Analysis 1A
Rose

Name KEY

Optional Extra Differentiation Practice

1. $\lim_{t\to 0}\frac{e^{\tan(x+t))}-e^{\tan x}}{t}=$

$\frac{d}{dx}\left[e^{\tan x}\right] = e^{\tan x}\cdot \sec^2 x$

2. $\lim_{h\to 0}\frac{\cos(x+h)^2-\cos x^2}{h}=\frac{d}{dx}\left[\cos(x^2)\right]$

$= -\sin(x^2)\cdot 2x$

3. $\lim_{x\to\frac{\pi}{3}}\frac{\sec x-2}{x-\frac{\pi}{3}}=\frac{d}{dx}\left[\sec x\right]\Big|_{x=\frac{\pi}{3}}$

$\sec x\tan x\Big|_{x=\frac{\pi}{3}}$

$= 2\sqrt{3}$

4. $\lim_{x\to 27}\frac{\log_3(\sqrt[3]{x})-1}{x-27}=\frac{d}{dx}\left[\log_3(\sqrt[3]{x})\right]\Big|_{x=27}$

$= \frac{1}{\ln 3}\cdot\frac{1}{x^{1/3}}\cdot\frac{1}{3}\frac{1}{x^{2/3}}\Big|_{x=27} = \frac{1}{x\ln 27}\Big|_{x=27}$

$= \frac{1}{27\ln 27} = \frac{1}{\ln 27^{27}}$

Directions: find the derivatives of the following functions, where *a, b,* and *k* are constants. Sometimes simplification prior to differentiation will make the work easier.

5. $f(x)=\frac{5x}{x^2-4}$

$f'(x)=\frac{5(x^2-4)-5x(2x)}{(x^2-4)^2}=\frac{5x^2-20-10x^2}{(x^2-4)^2}$

$=\frac{-5x^2-20}{(x^2-4)^2}=\frac{-5(x^2+4)}{(x^2-4)^2}$

6. $y=\frac{3t^5-t^2+6}{\sqrt{t}}$

$\Rightarrow y = 3t^{9/2}-t^{3/2}+6t^{-\frac{1}{2}}$

$\cancel{\frac{dy}{dx}} = \frac{27}{2}t^{7/2}-\frac{3}{2}t^{1/2}-3t^{-3/2}$

7. $f(x)=\tan^4(8x^3)$

$f'(x)=4\left[\tan(8x^3)\right]^3\sec^2(8x^3)\cdot 24x^2$

$96x^2\sec^2(8x^3)\tan^3(8x^3)$

8. $f(x)=\frac{1}{2\sin x\cos x}$

$f(x)=\frac{1}{\sin 2x} = \csc(2x)$

$f'(x)= -\csc(2x)\cot(2x)\cdot 2$

9. $f(x)=2^{-x}$

$f'(x)=\ln 2\cdot 2^{-x}\cdot(-1)$

$f'(x)=\frac{-\ln 2}{2^x}$

10. $f(x)=e^{\sin 3t}$ $\quad f'(x) = e^{\sin(3t)} \cdot \cos(3t) \cdot 3$

11. $f(x)=\cos(\arctan \pi x)$ $\quad = -\sin(\arctan \pi x) \cdot \frac{1}{(\pi x)^2+1} \cdot \pi$

$$= \frac{-\pi \sin(\arctan \pi x)}{\pi^2 x^2 + 1}$$

12. $f(x)=e^{3x}(x^2+7^x)$ $\quad f'(x) = e^{3x} \cdot 3(x^2+7^x) + e^{3x}(2x + \ln 7 \cdot 7^x)$

$$= e^{3x}(3x^2 + 3 \cdot 7^x + 2x + \ln 7 \cdot 7^x)$$

13. $f(x)=\ln(\sec(x^3))$ $\quad f'(x) = \frac{1}{\sec(x^3)} \cdot \sec(x^3)\tan(x^3) \cdot 3x^2$

$$= 3x^2 \tan(x^3)$$

14. $f(x)=\ln\left(\frac{e^{kx}}{b}\right)$

$f(x) = \ln(e^{kx}) - \ln b$

$f(x) = kx - \ln b$

$f'(x) = k$

15. $f(x)=\log_5(\tan x)$ $\quad f'(x) = \frac{1}{\ln 5} \cdot \frac{1}{\tan x} \cdot \sec^2 x$

$$f'(x) = \frac{\csc x \sec x}{\ln 5}$$

16. $f(x)=\frac{1}{8}\log_2(\csc x)$ $\quad f'(x) = \frac{1}{8} \cdot \frac{1}{\ln 2} \cdot \frac{1}{\csc x} \cdot -\csc x \cot x$

$$f'(x) = \frac{-\cot x}{8 \ln 2}$$

17. $f(x)=x^{a\cot x}$

$y = x^{a\cot x}$

$\ln y = a\cot x \cdot \ln x$

$\frac{1}{y} \cdot \frac{dy}{dx} = a(-\csc^2 x)\ln x + a\cot x \cdot \frac{1}{x}$

$\frac{dy}{dx} = a x^{a\cot x}\left[\frac{\cot x}{x} - \ln x \cdot \csc^2 x\right]$

18. $y=\frac{e^t - e^{-t}}{e^t + e^{-t}}$ $\quad \frac{dy}{dx} = \frac{(e^t+e^{-t})(e^t+e^{-t}) - (e^t-e^{-t})(e^t-e^{-t})}{(e^t+e^{-t})^2}$

$$= \frac{2-(-2)}{(e^t+e^{-t})^2} = \frac{4}{e^{2t}+2+\frac{1}{e^{2t}}} = \frac{4e^{2t}}{(e^{2t}+1)^2}$$

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