Integral as Net Change

Net Change Theorem:

The integral of a rate of change is the net change: $\displaystyle\int_a^b F'(x)dx = F(b) - F(a)$

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1. If $V(t)$ is the volume of water in gallons in a reservoir at time t in hours, then its derivative $V'(t)$ is the rate of change of the volume of water at time t. If $V'(t) > 0$, then more water is flowing into the reservoir than is flowing out of the reservoir at that time and $V'(t)$ measures the net rate at which water is flowing into the reservoir. If $V'(t) < 0$, then more water is flowing out of the reservoir than is flowing into the reservoir at that time and $V'(t)$ measures the net rate at which water is flowing out of the reservoir. What does $\displaystyle\int_{t_1}^{t_2} V'(t)dt$ represent? (include units)

2. If an object moves along a straight line so that its position at time t seconds is given by function $s(t)$, measured in feet, then its velocity is $v(t) = s'(t)$. So what does $\displaystyle\int_{t_1}^{t_2} v(x)dx$ represent?

3. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\displaystyle\int_5^{10} w'(t)dt$ represent?

4. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\displaystyle\int_0^{120} r(t)dt$ represent?

5. A honeybee population starts with 100 bees and increases at a rate of $n(t)$ bees per week. Write an expression involving an integral for the number of honeybees after 15 weeks.

6. Let $P(r) = \dfrac{6}{\sqrt{1+r}}$ on $[0, \infty)$ represent the population density of Centerville in thousands of people per square mile at a point r miles from the city center. Make of sketch of the graph of P below.

I. Congressional District A (CDA) covers all people living between 3 and 4 miles from the city center. Find the population of CDA.

II. Congressional District B (CDB) is being created by the Centerville Census Bureau. The borders are being drawn so that it extends from all points 4 miles from the center out to a certain distance. If they want the population of CDB to be the same as CDA, where do the draw the line for CDB's outer border? (Set up by hand, integrate by calculator)