

Three Dimensional Multiple Near-field Source Localization Based on MUSIC Algorithm to Increase the Localization Accuracy of Optimal Beamformer

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Abstract— In this paper the MUSIC source localization algorithm is applied to the near-field narrowband optimal beamformer to increase its localization accuracy and resolution capability. Optimal beamformer cannot identify closely spaced multiple near-field signals. MUSIC algorithm is an Eigen-decomposition based source localization technique. A three dimensional MUSIC algorithm is used with near-field optimal beamformer to correctly localize the three parameters (range, elevation and azimuthal angle) of multiple sources. The robustness of this proposed beamformer against the white Gaussian noisy environment is also examined. The Root Mean Square Error (RMSE) to localize the multiple near-field targets is also studied. The simulation results show that the MUSIC based optimal beamformer can easily sense the multiple closely spaced sources in the noisy environment with sharper radiation lobe using minimum number of snapshots and sensors.

Keywords—Beamformer; MUSIC; HPBW; RMSE; Near-field)

I. INTRODUCTION

Source localization nowadays becoming more important because of its widespread application in wireless communication. But most of the research have done in the past for the far-field analysis where the arriving sources are considered as plane wave front and only the elevation angle of the source is estimated. But in near-field case the sources are close to the array so the arriving wave front is not plane rather spherical. So a specific near-field study is required to estimate the radial distance, elevation and azimuthal angle of near-field spherical source. This research has focused on the three dimensional near-field localization study.

Various localization study have done by the previous researchers for the far-field, near-field and mixed cases. Maximum likelihood [1] and higher order cumulants [2-3] based near field localization study suffer from either high computational cost or higher complexity to compute higher order cumulants. A near-field higher order sub spaced based ESPRIT algorithm in [4] requires multidimensional search and suffer from aperture loss. A MUSIC and Capon DOA estimation algorithm for near-field case has proposed in [5] without mentioning the localization accuracy. A 3-D

parameter estimation using second order statistics for linear array discussed in [6] and the three dimensional near field analysis in [7] uses an efficient algorithm. These research mainly suffer from pairing and resolution problem or don't specifically observed the power spectrum curve and other assessment parameters of localization. MUSIC algorithm has the capability to sense multiple sources with high resolution but a three dimensional near-field localization study using MUSIC is very rare.

Optimal is an adaptive beamforming technique which is used to identify the wanted and unwanted signals in the noisy environment. But severe performance degradation of optimal beamformer occurs for identifying multiple closely spaced sources [8]. A robust near-field optimal beamformer in [9] uses variable loading technique to provide robustness against steering vector mismatch but it did not show any protection against multiple closely spaced source. So it is necessary to increase the toughness of optimal beamformer for identifying three parameter of multiple closely spaced near-field sources.

So the main objectives of this paper is to apply the MUSIC source localization technique to the optimal beamformer in order to increasing the localization accuracy of optimal beamformer for the localization of multiple closely spaced near-field sources. Power density curve and resolution capability will be analyzed with respect to the radial distance, elevation and azimuthal angle. RMSE for the localization scheme will also be measured against the Signal to Noise Ratio (SNR) and against the number of snapshots. Beamwidth of maximum radiation pattern lobe according to the sensing elements is also to be showed.

II. NEAR FIELD SIGNAL MODEL

A signal is defined as near-field if its radial distance $r < 2D^2/\lambda$ where D denotes maximum dimension of antenna known as antenna aperture and λ be the wavelength of signal [10]. Let consider a spherical near-field source $p(r_i, \theta_i, \phi_i)$ is to be localized by linear array of equispaced elements like that of Fig. 1. Assuming that the required time of i th source to reach at the l th element is [11]

$$\tau_{il}(r_i, \theta_i, \phi_i) = (|x_i - x_l| - |x_i|) / c \quad (1)$$

Where

$$x_i = r_i (\sin \theta_i \cos \phi_i + \sin \theta_i \sin \phi_i + \cos \theta_i) \quad (2)$$

be the position vector of i th source and c is the speed of propagation of spherical wave front. Then induced signal at l th element due to i th source is given by [11]

$$x_l(t) = m_i(t) * r_i / r_{il} * \exp(j2\pi f(t + \tau_{il})) + n_l(t) \quad (3)$$

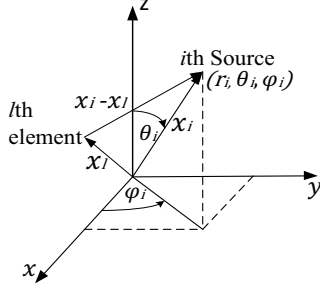


Fig.1: Near field coordinate system.

Here $m_i(t)$ be the modulated signal, r_{il} is the distance of i th source from l th element and $n_l(t)$ denotes for the noise environment at l th element.

Equation (3) can be reduced to

$$x_l(t) = m_i(t) * s_i(t) + n_l(t) \quad (4)$$

Here $s_i(t) = r_i / r_{il} * \exp(j2\pi f(t + \tau_{il}))$ is called near-field steering vector which is the function of three variable (range, elevation and azimuthal angle). Then array correlation matrix is defined as [12]

$$R = E[x_l(t) * x_l(t)^H] \quad (5)$$

It's assumed that the sources and noises are uncorrelated with each other. Then equation (5) can be reduced to [12]

$$R = p_i s_i s_i^H + \sigma_n^2 I \quad (6)$$

Here p_i denotes signals power, σ_n^2 is the noise variance and I stands for identity matrix. $E[\cdot]$ and H denotes for the expectation operator and complex conjugate transposition respectively.

III. OPTIMAL BEAMFORMER

Optimal beamformer can sense desired signals position and direction but suffer from accuracy and resolution problem while sensing multiple closely spaced signals. An adjustable weight is multiplied with each array element's induced signal. The weight w is the solution of following optimization problem.

minimize	$w^H R w$
$w = R^{-1} S_i / S_i^H R^{-1} S_i$	
Subject to	$w^H S_i = 1$

IV. MUSIC ALGORITHM

This algorithm search for the sources using following spectral equation [13]

$$P(r, \theta, \phi) = S_i^H / S_i * E_n^H * E_n * S_i^H \quad (7)$$

Here E_n is the noise subspace vector. E_n is calculated by decomposing of array correlation matrix. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_L$ be the eigen value of R and $E = [e_1, e_2, \dots, e_L]$ are corresponding eigen vector. The eigen vector corresponding to the maximum eigen value is defined as signal subspace. For the localization of r sources the signal and noise subspace vector is defined as

$$E_s = [e_1, e_2, \dots, e_r] \text{ and } E_n = [e_{r+1}, e_{r+2}, \dots, e_L] \quad (8)$$

This E_n value is used for spectral search. E_s can also be used in localization equation for searching signals peak.

V. PERFORMANCE EVALUATION

Multiple sources localization performance of optimal beamformer after applying MUSIC algorithm is investigated in this section. RMSE is one of the major assessment tool for source localization is also measured using following equation.

$$\text{RMSE} = \sqrt{(\sum_i^N (x_N - x_{\text{actual}})^2) / N} \quad (9)$$

Here x_N is the estimated results, x_{actual} is actual value and N be the number of independent Monte Carlo trials.

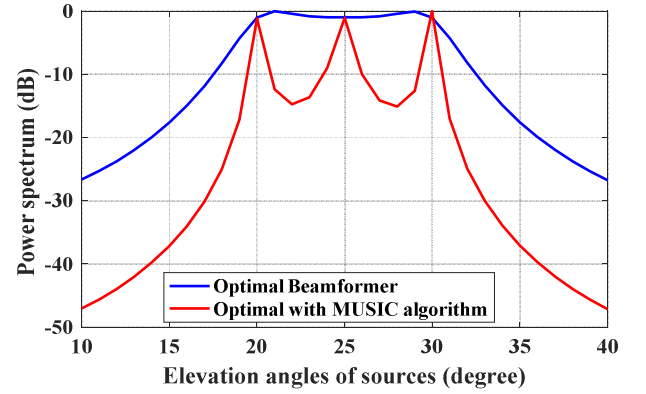


Fig. 2: Localization of elevation angle of three sources when signals focal point $(r, \theta, \phi) = (20\lambda, 20^\circ, 90^\circ)$, $(20\lambda, 25^\circ, 90^\circ)$ and $(20\lambda, 30^\circ, 90^\circ)$ with number of sensor = 40, SNR = 15dB, number of snapshot = 1000 and each signal power = 1.

Fig.2, Fig. 3 and Fig. 4 shows the localization performance of optimal beamformer before and after applying MUSIC algorithm. Elevation angle, azimuthal angle and the radial distance of three sources are localized in these three figures separately. One can observe from these figures that optimal beamformer is incapable to correctly sense closely spaced targets whereas the MUSIC based optimal beamformer perfectly localizes these sources using only 40 sensors and 15 dB SNR level.

The main disadvantages of optimal beamformer is that it cannot identify very closely targets i.e. it suffers from resolution problem. Table 1 shows that the optimal beamformer cannot sense multiple signals with angular spaced 7° or less.

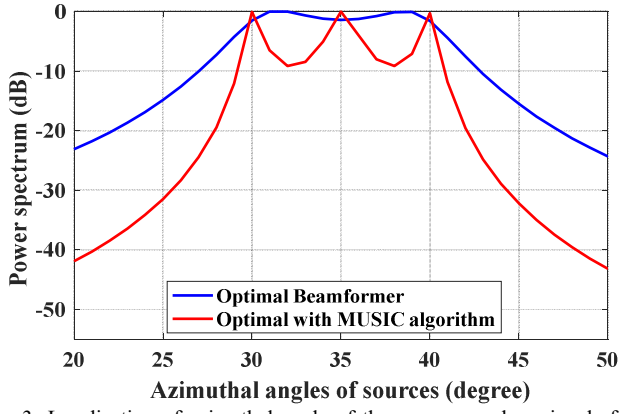


Fig. 3: Localization of azimuthal angle of three sources when signals focal point $(r, \theta, \phi) = (20\lambda, 10^\circ, 30^\circ)$, $(20\lambda, 10^\circ, 35^\circ)$ and $(20\lambda, 10^\circ, 40^\circ)$ with number of sensor = 40, SNR = 15dB, number of snapshot = 1000 and each signal power = 1.

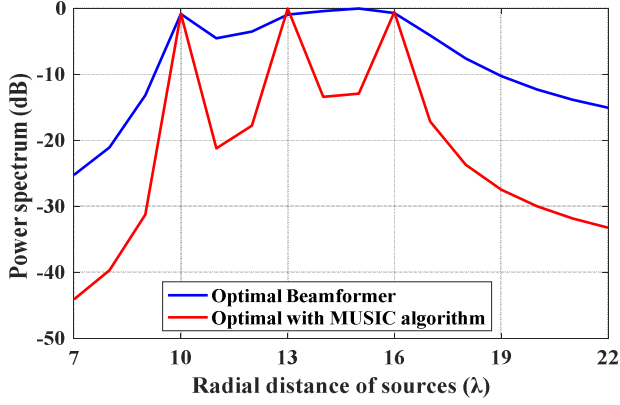


Fig. 4: Localization of range of three sources when signals focal point $(r, \theta, \phi) = (10\lambda, 30^\circ, 90^\circ)$, $(13\lambda, 30^\circ, 90^\circ)$ and $(16\lambda, 30^\circ, 90^\circ)$ with number of sensor = 40, SNR = 15dB, number of snapshot = 1000 and each signal power = 1.

Table 1: Resolution capability (in terms of elevation angle) of localizing three sources

Difference of elevation angle of three sources	Optimal Beamformer	MUSIC based optimal Beamformer
1°	Sense only 1 source	Sense 2 sources
$2^\circ, 3^\circ$	Sense 2 sources with 1° error	Sense 2 sources without error
$4^\circ, 5^\circ$	Sense 2 sources with 1° error	Perfectly sense 3 sources without any error
$6^\circ, 7^\circ$	Sense 3 sources: 2 with 1° error and 1 without error	Perfectly sense 3 sources without any error
8° or more	Perfectly sense 3 sources without any error	Perfectly sense 3 sources without any error

But the MUSIC based optimal has gained the ability to detect 4° spaced multiple targets. Another advantages of MUSIC algorithm over optimal is discussed in Table 2. MUSIC based optimal beamformer uses only 16 sensors to identify three sources without error whereas optimal requires 25 or more.

Optimal and MUSIC provides almost same RMSE for estimating multiple sources. RMSE for the identifying of elevation angles is studied in Fig. 5 and Fig. 6 and it's

concluded from these figure that RMSE of MUSIC based optimal is lower than the optimal beamformer. RMSE (azimuthal angle) and RMSE (radial distance) according to snapshot and SNR variation of optimal and MUSIC based optimal beamformer are exactly the same.

Table 2: Resolution capability (in terms no. of sensing elements) of localizing three sources

Number of sensing elements	Optimal Beamformer	MUSIC based optimal Beamformer
15 or less	Sense 2 sources with 1λ error	Sense 2 sources without error
16, 17	Sense 2 sources with 1λ error and non-uniform peaks	Sense 3 sources without error
18-24	Sense 2 sources with 1λ error and non-uniform peaks	Sense 3 sources without error and with sharper peaks
25 or more	Sense 3 sources without error	Sense 3 sources without error and with sharper peaks

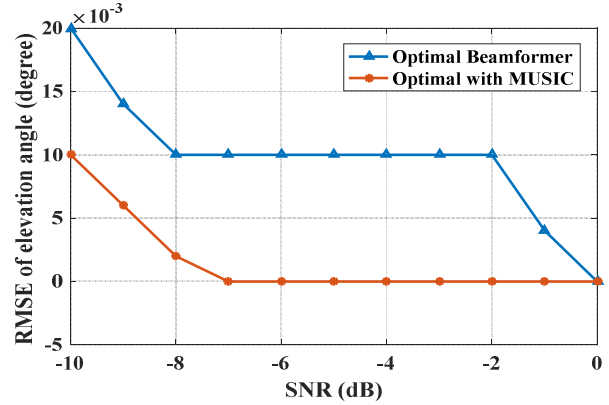


Fig. 5: RMSE of estimating elevation angle of sources with SNR variation when no. of sensor= 15 and no. of snapshot=50.

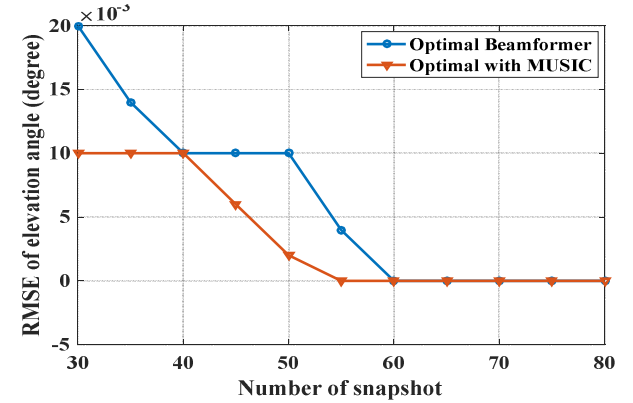


Fig. 6: RMSE of estimating elevation angle of sources with snapshot variation when no. of sensor= 15 and SNR = -10 dB.

Half-power beamwidth is another important measure for antenna localization performance which is defined as the separation between two points on the radiation pattern main lobe at which the signal power is half (-3dB point) of its maximum value.

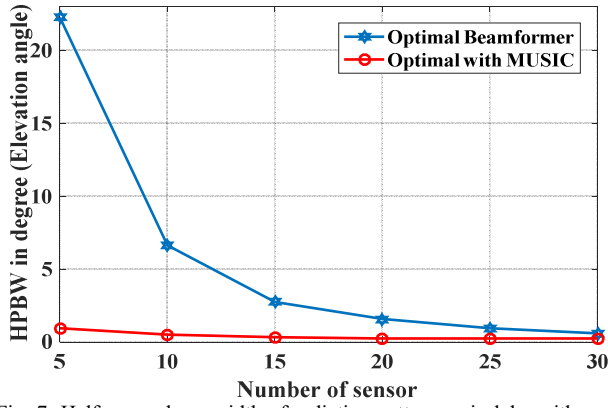


Fig. 7: Half-power beamwidth of radiation pattern main lobe with respect to the no. of sensor for the localizing of elevation angle of a single source.

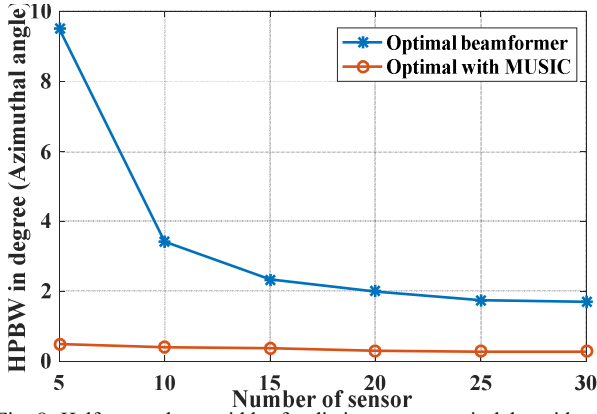


Fig. 8: Half-power beamwidth of radiation pattern main lobe with respect to the no. of sensor for the localizing of azimuthal angle of a single source.

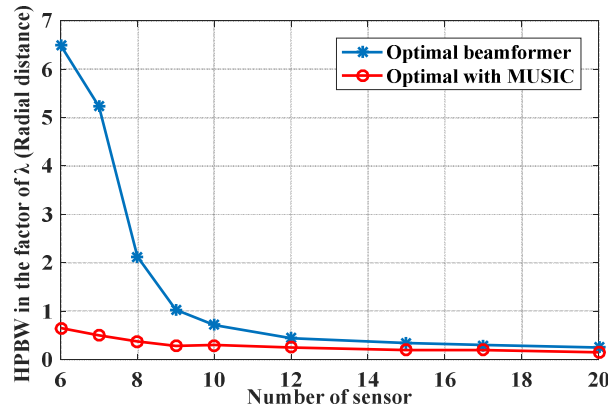


Fig. 9: Half-power beamwidth of radiation pattern main lobe with respect to the no. of sensor for the localizing of radial distance of a single source.

There exists a relationship between beamwidth and directivity. Lower the beamwidth, more sharper the radiation pattern and more possibility to localize the exact position of the sources. Fig. 7, Fig. 8 and Fig. 9 is the plot of HPBW of radiation pattern main lobe for the source localization by the beamformers. One can observe from these figures that using lower amount of sensing element both the beamformer

(optimal and MUSIC based optimal) can identify the sources but MUSIC based algorithm shows a lower beamwidth i.e. higher possibility to localize the sources using same number of sensors.

VI. CONCLUSION

This paper has applied the three dimensional MUSIC source localization algorithm to the optimal beamformer in order to increasing its localization performance. Three parameters (range, elevation and azimuthal angle) of a near field spaced source has estimated using a single processor and the error to sense these parameters according to SNR and snapshot variation has investigated where the MUSIC based optimal beamformer has provided lower RMSE. The proposed beamformer identifies much closed multiple sources with high directivity gain i.e. provides high resolution capability. HPBW of maximum radiation pattern of MUSIC based optimal beamformer has also showed.

REFERENCES

- [1] J. C. Chen, R. E. Hudson, and K. Yao, "Maximum-likelihood source localization and unknown sensor location estimation for wideband signals in the near-field," *IEEE Transactions on Signal Processing*, vol. 50, no. 8, pp. 1843–1854, 2002.
- [2] J. Liang and D. Liu, "Passive localization of near-field sources using cumulant," *IEEE Sensors Journal*, vol. 9, no. 8, pp. 953–960, 2009.
- [3] L. Zhou and D. Huang, "A new ESPRIT algorithm based on Toeplitz method for coherent signals," *Proceedings 2011 International Conference on Transportation, Mechanical, and Electrical Engineering (TMEE)*, pp. 1521–1524, 2011.
- [4] Y. Xu, H. Zhang and J. Zhan, "A Signal-Subspace-Based ESPRIT-Like Algorithm for Coherent DOA Estimation," *2009 5th International Conference on Wireless Communications, Networking and Mobile Computing*, pp. 1–4, 2009.
- [5] R. J. Weber and Y. Huang, "Analysis for Capon and MUSIC DOA estimation algorithms," *2009 IEEE Antennas and Propagation Society International Symposium*, North Charleston, pp. 1–4, SC, USA, 2009.
- [6] C.M Lee, K. S. Yoon, J. H. Lee, and K. K. Lee, "Efficient algorithm for localizing 3-D narrowband multiple sources," *IEEE Proc., Radar Sonar Navig.*, Vol. 148, No. 1, pp. 23–26, 2001.
- [7] X. Li, N. Liu, N. Jiang, X. Long and X. Jiao, "Second-Order Statistics-Based Multi-Parameter Estimation of Near-Field Acoustic Sources," *Applied Mechanics and Materials*, pp. 977–983, Nov.-2013.
- [8] R. Tota and M.S. Hossain, "Robust near-field narrowband beamformer against steering angle mismatch and distance error using diagonal loading technique," *International Journal of Innovative Science and Research Technology*, Vol. 6, Issue 5, pp. 991–996, May-2021.
- [9] M. R. Islam, L. C. Godara and M. S. Hossain, "Robust near field broadband beamforming in the presence of steering vector mismatches," *WAMICON IEEE Wireless & Microwave Technology Conference*, pp. 1–6, 2012.
- [10] B. D. Steinberg, "Principles of Aperture and Array System Design: Including Random and Adaptive Arrays," New York: Wiley, 1996.
- [11] M. Islam, L. Godara, and M. Hossain, "A computationally efficient near field broadband beamformer," in *Communications (LATINCOM), 2011 IEEE Latin American Conference on*, pp. 1–5, Oct. 2011.
- [12] L. C. Godara, "SMART ANTENNA," CRC Press LLC, Boca Raton, Florida-33431, 2004.
- [13] N. Le Bihan, S. Miron and J. I. Mars, "MUSIC Algorithm for Vector-Sensors Array Using Biquaternions," in *IEEE Transactions on Signal Processing*, vol. 55, no. 9, pp. 4523–4533, Sept. 2007.