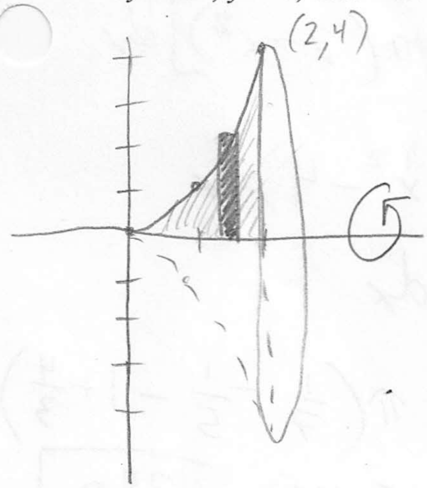
1. Find the volume of the solid formed by rotating about the x-axis the region bounded by $y = x^2$, $y = 0$, & $x = 2$.

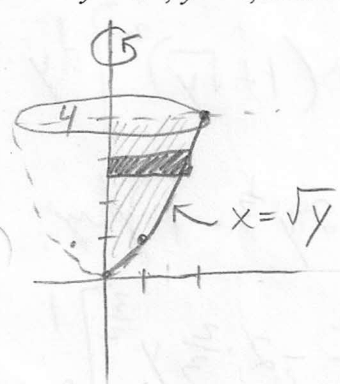


$$V \approx \sum_{i=1}^{n} \pi y^2 \Delta x$$

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \left[\left(x_i^*\right)^2 \right]^2 \Delta x$$

$$= \int_0^2 \pi x^4 \, dx = \pi \frac{x^5}{5} \Big]_0^2 = \boxed{\frac{32\pi}{5}}$$

2. Find the volume of the solid formed by rotating about the y-axis the region bounded by $y = x^2$, $y = 4$, & $x = 0$.

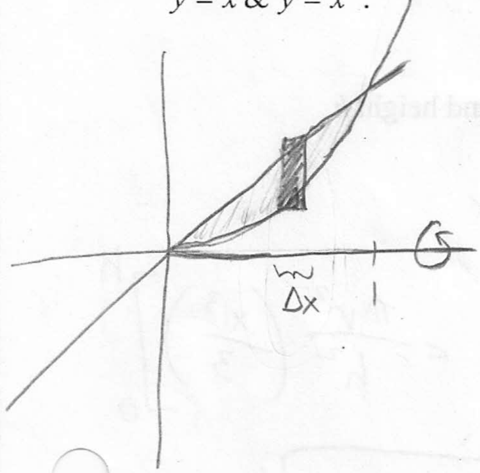


$$V \approx \sum_{i=1}^{n} \pi x^2 \Delta y$$

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} \pi \left(\sqrt{y_i^*}\right)^2 \Delta y$$

$$= \int_0^4 \pi y \, dy = \pi \frac{y^2}{2} \Big]_0^4 = \boxed{8\pi}$$

3. Find the volume of the solid formed by rotating about the x-axis the region bounded by $y = x$ & $y = x^2$.

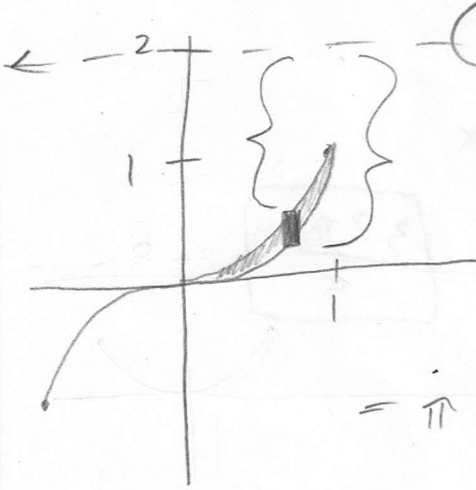


$$V \approx \sum_{i=1}^{n} \left[\pi (x_i)^2 - \pi \left(x_i^2\right)^2 \right] \Delta x$$

$$V = \int_0^1 \left(\pi x^2 - \pi x^4\right) dx$$

$$= \pi \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big]_0^1 = \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \boxed{\frac{2\pi}{15}}$$

4. Find the volume of the solid formed by rotating about the line $y = 2$ the region bounded by $y = x^2$ & $y = x^3$.



$$V=\int_0^1 \left[\pi(2-x^3)^2-\pi(2-x^2)^2\right]dx$$

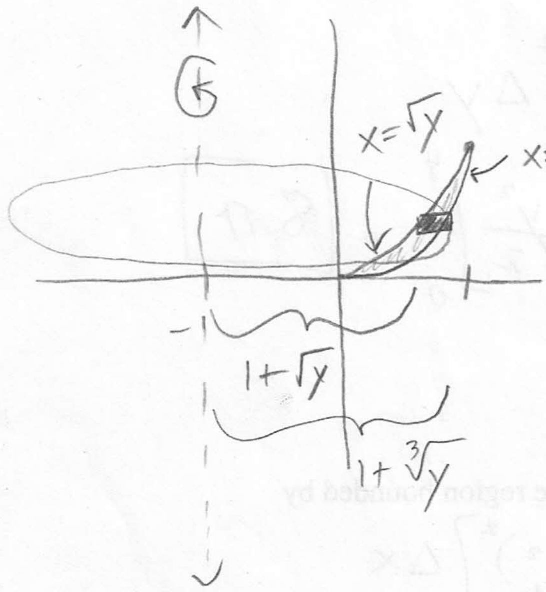
$$=\pi\int_0^1 (x^6-4x^3+4-x^4+4x^2-4)\,dx$$

$$=\pi\int_0^1 (x^6-x^4-4x^3+4x^2)\,dx$$

$$=\pi\left[\frac{x^7}{7}-\frac{x^5}{5}-x^4+\frac{4x^3}{3}\right]_0^1=\pi\left(\frac{1}{7}-\frac{1}{5}-1+\frac{4}{3}\right)$$

$$=\pi\left(\frac{15-21-105+140}{105}\right)=\pi\left(\frac{155-126}{105}\right)=\boxed{\frac{29\pi}{105}}$$

5. Find the volume of the solid formed by rotating about the line $x = -1$ the region bounded by $y = x^2$ & $y = x^3$.



$$V=\int_0^1\left[\pi(1+\sqrt[3]{y})^2-\pi(1+\sqrt{y})^2\right]dy$$

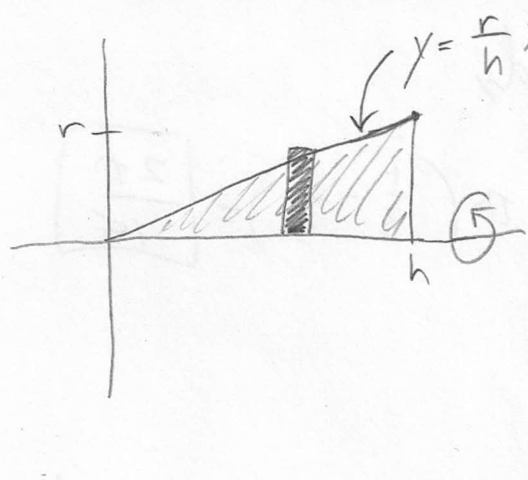
$$=\pi\int_0^1 \left(y^{2/3}+2y^{1/3}+1-y-2y^{\frac{1}{2}}-1\right)dy$$

$$=\pi\left[\frac{3}{5}y^{5/3}+2\cdot\frac{3}{4}y^{4/3}-\frac{y^2}{2}-2\cdot\frac{2}{3}y^{\frac{3}{2}}\right]_0^1$$

$$=\pi\left(\frac{3}{5}+\frac{3}{2}-\frac{1}{2}-\frac{4}{3}\right)=\pi\left(\frac{9}{15}+\frac{15}{15}-\frac{20}{15}\right)$$

$$=\boxed{\frac{4\pi}{15}}$$

6. Use calculus to derive the formula for the volume of a cone with radius r and height h.



$$V=\int_0^h \pi\left(\frac{r}{h}x\right)^2dx$$

$$=\frac{\pi r^2}{h^2}\int_0^h x^2\,dx=\frac{\pi r^2}{h^2}\left(\frac{x^3}{3}\right)\Big]_0^h$$

$$=\frac{\pi r^2}{h^2}\cdot\frac{h^3}{3}=\boxed{\frac{1}{3}\pi r^2 h}$$