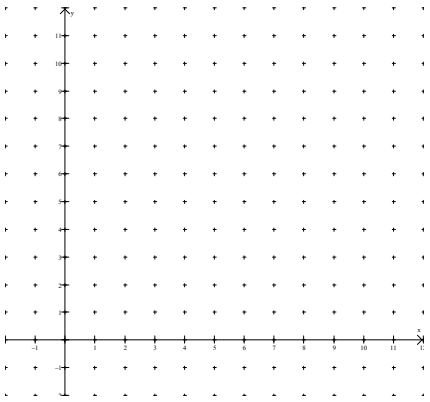
Magnet Analysis 1A Name_____
Rose

Review for Test on Integration

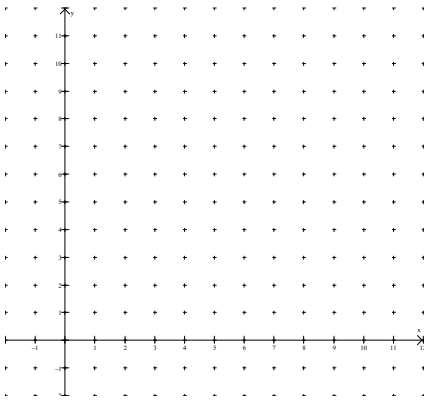
1. For the following function *f* indicated below with a table of values, estimate $\int_{-1}^{10} f(x)dx$ by using (a) the left rectangular method (b) the right rectangular method (c) the trapezoid method. For each method, include a sketch of the area you are computing and show work for partial credit.

x	-1	1	2	5	6	8	10
$f(x)$	1	3	7	2	10	5	0

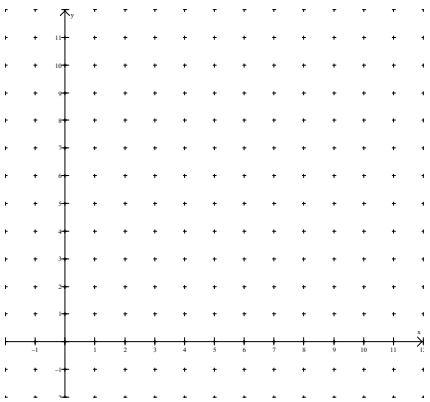
LRAM:



RRAM:



Trapezoid Method:



$$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

2. Using the definition of the integral as a limit of Riemann Sums, compute $\int_1^4 (2x^3 + 1)dx$. Include a picture and show all work.

3. Using the definition of the integral as a limit of Riemann Sums, interpret the following limit as an integral and then use the Fundamental theorem of calculus to evaluate it.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{\pi}{2n}\right)\sin\left(\frac{\pi}{3}+\frac{i\pi}{2n}\right)$$

4. Calculate the following using geometry: (Hint: make a picture!)

$$\int_0^5 \left(\,|x-3|-1\,\right)dx$$

$$\int_{-7}^{7} 15\sqrt{x}\,dx$$

$$\int_0^5 \sqrt{25-x^2}\,dx$$

$$\int_{-3}^{5} \big|\,|x|-5\big|dx$$

$$\int_{3\sqrt{2}}^{6} \sqrt{36-x^2}\,dx$$

5. $\dfrac{d}{dx}\left[\int_3^x \sqrt{t^2+1}\ dt\right]$

$\dfrac{d}{dx}\left[\int_3^{x^2} \tan t\ dt\right]$

$\dfrac{d}{dx}\left[\int_{-3}^{\cos x} t^4\ dt\right]$

$\dfrac{d}{dx}\left[\int_{\cos x}^{3^x} t^4\ dt\right]$

$\dfrac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)\ dt\right]$

6. Compute $\dfrac{d}{dx}\left[\left(x^2-2\right)\sin x + 2x\cos x\right]$ and use this formula to compute $\int_0^{\pi/2}\left(x^2-3\right)\left(\cos x+2\right)dx$

7. The velocity of an object moving along a straight line in ft/sec is determined by the equation $v(t) = 6(t+1)(t-2)$

Find the displacement over the interval $[-2, 3]$.

Find the total distance traveled over the interval $[-2, 3]$.

8. An object moving along a straight line has acceleration in m/s^2 given by $a(t) = 12t - 17$. On $[-1, 2]$, the object's position changes from -21 meters to 7.5 meters. Find the velocity and position at time $t = 1$.

9. Suppose we wish to compute $\int_0^1 \arctan x\,dx$

a) Using geometry and the definition of *arctan* as the inverse of *tan*, compute this integral.

b) Whether or not you were successful in part (a), call your result A.

Compute the following in terms of A:

$\int_{-\sqrt{3}}^{\sqrt{3}} \arctan x\,dx$ $\int_3^4 \arctan(x-3)\,dx$ $\int_0^1 (5\arctan x - 2)\,dx$

$\int_0^3 \arctan\left(\frac{x}{3}\right)dx$ $\int_0^1 \left(2\arctan x + x^2\right)dx$ $\int_{-1}^1 \arctan|x|\,dx$

10. Find the average value of $f(x) = \tan x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$