Magnet Analysis 1A
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Name KEY

Wallops Anti-differentiation Packet

1. $\frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}dx$

$u = 1+x^2$
$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{1}{2}\cdot 2u^{\frac{1}{2}} + C$

$= \sqrt{u} + C = \sqrt{1+x^2} + C$

2. $\int \frac{2x+3}{(x^2+3x+5)^4}dx$

$u = x^2+3x+5$
$du = (2x+3)dx$

$\int \frac{du}{u^4} = \int u^{-4}du$

$= -\frac{1}{3}u^{-3} + C = \frac{-1}{3(x^2+3x+5)^3} + C$

3. $\int \tan\theta \sec^2\theta\, d\theta$

$u = \tan\theta$
$du = \sec^2\theta\, d\theta$

$\int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\tan^2\theta + C$

4. $2\int \frac{dt}{2\sqrt{t}(1+\sqrt{t})^3}$

$u = 1+\sqrt{t}$
$du = \frac{1}{2}t^{-1/2}dt$

$2\int \frac{\frac{1}{2}t^{-1/2}}{(1+\sqrt{t})^3}$

$2\int \frac{du}{u^3} = 2\int u^{-3}du$

$= 2(-\frac{1}{2})u^{-2} + C$

$= \frac{-1}{(1+\sqrt{t})^2} + C$

5. $\int x^4 \sin x^5 dx$

$u = x^5$
$du = 5x^4dx$

$\frac{1}{5}\int \sin x^5 \cdot 5x^4dx$

$\frac{1}{5}\int \sin u\, du$

$\frac{1}{5}(-\cos u) + C = -\frac{1}{5}\cos(x^5) + C$

6. $\int \frac{x}{(1+x)^3}dx$

$u = 1+x$
$du = dx$
$x = u-1$

$\int \frac{u-1}{u^3}du$

$= \int (u^{-2} - u^{-3})du$

$= -\frac{1}{u} + \frac{1}{2}(u^{-2}) + C$

$= \frac{-1}{x+1} + \frac{1}{2(x+1)^2} + C$

7. $\int \frac{x}{(x+1)^3}dx$

Oops! Repeat

8. $\int \frac{\sin\theta}{\cos^2\theta}d\theta$

$u = \cos\theta$
$du = -\sin\theta\, d\theta$

$-\int \frac{-\sin\theta\, d\theta}{\cos^2\theta}$

$= -\int \frac{du}{u^2} = -\int u^{-2}du$

$= \frac{1}{u} + C = \frac{1}{\cos\theta} + C$

$\sec\theta + C$

9. $\int \frac{(\ln x)^4}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u^4\,du$

$\frac{1}{5}u^5 + C$

$= \frac{1}{5}(\ln x)^5 + C$

10. $\int x^2\sqrt{2x^3-1}\,dx$

$u = 2x^3 - 1$

$du = 6x^2\,dx$

$\frac{1}{6}\int 6x^2\sqrt{2x^3-1}\,dx$

$\frac{1}{6}\int u^{1/2}\,du = \frac{1}{6}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{9}u^{3/2} + C = \frac{1}{9}(2x^3-1)^{3/2} + C$

11. $\int \frac{\sin(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \sin u\,du$

$-\cos u + C$

$-\cos(\ln x) + C$

12. $\int \sqrt{x+4}\,dx$

$u = x + 4$

$du = dx$

$\int u^{1/2}\,du$

$\frac{2}{3}u^{3/2} + C$

$\frac{2}{3}(x+4)^{3/2} + C$

13. $\int 4\cos(6x)\,dx$

$u = 6x$

$du = 6\,dx$

$\frac{2}{3}\int \cos u\,du$

$\frac{2}{3}\sin u + C$

$\frac{2}{3}\sin 6x + C$

14. $\int \frac{x}{\sqrt{2x^2+1}}\,dx$

$u = 2x^2 + 1$

$du = 4x\,dx$

$\frac{1}{4}\int \frac{4x\,dx}{\sqrt{2x^2+1}}$

$\frac{1}{4}\int u^{-1/2}\,du$

$\frac{1}{4}\cdot 2u^{1/2} + C$

$\frac{1}{2}\sqrt{2x^2+1} + C$

15. $\int x^2\sin(4x^3+8)\,dx$

$\frac{1}{12}\int \sin(4x^3+8)\cdot 12x^2\,dx$

$u = 4x^3 + 8$

$du = 12x^2\,dx$

$\frac{1}{12}\int \sin u\,du$

$-\frac{1}{12}\cos u + C$

$-\frac{1}{12}\cos(4x^3+8) + C$

16. $\int \frac{x^2}{\sqrt[3]{2x^3+7}}\,dx$

$u = 2x^3 + 7$

$du = 6x^2\,dx$

$\frac{1}{6}\int \frac{6x^2\,dx}{\sqrt[3]{2x^3+7}}$

$\frac{1}{6}\int \frac{du}{\sqrt[3]{u}} = \frac{1}{6}\int u^{-1/3}\,du$

$\frac{1}{6}\cdot\frac{3}{2}u^{2/3} + C$

$\frac{1}{4}(2x^3+7)^{2/3} + C$

17. $\int x^2 e^{4x^3}\,dx$

$u = 4x^3$

$du = 12x^2\,dx$

$\frac{1}{12}\int 12x^2 e^{4x^3}dx$

$= \frac{1}{12}\int e^u\,du = \frac{1}{12}e^u + C$

$= \frac{1}{12}e^{4x^3} + C$

18. $\int \frac{e^{1/x}}{x^2}\,dx$

$u = \frac{1}{x}$

$du = \frac{-1}{x^2}dx$

$-\int e^{1/x}\left(-\frac{1}{x^2}dx\right)$

$= -\int e^u\,du$

$= -e^u + C$

$-e^{1/x} + C$

19. $\int (3x-2)^4\,dx$

$u = 3x-2$

$du = 3\,dx$

$\frac{1}{3}\int (3x-2)^4\, 3\,dx$

$\frac{1}{3}\int u^4\,du$

$= \frac{1}{3}\cdot\frac{1}{5}u^5 + C$

$= \frac{1}{15}(3x-2)^5 + C$

20. $\int \left[x + \frac{1}{(3x-1)^3}\right]dx$

$u = 3x-1$

$du = 3dx$

$\int x\,dx + \int (3x-1)^{-3}dx$

$\int x\,dx + \frac{1}{3}\int (3x-1)^{-3}\,3dx$

$\int x\,dx + \frac{1}{3}\int u^{-3}\,du$

$\frac{1}{2}x^2 + \frac{1}{3}\left(\frac{-1}{2}\right)u^{-2} + C$

$\frac{1}{2}x^2 - \frac{1}{6(3x-1)^2} + C$

21. $\int \frac{5x^2}{x^3-2}\,dx$

$u = x^3 - 2$

$du = 3x^2$

$\frac{5}{3}\int \frac{3x^2\,dx}{x^3-2}$

$\frac{5}{3}\int \frac{du}{u}$

$\frac{5}{3}\ln|u| + C$

$\frac{5}{3}\ln|x^3-2| + C$

22. $\int \frac{x^2}{x-1}\,dx$

$$\begin{array}{r}
x + 1 \\
x-1 \overline{)\, x^2 + 0x} \\
x^2 - x \\
\hline
x \\
x - 1 \\
\hline
1
\end{array}$$

$\int \left(x + 1 + \frac{1}{x-1}\right)dx$

$\frac{x^2}{2} + x + \ln|x-1| + C$

23. $\int \frac{(1+e^t)^2}{e^t}\,dt$

$\int \frac{1+2e^t+e^{2t}}{e^t}\,dt$

$\int (e^{-t}+2+e^t)\,dt$

$-e^{-t}+2t+e^t+C$

24. $\int \frac{2}{e^{-x}+1}\,dx$

$2\int \frac{1}{\frac{1}{e^x}+1}\,dx = 2\int \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$
$du = e^x dx$

$= 2\int \frac{du}{u}$

$= 2\ln|u| + C$

$2\ln|1+e^x| + C$

25. $\int \frac{1+\sin x}{\cos x}\,dx$

$\int \frac{1+\sin x}{\cos x} \cdot \frac{1-\sin x}{1-\sin x}\,dx$

$\int \frac{1-\sin^2 x}{\cos x(1-\sin x)}\,dx = \int \frac{\cos^2 x \, dx}{\cos x(1-\sin x)}$

$= \int \frac{\cos x\,dx}{1-\sin x}$

$u = 1-\sin x$
$du = -\cos x\,dx$

$= -\int \frac{du}{u} = -\ln|u| + C$

$= -\ln|1-\sin x| + C$

$= \ln\left|\frac{1}{1-\sin x}\right| + C$

$= \ln\left|\frac{1+\sin x}{\cos^2 x}\right| + C$

26. $\int \frac{-2x}{\sqrt{x^2-4}}\,dx$

$u = x^2-4$
$du = 2x\,dx$

$-\int \frac{du}{\sqrt{u}}$

$= -\int u^{-1/2}\,du$

$= -2u^{1/2} + C$

$= -2\sqrt{x^2-4} + C$

27. $\int \frac{2}{(2x-1)^2+4}\,dx$

$u = 2x-1$
$du = 2\,dx$

$\int \frac{du}{u^2+4}$

$\int \frac{du}{4\left(\frac{u^2}{4}+1\right)}$

$v = \frac{u}{2}$
$dv = \frac{1}{2}\,du$

$= \frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1} = \frac{1}{2}\int \frac{\frac{1}{2}\,du}{\left(\frac{u}{2}\right)^2+1}$

$= \frac{1}{2}\int \frac{dv}{v^2+1} = \frac{1}{2}\arctan v + C$

$= \frac{1}{2}\arctan\left(\frac{u}{2}\right) + C$

$= \frac{1}{2}\arctan\left(\frac{2x-1}{2}\right) + C$

29. $\int \frac{t}{\sqrt{1-t^4}}\,dt$

$u = t^2$
$du = 2t\,dt$

$\frac{1}{2}\int \frac{2t\,dt}{\sqrt{1-(t^2)^2}}$

$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\arcsin u + C$

$= \frac{1}{2}\arcsin(t^2) + C$

28. $\int \frac{1}{x\sqrt{4x^2-1}}\,dx$

$u = 2x$
$du = 2\,dx$

$\int \frac{1}{x\sqrt{(2x)^2-1}}$

$\int \frac{2\,dx}{(2x)\sqrt{(2x)^2-1}}$

$\int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec}|u| + C$

$= \operatorname{arcsec}|2x| + C$

30. $\int \frac{\sec^2 x}{4+\tan^2 x}\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$\int \frac{du}{u^2+4}$

$\int \frac{du}{4\left(\frac{u^2}{4}+1\right)}$

$v = \frac{u}{2}$
$dv = \frac{1}{2}\,du$

$\frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1} = \frac{1}{2}\int \frac{\frac{1}{2}\,du}{\left(\frac{u}{2}\right)^2+1}$

$= \frac{1}{2}\int \frac{dv}{v^2+1}$

$= \frac{1}{2}\arctan v + C$

$= \frac{1}{2}\arctan\frac{u}{2} + C$

$= \frac{1}{2}\arctan\left(\frac{\tan x}{2}\right) + C$

31. $\int \frac{\tan\left(\frac{2}{t}\right)}{t^2}\,dt$

$u = \frac{2}{t}$

$du = \frac{-2}{t^2}\,dt$

$-\frac{1}{2}\int \tan\left(\frac{2}{t}\right)\left(\frac{-2}{t^2}\,dt\right)$

$-\frac{1}{2}\int \tan u\,du$

$-\frac{1}{2}\int \frac{\sin u}{\cos u}\,du$

$v = \cos u$

$dv = -\sin u\,du$

$\frac{1}{2}\int \frac{-\sin u\,du}{\cos u} = \frac{1}{2}\int \frac{dv}{v}$

$= \frac{1}{2}\ln|v| + C = \frac{1}{2}\ln|\cos u| + C$

$= \frac{1}{2}\ln\left|\cos\left(\frac{2}{t}\right)\right| + C$

$= \ln\sqrt{\cos\left(\frac{2}{t}\right)} + C$

32. $\int \frac{3}{\sqrt{6x - x^2}}\,dx$

$3\int \frac{dx}{\sqrt{-(x^2 - 6x\quad)}} = 3\int \frac{dx}{\sqrt{-(x^2 - 6x + 9) + 9}}$

$= 3\int \frac{dx}{\sqrt{9 - (x-3)^2}}$

$u = x - 3$

$du = dx$

$= 3\int \frac{du}{\sqrt{9 - u^2}} = 3\int \frac{du}{\sqrt{9\left(1 - \frac{u^2}{9}\right)}}$

$= \int \frac{du}{\sqrt{1 - \left(\frac{u}{3}\right)^2}}$

$v = \frac{u}{3}$

$dv = \frac{1}{3}\,du$

$= 3\int \frac{\frac{1}{3}du}{\sqrt{1 - (u/3)^2}} = 3\int \frac{dv}{\sqrt{1 - v^2}}$

$= 3\arcsin v + C = 3\arcsin\left(\frac{u}{3}\right) + C$

$= 3\arcsin\left(\frac{x-3}{3}\right) + C$

33. $\int \frac{1}{(x-1)\sqrt{4x^2 - 8x + 3}}\,dx$

$\int \frac{1}{(x-1)\sqrt{4(x^2 - 2x + 1) - 4 + 3}}\,dx$

$\int \frac{1}{(x-1)\cdot 2\sqrt{(x-1)^2 - 1}}\,dx$

$u = x - 1$

$du = dx$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2 - 1}}$

$= \frac{1}{2}\operatorname{arcsec}|u| + C$

$= \frac{1}{2}\operatorname{arcsec}|x-1| + C$

34. $\int \frac{4}{4x^2 + 4x + 65}\,dx$

$\int \frac{4\,dx}{4(x^2 + x\quad) + 65}$

$4\int \frac{dx}{4\left(x^2 + x + \frac{1}{4}\right) + 64}$

$u = x + \frac{1}{2}$

$du = dx$

$\int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + 16}$

$\int \frac{du}{u^2 + 16} = \int \frac{du}{16\left(\frac{u^2}{16} + 1\right)}$

$= \frac{1}{16}\int \frac{du}{\left(\frac{u}{4}\right)^2 + 1}$

$v = \frac{u}{4}$

$dv = \frac{1}{4}\,du$

$= \frac{1}{4}\int \frac{\frac{1}{4}du}{\left(\frac{u}{4}\right)^2 + 1} = \frac{1}{4}\int \frac{dv}{v^2 + 1}$

$\frac{1}{4}\arctan v + C$

$= \frac{1}{4}\arctan\left(\frac{u}{4}\right) + C = \frac{1}{4}\arctan\left(\frac{x + \frac{1}{2}}{4}\right) + C$