Volume by Shells

Name KEY

1. Find the volume of the solid formed by rotating about the y-axis the region in quadrant 1 bounded by $y = 2x^2 - x^3$ & the x-axis.

$y = x^2(2-x)$

$y = -x^2(x-2)$

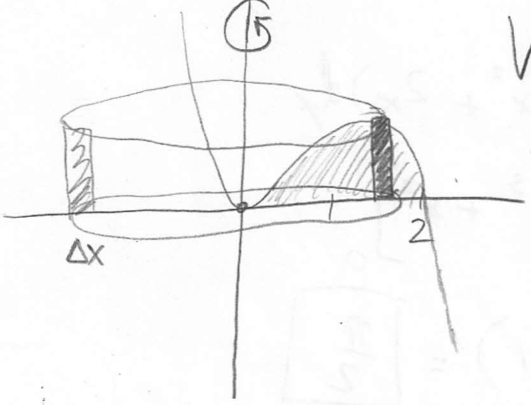
$$V \approx \sum_{i=1}^{n} (2\pi x_i^*)\, y\, \Delta x$$

$$V = \int_0^2 2\pi x(2x^2 - x^3)\,dx$$

$$= 2\pi \int_0^2 (2x^3 - x^4)\,dx$$

$$= 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5}\right]_0^2 = 2\pi\left(8 - \frac{32}{5}\right)$$

$$= 2\pi\left(\frac{8}{5}\right) = \boxed{\frac{16\pi}{5}}$$



2. Find the volume of the solid formed by rotating about the y-axis the region bounded by $y = x$ & $y = x^2$.

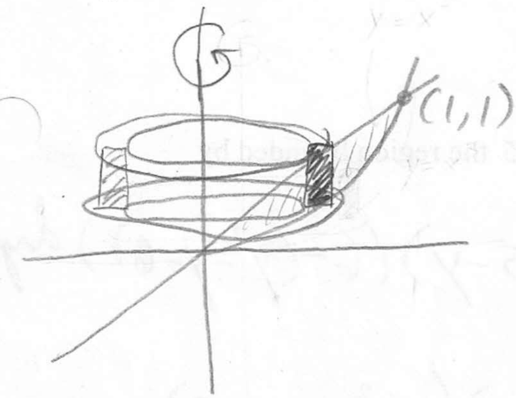
a) Use Shells:

$$V = \int_0^1 2\pi x(x - x^2)\,dx$$

$$= 2\pi \int_0^1 (x^2 - x^3)\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = \boxed{\frac{\pi}{6}}$$

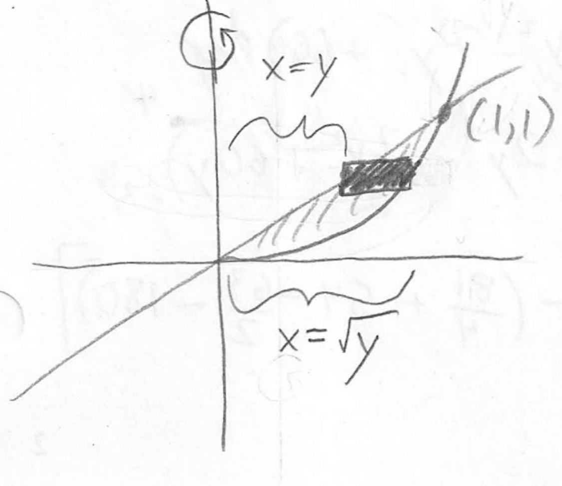


b) Use Washers:

$$V = \int_0^1 \left[\pi(\sqrt{y})^2 - \pi y^2\right] dy$$

$$= \pi \int_0^1 (y - y^2)\,dy$$

$$= \pi \left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1 = \boxed{\frac{\pi}{6}}$$



3. Find the volume of the solid formed by rotating about the line $x = 2$ the region bounded by $y = x - x^2$ & $y = 0$.

$y = x(1-x)$

$y = -x(x-1)$

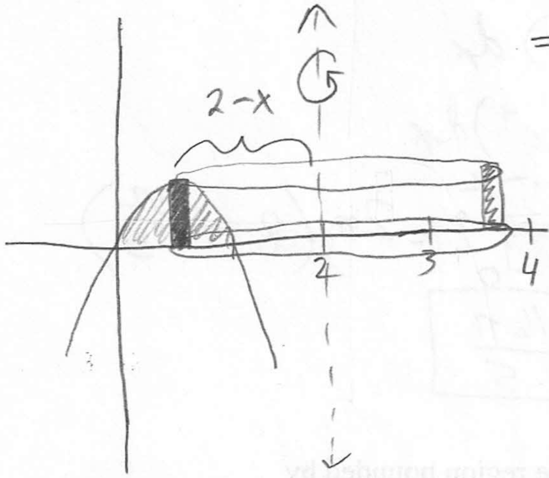
$$V = \int_0^1 2\pi(2-x)(x-x^2)\,dx$$

$$= 2\pi \int_0^1 (2x - 2x^2 - x^2 + x^3)\,dx$$

$$= 2\pi \int_0^1 (x^3 - 3x^2 + 2x)\,dx$$

$$= 2\pi \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1$$

$$= 2\pi\left(\frac{1}{4} - 1 + 1\right) = \boxed{\frac{\pi}{2}}$$



4. Find the volume of the solid formed by rotating about the line $y = 5$ the region bounded by $x = y^2 - y - 6$ & $x = 6$.

$x = (y-3)(y+2)$

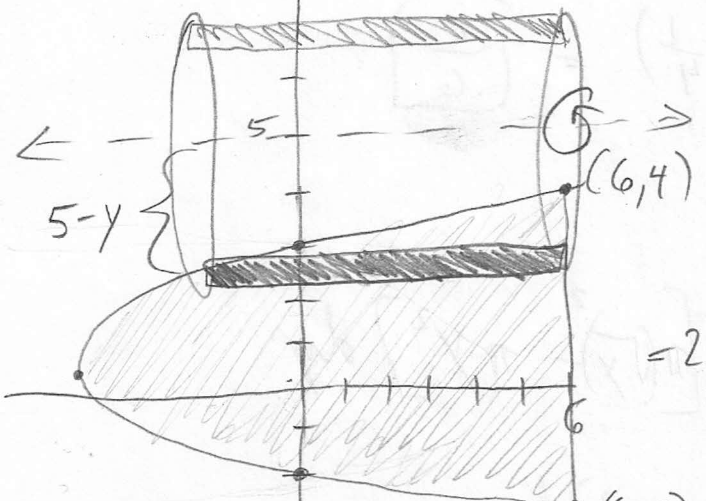
$$V = \int_{-3}^{4} 2\pi(5-y)(6-(y^2-y-6))\,dy$$

$$= \int_{-3}^{4} 2\pi(5-y)(-y^2+y+12)$$

$$= 2\pi \int_{-3}^{4} (-5y^2 + 5y + 60 + y^3 - y^2 - 12y)\,dy$$

$$= 2\pi \int_{-3}^{4} (y^3 - 6y^2 - 7y + 60)\,dy$$

$$= 2\pi\left(\frac{y^4}{4} - 2y^3 - \frac{7y^2}{2} + 60y\right)_{-3}^{4}$$



$x = 6$

$y^2 - y - 6 = 6$

$y^2 - y - 12 = 0$

$(y-4)(y+3) = 0$

$y = 4, -3$

$$= 2\pi\left[(64 - 128 - 56 + 240) - \left(\frac{81}{4} + 54 - \frac{63}{2} - 180\right)\right]$$

$$= 2\pi\left(\frac{1029}{4}\right) = \boxed{\frac{1029\pi}{2}}$$