

Absolute Value

Halil Yiğit KOÇHAN

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$$|x| = \begin{cases} x < 0, & -x \\ x > 0, & x \end{cases}$$

$$\left| \frac{-1}{2} \right| = \frac{1}{2}$$

$$|-x| = |x|$$

$$|x - y| = |y - x|$$

$$|x \times y| = |x| \times |y|$$

$$\left| \frac{x}{y} \right| = \frac{|x|}{|y|}$$

$$a > 0, |f(x)| = a \text{ ise } f(x) = a, f(x) = -a$$

$$a > 0, |f(x)| \leq a \text{ ise } -a \leq f(x) \leq a$$

$$a > 0, |f(x)| \geq 0 \text{ ise } f(x) \geq a, f(x) \leq -a$$

$$|x + y| \leq |x| + |y|$$

Problem 0.1 $-x > 0$ ve $y < 0$ ise $|x - y| + |x| + |y| = ?$

Sol.

$$x - y + x - y = 2x - 2y = 2(x - y)$$

□

Problem 0.2 $-x < 0 < y$ ise $\frac{x^2 + 2|xy| + y^2}{|y - x|} = ?$

Sol.

$$\frac{x^2 + y^2 - 2xy}{y - x} = \frac{(x - y)^2}{y - x} = y - x$$

□

Problem 0.3 $-a, b \in R, a - b = 2, a - |b - a| = 4$ ise $a + b = ?$

Sol.

$$\begin{aligned} |a - b| &= 2 & a - 2 &= 4 \\ |b - a| &= |a - b| = 2 & a &= 6 \\ & & 6 + 4 &= 10 \end{aligned}$$

□

Problem 0.4 $-x < 0 < y$ olmak üzere $\sqrt{9x^2} - \sqrt[3]{y^3} + \sqrt{(x-y)^2}$

Sol.

$$|3x| - y + |x - y| = -3x - y - x + y = -4x$$

□

Problem 0.5 $|x - 3| = 1905!$ ise x değerleri toplamı?

Sol.

$$\begin{aligned} x - 3 &= 1905!, & x &= 1905! + 3 & x - 3 &= -1905!, & x &= 1905 + 3 \\ & & x &= 1905 + 3 - 1905! + 3 & & & &= 6 \end{aligned}$$

□

Problem 0.6 $|x^2 - 16| = |x - 4|$ denklemini sağlayan x değerleri toplamı?

Sol.

$$\begin{aligned} |x - 4| \times |x + 4| &= |x - 4| \\ |x + 4| &= 1 & |x + 4| &= -1 \\ x + 4 &= 1 & x + 4 &= -1 \\ x &= -3 & x &= -5 \end{aligned}$$

□

Problem 0.7 $|x - 1| \leq 2, x + y - 3 = 0, y$ 'nin en büyük değeri?

Sol.

$$\begin{aligned} -2 &\leq x - 1 \leq 2 & x + y &= 3 \\ -1 &\leq x \leq 3 & x &= -1 \\ & & y &= 4 \end{aligned}$$

□

Problem 0.8 – $|\frac{2}{x-3}| \leq \frac{1}{2}$ eşitliğine sağlayan x değerleri toplamı?

Sol.

$$|x - \frac{3}{2}| \leq 2$$

$$\begin{array}{ll} -2 \leq x - \frac{3}{2} \leq 2 & -1 \leq x \leq 7 \\ -4 \leq x - 3 \leq 4 & [-1, 7] - \{3\} = 24 \end{array}$$

□

Problem 0.9 – $\frac{|x+1|-4}{|x|+2} \leq 0$ eşitliğini sağlayan x tamsayı değerlerinin toplamı?

Sol.

$$\begin{array}{ll} |x+1| - 4 \leq 0 & -4 \leq x+1 \leq 4 \\ |x+1| \leq 4 & -5 \leq x \leq 3 \end{array}$$

$$\sum x = 9$$

□

Problem 0.10 – $\sqrt{x^2 + 6x + 9} > 2$ eşitliğini x tamsayı değerleri toplamı? (sağlamayan $= \leq 2$)

Sol.

$$\begin{array}{ll} \sqrt{(x-3)^2} & -2 \leq x-3 \leq 2 \\ |x-3| \leq 2 & 1 \leq x \leq 5 \end{array}$$

$$\sum x = 15$$

□

Problem 0.11 – $|x+4| < |x+6|$, $\text{Ç.K} = ?$

Sol.

$$\begin{array}{ll} (x+4)^2 < (x+6)^2 & -4x < 20 \\ x^2 + 8x + 16 < x^2 + 12x + 36 & x > -5 \\ \text{Ç.K} = (-5, \infty) \end{array}$$

□

Problem 0.12 $-7 \leq |2x - 1| \leq 13$ eşitsizliğini sağlayan kaç x değeri vardır?

Sol.

$$7 \leq 2x - 1 \leq 13 \qquad 7 \leq -2 + 1 \leq 13$$

$$8 \leq 2x \leq 14 \qquad 6 \leq -2 \leq 12$$

$$4 \leq x \leq 7 \qquad -12 \leq 2x \leq -6$$

$$-6 \leq x \leq -3$$

$$\sum x = 8$$

□