Magnet Analysis 1A
Rose

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Name KEY

Area

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Approximate the area between the x-axis and the curve $y = -x^2 + 5$ from $x = 0$ to $x = 2$. Divide the interval $(0, 2)$ into 5 equal-width subintervals and approximate the area using rectangles that lie alternatively below and above the curve.

Lower Estimate:

$$S = \frac{2}{5}f\left(1\cdot\frac{2}{5}\right) + \frac{2}{5}f\left(2\cdot\frac{2}{5}\right) + \frac{2}{5}f\left(3\cdot\frac{2}{5}\right) + \frac{2}{5}f\left(4\cdot\frac{2}{5}\right) + \frac{2}{5}f\left(5\cdot\frac{2}{5}\right)$$

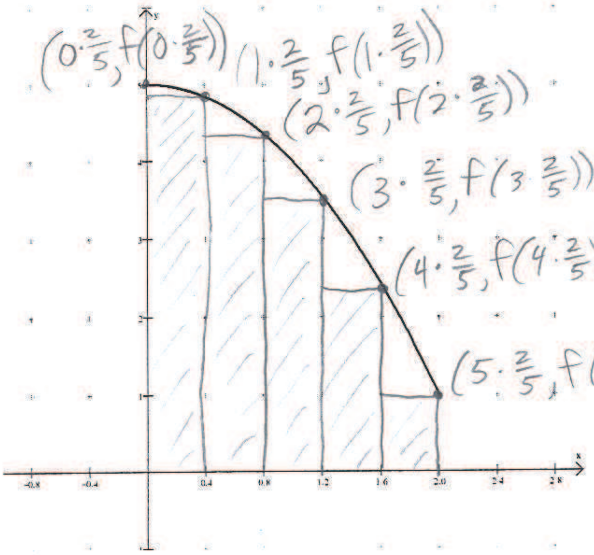
$$= \sum_{i=1}^{5} \frac{2}{5} f\left(\frac{2}{5}i\right) = \frac{2}{5}\sum_{i=1}^{5}\left[-\left(\frac{2}{5}i\right)^2 + 5\right]$$

$$= \frac{2}{5}\sum_{i=1}^{5} 5 - \frac{2}{5}\sum_{i=1}^{5}\frac{4}{5}i^2$$

$$= \frac{2}{5}(25) - \frac{8}{125}\sum_{i=1}^{5} i^2$$

$$= 10 - \frac{8}{125}\left(\frac{5\cdot 6\cdot 11}{6}\right)$$

$$= 10 - \frac{88}{25} = \frac{250-88}{25} = \boxed{\frac{162}{25}}$$



$$S = \frac{2}{5}f\left((1-1)\frac{2}{5}\right) + \frac{2}{5}f\left((2-1)\frac{2}{5}\right) + \frac{2}{5}f\left((3-1)\frac{2}{5}\right) + \frac{2}{5}f\left((4-1)\frac{2}{5}\right) + \frac{2}{5}f\left((5-1)\frac{2}{5}\right)$$

Upper Estimate:

$$= \sum_{i=1}^{5}\frac{2}{5}f\left((i-1)\frac{2}{5}\right) = \frac{2}{5}\sum_{i=1}^{5}\left(-\left[\frac{2}{5}(i-1)\right]^2 + 5\right)$$

$$\frac{2}{5}\sum_{i=1}^{5} 5 - \frac{2}{5}\sum_{i=1}^{5}\frac{4}{25}(i-1)^2$$

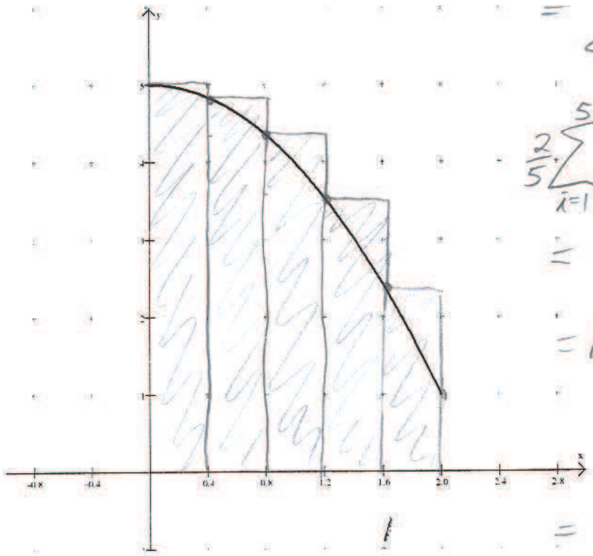
$$= \frac{2}{5}(25) - \frac{8}{125}\sum_{i=1}^{5}(i^2 - 2i + 1)$$

$$= 10 - \frac{8}{125}\sum_{i=1}^{5} i^2 + \frac{16}{125}\sum_{i=1}^{5} i - \frac{8}{125}\sum_{i=1}^{5} 1$$

$$10 - \frac{88}{25} + \frac{16}{125}\left(\frac{5\cdot 6}{2}\right) - \frac{8}{125}(5)$$

$$= 10 - \frac{88}{25} + \frac{48}{25} - \frac{8}{25}$$

$$= 10 - \frac{48}{25} = \frac{250-48}{25} = \boxed{\frac{202}{25}}$$

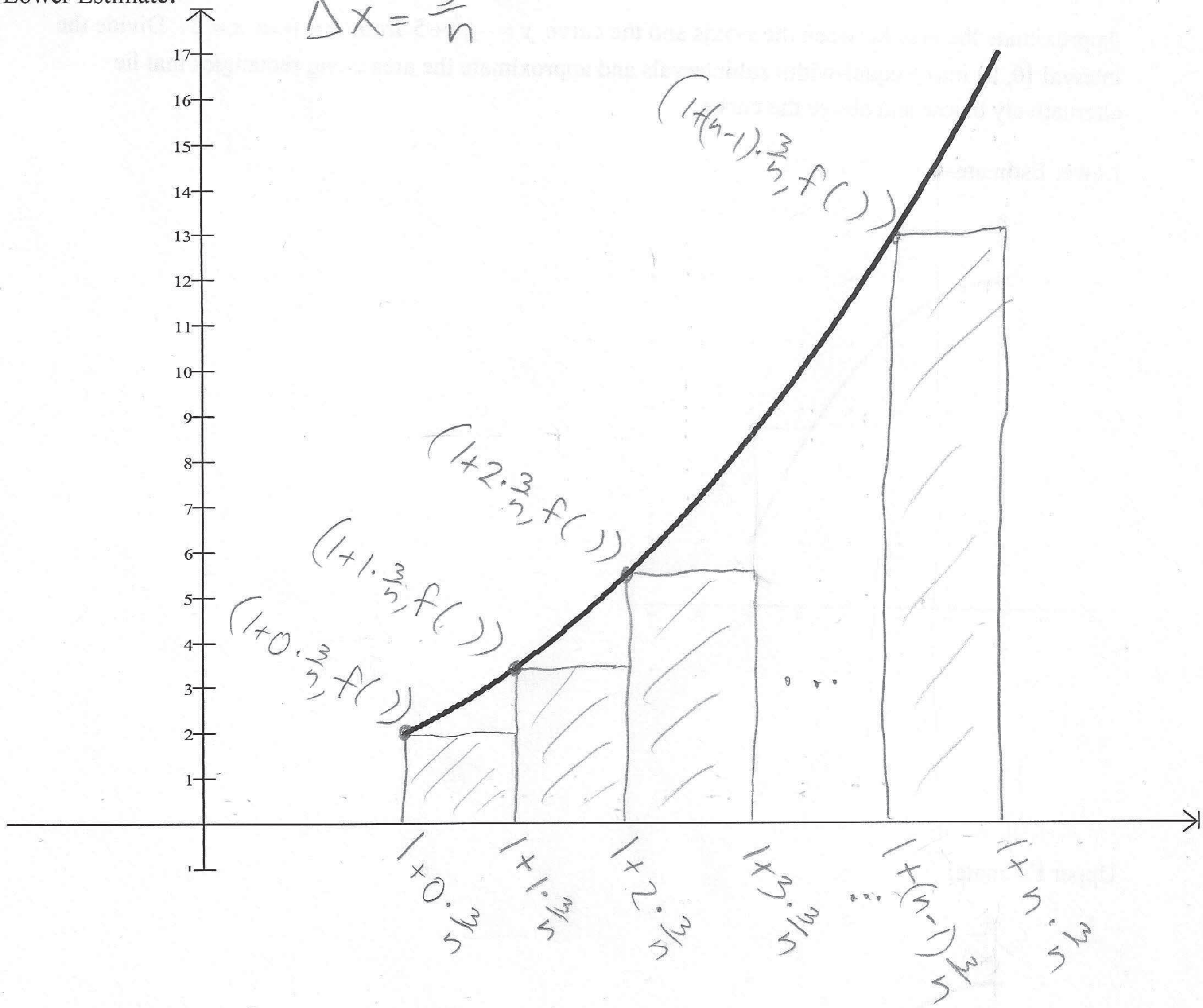


Find the area between the x-axis and the curve $y = x^2 + 1$ from $x = 1$ to $x = 4$. Divide the interval into n equal-width subintervals and approximate the area using rectangles that lie alternatively below and above the curve.

n subintervals

Lower Estimate:

$\Delta x = \frac{3}{n}$



$$\text{Area} = \frac{3}{n}\cdot f\left(1+0\cdot\frac{3}{n}\right) + \frac{3}{n}\cdot f\left(1+1\cdot\frac{3}{n}\right) + \frac{3}{n}\cdot f\left(1+2\cdot\frac{3}{n}\right) + \cdots + \frac{3}{n}\cdot f\left(1+(n-1)\frac{3}{n}\right)$$

$$= \frac{3}{n}\sum_{i=1}^{n} f\left(1+(i-1)\frac{3}{n}\right)$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3}{n}(i-1)+1\right)^2 + 1\right]$$

$$= \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9}{n^2}(i-1)^2 + \frac{6}{n}(i-1) + 1 + 1 \right]$$

$$= \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9}{n^2}(i^2-2i+1) + \frac{6}{n}(i-1) + 2 \right]$$

$$= \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9}{n^2} i^2 - \frac{18}{n^2} i + \frac{9}{n^2} + \frac{6}{n} i - \frac{6}{n} + 2 \right]$$

$$= \frac{3}{n} \sum_{i=1}^{n} \left[\frac{9}{n^2} i^2 + \left(\frac{6}{n} - \frac{18}{n^2}\right) i + \left(\frac{9}{n^2} - \frac{6}{n} + 2\right) \right]$$

$$= \frac{27}{n^3} \sum_{i=1}^{n} i^2 + \frac{3(6n-18)}{n^3} \sum_{i=1}^{n} i + \frac{3(9-6n+2n^2)}{n^3} \sum_{i=1}^{n} 1$$

$$= \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{3(6n-18)}{n^3} \cdot \frac{n(n+1)}{2} + \frac{3(9-6n+2n^2)}{n^3} \cdot n$$

$$= \frac{9}{2} \cdot \frac{2n^2+3n+1}{n^2} + \frac{(9n-27)(n+1)}{n^2} + \frac{6n^2-18n+27}{n^2}$$

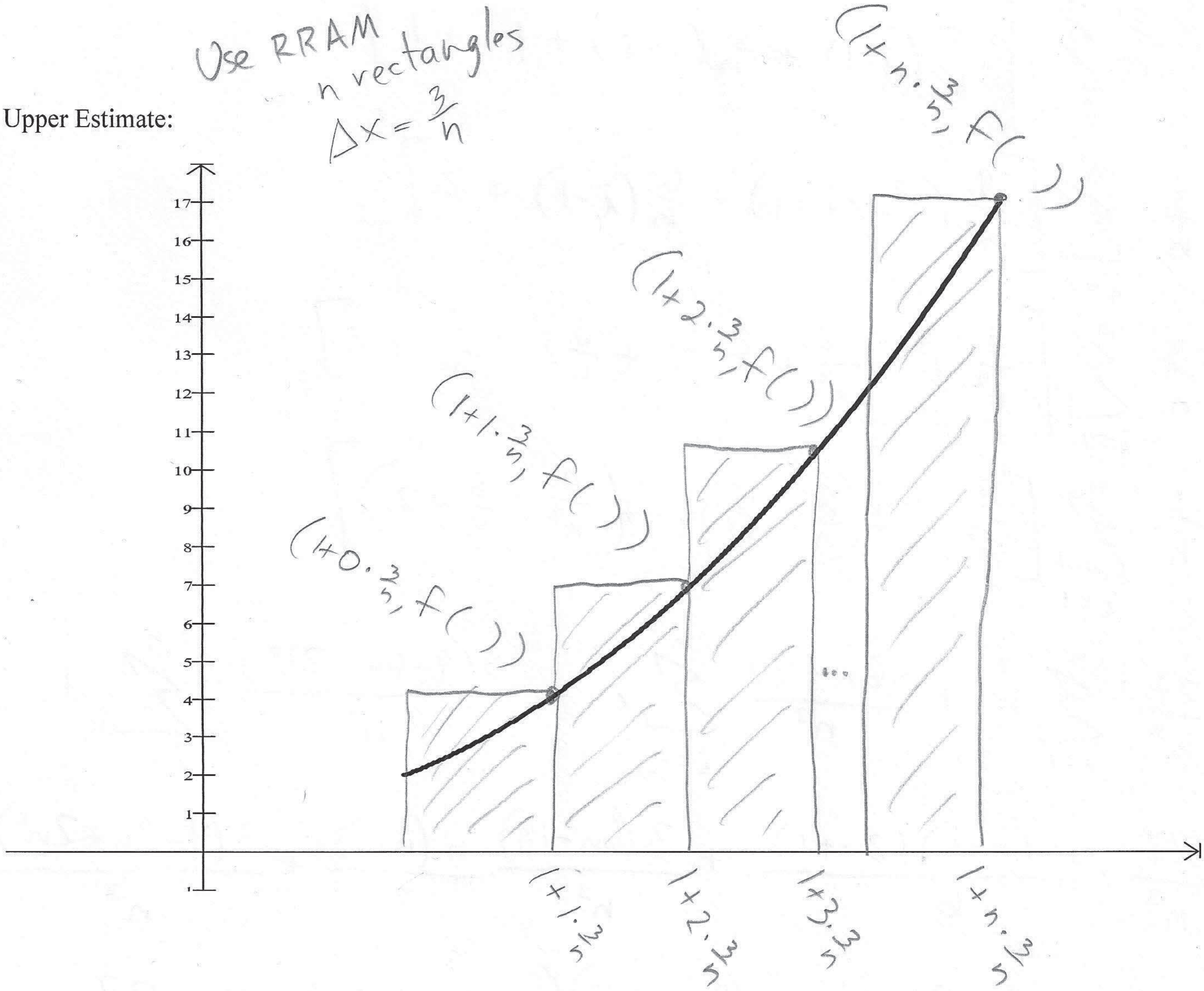
Area =

$$\lim_{n\to\infty} \left(\frac{9}{2} \cdot \frac{2n^2+3n+1}{n^2} + \frac{9n^2-18n-27}{n^2} + \frac{6n^2-18n+27}{n^2} \right)$$

$$= 9 + 9 + 6 = \boxed{24}$$

Upper Estimate:

Use RRAM
n rectangles
$\Delta x = \frac{3}{n}$



$$\text{Area} = \frac{3}{n}\cdot f\left(1+1\cdot\frac{3}{n}\right) + \frac{3}{n}\cdot f\left(1+2\cdot\frac{3}{n}\right) + \frac{3}{n}\cdot f\left(1+3\cdot\frac{3}{n}\right) + \cdots + \frac{3}{n} f\left(1+n\cdot\frac{3}{n}\right)$$

$$= \frac{3}{n}\sum_{i=1}^{n} f\left(1+i\cdot\frac{3}{n}\right) = \frac{3}{n}\sum_{i=1}^{n}\left[\left(\frac{3}{n}i+1\right)^2+1\right]$$

$$= \frac{3}{n}\sum_{i=1}^{n}\left[\frac{9}{n^2}i^2+\frac{6}{n}i+2\right]$$

$$= \frac{27}{n^3}\sum_{i=1}^{n} i^2 + \frac{18}{n^2}\sum_{i=1}^{n} i + \frac{6}{n}\sum_{i=1}^{n} 1$$

$$= \frac{27}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{18}{n^2}\,\frac{n(n+1)}{2} + \frac{6}{n}\cdot n$$

$$= \frac{9}{2}\,\frac{2n^2+3n+1}{n^2} + 9\cdot\frac{n+1}{n} + 6$$

As $n\to\infty$, $\Rightarrow\ 9+9+6 = \boxed{24}$