

NOTES FOR CS/MATH 240, SPRING 2025

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1 Propositions and Logical Operations

Logic is the study of formal reasoning. A statement in logic always has a well-defined meaning.

1.1 Applications of Logic

- In Mathematics, logic is used to prove theorems.
- In Computer Science, logic is used in areas such as AI and in designing digital circuits.
- In Medicine, logic precisely specifies the conditions under which a particular diagnosis applies.

1.2 Elements of Logic

1.2.1 Propositions

A **proposition** is a statement that must be either true or false (truth value).

Examples:

- “*There are an infinite number of prime numbers.*” (Truth value: True)
- “*17 is an even number.*” (Truth value: False)

1.2.2 Proposition Variables

Variables like p , q , and r can be used to denote propositions.

Examples:

- p : January has 31 days.
- q : February has 33 days.

1.3 Logical Operations

1.3.1 Compound Proposition

It will only be true if both p and q are true.

Operator: \wedge (and)

Example: $p \wedge q$ (January has 31 days and February has 33 days)

Truth Value: False

1.3.2 Disjunction Proposition

Operator: \vee (or)

Example: $p \vee q$ (January has 31 days or February has 33 days)

Truth Value: True

1.3.3 Exclusive Or (XOR)

Operator: \oplus

Example: $p \oplus q$ (January has 31 days or February has 33 days, but not both)

Truth Value: True

1.3.4 Negation

Operator: \neg

Example: $\neg q$ (February has 33 days)

Truth Value: True