1 Von Mises distribution

Intuitively, the Von Mises distribution [**?**] is a simple approximation for the normal distribution on a circle (known as the *wrapped normal distribution*). The Von Mises probability density function is defined by the following equation:

$$P(x) = \frac{e^{b\cos(x-a)}}{2\pi I_0(b)}$$

where $I_0(x)$ is the modified Bessel function of the first kind; the Von Mises cumulative density function has no closed form.

The mean $\mu = a$ (intuitively, the angle that the distribution clustered around), and the circular variance $\sigma^2 = 1 - \frac{I_1(b)}{I_0(b)}$ (intuitively, b is the "concentration" parameter). Therefore, as $b \to 0$, the distribution becomes uniform; as $b \to \infty$, the distribution becomes normal with $\sigma^2 = 1/b$.

2 Von Mises-Fisher distribution

The Von-Mises Fisher distribution is the generalization of the Von Mises distribution to n-dimensional hyperspheres. It reduces to the Von-Mises distribution with $n = 2$. The probability density function is defined by the following equation:

$$f_p(\boldsymbol{x}; \boldsymbol{\mu}, \kappa) = C_p(\kappa)\exp(\kappa\boldsymbol{\mu}^T\boldsymbol{x})$$

where

$$C_p(\kappa) = \frac{\kappa^{p/2-1}}{(2\pi)^p I_{p/2-1}(\kappa)}$$

and intuitively is an approximation for the normal distribution on the hypersphere.

Example code for constructing random (Von Mises-Fisher distributed) points on a sphere can be found in the Appendix, at section **??**.