Magnet Analysis 1A
Rose

2/22/2010

Name KEY

Limits Review III

1. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

$$=\frac{12}{4}=3$$

2. $\lim\limits_{x\to\infty}\dfrac{1}{3^{-x}}$

$$\lim_{x\to\infty}3^x=\infty$$

3. $\lim\limits_{x\to\infty}\sin^2 x$

$$\lim_{x\to\infty}\left(\frac{1-\cos 2x}{2}\right)$$

$$\lim_{x\to\infty}\left[\frac{1}{2}-\frac{1}{2}\cos(2x)\right]$$

so d.n.e.

4. $\lim\limits_{x\to a^+}\dfrac{|x-a|}{x^2-a^2}$

$$\boxed{\begin{array}{l}\text{Since } x\to a^+,\\ x>a\\ \text{so } |x-a|=x-a\end{array}}$$

$$=\lim_{x\to a^+}\frac{\cancel{x-a}}{\cancel{(x-a)}(x+a)}$$

$$=\frac{1}{2a}$$

Since $x\to a^-$, $x<a$
so $|x-a|=-(x-a)$

5. $\lim\limits_{x\to a^-}\dfrac{|x-a|}{x^2-a^2}$

$$=\lim_{x\to a^-}\frac{-\cancel{(x-a)}}{\cancel{(x-a)}(x+a)}$$

$$=\frac{-1}{2a}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

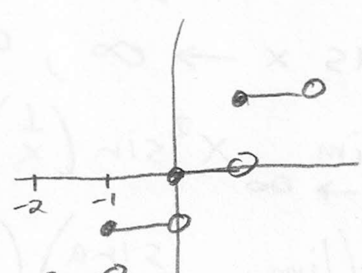
6. $\lim\limits_{h\to 0}\dfrac{(1+h)^6-1}{h}$

$$=\lim_{h\to 0}\frac{(h^6+6h^5+15h^4+20h^3+15h^2+6h\cancel{+1})\cancel{-1}}{h}$$

$$=\lim_{h\to 0}\left(h^5+6h^4+15h^3+20h^2+15h+6\right)$$

$$=6$$

7. $\lim\limits_{x\to-\frac{1}{2}}\lfloor 3x\rfloor=-2$



8. $\lim\limits_{x\to\infty} \dfrac{3x+4}{\sqrt{2x^2-5}} \left(\dfrac{\frac{1}{x}}{\frac{1}{x}}\right)$

$$\lim_{x\to\infty} \frac{(3x+4)\left(\frac{1}{x}\right)}{\sqrt{2x^2-5}\,\sqrt{\frac{1}{x^2}}}$$

$$\lim_{x\to\infty} \frac{3+\frac{4}{x} \to 0}{\sqrt{2-\frac{5}{x^2}} \to 0}$$

$$= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

9. $\lim\limits_{x\to\infty} \dfrac{\sqrt{1+x}}{\sqrt{9x-1}} \dfrac{\sqrt{\frac{1}{x}}}{\sqrt{\frac{1}{x}}}$

$$\lim_{x\to\infty} \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{9-\frac{1}{x}}} \quad \frac{1}{x}\to 0$$

$$= \frac{1}{3}$$

---

10. $\lim\limits_{x\to\infty} \left(\sqrt{x^2+3x}-x\right) \left(\dfrac{\sqrt{x^2+3x}+x}{\sqrt{x^2+3x}+x}\right)$

$$\lim_{x\to\infty} \frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x} = \lim_{x\to\infty} \frac{3x}{\sqrt{x^2+3x}+x} \left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{x^2+3x}\sqrt{\frac{1}{x^2}}+1} = \lim_{x\to\infty} \frac{3}{\sqrt{1+\frac{3}{x}}+1} = \frac{3}{2}$$

$$\frac{3}{x} \to 0$$

---

11. $\lim\limits_{x\to\infty} \left(2x-\sqrt{4x^2-5x}\right) \left(\dfrac{2x+\sqrt{4x^2-5x}}{2x+\sqrt{4x^2-5x}}\right)$

$$\lim_{x\to\infty} \frac{4x^2-(4x^2-5x)}{2x+\sqrt{4x^2-5x}} = \lim_{x\to\infty} \frac{5x}{2x+\sqrt{4x^2-5x}} \left(\frac{\frac{1}{x}}{\frac{1}{x}}\right)$$

$$= \lim_{x\to\infty} \frac{5}{2+\sqrt{4x^2-5x}\sqrt{\frac{1}{x^2}}} = \lim_{x\to\infty} \frac{5}{2+\sqrt{4-\frac{5}{x}}} = \frac{5}{4}$$

$$\frac{5}{x} \to 0$$

12. $\lim\limits_{x\to\infty} x^2 \sin\left(\dfrac{1}{x}\right)$

let $a = \frac{1}{x}$

as $x \to \infty$, $a \to 0^+$, so rewrite the limit

$$\lim_{x\to\infty} x^2 \sin\left(\frac{1}{x}\right) = \lim_{a\to 0^+} \frac{1}{a^2}\cdot \sin a$$

$$= \left(\lim_{a\to 0^+} \frac{\sin a}{a}\right)\left(\lim_{a\to 0^+} \frac{1}{a}\right) = 1\cdot\infty = \infty$$