# Math 690 F2017: Topics in Data Analysis and Computation
# Homework 6

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1. Using concentration argument, finish the proof of the upper bound of the Johnson-Lindenstrauss lemma. The proof is given in [DG03].

2. Study the concentration of $\lambda_2$, the second smallest eigenvalue, of the normalized graph laplacian of an Erdos-Renyi random graph $G(n,p)$, i.e. the graph has $n$ nodes and the probability of $A_{ij} = 1$ equals $p \in (0,1)$. We know that when the graph is connected (which happens almost surely if $p > (1+\varepsilon)\frac{\log n}{n}$, as proved in the classical work of Erdos and Renyi in 1960), $0 < \lambda_2 < 2$.

   (1) Let $p$ be fixed constant, and $n$ increases. What is the limiting statistics of $\lambda_2$ like? (Hint: $\lambda_2 \to a$ for some constant $a$, and after properly centering and rescaling $(\lambda_2 - a)$ converges to a limiting distribution.)

   (2) What happens if $p$ decreases with $n$, e.g. $p = \alpha \frac{\log n}{n}$ for $\alpha > 1$? Numerically observe the distribution of $\lambda_2$ in this case.