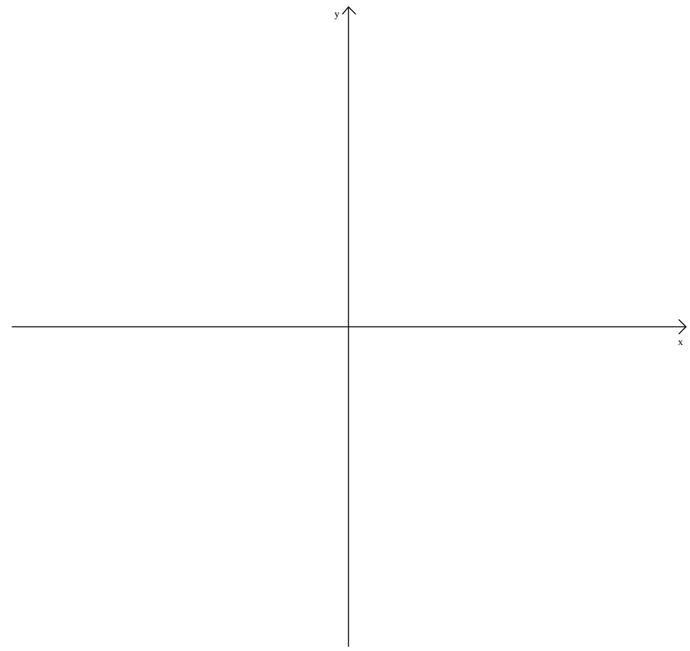
Magnet Analysis 1A
Rose

Name________________________

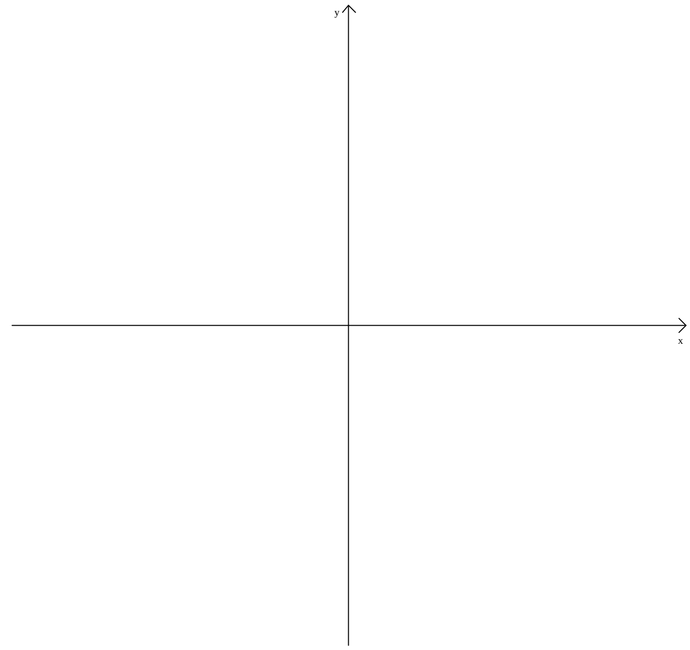
Applications of Differentiation Packet 2014

1. Complete the chart below and use it to make a sketch of the function $f(x) = x \ln x$



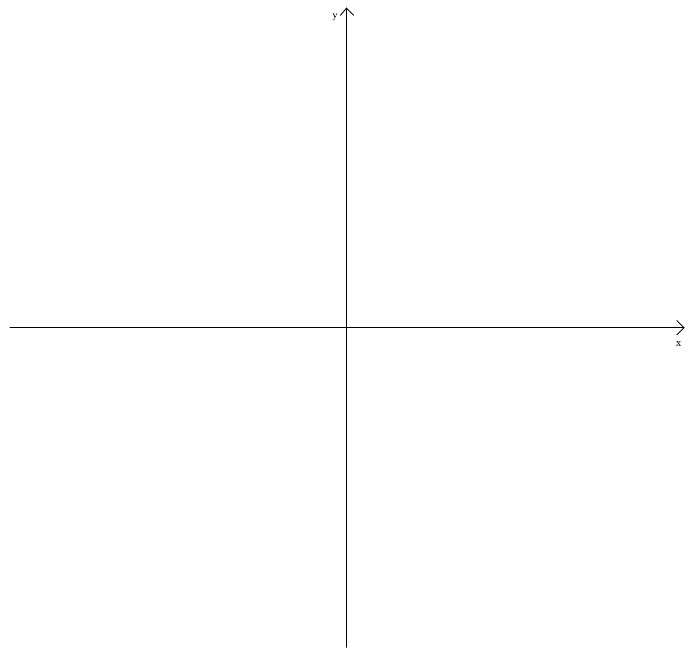
intervals where *f* is increasing	intervals where *f* is decreasing	*x*-coordinate of relative maxima
x-coordinate of relative minima	intervals where *f* is concave up	intervals where *f* is concave down
x-coordinate of points of inflection	$\lim_{x \to 0^+} f(x) =$ $\lim_{x \to \infty} f(x) =$	

2. Complete the chart below and make a sketch of the function $f(x) = 2\cos x - \cos 2x$ on $[-2\pi, 2\pi]$. [Note: don't bother finding concave up and concave down intervals or points of inflection for this problem.]



intervals where *f* is increasing	intervals where *f* is decreasing	*x*-coordinate of relative maxima
x-coordinate of relative minima		

3. Complete the chart below and use it to make a sketch of the function $f\left(x\right)=x^2e^x$



intervals where *f* is increasing	intervals where *f* is decreasing	*x*-coordinate of relative maxima
x-coordinate of relative minima	intervals where *f* is concave up	intervals where *f* is concave down
x-coordinate of points of inflection	$\lim\limits_{x\to\infty} f\left(x\right)=$ $\lim\limits_{x\to-\infty} f\left(x\right)=$	

4. A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can. In other words, find the dimensions that will produce a minimal surface area for a cylinder of fixed volume 1 L. Warning: be careful with unit. 1 L = $1000 cm^3$.

5. A right circular cylinder is inscribed in a sphere of radius *R*. Find the largest possible volume of such a cylinder.

6. A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach a point B, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row 6 km/h and run 8 km/hr, where should he land to reach B as soon as possible? (Don't forget to consider endpoints.)

7. A cone-shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges. See the figure below. (Note: R is a constant.) Let r and h be the radius and height, respectively, of the cone. Express r and h in terms of θ and R. Find the height h which maximizes the volume of the cup and find the measure (in radians) of the angle to be cut out. Then find the maximum capacity of such a cup. (Hint: let θ be the angle of the sector that remains as shown below.)

8. A particle moves along a line so that its position at any time t is given by the function $s(t) = t^2 - 3t + 2$, where s is measured in meters and t is measured in seconds. Consider values of t on $(-\infty, \infty)$.

a. Find the position at time 5 seconds.

b. Find the average velocity of the particle from $t = 0$ to $t = 5$.

c. Find the instantaneous velocity of the particle when $t = 4$.

d. Find the acceleration of the particle when $t = 4$.

e. At what values of t does the particle change direction?

f. Where is the particle when s is a minimum?

g. When is the particle moving toward the origin?

h. When is the particle moving away from the origin?

9. A particle moves along a line so that its position at any time t is given by the function $s(t) = t^3 - 6t^2 + 9t - 5$, where s is measured in meters and t is measured in seconds.

a. Find the position of the particle at time 5 seconds.

b. Find the average velocity of the particle from $t = 0$ to $t = 5$.

c. Find the instantaneous velocity of the particle when $t = 4$.

d. At what values of t does the particle change direction?

e. When is the particle moving left?

f. When is the particle moving right?

g. When is the particle speeding up?

h. When is the particle slowing down?

10. A particle moves along a line so that its position at any time t is given by the function $s(t) = (t-2)^2(t-4)$, where s is measured in meters and t is measured in seconds.

a. Find the instantaneous velocity at any time t.

b. At what values of t does the particle change direction?

c. When is the particle moving left?

d. When is the particle moving right?

e. When is the particle moving toward the origin?

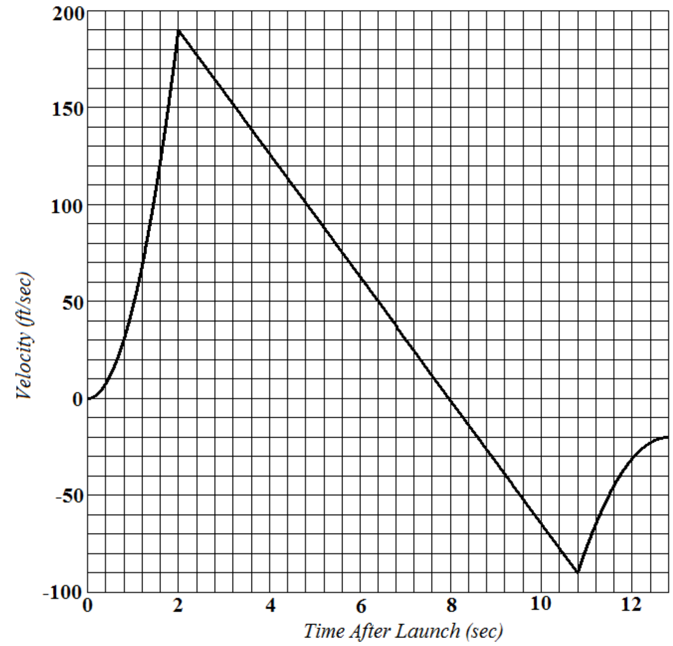
f. Find the acceleration at any time t.

g. When is the particle speeding up?

h. When is the particle slowing down?

11. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket coasts upward for a while and then begins to fall. A small explosive charge pops out a parachute shortly after the rocket starts downward. The parachute slows the rocket to keep it from breaking when it lands. The graph below shows velocity data from the flight.

Write a paragraph explaining the shape of the graph in terms of the situation described above. Note the sign, slope, and concavity of the velocity in your explanation.



a. How fast was the rocket climbing when the engine stopped? For how many seconds did the engine burn?

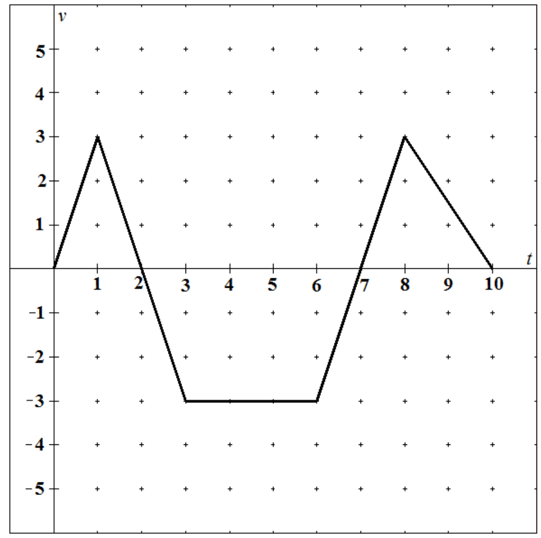
b. When did the rocket reach its highest point? What was its velocity then?

c. When did the parachute pop out? How fast was the rocket falling then?

d. How long did the rocket fall before the parachute opened?

e. When was the rocket's acceleration greatest? When was acceleration constant?

12. The graph given below shows the velocity, v, in meters per second of a particle moving along a coordinate line at time t seconds.

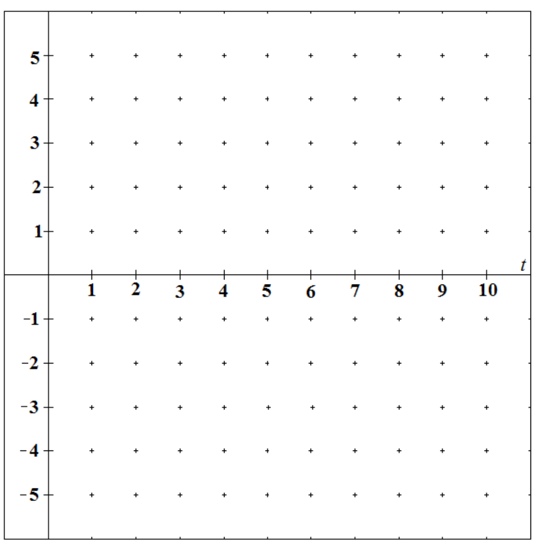


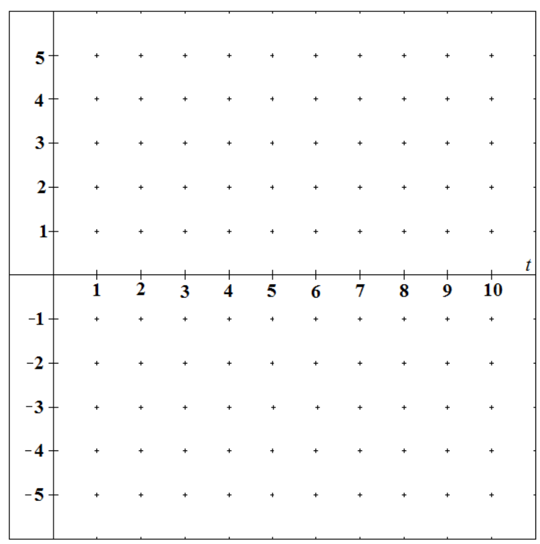
a. When does the particle change direction?

b. When is the particle slowing down? When is it speeding up?

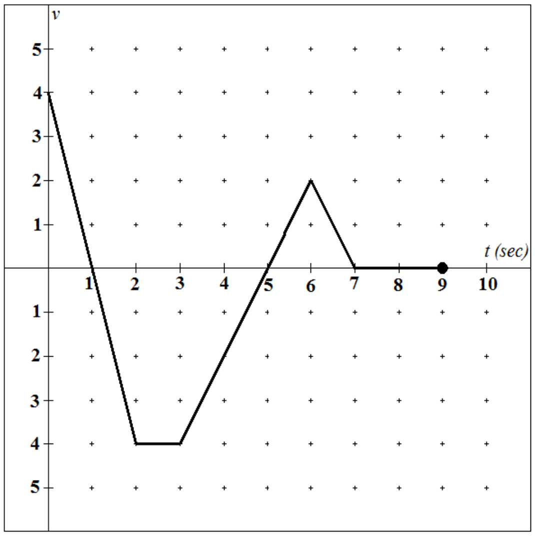
c. Graph the speed of the particle on the interval $[0,10]$

d. Graph the acceleration of the particle, when defined.





13. The figure below shows the velocity, *v*, of a particle moving on a coordinate line.



a. When is the particle moving to the right?

b. Moving to the left?

c. Speeding up?

d. Slowing down?

e. When is the particle's acceleration positive? Negative? Zero?

f. When does the particle move at its greatest speed?

g. When does the particle stand still for more than an instant?