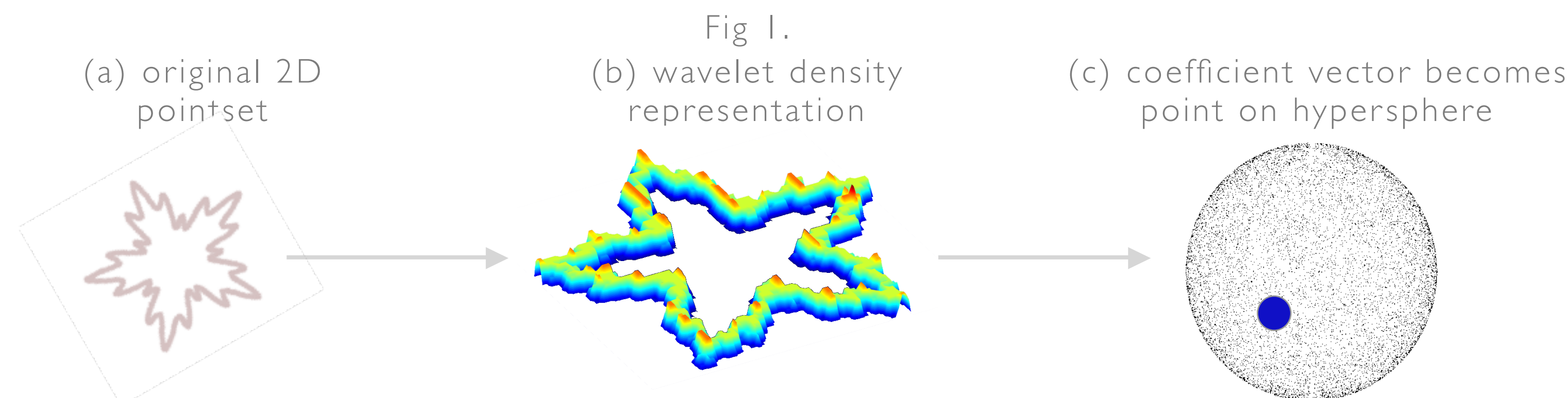# SHAPE RETRIEVAL ON THE
WAVELET DENSITY HYPERSHPERE

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PROBLEM
ABSTRACT

(a) original 2D pointset

Fig 1.
(b) wavelet density representation

(c) coefficient vector becomes point on hypersphere

With the advancement in 3D modeling technology, understanding shape is more important than ever. Shape retrieval is the problem of searching for similar shapes in a database given a query shape, similar to search engines for text.

In our novel approach, we represent shapes as probability densities and use the intrinsic geometry of this space to match similar shapes. Specifically, we expand the square-root of the density in a multiresolution wavelet basis. Under this model, each density (of a corresponding shape) is mapped to a point on unit hypersphere, where the angle between a pair of points can be used as a measure of similarity.

OUR CONTRIBUTIONS

- Optimized performance of 2D multiresolution wavelet density estimator

- Improved shape similarity metric using linear assignment and multiresolution wavelets

- Implemented hierarchical clustering algorithm on high-dimensional unit hypersphere and analyzed algorithmic complexity

- Time complexity proof of hierarchical retrieval scheme

- Experimental validations across multiple datasets, with empirical results competitive with state of the art

APPROACH
WAVELET DENSITY ESTIMATION

Wavelets are crucial mathematical functions that form an orthonormal basis for probability density functions. Given a point-set representation of a shape, we use a constrained maximum likelihood approach to estimate the coefficients of the wavelet density basis expansion in eq. (1).

$$\sqrt{p(\mathbf{x})} = \sum_{j_0,\mathbf{k}} \alpha_{j_0,\mathbf{k}} \phi_{j_0,\mathbf{k}}(\mathbf{x}) + \sum_{j \geq j_0,\mathbf{k}}^{j_1} \sum_{w=1}^{3} \beta_{j,\mathbf{k}}^{w} \psi_{j,\mathbf{k}}^{w}(\mathbf{x}) \quad (1)$$

APPROACH & RESULTS
2D WDE OPTIMIZATION

Wavelet density estimation (WDE) is a computationally expensive task, but we improved the speed by orders of magnitude through parallelization. We had significant improvements in both initializing the wavelet coefficients (alpha and beta in equation 1) and in our negative log likelihood based optimization process.

100× FASTER

Fig 2. SAMPLE SIZE v. RUNTIME for WDE SYM4 LEVELS = 2-4, DOMAIN = [-1 1]

TIME (hours)

	before	after
SHREC11 (600)	7.2	0.13
MPEG7 (1400)	16.8	0.33
BROWN (99)	1.2	0.02

DATABASE (# of shapes)

APPROACH & RESULTS
HIERARCHICAL SHAPE RETRIEVAL

Hierarchical clustering uses different levels of abstraction to group similar shapes together. Because of the hierarchical tree structure, retrieval speed and accuracy are improved.

We used spherical form of k-means clustering on the hypersphere for each level of clustering, and create a recursive tree structure where the means of one level form the children of the higher level.
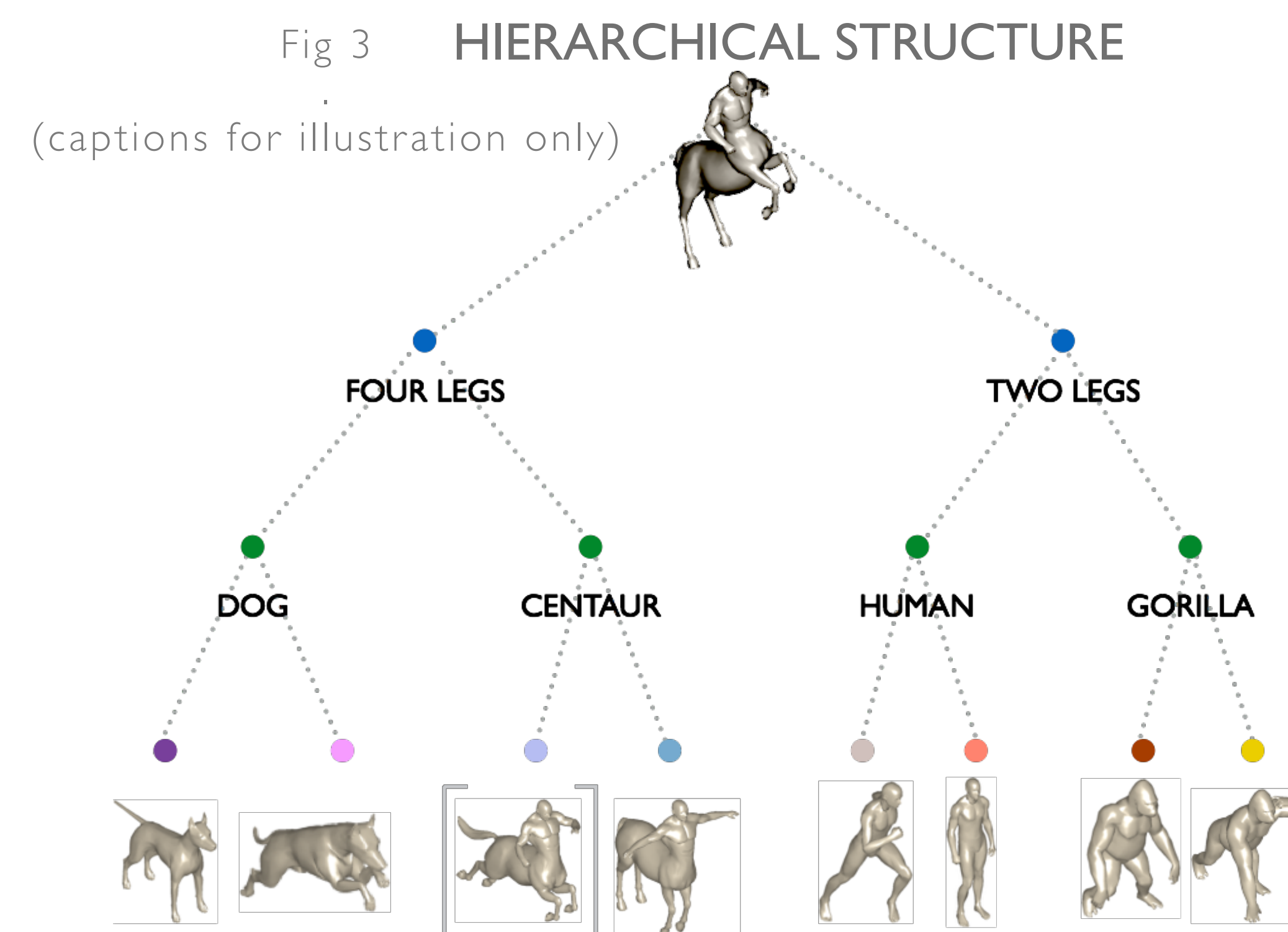
Fig 3. **HIERARCHICAL STRUCTURE**
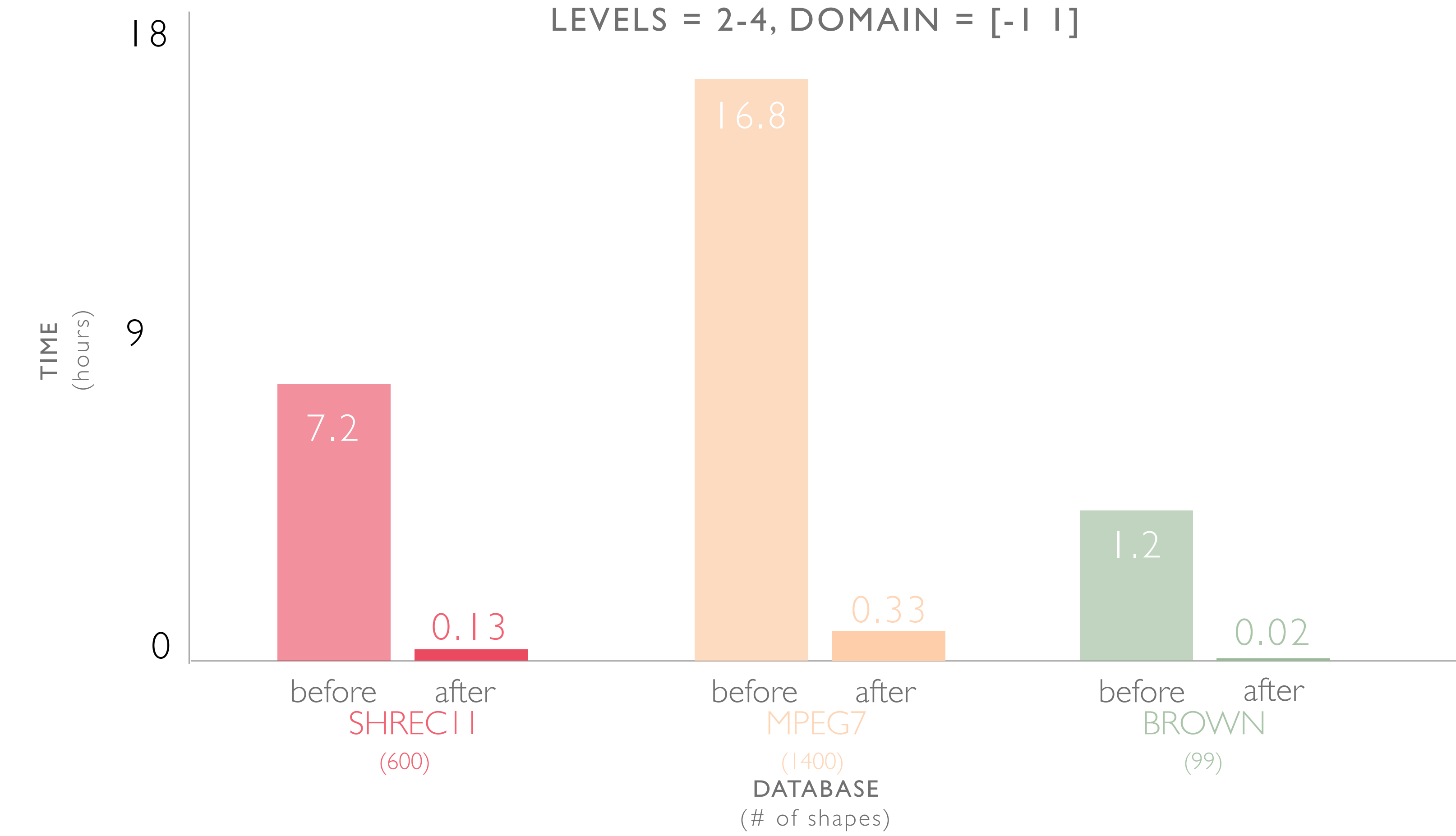
(captions for illustration only)

FOUR LEGS   TWO LEGS

DOG   CENTAUR   HUMAN   GORILLA

Table 1. **ACCURACY**

	SHREC11	MPEG7	BROWN
w/o HIERARCHY	20.7%	75.3%	51.8%
w/ HIERARCHY.	99.7%	99.3%	83.0%

REFERENCES

- Adrian M Peter and Anand Rangarajan. Maximum likelihood wavelet density estimation with applications to image and shape matching. IEEE Transactions on Image Processing, 17(4):458–468, 2008.

- Adrian M. Peter, Anand Rangarajan, and Jeffrey Ho. Shape lane rouge: Sliding wavelets for indexing and retrieval. In Computer Vision and Pattern Recognition, 2008. CVPR 2008. IEEE Conference on, pages 1–8. IEEE, 2008.

APPROACH & RESULTS
LINEAR ASSIGNMENT

Linear assignment warps two shapes closer together to reduce non-rigid differences between them. Because shapes easily warp when they're close together, we reduce the distance between similar shapes while keeping dissimilar shapes far apart. This makes our dissimilarity metric more robust to small deformations, which improve the accuracy of shape retrieval.

We have an objective function that balances warping the shapes together with regularizing the amount of warping allowed. We decrease the amount of warping by increasing the parameter λ.

Fig 4. **WARPING FROM SOURCE TO TARGET**
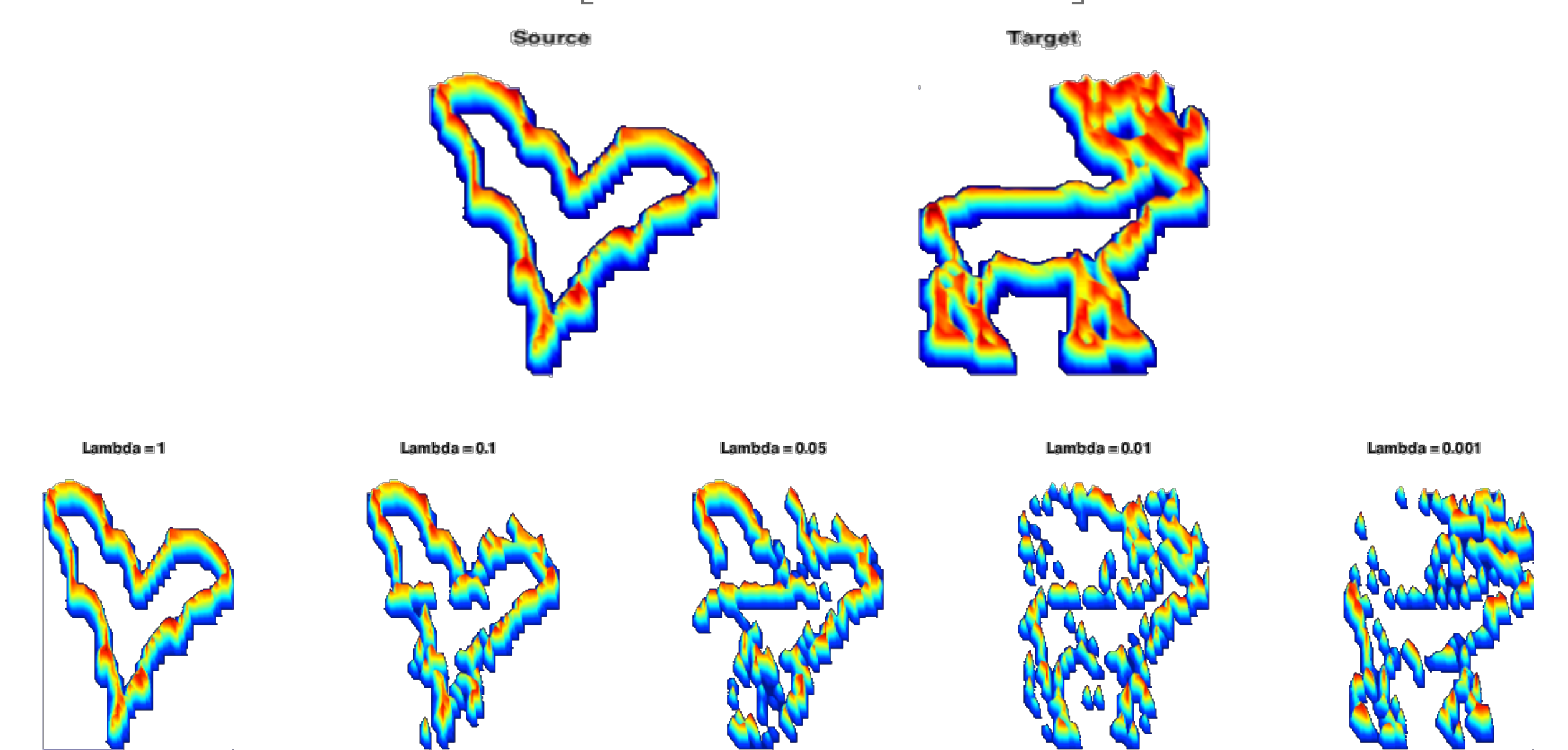$\lambda = [\ 1, 0.1, 0.05, 0.01, 0.001]$

Source   Target

Lambda =1   Lambda =0.1   Lambda =0.05   Lambda =0.01   Lambda =0.001

Table 2. **ACCURACY**

	SHREC11	MPEG7	BROWN
w/o LINEAR ASSGN.	xx.x%	xx.x%	xx.x%
w/ LINEAR ASSGN.	xx.x%	xx.x%	xx.x%

- Vinh Nguyen. Gene clustering on the unit hypersphere with the spherical k-means algorithm: coping with extremely large number of local optima. In World Congress in Computer Science, Computer Engineering, and Applied Computing (Hamid R. Arabnia and Youngsong Mun 14 July 2008 to 17 July 2008), pages 226–233. CSREA Press, 2008.