Analysis 1A Name_____
Rose

Review for Limits Test

1. Find and justify the following limits

$$\lim_{x\to 0} \arctan\left(-e^{-x}\right)$$

$$\lim_{x\to \infty} \arctan\left(-e^{-x}\right)$$

$$\lim_{x\to -\infty} \arctan\left(-e^{-x}\right)$$

2. $\lim_{x\to 2} \dfrac{\ln\left((x-1)^{3x^2}\right)}{\ln(x-1)} =$

3. Rederive $\lim_{x\to 0} \dfrac{\cos x - 1}{x}$

4. Prove using an ε, δ proof that $\lim_{x \to -5} \left(\frac{3}{5}x - 4 \right) = -7$. First make a graph of the function, labelling ε and δ on your picture. Please show your work as you find δ, but clearly mark the point at which your proof begins by writing something like "proof begins here."

5. Determine all values of the constant *a* such that $\lim_{x \to 0} f(x)$ exists where

$$f(x) = \begin{cases} a^2 - 2, & x < 0 \\ \dfrac{ax}{\tan x}, & x > 0 \end{cases}$$

6. In proving that $\lim_{x\to -3} 2^{-x} = 8$, find the largest possible δ, given an $\varepsilon = .01$. (You do not need to do the proof; just find the δ.)

7. Prove $\lim_{x\to 0} \sin^2 x \cdot \cos\left(\frac{1}{x}\right) = 0$. (Hint: use the Squeeze Theorem.)

8. $\lim_{x\to 1^+} \frac{1-\sqrt[3]{x}}{x-1} =$

9. Find analytically: $\lim_{\Delta x \to 0} \dfrac{\sin\left[\dfrac{\pi}{6} + \Delta x\right] - \dfrac{1}{2}}{\Delta x}$

10. Determine constants *b* and *c* so that $f(x) = \begin{cases} x+1 & 1 < x < 3 \\ x^2 + bx + c & |x-2| \geq 1 \end{cases}$ is continuous everywhere.

11. $\lim_{x \to \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 - x} =$

12. $\lim_{x\to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right)$

13. $\lim_{x\to\left(\frac{-\pi}{2}\right)^-} \frac{\sec x}{x}$

14. $\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$

15. Let $f(x)=\begin{cases} x, & x \text{ is irrational} \\ 0, & x \text{ is rational} \end{cases}$. Show that $\lim_{x\to 0} f(x)=0$. (Hint: use the Squeeze Theorem.)

16. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

17. Find and justify all horizontal and vertical asymptotes of $f(x)=\dfrac{x^3+1}{x^3+x}$. Make a graph.

18. Find and justify all horizontal and vertical asymptotes of $f(x)=\dfrac{\sqrt{2x^2+1}}{3x-5}$. Make a graph.