Magnet Analysis 1A Name_____
Rose

Final Exam Review -- 2013

1. $\lim_{x\to 3}\dfrac{x^2-9}{5x-x^2-6}$

2. $\lim_{x\to 0^+}\dfrac{e^{1+x}-e}{x}$

3. $\lim_{h\to 0}\dfrac{\sin\left(\dfrac{\pi}{6}+h\right)-\sin\dfrac{\pi}{6}}{h}$

4. $\lim_{x\to 2}\dfrac{x^2+2x-8}{x-2}$

5. $\lim_{x\to 0}\dfrac{5x}{3\sin x\cos 4x}$

6. $\lim_{h\to 0}\dfrac{(3+h)^2-(3-h)^2}{2h}$

7. $\lim_{x\to \frac{\pi}{2}}\dfrac{\sin x}{x}$

8. $\lim_{x\to a}\dfrac{\sqrt{x}-\sqrt{a}}{x-a}$

Find $\frac{dy}{dx}$ for the following

9. $y = 5(2x^3 + 1)^2 + 5\arctan(2x)$

10. $y = (2x+1)^{\sin x}$

11. $y = 4(3x+5)\cos^2 x$

12. $y = \frac{5}{\sqrt{x^3+5}}$

13. $y = \int_{\pi}^{\cos x} \frac{dt}{(t^2+1)^{2/3}}$

Given that f is an even function and $\int_0^5 f(x)dx = 8$ and g is an odd function and $\int_0^5 g(x)dx = 4$, evaluate each of the following if possible:

14. $\int_{-5}^{5} [f(x) + g(x)]dx$

15. $\int_{-5}^{5} |g(x)|dx$

16. $\int_{-5}^{5} [3f(x) + x^2]dx$

17. $\int_0^5 \frac{f(x)}{g(x)}dx$

18. $\int_{-5}^{5} [f(x) + 4]dx$

19. Let $f(x)=\int_{3}^{5x^2}\frac{dt}{\sqrt[3]{t^2+1}}$

Find $f\left(\frac{1}{2}\right)$ [Use calculator]

Find $f'(2)$

20. Suppose that *f* is continuous on $[-5,5]$ and has the following properties: $f(0)=2$, $f(3)=-2$, $f(5)=1$, $f''(x)>0$ on $(-5,0)$ and $(1.5,5)$ only, f is decreasing when $x<3$, and *f* is increasing when $x>3$. Sketch a possible graph of *f*.

21. Given $f(x)=\ln(3x^2+1)$, use the linearization of *f* at $x=1$ to estimate $f(1.2)$

22. Let $x^2 - 4xy + y^2 = 3$

a. Find $\frac{dy}{dx}$

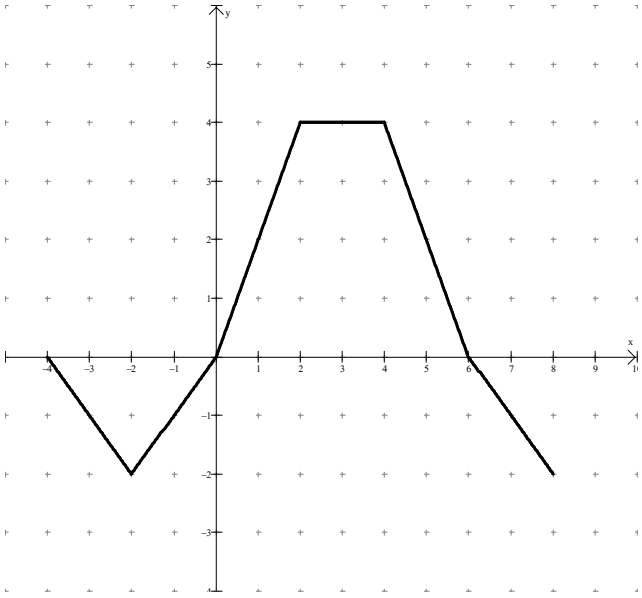
b. Find and justify any points on the curve where the tangent line is horizontal or vertical, if they exist.

23. Let $f(x) = 3x^3 - 5x^2 + 2x + 2$

a. Find the value of "c" guaranteed by the Mean Value Theorem on the interval $[-1, 2]$.

b. Find the average value of f on $[-1, 2]$.

24. The graph of a function *f* is given below. Let $g(x) = \int_0^x f(t)dt$



a. Find each of the following: $g(0)$, $g(2)$, $g(-2)$

b. Find all values of *x* for which *g* has a relative minimum on the open interval $(4, 8)$. Justify your answer.

c. Write an equation of the tangent line to the graph of *g* at $x = 2$.

d. Find the *x*-coordinate of each point of inflection of the graph of *g* on the open interval $(-4, 8)$. Justify your answer.

25. Chemicals from a storage tank are leaking into a pond. The rate of flow is measured at intervals and is recorded in the table below: (*t* is measured in hours; *R(t)* in gallons per hour)

T	0	2	4	6	8	10	12	14	16
R(t)	40	38	36	30	26	18	8	6	3

a. Make a sketch of the data to represent the rate of flow as a function of time.

b. Write an integral expression that represents the total amount of chemical that entered the pond during the 16-hour period.

c. Estimate the total volume of chemicals that entered the pond using LRRAM (n = 8), RRAM (n = 8), Midpoint Rule (n = 4), and Trapezoidal Rule (n = 4)

26. The acceleration of a particle is given by $a(t) = 2 + 5\sqrt{t}$ $ft/s^2; t \geq 0$. $v(0) = 6$

a. What is the velocity of the object at $t = 9$ seconds?

b. What is the total distance covered by the object during the first 9 seconds?

27. Let $f(x) = 3\sin\left(\frac{x}{3}\right) + 1$ and $g(x) = x^2 - 8x + 10$

a. Sketch the graphs of *f* and *g* and find their points of intersection by calculator.

b. Find the area enclosed by the graphs of *f* and *g*.

c. Find the volume of the solid whose base is the region enclosed by the graphs of *f* and *g* such that cross sections perpendicular to the *x*-axis are quarter circles.

d. Find the volume of the solid formed by rotating about the line $y = 5$ the region enclosed by the graphs of *f* and *g*.

28. Set up but do not evaluate one or more integral expressions that could be used to find the area between the curve $y = 3x^3 + 5x^2 - 2x$ and the *x*-axis.

29. Evaluate each antiderivative:

a. $\int \left(xe^{x^2-1} + \cos 3x \right) dx$

b. $\int \left[\sin^3(4x)\cos(4x) \right] dx$

c. $\int \left[5\sin(3x)e^{\cos 3x} \right] dx$

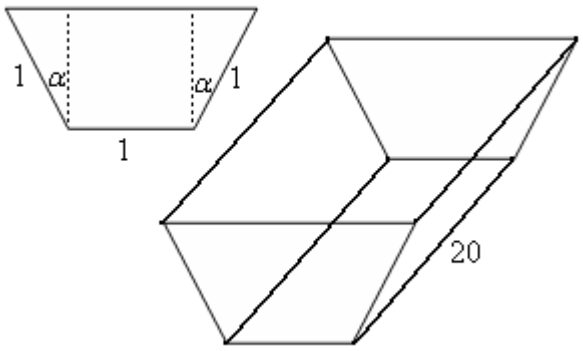
d. $\int \left[x\cos(3x^2) \right] dx$

e. $\int \left[x^2 \sec^2(x^3) \right] dx$

f. $\int_0^1 \frac{5x^2}{8+3x^3}\, dx$

(This is the trough page)

30. The trough in the figure below is to be made with the dimensions shown. Only the angle α can be varied. What value of α will maximize the trough's volume.



31. Water trough is 12 feet long and its cross section is an equilateral triangle with sides 2 feet long. Water is being pumped into the trough at a rate of 3 cubic feet per minute. How fast is the water level rising when the depth of the water is 6 inches?

32. Find the equations of the tangent lines to the curve $y = x^3 - 9x$ through the point $(1, -9)$.

33. An object moves along the *x*-axis according to the following position function which describes its *x*-coordinate (in meters) at time *t* seconds: $s(t) = 3t^5 - 5t^4$ Consider values of *t* on the interval $(-\infty, \infty)$. Make clear, labeled number lines below for position, velocity, and acceleration. Include units in your answer when appropriate.

a. On what time intervals is the object moving to the left?

b. What is the average velocity of the object on the time interval $[-1, 2]$?

c. On what time intervals is the object moving away from the origin?

d. On what time intervals is the object slowing down? (speed is a scalar)

e. What is the total distance travelled on the interval $[-1, 2]$?

Using 10 subintervals, approximate $\int_1^4 e^{x^2}\,dx$ using:

(a) a right endpoint approximation (b) midpoint rule (c) trapezoid rule (d) Simpson's rule

34. In approximating $\int_1^5 \frac{1}{x}\,dx$, how large should we take n in order to guarantee that our answer is accurate to within 0.0001, using (a) left endpoint approx. (b) trap rule (c) midpoint rule (d) Simpson's rule ?

35. A curve C is defined by the parametric equations $\begin{aligned} x &= t^2 \\ y &= t^3 - 3t \end{aligned}$.

a) Show that C crosses itself at $(3, 0)$ by finding two values of *t* that place the curve there.

b) Find $\frac{dy}{dx}$ and use it to give the equations of the two tangent lines to C at $(3, 0)$

c) Give the coordinates of all points where C has a horizontal tangent line.

d) Give the coordinates of all points where C has a vertical tangent line.

e) Find all intervals where C is concave up or concave down.