

# K-means clustering

$k$ -means clustering is a simple and popular algorithm for the *clustering problem*, the task of grouping a set of observations so that a group is “similar” within itself and “dissimilar” to other groups.  $k$ -means partitions  $n$  observations into  $k$  clusters, with each observation belonging to the nearest mean of the cluster. This problem is NP-hard in general, but there are heuristics which guarantee convergence to a local optimum.

The standard heuristic (known as *Lloyd’s algorithm*) is the following:

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**Algorithm 1** Lloyd’s algorithm for  $k$ -means clustering

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1: generate an initial set of  $k$  means
2: while not converged do
3:   assign all data points to nearest Euclidean-distance mean
4:   calculate new means to as the centroids of the observations in the cluster
5: end while
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There is a choice of initialization method. The *Forgy method* randomly picks  $k$  observations as initial means, while the *Random Partition method* randomly picks a cluster for each observation.

The Lloyd’s algorithm is a heuristic, so it does not guarantee a global optimum. Furthermore, there exists sets of points in which it converges in exponential time. However, it has been shown to have a smoothed polynomial running time, and in practice converges quickly.

## References