NOTES FOR LINEAR ALGEBRA 341, SPRING 2025

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1. LOGIC

Logic is the study of formal reasoning. A statement in logic always has a well-defined meaning.

1) Propositions and Logical Operations

Applications of Logic

- In Mathematics, logic is used to prove theorems.
- In Computer Science, logic is used in areas such as AI and in designing digital circuits.
- In Medicine, logic precisely specifies the conditions under which a particular diagnosis applies.

Elements of Logic

PROPOSITIONS

A **proposition** is a statement that must be either true or false (truth value).

Examples:

- *"There are an infinite number of prime numbers."* (Truth value: True)
- *"17 is an even number."* (Truth value: False)

PROPOSITION VARIABLES

Variables like p, q, and r can be used to denote propositions.

EXAMPLES:

- p: January has 31 days.
- q: February has 33 days.

Logical Operations

COMPOUND PROPOSITION

It will only be true if both p and q are true.
Operator: $\wedge$ (and)
Example: $p \wedge q$ (January has 31 days and February has 33 days)
Truth Value: False

DISJUNCTION PROPOSITION It will be true if either p and q are true or both true.
Operator: $\vee$ (or)

Example: $p \vee q$ (January has 31 days or February has 33 days)
Truth Value: True

Exclusive Or (XOR) **Operator:** $\oplus$

Example: $p \oplus q$ (January has 31 days or February has 33 days, but not both)
Truth Value: True

Negation **Operator:** $\neg$

Example: $\neg q$ (February has 33 days)
Truth Value: True