Magnet Analysis 1A Name______________________
Rose 2/13/2008

The Limit of a Product of Two Functions

Suppose that $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$ where L and K are nonzero real numbers. We then have a theorem that says that $\lim_{x\to a} fg(x) = LK$.

But there are many other special cases to consider. A limit may be a positive real number, a negative real number, zero, infinity, negative infinity, or fail to exist. There are thus 21 separate cases to consider. Fill in the chart below, marking the limit as “indeterminate” if more information is needed.

$\lim_{x\to a} f(x) =$	$\lim_{x\to a} g(x) =$	$\lim_{x\to a} fg(x) =$
$L > 0$	$K > 0$	LK
$L > 0$	$K < 0$	LK
$L < 0$	$K < 0$	LK
$L > 0$	∞	∞
$L > 0$	$-\infty$	$-\infty$
$L < 0$	∞	$-\infty$
$L < 0$	$-\infty$	∞
0	0	0
0	$K > 0$	0
0	$K < 0$	0
0	∞	Indeterminate
0	$-\infty$	Indeterminate
∞	∞	∞
∞	$-\infty$	$-\infty$
$-\infty$	$-\infty$	∞
∞	d.n.e.	Indeterminate
$-\infty$	d.n.e.	Indeterminate
0	d.n.e.	Indeterminate
$L > 0$	d.n.e.	d.n.e.
$L < 0$	d.n.e.	d.n.e.
d.n.e.	d.n.e.	Indeterminate

For each of the limits you marked “indeterminate”, explain on the back by giving examples of functions *f* and *g* such that the limit *fg* is not determined by the information in the chart.

The $0 \cdot \infty$ indeterminate form:

$\lim_{x\to 0} x = 0$	$\lim_{x\to 0} \frac{1}{x^2} = \infty$	$\lim_{x\to 0} x \cdot \frac{1}{x^2}$ d.n.e.
$\lim_{x\to 0} x^3 = 0$	$\lim_{x\to 0} \frac{1}{x^2} = \infty$	$\lim_{x\to 0} x^3 \cdot \frac{1}{x^2} = 0$
$\lim_{x\to 0} x^2 = 0$	$\lim_{x\to 0} \frac{1}{x^4} = \infty$	$\lim_{x\to 0} x^2 \cdot \frac{1}{x^4} = \infty$
$\lim_{x\to 0} -x^2 = 0$	$\lim_{x\to 0} \frac{1}{x^4} = \infty$	$\lim_{x\to 0} \left(-x^2\right) \cdot \frac{1}{x^4} = -\infty$
$\lim_{x\to 0} 3x^2 = 0$	$\lim_{x\to 0} \frac{1}{x^2} = \infty$	$\lim_{x\to 0} 3x^2 \cdot \frac{1}{x^2} = 3$

The $0 \cdot -\infty$ indeterminate form:

(very similar to above)

The $\infty \cdot \text{d.n.e.}$ indeterminate form:

$\lim_{x\to 0} \frac{1}{x^2} = \infty$	$\lim_{x\to 0} \frac{1}{x}$ d.n.e.	$\lim_{x\to 0} \frac{1}{x} \cdot \frac{1}{x^2}$ d.n.e.
$f(x) = \begin{cases} 1, & x < 0 \\ 2, & x > 0 \end{cases}$ $\lim_{x\to 0} f(x)$ d.n.e.	$\lim_{x\to 0} \frac{1}{x^2} = \infty$	$\lim_{x\to 0} f(x) \cdot \frac{1}{x^2} = \infty$

The $-\infty \cdot \text{d.n.e.}$ indeterminate form:

(very similar to above)

The $0 \cdot \text{d.n.e.}$ indeterminate form:

$\lim_{x\to 0} x = 0$	$\lim_{x\to 0} \frac{1}{x}$ d.n.e.	$\lim_{x\to 0} x \cdot \frac{1}{x} = 1$
$\lim_{x\to 0} x = 0$	$\lim_{x\to 0} \frac{1}{x^3}$ d.n.e.	$\lim_{x\to 0} x \cdot \frac{1}{x^3} = \infty$
$\lim_{x\to 0} -x = 0$	$\lim_{x\to 0} \frac{1}{x^3}$ d.n.e.	$\lim_{x\to 0} (-x) \cdot \frac{1}{x^3} = -\infty$
$\lim_{x\to 0} x = 0$	$\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ d.n.e.	$\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right) = 0$

The $\text{d.n.e.} \cdot \text{d.n.e.}$ indeterminate form:

$\lim_{x\to 0} \frac{1}{x}$ d.n.e.	$\lim_{x\to 0} \frac{1}{x}$ d.n.e.	$\lim_{x\to 0} \frac{1}{x^2} = \infty$
$\lim_{x\to 0} \frac{1}{x}$ d.n.e.	$\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ d.n.e.	$\lim_{x\to 0} \frac{1}{x} \cdot \sin\left(\frac{1}{x}\right)$ d.n.e.