Math 690 F2017: Topics in Data Analysis and Computation
Homework 5

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1. (Non-local means) Apply non-local means to the problem of (patch based) image denoising, and consider the following modifications

   (i) Self-tuning of the σ in heat kernel: let

   $$W_{ij} = \exp\{-\frac{\|x_i - x_j\|^2}{2\sigma_i\sigma_j}\},$$

   where σ_i equals the distance of the k-the nearest neighbor of data point x_i. The parameter k is set to be a constant, e.g. $k = 10$.

   (ii) The combination of both local and non-local means: let

   $$W_{ij} = \exp\{-\frac{\|x_i - x_j\|^2}{2\sigma_i\sigma_j}\} \cdot \exp\{-\frac{\|u_i - u_j\|^2}{2\sigma^2}\}$$

   where u_i stands for the position of the center pixel of the patch on the 2D grid.

   What are the effects of these modifications? Compare the performance with e.g. PCA.

2. ($\mathbb{Z}_2$ synchronization) The problem is to recover some arbitrary signs $\{z_i\}_{i=1}^n$ on n nodes, $z_i = \pm 1$, from $\frac{n(n-1)}{2}$ noise-corrupted observations: when $i < j$,

   $$G_{ij} = \begin{cases} z_i z_j, & \text{with prob. } p, \\ W_{ij}, & \text{with prob. } 1-p, \end{cases}$$

   and $G_{ij} = G_{ji}$, $G_{ii} = 1$, and W is a symmetric matrix consisting of i.i.d. random signs, i.e. $W_{ij} = \pm 1$ with prob. $\frac{1}{2}$. Apply spectral methods, i.e. using the eigenvector associated with the largest eigenvalue, to the problem, and study how the successful sign recovery depends on the correct-observation rate p and the number of nodes n.