ENVIRONMENTAL NOISE CONTROL
The 2005 Congress and Exposition on Noise Control Engineering
07-10 August 2005 - Rio de Janeiro - Brazil

Estimation of Partial Area Sound Power Data with Beamforming

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Abstract Nearfield Acoustical Holography (NAH) provides accurate calibrated maps of any acoustical parameter near the source, in particular of sound intensity, from which partial area sound power data can be obtained. Beamforming, on the other hand, is basically a far-field measurement technique that estimates, how much of the pressure at the array position is incident from different directions. No calibrated source data are obtained. Beamforming is, however, being used more and more for general noise source identification, because measurements are fast and provide good resolution at the high frequencies. In such applications, calibrated source maps that can be compared with the output from intensity measurements or NAH are important. Beamformer arrays with uniform element spacing can be used at measurement distances down to around 0.6 times the array diameter, which means that the array will pick up the major part of the radiation from the source area under the array. The paper derives a simple mathematical expression for a factor to scale the beamformed maps as sound intensity in such a way that area integration provides good estimates of partial area sound power data. The accuracy of the method is analyzed through numerical simulations and practical measurements.

1. INTRODUCTION TO DELAY-AND-SUM BEAMFORMING WITH INFINITE FOCUS DISTANCE

This section gives a short introduction to the theory of Beamforming, which is needed for the subsequent derivation of the scaling.

As illustrated in Figure 1, we consider a planar array of M microphones at locations $\mathbf{r}_m$ $(m = 1,2,...,M)$ in the xy-plane of our coordinate system. When such an array is applied for Delay-And-Sum Beamforming, the measured pressure signals p_m are individually weighted and delayed, and then all signals are summed, [1]:

$$b(\boldsymbol{\kappa},t) = \frac{1}{M}\sum_{m=1}^{M} w_m\, p_m(t-\Delta_m(\boldsymbol{\kappa})) \tag{1}$$

The individual time delays Δ_m are chosen with the aim of achieving selective directional sensitivity in a specific direction, characterized here by a unit vector $\boldsymbol{\kappa}$. This objective is met by adjusting the time delays in such a way that signals associated with a plane wave, incident from the direction $\boldsymbol{\kappa}$, will be aligned in time before they are summed. Geometrical considerations (see Figure 1) show that this can be obtained by choosing:

$$\Delta_m = \frac{\boldsymbol{\kappa} \cdot \mathbf{r}_m}{c} \tag{2}$$

where c is the propagation speed of sound. Signals arriving from other far-field directions will not be aligned before the summation, and therefore they will not coherently add up.

The frequency domain version of formula (1) for the Delay-And-Sum Beamformer output is:

$$B(\boldsymbol{\kappa},\omega) = \frac{1}{M}\sum_{m=1}^{M} w_m\, P_m(\omega)\, e^{-j\omega\Delta_m(\boldsymbol{\kappa})} = \frac{1}{M}\sum_{m=1}^{M} w_m\, P_m(\omega)\, e^{j\mathbf{k}\cdot\mathbf{r}_m} \tag{3}$$

Here, ω is the temporal angular frequency, $\mathbf{k} \equiv -k\boldsymbol{\kappa}$ is the wave number vector of a plane wave incident from the direction $\boldsymbol{\kappa}$ in which the array is focused – see Figure 1 – and $k = \omega/c$ is the wave number. In equation (3) an implicit time factor equal to $e^{j\omega t}$ is assumed.

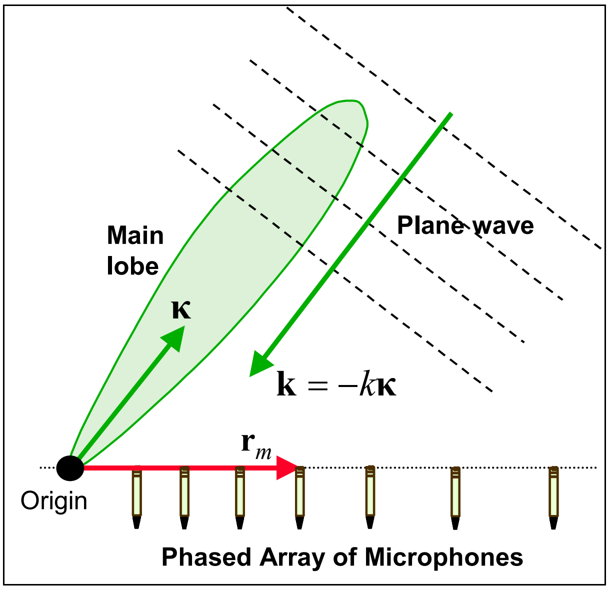

Figure 1: Illustration of a phased microphone array, a directional sensitivity represented by a mainlobe, and a Plane wave incident from the direction of the mainlobe.

Through our choice of time delays $\Delta_m(\boldsymbol{\kappa})$, or equivalently of the "preferred" wave number vector $\mathbf{k} \equiv -k\boldsymbol{\kappa}$, we have "tuned" the beamformer on the far-field direction $\boldsymbol{\kappa}$. Ideally we would like to measure only signals arriving from that direction, in order to get a perfect localization of the sound source. To investigate, how much "leakage" we will get from plane waves incident from other directions, we assume now a plane wave incident with a wave number vector $\mathbf{k}_0$ different from the preferred $\mathbf{k} \equiv -k\boldsymbol{\kappa}$. The pressure measured by the microphones will then ideally be:

$$P_m(\omega) = P_0\, e^{-j\mathbf{k}_0\cdot\mathbf{r}_m} \tag{4}$$

which according to equation (3) will give the following output from the beamformer:

$$B(\boldsymbol{\kappa},\omega)=\frac{P_0}{M}\sum_{m=1}^{M} w_m\, e^{j(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}_m} \equiv P_0\, W(\mathbf{k}-\mathbf{k}_0) \tag{5}$$

Here, the function W

$$W(\mathbf{K}) \equiv \frac{1}{M}\sum_{m=1}^{M} w_m\, e^{j\mathbf{K}\cdot\mathbf{r}_m} \tag{6}$$

is the so called Array Pattern. It has the form of a 2D spatial Fourier transform of a weighting function w, which consists of delta functions of strength w_m at the microphone positions. In the following we will assume all weights w_m to equal one. Because the microphone positions $\mathbf{r}_m$ have z-coordinate equal to zero, the Array Pattern is independent of K_z. We therefore consider the Array Pattern W only in the (K_x, K_y) plane, and when W is used, as in equation (5), the 3D wavenumber vector is projected onto the (K_x, K_y) plane. In that plane, W has an area with high values around the origin with a peak value equal to 1.0 at $(K_x, K_y) = (0,0)$. According to equation (5), this peak represents the high sensitivity to plane waves coming from the direction $\boldsymbol{\kappa}$, in which the array is focused. Figure 1 contains an illustration of that peak, which is called the *mainlobe*. Other directional peaks, which are called *sidelobes*, will cause waves from such directions to leak into the measurement of the mainlobe direction $\boldsymbol{\kappa}$, creating so-called Ghost sources or *Ghost images*. From equation (5) it is clear that at a given frequency only the part of the Array Pattern $W(\mathbf{K})$ with $\left[\mathbf{K}\right] \leq 2k$ is "visible". The *Maximum Sidelobe Level* (MSL) at a given frequency is defined as the ratio between the highest sidelobe and the mainlobe.

In the expression (5) for the response to a plane wave, notice that the output is exactly equal to the amplitude P_0 of the plane wave, when the array is focused towards to direction of incidence of the plane wave, i.e. when $\mathbf{k} = \mathbf{k}_0$. So the beamformer is estimating, how much of the sound pressure at the array location is incident from a given direction.

For stationary sound fields it is natural to operate with the matrix of cross spectra between the microphones, which provides a better average representation of the stationary phenomena. Exclusion of the auto-spectra offers the possibility of reducing the influence of noise in the individual measurement channels, and it turns out that it also often reduces the sidelobe level, [2]. For the derivation of the sound intensity scaling we will, however, not use the Cross-spectral formulation. But the scaling holds for the Cross-spectral formulation as well, as long as it is scaled in such a way that the response to an incident plane wave is equal to the squared amplitude of the wave. The formulation in reference [2] is scaled that way, and it uses focusing at a finite distance. The validity of the intensity scaling in combination with the Cross-spectral Beamformer with finite focus distance is investigated both through simulations and practical measurements in subsequent sections.

From the literature it is known that the size and shape of the mainlobe of the array pattern is determined almost entirely by the size and overall shape of the array, [1], [2], while the sidelobes are highly affected by the actual positions of the microphones. The shape of the mainlobe is usually close to the mainlobe from a "continuous aperture" of the same shape as the array or, equivalently, a very densely populated array covering the same area. For circular array geometry, the equivalent continuous aperture has the following array pattern:

$$\bar{W}(K) = 2\frac{J_1(\frac{1}{2}KD)}{\frac{1}{2}KD}, \qquad K \equiv |\tilde{\mathbf{K}}| \tag{7}$$

where D is the diameter of the aperture (or equivalently of the array), J_1 is the Bessel function of order 1, and $\tilde{\mathbf{K}}$ is the projection of $\mathbf{K}$ onto the (K_x, K_y) plane. What we have achieved is a general approximation for the shape of the mainlobe, which is independent of the specific positioning of the microphones,

$$W(\mathbf{K}) \approx \bar{W}(|\tilde{\mathbf{K}}|) \quad \text{for} \quad |\tilde{\mathbf{K}}| \le K_1 \tag{8}$$

Here, K_1 is the first null of the aperture array pattern, $\bar{W}(K_1) = 0$, given by

$$\tfrac{1}{2}K_1 D = \xi_1 \approx 3.83 \tag{9}$$

ξ_1 being the first null of the Bessel function of the first order.

2. DERIVATION OF THE SCALING FACTOR

For the derivation we now assume a single monopole point source on the array axis at so large distance L that the amplitude and phase of the pressure is practically constant across the array area. Thus, for the array the sound field is a plane wave with amplitude P_0 incident with wave number vector $\mathbf{k}_0 = -k\hat{z}$, where $\hat{z}$ is the unit vector in the z-direction. The sound power P_a radiated by the monopole is then

$$P_a = 4\pi L^2 \cdot I = 4\pi L^2 \cdot \frac{|P_0|^2}{2\rho c} = 2\pi L^2 \cdot \frac{|P_0|^2}{\rho c} \tag{10}$$

where I is the sound intensity at the position of the array and ρ is the density of the medium.

From equation (5) we get for the output from the Delay-And-Sum beamformer

$$B(\boldsymbol{\kappa}) = P_0 W(\mathbf{k} - \mathbf{k}_0) = P_0 W(-k\boldsymbol{\kappa} + k\hat{z}) \tag{11}$$

where the known values of the two wave number vectors have been inserted. In order to use the approximation (8) for the mainlobe of the array pattern, we need to project the wave number vectors onto the xy-plane, which leads to:

$$B(\boldsymbol{\kappa}) \approx P_0 \bar{W}\left(k\sin(\theta)\right) \quad \text{for} \quad |k\sin(\theta)| \le K_1 \tag{12}$$

θ being the angle from the array axis (the z–axis) to the focus direction $\boldsymbol{\kappa}$.

The Beamformer is now used to create a source map in the plane $z = L$. Each position on this source plane is described by its distance R to the z-axis and its azimuth angle ϕ. Assuming relatively small angles from the z-axis we can use the approximation:

$$R = L\tan(\theta) \approx L\sin(\theta) \tag{13}$$

Use of equation (13) in (12) leads to the following approximate expression for the "mainlobe" of the beamformed map on the source plane:

$$B(R,\phi) \approx P_0 \bar{W}\left(kR/L\right) \quad \text{for} \quad R \le K_1 L/k \equiv R_1 \tag{14}$$

By the use of equation (9), we get for the radius R_1 of the mainlobe on the source plane

$$R_1 \equiv \frac{K_1 L}{k} = \frac{2L}{kD}\xi_1 \approx 1.22\frac{L\lambda}{D} \quad (15)$$

The scaling factor α needed to obtain the intensity scaled beamformer output B_I,

$$B_I(R,\phi) = \alpha \cdot \left|B(R,\phi)\right|^2 \quad (16)$$

is now defined in such a way that the integral of $B_I(R,\phi)$ over the mainlobe equals half of the radiated sound power P_a, i.e. the power radiated into the hemisphere containing the array:

$$\tfrac{1}{2}P_a = \int_0^{R_1}\int_0^{2\pi} \alpha \left|B(R,\phi)\right|^2 R\,dRd\phi = 2\pi\,\alpha\left|P_0\right|^2 \int_0^{R_1} \bar{W}^2(kR/L)\,R\,dR \quad (17)$$

Use of equation (7), substitution with the variable

$$u \equiv \frac{kR}{L}\frac{D}{2} = \frac{kD}{2L}R \quad (18)$$

for R in equation (17) and application of the relation (15) leads to

$$\tfrac{1}{2}P_a = 2\pi\alpha\left|P_0\right|^2 \int_0^{\xi_1}\left[2\frac{J_1(u)}{u}\right]^2\left(\frac{2L}{kD}\right)^2 u\,du = \alpha\cdot 32\pi\left(\frac{\left|P_0\right|L}{kD}\right)^2\cdot\Gamma \quad (19)$$

with

$$\Gamma \equiv \int_0^{\xi_1}\left[\frac{J_1(u)}{u}\right]^2 u\,du \approx 0.419 \quad (20)$$

The scaling factor can finally be obtained through use of the expression (10) for the sound power in equation (19):

$$\alpha = \frac{1}{32\Gamma}\frac{(kD)^2}{\rho c} \approx \frac{2.94}{\rho c}\left(\frac{D}{\lambda}\right)^2 \quad (21)$$

Clearly, the scaling factor is proportional to the square of the array diameter measured in wavelengths. This is natural, because the un-scaled beamformer output $B(\hat{z})$ with the array focused towards the point source is basically independent of array geometry, but the width of the mainlobe is inverse proportional to the array diameter measured in wavelengths, ref. equation (15). To maintain the area-integrated power with increasing array diameter, the scaling factor must have the mentioned proportionality.

3. EVALUATION OF ERROR

The major principle of the scaling is that area integration of the scaled output must provide a good estimate of the sub-area sound power. For that reason it is natural to use the term "Sound Intensity Scaling" about the method. The scaling is defined for a single omni-directional point source in such a way that area integration of the peak created by the mainlobe equals the known radiated power from the point source. So by this definition the total power will be within the mainlobe radius from the source position, and integration over a larger area will cause an overestimation of the sound power. One reason for choosing this definition is that only the mainlobe has a form that depends only on the array diameter and not on all microphone positions. Other choices would be somewhat arbitrary, require integration over a larger area to get the total power and would need the scaling factor to depend on the particular set of microphone positions. The influence of the sidelobes on the power integration is a weakness, which cannot be avoided: If the mainlobe is rather narrow and sound power integration is performed over an area much larger than the size of the mainlobe on the source plane, then the level of sidelobes often present in beamforming can contribute significantly to the power integration and cause a significant over-estimation of the sound power. The solution adopted to avoid this significant over-estimation is to use only a finite dynamic range of sound intensity in the area integration, typically around 10 dB. The applied dynamic range, however, should depend on the MSL of the array.

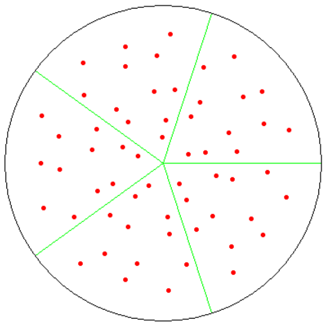

Figure 2: Geometry of 60-channel Sector Wheel Array used for simulations and measurements. The diameter of the enclosing circle is 120cm, so the array itself has a diameter around 1 meter. The array geometry is optimized for both Beamforming and for SONAH acoustical holography, ref. [3] for details.

The scaling was derived for a single omni-directional point source on the array axis. Beyond that we have assumed the monopole to be so far away from the array that its sound field has the form of a plane wave across the array. Thus, we have assumed the source to be in the far-field region relative to the array. The second important assumption introduced in equations (13-14) above is that the mainlobe covers a relatively small solid angle. To investigate the effect of the last two assumptions, a series of simulations have been performed with the 60-channel Sector Wheel Array of Figure 2 and with a single monopole point source at different distances on the array axis, operating at different frequencies. The beamforming calculation has been performed with two different beamformers:

1. A Delay-And-Sum beamformer focused at the finite source distance, but without any amplitude compensation, [2].

2. The Cross-spectral beamformer with exclusion of Auto-spectra described in reference [2]. This method compensates for the amplitude variation across the array of the sound pressure from a monopole on the source plane.

The output has then been scaled as sound intensity through multiplication with the scaling factor α of equation (21), and finally the sound power has been estimated by integration over a circular area with radius equal to R_1 (ref. equation (15)) around the array axis.

Figure 3 shows the ratio between the estimated and the exact sound power in Decibel for the case of the Delay-And-Sum beamformer. At 1000 Hz the mainlobe (and therefore the hot spot generated around the source position on the array axis) covers an angle of approximately 24º from the array axis. This will introduce a significant error in equations (13-14) and therefore an error in the estimated sound power, even when the source distance is relatively large. But since the resolution is anyway very poor below 1000 Hz, the introduction of the scaling error is less significant. The error increases quickly for distances smaller than approximately 1 meter, which is the approximate diameter of the array. Here, the assumption of the source being in the far-field region relative to the array certainly does not hold. But fortunately the error does not get worse than approximately 0.6 dB for distances down to 0.6 times the array diameter. To achieve the best possible resolution it is desirable to use the array at the shortest possible measurement distance.

The chosen array, shown in Figure 2, applies for SONAH holography below approximately 1300 Hz, [3], and in that frequency range SONAH can provide much better resolution than beamforming. So the array will typically be used for beamforming only above 1300 Hz.

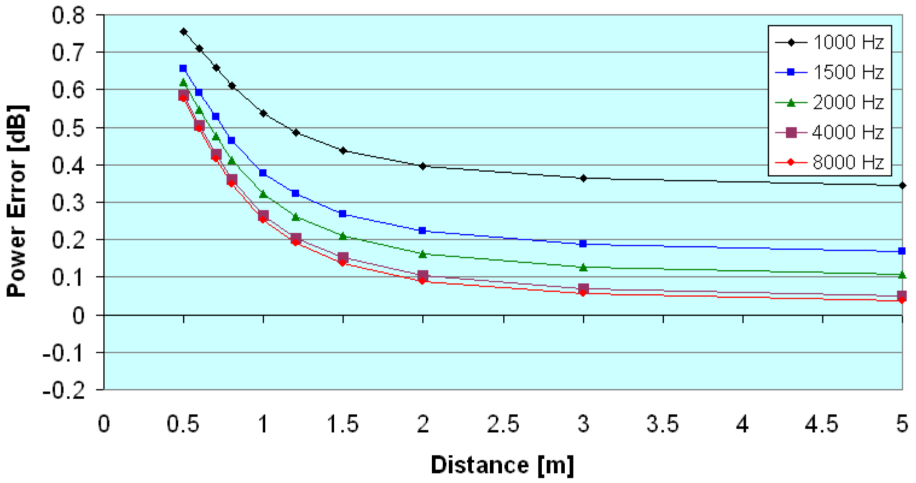

Figure 3: Difference in Decibel between estimated and true Sound Power. The estimated value is from an Intensity scaled Delay-And-Sum Beamformer. The source is a monopole on the array axis.

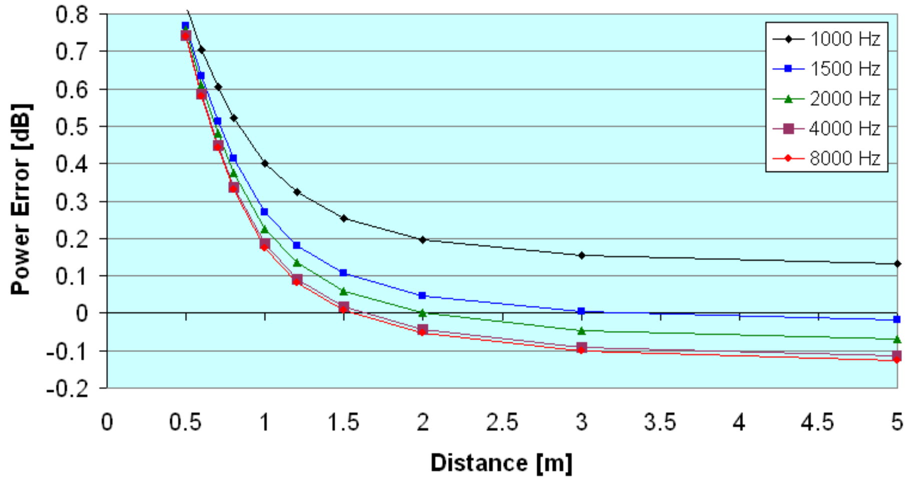

Figure 4: Difference in Decibel between estimated and true Sound Power. The estimated value is from an Intensity scaled Cross-spectral Beamformer with exclusion of Auto-spectra. The source is a monopole on the array axis.

Figure 4 shows the difference between the estimated and the exact sound power in Decibel for the case of the Cross-spectral beamformer with exclusion of Auto-spectra. This algorithm is implemented in Brüel & Kjær's Stationary and Quasi-stationary Beamforming calculation software, and therefore it has been used for the measurements presented in the present paper. Comparison of Figure 3 and Figure 4 shows that in general the Cross-spectral algorithm produces smaller errors then the Delay-And-Sum algorithm, except when the measurement distance gets shorter than around 0.6 meter.

The error associated with the assumption of a narrow mainlobe in equations (13-14) could be avoided, but it would complicate the correction, and the level of error indicated in Figure 3 and 4 is small compared to for example the influence of coherent source distributions.

It is of course also important to consider, how the sound intensity scaling works for more realistic source distributions than a single monopole. Consider first the case of several omni-directional, but mutually incoherent point sources in the source plane. The incoherent sources will contribute independently to the Cross-spectral matrix between the microphones, i.e. the matrix will be the sum of elementary matrices related to each one of the point sources. If a Cross-spectral Beamformer is used, then the (power) output is equal to the sum of contributions from the elementary matrices, meaning that the incoherent partial sources contribute additively to the Beamformer (power) output. Since they contribute also additively to the Sound Power, the conclusion is that the intensity scaling will hold for a set off incoherent monopole point sources.

When there is full or partial coherence between a set of monopole sources, the radiation from the total set of sources is no longer omni-directional, which will introduce an error that cannot be compensated: The array covers only a certain part of the 2π solid angle for which the sound power is desired. For angles not covered by the array we do not know the radiation and therefore we cannot know the sound power.

4. MEASUREMENTS

A series of measurements were performed on two small loudspeakers in order to test the accuracy of the scaling and the associated sound power determination. Measurements were taken with the 60-element Sector Wheel Array of Figure 2 at 55 cm distance from the loudspeakers for Beamforming (BF) processing. Brüel & Kjær Type 4935 array microphones were used in the array. Three different excitations were measured: 1) A single speaker excited by white random noise. 2) Incoherent white-noise excitation of the two speakers (same level on both speakers). 3) The same white-noise excitation on both speakers, i.e. coherent excitation. For each of these three excitations, a scan was performed approximately 7 cm from the two loudspeakers with a B&K sound intensity probe Type 3599 with 12 mm spacer. The two speakers were identical small PC units with drivers of 7 cm diameter, and they were mounted with 17 cm between the centers of the drivers, see Figure 5. The Beamforming processing was performed with the Cross-spectral algorithm with exclusion of Auto-spectra, [2].

Figure 5 shows resulting 1/3-octave intensity maps for the measurements with only the rightmost speaker excited. The sound intensity scaled Beamformer maps in the first row have been estimated in the source plane over an area of approximate size 80cm x 80cm. The second row shows the sound intensity measured 7 cm from the plane of the speakers over an area of size 36cm x 21cm. All plots show a 15 dB dynamic range from the maximum level, with 1.5 dB steps between the colours. Yellow/green colours represent outwards intensity and blue colours represent inwards intensity.

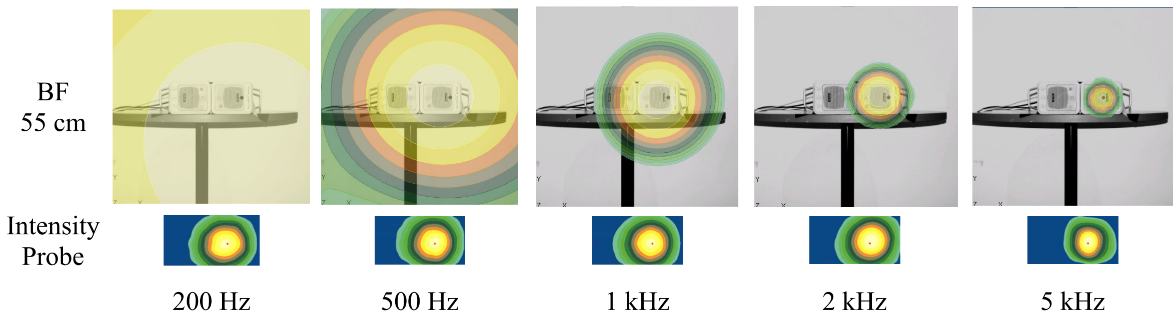

Figure 5: 1/3-octave sound intensity maps. The two rows represent Beamforming (BF) from 55 cm distance and measurement with a sound intensity probe at 7 cm distance. The displayed dynamic range is 15 dB.

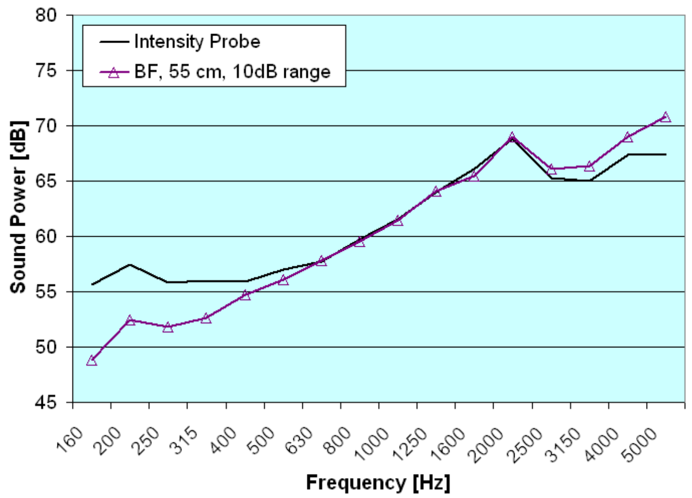

Figure 6: 1/3-octave sound power spectra for the single speaker measurement. The results obtained with Intensity Probe and Beamforming are compared.

Figure 6 shows the 1/3-octave sound power spectra for the single loudspeaker obtained with intensity probe and Beamforming. For the Intensity Probe measurement, power integration has been performed over the entire mapping area (see Figure 5). For Beamforming the sound power integration has been performed also over the full mapping area, but in order to reduce the influence of sidelobes only a 10 dB range of intensity data have been used (i.e., data points where the level is at least 10 dB below Peak level were ignored). The effect of this dynamic range limitation has been verified to be very close to the effect of limiting the integration area to cover only the mainlobe area. Above 500 Hz this leads to a good estimate of the sound power, apart from a small overestimation at the highest frequencies. The main reason for this overestimation is the directivity of the speaker towards to array at these frequencies. The strong underestimation below 500 Hz is due to the fact that the mainlobe is larger than the mapping area, so a significant part of the mainlobe is outside the map and therefore included in the power integration, see Figure 5.

The results with equal but incoherent excitation of the two speakers are very similar to the results with only one loudspeaker excited. The sound power spectra all increase by approximately 3 dB over the major part of the frequency range, but the differences between the spectra remain unchanged. The data are not shown here.

Equal but coherent in-phase excitation of the two loudspeakers will, on the other hand, cause the radiation to deviate more from being omni-directional, which violates the assumptions on which the intensity scaling of Beamformer maps are based. Figure 7 depicts the 1/3-octave sound power spectra obtained using intensity probe and Beamforming with identical excitation of the two speakers. Comparing with the case of only a single speaker being excited, the scaled Beamformer map shows additional deviation in the frequency range from 1 kHz to 2 kHz. In that frequency range the distance between the two speakers is between half a wavelength and one wavelength, which will focus the radiation in the directions covered by the array. But the deviation remains within approximately 2 dB from the power spectrum obtained with the sound intensity probe.

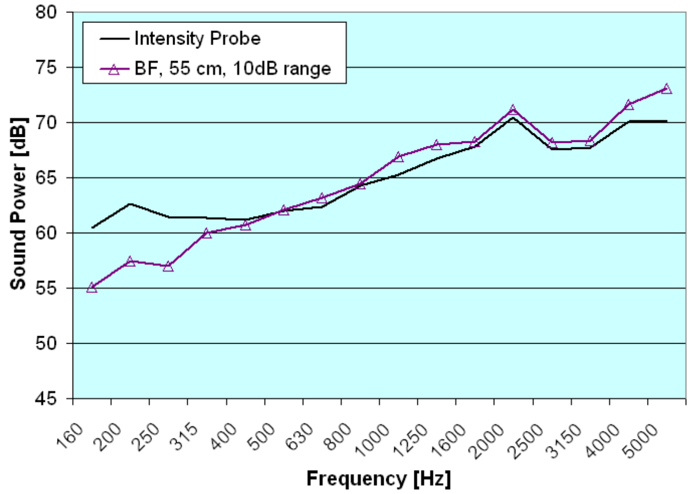

Figure 7: 1/3-octave sound power spectra for the case of the two speakers being excited with the same white noise signal. Results obtained with Intensity Probe and Beamforming are compared.

5. SUMMARY

A simple mathematical expression for a scaling factor has been derived to scale the output from a Delay-And-Sum Beamformer in such a way that area integration of the scaled output provides good estimates of the sub-area sound power. For that reason it is natural to use the term "Sound Intensity Scaling" about the method. The derivation was performed looking at a single monopole point source in the far-field region, and assuming the mainlobe of the beamformer to cover only a small solid angle. An evaluation has been given then of the errors introduced by the far-field assumption and the assumption of a narrow mainlobe. This has been done both for Delay-And-Sum processing and for the Cross-spectral algorithm with exclusion of Auto-spectra described in reference [2]. Within the main frequency range of the chosen 60-element array, the error does not exceed approximately 0.6 dB when the measurement distance is larger than 0.6 times the array diameter. Loudspeaker measurements confirmed the good accuracy in connection with omni-directional sources.

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