

How Accurate can a Stereovision Measurement Be?

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Abstract—What is the best accuracy that can be achieved with two cameras in a stereovision arrangement? Stereovision allows 3D measurement. Such arrangements are used more and more and can be a cost effective way of taking complex measurements. However, it turns out that the complex relationship between the parameters on which the result depends makes it difficult to judge the accuracy that can be achieved. This paper reviews some of the literature on stereovision accuracy using CCD cameras. It then derives a limit for the accuracy based on the current state of the art technology. The limit is analytically derived from the pin hole camera model and incorporates both the intrinsic and extrinsic camera parameters as well as the quantization error. Finally, the paper explores the influence of the uncertainty of all the camera parameters as well and the quantization uncertainty. The accuracy model derived in the paper will be useful for practitioners wishing to determine the best achievable accuracy of a particular stereo camera arrangement.

Keywords—stereovision; accuracy; uncertainty, law of error propagation, CCD.

I. INTRODUCTION

There is a complex relationship between the parameters on which a stereovision measurement depends. The relationship is based on the pinhole model of cameras and may even include nonlinear effects caused by lens distortion. The accuracy of the measurement naturally depends on the combination of the system parameters. However, because of the complex relationship, determining the measurement accuracy is not simple and may require expensive and time consuming experiments. Also, if the system setup is changed, e.g. the focal length of one of the cameras is changed, it is essentially a new measurement system which needs to be recalibrated and will have new error characteristics. It will be difficult for practitioners using such systems to have confidence in the measurement result. Error models including all the system parameters and uncertainties do not exist in the open literature and the series of ISO standards dealing with 3D measurement systems (ISO 10360-1 to 10360-7) does not yet include something for camera based systems.

This paper sets out to first answer the fundamental question, i.e. what is the best possible accuracy that can be achieved with a particular system configuration using the current state of the art technology? This level of accuracy would obviously only be possible in the very best metrology

laboratories in the world. Therefore, the paper uses the error model derived to answer the first question, to explore the effect of uncertainty of the system parameters for more general systems.

This introductory section of the paper will describe the camera model used throughout the paper. It will also give a brief overview of some of the literature dealing with stereovision accuracy. The error model is derived in section II. Section III deals with the question of the best achievable accuracy and section IV explores the error model in more general terms showing the effect of the system parameters and uncertainties on the measurement error. The paper concludes with section V.

A. Camera Model

This part of the paper is mostly based on a previous paper by the author [1], which follows the theory described in [2]; however a description of the camera model is necessary to understand the error model following later.

The basic pinhole camera model is illustrated in Fig. 1. The camera is modelled with optical centre at \mathbf{O}_c . The plane defined by the axes x, y is the focal plane. The image or retinal plane is defined by the axis u, v . For digital cameras, this plane will be discretized according to the pixel distribution on the CCD. These two planes are parallel to each other and are separated a distance f called the focal length of the camera. Real world points are mapped to the retinal plane along a ray through \mathbf{O}_c . The world point is \mathbf{M} and its image is \mathbf{m} . The optical axis, the z axis, goes through \mathbf{O}_c and the image centre \mathbf{c} , and is perpendicular to the focal and retinal planes.

A new coordinate system for the real world can be chosen as (\mathbf{O}_c, x, y, z) and an image coordinate system is (\mathbf{c}, u, v) . Then the projection of the world point onto the image plane is as follows.

This can be written in matrix form as

$$\begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} \quad (1)$$

where

$$u = \frac{U}{S}, v = \frac{V}{S} \quad (2)$$

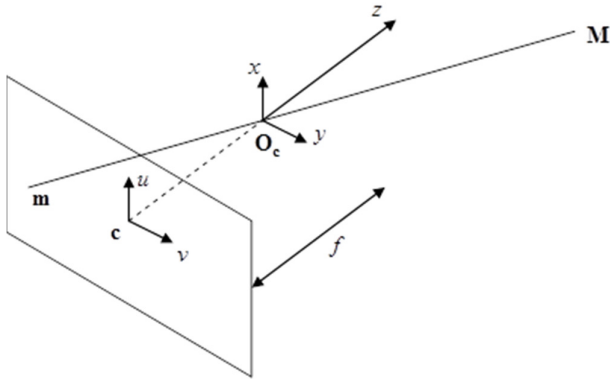


Fig. 1. Pinhole camera model [1]

Equation (2) is a very basic form of the camera matrix, which projects 3D homogeneous world points onto 2D homogeneous image points. This can now be rewritten in the form

$$\mathbf{x} = \mathbf{P}\mathbf{X} \quad (3)$$

The camera matrix in (2) is very basic. For example, normally the image centre is the top left corner, not the middle of the image, the pixels may not be square, etc. However, for the purpose of investigating the accuracy of the measurements, this simplified model will suffice for one of the cameras.

A more general camera matrix will be

$$\begin{bmatrix} -\alpha_u f & s & p_u & 0 \\ 0 & -\alpha_v f & p_v & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (4)$$

where α_u and α_v are scaling factors taking into account the fact that the CCD pixels may not be square; s is a skewness parameter and p_u and p_v are the offsets of the principal point from the image centre. Since this paper focusses on finding the best possible accuracy, it will assume that the principal point is exactly on the image centre, that the pixels are square and therefore $\alpha_u = \alpha_v = 1$ and that the skewness can be neglected which means that $s = 0$. The camera matrix in (1) will therefore be used for the first camera.

The second camera will be some distance from the first camera in the xz plane and rotated about the y axis for the purpose of this investigation. With this simplified system model, the main parameters, i.e. focal length, distance between cameras and the angle between viewing directions can be investigated. It is further assumed that the two cameras are

identical. The rotation and translation of the second camera can be incorporated in the camera matrix as follows:

$$\begin{bmatrix} U \\ V \\ S \end{bmatrix} = \begin{bmatrix} -f & 0 & 0 & 0 \\ 0 & -f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & -RC \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (5)$$

Here, R is a 3x3 rotation matrix, C is a column vector of 3 elements containing the second camera's position and $\mathbf{0}$ is a row vector of 3 elements, all zero. The product of the first two matrices of the right hand side of equation 5 is the camera matrix of the second camera.

The rotation matrix is

$$R = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \quad (6)$$

Thus, the rotation depends on a single variable, θ the rotation angle about the y axis. If the distance between the camera and the object is d , then C can be expressed as (see Fig. 2)

$$C = \begin{bmatrix} d \sin \theta \\ 0 \\ d - d \cos \theta \end{bmatrix} \quad (7)$$

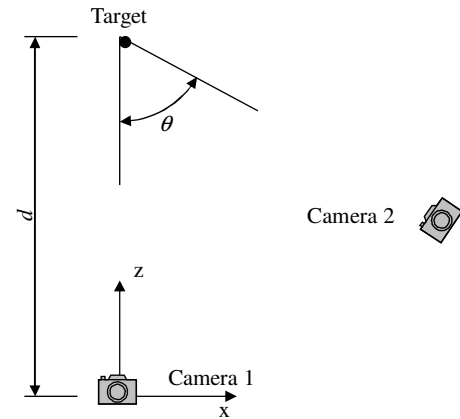


Fig. 2. Stereovision general layout used for the error model

Here, it will be assumed that the two camera's internal parameters are the same; therefore their focal lengths will be the same. The 3D world position thus depends on f, d, θ , and the image coordinates of the two images, (u_1, v_1) and (u_2, v_2) .

The nonlinear effect of lens distortion is ignored. For the ideal accuracy, it will be assumed here that lens distortion is negligible.

The 3D coordinate can then be calculated from [2]

$$\mathbf{AX}=\mathbf{0} \quad (8)$$

where

$$\mathbf{A} = \begin{bmatrix} u_1 \mathbf{p}_1^{3T} & -\mathbf{p}_1^{1T} \\ v_1 \mathbf{p}_1^{3T} & -\mathbf{p}_1^{2T} \\ u_2 \mathbf{p}_2^{3T} & -\mathbf{p}_2^{1T} \\ v_2 \mathbf{p}_2^{3T} & -\mathbf{p}_2^{2T} \end{bmatrix} \quad (9)$$

The subscripts 1 and 2 refer to the first and second camera sets and the superscripts refer to the i^{th} row of the camera matrix. Equation (8) can be solved in various ways. Equation (8) is usually solved with a least squares approach. In this paper a straight forward triangulation approach is followed, i.e the intersection of the rays from the pixel position through the camera principal point is found. The solution is given later in the paper.

B. Literature Review

There are many attempts to understand the accuracy of stereovision measurements. Some focus predominantly on empirical studies, such as [3]. These offer valuable data, but it is difficult to use the results in a predictive way beyond the range of the investigation.

There are a number of studies on camera calibration accuracy [4,5,6,7]. Salvi et al. [4] compared the performance of various existing calibration algorithms. They did not derive an analytical expression for the accuracy. González et al. [5] also studied various camera calibration methods using a real camera as well as simulation results. They studied the effect on intrinsic and extrinsic camera parameters and showed that these parameters are very sensitive to noise. However, even with unstable camera parameters, good triangulation could still be achieved, although triangulation accuracy was not the main focus of their paper. Using computer simulations as well as real world experiments, Sun et al. [6,7] studied the influence of calibration fiducial point quantity, uncertainty of fiducial point position, pixel coordinate noise and various camera models. Kopparapu and Corke [8] also studied the effect of noise on camera parameters. These studies show the dependence on the algorithm being used. This paper will not consider calibration algorithms and will simply assume that the camera parameters are known with a certain level of uncertainty.

In the 1980's the effect of sensor resolution, i.e. quantization error, was studied. These studies [9,10,11] looked at the sensor resolution and showed what the influence of different camera parameters is on accuracy. They considered focal length [9] for two cameras with parallel principal axes, camera alignment assuming uniform error distribution [10] and finally a canonical camera with Gaussian error distribution [11]. These papers showed what the best possible accuracy

could be if all the camera parameters were exactly known. The accuracy strongly depends on the camera parameters and sensor resolution.

As this paper will show, sensor resolution is one of the most significant limiting factors for stereovision accuracy. Recent research on sensor development showed that the current CCD technology is approaching a photometric limit. Sensors with pixels of 1.1 μm are in production [12], with the limit for indoor photography being 0.9 μm . This is not a manufacturing limit, but a result of the required number of photons that can be practically sensed by the pixel. At these small sizes, sensor noise plays a significant role. Schöberl et al. [12] sets the practical minimum pixel size at 1.35 μm .

Rodriguez et al. [13] studied the depth resolution of stereovision systems based on the quantization effects. They showed the dependence of the depth resolution on the other camera parameters. Mandelbaum et al. [14] studied the same effect using a fixed-size confidence interval approach. More recently, Son et al. [15] derived an analytical model for the depth resolution based on the pinhole camera parameters. They showed that the resolvable depth is approximately linearly dependent on the camera distance and inversely proportional to the aperture diameter of the camera objective. Their inclusion of the aperture diameter is interesting. Few studies on error modelling include that. Schöberl et al. [12] also showed that the minimum quantization is dependent on this diameter. Unfortunately, this paper also does not include the influence of aperture diameter.

There are some attempts at a more comprehensive treatment of all the interdependent parameters' influence on accuracy. Trobina [16] and Yang and Wang [17] studied structured light systems. In structured light systems, one of the cameras will be replaced by a calibrated structured light source. Trobina followed an analytical approach, but studied the accuracy only at one point in the measurement domain. He also did not show the accuracy's dependence on the camera parameters. Yang and Wang pointed out that system modelling may contribute to the error. The pinhole model is an approximation and so is the models used to correct for lens distortion. They also pointed out that image processing, used to identify the projection of the target in the image plane, can contribute significantly. In this paper, these effects are not considered, firstly because the aim is to find the best possible accuracy and secondly because the effect image processing is implicitly captured by assigning an uncertainty to the pixel coordinates. Zhao and Nandhakumar [18] showed the accuracy's dependence on camera alignment errors for two cameras with parallel principal axes. They also included the effects of non-parallel CCD arrays and lens distortion.

From the literature a lot can be learned about the effects of the parameters of a stereovision setup. However, no comprehensive analytical model that included all the system parameters and their uncertainties could be found. The model proposed in this paper is a step closer to such a comprehensive model.

II. ERROR MODELLING

The *Law of Error Propagation* [20] can be used to show how the uncertainty of dependent variables propagates to the uncertainty of independent variables. Its use in computer vision is not unusual [21]. The law simply states that

$$\Lambda_F = J(\bar{x})\Lambda_x J(\bar{x})^T \quad (10)$$

where Λ_x is the covariance matrix of the independent variables, \bar{x} is the vector of independent variables, $J(\bar{x})$ is the Jacobian of the system and Λ_F is the covariance matrix of the dependent variables.

As mentioned in section I.A, the position of the triangulated point can be found with the following equations.

$$Z = -\frac{fd(u_2 - f \sin \theta - u_2 \cos \theta)}{(u_1 u_2 + f^2) \sin \theta + f(u_2 - u_1) \cos \theta} \quad (11)$$

$$Y = -\frac{dv_2(u_1 + f \sin \theta - u_1 \cos \theta)}{(u_1 u_2 + f^2) \sin \theta + f(u_2 - u_1) \cos \theta} \quad (12)$$

$$X = \frac{du_1(u_2 - f \sin \theta - u_2 \cos \theta)}{(u_1 u_2 + f^2) \sin \theta + f(u_2 - u_1) \cos \theta} \quad (13)$$

From these equations, the Jacobian can be found. It is a 3×7 matrix for which an analytical solution is clearly possible. However, due to the space limitation of a paper such is this, it is not given here.

The covariance matrix of the independent variables captures their uncertainties. In principle, these variables should be uncorrelated. Therefore Λ_x will be a diagonal matrix with the variance of each independent variable on the diagonal. In practice, due to the calibration methods commonly in used, there may be some correlation between the parameters.

With this, it is now straight forward to calculate Λ_F using (10). It will be a 3×3 covariance matrix with the variances of X , Y and Z , σ_x^2 , σ_y^2 and σ_z^2 respectively, on the diagonal. Again, an analytical solution is possible, but too large to include in this paper. The analytical solution shows that X , Y and Z are correlated. Using (10) through (13) it is therefore possible to find a closed form solution of the coordinates and their standard deviations.

In this paper, the measurement error is then defined as

$$\varepsilon = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (14)$$

The model can be used to estimate the measurement error for any target using any combination of the camera parameters for the camera model in section I.A.

III. ACHIEVABLE ACCURACY

What is the best accuracy that can be achieved with a stereovision system? It is clear that the answer depends on the camera parameters. If the camera parameters are determined using the best available measurement technology and answer could be given for the current state-of-the-art technology.

Length uncertainty can be obtained from comparisons done between different national metrology institutes. Table 1 gives measurement accuracies of various countries based on gauge

block length comparisons as obtained from the BIPM CMC database [22]. Gauge block measurements is not the most accurate length measurements, but the measurement does capture some of the difficulties of the optical setup. The standard for gauge blocks up to 1000 mm in length is used since. A distance of 400 mm from the object to the cameras was arbitrarily chosen to demonstrate the use of the error model. Note that the length uncertainty is length dependent; therefore Table 1 gives the expanded uncertainty at the maximum range as well as at the chosen distance of 400 mm for illustrative purposes. The CMC database reports the expanded uncertainty with a coverage factor of 2. Therefore, the standard deviation is found by dividing the values in Table 1 by two [20]. From the table, the best length uncertainty (after dividing by the coverage factor) is 24 nm and for angles it is $0.005''$. These two values will be used to demonstrate the use of the error model.

TABLE I. LENGTH AND ANGLE MEASUREMENT ACCURACY OF VARIOUS COUNTRIES [22]. THE EXPANDED UNCERTAINTY IS SHOWN WITH A COVERAGE FACTOR OF $k=2$.

Gauge block length [mm]	Length Expanded Uncertainty [nm]		Angle Expanded Uncertainty [arc seconds]
	1000	400	
Brazil	102	88	0.3
Germany	88	48	0.01
UK	132	82	0.03
Switzerland	200	98	0.01

The best practical resolution has already been shown to be $1.35 \mu\text{m}$ in section I.B. Other researchers have followed by setting the quantization uncertainty at $1/2$ pixel, i.e. $0.68 \mu\text{m}$.

Given these uncertainties and a camera setup as in Fig. 2, with $d=400$ mm, $f=25$ mm and $\theta=\pi/2$ and also assuming that the target is on the principal axes of both cameras, the error model predicts an uncertainty of $18.7 \mu\text{m}$. If the uncertainties of d , f and θ are set to zero, i.e. giving only the quantization error, the model predicts essentially the same uncertainty (the values only differ at the 11th decimal). This shows that with the best calibration achievable with current technology, the uncertainties of the camera parameters in this setup will not play a significant role and the quantization error will dominate. However, calibrating the camera parameters to this level of uncertainty is unlikely in practice (with current technology).

The accuracies calculated above are only indicative. It is very dependent on the camera parameters, their uncertainties and the target position. The rest of this section studies the effect of the parameters on the accuracy, still assuming the best achievable uncertainties. In what follows, the graphs were calculated with the following parameters: $d=400$ mm, $f=25$ mm, $\theta=\pi/2$, $u_1=v_1=u_2=v_2=0 \mu\text{m}$ (rather than working with the CCD sensor position in pixel coordinates, a metric coordinate is used here, the effect of the finite size of the pixel is taken into account in the pixel uncertainty), $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005''$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68 \mu\text{m}$.

The effect of the distance from the camera center to the target shows a strong linear trend (Fig. 3).

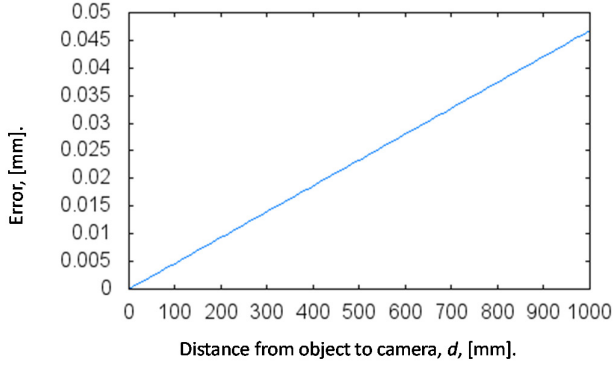


Fig. 3. Relationship between the distance from the object to the camera and measurement accuracy for the described setup. ($\theta=90^\circ$, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μm , $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μm)

The angle between the cameras has a strong influence on the accuracy (Fig. 4). The error rises asymptotically towards 0° and 180° . The angle depicted here is actually the angle between the two rays from the target to the camera centers. Fig. 4 shows an optimal accuracy at 90° , however there is only one such point in the workspace. This effect is explored further later in this section when the position in the workspace is studied.

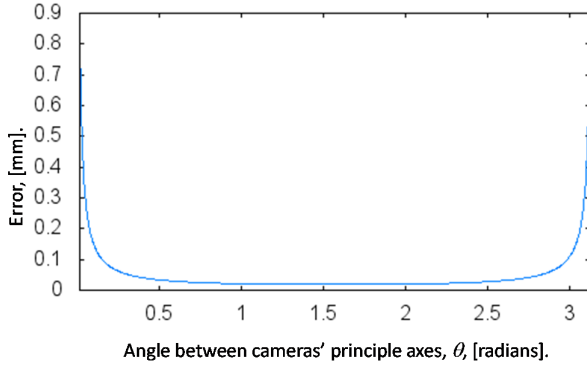


Fig. 4. Relationship between the angle between the cameras' principal axes and measurement accuracy for the described setup. ($d=400$ mm, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μm , $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μm)

The focal length shows a strong nonlinear influence on the accuracy with an asymptotic increase towards 0 mm. It approaches a near 0 mm accuracy as the focal length increases. As the focal length increases the camera's field of view (FOV) decreases, thereby decreasing the quantization error. With the correct setup it will be possible to reach the diffraction limit for optical systems, which is roughly 0.4 μm [23] for white light photography. This is the best achievable accuracy of a stereovision system.

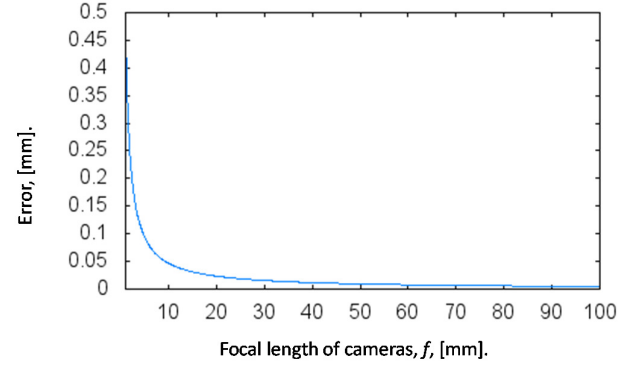


Fig. 5. Relationship between the focal length and the measurement accuracy for the described setup. ($\theta=90^\circ$, $d=400$ mm, $u_1=u_2=v_1=v_2=0$ μm , $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μm)

It is well known that the quantization error is significant. Fig. 6 shows a strong linear increase of the accuracy with an increase in pixel size.

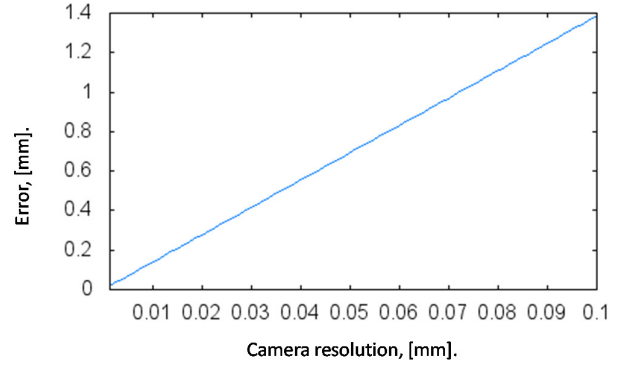


Fig. 6. Relationship between the CCD resolution and measurement accuracy for the described setup. ($\theta=90^\circ$, $d=400$ mm, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μm , $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$)

The above results are all valid for the point in space where the principal axes intersect (point P in Fig. 7). It was already shown that there is a strong variation in accuracy throughout the observable space [1,24]. The proposed model confirms this trend. Fig. 7 is a setup used to investigate the distribution of the error in the observable volume. The error was estimated for all the points on the plane EFGH. The normal vector to this surface is perpendicular to the bisection of the principal axes of the two cameras. Fig. 8 shows the magnitude of the errors.

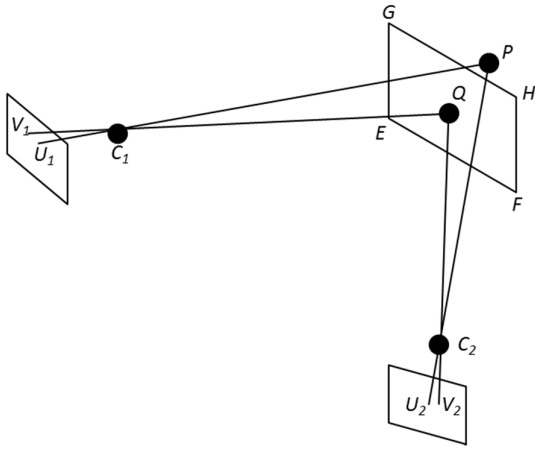


Fig. 7. Configuration to test error distribution in observable volume.

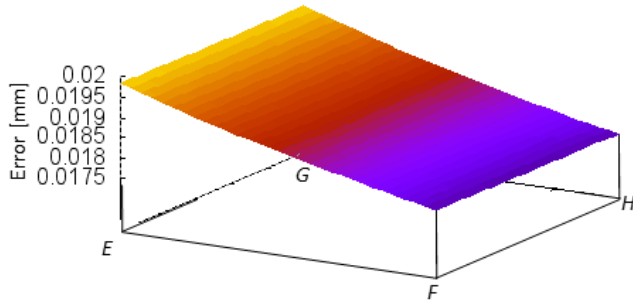


Fig. 8. Distribution of errors of all the points on plane EFGH in Fig. 7. ($\theta=90^\circ$, $d=400$ mm, $f=25$ mm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μ m, $\sigma_d=\sigma_f=24$ nm)

IV. EFFECT OF PARAMETER UNCERTAINTY

The previous section showed how the errors will vary for the key parameters in a stereovision measurement assuming the best estimates of these parameters. This section shows the effect of the uncertainty in these parameters.

Accuracy as influenced by the uncertainty in θ (Fig.9), d (Fig. 10) and f (Fig. 11) are shown below. All three figures show a similar strong dependence on the uncertainty of these parameters.

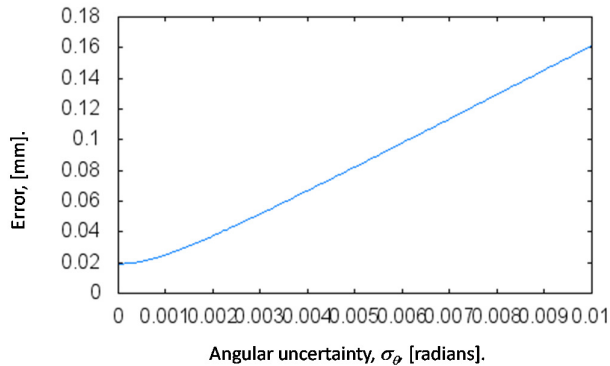


Fig. 9. Effect of uncertainty in θ . ($\theta=90^\circ$, $d=400$ mm, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μ m, $\sigma_d=\sigma_f=24$ nm, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μ m)

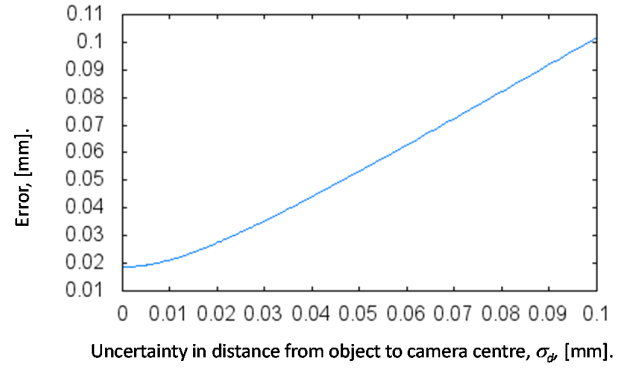


Fig. 10. Effect of uncertainty in d . ($\theta=90^\circ$, $d=400$ mm, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μ m, $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μ m)

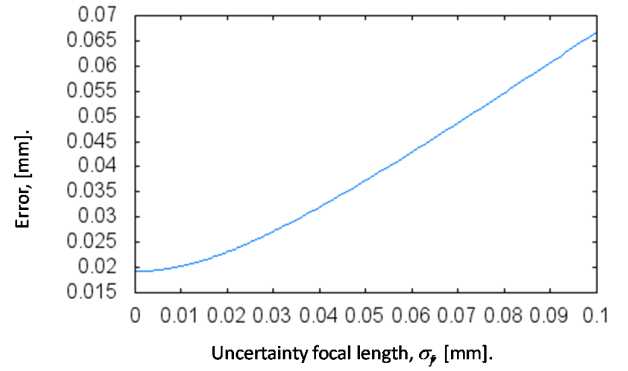


Fig. 11. Effect of uncertainty in f . ($\theta=90^\circ$, $d=400$ mm, $f=25$ mm, $u_1=u_2=v_1=v_2=0$ μ m, $\sigma_d=\sigma_f=24$ nm, $\sigma_\theta=0.005^\circ$, $\sigma_{u1}=\sigma_{v1}=\sigma_{u2}=\sigma_{v2}=0.68$ μ m)

V. CONCLUSION

The best achievable accuracy with a stereovision setup as illustrated in Fig. 2 is strongly limited by the CCD resolution. The paper shows that if the camera parameters are determined with the best current achievable measurement uncertainty, then the quantization error dominates. The available literature suggests that there is a limit to the CCD resolution for practical indoor photography and that current manufacturing technology is already close to this limit. Of course, the accuracy cannot be better than the diffraction limit. If the camera parameters are determined with less accurate methods, then their uncertainties becomes significant. The model presented here is useful since it includes all the significant parameters of a typical stereovision setup. It can be used to estimate the best achievable accuracy of any similar setup. The model is easy to implement in a symbolic processor such as wxMaxima.

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