Magnet Analysis 1A

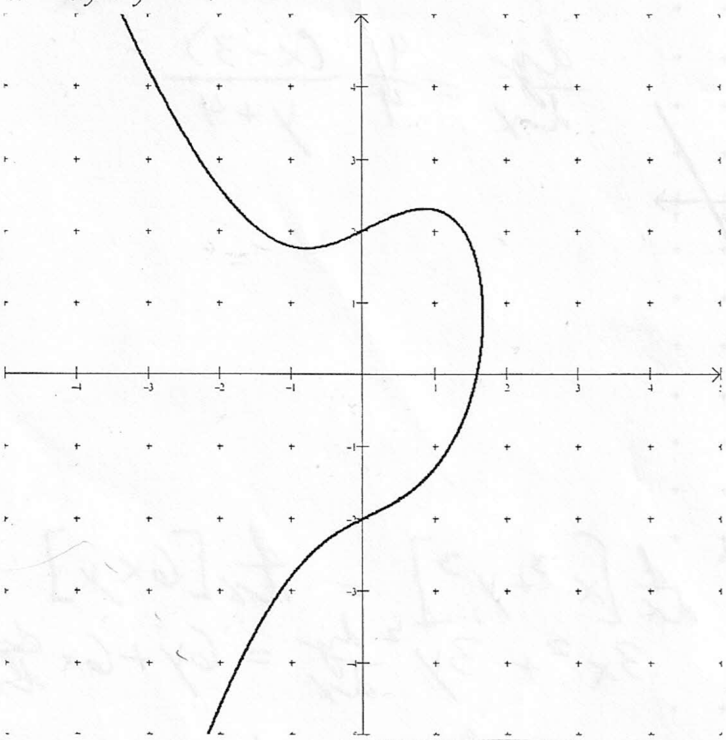
3/3/2009

Name KEY

Implicit Differentiation

For each function y, defined implicitly in terms of x, find $\frac{dy}{dx}$:

$x^3 - xy + y^2 = 4$



$$\frac{d}{dx}\left[x^3 - xy + y^2\right] = \frac{d}{dx}\left[4\right]$$

$$3x^2 - \left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$3x^2 - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

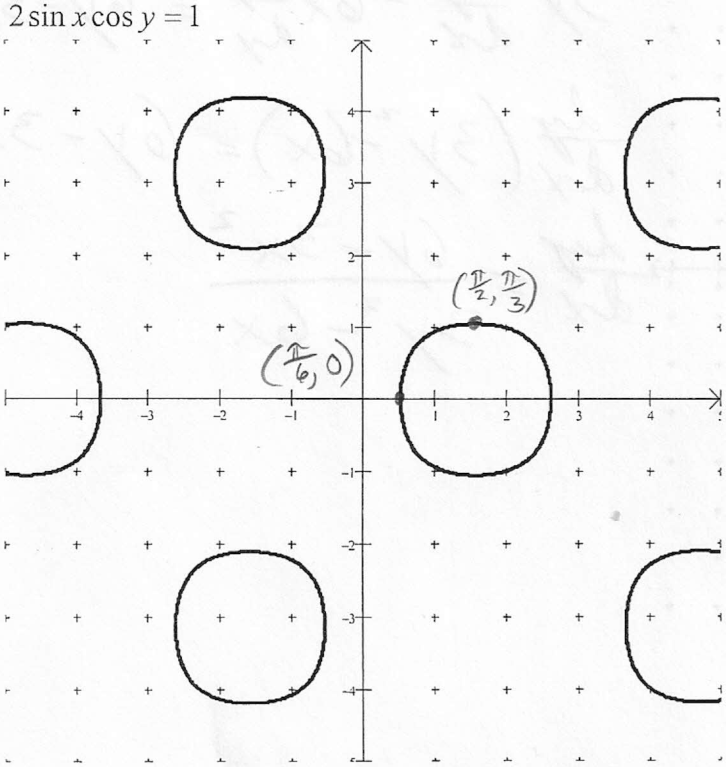
$$3x^2 - y = x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$\frac{dy}{dx}(x - 2y) = 3x^2 - y$$

$$\frac{dy}{dx} = \frac{3x^2 - y}{x - 2y}$$

$$\left.\frac{dx}{dx}\right|_{(0,2)} = \frac{1}{2} \checkmark$$

$2 \sin x \cos y = 1$



$$\frac{d}{dx}\left[\sin x \cos y\right] = \frac{d}{dx}\left[\frac{1}{2}\right]$$

$$\cos x \cos y + \sin x\left(-\sin y \cdot \frac{dx}{dx}\right) = 0$$

$$\cos x \cos y = \sin x \sin y \frac{dy}{dx}$$

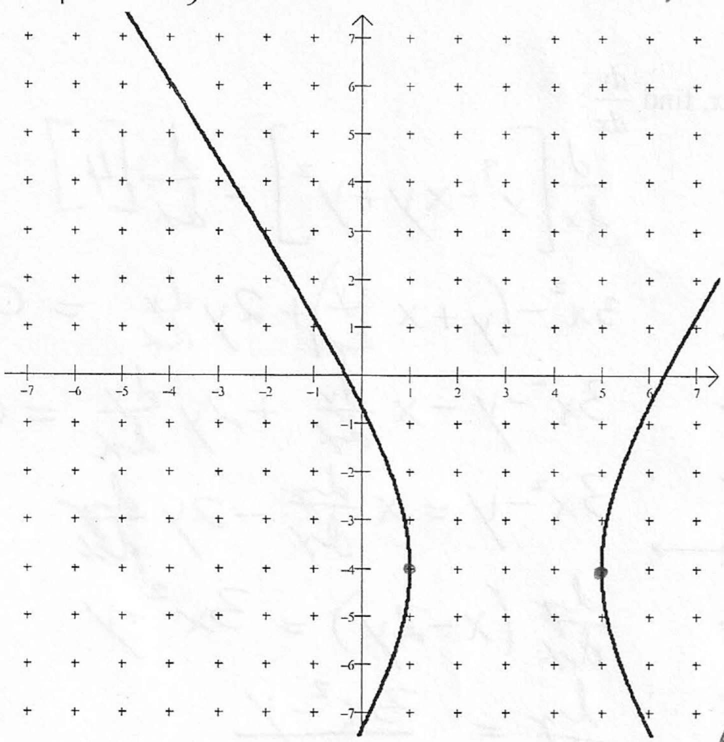
$$\frac{dy}{dx} = \cot x \cot y$$

$$\frac{(x-3)^2}{4} - \frac{(y+4)^2}{9} = 1 \implies 9(x-3)^2 - 4(y+4)^2 = 36$$

$$18(x-3) - 8(y+4)\frac{dy}{dx} = 0$$

$$18(x-3) = 8(y+4)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{9}{4}\,\frac{(x-3)}{y+4}$$



Folium of Descartes

$x^3 + y^3 - 6xy = 0$

$$\frac{d}{dx}\left[x^3 + y^3\right] = \frac{d}{dx}[6xy]$$

$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 6x) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$