$$\begin{array}{r} x+9 \\ x^2+2\,\overline{\big)\,x^3+9x^2+\quad+1} \\ \underline{x^3\qquad+2x} \\ 9x^2-2x+1 \\ \underline{9x^2\qquad+18} \\ -2x-17 \end{array}$$

5. $\int \frac{x^3+9x^2+1}{x^2+2}dx = \int (x+9)dx - \int \frac{2x+17}{x^2+2}$

$\frac{x^2}{2}+9x-\int\frac{2x}{x^2+2}dx - 17\int\frac{1}{x^2+2}dx$

$\frac{x^2}{2}+9x-\ln|x^2+2| - \frac{17\sqrt{2}}{2}\int\frac{\frac{1}{\sqrt{2}}\,1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}dx$

$u=\frac{x}{\sqrt{2}}$

$du=\frac{1}{\sqrt{2}}dx$

$\boxed{\frac{x^2}{2}+9x-\ln|x^2+2|-\frac{17}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right)+C}$

6. $\int \frac{dx}{\sqrt{-16x^2+8x+3}}$

$u=2x-\frac{1}{2}$

$du=2dx$

$\int\frac{dx}{\sqrt{-16(x^2-\frac{1}{2}x+\frac{1}{16})+3+1}} = \int\frac{dx}{\sqrt{4-16(x-\frac{1}{4})^2}} = \frac{1}{2}\int\frac{2\,dx}{\sqrt{4[1-(2x-\frac{1}{2})^2]}}$

$=\frac{1}{4}\int\frac{du}{\sqrt{1-u^2}} = \boxed{\frac{1}{4}\arcsin(2x-\tfrac{1}{2})+C}$

7. $\int\frac{\sqrt{x-2}}{x+1}dx = \int\frac{u(2u\,du)}{u^2+3} = 2\int\frac{u^2\,du}{u^2+3}$

$x=u^2+2,\ x+1=u^2+3$

$u^2=x-2$

$u=\sqrt{x-2}$

$du=\frac{1}{2\sqrt{x-2}}dx$

$dx=2\sqrt{x-2}\,du$

$dx=2u\,du$

$=2\int\left(1-\frac{3}{u^2+3}\right)du = 2u-6\int\frac{d}{u^2+3}$

$=2\sqrt{x-2}-2\sqrt{3}\int\frac{du\ \frac{1}{\sqrt{3}}}{\left(\frac{u}{\sqrt{3}}\right)^2+1}$

$v=\frac{u}{\sqrt{3}}$

$dv=\frac{1}{\sqrt{3}}du$

$$\begin{array}{r} 1 \\ u^2+3\,\overline{\big)\,u^2\quad} \\ \underline{u^2+3} \\ -3 \end{array}$$

$=\boxed{2\sqrt{x-2}-2\sqrt{3}\arctan\sqrt{\frac{x-2}{3}}+C}$

8. $\int \sec x\,dx$

$\int \sec x\,dx\left(\frac{\sec x+\tan x}{\sec x+\tan x}\right) = \int\frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}$

$=\int\frac{du}{u} = \boxed{\ln|\sec x+\tan x|+C}$

$u=\sec x+\tan x$

$du=\sec x\tan x+\sec^2 x$

9. $\int \sin^2 x\,dx$

$\int\frac{1-\cos 2x}{2}dx = \int\frac{1}{2} - \frac{1}{2}\frac{1}{2}\int 2\cos 2x\,dx$

$=\boxed{\frac{1}{2}x-\frac{\sin 2x}{4}+C}$