Jakob Horvath
u1092049

N	$p = 2$	$p = 3$	$p = 4$	$p = 5$	$p = 6$	$p = 8$
17	0.3333333333	0.25	0.2000020345	0.1666717529	0.142867287	0.1111393299
33	0.3333333333	0.25	0.2000001272	0.1666669846	0.1428577782	0.1111128872
65	0.3333333333	0.25	0.2000000079	0.1666666865	0.1428571826	0.1111112223
129	0.3333333333	0.25	0.2000000005	0.1666666679	0.1428571453	0.1111111181
257	0.3333333333	0.25	0.2	0.1666666667	0.142857143	0.1111111115
513	0.3333333333	0.25	0.2	0.1666666667	0.1428571429	0.1111111111

The composite Simpson formula performed very well with x^p , as can be seen above. Lower level exponents resulted in more accuracy at small N than higher level exponents. This is most likely due to the increase in peaks and valleys that are observed in exponential functions as p becomes larger.

N	$1 + sin(x) * cos(2x/3) * sin(4x)$
17	6.311734901
33	6.305321316
65	6.305178204
129	6.30517101
257	6.305170584
513	6.305170557

Once $N = 65$, Simpson's Rule produces a fairly accurate measure of $1 + sin(x) * cos(2x/3) * sin(4x)$ from 0 to 2Π . $N = 17$ is the only outlier in terms of accuracy of the estimated integral, and would not suffice in most circumstances.

tol	Q	fcount	Time (seconds)
1.00E-07	0.5221864391	813	0.0110301
1.00E-08	0.5315944606	1365	0.0105523
1.00E-09	0.5315944526	2089	0.0166355
1.00E-10	0.531594452	3177	0.0322795
1.00E-11	0.5315944519	5013	0.0403689
1.00E-12	0.5315944519	7841	0.0604422
1.00E-13	0.5315944519	12241	0.0930336
1.00E-14	0.5315944519	19545	0.1482685

As the tolerance became more strict, the number of function calls to $cos(x^3)^{200}$ inside quadtx increased, as well as the time it took to obtain the desired result.

tol	Q	fcount
1.00E-07	0.5315944523	4101
1.00E-08	0.5315944522	7213
1.00E-09	0.5315944519	12709
1.00E-10	0.5315944519	22285
1.00E-11	0.5315944519	38645
1.00E-12	0.5315944519	69497
1.00E-13	0.5315944519	125205

Changing the error to be less than $tol(h/(b-a))$ certainly resulted in more function evaluations. It should also be noted that the last output could not be determined, because the sheer amount of necessary recursions caused Matlab to run out of memory.

N	Simpson	Error Estimate
33	0.4309196626	---
65	0.5242063043	0.006219109444
129	0.5614196896	0.002480892355
257	0.5391595661	0.001484008234
513	0.5287490207	0.0006940363631
1025	0.5315268309	0.0001851873487

quadtx	fcount
0.4313135523	21
0.4313135523	21
0.4312136392	25
0.4312136392	25
0.4391010393	73

Using an amount of intervals that ranged into the thousands, Simpson's Rule was able to eventually create somewhat accurate output. Plugging in $(1/15)(Simp(h/2) - Simp(h))$ for quadtx's tol parameter did not result in precise measurements for the integral. However, the accuracy still increased a tiny percentage when Simpson's Rule was able to make use of larger N values (more intervals), and the fcount reflects this as well.