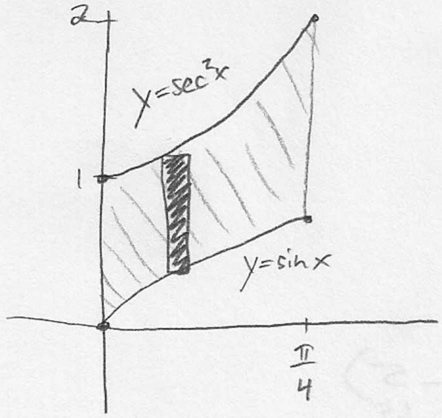
Magnet Analysis 1A
Rose

Name KEY

Area between two Curves (Area is always positive, like distance, speed, volume, etc)

1. Find the area of the region between $y = \sec^2 x$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{4}$. (Hint: $y = \sec^2 x$ doesn't look terribly different from $y = \sec x$ on this interval; point-plot).



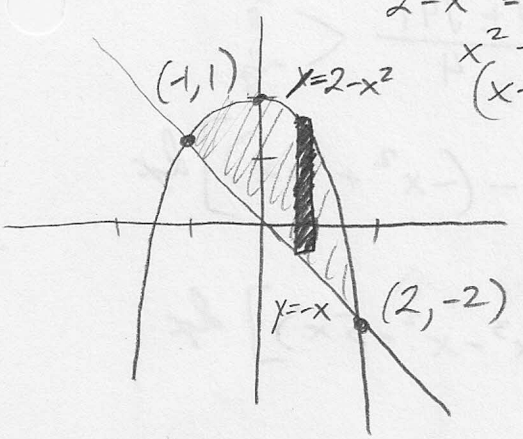
$$\int_0^{\frac{\pi}{4}} (\sec^2 x - \sin x)\,dx$$

$$\Big[\tan x + \cos x\Big]_0^{\frac{\pi}{4}}$$

$$\left(1 + \frac{\sqrt{2}}{2}\right) - (0 + 1)$$

$$\boxed{\frac{\sqrt{2}}{2}}$$

2. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.



$$2 - x^2 = -x$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, -1$$

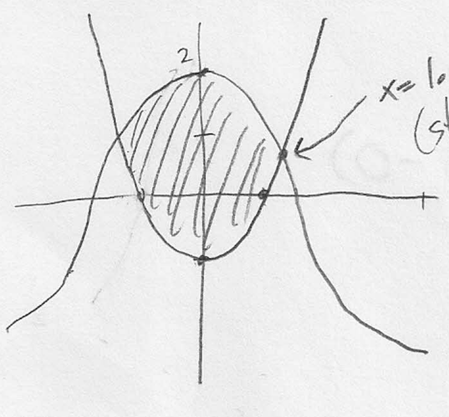
$$\int_{-1}^{2} \left[(2-x^2) - (-x)\right] dx$$

$$\int_{-1}^{2} (-x^2 + x + 2)\,dx$$

$$\left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2} = \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$-\frac{8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2 = \boxed{\frac{5}{2}}$$

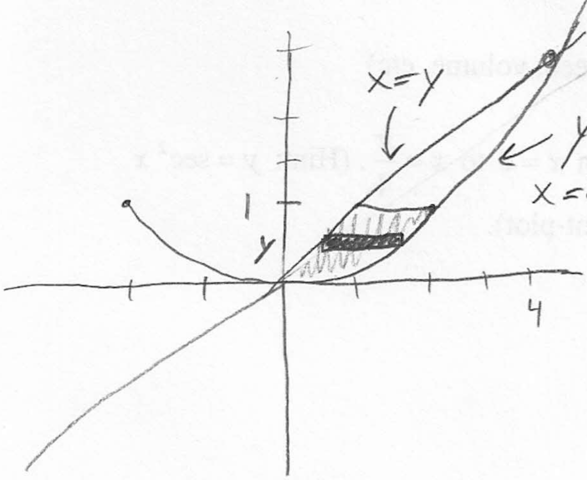
3. Find the area of the region enclosed by the graphs of $y = 2\cos x$ and $y = x^2 - 1$. (Hint: use a calculator for this one. Graph the curves and find their points of intersection, store the values, and integrate numerically via calculator.)



$$2\int_0^a \left(2\cos x - (x^2 - 1)\right) dx$$

$$= \boxed{4.994}$$

4. Find the area in the first quadrant enclosed by the curves $y = x$ and $y = \frac{x^2}{4}$ and below the line $y = 1$.



$$\int_0^1 \left[(2\sqrt{y}) - y\right] dy$$

$$\left.\frac{4}{3} y^{3/2} - \frac{y^2}{2}\right]_0^1$$

$$\frac{4}{3} - \frac{1}{2} = \boxed{\frac{5}{6}}$$

$-x(x-3)$

5. Find the area enclosed by the curves $y = -x^2 + 3x$ and $y = 2x^3 - x^2 - 5x$.

$x(2x^2 - x - 5)$

$$-x^2 + 3x = 2x^3 - x^2 - 5x$$

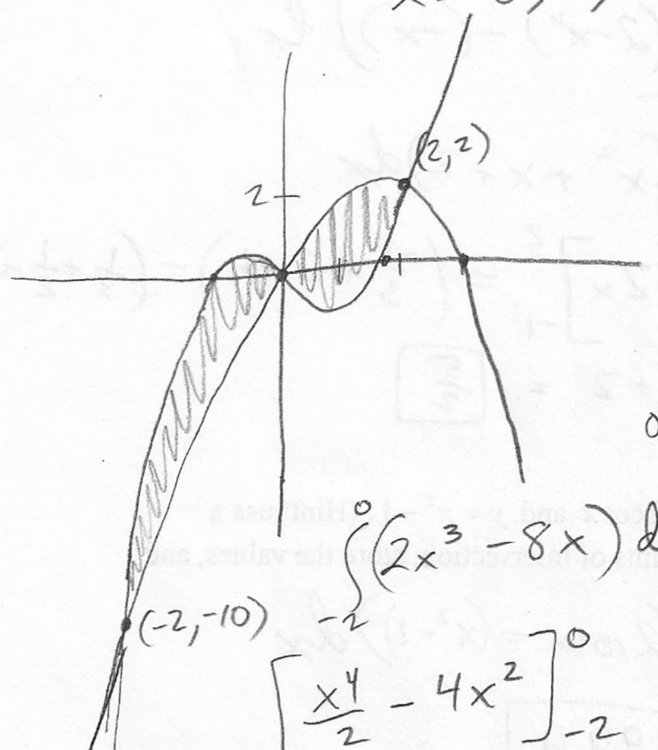
$$0 = 2x^3 - 8x$$

$$2x(x^2 - 4) = 0$$

$$x = 0, 2, -2$$

$$x = \frac{1 \pm \sqrt{1 - 4(2)(-5)}}{4}$$

$$= \frac{1 \pm \sqrt{41}}{4} < \begin{matrix} \frac{15}{8} \\ -\frac{11}{8} \end{matrix}$$



$$\int_{-2}^{0} \left[(2x^3 - x^2 - 5x) - (-x^2 + 3x)\right] dx$$

$$+$$

$$\int_{0}^{2} \left[(-x^2 + 3x) - (2x^3 - x^2 - 5x)\right] dx$$

$$\int_{-2}^{0} (2x^3 - 8x)\, dx + \int_{0}^{2} (8x - 2x^3)\, dx$$

$$\left[\frac{x^4}{2} - 4x^2\right]_{-2}^{0} + \left[4x^2 - \frac{x^4}{2}\right]_0^2$$

$$(0-0) - (8-16) + (16-8) - (0-0)$$

$$8 + 8 = \boxed{16}$$