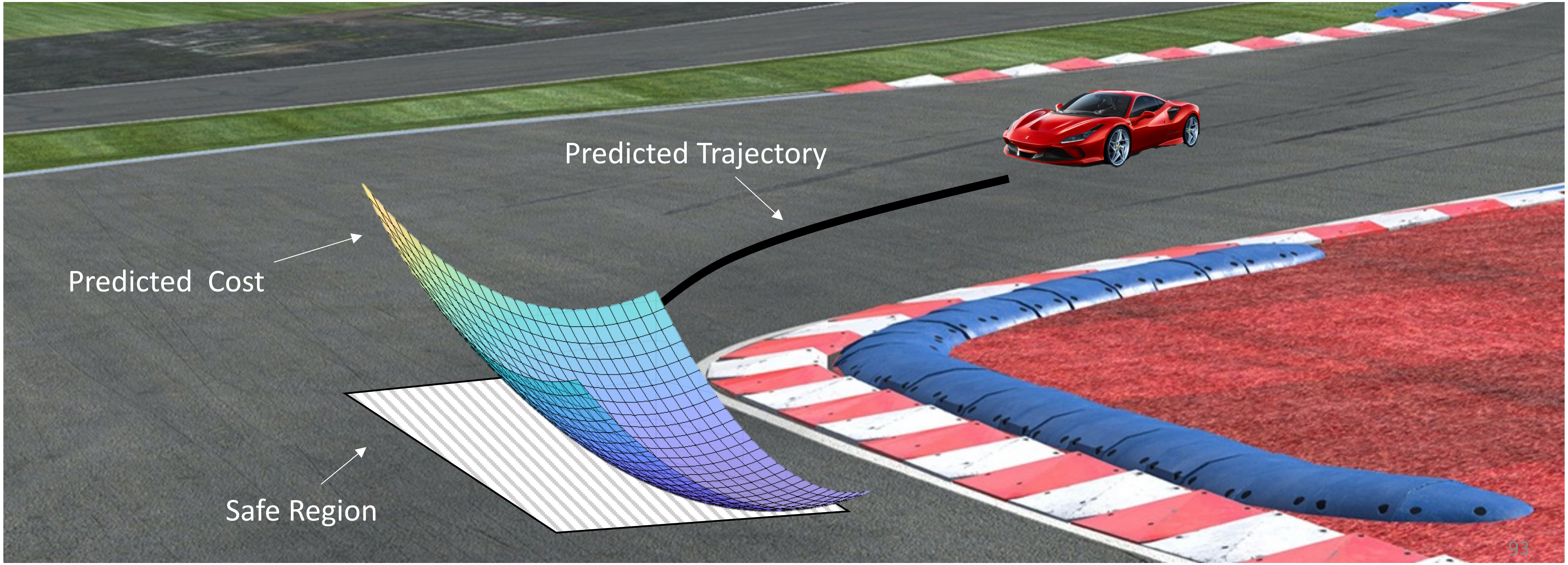
Data Point Selection



The key components

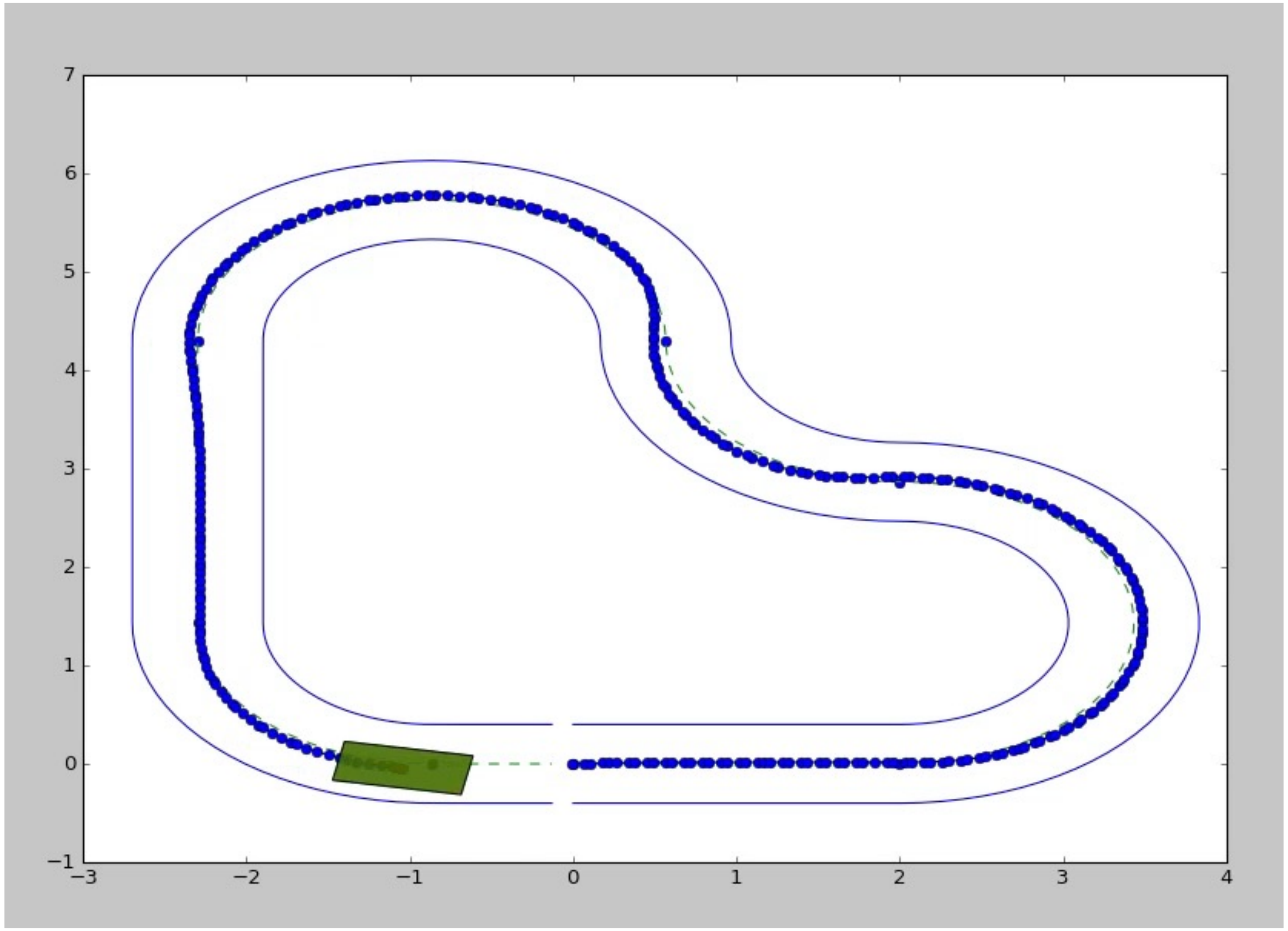
- Predicted trajectory given by prediction model
- Predicted cost estimated by value function approximation
- Safe region estimated by the safe set



Do you need the safe set? – Yes

LMPC without Invariant Set

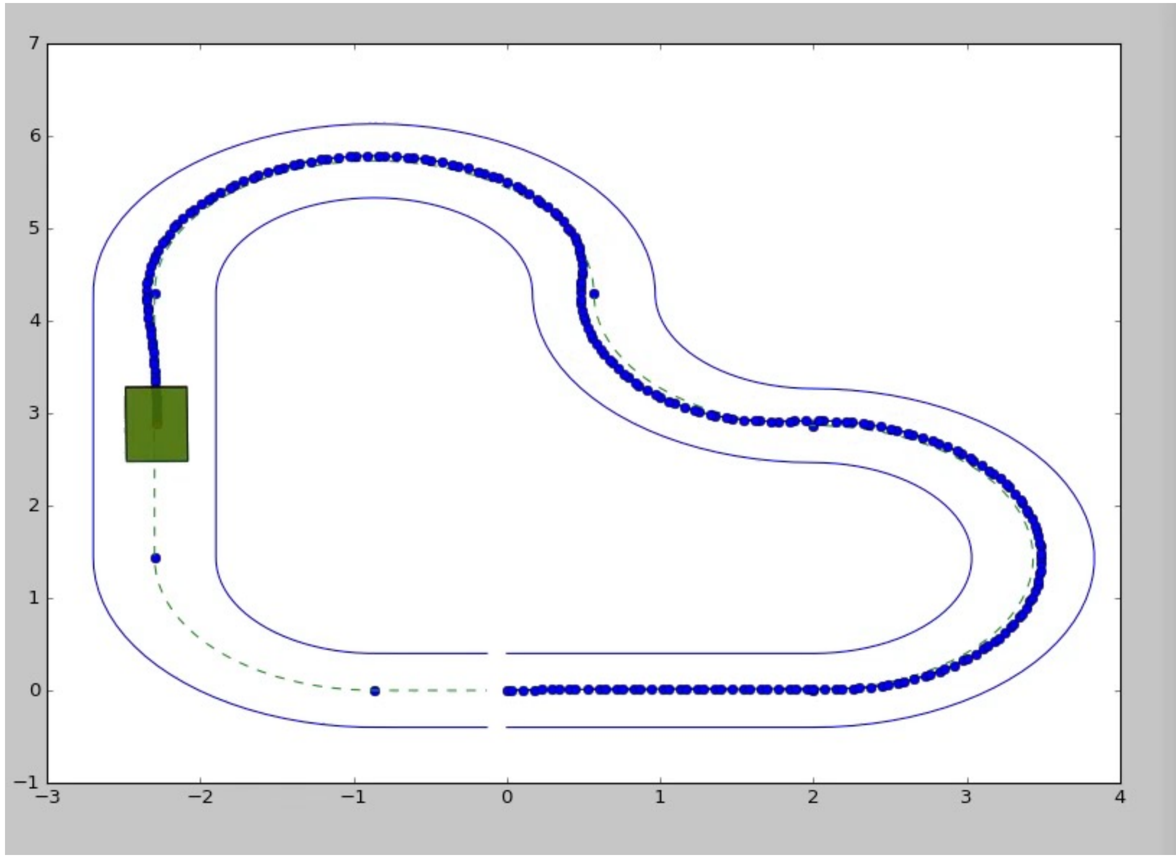
The controller extrapolates the value function on the Vx dimension

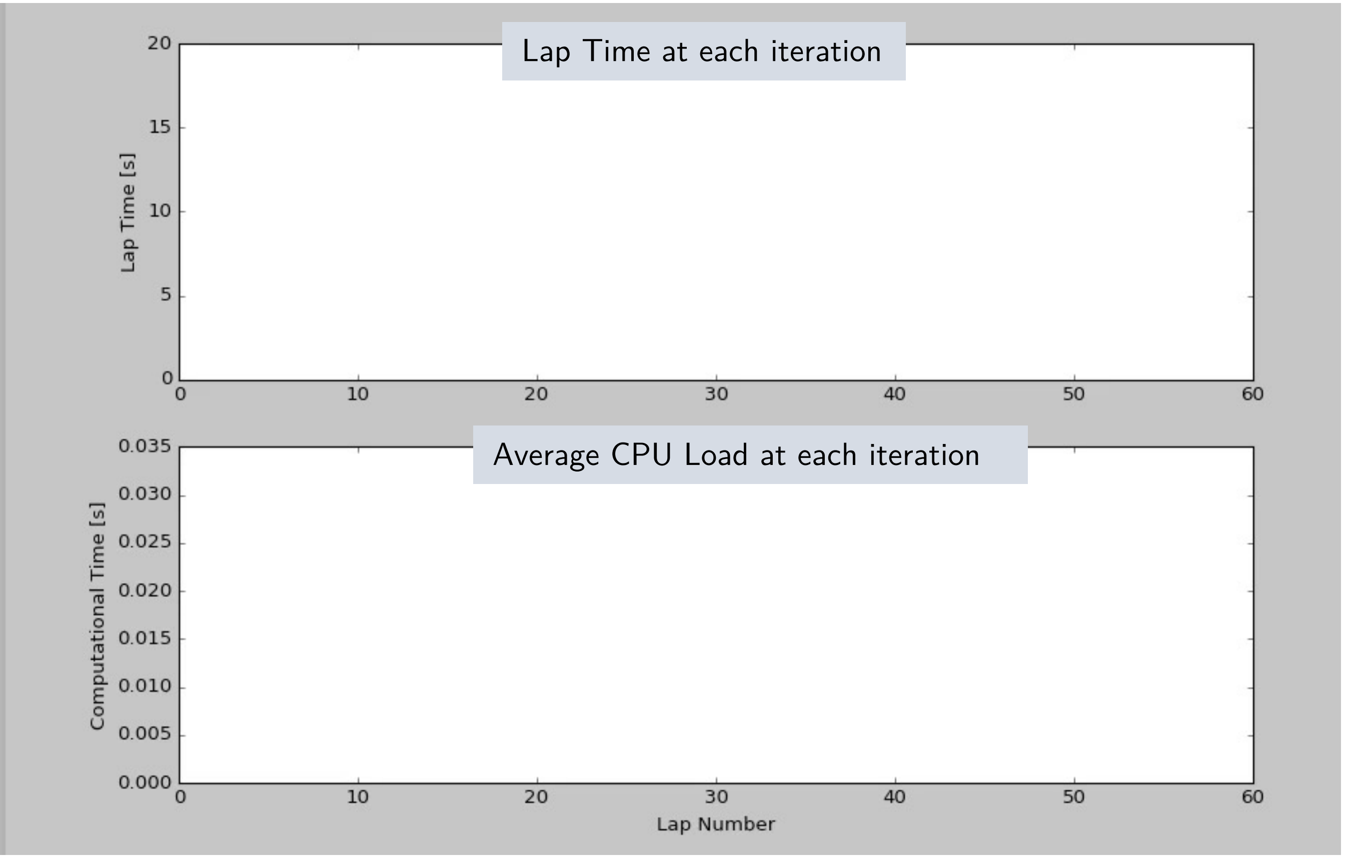


Do you need to Predict to Learn? Yes

When the LMPC horizon is N = 1 the controller

- solves the Bellman equation using the value function approximation
- does not explore the state space as it cannot plan outside the safe set





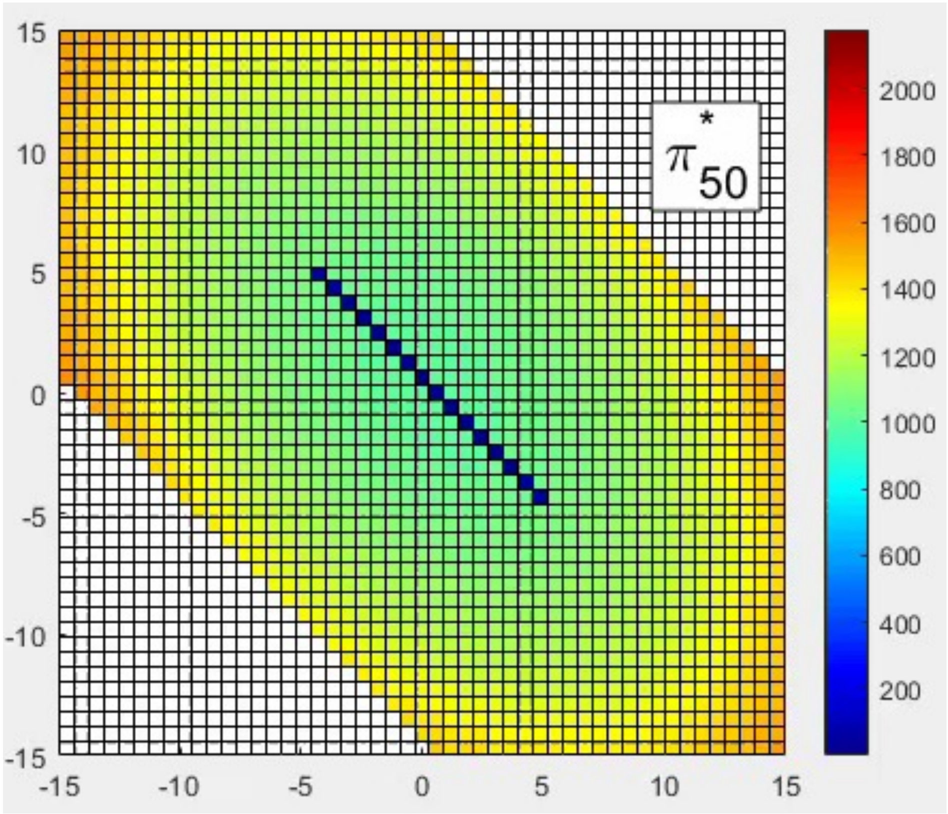
Comparison with Approximate DP (aka RL)

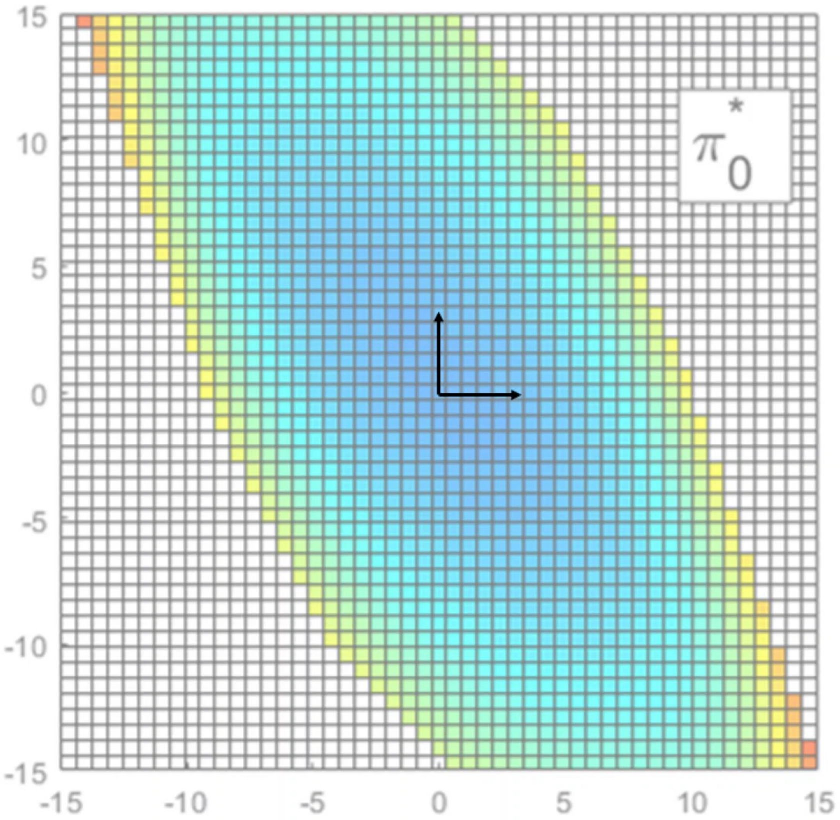
- Some references:
  - Bertsekas paper connecting MPC and ADP [1], books on RL and OC [2,3]
  - Lewis and Vrabie survey [4]
  - Recht survey [5]

- LMPC highlights
  - Continuous state and action formulation
  - Constraints satisfaction and Sampled Safe Sets
  - Terminal constructed locally based on cost/model driven exploration
  - Terminal at stored state is "exact" and upperbounds at intermediate points

[1] D. Bertsekas, "Dynamic programming and suboptimal control: A survey from ADP to MPC." European Journal of Control 11.4-5 (2005)
[2] D. Bertsekas, "Reinforcement learning and optimal control." Athena Scientific, 2019.
[3] D. Bertsekas, "Distributed Reinforcement Learning" http://web.mit.edu/dimitrib/www/RL_2_Rollout_&_PI.pdf
[4] F. Lewis, Frank, and D. Vrabie. "Reinforcement learning and adaptive dynamic programming for feedback control." IEEE circuits and systems magazine 9.3 (2009)
[5] R. Benjamin. "A tour of reinforcement learning: The view from continuous control." Annual Review of Control, Robotics, and Autonomous Systems 2 (2019)

Forward Value Iteration





Dynamic Programming:

- Gridding, global properties
- Backward, one-step iteration

LMPC:

- No Gridding, local properties
- Forward, multi-step prediction
- LICQ required for optimality

Model Learning in MPC

A short and non-comprehensive summary

The complexity of the prediction model

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Linear

$$x_{k+1} = Ax_k + Bu_k + w_k$$

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Linear

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Gray Box

known unknown

$$x_{k+1} = f(x_k, u_k) + g(x_k, u_k) + w_k$$

The complexity of the prediction model

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Gray Box

$$x_{k+1} = \underbrace{f(x_k, u_k)}_{\text{known}} + \underbrace{g(x_k, u_k)}_{\text{unknown}} + w_k$$

Black Box

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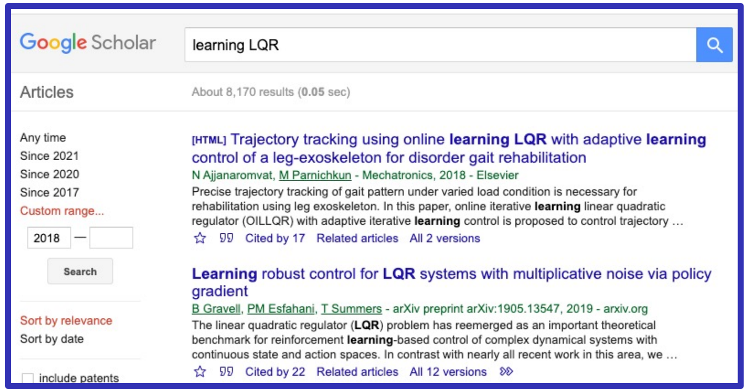

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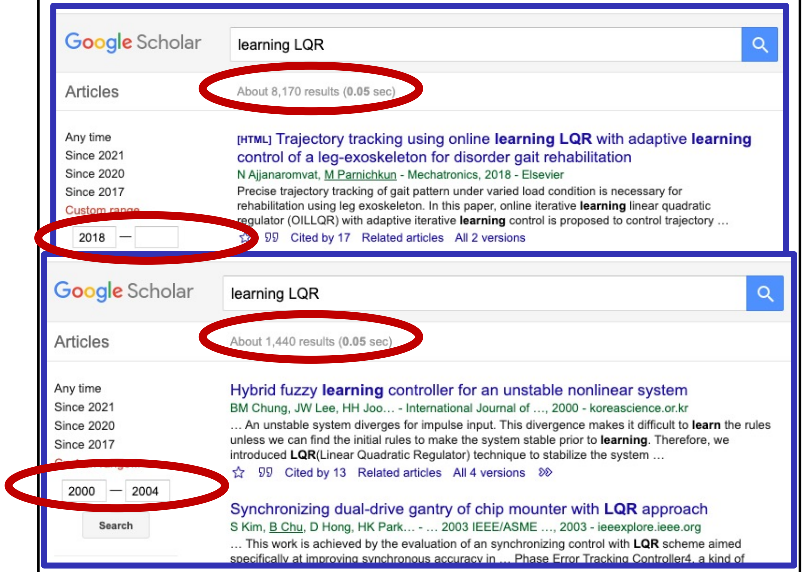

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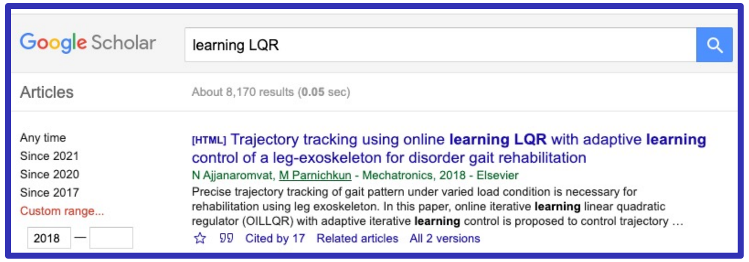

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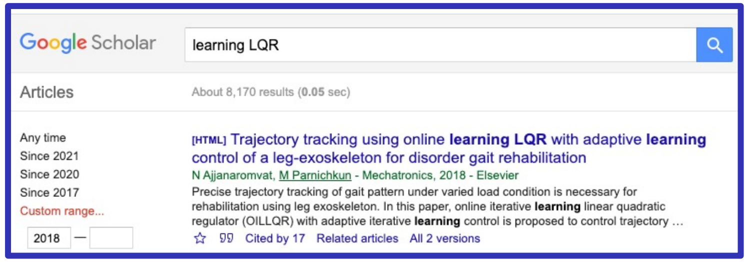

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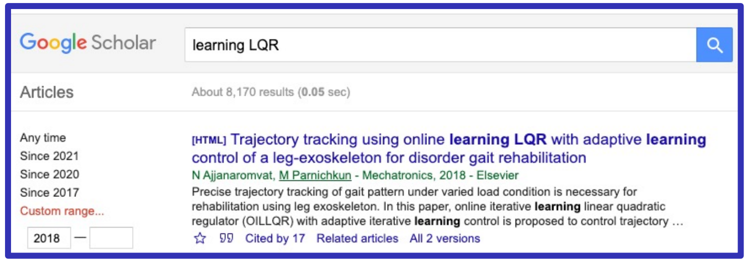

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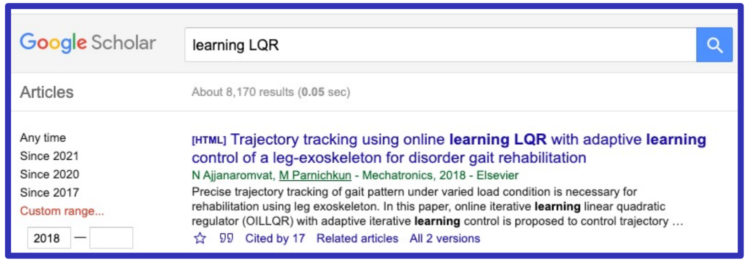

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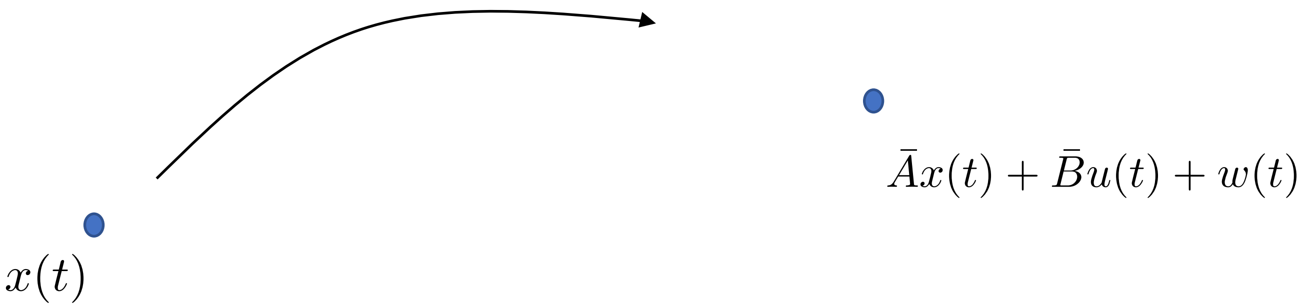

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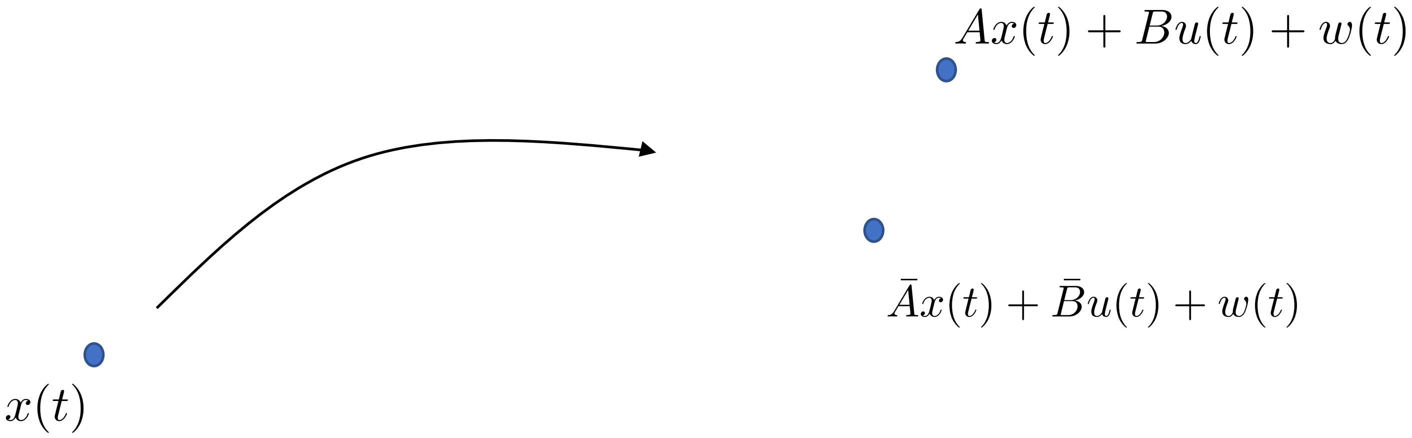

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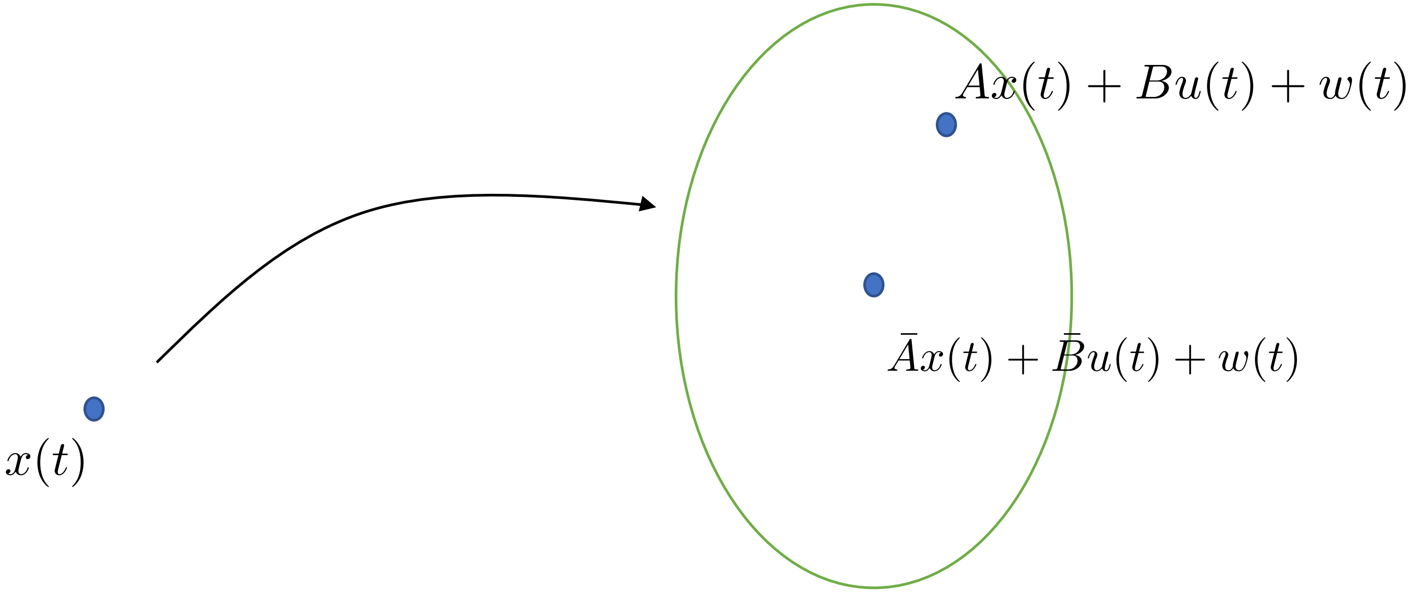

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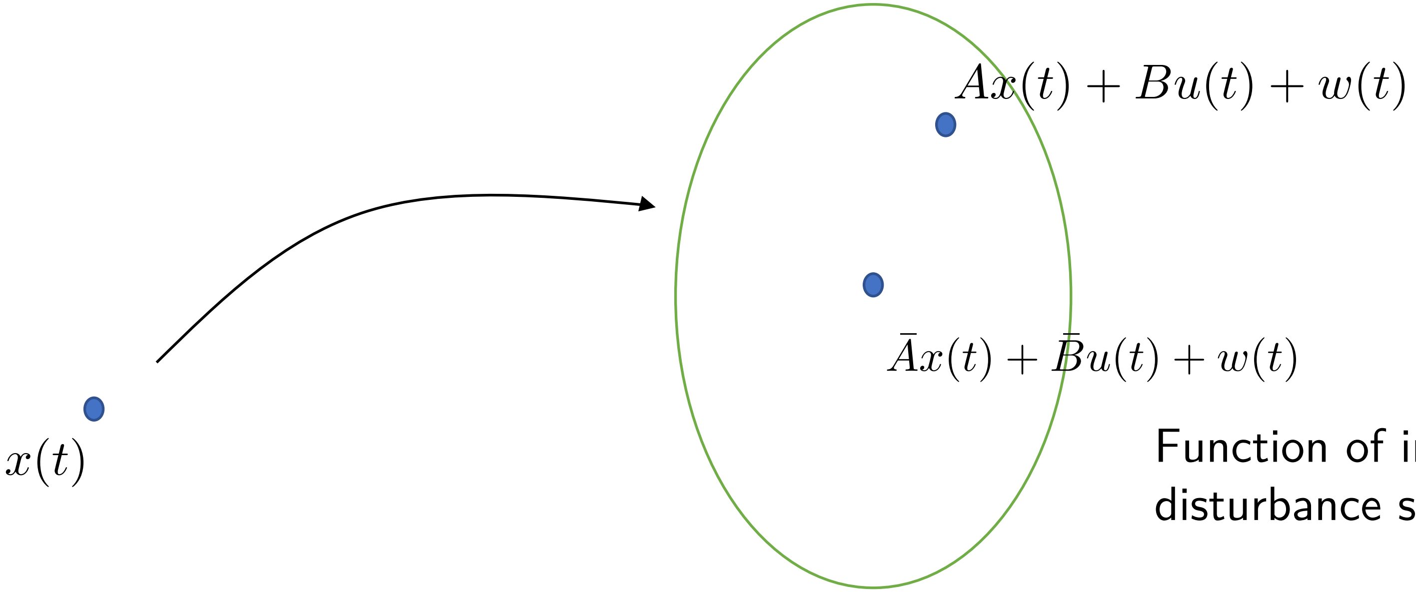

Function of initial condition, applied input, disturbance support and the identification error:

$$||A - \bar{A}|| \leq \epsilon_A$$
$$||B - \bar{B}|| \leq \epsilon_B$$

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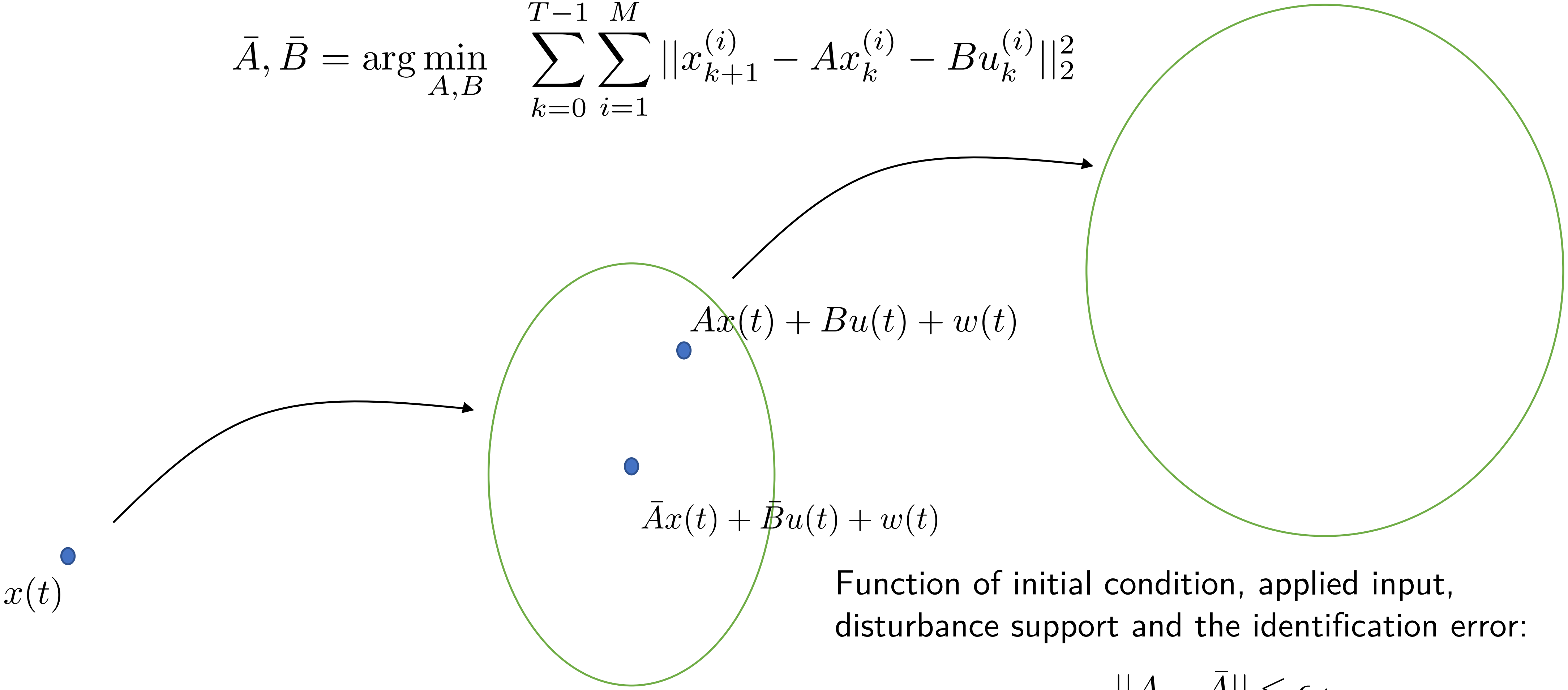

Function of initial condition, applied input, disturbance support and the identification error:

$$||A - \bar{A}|| \leq \epsilon_A$$
$$||B - \bar{B}|| \leq \epsilon_B$$

Marco C., Campi, and Erik Weyer. "Finite sample properties of system identification methods." *IEEE Transactions on Automatic Control* 47.8 (2002): 1329-1334.

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Learning Linear Models: Multi-Step Prediction

Enrico Terzi, Lorenzo Fagiano, Marcello Farina, Riccardo Scattolini. Learning multi-step prediction models for receding horizon control. In2018 European Control Conference (ECC) 2018 Jun 12 (pp. 1335-1340). IEEE.

Learning Linear Models: Multi-Step Prediction

$$\bar{A}, \bar{B}_0, \ldots, \bar{B}_N = \arg \min_{A, B_0, \ldots, B_N} \quad \sum_{k=0}^{T-1} \sum_{i=1}^{M} ||x_{k+N+1}^{(i)} - Ax_k^{(i)} - \sum_{j=0}^{N} B_j u_{k+j}^{(i)}||_2^2$$

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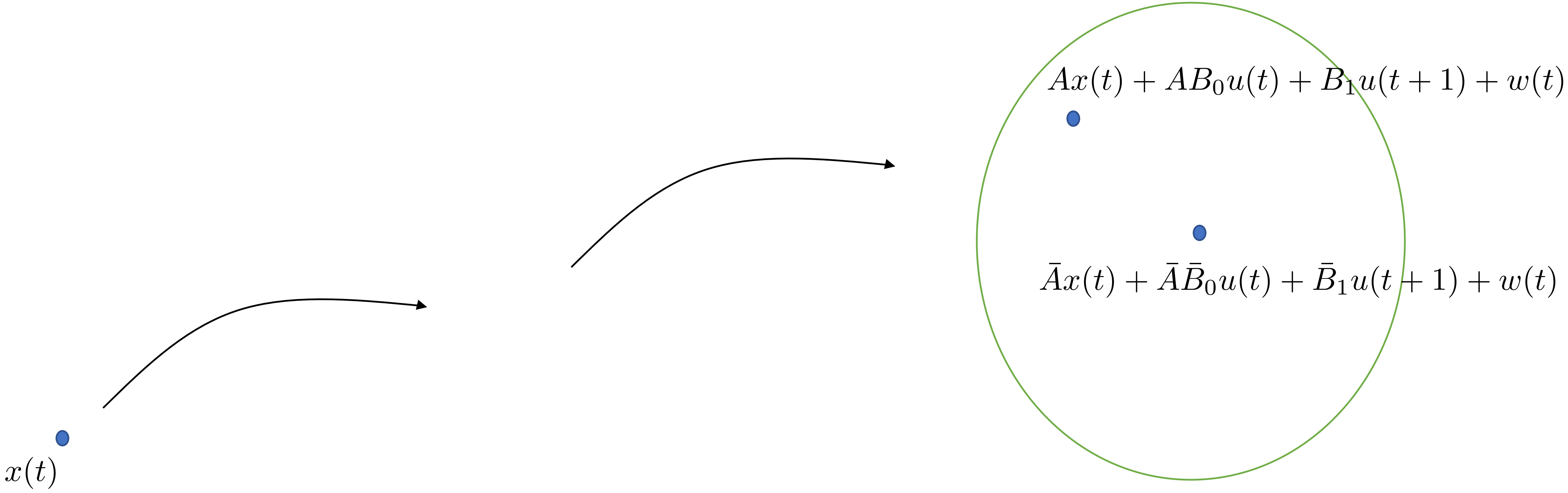

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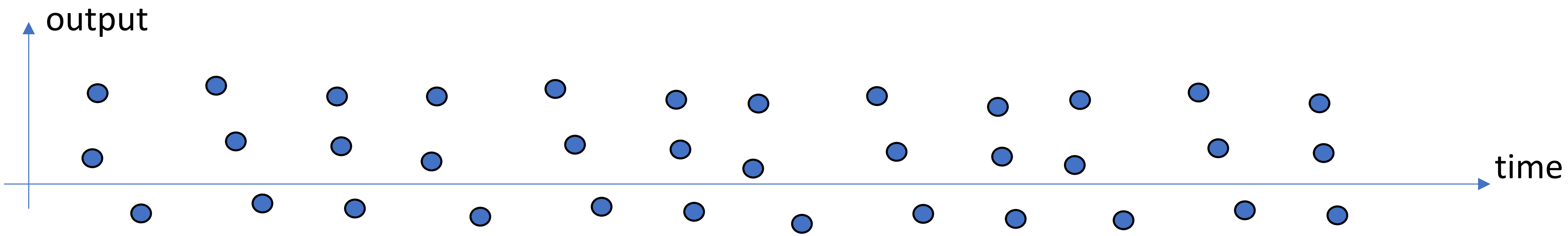

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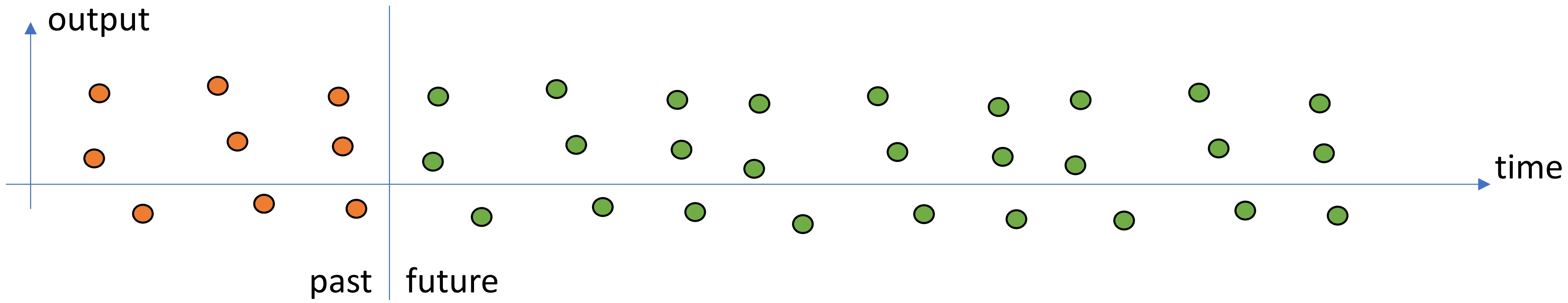

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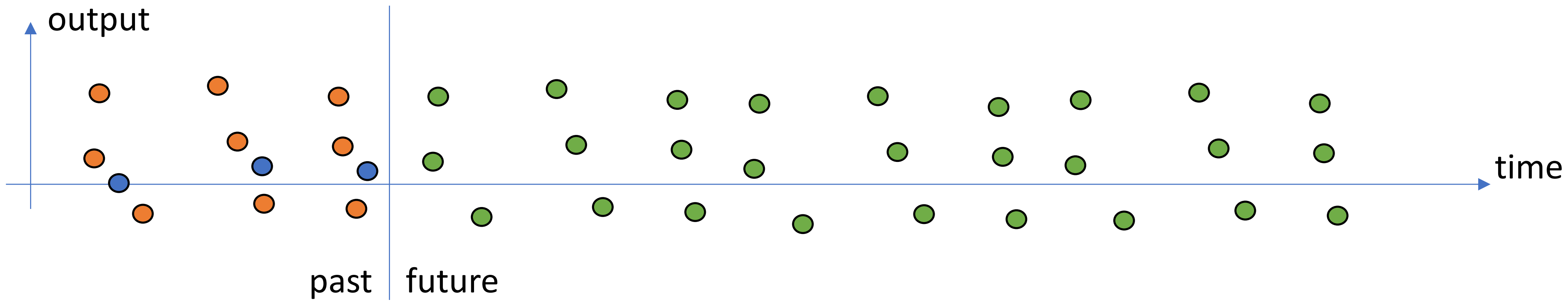

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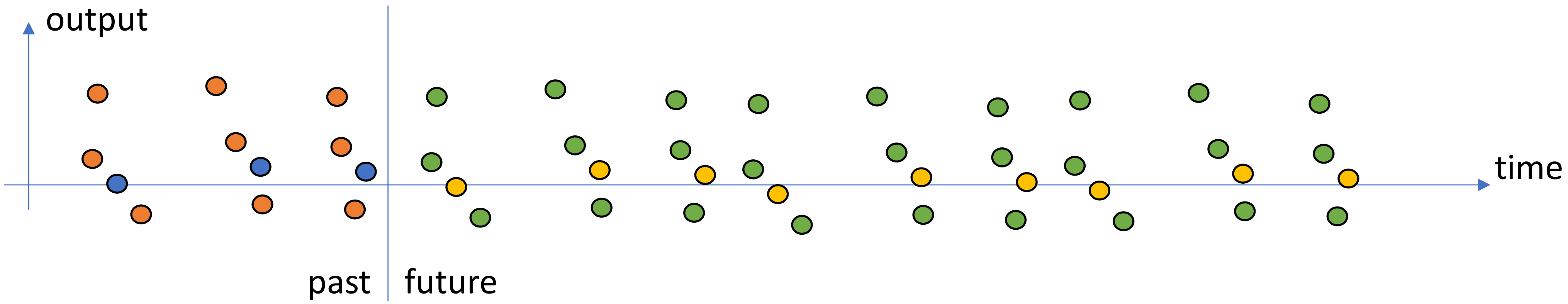

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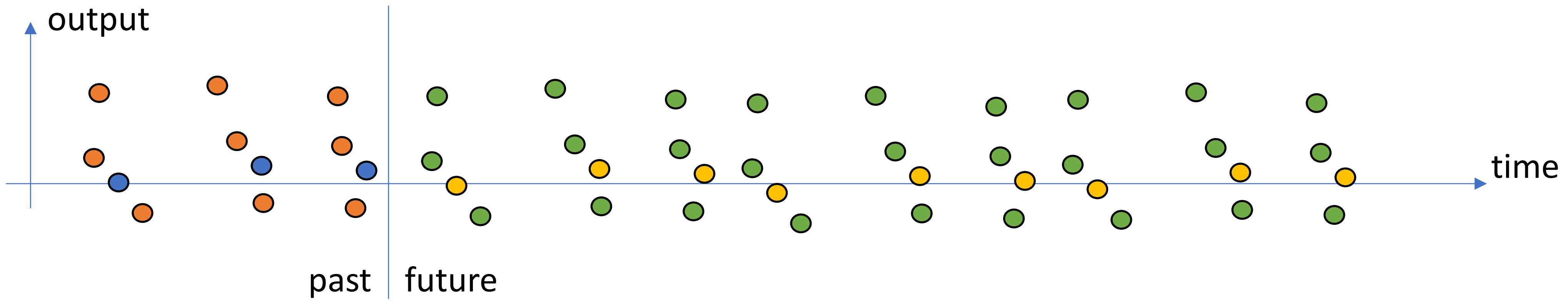

Construct the Hankel Matrix using Data

$$\mathscr{H}_L(u) := \begin{pmatrix} u_1 & \cdots & u_{T-L+1} \\ \vdots & \ddots & \vdots \\ u_L & \cdots & u_T \end{pmatrix}$$

$$\begin{pmatrix} U_\mathrm{p} \\ U_\mathrm{f} \end{pmatrix} := \mathscr{H}_{T_\mathrm{ini}+N}(u^\mathrm{d}), \quad \begin{pmatrix} Y_\mathrm{p} \\ Y_\mathrm{f} \end{pmatrix} := \mathscr{H}_{T_\mathrm{ini}+N}(y^\mathrm{d}),$$

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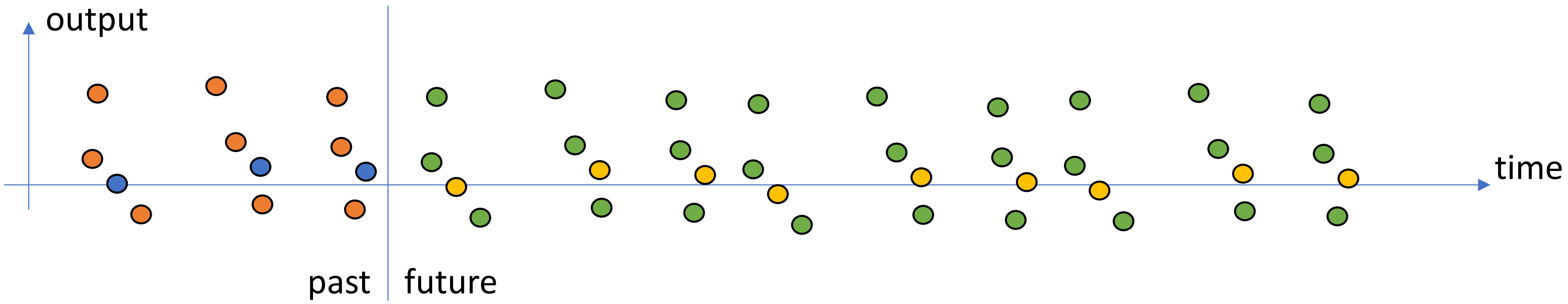

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Replace the model

$$\begin{aligned} \underset{g,u,y}{\text{minimize}} \quad & \sum_{k=0}^{N-1} \left(\|y_k - r_{t+k}\|_Q^2 + \|u_k\|_R^2 \right) \\ \text{subject to} \quad & \begin{pmatrix} U_\mathrm{p} \\ Y_\mathrm{p} \\ U_\mathrm{f} \\ Y_\mathrm{f} \end{pmatrix} g = \begin{pmatrix} u_\mathrm{ini} \\ y_\mathrm{ini} \\ u \\ y \end{pmatrix}, \\ & u_k \in \mathcal{U}, \ \forall k \in \{0, \ldots, N-1\}, \\ & y_k \in \mathcal{Y}, \ \forall k \in \{0, \ldots, N-1\}. \end{aligned}$$

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The complexity of the prediction model

Linear

$$x_{k+1} = Ax_k + Bu_k + w_k$$

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Gray Box

known

unknown

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Black Box

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Kendall Lowrey, Aravind Rajeswaran, Sham Kakade, Emanuel Todorov, and Igor Mordatch. "Plan online, learn offline: Efficient learning and exploration via model-based control." in Proc. Int. Conf. Mach. Learn., 2019.

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Gray Box Model

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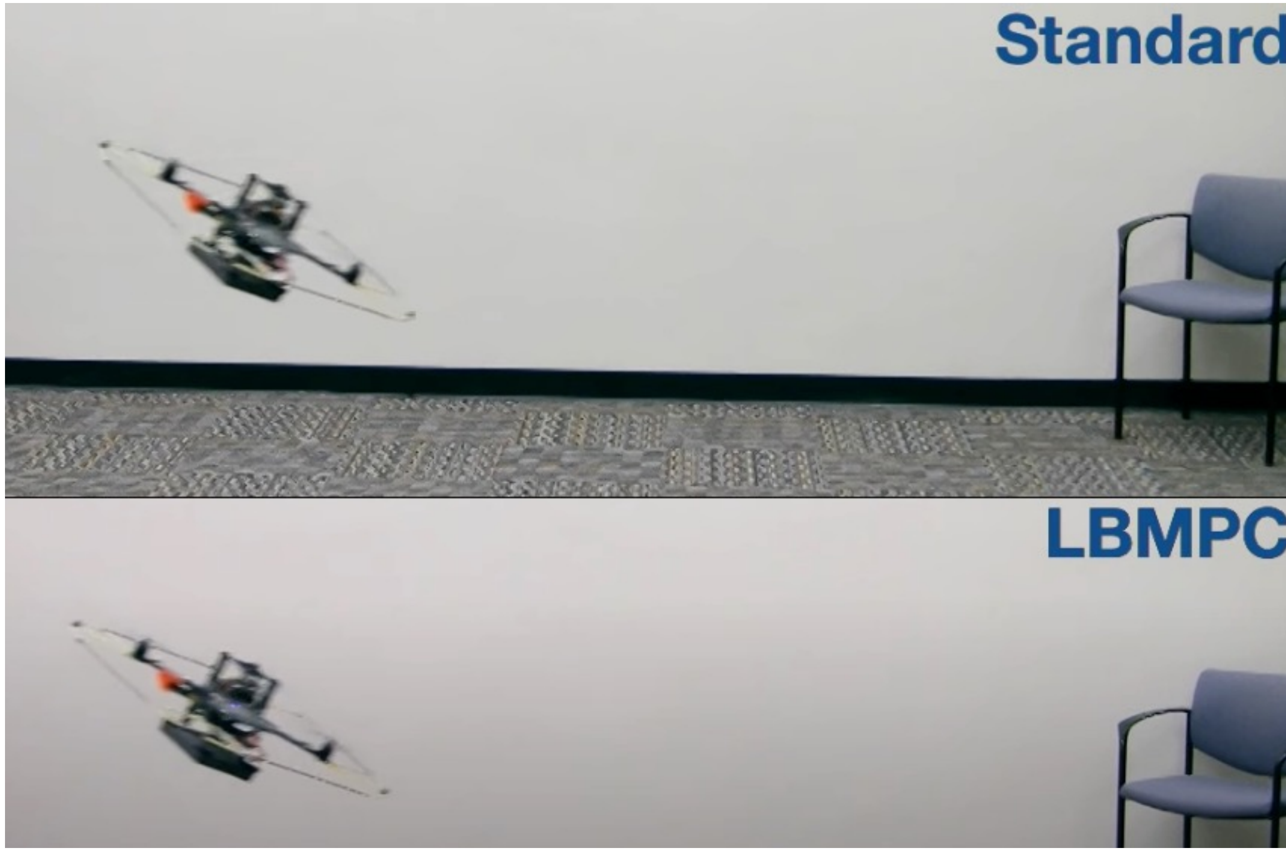

Juraj Kabzan,, Lukas Hewing, Alexander Liniger, and Melanie N. Zeilinger. "Learning-based model predictive control for autonomous racing." *IEEE Robotics and Automation Letters* 4, no. 4 (2019): 3363-3370.

Bouffard, Patrick, Anil Aswani, and Claire Tomlin. "Learning-based model predictive control on a quadrotor: Onboard implementation and experimental results." In *2012 IEEE International Conference on Robotics and Automation*, pp. 279-284. IEEE, 2012.

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Deisenroth, Marc, and Carl E. Rasmussen. "PILCO: A model-based and data-efficient approach to policy search." In *Proceedings of the 28th International Conference on machine learning (ICML-11)*, pp. 465-472. 2011.

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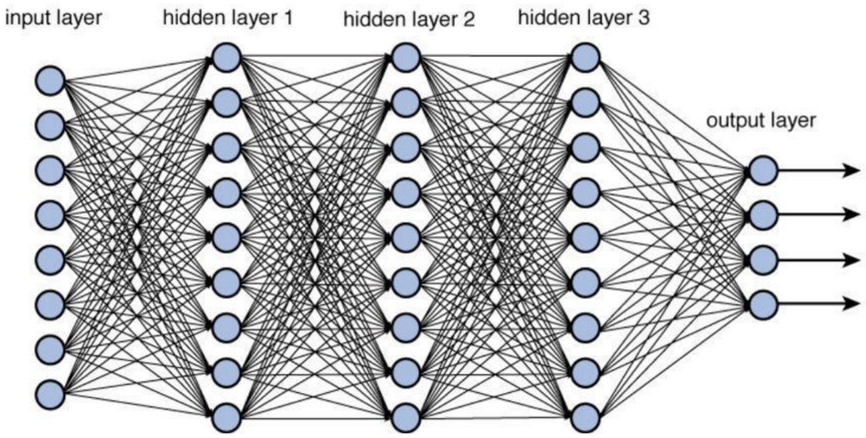
Black Box Model



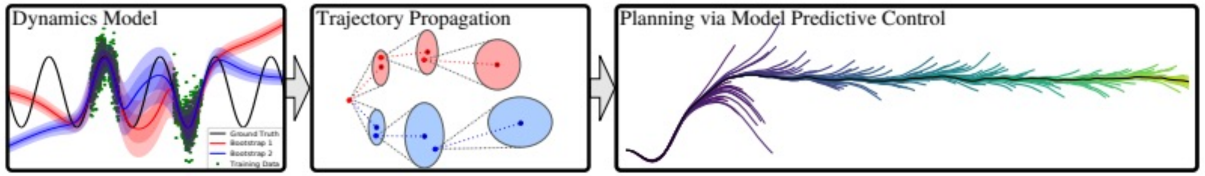


Figure 1: Our method (PE-TS): **Model**: Our probabilistic ensemble (PE) dynamics model is shown as an ensemble of two bootstraps (bootstrap disagreement far from data captures epistemic uncertainty: our subjective uncertainty due to a lack of data), each a probabilistic neural network that captures aleatoric uncertainty (inherent variance of the observed data). **Propagation**: Our trajectory sampling (TS) propagation technique uses our dynamics model to re-sample each particle (with associated bootstrap) according to its probabilistic prediction at each point in time, up until horizon T. **Planning**: At each time step, our MPC algorithm computes an optimal action sequence, applies the first action in the sequence, and repeats until the task-horizon.

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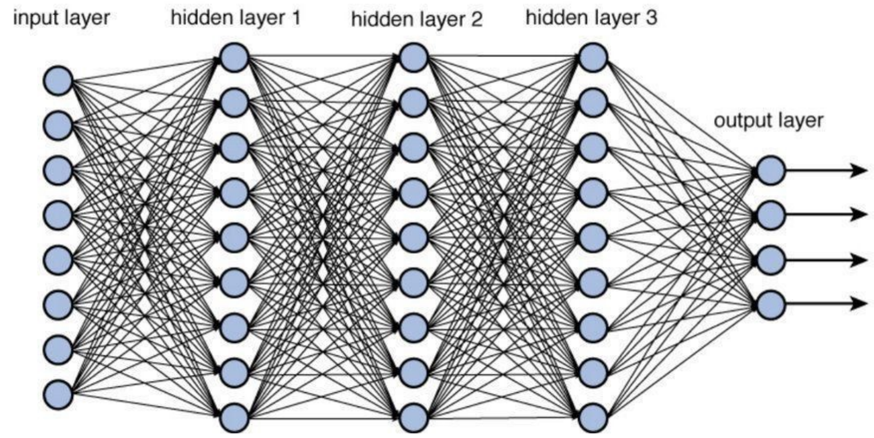

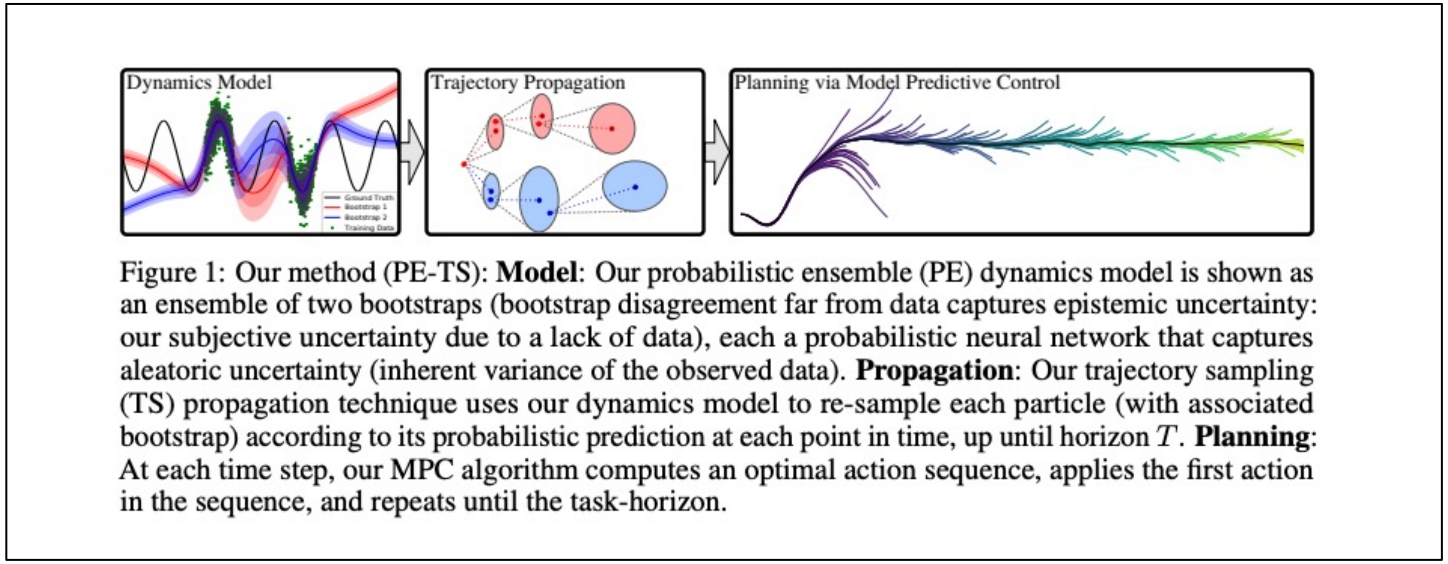

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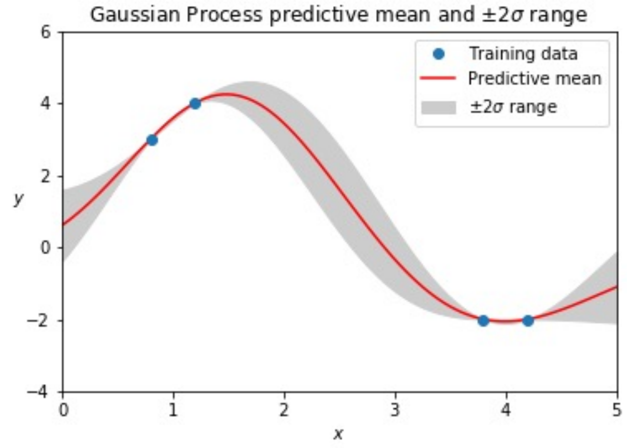

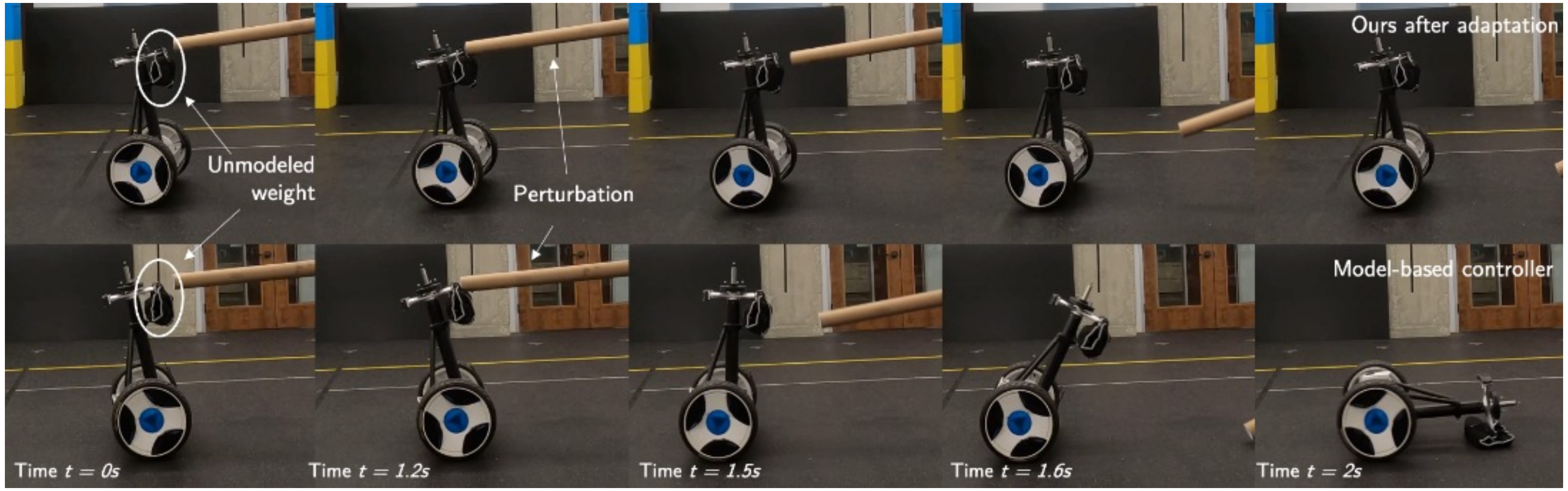

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Planning under uncertainty

Optimizing over control policies

Why is planning in uncertain environments harder?

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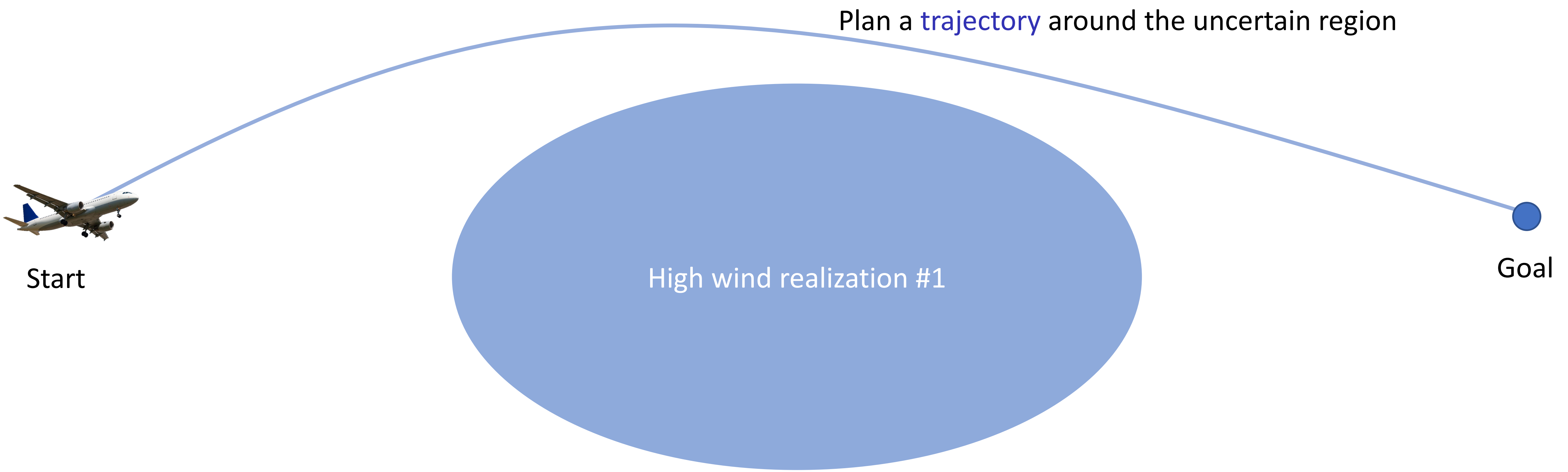

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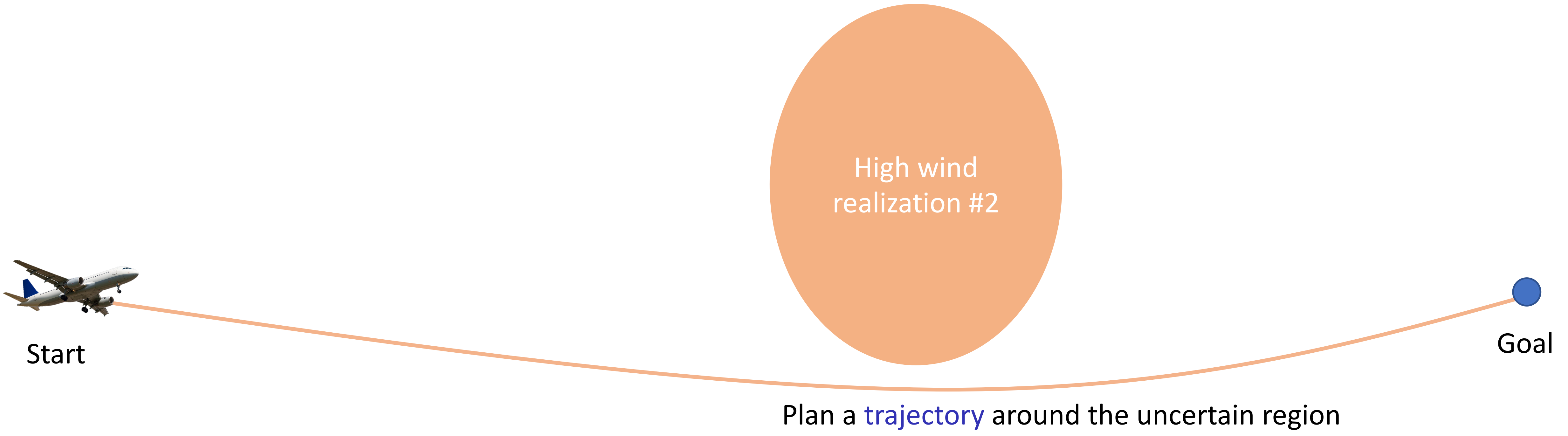

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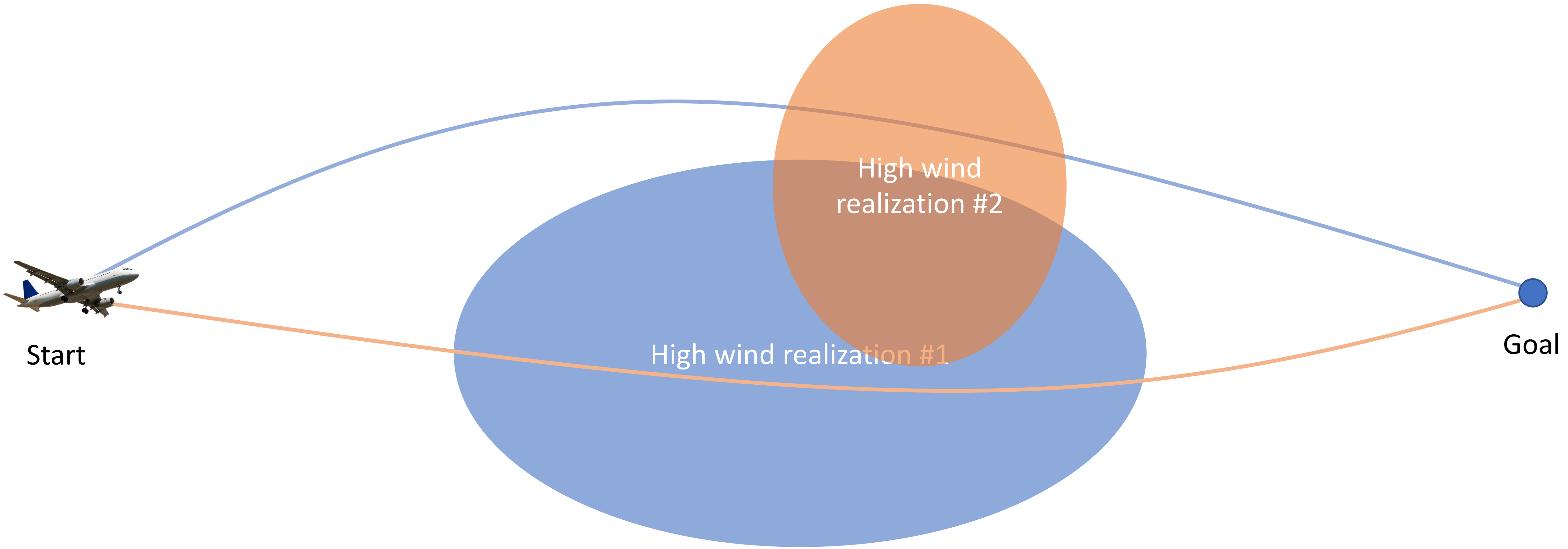
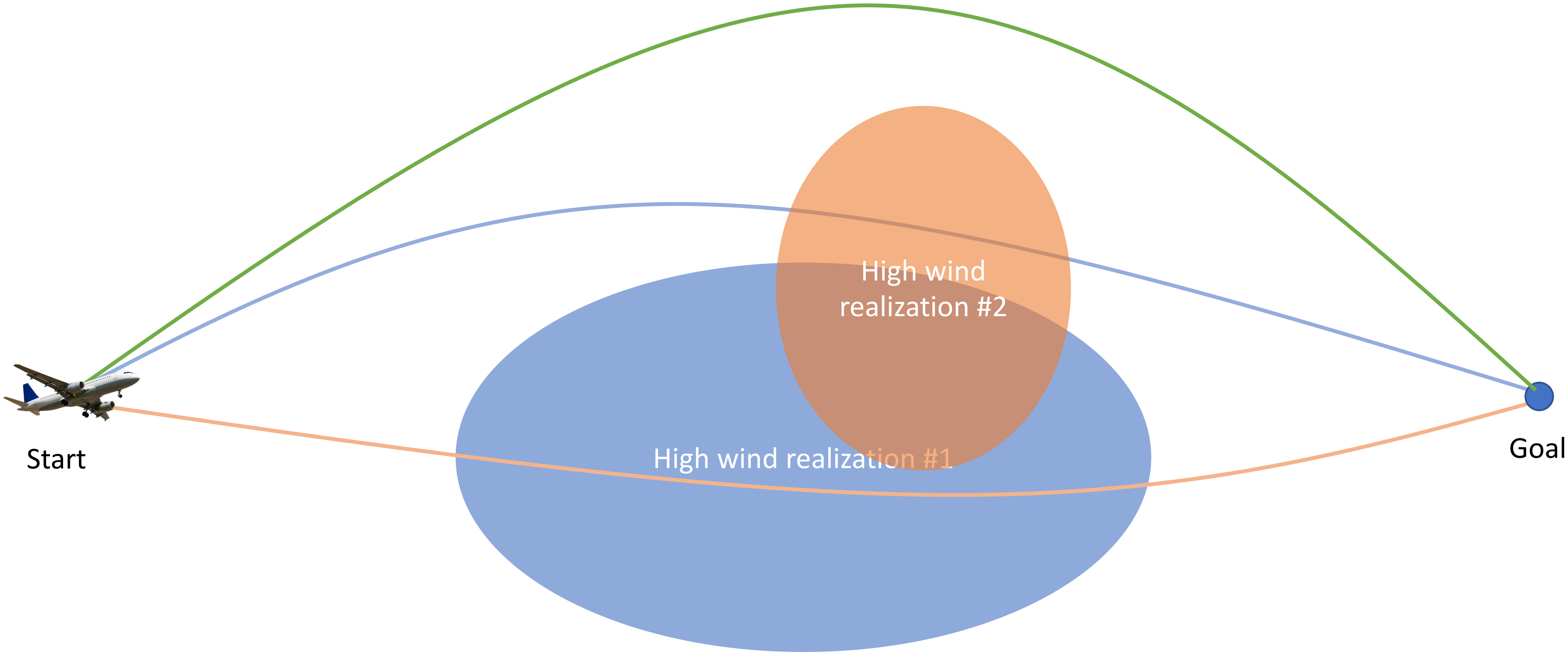


Why is planning in uncertain environments harder?

Plan a trajectory around the both uncertain regions



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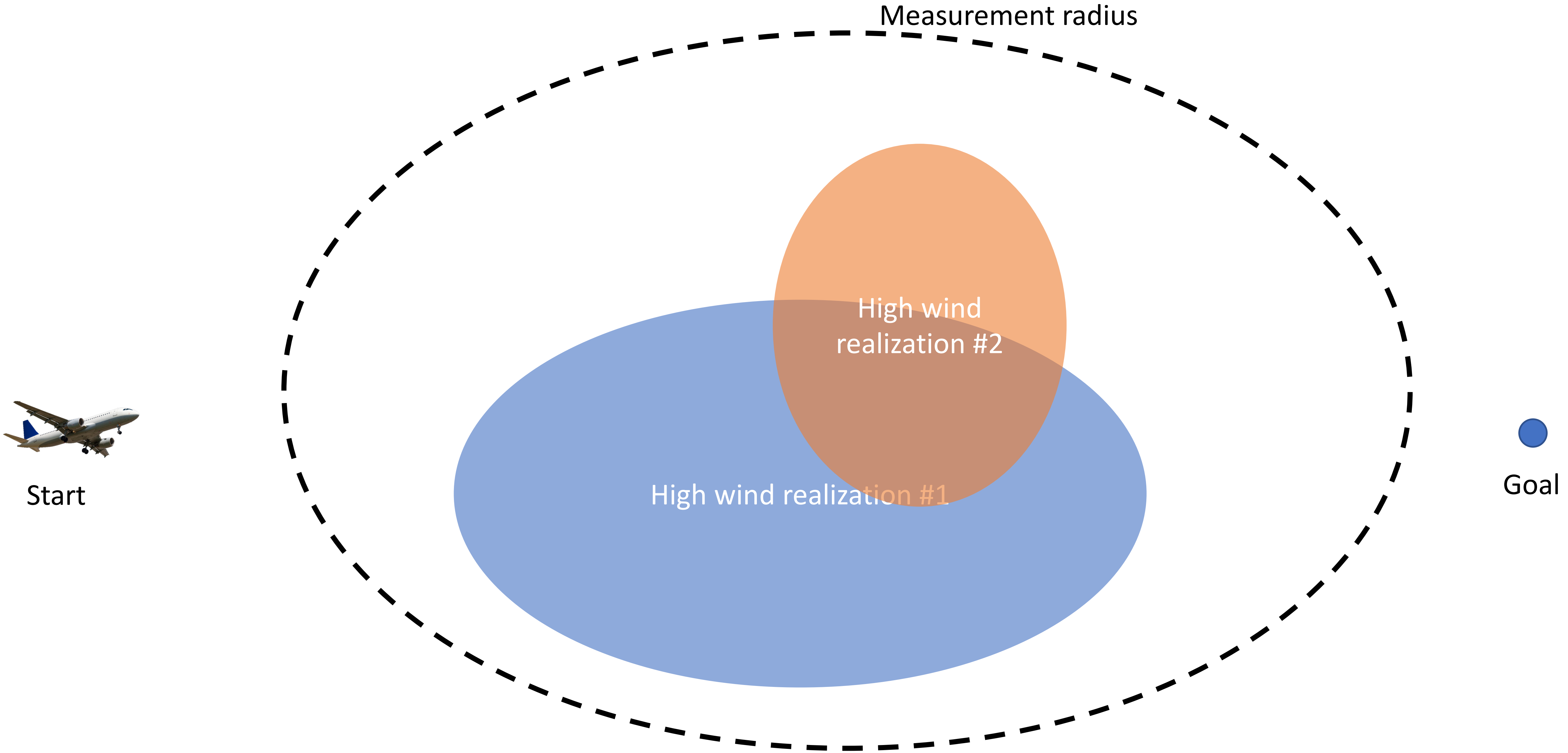
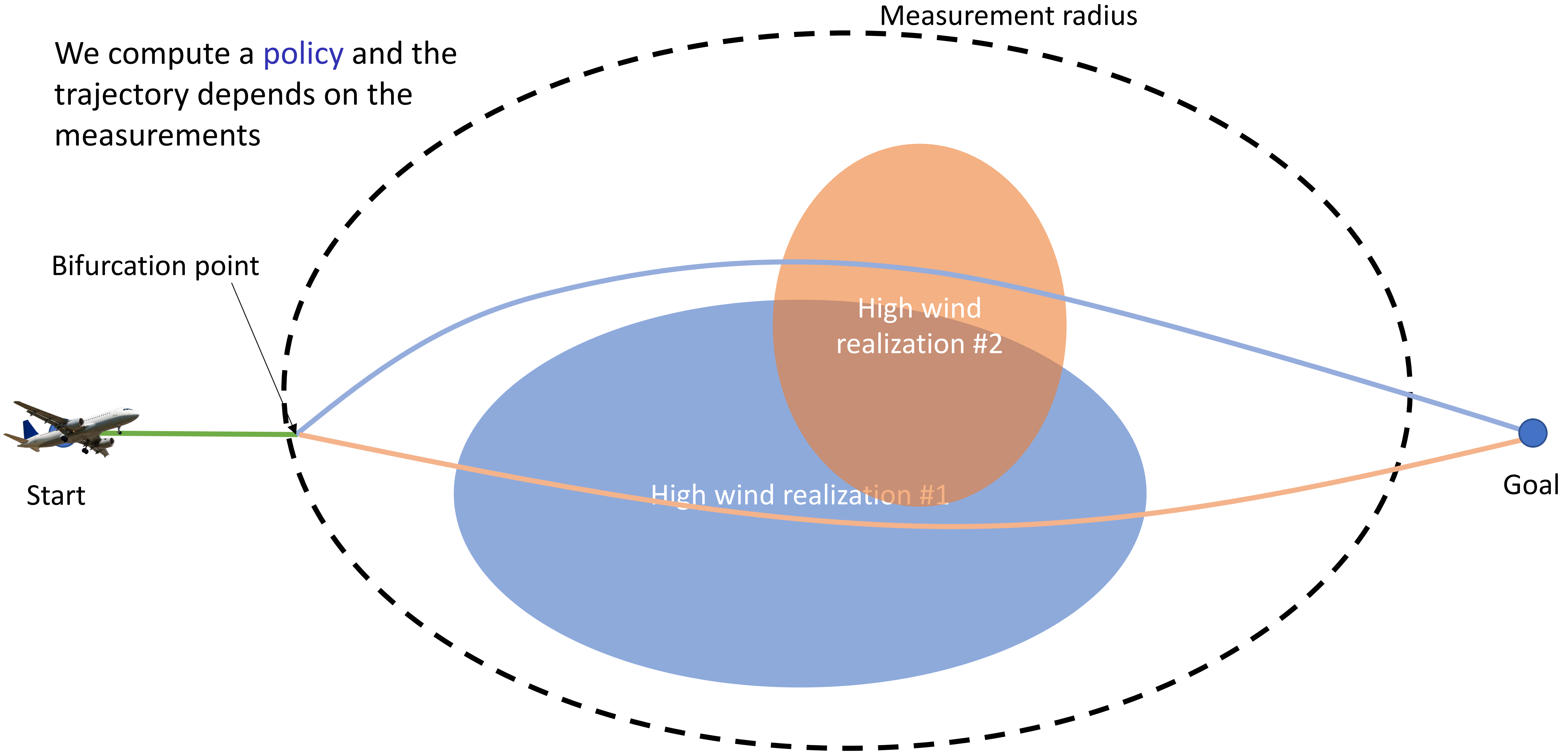


Why is planning in uncertain environments harder?

We compute a policy and the trajectory depends on the measurements



Problem Formulation

True system dynamics: $x_{k+1} = Ax_k + Bu_k + w_k$

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Nominal model: $\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}\bar{u}_k + w_k$ where $||A - \bar{A}|| \leq \epsilon_A, ||B - \bar{B}|| \leq \epsilon_B$

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Robust constraints:

Given a control policy $\pi : \mathbb{R}^n \rightarrow \mathbb{R}^d$ designed using the nominal model, we want to guarantee that the closed-loop system

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satisfies $x_k \in \mathcal{X}, \pi(x_k) \in \mathcal{U}$ for all $w_k \in \mathcal{W}$.

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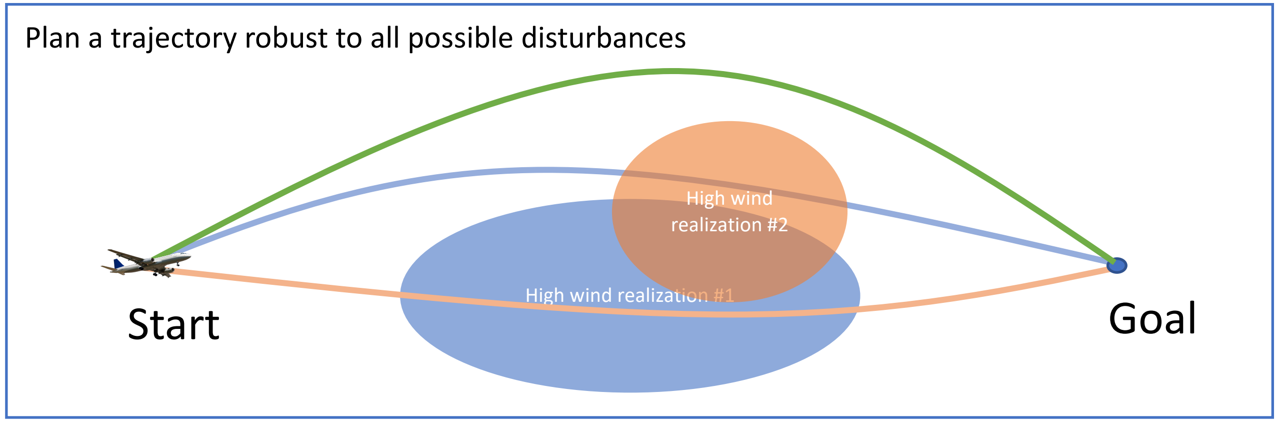

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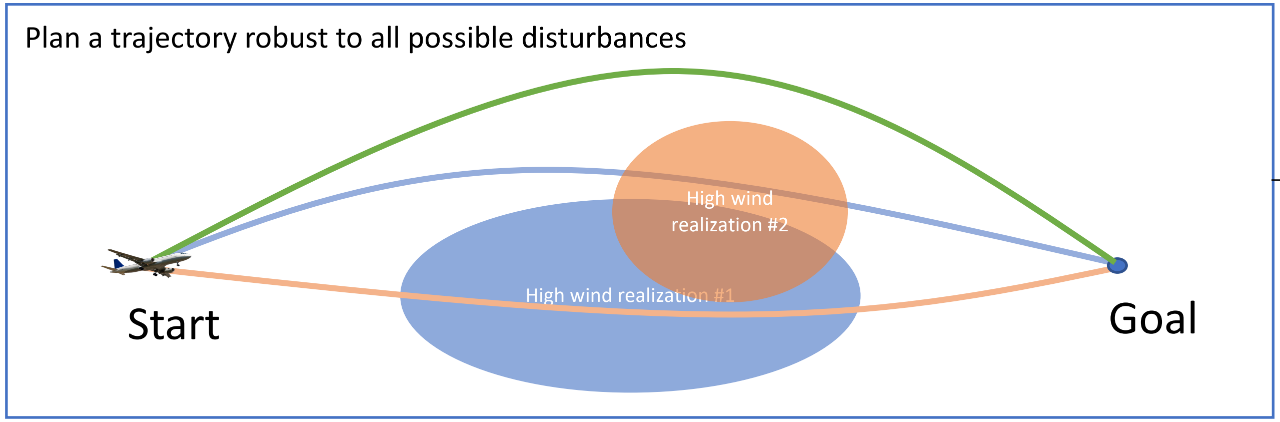

Control action $u(t)$

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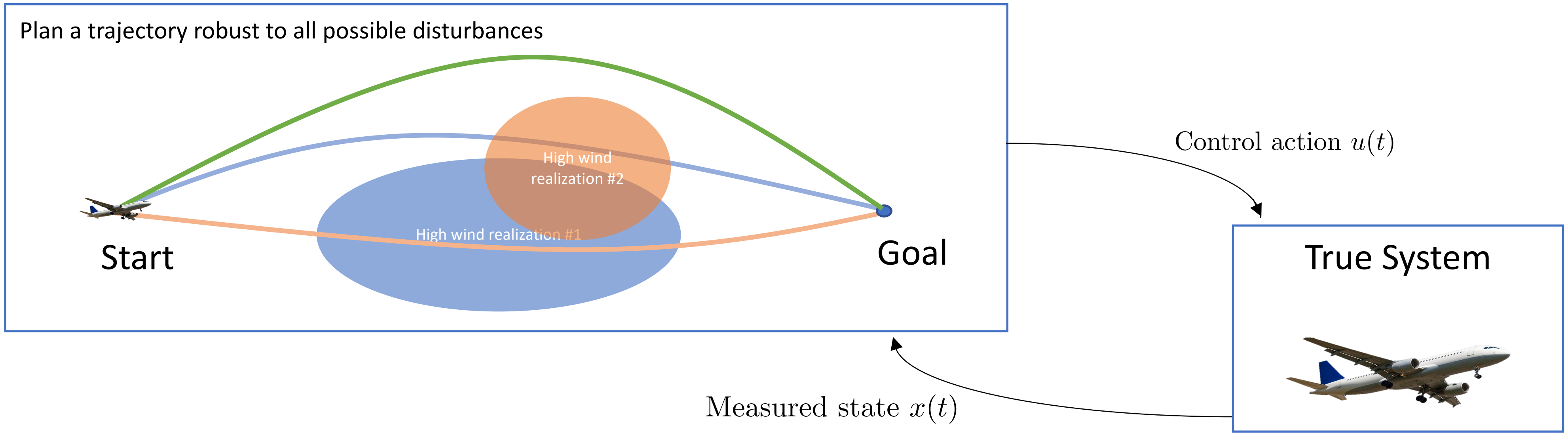
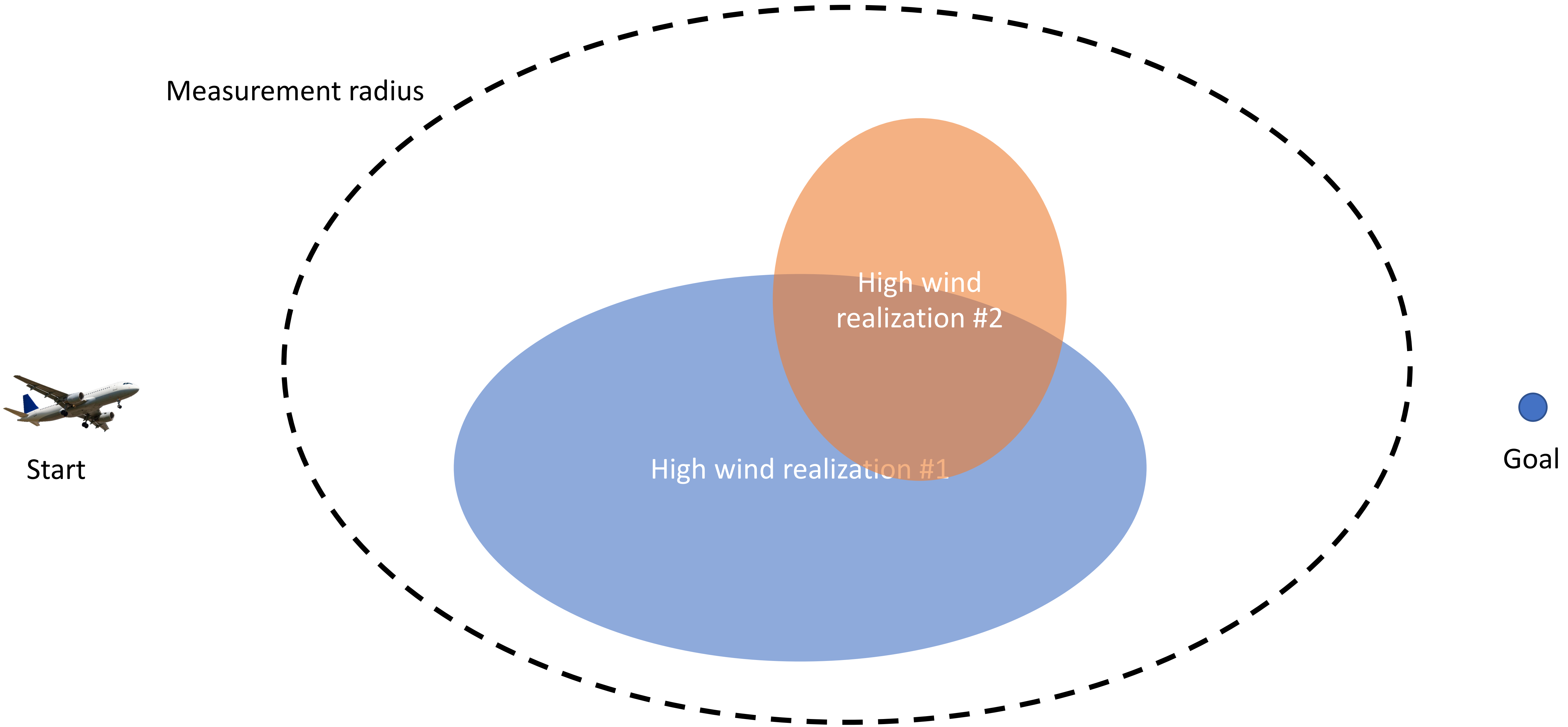


Can we leverage the MPC approach?



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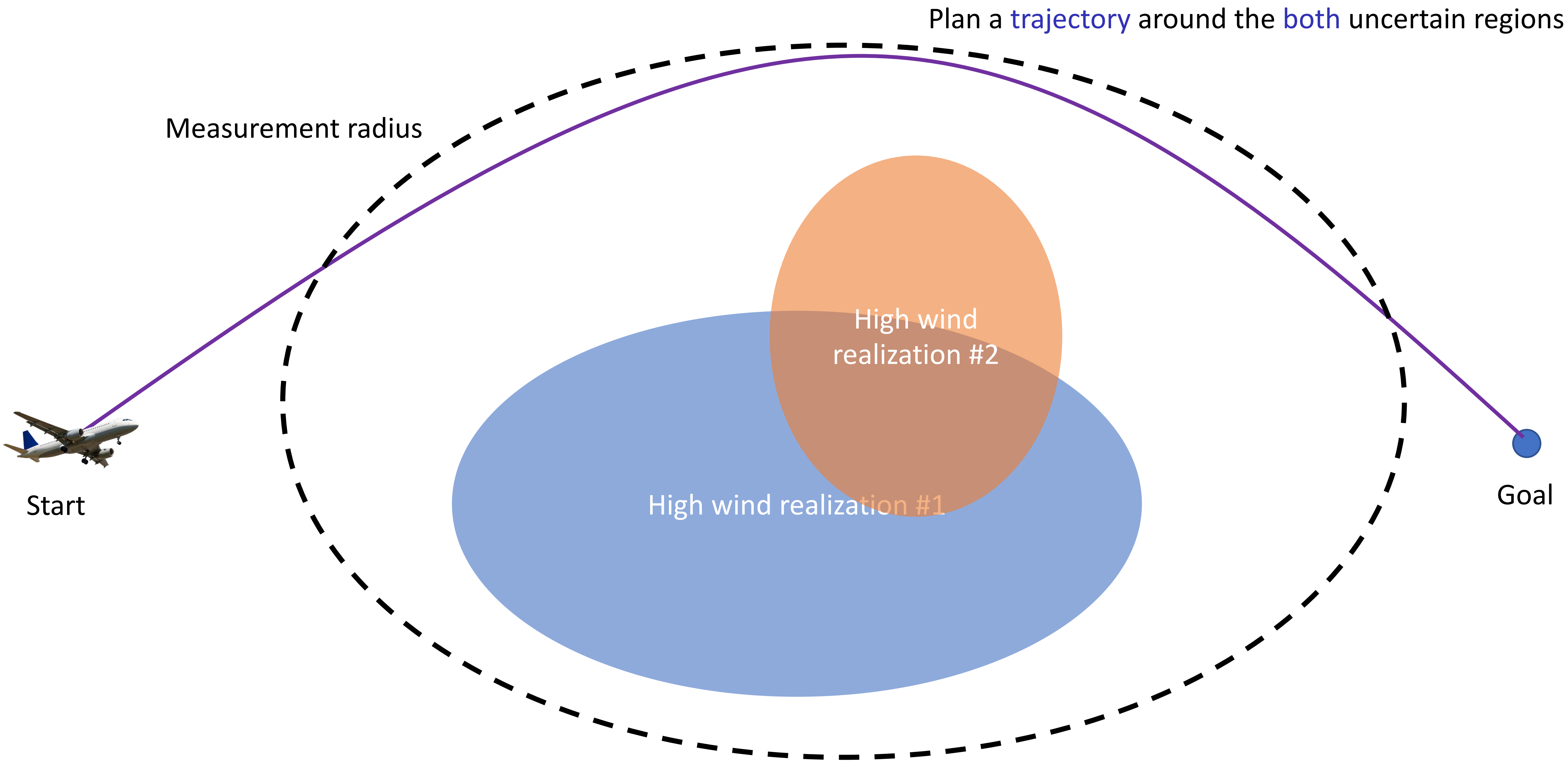

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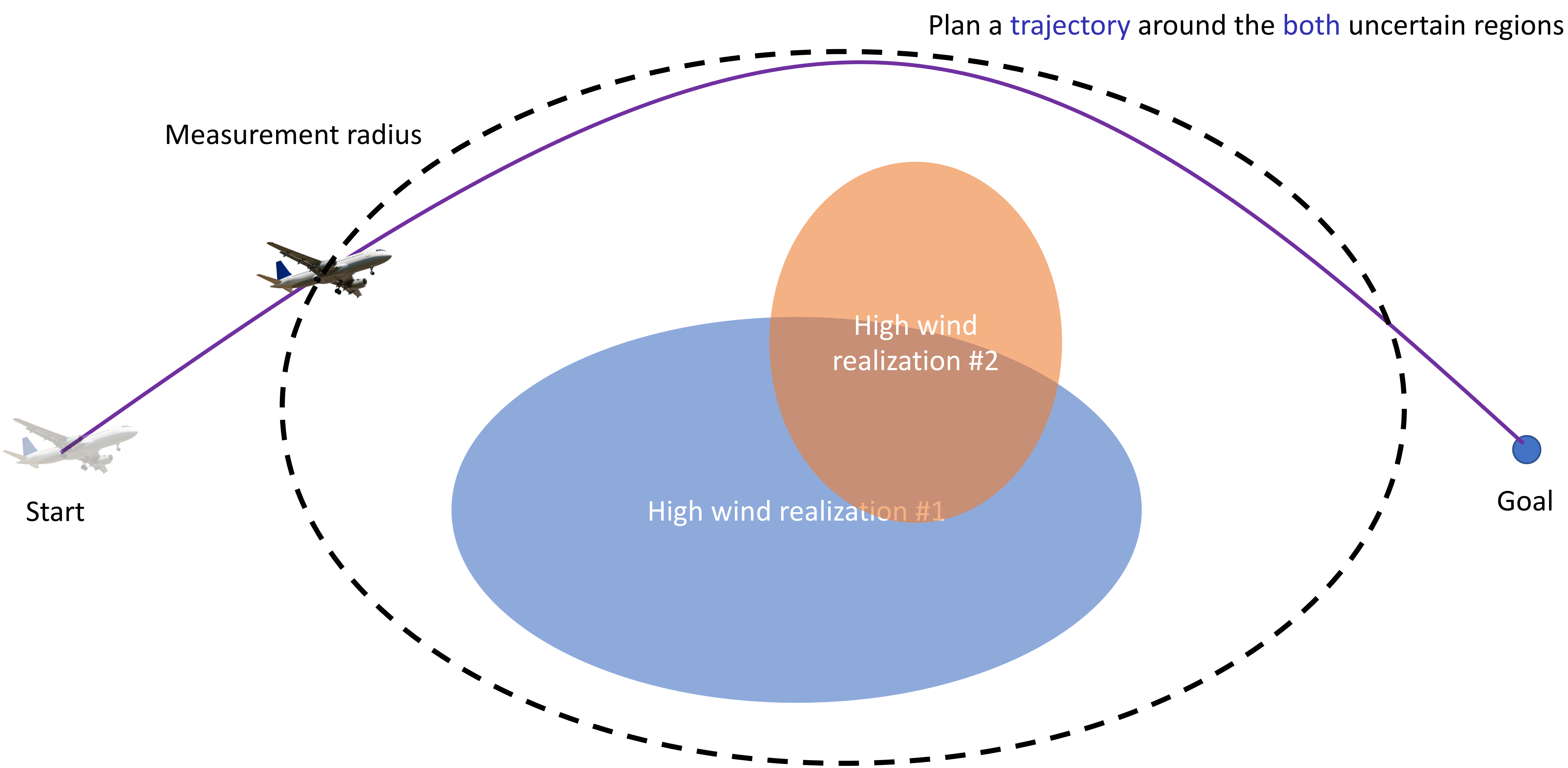

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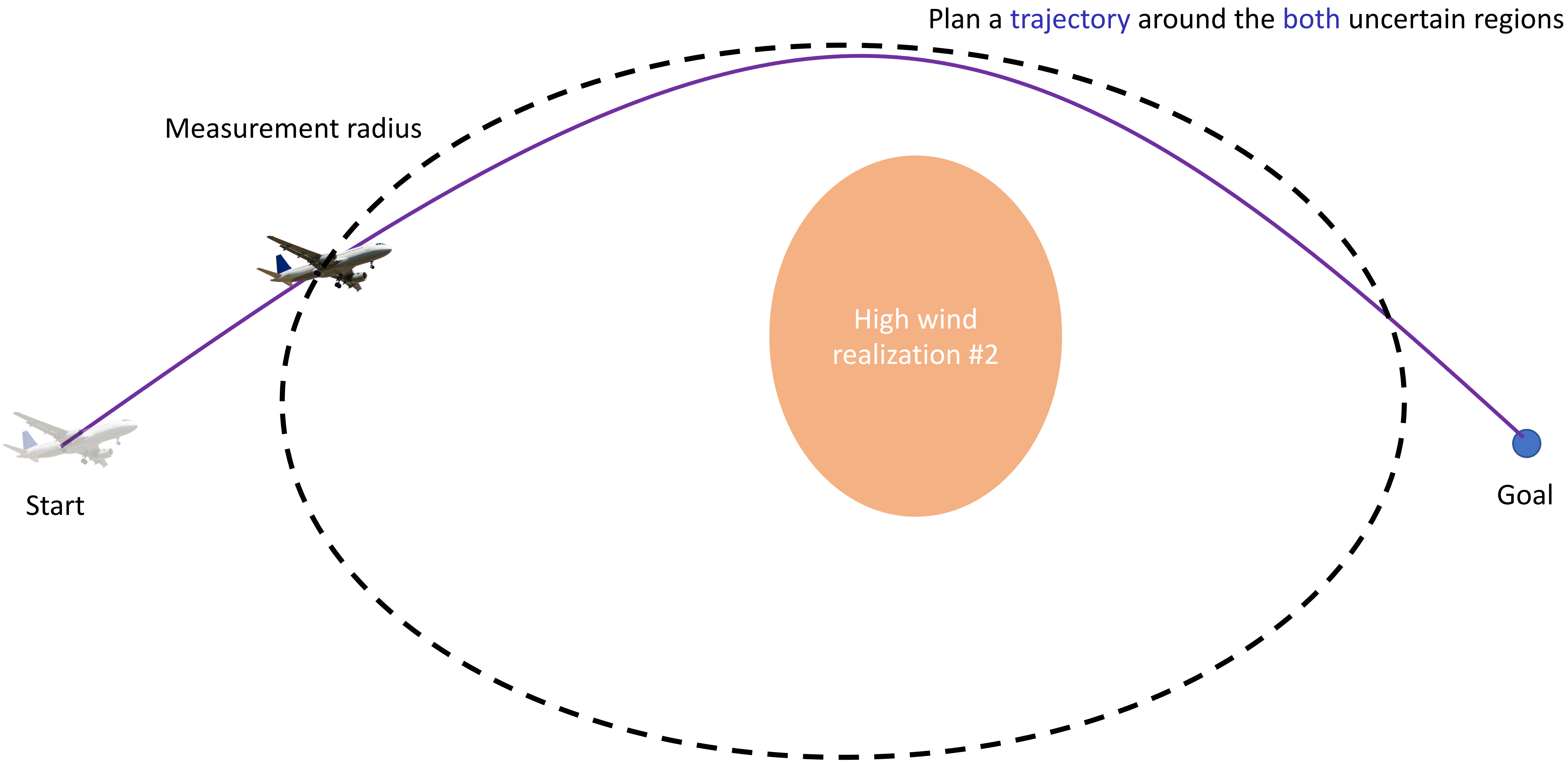

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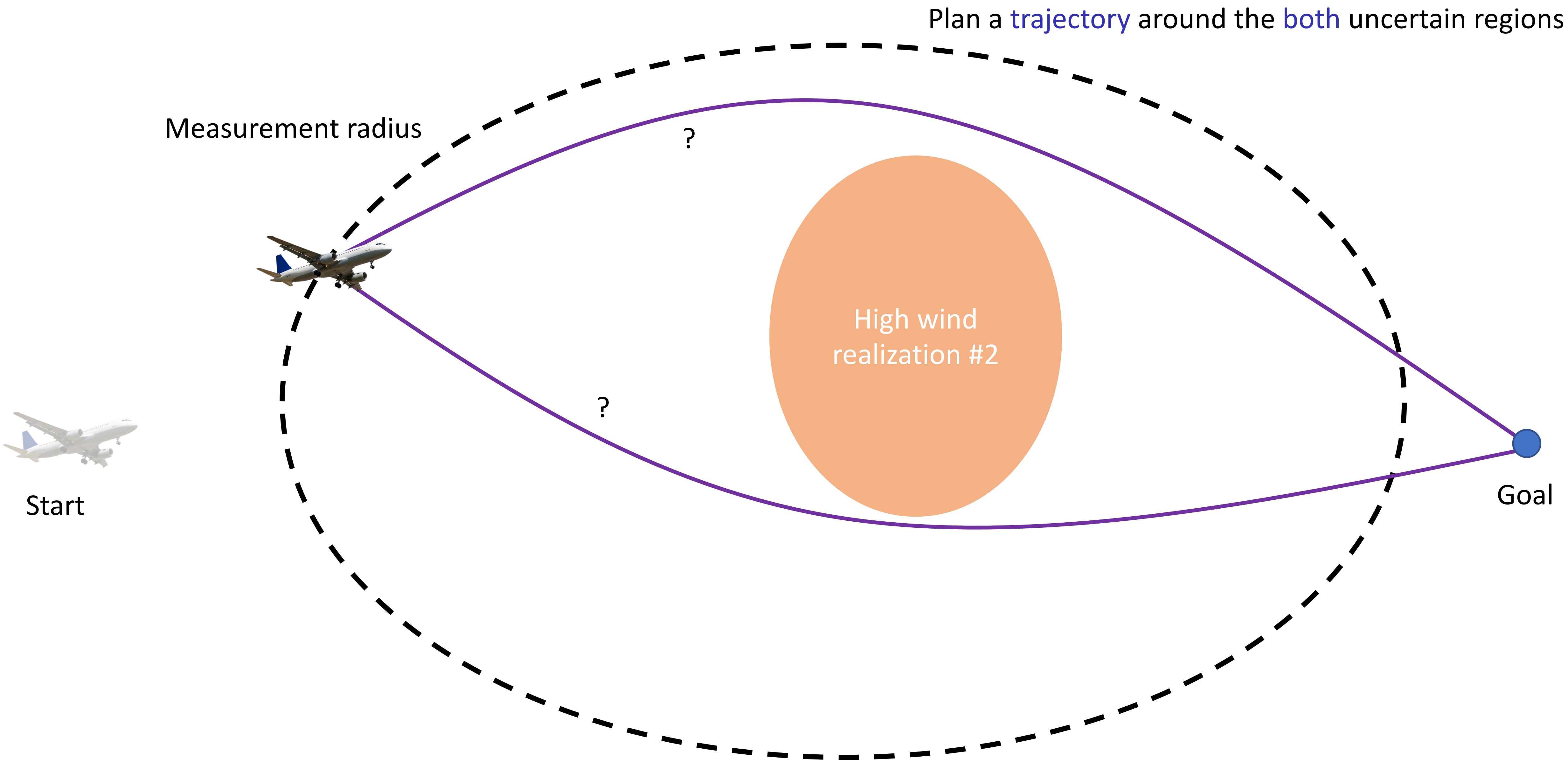

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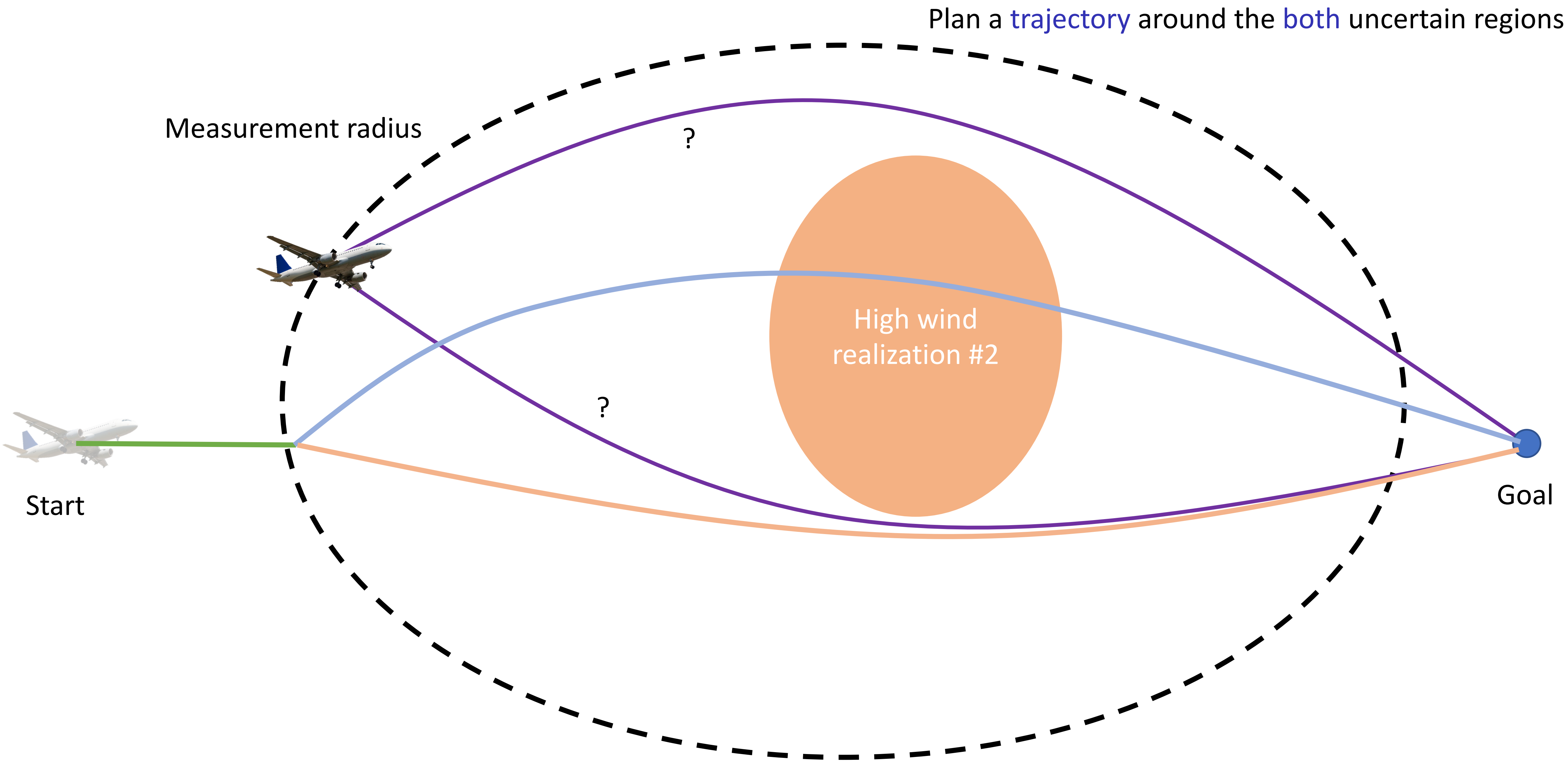


Why should I care?

Why should I care?

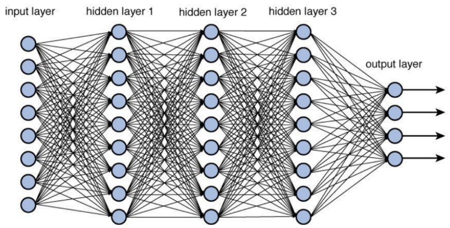
Model-based RL

Why should I care?

Model-based RL

Step 1: Learn a Model

(x_k, u_k)



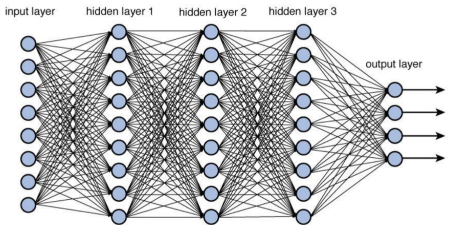
x_{k+1}

Why should I care?

Model-based RL

Step 1: Learn a Model

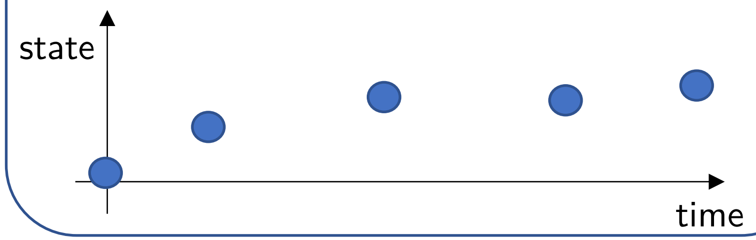
(x_k, u_k)



x_{k+1}

Step 2: Receding horizon planning

$$\min \mathbb{E}_{\substack{a_t \sim \pi \\ s_t \sim p}} [\textstyle\sum_{t=0}^{H} \gamma^t c(a_t, s_t)]$$

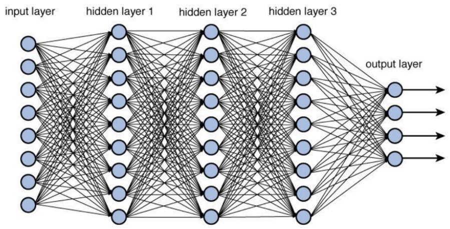


Why should I care?

Model-based RL

Step 1: Learn a Model

(x_k, u_k)



x_{k+1}

Step 2: Receding horizon planning

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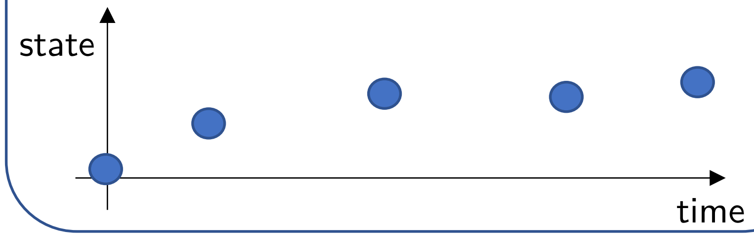
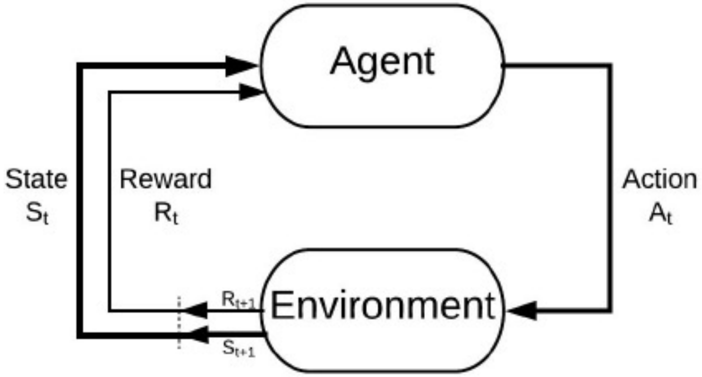
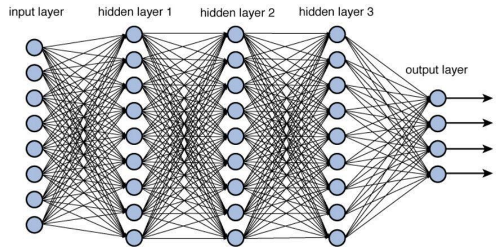


Model-free RL

Step 1: Learn a policy



x_k



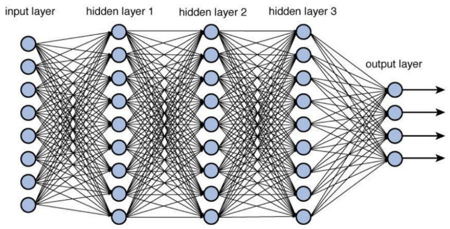
u_k

Why should I care?

Model-based RL

Step 1: Learn a Model

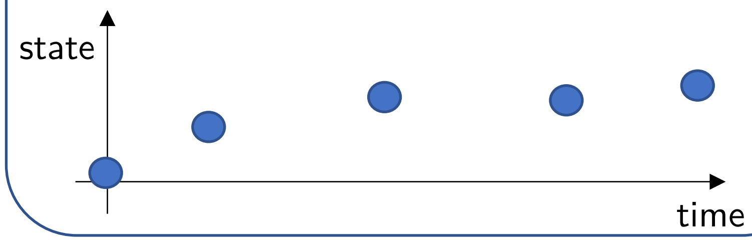
(x_k, u_k)



x_{k+1}

Step 2: Receding horizon planning

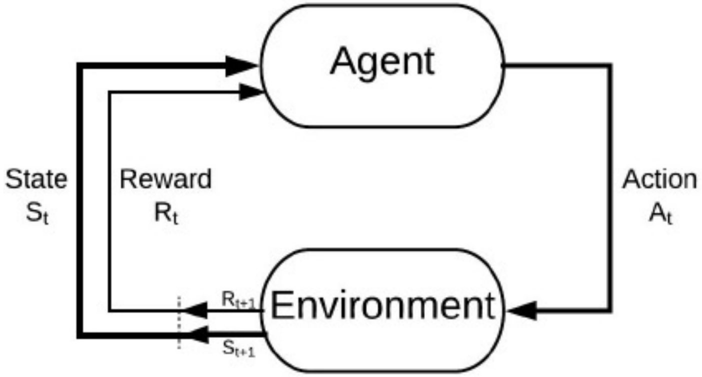
$$\min \mathbb{E}_{\substack{a_t \sim \pi \\ s_t \sim p}} [\textstyle\sum_{t=0}^{H} \gamma^t c(a_t, s_t)]$$



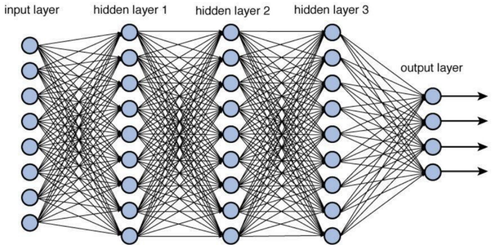
Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



x_k



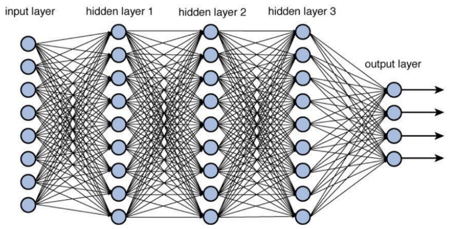
u_k

Why should I care?

Model-based RL

Step 1: Learn a Model

(x_k, u_k)



x_{k+1}

Step 2: Receding horizon planning

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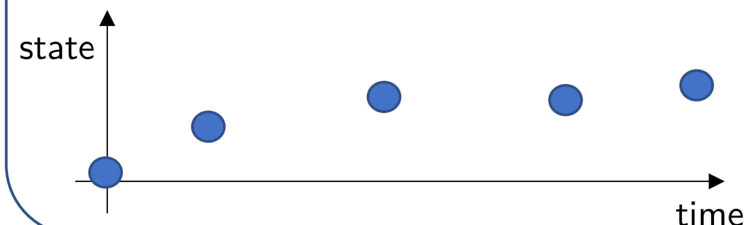
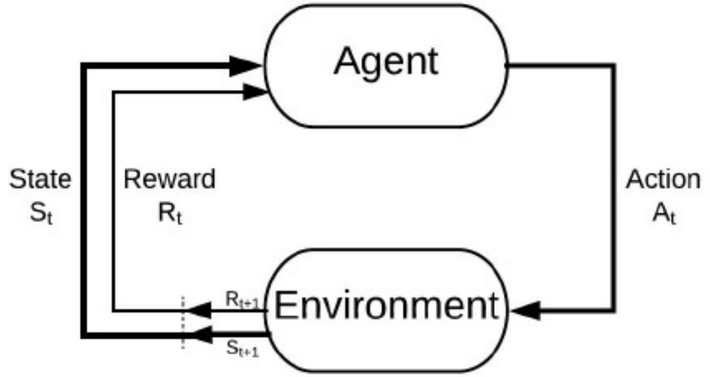


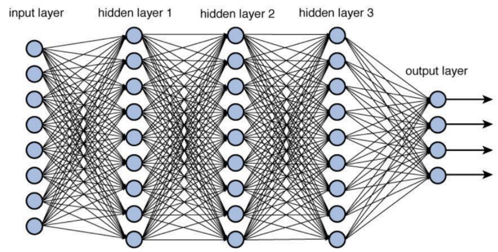
Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



x_k



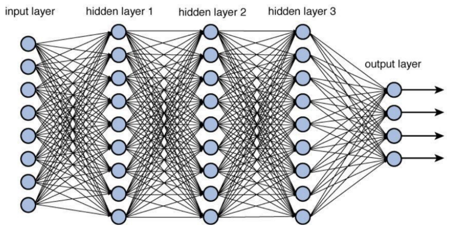
u_k

What can go wrong with model-based RL?

Why should I care?

Model-based RL

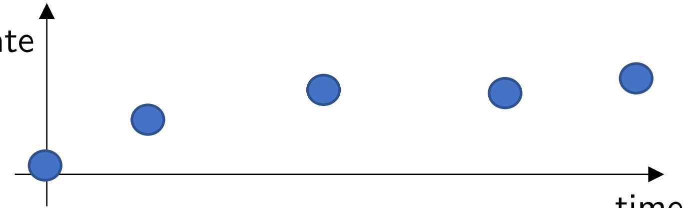
Step 1: Learn a Model



(x_k, u_k) x_{k+1}

Step 2: Receding horizon planning

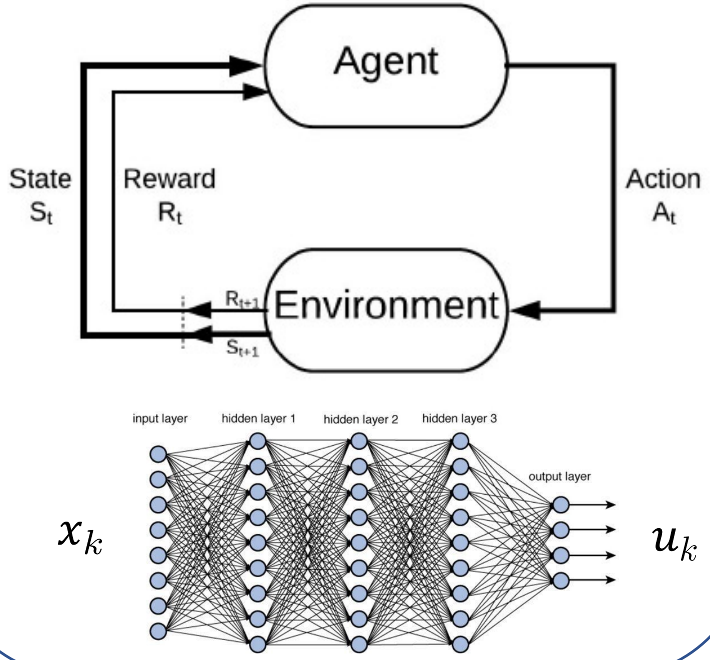
$$\min \mathbb{E}_{\substack{a_t \sim \pi \\ s_t \sim p}} [\textstyle\sum_{t=0}^{H} \gamma^t c(a_t, s_t)]$$



Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



x_k u_k

What can go wrong with model-based RL?

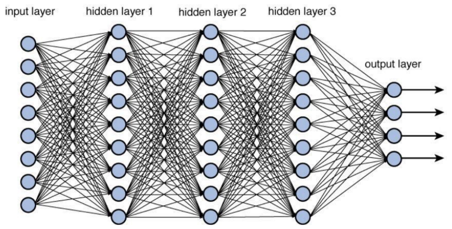
- The prediction horizon is short → need a value function and safe set

Why should I care?

Model-based RL

Step 1: Learn a Model

(x_k, u_k)



x_{k+1}

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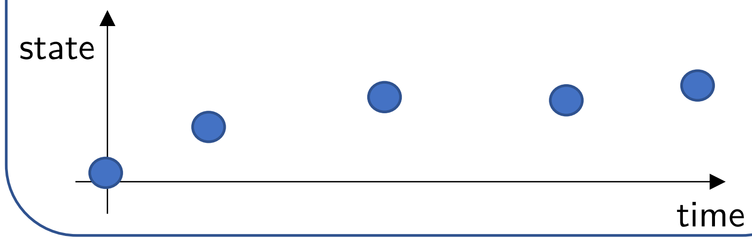
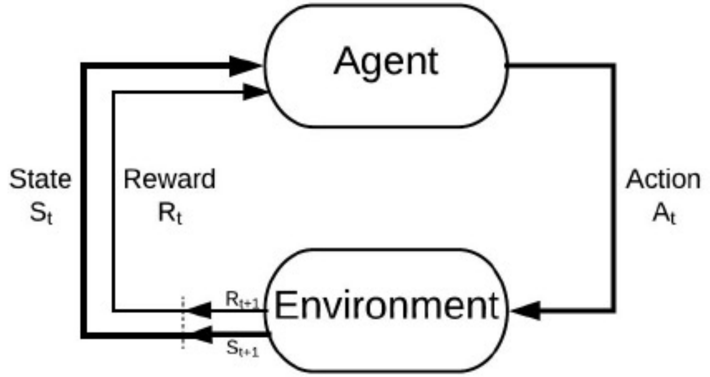


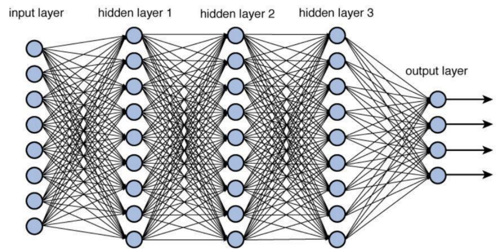
Which one has better asymptotic performance?

Model-free RL

Step 1: Learn a policy



x_k



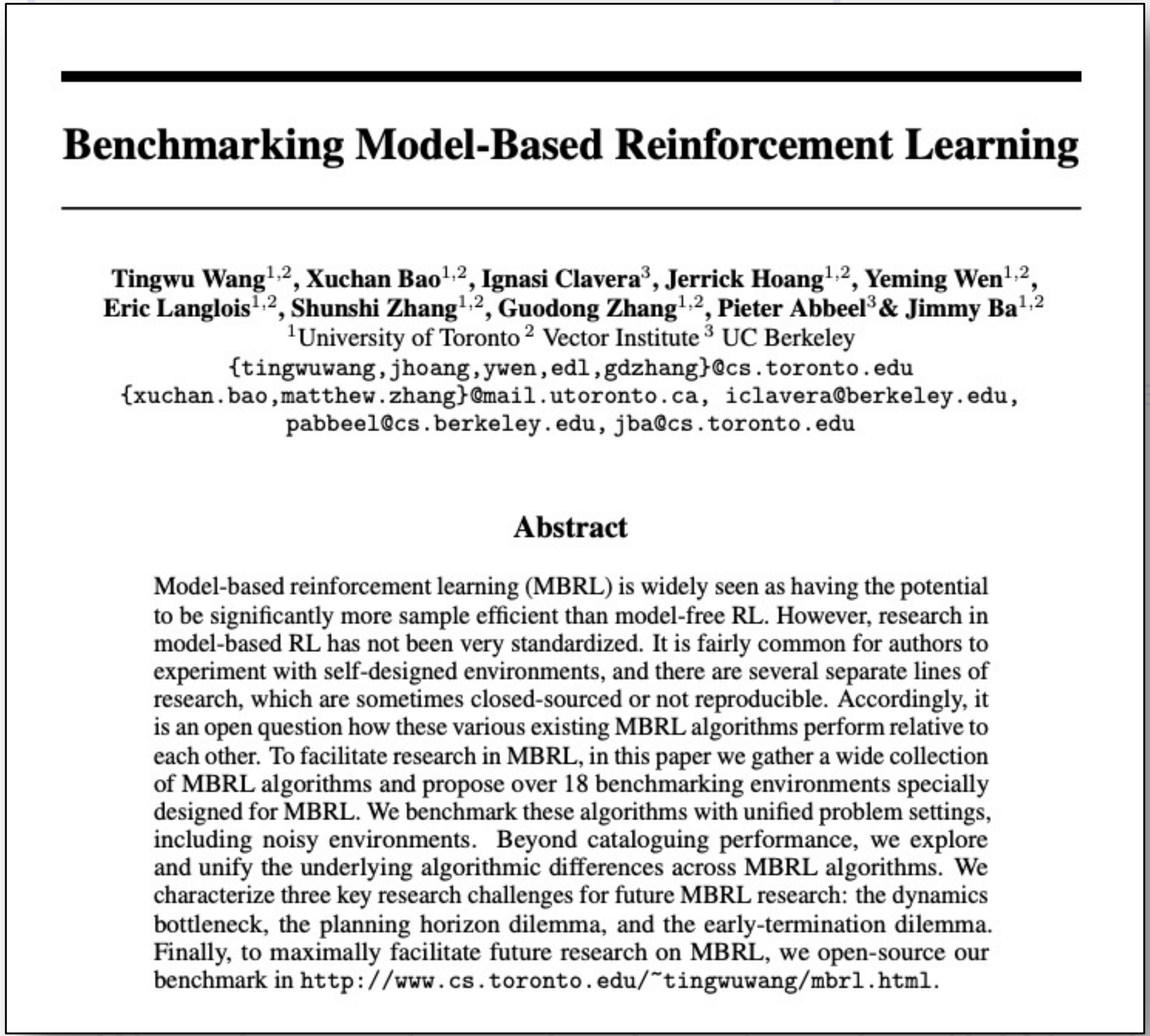
u_k

What can go wrong with model-based RL?

- The prediction horizon is short → need a value function and safe set
- Planning trajectories is suboptimal! We should plan over policies

Why should I care?

Model-based RL



Benchmarking Model-Based Reinforcement Learning

Tingwu Wang[1,2], Xuchan Bao[1,2], Ignasi Clavera[3], Jerrick Hoang[1,2], Yeming Wen[1,2], Eric Langlois[1,2], Shunshi Zhang[1,2], Guodong Zhang[1,2], Pieter Abbeel[3] & Jimmy Ba[1,2]
[1]University of Toronto [2] Vector Institute [3] UC Berkeley
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{xuchan.bao,matthew.zhang}@mail.utoronto.ca, iclavera@berkeley.edu,
pabbeel@cs.berkeley.edu, jba@cs.toronto.edu

Abstract

Model-based reinforcement learning (MBRL) is widely seen as having the potential to be significantly more sample efficient than model-free RL. However, research in model-based RL has not been very standardized. It is fairly common for authors to experiment with self-designed environments, and there are several separate lines of research, which are sometimes closed-sourced or not reproducible. Accordingly, it is an open question how these various existing MBRL algorithms perform relative to each other. To facilitate research in MBRL, in this paper we gather a wide collection of MBRL algorithms and propose over 18 benchmarking environments specially designed for MBRL. We benchmark these algorithms with unified problem settings, including noisy environments. Beyond cataloguing performance, we explore and unify the underlying algorithmic differences across MBRL algorithms. We characterize three key research challenges for future MBRL research: the dynamics bottleneck, the planning horizon dilemma, and the early-termination dilemma. Finally, to maximally facilitate future research on MBRL, we open-source our benchmark in http://www.cs.toronto.edu/~tingwuwang/mbrl.html.

has better
erformance?

Model-free RL

Step 1: Learn a policy

value function and safe set

▶ Planning trajectories is suboptimal! We should plan over policies

Why should I care?

Model-based RL

Model-free RL

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Model-Based Reinforcement Learning via Meta-Policy Optimization

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John Schulman
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Pieter Abbeel
UC Berkeley, Covariant.AI

Abstract: Model-based reinforcement learning approaches carry the promise of being data efficient. However, due to challenges in learning dynamics models that sufficiently match the real-world dynamics, they struggle to achieve the same asymptotic performance as model-free methods. We propose Model-Based Meta-Policy-Optimization (MB-MPO), an approach that foregoes the strong reliance on accurate learned dynamics models. Using an ensemble of learned dynamic models, MB-MPO meta-learns a policy that can quickly adapt to any model in the ensemble with one policy gradient step. This steers the meta-policy towards internalizing consistent dynamics predictions among the ensemble while shifting the burden of behaving optimally w.r.t. the model discrepancies towards the adaptation step. Our experiments show that MB-MPO is more robust to model imperfections than previous model-based approaches. Finally, we demonstrate that our approach is able to match the asymptotic performance of model-free methods while requiring significantly less experience.

Keywords: Reinforcement Learning, Meta-Learning, Model-Based, Model-Free

▶ Planning trajectories is suboptimal! We should plan over policies

Why should I care?

Model-based RL

Model-free RL

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Keywords: Reinforcement Learning, Meta-Learning, Model-Based, Model-Free

- Planning trajectories is suboptimal! We should plan over policies

How do we fix this problem?

How do we fix this problem?

How do we fix this problem?

1136 IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 43, NO. 8, AUGUST 1998

Min–Max Feedback Model Predictive Control for Constrained Linear Systems

P. O. M. Scokaert and D. Q. Mayne

Abstract—Min–max feedback formulations of model predictive control are discussed, both in the fixed and variable horizon contexts. The schemes the authors discuss introduce, in the control optimization, the notion that feedback is present in the receding-horizon implementation of the control. This leads to improved performance, compared to standard model predictive control, and resolves the feasibility difficulties that arise with the min–max techniques that are documented in the literature. The stabilizing properties of the methods are discussed as well as practical implementation details.

Index Terms—Feedback, min–max optimization, model predictive control.

I. Introduction

Model predictive control (MPC) is a control methodology that is becoming mature. The basic aspects of the method are well understood, and stabilizing formulations of the control law are documented in the literature, both for linear and nonlinear processes [1], [2].

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Automatica 42 (2006) 523–533

automatica

Optimization over state feedback policies for robust control with constraints

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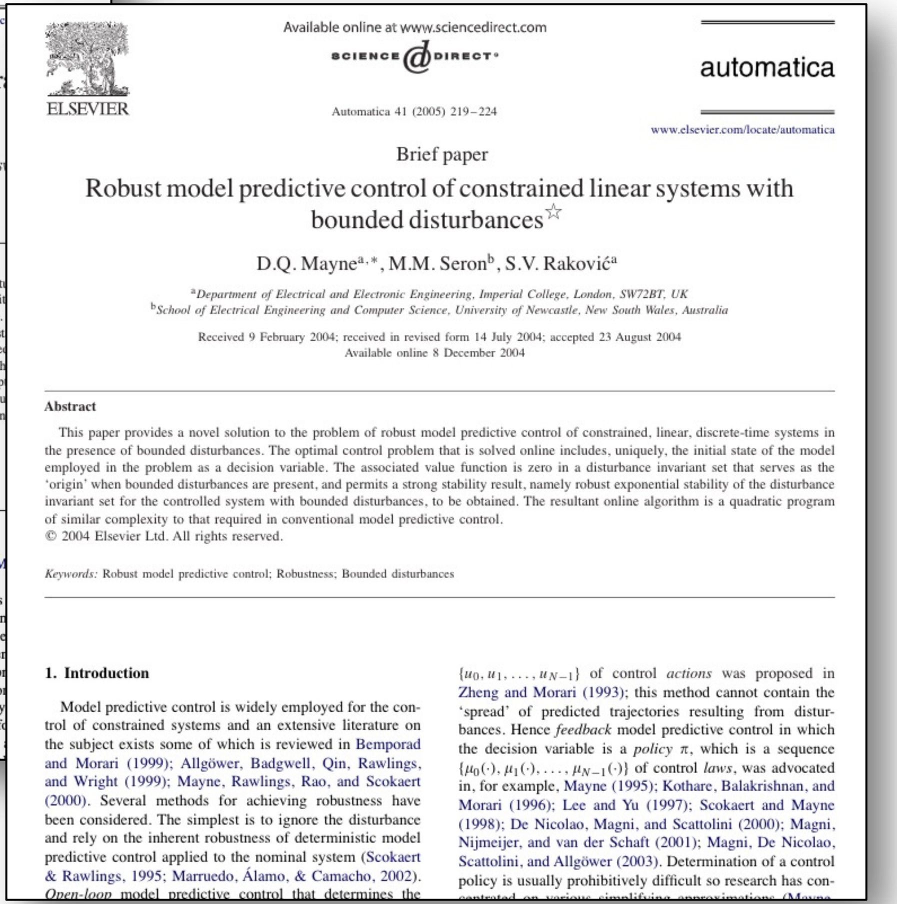
Received 15 March 2005; received in revised form 8 June 2005; accepted 25 August 2005

Abstract

This paper is concerned with the optimal control of linear discrete-time systems subject to unknown but bounded state disturbances and mixed polytopic constraints on the state and input. It is shown that the class of admissible affine state feedback control policies with knowledge of prior states is equivalent to the class of admissible feedback policies that are affine functions of the past disturbance sequence. … © 2006 Elsevier Ltd. All rights reserved.

Keywords: Robust control; Constraint satisfaction; Robust optimization; Predictive control; Optimal control



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Automatica 41 (2005) 219–224

automatica

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Brief paper

Robust model predictive control of constrained linear systems with bounded disturbances

D.Q. Mayne[a,*], M.M. Seron[b], S.V. Raković[a]

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Received 9 February 2004; received in revised form 14 July 2004; accepted 23 August 2004

Available online 8 December 2004

Abstract

This paper provides a novel solution to the problem of robust model predictive control of constrained, linear, discrete-time systems in the presence of bounded disturbances. The optimal control problem that is solved online includes, uniquely, the initial state of the model employed in the problem as a decision variable. The associated value function is zero in a disturbance invariant set that serves as the 'origin' when bounded disturbances are present, and permits a strong stability result, namely robust exponential stability of the disturbance invariant set for the controlled system with bounded disturbances, to be obtained. The resultant online algorithm is a quadratic program of similar complexity to that required in conventional model predictive control.

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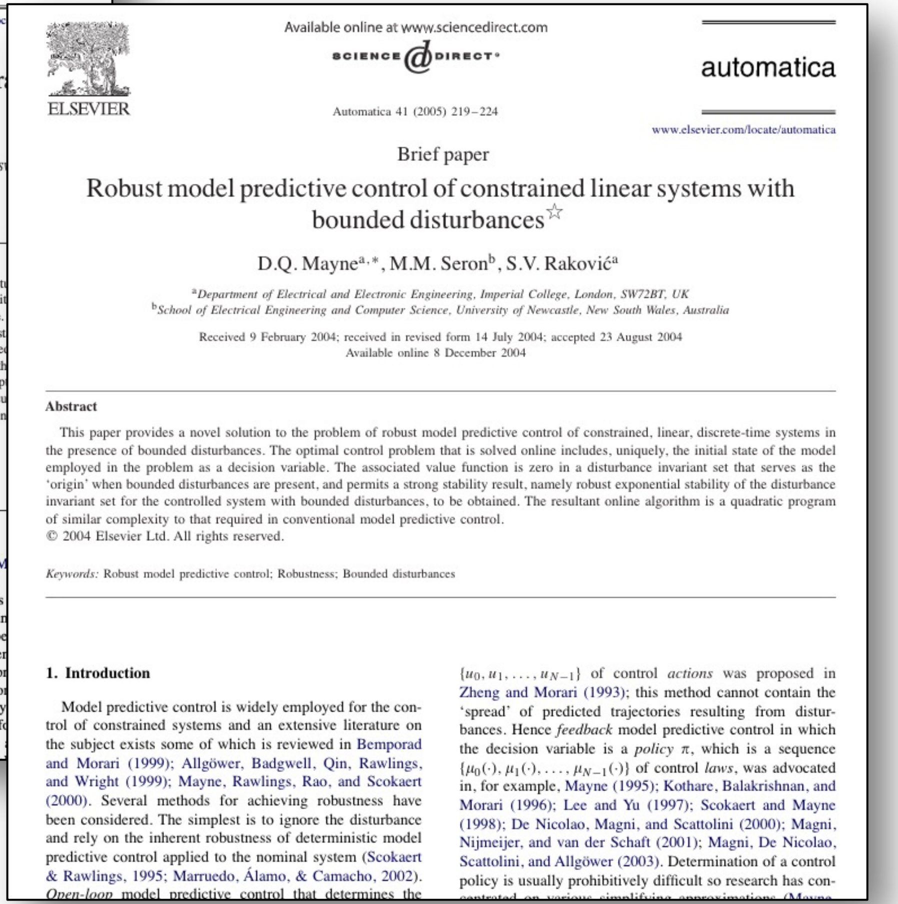
Keywords: Robust model predictive control; Robustness; Bounded disturbances

The problem is NP-hard!

How do we fix this problem?

The problem is NP-hard!



An elegant approximation

How do we fix this problem?

Statement 2: Under the assumptions of this theorem for any $0 \le t_0 \le t$

$$\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}}(t_0) \ge \Phi_{\tilde{u}^*}(t) - \Phi_{\tilde{u}^*}(t_0) = -\frac{1}{h}\Delta\vartheta^T(t)L^{-1}\Delta\vartheta(t) + o_\omega(h)$$

where

$$\Phi_{\tilde{u}}(t) := \int_0^t (2\tilde{u}^T(\tau)d\vartheta(\tau) + \tilde{u}^T(\tau)L\tilde{u}(\tau)d\tau) \quad (20)$$

$$\Delta\vartheta(t) := B_{u0}^T P[B_{u0}(u_{comp}(t) - x^*(t))h + (K + A_0C_0^+)\Delta y(t)], \quad \Delta y(t) = y(t) - y(t-h)$$

and this minimum is reachable for

$$\tilde{u}(t)dt = \tilde{u}^*(t)dt := -L^{-1}B_{u0}^T P[B_{u0}(u_{comp}(t) + x^*(t))dt + (K + A_0C_0^+)dy(t)].$$

As a result, we have: $d\Phi_{\tilde{u}^*}(t) \le 0$ (in symbolic form).

Proof of Statement 2: Using the Euler–Maruyama's formula [3], [9] we obtain the following relation: $(h := t - t_0 \to 0)$: $\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}}(t_0) = 2\tilde{u}^T(t)\Delta\vartheta(t) + \tilde{u}^T(t)L\tilde{u}(t)h + o_\omega(h)$.

Minimizing then the right side for each fixed t, we derive

$$\min_{\tilde{u}(t)}[\Phi_{\tilde{u}}(t) - \Phi_{\tilde{u}}(t_0)] = \Phi_{\tilde{u}^*}(t) - \Phi_{\tilde{u}^*}(t_0) = -\frac{1}{h}\Delta\vartheta^T(t)L^{-1}\Delta\vartheta(t) + o_\omega(h) \le o_\omega(h). \quad (21)$$

Hence, taking into account the definition (20), we have $d\Phi_{\tilde{u}^*}(t) \le 0$. □

Rewriting (18) in differential form and taking into account that P is the solution of the Riccati equation (see Assumption 4 of this theorem) we finally derive

$$dV(e(t)) \overset{a.s.}{\le} [\varphi(t) + I(t)]dt + S^T(t)dw(t) - e^T(t)Q_e e(t)dt.$$

According to the Kronecker lemma, the Large Number law for martingale, and Skorohod lemma (see [2] and [4]), we obtain the result of the theorem. Details of this proof can be found in [13].

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Min–Max Feedback Model Predictive Control for Constrained Linear Systems

P. O. M. Scokaert and D. Q. Mayne

Abstract—Min–max feedback formulations of model predictive ... are discussed, both in the fixed and variable horizon contexts. The ... schemes the authors discuss introduce, in the control optimizati... notion that feedback is present in the receding-horizon implemen... of the control. This leads to improved performance, compared to sta... model predictive control, and resolves the feasibility difficulties tha... with the min–max techniques that are documented in the liter... The stabilizing properties of the methods are discussed as well a... practical implementation details.

Index Terms—Feedback, min–max optimization, model predicti... trol.

I. INTRODUCTION

Model predictive control (MPC) is a control methodolog... is becoming mature. The basic aspects of the method are... well understood, and stabilizing formulations of the control la... documented in the literature, both for linear and nonlinear pro... [1], [2].

The MPC strategy optimizes an open-loop control sequen... each sample, to minimize a nominal cost function, subject to... state and input constraints. The optimization is usually bas... the assumption that the process model is exact and that ... disturbances are constant. Because the control law ignores the ... of possible future changes in disturbance and model mis... closed-loop performance can be poor [3], with likely violati... the constraints, when disturbances or model mismatch are pres...

Some formulations of MPC have been proposed that address ... issues [4], [5]. These methods rely on a min–max optimizat... predicted performance. However, these formulations optimize a... control profile over all possible disturbance (or model mis...

The problem is NP-hard!

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Optimization over state feedback policies for robust control with constr...

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Abstract

This paper is concerned with the optimal control of linear discrete-time systems subject to unknown but bounded state distu... mixed polytopic constraints on the state and input. It is shown that the class of admissible affine state feedback control policies wi... of prior states is equivalent to the class of admissible feedback policies that are affine functions of the past disturbance sequence. ... that a broad class of constrained finite horizon robust and optimal control problems, where the optimization is over affine st... policies, can be solved in a computationally efficient fashion using convex optimization methods. This equivalence result is used ... robust receding horizon control (RHC) state feedback policy such that the closed-loop system is input-to-state stable (ISS) and th... are satisfied for all time and all allowable disturbance sequences. The cost to be minimized in the associated finite horizon op... problem is quadratic in the disturbance-free state and input sequences. The value of the receding horizon control law can be calcu... sample instant using a single, tractable and convex quadratic program (QP) if the disturbance set is polytopic, or a tractable secon... program (SOCP) if the disturbance set is given by a 2-norm bound.

Keywords: Robust control; Constraint satisfaction; Robust optimization; Predictive control; Optimal control

1. Introduction

This paper is concerned with the control of constrained discrete-time linear systems that are subject to additive, but bounded disturbances on the state. The main aim is to provide results that allow for the efficient computation of an optimal and stabilizing state feedback control policy that ensures a given set of state and input constraints are satisfied for all time, despite the presence of the disturbances. This is a problem that has been studied for some time now in the optimal control literature (Bertsekas & Rhodes, 1973) and a number of different

& Diaz-Bobillo, 1995) or predictive control (M... 2002; Mayne, Rawlings, Rao, & Scokaert, 2000).

It is generally accepted that if disturbances ... accounted for in the formulation of a constrain... control problem, then the optimization has to be ... admissible state feedback policies, rather than open... sequences, otherwise infeasibility and instability pr... occur (Mayne et al., 2000). However, optimization ... trary (nonlinear) feedback policies is particularly ... constraints have to be satisfied. Current proposals fo... this using finite dimensional optimization, such ...

An elegant approximation

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Brief paper

Robust model predictive control of constrained linear systems with bounded disturbances☆

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Abstract

This paper provides a novel solution to the problem of robust model predictive control of constrained, linear, discrete-time systems in the presence of bounded disturbances. The optimal control problem that is solved online includes, uniquely, the initial state of the model employed in the problem as a decision variable. The associated value function is zero in a disturbance invariant set that serves as the 'origin' when bounded disturbances are present, and permits a strong stability result, namely robust exponential stability of the disturbance invariant set for the controlled system with bounded disturbances, to be obtained. The resultant online algorithm is a quadratic program of similar complexity to that required in conventional model predictive control.

Keywords: Robust model predictive control; Robustness; Bounded disturbances

1. Introduction

Model predictive control is widely employed for the control of constrained systems and an extensive literature on the subject exists some of which is reviewed in Bemporad and Morari (1999); Allgöwer, Badgwell, Qin, Rawlings, and Wright (1999); Mayne, Rawlings, Rao, and Scokaert (2000). Several methods for achieving robustness have been considered. The simplest is to ignore the disturbance and rely on the inherent robustness of deterministic model predictive control applied to the nominal system (Scokaert & Rawlings, 1995; Marruedo, Álamo, & Camacho, 2002). Open-loop model predictive control that determines the

$\{u_0, u_1, \ldots, u_{N-1}\}$ of control *actions* was proposed in Zheng and Morari (1993); this method cannot contain the 'spread' of predicted trajectories resulting from disturbances. Hence *feedback* model predictive control in which the decision variable is a *policy* π, which is a sequence $\{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot)\}$ of control *laws*, was advocated in, for example, Mayne (1995); Kothare, Balakrishnan, and Morari (1996); Lee and Yu (1997); Scokaert and Mayne (1998); De Nicolao, Magni, and Scattolini (2000); Magni, Nijmeijer, and van der Schaft (2001); Magni, De Nicolao, Scattolini, and Allgöwer (2003). Determination of a control policy is usually prohibitively difficult so research has con...

What people do in practice