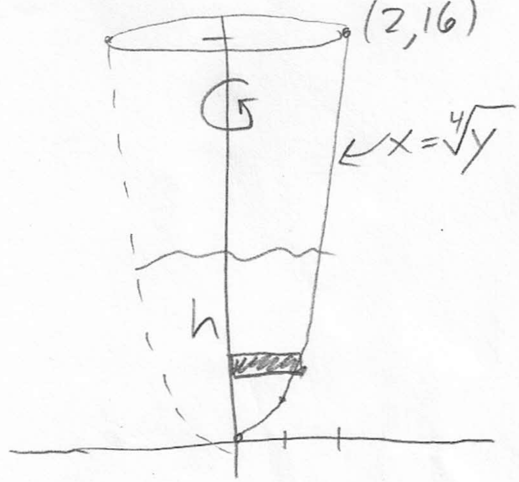
Magnet Analysis 1A
Rose 5/28/2009

Name KEY

Volume, Related Rates, Fundamental Theorem of Calculus Problem

The curve $y = x^4$ from $x = 0$ to $x = 2$ is rotated about the y-axis to form a bowl-shaped water tank. Water is being pumped into this tank at the rate of 10 ft^3/min. If the tank is empty at time $t = 0$, at what rate is the height of the water level in tank changing after 5 seconds?



Let V = Volume of water in tank

Let h = height of water

$V|_{t=0} = 0$

$\frac{dV}{dt} = 10$ ft³/min, Want: $\left.\frac{dh}{dt}\right|_{t=\frac{1}{12}}$

$$V(h) = \int_0^h \pi\left(\sqrt[4]{y}\right)^2 dy$$

$$\boxed{V = \pi \int_0^h \sqrt{y}\, dy}$$

$$\frac{dV}{dt} = \frac{d}{dt}\left[\pi \int_0^h \sqrt{y}\, dy\right]$$

$$\frac{dV}{dt} = \pi\sqrt{h}\,\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{10}{\pi\sqrt{h}}$$

$$\left.\frac{dh}{dt}\right|_{t=\frac{1}{12}} = \frac{10}{\pi\left(\frac{5}{4\pi}\right)^{1/3}} \Longrightarrow$$

Need: $h|_{t=\frac{1}{12}}$

at $t = \frac{1}{12}$, $V = \frac{5}{6}$ ft³/min

$$\frac{5}{6} = \pi \int_0^h \sqrt{y}\, dy$$

$$\frac{5}{6} = \pi \cdot \frac{2}{3} \cdot h^{3/2}$$

$$h^{3/2} = \frac{5}{\cancel{6}_2} \cdot \frac{\cancel{3}}{2} \cdot \frac{1}{\pi} = \frac{5}{4\pi}$$

$$h = \left(\frac{5}{4\pi}\right)^{2/3}$$

$$2^1 \cdot 5^1 \cdot \pi^{-1} \cdot 4^{1/3} \cdot \pi^{1/3} \cdot 5^{-1/3}$$

$$\frac{2^{5/3}\, 5^{2/3}}{\pi^{2/3}} = \boxed{2 \cdot \left(\frac{10}{\pi}\right)^{2/3}}$$