Magnet Analysis 1A

Name _____

Volume by Cross Sections (Slices)

Quick Review 7.3

In Exercises 1–10, give a formula for the area of the plane region in terms of the single variable x.

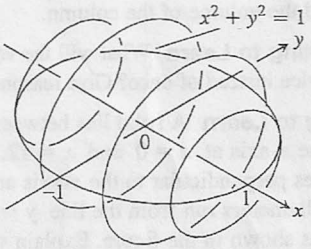
1. a square with sides of length x
2. a square with diagonals of length x
3. a semicircle of radius x
4. a semicircle of diameter x
5. an equilateral triangle with sides of length x
6. an isosceles right triangle with legs of length x
7. an isosceles right triangle with hypotenuse x
8. an isosceles triangle with two sides of length $2x$ and one side of length x
9. a triangle with sides $3x$, $4x$, and $5x$
10. a regular hexagon with sides of length x

Section 7.3 Exercises

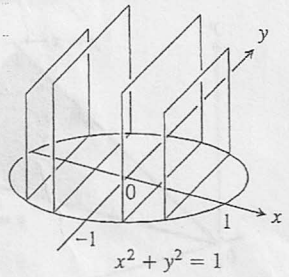
In Exercises 1 and 2, find a formula for the area $A(x)$ of the cross sections of the solid that are perpendicular to the x-axis.

1. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$.

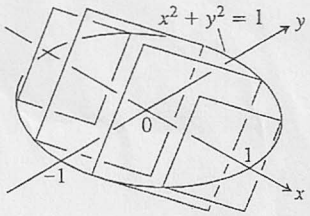
(a) The cross sections are circular disks with diameters in the xy-plane.



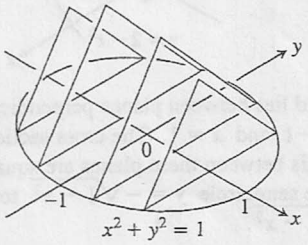
(b) The cross sections are squares with bases in the xy-plane.



(c) The cross sections are squares with diagonals in the xy-plane. (The length of a square's diagonal is $\sqrt{2}$ times the length of its sides.)

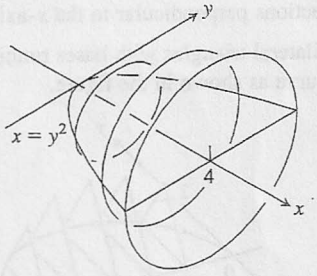


(d) The cross sections are equilateral triangles with bases in the xy-plane.

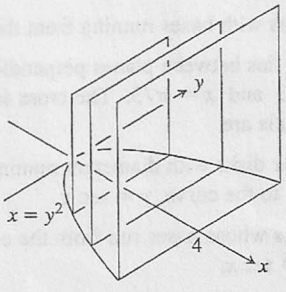


2. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the x-axis between these planes run from $y = -\sqrt{x}$ to $y = \sqrt{x}$.

(a) The cross sections are circular disks with diameters in the xy-plane.



(b) The cross sections are squares with bases in the xy-plane.



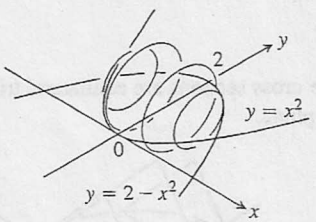
(c) The cross sections are squares with diagonals in the xy-plane.

(d) The cross sections are equilateral triangles with bases in the xy-plane.

In Exercises 3–10, find the volume of the solid analytically.

3. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the axis on the interval $0 \le x \le 4$ are squares whose diagonals run from $y = -\sqrt{x}$ to $y = \sqrt{x}$.

4. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$.

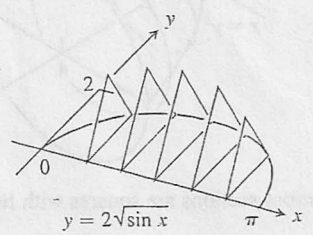


5. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes are squares whose bases run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$.

6. The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis between these planes are squares whose diagonals run from the semicircle $y = -\sqrt{1 - x^2}$ to the semicircle $y = \sqrt{1 - x^2}$.

7. The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are

(a) equilateral triangles with bases running from the x-axis to the curve as shown in the figure.



(b) squares with bases running from the x-axis to the curve.

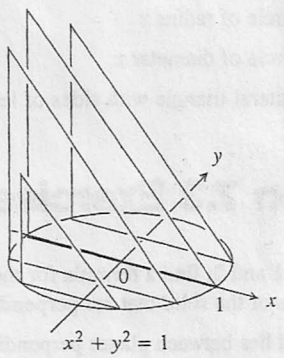
8. The solid lies between planes perpendicular to the x-axis at $x = -\pi/3$ and $x = \pi/3$. The cross sections perpendicular to the x-axis are

(a) circular disks with diameters running from the curve $y = \tan x$ to the curve $y = \sec x$.

(b) squares whose bases run from the curve $y = \tan x$ to the curve $y = \sec x$.

9. The solid lies between planes perpendicular to the y-axis at $y = 0$ and $y = 2$. The cross sections perpendicular to the y-axis are circular disks with diameters running from the y-axis to the parabola $x = \sqrt{5}y^2$.

10. The base of the solid is the disk $x^2 + y^2 \le 1$. The cross sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk.



11. *A Twisted Solid* A square of side length s lies in a plane perpendicular to a line L. One vertex of the square lies on L. As this square moves a distance h along L, the square turns one revolution about L to generate a corkscrew-like column with square cross sections.

(a) Find the volume of the column.

(b) **Writing to Learn** What will the volume be if the square turns twice instead of once? Give reasons for your answer.

12. **Writing to Learn** A solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 12$. The cross sections by planes perpendicular to the x-axis are circular disks whose diameters run from the line $y = x/2$ to the line $y = x$ as shown in the figure. Explain why the solid has the same volume as a right circular cone with base radius 3 and height 12.