Jakob Horvath
u1092049

Actual b	Jacobi b	Gauss-Seidel b
0.000244140625	0.0001644935137	0.0002439960202
0.000244140625	6.56E-05	0.0002438278424
0.000244140625	-1.25E-05	0.0002437123469
0.000244140625	-0.0001008210359	0.0002436927973
0.000244140625	0.0002370424624	0.000243773668
0.000244140625	5.42E-05	0.0002439253236
0.000244140625	0.0001225977768	0.0002441493542
0.000244140625	0.0002470341435	0.000244140625

Using the parameter **c** to check for convergence, the Gauss-Seidel method clearly works better than the Jacobi method. Gauss-Seidel is able to narrow down it’s results to be within limit given the tolerance of 1.0e-5. The last row matches the actual vector **b**’s last value exactly.

N=16				N=32				N=64			
tol=1.e-3	tol=1.e-5	tol=1.e-7	tol=1.e-9	tol=1.e-3	tol=1.e-5	tol=1.e-7	tol=1.e-9	tol=1.e-3	tol=1.e-5	tol=1.e-7	tol=1.e-9
itr=2	itr=11	itr=11	itr=11	itr=2	itr=11	itr=11	itr=11	itr=2	itr=2	itr=11	itr=11
7.48E-06	1.18E-05	1.18E-05	1.18E-05	4.68E-07	7.35E-07	7.35E-07	7.35E-07	2.92E-08	2.92E-08	4.59E-08	4.59E-08
2.79E-06	7.43E-06	7.43E-06	7.43E-06	1.74E-07	4.65E-07	4.65E-07	4.65E-07	1.09E-08	1.09E-08	2.91E-08	2.91E-08
5.17E-07	3.68E-06	3.68E-06	3.68E-06	3.23E-08	2.34E-07	2.34E-07	2.34E-07	2.02E-09	2.02E-09	1.46E-08	1.46E-08
-1.98E-07	1.11E-06	1.11E-06	1.11E-06	-1.24E-08	7.87E-08	7.87E-08	7.87E-08	-7.75E-10	-7.75E-10	4.92E-09	4.92E-09
-2.56E-07	2.62E-07	2.62E-07	2.62E-07	-1.60E-08	-3.06E-09	-3.06E-09	-3.06E-09	-9.99E-10	-9.99E-10	-1.91E-10	-1.91E-10
-1.49E-07	-2.17E-07	-2.17E-07	-2.17E-07	-9.32E-09	-3.20E-08	-3.20E-08	-3.20E-08	-5.83E-10	-5.83E-10	-2.00E-09	-2.00E-09
-5.84E-08	-3.13E-07	-3.13E-07	-3.13E-07	-3.65E-09	-3.22E-08	-3.22E-08	-3.22E-08	-2.28E-10	-2.28E-10	-2.01E-09	-2.01E-09
-1.32E-08	-1.30E-06	-1.30E-06	-1.30E-06	-8.24E-10	-2.18E-08	-2.18E-08	-2.18E-08	-5.15E-11	-5.15E-11	-1.36E-09	-1.36E-09
1.81E-09	-1.32E-06	-1.32E-06	-1.32E-06	1.13E-10	-1.10E-08	-1.10E-08	-1.10E-08	7.07E-12	7.07E-12	-6.90E-10	-6.90E-10
3.83E-09	-1.62E-06	-1.62E-06	-1.62E-06	2.39E-10	-3.75E-09	-3.75E-09	-3.75E-09	1.50E-11	1.50E-11	-2.34E-10	-2.34E-10
-1.67E-07	5.79E-07	5.79E-07	5.79E-07	1.49E-10	-1.09E-10	-1.09E-10	-1.09E-10	9.34E-12	9.34E-12	-6.79E-12	-6.79E-12
-1.64E-06	2.09E-06	2.09E-06	2.09E-06	6.11E-11	1.07E-09	1.07E-09	1.07E-09	3.82E-12	3.82E-12	6.67E-11	6.67E-11
-3.54E-06	6.84E-06	6.84E-06	6.84E-06	1.53E-11	1.04E-09	1.04E-09	1.04E-09	9.57E-13	9.57E-13	6.51E-11	6.51E-11
2.79E-05	9.15E-06	9.15E-06	9.15E-06	-5.55E-13	6.03E-10	6.03E-10	6.03E-10	-3.47E-14	-3.47E-14	4.06E-11	4.06E-11
-2.12E-05	1.73E-05	1.73E-05	1.73E-05	-3.22E-12	8.40E-10	8.40E-10	8.40E-10	-2.01E-13	-2.01E-13	1.83E-11	1.83E-11
1.53E-05	1.53E-05	1.53E-05	1.53E-05	-2.16E-12	-2.33E-09	-2.33E-09	-2.33E-09	-1.35E-13	-1.35E-13	4.87E-12	4.87E-12
0	0	0	0	-9.22E-13	4.63E-09	4.63E-09	4.63E-09	-5.76E-14	-5.76E-14	-8.19E-13	-8.19E-13
0	0	0	0	-2.50E-13	-1.49E-09	-1.49E-09	-1.49E-09	-1.56E-14	-1.56E-14	-2.11E-12	-2.11E-12
0	0	0	0	-6.62E-15	-4.41E-09	-4.41E-09	-4.41E-09	-4.14E-16	-4.14E-16	-1.64E-12	-1.64E-12
0	0	0	0	4.06E-14	-9.08E-09	-9.08E-09	-9.08E-09	2.54E-15	2.54E-15	-8.69E-13	-8.69E-13
0	0	0	0	2.94E-14	1.94E-08	1.94E-08	1.94E-08	1.84E-15	1.84E-15	-3.15E-13	-3.15E-13
0	0	0	0	1.31E-14	1.85E-08	1.85E-08	1.85E-08	8.17E-16	8.17E-16	-4.03E-14	-4.03E-14
0	0	0	0	3.78E-15	1.26E-08	1.26E-08	1.26E-08	2.36E-16	2.36E-16	4.86E-14	4.86E-14
0	0	0	0	2.74E-16	-5.92E-08	-5.92E-08	-5.92E-08	1.71E-17	1.71E-17	5.21E-14	5.21E-14
0	0	0	0	-4.86E-16	-7.16E-08	-7.16E-08	-7.16E-08	-3.03E-17	-3.03E-17	3.18E-14	3.18E-14
0	0	0	0	-3.84E-16	-9.73E-08	-9.73E-08	-9.73E-08	-2.40E-17	-2.40E-17	1.35E-14	1.35E-14
0	0	0	0	-1.06E-08	3.63E-08	3.63E-08	3.63E-08	-1.11E-17	-1.11E-17	3.26E-15	3.26E-15
0	0	0	0	-1.02E-07	1.29E-07	1.29E-07	1.29E-07	-3.40E-18	-3.40E-18	-6.67E-16	-6.67E-16
0	0	0	0	-2.21E-07	4.27E-07	4.27E-07	4.27E-07	-3.71E-19	-3.71E-19	-1.37E-15	-1.37E-15
0	0	0	0	1.75E-06	5.71E-07	5.71E-07	5.71E-07	3.47E-19	3.47E-19	-9.70E-16	-9.70E-16
0	0	0	0	-1.32E-06	1.08E-06	1.08E-06	1.08E-06	3.03E-19	3.03E-19	-4.63E-16	-4.63E-16
0	0	0	0	9.54E-07	9.54E-07	9.54E-07	9.54E-07	1.47E-19	1.47E-19	-1.43E-16	-1.43E-16
0	0	0	0	0	0	0	0	4.73E-20	4.73E-20	-4.79E-18	-4.79E-18
0	0	0	0	0	0	0	0	6.59E-21	6.59E-21	3.05E-17	3.05E-17
0	0	0	0	0	0	0	0	-3.80E-21	-3.80E-21	2.59E-17	2.59E-17
0	0	0	0	0	0	0	0	-3.72E-21	-3.72E-21	1.37E-17	1.37E-17
0	0	0	0	0	0	0	0	-1.88E-21	-1.88E-21	4.88E-18	4.88E-18
0	0	0	0	0	0	0	0	-6.29E-22	-6.29E-22	7.12E-19	7.12E-19
0	0	0	0	0	0	0	0	-1.19E-22	-1.19E-22	-5.69E-19	-5.69E-19

0	0	0	0	0	0	0	0	6.62E-23	6.62E-23	-6.19E-19	-6.19E-19
0	0	0	0	0	0	0	0	2.65E-23	2.65E-23	6.54E-17	6.54E-17
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-1.34E-15	-1.34E-15
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	7.19E-15	7.19E-15
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	2.11E-14	2.11E-14
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-1.51E-13	-1.51E-13
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-2.95E-12	-2.95E-12
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	3.43E-11	3.43E-11
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-1.50E-10	-1.50E-10
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	2.90E-10	2.90E-10
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-9.12E-11	-9.12E-11
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-2.74E-10	-2.74E-10
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-5.67E-10	-5.67E-10
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	1.21E-09	1.21E-09
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	1.15E-09	1.15E-09
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	7.90E-10	7.90E-10
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-3.70E-09	-3.70E-09
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-4.47E-09	-4.47E-09
0	0	0	0	0	0	0	0	6.62E-24	6.62E-24	-6.08E-09	-6.08E-09
0	0	0	0	0	0	0	0	-6.62E-10	-6.62E-10	2.27E-09	2.27E-09
0	0	0	0	0	0	0	0	-6.40E-09	-6.40E-09	8.08E-09	8.08E-09
0	0	0	0	0	0	0	0	-1.38E-08	-1.38E-08	2.67E-08	2.67E-08
0	0	0	0	0	0	0	0	1.09E-07	1.09E-07	3.57E-08	3.57E-08
0	0	0	0	0	0	0	0	-8.27E-08	-8.27E-08	6.75E-08	6.75E-08
0	0	0	0	0	0	0	0	5.96E-08	5.96E-08	5.96E-08	5.96E-08

Using matrix ranks of 16, 32, and 64 results in **b** vectors with individual values around 1.53e-5, 9.54e-7, and 5.96e-8 respectively. Tolerance levels used were 1.e-3, 1.e-5, 1.e-7, and 1.e-9. As can be seen from the lengthy table above, the Gauss-Seidel method struggled to produce accurate estimates when asked to deal with larger matrices than the previous problem. When n=16, the estimate for **b** was reasonable, but n=32 and n-64 gave large errors in the middle range of the estimates. Interestingly, values near the ends of the vectors were always close to error-free. In addition, the number of iterations was always either 2 or 11, with higher tolerances resulting in more accuracy, as is expected.

N=16			N=32			N=64		
tol=1.e-3	tol=1.e-4	tol=1.e-5	tol=1.e-3	tol=1.e-4	tol=1.e-5	tol=1.e-3	tol=1.e-4	tol=1.e-5
itr=2	itr=2	itr=2	itr=2	itr=2	itr=2	itr=2	itr=2	itr=2
6.27E-06	6.27E-06	6.27E-06	3.92E-07	3.92E-07	3.92E-07	2.45E-08	2.45E-08	2.45E-08
2.07E-06	2.07E-06	2.07E-06	1.30E-07	1.30E-07	1.30E-07	8.10E-09	8.10E-09	8.10E-09
2.96E-07	2.96E-07	2.96E-07	1.85E-08	1.85E-08	1.85E-08	1.16E-09	1.16E-09	1.16E-09
-1.84E-07	-1.84E-07	-1.84E-07	-1.15E-08	-1.15E-08	-1.15E-08	-7.18E-10	-7.18E-10	-7.18E-10
-1.89E-07	-1.89E-07	-1.89E-07	-1.18E-08	-1.18E-08	-1.18E-08	-7.37E-10	-7.37E-10	-7.37E-10
-1.00E-07	-1.00E-07	-1.00E-07	-6.28E-09	-6.28E-09	-6.28E-09	-3.92E-10	-3.92E-10	-3.92E-10
-3.62E-08	-3.62E-08	-3.62E-08	-2.26E-09	-2.26E-09	-2.26E-09	-1.41E-10	-1.41E-10	-1.41E-10
-7.00E-09	-7.00E-09	-7.00E-09	-4.38E-10	-4.38E-10	-4.38E-10	-2.74E-11	-2.74E-11	-2.74E-11
1.75E-09	1.75E-09	1.75E-09	1.10E-10	1.10E-10	1.10E-10	6.85E-12	6.85E-12	6.85E-12
2.53E-09	2.53E-09	2.53E-09	1.58E-10	1.58E-10	1.58E-10	9.87E-12	9.87E-12	9.87E-12
-7.48E-08	-7.48E-08	-7.48E-08	9.10E-11	9.10E-11	9.10E-11	5.69E-12	5.69E-12	5.69E-12
-7.37E-07	-7.37E-07	-7.37E-07	3.50E-11	3.50E-11	3.50E-11	2.18E-12	2.18E-12	2.18E-12
-1.03E-06	-1.03E-06	-1.03E-06	7.90E-12	7.90E-12	7.90E-12	4.94E-13	4.94E-13	4.94E-13
1.98E-05	1.98E-05	1.98E-05	-8.19E-13	-8.19E-13	-8.19E-13	-5.12E-14	-5.12E-14	-5.12E-14
2.35E-05	2.35E-05	2.35E-05	-2.00E-12	-2.00E-12	-2.00E-12	-1.25E-13	-1.25E-13	-1.25E-13
1.53E-05	1.53E-05	1.53E-05	-1.24E-12	-1.24E-12	-1.24E-12	-7.77E-14	-7.77E-14	-7.77E-14
0	0	0	-5.04E-13	-5.04E-13	-5.04E-13	-3.15E-14	-3.15E-14	-3.15E-14
0	0	0	-1.27E-13	-1.27E-13	-1.27E-13	-7.91E-15	-7.91E-15	-7.91E-15
0	0	0	2.36E-15	2.36E-15	2.36E-15	1.48E-16	1.48E-16	1.48E-16
0	0	0	2.42E-14	2.42E-14	2.42E-14	1.51E-15	1.51E-15	1.51E-15
0	0	0	1.63E-14	1.63E-14	1.63E-14	1.02E-15	1.02E-15	1.02E-15
0	0	0	6.94E-15	6.94E-15	6.94E-15	4.34E-16	4.34E-16	4.34E-16
0	0	0	1.89E-15	1.89E-15	1.89E-15	1.18E-16	1.18E-16	1.18E-16
0	0	0	7.55E-17	7.55E-17	7.55E-17	4.72E-18	4.72E-18	4.72E-18
0	0	0	-2.82E-16	-2.82E-16	-2.82E-16	-1.76E-17	-1.76E-17	-1.76E-17
0	0	0	-2.07E-16	-2.07E-16	-2.07E-16	-1.30E-17	-1.30E-17	-1.30E-17
0	0	0	-4.77E-09	-4.77E-09	-4.77E-09	-5.78E-18	-5.78E-18	-5.78E-18
0	0	0	-4.61E-08	-4.61E-08	-4.61E-08	-1.69E-18	-1.69E-18	-1.69E-18
0	0	0	-6.45E-08	-6.45E-08	-6.45E-08	-1.42E-19	-1.42E-19	-1.42E-19
0	0	0	1.24E-06	1.24E-06	1.24E-06	1.99E-19	1.99E-19	1.99E-19
0	0	0	1.47E-06	1.47E-06	1.47E-06	1.61E-19	1.61E-19	1.61E-19
0	0	0	9.54E-07	9.54E-07	9.54E-07	7.53E-20	7.53E-20	7.53E-20
0	0	0	0	0	0	2.33E-20	2.33E-20	2.33E-20
0	0	0	0	0	0	2.79E-21	2.79E-21	2.79E-21
0	0	0	0	0	0	-2.17E-21	-2.17E-21	-2.17E-21
0	0	0	0	0	0	-1.93E-21	-1.93E-21	-1.93E-21
0	0	0	0	0	0	-9.76E-22	-9.76E-22	-9.76E-22

0	0	0	0	0	0	-3.08E-22	-3.08E-22	-3.08E-22
0	0	0	0	0	0	-5.29E-23	-5.29E-23	-5.29E-23
0	0	0	0	0	0	3.97E-23	3.97E-23	3.97E-23
0	0	0	0	0	0	6.62E-24	6.62E-24	6.62E-24
0	0	0	0	0	0	6.62E-24	6.62E-24	6.62E-24
0	0	0	0	0	0	6.62E-24	6.62E-24	6.62E-24
0	0	0	0	0	0	-1.99E-23	-1.99E-23	-1.99E-23
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-6.62E-24	-6.62E-24	-6.62E-24
0	0	0	0	0	0	-2.98E-10	-2.98E-10	-2.98E-10
0	0	0	0	0	0	-2.88E-09	-2.88E-09	-2.88E-09
0	0	0	0	0	0	-4.03E-09	-4.03E-09	-4.03E-09
0	0	0	0	0	0	7.75E-08	7.75E-08	7.75E-08
0	0	0	0	0	0	9.18E-08	9.18E-08	9.18E-08
0	0	0	0	0	0	5.96E-08	5.96E-08	5.96E-08

Once again using matrix ranks of 16, 32, and 64, along with new tolerances of 1.e-3, 1.e-4, and 1.e-5, we see that using under-relaxation helped reduce the number of iterations for each trial to be only 2. However, the accuracy went down considerably due to this increase in speed. After testing variations of omega, I settled on a value of 0.45. As before, the ends of the vector **b** estimates display reasonable values, but the rest tend to drift very far off from the actual result.