## **Introduction to Limits**

1. 
$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x+1)(x^2 + x + 1)}{x + 1}$$

$$= \lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x+1)(x+1)}{x - 1} = \lim_{x \to 1} \frac{(x+1)(x+1)}{x - 1} = \lim_{x \to 1} \frac{(x+1)(x+1)}{x - 1} = 2$$

2. 
$$\lim_{x \to 0} \frac{5\sin 2x}{x} \cdot \frac{2}{2}$$

$$= 10 \lim_{x \to 0} \frac{\sin(2x)}{(2x)}$$

$$= 10 \cdot 1 = 10$$

3. 
$$\lim_{x \to 2} \frac{1 - \sqrt{x-1}}{x-2} \left( \frac{1 + \sqrt{x-1}}{1 + \sqrt{x-1}} \right)$$

$$= \lim_{x \to 2} \frac{1 - (x-1)}{(x-2)(1 + \sqrt{x-1})}$$

$$= \lim_{x \to 2} \frac{2 + x}{x - 2(1 + \sqrt{x-1})}$$

$$= \lim_{x \to 3} \frac{-1}{\sqrt{x+6-3}}$$

$$\lim_{x \to 3} \frac{x-1}{\sqrt{x+6-3}} \stackrel{?}{\sim} \frac{2}{\sqrt{x+6-3}}$$

$$\lim_{x \to 3^{+}} \frac{x-1}{\sqrt{x+6-3}} \stackrel{?}{\sim} \frac{2}{\sqrt{x+6-3}} = -\infty$$

$$\lim_{x \to 3^{+}} \frac{x-1}{\sqrt{x+6-3}} \stackrel{?}{\sim} \frac{2}{\sqrt{x+6-3}} = \infty$$

$$|\lim_{x \to 3^{+}} \frac{x-1}{\sqrt{x+6-3}} \stackrel{?}{\sim} \frac{2}{\sqrt{x+6-3}} = \infty$$

5. 
$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x + 1)(x + 1)}{x + 1}$$
$$= 2$$

6. 
$$\lim_{x \to \pi^+} \csc x = -\infty$$

7. 
$$\lim_{x \to 4} \frac{\sqrt{x-2}}{x-4} = \lim_{x \to 4} \frac{\sqrt{x-2}}{(x-2)(\sqrt{x+2})}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{x+2}} = \frac{1}{4}$$