Magnet Analysis 1A
Rose

Name KEY

Integral as Net Change

Net Change Theorem:

The integral of a rate of change is the net change: $\int_a^b F'(x)dx = F(b) - F(a)$

1. If $V(t)$ is the volume of water in a reservoir at time *t*, then its derivative $V'(t)$ is the rate of change of the volume of water at time *t*. If $V'(t) > 0$, then more water is flowing into the reservoir than is flowing out of the reservoir at that time and $V'(t)$ measures the net rate at which water is flowing into the reservoir. If $V'(t) < 0$, then more water is flowing out of the reservoir than is flowing into the reservoir at that time and $V'(t)$ measures the net rate at which water is flowing out of the reservoir.

   What does $\int_{t_1}^{t_2} V'(t)dt$ represent?

   the net change in volume of water in the reservoir from time t_1 to time t_2 in gallons.

2. If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$. So what does $\int_{t_1}^{t_2} v(x)dx$ represent?

   the change in position from t_1 to t_2 in feet

3. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)dt$ represent?

   the total increase in weight of the child in pounds from age 5 to age 10

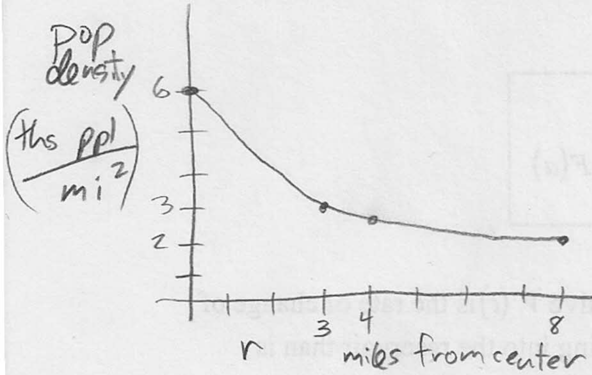
4. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time *t*, what does $\int_0^{120} r(t)dt$ represent?

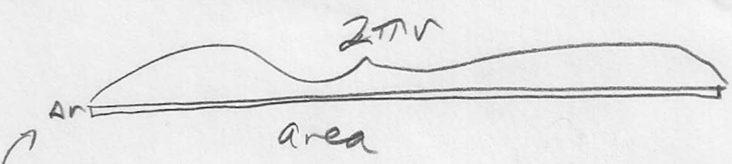
   the total gallons of oil leaked from the tank in the first two hours

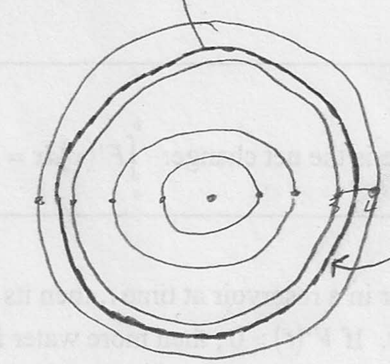
5. A honeybee population starts with 100 bees and increases at a rate of $n(t)$ bees per week. Write an expression involving an integral for the number of honeybees after 15 weeks.

   $100 + \int_0^{15} n(t)\,dt$

6. Let $P'(r) = \dfrac{6}{\sqrt{1+r}}$ on $[0, \infty)$ represent the population density of Centerville in thousands of people per square mile at a point r miles from the city center. Make of sketch of the graph of P' below.







pop density × area = pop

$\left(\dfrac{\text{thds ppl}}{\text{mi}^2}\right)\left(\text{mi}^2\right)$ thds ppl

people living here

$\dfrac{6}{\sqrt{1+r}}(2\pi r)\Delta r$

I. Congressional District A (CDA) covers all people living between 3 and 4 miles from the city center. Find the population of CDA.

$$\lim_{\Delta r \to 0} \sum \left(\frac{6}{\sqrt{1+r}} 2\pi r \Delta r\right)$$

$$12\pi \int_3^4 \frac{r}{\sqrt{1+r}}\,dr$$

$u = 1+r$
$du = dr$
$r = u-1$

$$12\pi \int_4^5 \frac{u-1}{\sqrt{u}}\,du = 12\pi \int_4^5 \left(u^{1/2} - u^{-1/2}\right) du$$

$$12\pi \left(\tfrac{2}{3} u^{3/2} - 2u^{1/2}\right)_4^5 =$$

$$8\pi\left(u^{3/2} - 3\sqrt{u}\right)_4^5$$

$$8\pi \sqrt{u}\,(u-3)\Big|_4^5$$

$$8\pi\left(\sqrt{5}(2) - 2(1)\right) = 16\pi(\sqrt{5}-1)$$

II. Congressional District B (CDB) is being created by the Centerville Census Bureau. The borders are being drawn so that it extends from all points 4 miles from the center out to a certain distance. If they want the population of CDB to be the same as CDA, where do the draw the line for CDB's outer border?

CDB Pop = CDA Pop =

$$12\pi \int_4^x \frac{r}{\sqrt{1+r}}\,dr = 12\pi \int_3^4 \frac{r}{\sqrt{1+r}}\,dr$$

$$8\pi \sqrt{r+1}\,(r-2)\Big]_4^x = 8\pi\sqrt{r+1}\,(r-2)\Big]_3^4$$

$$\sqrt{x+1}\,(x-2) - \sqrt{5}(2) = 2\sqrt{5} - 2$$

$$\sqrt{x+1}\,(x-2) = 4\sqrt{5} - 2$$

$$(x+1)(x^2-4x+4) = 80 - 16\sqrt{5} + 4$$

$$x^3 - 3x^2 + 4 = 80 - 16\sqrt{5} + 4$$

$$x^3 - 3x^2 + 16\sqrt{5} - 80 = 0$$

$$\boxed{x = 4.867}$$