Limits Review II

1. $\lim\limits_{x \to 3} \dfrac{|x-3|}{x-3}$ d.n.e.

$\lim\limits_{x \to 3^-} \dfrac{|x-3|}{x-3} = -1$

$\lim\limits_{x \to 3^+} \dfrac{|x-3|}{x-3} = 1$

2. $\lim\limits_{x \to \infty} \sqrt{x^2+2} - x \left(\dfrac{\sqrt{x^2+2}+x}{\sqrt{x^2+2}+x} \right)$

$\lim\limits_{x \to \infty} \dfrac{2}{\sqrt{x^2+2}+x} \left(\dfrac{\frac{1}{x}}{\frac{1}{x}} \right)$

$\lim\limits_{x \to \infty} \dfrac{\frac{2}{x}}{\sqrt{1+\frac{2}{x^2}}+1} = 0$

3. $\lim\limits_{x \to \infty} \dfrac{\sqrt{3x^2+1}}{4x-2} \left(\dfrac{\frac{1}{x}}{\frac{1}{x}} \right)$

$\lim\limits_{x \to \infty} \dfrac{\sqrt{\frac{1}{x^2}}\sqrt{3x^2+1}}{4-\frac{2}{x}}$

$= \lim\limits_{x \to \infty} \dfrac{\sqrt{3+\frac{1}{x^2}}}{4-\frac{2}{x}} = \dfrac{\sqrt{3}}{4}$

when $x > 0$

$\dfrac{1}{x} = \dfrac{1}{\sqrt{x^2}}$

4. $\lim\limits_{x \to 25} \dfrac{\sqrt{x}-5}{x-25} = \lim\limits_{x \to 25} \dfrac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)}$

$\lim\limits_{x \to 25} \dfrac{1}{\sqrt{x}+5} = \dfrac{1}{10}$

5. $\lim\limits_{x \to 0} \dfrac{(x+3)^3 - 27}{x} = \lim\limits_{x \to 0} \dfrac{x^3+9x^2+27x+27-27}{x}$

$= \lim\limits_{x \to 0} (x^2+9x+27) = 27$

6. $\lim\limits_{x \to -\infty} \dfrac{2x^2+3x+5}{3x^2+4x-7} \left(\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} \right)$

$\lim\limits_{x \to \infty} \dfrac{2+\frac{3}{x}+\frac{5}{x^2}}{3+\frac{4}{x}-\frac{7}{x^2}} = \dfrac{2}{3}$

7. $\lim\limits_{x \to 3} \dfrac{x^2-9}{x^2-x-6} = \lim\limits_{x \to 3} \dfrac{(x-3)(x+3)}{(x-3)(x+2)}$

$= \lim\limits_{x \to 3} \dfrac{x+3}{x+2} = \dfrac{6}{5}$

8. $\lim\limits_{x \to 1^+} \begin{cases} 4-x^2, & x \le 1 \\ 3x-1, & x > 1 \end{cases}$

$= \lim\limits_{x \to 1^+} (3x-1) = 2$

1

9. $\lim\limits_{x \to \infty} e^{-x} \sin x$

$-1 \le \sin x \le 1$

$-e^{-x} \le e^{-x} \sin x \le e^{-x}$

So by Squeeze thm,

$\lim\limits_{x \to \infty} e^{-x} = 0$

$\lim\limits_{x \to \infty} -e^{-x} < 0$

$\lim\limits_{x \to \infty} e^{-x} \sin x = 0$

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10. $\lim\limits_{x \to -\infty} \arctan x$

$= -\dfrac{\pi}{2}$

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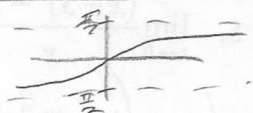
11. $\lim\limits_{x \to -\infty} \dfrac{x+3}{\sqrt{6x^2+5}} \left(\dfrac{\frac{1}{x}}{\frac{1}{x}} \right)$

when $x < 0$

$\dfrac{1}{x} = -\sqrt{\dfrac{1}{x^2}}$

$\lim\limits_{x \to -\infty} \dfrac{1 + \frac{3}{x}}{-\sqrt{\frac{1}{x^2}}\sqrt{6x^2+5}}$

$= \lim\limits_{x \to \infty} \dfrac{1 + \frac{3}{x}\to 0}{-\sqrt{6 + \frac{5}{x^2}}}$

$= \dfrac{-1}{\sqrt{6}} = \dfrac{-\sqrt{6}}{6}$

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12. $\lim\limits_{x \to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x} = \lim\limits_{x \to 0} \dfrac{4-(x+4)}{4(x+4)x}$

$= \lim\limits_{x \to 0} \dfrac{-x}{4x(x+4)} = \lim\limits_{x \to 0} \dfrac{-1}{4(x+4)}$

$= -\dfrac{1}{16}$

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13. $\lim\limits_{x \to 1^-} \cot(\ln x)$

$= \lim\limits_{x \to 1^-} \cot(0^-)$

$= -\infty$

$y = \ln x$

$y = \cot x$

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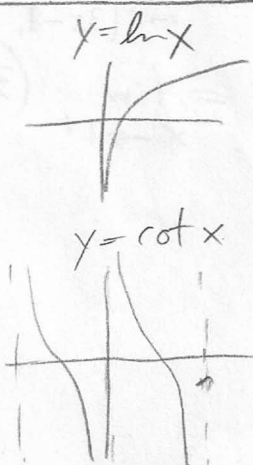
14. $\lim\limits_{x \to \infty} \arctan\left(\dfrac{x^{4/5} + x^{5/4} + 1}{x^{8/7} + x^{7/8}} \right) =$

$\lim\limits_{x \to \infty} \arctan(\infty) = \dfrac{\pi}{2}$

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15. $\lim\limits_{x \to \frac{\pi}{4}^-} \ln x \sec(2x)$

$\dfrac{\pi}{4} < 1$ so $\ln\left(\dfrac{\pi}{4}\right) < 0$
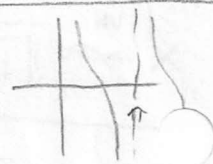
$\lim\limits_{x \to \frac{\pi}{4}^-} \ln x \sec(2x)$

$= (\text{something neg}) \infty$

$= -\infty$

$y = \ln x$

$y = \sec(2x)$

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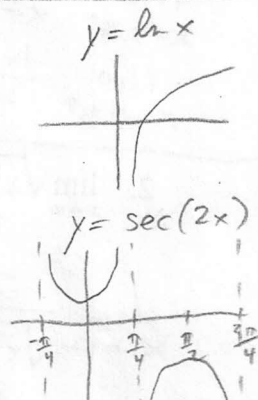
16. $\lim\limits_{x \to \pi} \cot x$ d.n.e.

$\lim\limits_{x \to \pi^-} \cot x = -\infty$

$\lim\limits_{x \to \pi^+} \cot x = \infty$

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17. $\lim\limits_{x \to \infty} \dfrac{2 - 11x^3}{4x^2 + 5x + 3} \left(\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} \right)$

$\lim\limits_{x \to \infty} \dfrac{-11x + \frac{2}{x^2}}{4 + \frac{5}{x} + \frac{3}{x^2}} \approx -11x = -\infty$

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18. $\lim\limits_{x \to 0} (2^x - \arcsin x - 4)$

$= 1 - 0 - 4 = -3$

2.