Analysis 1A
Rose

Name KEY

Differentiation Review

Directions: find the derivatives of the following functions, where *a, b,* and *k* are constants. Sometimes simplification prior to differentiation will make the work easier.

1. $f(x)=\dfrac{5}{(b^2-x^2)^2}$ $= 5(b^2-x^2)^{-2}$ $f'(x) = -10(b^2-x^2)^{-3}(-2x)$ $= \boxed{\dfrac{20x}{(b^2-x^2)^3}}$

2. $y = xe^{\tan x}$

$\dfrac{dy}{dx} = e^{\tan x} + xe^{\tan x}\cdot\sec^2 x = \boxed{e^{\tan x}(1+x\sec^2 x)}$

3. $f(x)=\arctan(3x^2+1)$

$f'(x) = \dfrac{1}{(3x^2+1)^2+1}\cdot 6x = \boxed{\dfrac{6x}{9x^4+6x^2+2}}$

4. $f(x)=a^{x^3-x}$

$f'(x) = \boxed{\ln a\cdot a^{x^3-x}(3x^2-1)}$

5. $f(x)=\dfrac{\sin(5-x)}{x^2}$

$f'(x) = \dfrac{\cos(5-x)(-1)x^2-\sin(5-x)(2x)}{(x^2)^2} = \boxed{\dfrac{-(x\cos(5-x)+2\sin(5-x))}{x^3}}$

6. $y=\ln\left(\cos\left(\dfrac{x}{k}\right)\right)$

$\dfrac{dy}{dx} = \dfrac{1}{\cos(\frac{x}{k})}\cdot -\sin(\frac{x}{k})\cdot\frac{1}{k} = \boxed{-\frac{1}{k}\tan(\frac{x}{k})}$

7. $y=\dfrac{x^3}{8}(2\ln x-1)$

$\dfrac{dy}{dx} = \frac{1}{8}\left[3x^2(2\ln x-1)+x^3\cdot\frac{2}{x}\right] = \frac{1}{8}\left[6x^2\ln x - x^2\right] = \boxed{\frac{x^2}{8}\left[6\ln x-1\right]}$

8. $f(x)=(\cos(x^2+3))^{100}$

$f'(x) = 100\cos^{99}(x^2+3)\cdot(-\sin(x^2+3))(2x) = \boxed{-200x\sin(x^2+3)\cos^{99}(x^2+3)}$

9. $f(x) = \dfrac{x}{\csc^2 x} = x\sin^2 x$

$f'(x) = \sin^2 x + x\,2\sin x\cos x = \boxed{\sin x(\sin x + 2x\cos x)}$

10. $f(x) = \ln\left(e^{ax^2-b}\right) = ax^2 - b$

$f'(x) = \boxed{2ax}$

11. $f(x) = \log_3 \sqrt{\sin x} = \log_3 (\sin x)^{\frac{1}{2}} = \frac{1}{2}\log_3(\sin x)$

$f'(x) = \dfrac{1}{2\ln 3}\cdot\dfrac{1}{\sin x}\cdot\cos x = \boxed{\dfrac{\cot x}{\ln 9}}$

12. $f(x) = \arcsin\left(e^{3x}\right)$

$f'(x) = \dfrac{1}{\sqrt{1-(e^{3x})^2}}\cdot e^{3x}\cdot 3 = \boxed{\dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}}$

13. $y = (\tan x)^x$

$\ln y = x\ln(\tan x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \ln(\tan x) + x\,\dfrac{1}{\tan x}\sec^2 x$

$\boxed{\dfrac{dy}{dx} = (\tan x)^x\left[\ln(\tan x) + x\sec x\csc x\right]}$

14. $y = (x+1)^{\sin x}$

$\ln y = \sin x\ln(x+1)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos x\ln(x+1) + \sin x\cdot\dfrac{1}{x+1}$

$\dfrac{dy}{dx} = (x+1)^{\sin x}\left[\cos x\ln(x+1) + \dfrac{\sin x}{x+1}\right]$

15. $x^3 - 4x^2 y + y^2 = 17$ Find $\dfrac{dy}{dx}$

$3x^2 - 8xy - 4x^2\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(2y - 4x^2) = 8xy - 3x^2$

$\boxed{\dfrac{dy}{dx} = \dfrac{8xy - 3x^2}{2y - 4x^2}}$

16. $\cos(xy) = x - 2y$ Find $\dfrac{dy}{dx}$

$-\sin(xy)\left[y + x\dfrac{dy}{dx}\right] = 1 - 2\dfrac{dy}{dx}$

$-y\sin(xy) - x\sin(xy)\dfrac{dy}{dx} = 1 - 2\dfrac{dy}{dx}$

$\dfrac{dy}{dx}(2 - x\sin(xy)) = 1 + y\sin(xy)$

$\boxed{\dfrac{dy}{dx} = \dfrac{1 + y\sin(xy)}{2 - x\sin(xy)}}$