

Radical Numbers

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1 Köklü Sayılar

$$\sqrt[n]{a^m} = \frac{a^m}{n}$$

$$\sqrt{a^2} = a$$

$$\sqrt[3]{a^3} = a$$

$$\sqrt[m]{\sqrt[n]{\sqrt[x]{\sqrt[a]{a}}}} = \sqrt[m \cdot n \cdot x]{a}$$

$$\sqrt[n]{x} \times \sqrt[n]{y} = \sqrt[n]{x \times y}$$

$$\sqrt{a + b + 2\sqrt{a \times b}} = \sqrt{a} + \sqrt{b}$$

$$\sqrt{a + b - 2\sqrt{a \times b}} = \sqrt{a} - \sqrt{b}$$

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$$

Problem 1.1 – $\sqrt{5 + 2\sqrt{6}} = ?$

Sol.

$$\sqrt{3} + \sqrt{2}$$

□

1.1 Eşlenik

$$\sqrt{x} - > \sqrt{x}$$

$$\sqrt{x + y} = \sqrt{x + y}\sqrt{x} + \sqrt{y} = \sqrt{x} - \sqrt{y}$$

Problem 1.2 – $\sqrt{10 - \sqrt{31 + \sqrt{21} + \sqrt{19} - \sqrt{9}}} = ?$

Sol.

$$\sqrt{10 - 6} = \sqrt{4} = 2$$

□

Problem 1.3 – $\sqrt{18} + \sqrt{50} + \sqrt{72} + \sqrt{8} = ?$

Sol.

$$3\sqrt{2} + 5\sqrt{2} + 6\sqrt{2} - 2\sqrt{2} = 12\sqrt{2} = \sqrt{288}$$

□

Problem 1.4 – $\sqrt{2 + \sqrt{a}} = 3, \sqrt{30 + \sqrt{27 + \sqrt{b}}} = 6$ ise $b - a = ?$

Sol.

$$\begin{aligned} 2 + \sqrt{a} &= 9 & 30 + \sqrt{27 + \sqrt{b}} &= 36 \\ \sqrt{a}^2 &= 7^2 & (27 + \sqrt{b})^2 &= 6^2 \\ a &= 49 & \sqrt{b}^2 &= 9^2 \\ & & b &= 81 \\ b - a &= 81 - 49 = 32 \end{aligned}$$

□

Problem 1.5 – $\frac{\sqrt{0,49} - \sqrt{64} + \sqrt{1,69}}{\sqrt{5} - \sqrt{2}} = ?$

Sol.

$$\frac{0,7 - 8 + 13}{\sqrt{5} - \sqrt{2}} = \frac{-6}{(\sqrt{5} - \sqrt{2})} \quad \frac{-6\sqrt{5} - 6\sqrt{2}}{3} = -2\sqrt{5} - 2\sqrt{3}$$

□

Problem 1.6 – $\frac{3+\sqrt{3}}{3-\sqrt{3}} + \frac{3-\sqrt{3}}{3+\sqrt{3}} = ?$

Sol.

$$\frac{9 + 3\sqrt{3} + 3\sqrt{3} + 9 - 3\sqrt{3} - 3\sqrt{3} + 3}{6} = \frac{24}{6} = 4$$

□

Problem 1.7 – $\sqrt{42 + \sqrt{42 + \sqrt{42} + \dots}} = ?$ (x) ($x = \sqrt{42 + \sqrt{42 + \dots}}$)

Sol.

$$\begin{aligned} \sqrt{42 + x^2} &= x^2 & 42 &= x(x - 1) \\ 42 + x &= x^2 & 7 & 6 \\ 42 &= x^2 - x & & \\ x &= 7 & & \end{aligned}$$

□

$$\textbf{Problem 1.8} - \frac{3}{\sqrt{3}-1} - \frac{1}{\sqrt{4+2\sqrt{3}}} = ?$$

Sol.

$$\frac{3}{\sqrt{3}-1} - \frac{1}{\sqrt{3}+1} = \frac{2\sqrt{3}+4}{2} = \sqrt{3}+2$$

$$\frac{3\sqrt{3}+3-\sqrt{3}+1}{2}$$

□

$$\textbf{Problem 1.9} - x, y \in R+, \frac{x\sqrt{y}+y\sqrt{x}}{\sqrt{x}+\sqrt{y}} = 5; x \times y = ?$$

Sol.

$$\frac{x\sqrt{x \times y} + y \times x - x \times y - y\sqrt{x \times y}}{x - y} = 5$$

$$5(x - y) = \sqrt{x \times y(x - y)}$$

$$5x - 5y = x\sqrt{x \times y - y\sqrt{x \times y}}$$

$$\sqrt{x \times y}^2 = 5^2$$

$$x \times y = 25$$

□

$$\textbf{Problem 1.10} - x, y \in R, y = \sqrt{x-5} + \sqrt{2-x} \text{ ise } x \text{ hangi aralıkta deęer alır?}$$

Sol.

$$x - 5 \geq 0, \quad x \geq 5$$

$$2 - x \geq 0, \quad x \leq 2$$

$$(-\infty, 2] \cup [5, \infty)$$

$$\sqrt{5}, \sqrt{7}, \sqrt{3}, \sqrt{3} < \sqrt{5} < \sqrt{7}$$

$$a = \sqrt[3]{2} = \sqrt[12]{16}$$

$$b = \sqrt[4]{3} = \sqrt[12]{27}$$

$$c = \sqrt[6]{7} = \sqrt[12]{49}$$

$$c > b > a$$

□