Danmarks Tekniske Universitet

Special course report: Numerical modeling of the acoustics inside MEMS microphones

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Contents

1 Introduction

As a summary of the corresponding special course, this report aims to demonstrate the author's understanding of the learning objectives by trying to explain the micro-acoustic theory and practicing the experiment using Comsol. The main content of the special course is structured according to the following learning objectives, where the most basic topic is to understand the theory of thermoviscous loss, and then to practice the construction of several different configurations of MEMS models, and finally, abstract/interpreting the results with lumped model, and try to optimize the original model structure with the understanding of the micro-acoustic theory behind.

The learning objective as below.

- Understand and explain viscous and thermal losses in microacoustics. section 1
- Prepare, understand, and interpret lumped element and transfer matrix methods (LEM and TMM) of small devices on the MEMS-scale. section 5
- Construct and interpret numerical models of (thermo-viscous) acoustics in small devices using commercial software. The models may be based on the Finite Element Method (FEM) and/or the Boundary Element Method (BEM). section 2 /section 3 /section 4
- Investigate and propose modifications to the modeled devices that can improve their performance. section 6

1.1 Theory

The micro-acoustic is a special case of general acoustics with its main purpose the study of sound at relatively small scale (related with frequency). The mainly considered problem in micro-acoustic area, i.e., the thermoviscous loss, is the general characteristic of sound transmission but normally ignored under a relatively large scale, for instance, speech transmission in a room. With that in mind, the overall goal of micro-acoustic study of the properties of sound under specific conditions, e.g., sound pressure or sound velocity at certain position under frequency, the average pressure response to frequency at a membrane, or the temperature and density perturbation at the whole geometry, etc. One important thing makes the micro-acoustic becomes a separate field is the energy loss. As energy considerations are of enormous practical importance in acoustic[1], I will try to describe it to demonstrate my understanding. The following is the interpretation including what is the sound energy, where the energy lost to, and how the energy lost.

- **What is sound energy** Source of sound emit sound power, and sound field are also energy field in which potential and kinetic energy are generated transmitted and dissipated [1]. Due to the conservation of energy, the energy lost only if it's going out of system or become other energy form, as energy conservation equation from the first law of thermodynamics,

$$dE_t = dQ + dW \tag{1}$$

  where dQ is the heat added to the system, dW is the work done on the system, and dEt is the increment in the total energy of the system [2]. If we look at this equation reversely, the sound energy decrement dE_t as the sum of the heat dissipation Q and the the work done by the system.

  For a more specific definition, the system is enclosed by boundary through which energy and mass may enter or leave the system. If considering the air with the continuous property as a sole system, the surface between itself and another material, metal, water, or plastic, can be seen as its "boundary". Energy crossing through the surface becomes the sound wave propagation's lost energy.

- **Where the energy lost to** The energy lost either through work or heat. The most common energy loss is the work done by the system through the boundary, e.g., the sound energy in a room transfer out through window, curtain, or the door. Such energy transfer depend on the impedance of boundary. On the other hand, the normally ignored heat dissipation depend on the difference of air and boundary heat capacity. As the heat energy must flow in proportion to the product of the temperature gradient and the thermal conductivity of the medium. [3]. Assuming the material contact with air has a much larger heat capacity, it is more dominating on the overall temperature. Since the temperature is continues, the thermal energy will flow to the media with higher thermal conductivity. The boundary normally assumed as the isothermal since it's thermal capacity is much larger then the air, it's temperature is constant despite the heat transferred from the air.

- **How the energy lost** Close to the boundary, the organized motion of sound is converted into the disorganized motion of heat.[4]. At the contact of the boundary and air, where the air particles close to the boundary are slowed down or caught due to more significant boundary stress, we can simply say the solid is more "sticky" than the air. At the constant temperature, the 'viscosity' is proportional to the shear stress, with the coefficient of dynamic viscosity, μ [3]. The shear stress, as the friction's work done to slow and disorganized the air particles, dissipate fluid kinetic energy into heat. [5].

In summery, the thermoviscous loss theory extends the fundamental acoustic theory, the introduce of sound energy lost brings together with the temperature and density variation during wave transfer, as well as the neglected friction and heat capacity difference between the boundary and air.

The consequence of thermoviscous depends on the practice scenario. The effect of thermal viscosity loss will be too significant to ignore at the micrometer level tubes or porous material and the MEMS size microphone. Such effects include the energy loss together with pressure and velocity drop, which cannot predict with the fundamental wave equation due to its presumption, such as in-compressible air and energy conservation. Like the Figure 1 shows, the thermoviscous caused the velocity profile significantly deformed, compared with the Non-thermoviscous wave equation.

The full set of thermoviscous loss equation as shown below, its contained more property of air behavior then the fundemental wave equation. The mainly construction is three set of conservative quantity: the mass, momentum and energy. Worth to mention that the conservation of momentum also been refered as the Navior-strokes equation. This report is not intened to derive or interperating such equation- it's still unsolved- instead, abstractly understand the concept of equation under the specific situation at MEMS size and audible frequency range.

Conservation of mass,

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \vec{V}) = 0 \tag{2}$$

where rho is density, V is velocity. If assume the air is in-compressible for a constant density, the equation becomes the conservation of volume.

Conservation of momentum,

$$\rho\frac{DV}{Dt} = -\nabla p + \mu\nabla^2 V \tag{3}$$

Where μ is the dynamic viscosity. This equation is simplified for easily understanding with assumption: incompressible fluid, constant mu and no gravity. The equation developed from Newton's Second Law, the left side represent the inertial force, the right side represent the force exerted on the fluid particle per unit volume, including the pressure output and the

viscous force.
Conservation of energy

$$\rho[\frac{\partial h}{\partial t} + \nabla \cdot (hV)] = -\frac{\partial p}{\partial t} + \nabla \cdot (k \nabla T) + \phi \tag{4}$$

where h is enthalpy and k is thermal conductivity, and ϕ is the heat dissipation. This equation is developed from Equation 1, where the left side represent the total energy of system, where enthalpy approximately proportional to the air temperature; and the decreasing of velocity will lead to the decreasing of system energy. The $-\frac{\partial p}{\partial t}$ is the work done by pressure; $\nabla \cdot (k \nabla T)$ represent the heat flux inside of system; and the ϕ is the heat dissipation, as predescribed related with the boundary condition and temperature.

The equation above is only the simplified version, if we step even further by assuming the wave only transfer inside a simple straight duct without the nonlinear backward wave. The equation can be once more simplified by separating the wave preparation to forward direction and compression diction. The divergence, laplace opreator, gradient, and velocity vector disappear, and a pseudo wave equation comes below.

$$\frac{\partial^2 p}{\partial x^2} - k^2 \Gamma^2 p = 0 \tag{5}$$

The additional propagation constant Γ describe the pressure and velocity profile by indicating the shear wave number s. Propagation constant,

$$\Gamma = \sqrt{\frac{\gamma}{n(s\sigma)B(s)}} = \sqrt{\frac{\gamma + (\gamma - 1)B\left(s_t\right)}{B(s)}} \tag{6}$$

Polytropic constant,

$$n(s\sigma) = \left[1 + \left[\frac{\gamma - 1}{\gamma}\right] B(s_t)\right]^{-1} \tag{7}$$

The $B(s)$ and $B(s_t)$ in above equation represents the geometry function[6], which determined the velocity profile in terms of different boundary shape. The geometry function represents the velocity or thermal profile depend on the input boundary layer type- the shear wave number or the thermal wave number. The s in Equation 8 represents the shear wave number, represents the velocity profile, and the s_t represents the thermal wave number. As Figure 1, the left demonstrates the low-frequency velocity profile derived from the Equation 8, where the velocity is dominated by the "drag" from the boundary, and the right plot demonstrates the high-frequency situation, where the viscous effect by the boundary is still evident but the flow is not affected by the boundary in the center.

$$B_A = \frac{J_2(si\sqrt{i})}{J_0(si\sqrt{i})}; s = l\sqrt{\frac{\omega\rho}{\mu}}; s_t = l\sqrt{\frac{\omega\rho Pr}{\mu}} \tag{8}$$

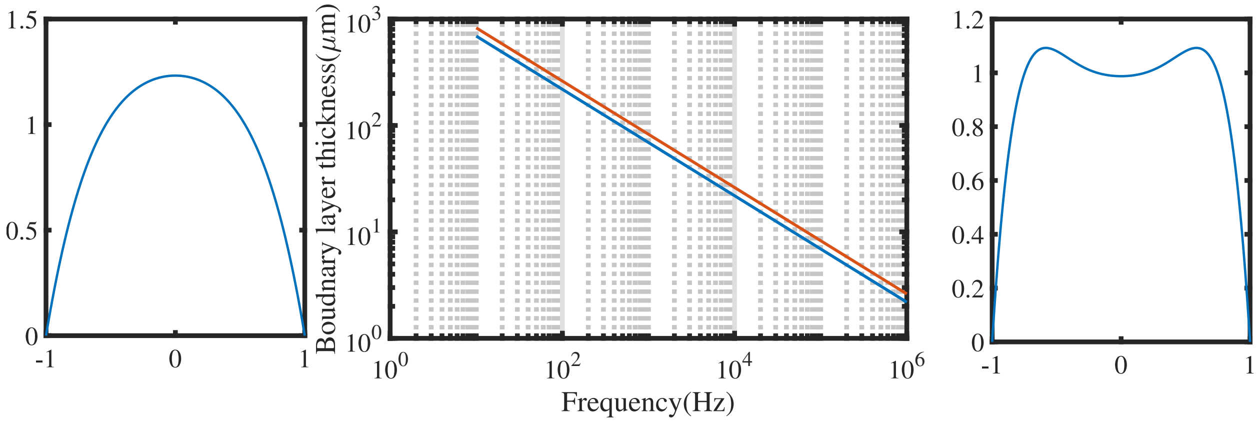

Figure 1: The frequency response of boundary layer thickness. The left and right plot represents the viscosity dominated and pressure dominated velocity profile individually

$$\delta_v = \sqrt{\frac{\mu}{\pi f \rho_0}}; \delta_{th} = \sqrt{\frac{k}{\pi f \rho_0 C_p}} \tag{9}$$

The frequency response of the boundary layer thickness as Figure 1 is derived by Equation 9, where the δ_v and δ_{th} represents the viscous and thermal boundary layer thickness individually. Physically, they represent the thickness from the zero velocity/boundary temperature point(if applying the no-slip and isothermal boundary condition) to the "peak" as in the Figure 1.

It's also worth to mention that the boundary effect in terms of velocity and temperature gradient is not completely unrelated. As Equation 8, their ratio is equal to the Prandtl number Pr, which is the ratio of momentum diffusivity to thermal diffusivity.

$$s_t = s * \sqrt{Pr}; \mathrm{Pr} = \frac{\nu}{\alpha} = \frac{c_p \mu}{k} \tag{10}$$

1.2 Real-world MEMS microphone structure

Before start the constructing the model, it's necessary to introducing a typically real-world MEMS microphone, based on such structure, the following model is only valid.

1.3 General process of Comsol model build

The build of Comsol model in all the following experiment is pretty similar. A typical process including the following steps:

- Build the geometry: The dimensions were refer to [7]. To facilitate comparison of results, most models use similar dimensional parameters, as Table 1

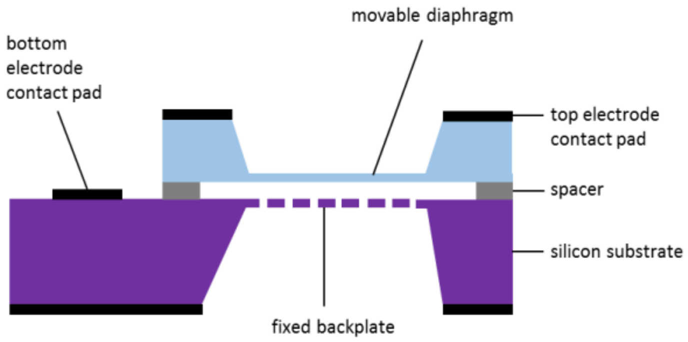

Figure 2: The back membrane structure[4], this structure is referenced by section 2 to aviod introduce the membrane structure too early.

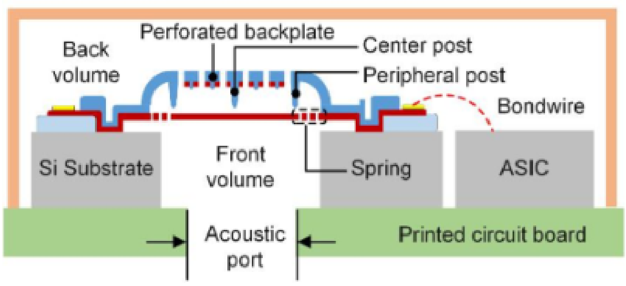

Figure 3: The front membrane structure[7], this structure is referenced by section 3 and section 4, the membrane is simulated by a membrane muti-physics component.

- Material: 'Air' and Polycrystalline Si' -used as membrane material -applied in all experiment, the material properites of Polycrystalline Si as Table 1.
- Set physics component & muti-physics components: Pressure acoustic include simple wave equation, Thermal Viscous acoustics include full set of thermal-viscous equation and Membrane -in short PA, TA,
- Set solver: frequency range set from 10 to 1e6 with exponential step $10^{0.1}$, to make the data point equally distributed on the logarithmic scale frequency axis.
- Mesh size: as in Figure 1, the velocity profile under visous is a parabolic-like curve. In order to minimize the error, the size of single mesh is recommended not be greater than one-sixth of the boundary layer thickness for a good approximation. But the increasing of mesh quality will also consume much computational resource, such problem becomes significant when performing 3D model simulations. The boundary layer mesh and DTU HPC was used at 3D model simulation to overcome such problem. However since the 2D mesh does not require to much computer resource, the strategy is simply increasing the mesh quality up to $0.2\mu m$

notation	value	description
r_d	425um	diaphragm radius, um
r_a	8.25um	backplate hole radius, um
t_d	1.4um	Thickness of diaphragm
g_a	4um	airgap
Xelem	24um	x-axis of first hole
Yelem	$24 * sqrt(3)[um]$	y-axia of first hole
m_dia	1.8431e-9 kg	mass of diaphragm
beta_dia_eff	0.5	area coeficient
alpha_dia_eff	0.2	mass coeficient
A_dia	5.6745e-7 m^2	area of diaphragm
A_dia_eff	2.8373e-7m^2	acoustic diaphragm
m_dia_eff	0.006496 kg/m^2	acoustic mass
c_dia_eff	8.5118e-7 $m^2 \cdot s^2/kg$	acoustic complience
P_ini_db	65dB	input pressure
membrane material: Polycrystalline silicon		
E1	160e9pa	Young's modulus
rho1	2320kg/m^3	Density
nu1	0.22	Poisson's ratio

Table 1: input parameter

2 2D Experiment

The first experiment will start with a simple 2-D model without a membrane, referencing the structure in Figure 2. As the first stage of the experiment, the motivation of this experiment is to exercise the skill of Comsol model build with observing the velocity profile and pressure gradient. The result can be easily compared with the theory since the structure is simple. Also, by the fact that comsol provides two alternative ways (pressure acoustic physics with additional VT boundary impedance or full VT physics) to calculate the thermal viscosity loss, it is necessary to compare these two methods in a simple model in order to arrive at the best setting.

As for the Pre-processing, compare the the most common 'front membrane' structure as Figure 3, this model has been up-reversed to avoid including the membrane; the wave comes from the front chamber and reach the backplate first. To simplify the model, the membrane at the top has been defined as an wall with impedance.

The pressure gradient as in Figure 6. In the pressure acoustic physics, only the Helmholtz wave equation was applied, hence the pressure has been treated as a independent variable. The pressure perturbation is only affected by wave transform. At the MEMS microphone size, the length of the whole model is approximately 450μm, yet the wavelength equals to 11433μm even up to 30kHz. Based on the long-wave approximation, we can assume the pressure will perform uniformly in the whole structure. This assumption is obviously applied for all the following experiments.

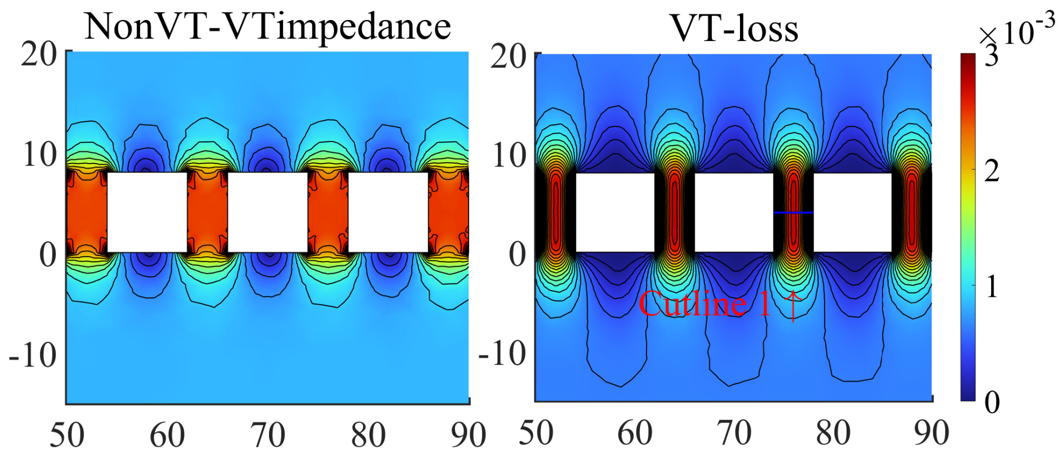

Figure 4: The Comparison on Velocity profile, the additional VT impedance located between the holes, have pretty close prediction to the exact VT loss. The VT impedance can be applied with only velocity to reduce the calculation time.

Despite the addition of boundary layer impedance at the boundary, the pressure is not affected by the impedance. However, since the velocity profile is the response by the pressure acting onto the impedance, a velocity diversity is clear as in Figure 4.

About the VT physics, the result shows a good agreement with the theories; the pressure and velocity gradient in Figure 6 demonstrate the differ happened along the cut-line. Similar to the theory, the velocity shows a typical viscous-dominated waveform. The drop of pressure gradient also agrees with theories as the absolute pressure amplitude should decrease when passing through the porous structure.

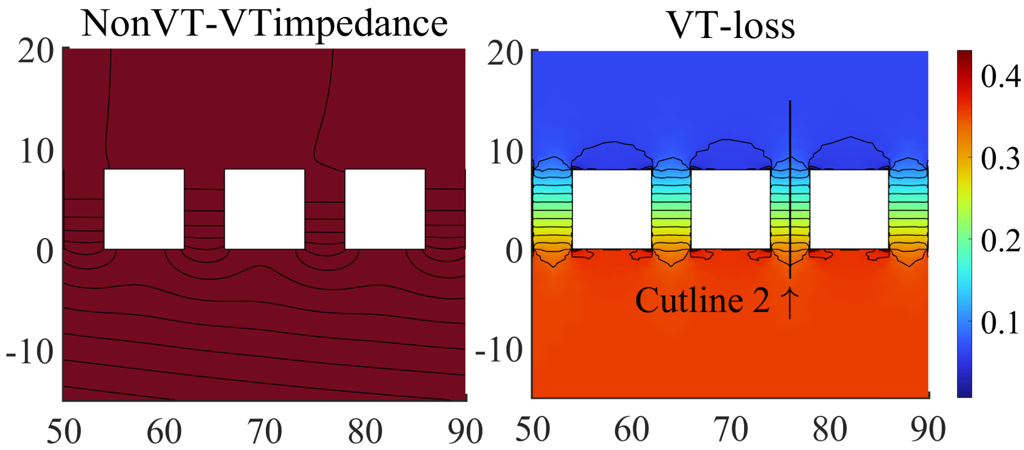

Figure 5: Comparison between NonVT and VT on pressure profile, the tradtional pressure acoustic adding an additional impedance to simulate the VT loss, but unable to generate significant difference to the pressure.

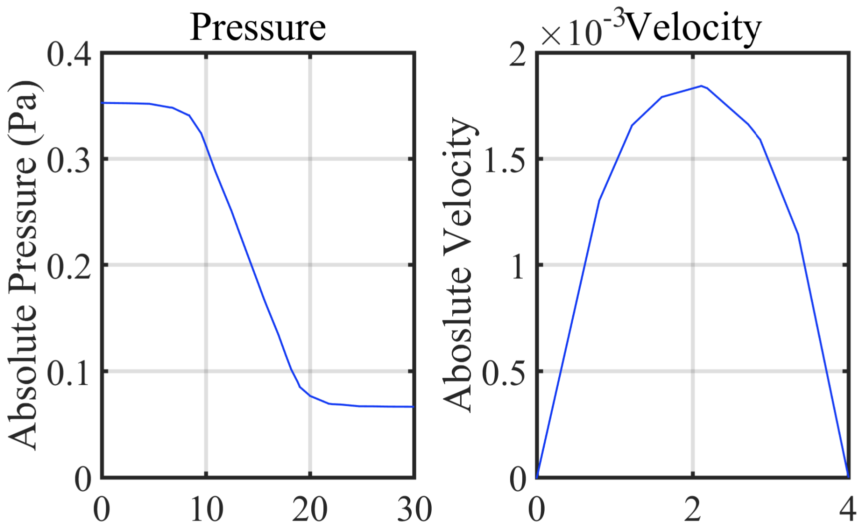

Figure 6: Result of Cutline in Figure 5 and Figure 4. The left plot shows the pressure drop and equivalent length; the right plot shows the vertical velocity profile between two boundaries; the velocity equals zero with contact with the boundary. Such shape of velocity profile represents the small Reynolds number and the viscous force dominance. [8]

3 2D axisymmetric Experiment

3.1 corner refinement

As the demonstrate before, the 2D model- as a simple practice -match to the theory. The proprieties of PA and TA can also been judged by pressure and velocity profile. But the removed membrane is not mimetic enough to reality. This model's motivation is to give the membrane result with PA and TA individually, and gives comparable result for following 3D model to validate.

When following the general modeling steps as mentioned before, the singularity effect has been observed. The singularity point is a common error point in FEM method, as the value near to the corner will abnormally high. With refine the mesh near to the corner, the singularity can be solved as Figure 7.

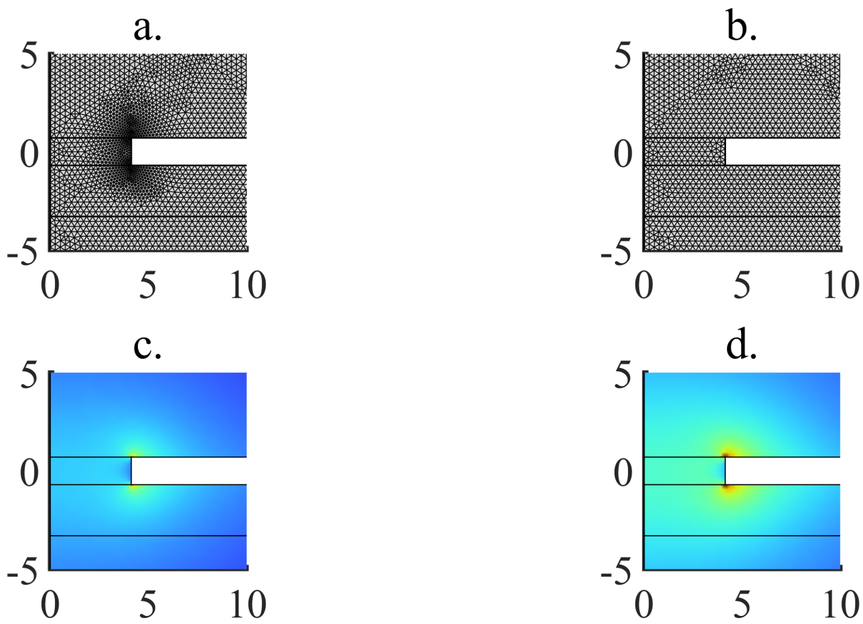

Figure 7: Corner effect to the singularity result; figure (a)&(c): with corner refinement, the singularity effect is insignificant; figure(b)&(d). Without defining the corner refinement, the singularity effect is significant and could affect the result. (Frequency = 100Hz). The figure (c)(d) plot in the same color range.

The setting of mesh size is determined by wave length λ and boundary layer thickness s. At least 6 mesh point should been included inside one wave length as well as the boundary layer thickness to maintain simulation accuracy. The maximum frequency is 30kHz, hence the size of mesh should be

The membrane deformation as Figure 8, it's clear that the membrane were more damped with full set of VT loss. Since the membrane deformation can also been consideGreen as the reaction to pressure, the decrease of pressure in full VT set as Figure 5 can affect

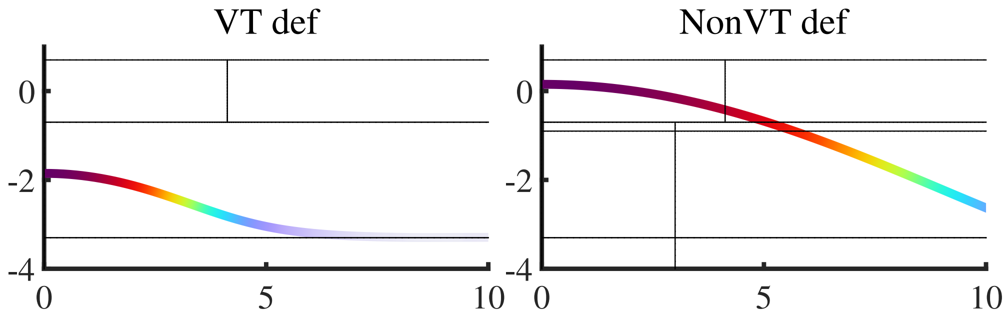

Figure 8: Membrane displacement amplitude. Pressure acoustic were applied except a $4\mu m$ VT physical layer coveGreen the membrane. The deformation in the figure is enlarged with scale factor = 1500.

the deformation amplitude. However, since the pressure acoustic physics not including the pressure drop, the deformation of membrane only depend on the stiffness of membrane itself.

The pressure acoustic physics and full set of VT loss show a huge difference in membrane deformation, then it's worthy to investigate if it's accurate enough to apply the boundary layer impedance in pressure acoustic. An additional experiment has been designed to validate as below.

3.2 Feasibility of Pressure acoustic with VT loss impedance

As performed in the first experiment, we used two methods in the simulation- the PA, in short of pressure acoustic physics with VT impedance, and the full set of VT loss physics. In order to demonstrate the differences of these two methods, three validation experiments has been done with structure as Figure 9. The vertical divided PA/TA experiment, the horizontally divided PA/TA experiment, and the full TA experiment as a reference. The two validation experiments has been proactively divided into 2 domains. One model is divided in the middle as the left figure, which designed to demonstrate the irGreenucible boundary layer effects in pressure acoustics. The middle validation experiment demonstrates the normally combine physics set, which has been believed to Greenuce computing time without too much sacrifice of accuracy. And the right model is the full set of TA physics.

The Figure 9 shows a huge difference in PA and TA result to the membrane. The possible reason might be that the assumption of pressure dominant is incorrect - the wave equation assumes the pressure dominates the wave transfer - the comsol multiphysics feeds the to membrane without pressure loss. Adding TA in the middle figure can temporally solve such a problem. Still, since the boundary layer at low frequency is thicker than the additional VT layer thickness, the result shows the lack of damping.

In a high-frequency situationFigure 10, the amplitude of membrane vibration becomes entirely close no matter which we chose dividing method. Since the thermal viscous boundary thickness at 3000Hz is approximately 10-20μm, the wave becomes pressure dominated at the center of membrane, which the classical wave equation is good enough to simulate the membrane movement.

However, if we compare the full VT physics set with the others, the the VT physics gives

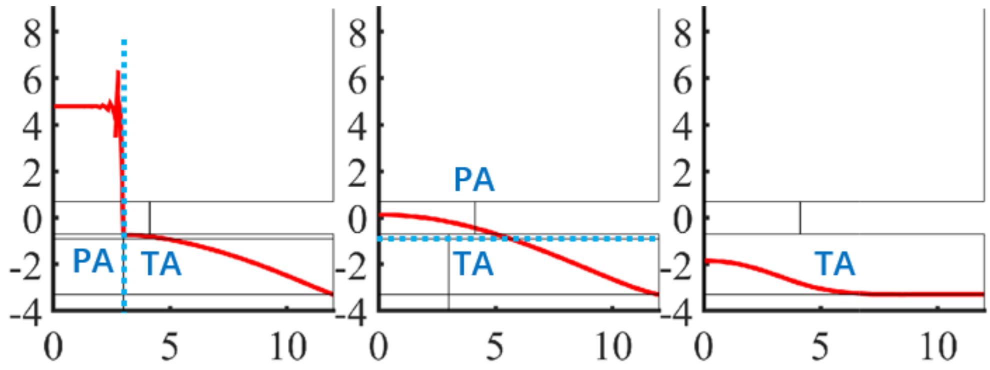

Figure 9: Deformation of the membrane at 100Hz. Left pic: Membrane displacement has instant increasing due to jumping from TA physics to PA physics. Middle pic: Combined PA and TA, the membrane deformation shape shows gradient but is inaccurate. Right pic: Full TA set, as the reference of membrane deformation.

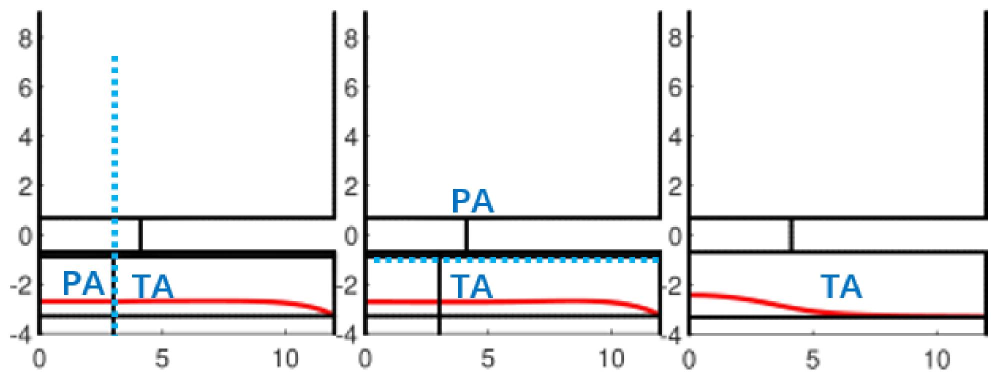

Figure 10: Deformation of the membrane at 3000Hz. The order of the subgraphs is as described previously.

a different membrane deformation shape -which is more concentrated at middle -Though the absolute membrane vibration amplitude is pretty close. It's probably due to the effect we observed from previous 2D experiment as Figure 4, the effective length of the slit is longer, under the full VT loss physics.

In the conclusion of this chapter, considering the accuracy of simulation, though the PA&TA combined method is more computationally efficient, the following model will only use the TA physic for a more reliable result.

3.3 Frequency response

Combine all the aforementioned model setting together, the frequency response of membrane deformation curve can be derived, as the blue curve in Figure 11. The frequency range were derived begin with 10Hz up to 1000kHz. Typically MEMS microphones do not operate in the range of up to 1000 kHz, such choice of range only for observing the resonance at 600kHz.

The Green vertical dashed line with denoted number in the figure represents the boundary layer thickness at the corresponding frequency. The blue line represents the total thermo-viscous power dissipation density, corresponding with the secondary y-axis which in the logarithmic scale.

The total VT power dissipation density before 30kHz is increasing exponentially, which is similar with the decreasing trending of boundary layer thickness as Figure 1. After the 30kHz, the boundary layer thickness becomes less then $4\mu m$, the VT power dissipation reach to the peak and begin the decreasing until the first resonance frequency around 600kHz, afterwards the decreasing continued since the boundary layer thickness is already low enough, and the wave propagation becomes dominated by pressure. The increasing and decreasing trending both following the boundary layer gradient, before the gradient full established

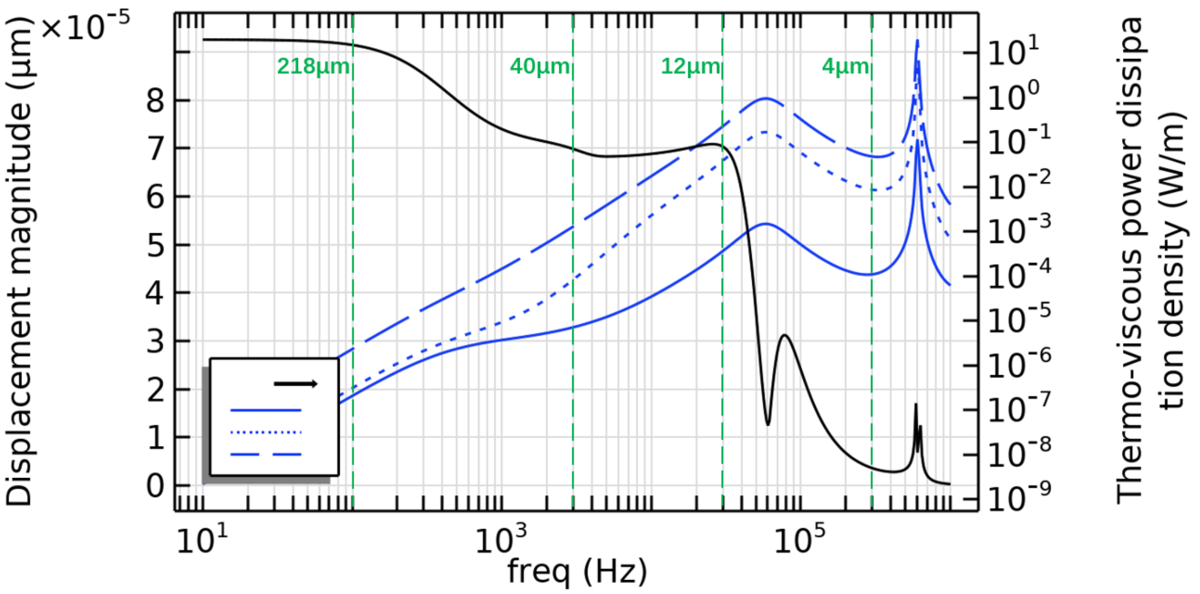

Figure 11: The average membrane deformation response to the frequency. The blue curve: membrane thermo-viscous power dissipation density, where the slit loss is the dashed line, the air gap boundary loss is the doted line, and the total loss is the the solid line; The black curve: membrane displacement response curve. Noted that the secondary y-axis is logarithmic scale

Comparing the general dimension of the subject MEMS microphone with the boundary layer thickness, as shows in the Table 2, The thickness of the boundary layer corresponding to the critical frequencies mentioned above is either barely equal to the length in general or directly equal to the radius. It's obvious that when the thickness of the boundary layer is gradually close to these critical dimensions, a significant change in the trend of the membrane displacement curve is produced. We can understand the frequency by using the lumped circuit model. All the elements of the model at low frequency acts like damper, with the decreasing of boundary layer, they slightly transforms from pure damper to the acoustics spring or acoustic mass. The acoustic spring property in front chamber, back chamber and the air gap will gives a phase shift to the sound pressure, opposite with the phase shift from the acoustic mass of the slit. The further discussion will take place in the section 5.

When frequency is lower then 100Hz, the boundary layer thickness is higher then the largest length of the model, the entire model can be assumed be dominated by damper. As Figure 12, the velocity profile at 100Hz is viscous dominated and in the same phase. When the frequency from 100Hz to 3000Hz, few volume at front and back chamber were more dominated by the compression of air, which gives a phase shift of front chamber and membrane. From 3000Hz to 30kHz, the pressure at air gap becomes more dominating than viscousity, where the more compression behavior at airgap gives more phase shift between the air gap and the slit, with which the phase shift reaches to 90 degree, a deep at 60kHz were generated in Figure 11. After 30kHz, the pressure becomes more dominatin at slit, which makes it perform like a air mass, with more then 90 degree phase shift then the airgap.

Before 100Hz, when the boundary layer thickness significantly higher than 218μm, the displacement magnitude will not affected by frequency change. From 100Hz to 3000Hz, though the boundary layer still enough to cover the whole membrane, But they are not thick enough to cover the entire model, hence membrane displacement decreasing.

Frequency	Boundary layer thickness(μm)	Corresponding Subject	Diameter(μm)
100Hz	218	Longitudinal length	450
3000Hz	40	Front chamber length	120
30kHz	12	Front chamber radius	12
300kHz	4	Slit radius	4

Table 2: Frequency-dependent boundary layer and model diameter comparison table

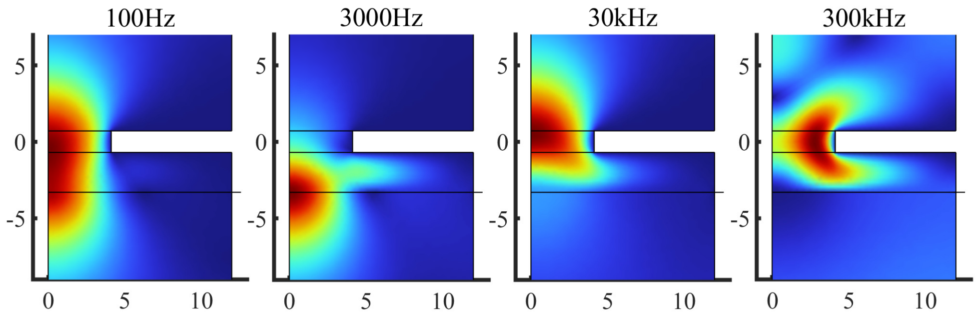

Figure 12: The Instantaneous acoustic velocity profile with 100Hz(218μm), 3000Hz(40μm), 30kHz(12μm), and 300kHz(4μm). The

4 3D Experiment

4.1 Vacuum model for membrane.

Before moving to the 3D model, it's worthy to investigate more on the membrane resonance properties. Since the size of the membrane is already large enough, and it is very thin, we need to know its resonant frequency to be sure that the following results will not be disturbed by the resonance of the membrane. In real world experiment, one measurement is vibrating the subject MEMS microphone inside the vacuum to avoid the interference from air[7]. Such method becomes easier to applied in Comsol world, as we only need to remove the air domain and gives the membrane an stimuli. The calculation only focus on the eigenfrequency of the membrane, hence then a Comsol build-in eigenfrequency solver can be used to reduce the computation time. The model is based on an application example on Comsol library, with few dimension and material properties changes.

The first 3 eigenfrequency as in Table 3. For a better demonstration, the shape membrane at third model as in Figure 13. Since the first mode is already up to 180kHz high. It's clear that the eigenfrequency of membrane will not affect the membrane displacement response at normal frequency range. The following experiment about the membrane displacement will only considering the membrane response to sound wave.

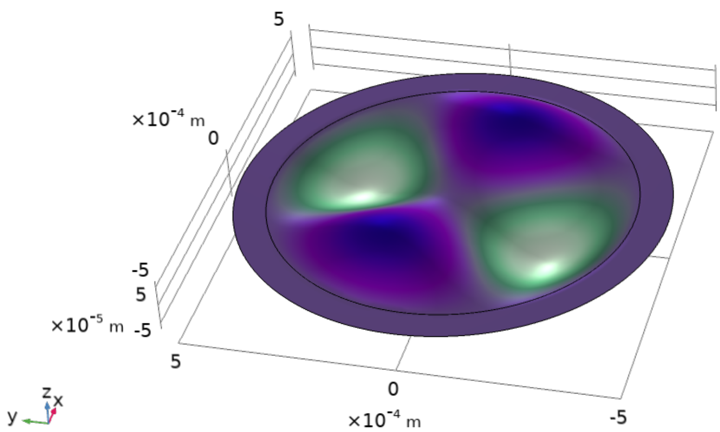

Figure 13: The mode shape of membrane at 399kHz

Eigenfrequency (Hz)	Mode
1.8697E5	1
2.9793E5	2
3.9939E5	3

Table 3: Eigenfrequency of the individual membrane, calculated by Comsol eigenfrequency solver, membrane dimension and properties follow the Table 1, which diameter = $850\mu m$, thickness = $4\mu m$

4.2 3-D model and mesh

As described before, the 3D model builds with full VT physics. For the sake of computing time, Only one twelfth of the whole structure were modeled. The general model structure, like a piece of cheese, as shown in Figure 14. The cut side of the cheese set to symmetry, the rest of the outer boundary set to wall. About the inside surface, the pressure applied to the lowest surface, where the deformation surface at Figure 14 represents the membrane, which is deforming under 65dB 1000Hz sound. The porous backplate over the membrane and the slits arrangement leads to this specific membrane deformation. The overall size of the structure follows theTable 1. The mesh also shown as in Figure 14, the mesh size in the front and back chamber set as more coarse, as the sound wave behavior at these area is simple until frequency reach around 30kHz. At 30kHz, the 12μm boundary layer thickness become lower then the maximum element size 19μm, which will cause the velocity gradient can not be well represented. Hence the boundary layer mesh were introduced to avoid error. Also as figure shows, the mesh is more refined in the center then the rest of model since the air behavior is more complicate and the membrane is normally deformed like a conical with diameter 8.25μm. The degree of freedom = 10487853 for such mesh set, approximately requires 65GB memory in total [9]. Hence the author's laptop is unable to running this simulation. The job were uploaded onto DTU HPC, where 64 cores and 160GB memory (I must confess that it was a bit overshot) were required for calculation. The Linux job script as shown in Appendix B, the total running time around 9 hours.

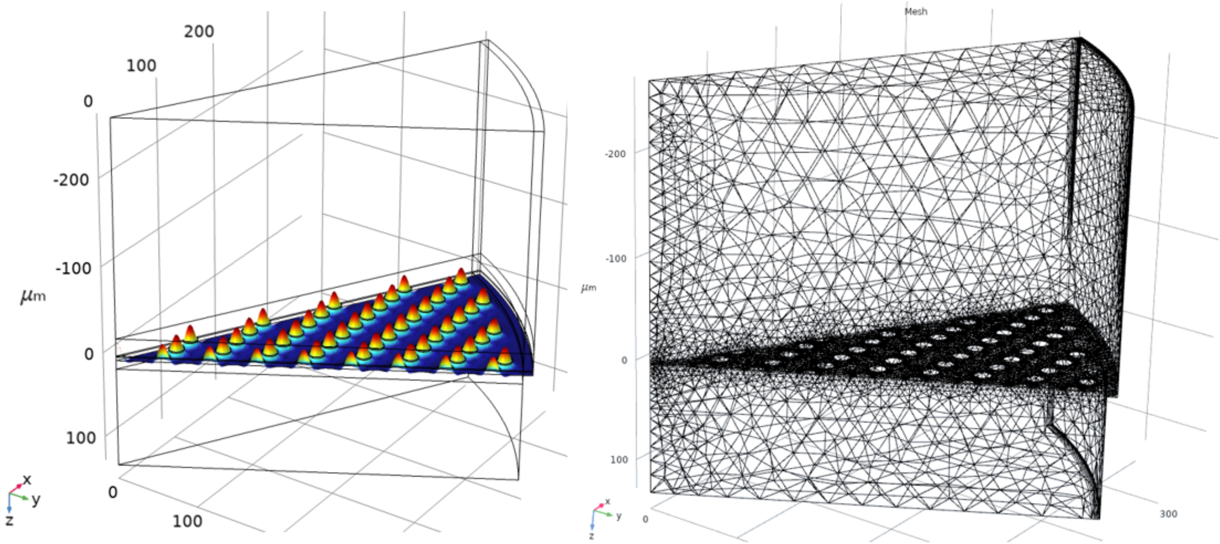

Figure 14: The membrane deformation magnitude and mesh. About the membrane deformation: Frequency = 1000Hz; Deformation scale factor = 1.5e7; color data from deformation magnitude. The model applied the pressure of 65dB at the lowest surface. About the mesh: element number = 119807, minimum element size = 0.55μm, maximum element size = 19μm (only at front and back chamber), boundary layer number = 12, thickness of first layer = 0.2μm,

The velocity profile around membrane at corresponding frequency as Figure 16 are basically the same as the 2D experiments. It's worth to note that at the profile at the central point (where x = 0) is different compare to others.

The shape of membrane deformation shows a good agreement compared to the previous 2D axial symmetric experiment, but the magnitude of membrane displacement at low frequency is not matched well. Below 100Hz, the membrane deformation is up to 16e-5 μm with a rapid decreasing trending, which is twice to the previous result. Through the thermo-viscous power dissipation response is pretty close to the boundary

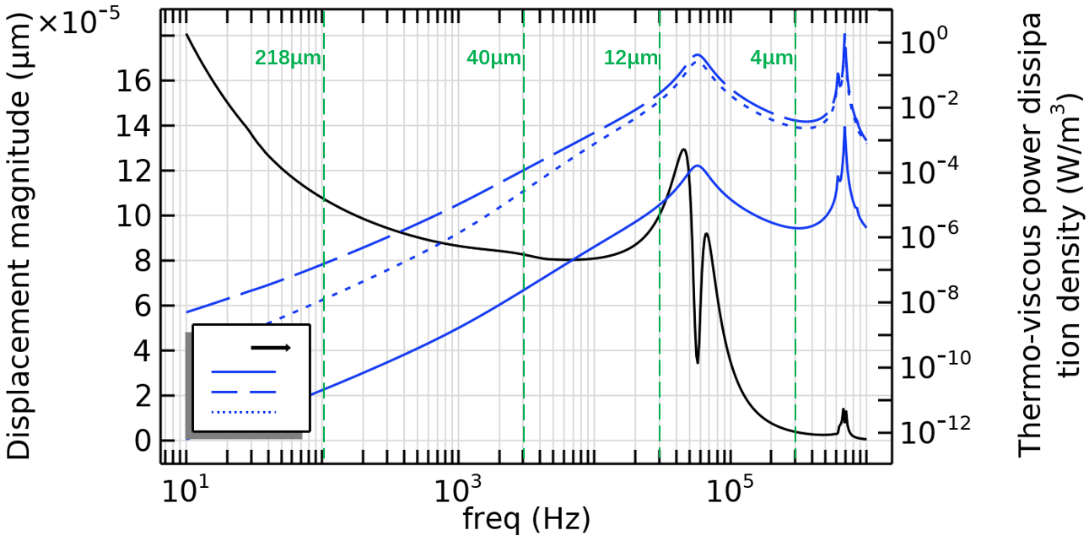

Figure 15: The average membrane deformation response to the frequency -3D large model

Same as what we observed in 2D axial symmetry experiment, the peak and deep also caused by the boundary layer thickness decreasing. As in Figure 15 and Table 2, the boundary layer thickness match to the model geometry diameter in terms of membrane deformation response. Such a property allows us to correlate the deformation response of the membrane with the geometry of the model.

5 Lumped model

Based on the observed result in section 3 and section 4, the lumped model structure could be roughly deduced. As a introduction, the lumped element simplifies the description of the behaviour of spatially distributed physical systems in to discrete entities that approximate the behaviour of the distributed system under certain assumptions [10]. In a short word, the lumped element model can gives an analogy of the MEMS model to electric circuit with compacting it's essential features.

As the most basic scenario, where we apply the pressure at beginning of an open air section, read the response at the end of the section. At such scenario, a simple lumped element analogies can applied to translate the acoustic system to electric or mechanic system.

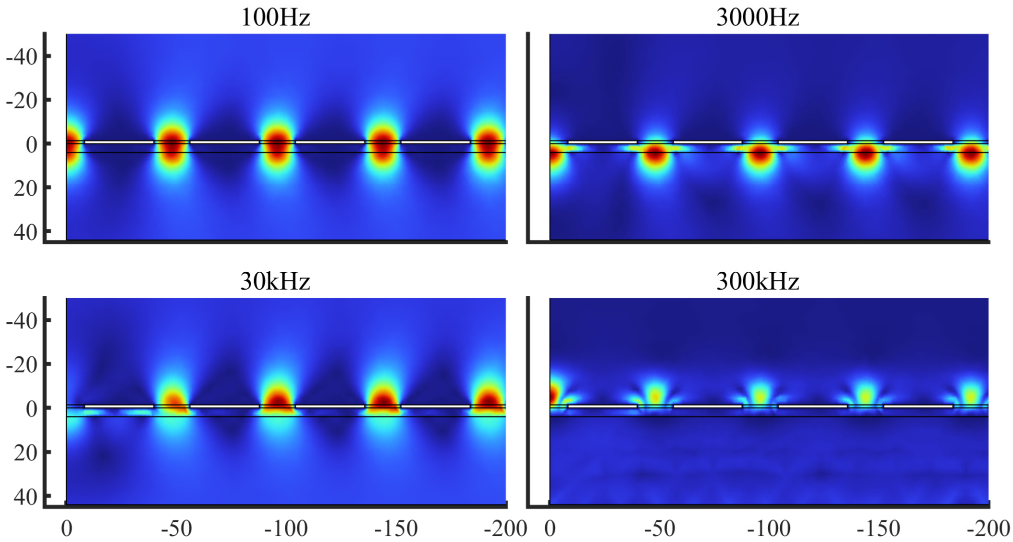

Figure 16: The velocity profile of 3D model. Compare to the 2D axial symmetry model, the profile shows the similar shape.

From acoustic to electric circuit, it is common practice to analogise pressure to voltage, and since the acoustic velocity is the response of the pressure applied, it can be analogise to current. At electric circuit, there are three elements to excite the current response by voltage: inductor, resistor, and capacitor. The inductor leads to a positive phase since current lags the voltage in an inductive circuit; The phase is negative for a capacitive circuit since the current leads the voltage. The analogy from acoustic to electric can be set: apply a pressure with certain frequency onto a acoustic volume. At low frequency the wave length is much larger then the volume, pressure can be seen as constant along the whole volume. hence the volume velocity responses obey this intrinsic inertial force, leading a positive phase like a indutor; When the frequency is high enough to make the wave length small to the volume, the volume velocity responses to the pressure with the compression force, with a phase lag in the output end like a capacitor. And the resistor used to represent the thermoviscous loss, since their both cause the energy transform to heat without phase lagging. An acoustic analogy to machinery would make things easier to understand, at the high frequency the volume is compressed by pressure like a spring; at the low frequency situation as the volume is resist the pressure by inertial force like a mass. And the damper used to represent the energy loss.

It's worth to note that the element property is depend on frequency, hence their characteristic will change with the frequency. For example, a section of air may dominated by thermalviscous loss at low frequency like a damper, but act like a acoustic spring at high frequency where the thermalviscous loss could be ignored. More generally say, the acoustic element characteristic is transformed from mass to spring, high damping to low damping when frequency increases. As we can expect, two values are important in this change of characteristic, they are the wavelength and the boundary layer thickness. A more detailed description of each part of the 3D model is as follows.

- **Front and back chamber** The front chamber acts like a spring-damper element at low frequency, and its damping effect decreases with increasing frequency. The critical frequency around 25Hz, where the boundary layer thickness is lower then the front chamber radius.

- **Air gap** The air gap also act like a spring-damper element, dominated by damping until the boundary layer thickness is reduced to 290kHz equal the length of the air gap, the damping effect begin attenuating.

- **Back plate slits** The slits at the back plate act like a mass-damper element, the damping effect dominated until 280kHz, the boundary layer thickness is lower than the slit radius.

6 Optimization

The optimization section is not meant to find the best performance geometry, but more like a study for elements parameter's influence to the frequency response. The air gap thickness, back plate thickness, and back plate slits radius will be changed with specified combinations as Table 4.

Element name	Parameter	Value	Combinations
Slit	radius, r_s	[1,5,9]	with t_a and t_b
Air gap	thickness, t_a	[10,20]	with r_s
backplate	thickness, t_b	[10,20]	with r_s

Table 4: The sweep parameter setting.

The parameter sweep result as shown in Figure 17 and Figure 18. After classifying the cluttered curves, we can obtain the following three types of response modes:

- **Damping response** when the slit radius = 1 μm, the membrane displacement will significantly reduced. As the back plate with such small slit makes the air gap more like a stiff damper, where the front chamber as a spring is difficult to establish an effective pressure difference between the two sides of the membrane. Hence the membrane response to the pressure only shows a decrease trending along with the frequency. The spring-damper system will be less dominated by damping if we increasing the air gap thickness, as in Figure 18.

- **Effect of back plate thickness** Compare the Figure 17 and Figure 18, we can observed that the decreasing of the back plate thickness will increasing the pseudo resonance frequency whatever the value of other parameter. Assuming the effect were caused by Helmholtz resonator is not valid, since the predicted resonance frequency at such geometry set is up to 192kHz with back plate thickness = 20 μm, the other parameter set will cause the prediction even higher. It is possible that the rationale behind this phenomenon can be explained with the help of lumped model and the

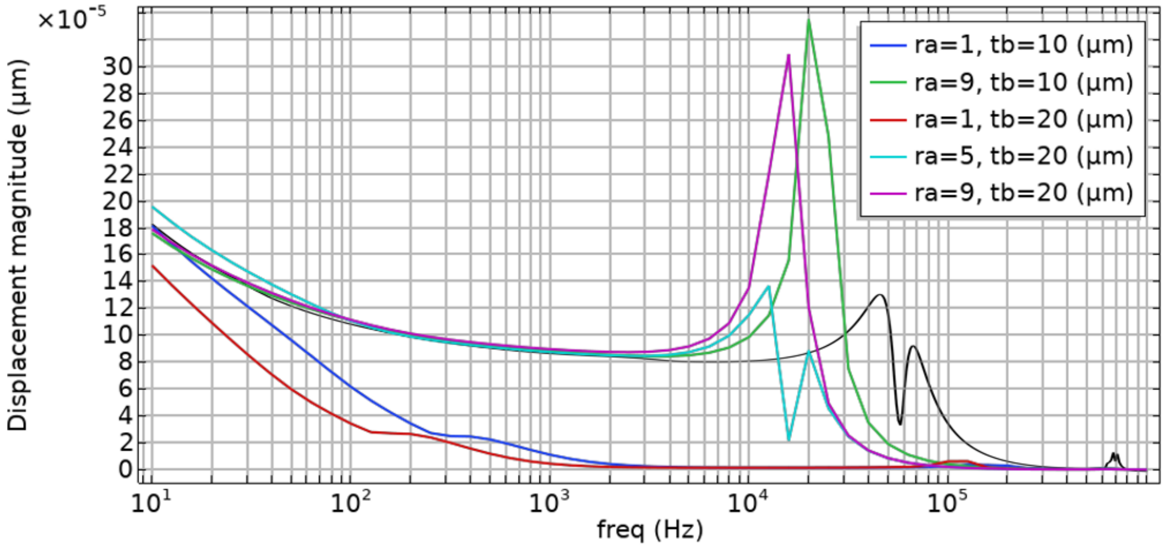

Figure 17: The frequency response of membrane displacement, parameter combination of slit radius and back plate thickness. where the color curve is the optimization curve with denoted combination; the black curve is the original membrane frequency response.

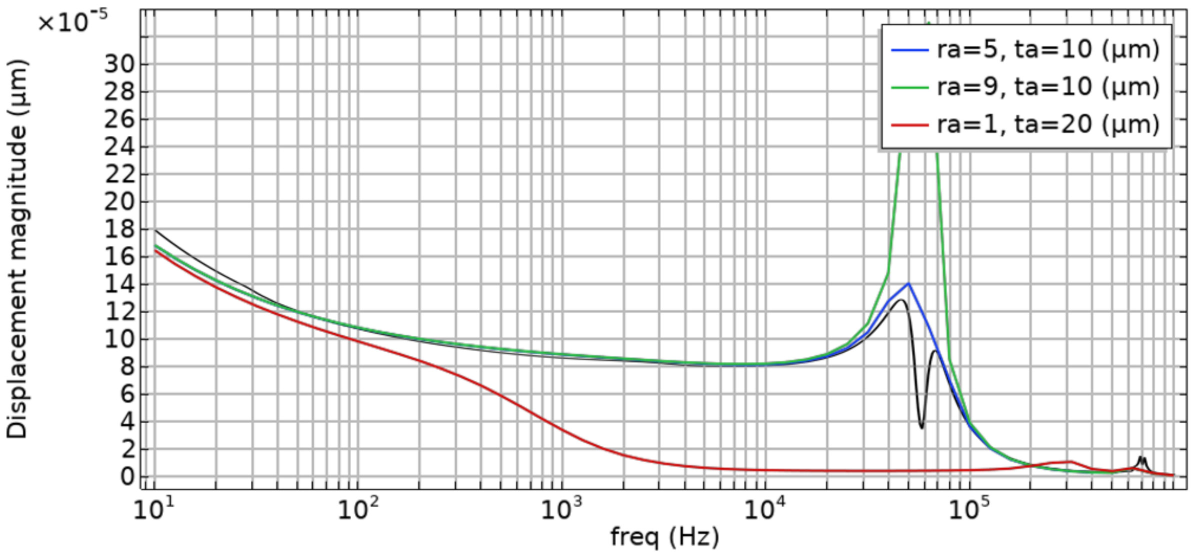

Figure 18: The frequency response of membrane displacement, parameter combination of slit radius and air gap thickness. where the color curve is the optimization curve with denoted combination; the black curve is the original membrane frequency response. The parameter combination ra=1,ta=10 and ra=5/9,ta=20 gives the exactly same result so the repetition has been omitted.

translation of element characteristic in previous chapter, but this report is already too long to discuss more detail about such hypothesis. The only conclusion could be draw here is the back plate thickness will affect the peek point of membrane displacement.

- **Effective length of slit** An other observation can be made from Figure 18. At the same slit radius and back plate thickness, increasing the air gap thickness from $1.4\mu m$ to $10\mu m$ will remove the downward peek of the curve. Without going into more proof and discussion, it can be hypothesized that the slit mass hit the membrane if the air gap thickness is not longer than the slit effective length. At certain frequency, the membrane vibration is in the anti-phase of the slit mass, the center of membrane will be "hit" back and reducing the average displacement magnitude. Such pattern can be seen in the COMSOL simulation result but will excluded in this report to avoid more discussion.

7 Conclusion

This report demonstrate and try to explain 3 experiments conducted in COMSOL, Although not given a significant contribution to academic research in this area -clearly no one expect that- I have still gained a great amount of theoretical and practical knowledge in this special course study. I gradually learnt how to draw the conclusion by combine the result and theory, instead of only operate the software. As described in this report, some unresolved problems still left and wait for further study. Hence, the potential future research direction could be:

- Get clear and try to proving the hypothesis of the element effect.
- Give a more detailed optimization study and try to find the optimal geometry.
- Construct the lumped element model and two-port model, try to abstract and explain the result.

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8 Appendix A

Livelink to Matlab

The Livelink from Comsol to Matlab gives a convenient way to export results and modify the fontsize. Livelink to Matlab generates most figures in this report. A simplified script is shown below.

```
%% Presetting
BeginMatlabComsol; % Auto preset for Livelink environment, see appendix A for script.

mphlaunch; % Launch the server.

mphopen; % Open and load the script onto server

defaultFrontsize; % Modify the default matlab frontsize setting, all figures plot will zoom to same linewidth as latex file and match the frontsize.


%% Plot figure
figure(1);

subplot(1,2,1);
mphplot(model,'pg13'); % mphplot returns the specific figure 'pg13' in server, generated by previous steps.
title('Pressure')
ylabel('Absolute Pressure (Pa)')
subplot(1,2,2)
Goodmphplot(model,'pg12',lline) % The Goodmphplot gives an additional input parameter to modify the linewidth of 2D plot, see appendix B for script.
title('Velocity')
ylabel('Aboslute Velocity')

saveas(1, ['fig' num2str(1)],'epsc') % Auto save the figure to .eps file.

% Note: add all figures onto overleaf only with 1\textwidth.
```

```
%% Lunch Server
currentdir = pwd;
cd('C:\Program Files\COMSOL\COMSOL60\Multiphysics\bin\win64');
system('comsolmphserver.exe &');
currentdir;

%% Bulid connection
```

```

currentdir = pwd;
cd('C:\Program Files\COMSOL\COMSOL60\Multiphysics\mli');
mphstart(2036);
currentdir;
```

9 Appendix B

Job script for HPC

```
#!/bin/sh
#BSUB -J Com_wrench_par_shared
#BSUB -W  40:00
#BSUB -R "rusage[mem=40GB]"
#BUBS -M 40GB
#BSUB -R "select[model == XeonGold6226R]&&[mem>450GB]"
#BSUB -n 4
#BSUB -R "affinity[core(16)]"
#BSUB -R "span[ptile=1]"

#BSUB -o Output_%J.out
#BSUB -e Error_%J.err

module load comsol
unset JAVA_TOOL_OPTIONS
export I_MPI_HYDRA_BOOTSTRAP=lsf

comsol batch -nn $LSB_DJOB_NUMPROC -np $OMP_NUM_THREADS -inputfile ~/comsol
    /3MEMS_comsol56.mph -outputfile ~/public_html/3MEMS_comsol56_out_geo.mph
   -tmpdir $HOME/comsoltmp
```