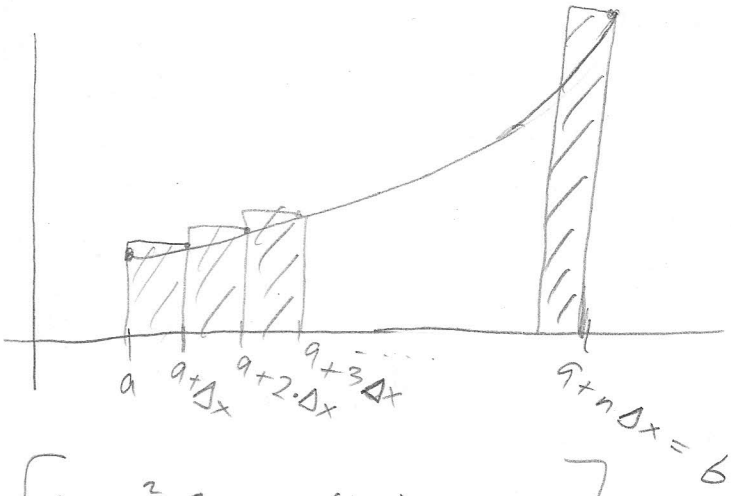
Magnet Analysis 1A
Rose

Name KEY

Integral of x^2

$$\int_a^b x^2\,dx =$$

$$\Delta x = \frac{b-a}{n}$$



$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) f\left(a + \frac{b-a}{n} i\right)$$

$$\left(\frac{b-a}{n}\right)\sum_{i=1}^{n}\left(a + \frac{b-a}{n} i\right)^2 = \frac{b-a}{n} \sum_{i=1}^{n}\left[\left(\frac{b-a}{n}\right)^2 i^2 + 2a\left(\frac{b-a}{n}\right) i + a^2\right]$$

$$= \frac{(b-a)^3}{n^3} \sum_{i=1}^{n} i^2 + \frac{2a(b-a)^2}{n^2} \sum_{i=1}^{n} i + \frac{a^2(b-a)}{n} \sum_{i=1}^{n} 1$$

$$= \frac{(b-a)^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{2a(b-a)^2}{n^2} \cdot \frac{n(n+1)}{2} + \frac{a^2(b-a)}{n} \cdot n$$

$$\lim_{n\to\infty}\left[\frac{(b-a)^3(2n^2+3n+1)}{6n^2} + \frac{a(b-a)^2(n+1)}{n} + a^2(b-a)\right]$$

$$= \frac{(b-a)^3}{3} + a(b-a)^2 + a^2(b-a)$$

$$= (b-a)\left[\frac{(b-a)^2}{3} + a(b-a) + a^2\right] = \frac{b-a}{3}\left[(b-a)^2 + 3a(b-a) + 3a^2\right]$$

$$= \frac{b-a}{3}\left[b^2 - 2ab + a^2 + 3ab - 3a^2 + 3a^2\right] = \frac{b-a}{3}\left[b^2 + ab + a^2\right]$$

$$= \frac{b^3 - a^3}{3} = \boxed{\frac{b^3}{3} - \frac{a^3}{3}}$$