The discrete geometric origin of elementary particles

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Abstract

This work introduces a geometric model based on octahedral symmetry to explain the experimentally measured ratio of neutrino squared-mass differences, which is close to the value of 33.

Introduction

The Standard Model of particle physics, established in the last century, is a fundamental theory that has successfully addressed numerous problems and profoundly influenced modern science. Despite its remarkable achievements, the Standard Model also presents significant unresolved issues, such as the origins of quark and lepton masses, the neutrino mass-generation mechanism, quark confinement, mixing angles, and CP violation. These limitations indicate the necessity for new theoretical frameworks, as many physical quantities within the Standard Model cannot be determined from first principles alone and currently rely on experimental input. While its overall framework is robust, obtaining constraints on these quantities from other perspectives is essential to enhance the theory's predictive power.

Motivated by the study of the Riemann Hypothesis—a problem of profound difficulty—we note a potential connection with quantum gravity. The primes studied in the Riemann Hypothesis exhibit a loss of directional properties at a certain level, analogous to the fundamental absence of directionality at the smallest scales of spacetime. Guided by the duality between physical reality and mathematical theory, we propose a novel multipoint duality, distinct from conventional two-point duality. Within a new framework combining octahedral geometry and this duality, we generalize the Higgs field to a quaternion representation. This approach allows for a more unified description of Majorana and Dirac neutrinos. Notably, the theory introduces no free parameters and geometrically predicts the Weinberg angle as well as a ratio of neutrino squared-mass differences approximately equal to 33.

The Mystery of Neutrino Mass

Duality

The set of natural numbers, Z, is fundamentally structured by addition. Selecting a unit element and iterating addition generates all elements of Z, which are equally spaced and exhibit a clear, increasing direction. Furthermore, both addition and multiplication in Z are commutative, forming the foundation for all linear structures. This set exemplifies the compatibility between directionality and distributional simplicity.

Consider an infinite set A whose elements are all drawn from Z. A natural question arises: can we find a function that establishes a correspondence between A and Z? The existence of such a function would indicate that A inherits the properties of Z, maintaining the compatibility of directionality with quantifiable distribution.

When we examine the set of prime numbers, P, we find that no analytic function with well-behaved properties can map it to Z. This demonstrates that within P, directionality and distributional simplicity are incompatible. Just as the local arrangement of primes in finite intervals is unpredictable and can only be described statistically in the limit, the two properties cannot coexist—yet they are not independent.

This suggests a form of duality: when the distribution is sufficiently simple, the primes must reside in a regime where directionality cannot be defined. This is reflected in the Riemann zeta function, where all non-trivial zeros lie on the critical line $Re(s) = 1/2$, yet they are infinite in number. If we interpret the real part as distributional simplicity and the imaginary part as direction, summing over all zeros yields a net result of zero direction, implying an overall absence of orientability.

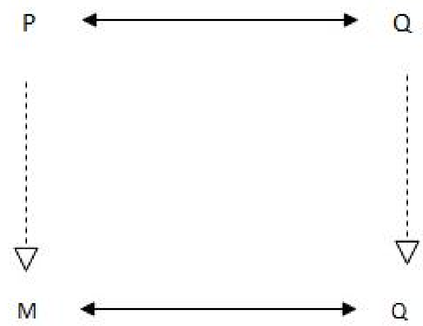
Returning to theoretical physics, it is essential to distinguish between physical reality and physical prediction. Physical reality is closely tied to experimental observation, while physical prediction is inherently mathematical. The fact that many quantities within the Standard Model cannot be theoretically predicted suggests that its underlying mathematical framework remains approximate—ultimately because its calculations rely fundamentally on addition and linear structures. This reveals that current mathematical formulations have not fundamentally transcended linearity and commutativity.

If a grand unified theory were fully nonlinear and represented an ultimate description of physics, it would imply that, in extreme regimes, physics and mathematics become dual. Consider now the case of quantum gravity. This theory posits that space possesses a minimal nonzero scale, beyond which further subdivision is meaningless. Scale itself embodies a form of directionality; thus, the existence of a fundamental length indicates a loss of directional definability at the deepest level. This parallels the behavior of prime numbers, suggesting that the mathematics describing primes may be dual to that of quantum gravity on certain scales.

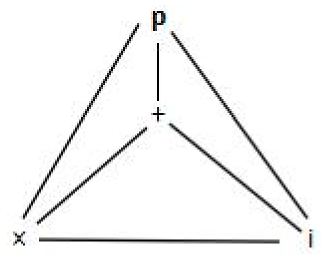
Among available tools, the zeta function stands out as a particularly useful—though still incompletely understood—mathematical object for such explorations. The energy and length scales of the Standard Model remain far below those relevant to quantum gravity. Consequently, using the zeta function to study certain physical questions will inevitably introduce corrections, potentially leading to better alignment with experimental observations.

High-dimensional duality

A common duality is the duality between two objects, as shown in the figure below



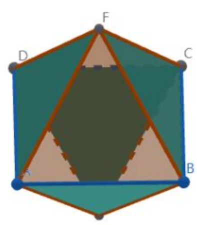
Downwards we indeed obtain a new set of dual relationships, which represent the transitivity of relations. The paradigm of modern science is seeking deeper dualities or correspondences, but if the world is not merely two-dimensional dualities, it will form a closed structure, as shown in the figure below.



Among them, real numbers, imaginary numbers, addition, and multiplication form a tetrahedral dual relationship, with each element being indispensable. This results in the inability to return to the starting point when choosing a fixed direction of movement, indicating that this duality possesses the topological characteristics of discrete geometry. The genus of a regular tetrahedron is 0, and its Euler characteristic is 2. Naturally, we consider that there are four other regular polyhedra that meet the same conditions, which extends to four dual relationships. In terms of physical reality, tetrahedral duality can represent four - dimensional spacetime, while the other dualities can represent internal physical spaces.

Octahedral duality and particle structure

The octahedron is relatively simple and has a non-trivial structure, capable of accommodating four types of electroweak bosons, electrons, and neutrinos. Below I will present their structure



This describes the structure of the electroweak gauge bosons within an octahedral framework, where the four distinct faces correspond—after symmetry breaking—to the photon, W^+, W^-, and Z bosons. In an octahedral geometry, particles and antiparticles are related by mirror symmetry. Notably, no fixed plane within the octahedron remains invariant under this mirror operation, implying that under conditions of high symmetry, none of the four bosons possesses a distinct antiparticle counterpart.

The emergence of the W bosons can be understood as a consequence of primordial symmetry, under which the spacetime background itself is symmetric. Upon embedding into four-dimensional spacetime, however, this background symmetry is broken, thereby inducing a corresponding breaking in the symmetry of the electroweak bosons. This breaking mechanism should be governed by differences in the geometric parameters shared between the tetrahedral and octahedral structures. In electroweak theory, a key parameter quantifying such symmetry breaking is the Weinberg angle.

$$\sin^2\omega = \sin\frac{\alpha}{2} - \sin\frac{\beta}{2}$$

Here, α is the octahedral dihedral angle and β is the tetrahedral dihedral angle, which can be obtained through calculation

$$\sin^2\omega = \frac{\sqrt{2}}{\sqrt{3}} - \frac{1}{\sqrt{3}}$$

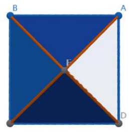
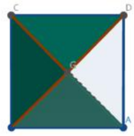
The approximate value of this parameter is 0.2391, but this is only the breaking parameter. The Weinberg angle is also related to the mass ratio of the Z boson to the W boson, meaning that the Higgs mechanism will also produce a correction to it, so the small discrepancy between the currently calculated value and the experimental value is completely acceptable

Higgs mechanism

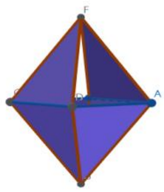
In the octahedral model, one face is selected as the reference plane. The direction associated with the Higgs mechanism is taken to be perpendicular to this plane. As a result, particles confined to the reference plane remain massless, while the other three gauge bosons acquire mass. In what follows, we will address the origin of neutrino mass and extend the description of the Higgs mechanism accordingly. The corresponding mathematical formulation will then be clearly presented.

Neutrinos and electrons

Since leptons exist in three generations, and including antiparticles there are a total of 12 states, they cannot be directly accommodated within a single octahedral representation. Nevertheless, the left-handed doublets formed by neutrinos and electrons in each generation suggest a complementary relationship. This structure can be derived by examining the same underlying arrangement from distinct orientations—a description that naturally requires six faces, as illustrated in the accompanying diagram.

Majorana neutrinos also require six faces, and such a structure prevents the neutrino from finding a unified mirror symmetry direction and remaining invariant, as shown in the figure below



These are the three generations of leptons, and now that we have completed the theoretical framework, we will move on to the mathematical section

Higgs field and quaternions

The electroweak Higgs field is usually defined as a complex doublet

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}$$

However, high-energy situations may involve not only changes in the number of heavy states but also changes in the number system. In the number system corresponding to an octahedron, quaternions can serve as the base number system, with the maximum Higgs field being a quaternion quartet, but the electroweak theory does not require so many states, so let's first define a Higgs singlet

$$\psi = \psi_0 + \psi_1 i + \psi_2 j + \psi_3 k$$

Here ijk are the quaternion units, satisfying

$$i^2 = j^2 = k^2 = -ijk = -1$$

This quaternion monad can be represented as a 2*2 complex matrix

$$\psi = \begin{pmatrix} \psi_0 + i\psi_3 & \psi_2 + i\psi_1 \\ -\psi_2 + i\psi_1 & \psi_0 - i\psi_3 \end{pmatrix}$$

Through the Pauli matrices, this naturally forms an SU(2)_L doublet, but at this point, obtaining the Lagrangian requires all fields to be extended to quaternions, which is unrealistic. Even if it could be extended, it would only describe the behaviour of particles at high energy. For Majorana neutrinos, their mass can be extremely high, so conventional methods cannot be used, but duality can.

Majorana-Dirac duality

Assuming that Majorana neutrinos and Dirac neutrinos are just different manifestations of the same particle at high and low energies, we can obtain a kind of duality in which both are indispensable. However, from polyhedral duality, we know that duality corresponds to an invariant; discrete duality is a topological invariant and continuous duality corresponds to a universal equation. We know that this equation is the Euler-Lagrange equation, but at this point the total field must have high-energy characteristics, indicating a quaternionic multiplet. Through the Euler-Lagrange equation, we obtain the equations of motion for quaternions. Quaternions can be projected back to complex numbers, at which point the equation should be the sum of the complex motion equation of the Majorana neutrino, the Dirac neutrino equation, and the electron equation. This principle guides our computational direction.

Ratio of neutrino mass variance

The neutrino field in high-energy situations must possess both Majorana and Dirac characteristics; we consider a quaternion doublet.

$$\Phi = \begin{pmatrix} \psi_M \\ \psi_D \end{pmatrix}$$

The Majorana neutrino is its own antimatter; in the quaternion system, the antimatter conjugation differs from complex conjugation, but we believe that the Majorana neutrino remains invariant under this stronger conjugation as well

$$\psi_M = \bar{\psi}_M \qquad \psi_D \neq \bar{\psi}_D$$

Definition

$$\psi = \psi_M \Phi$$

Because this quaternion conjugation preserves the modulus, the upper part of this doublet is constant, and its dynamic properties are determined by the lower part

$$\psi = \bar{\psi}_M \psi_D$$

Now consider this single-mode Yukawa coupling

$$\zeta_Y = -g\bar{\psi} Y \phi \psi$$

Here g is the coupling constant and Y is a 3*3 quaternion matrix. According to the standard Higgs mechanism, the Higgs field selects a particular direction

$$\phi = \frac{\nu}{\sqrt{2}}$$

Here v is a quaternion in a special direction, and the Yukawa coupling becomes

$$\zeta_Y = -g\bar{\psi} M \psi$$

M is a raw matrix containing mass information, and it can be divided into two parts

$$M = A_0 + A_1$$

The Yukawa coupling is decomposed into two terms

$$\zeta_{Y_0} = -g\bar{\psi}_D \psi_M A_0 \psi_M \bar{\psi}_D \qquad\qquad \zeta_{Y_1} = -g\bar{\psi}_D \psi_M A_1 \psi_M \bar{\psi}_D$$

among them

$$A_0 \psi_M = \psi_M A_0 \qquad\qquad A_1 \psi_D = \psi_D A_1$$

$$A_1 \psi_M = \kappa \psi_M \qquad\qquad A_0 \psi_D = \lambda \psi_D$$

This indicates that the Majorana state only couples with the A1 part, while the Dirac state only couples with the A0 part, which can be seen as the source of their mass difference. The second line is an equation similar to the eigenvalue equation, with all factors being quaternions, further

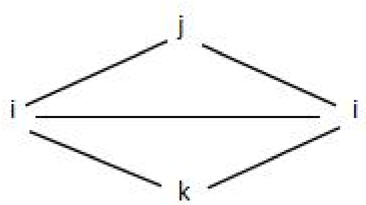
$$\zeta_{Y_0} = -g\bar{\psi}_D \lambda \psi_D$$

$$\zeta_{Y_1} = -g\bar{\psi}_D \psi_M \kappa \psi_M \psi_D$$

Now let's consider

$$\lambda\bar{\lambda} = \lambda^2 = \lambda_i^2 + \lambda_k^2 + \lambda_j^2$$

This indicates that the sum of the squares of the three quaternion components is restricted. Since we ultimately want to revert to complex numbers, but now i, j, k are on equal footing, we can write three transformations, given by the figure below and its rotation



Returning from different directions to a complex number yields different results, similar to parallel displacement, each transformation corresponds to a type of curvature, and the sum of the three curvatures is zero, which is what we know as the Bianchi identity

$$R_{12} + R_{23} + R_{31} = 0$$

The ratio of each variation satisfies

$$\sin\theta + \cos\theta + \sin\phi = 0$$

What should truly remain unchanged is the quality matrix M and a quantity corresponding to the total location. Since there are only three directions, it cannot provide all the information about this invariant, so a term similar to the real part should be added.

$$\lambda^2{}_M = \mu\lambda^2{}_0 + \lambda^2{}_i + \lambda^2{}_j + \lambda^2{}_\kappa$$

Since the orthogonal k direction becomes 0 when returning to the frequency domain, we obtain

$$\sin\phi = -1 \;\; \sin\theta = 1 \quad \cos\theta = 0$$

The mathematical definition of the projection operator P will be given below, and the 2*2 complex matrix representation of the quaternion Q is

$$Q = \begin{pmatrix} q_0 + iq_3 & q_2 + iq_1 \\ -q_2 + iq_1 & q_{0-} iq_3 \end{pmatrix}$$

$$P[Q] = Q.\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} q_0 + iq_3 \\ -q_2 + iq_1 \end{pmatrix}$$

Due to theoretical constraints, when the projection transformation acts on the field returning to the complex numbers, gauge symmetry must be preserved. Let's first consider the Higgs field doublet and the overall SU(2) transformation in the quaternion system：Q----UQ

$$\phi = P[Q] \qquad\qquad \phi^{\sim} = P[UQ] = UQ.\begin{pmatrix} 1 \\ 0 \end{pmatrix} = U\left(Q\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) = U\phi$$

$$\phi^+\phi = \sum q_i^2$$

This satisfies the requirements of the Higgs doublet.

Calculation of quality variance and determination of quality order

The Lagrangian of the neutrino is

$$\zeta = i\Upsilon^\mu \bar{\psi}\partial_\mu\psi - g\bar{\psi}Y\phi\psi$$

By substituting into the Euler–Lagrange equation combined with the Higgs mechanism

$$i\Upsilon^\mu\partial_\mu\psi - gM\psi = 0$$

$$i\Upsilon^\mu\left[\partial_\mu\psi_M.\bar{\psi}_D + \psi_M.\partial_\mu\bar{\psi}_D\right] - gM\psi_M\bar{\psi}_D = 0$$

Since there are at most three orthogonal directions and the M direction has invariance, it indicates that it has two degrees of freedom, meaning only left- and right-handed single states, further simplification gives

$$i\Upsilon^{\mu}[\partial_{\mu}\nu_R+\partial_{\mu}\nu_L]\bar{\psi}_D+i\Upsilon^{\mu}\psi_M[\partial_{\mu}\nu_{0R}+\partial_{\mu}\nu_{0L}+\partial_{\mu}e_{0L}]-gM\psi_M\bar{\psi}_D=0$$

Expand the quality items

$$-gM\psi_M\bar{\psi}_D=-g\psi_M\bar{\lambda}_{[ijk]}\bar{\psi}_D-g\kappa_{[ijk]}\psi_M\bar{\psi}_D$$

Compared with the standard equation

$$M_L\Leftrightarrow\kappa_i \qquad M_R\Leftrightarrow\kappa_j$$

$$M_{DL}\Leftrightarrow\lambda_i \qquad M_{DR}\Leftrightarrow\lambda_j \qquad M_{eL}\Leftrightarrow\kappa_j$$

An eigenvalue of the Majorana state corresponds to an electron, which indicates that the mass of Majorana particles and electrons is much greater than that of Dirac neutrinos

Since the following formula is a constant

$$\lambda^2{}_M=\mu\lambda^2{}_0+\lambda^2{}_i+\lambda^2{}_j+\lambda^2{}_\kappa$$

It is also possible to define a norm of a similar form when returning to low energy

$$M^2=m_1{}^2+m_2{}^2+m_3{}^2$$

Assume that the masses of the three neutrinos satisfy a linear relationship

$$m_3=\alpha m_1+\beta m_2$$

Make

$$M^2=(\alpha^2+1)m_1{}^2+(\beta^2+1)m_2{}^2+2\alpha\beta m_1m_2$$

$$\lambda^2{}_M=\left(\frac{\lambda^2{}_0}{\lambda^2{}_i}+1\right)\lambda^2{}_i+\left(\frac{\lambda^2{}_0}{\lambda^2{}_j}+1\right)\lambda^2{}_j+2\lambda^2{}_0$$

With μ equal to 4, due to the difference between high and low energies, we cannot directly equate the corresponding terms, but the following relationship should remain unchanged,

$$\alpha m_1=\beta m_2$$

This illustrates the mass difference between the two types of neutrinos, but we still need a relation, which comes from the fact that when the neutrino mass is extremely small, the masses of the two types of neutrinos are almost identical, which is relatively simple

$$\lambda^2{}_M=\left(\frac{\lambda^2{}_i}{\lambda^2{}_0}+1\right)\lambda^2{}_0+\left(\frac{\lambda^2{}_j}{\lambda^2{}_0}+1\right)\lambda^2{}_0+2\lambda^2{}_0$$

We normalise λ0 to obtain

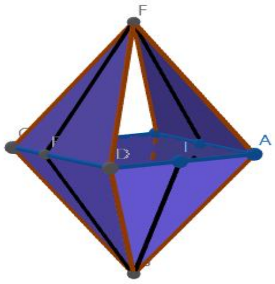
$$\lambda^2{}_M=\left(\frac{\lambda^2{}_i}{\lambda^2{}_0}+1\right)\lambda^2{}_0+\left(\frac{\lambda^2{}_j}{\lambda^2{}_0}+1\right)\lambda^2{}_0+2\lambda^4{}_0$$

$$\lambda^2{}_i = 2\lambda^2{}_j = 2\lambda^2{}_0 \qquad\qquad \alpha^2 + \beta^2 = 3$$

We need to maintain the difference in quality while ensuring that the quality is sufficiently low, so the following two equations need to hold simultaneously

$$\frac{m_1}{m_2} = \frac{\beta}{\alpha} \qquad\qquad \alpha^2 + \beta^2 = 3$$

To obtain the ratio, we return to the geometric structure of neutrinos. Due to the Majorana-Dirac duality, asymmetry in mass coupling at high energy inevitably affects low energy, so this ratio comes from the geometric structure of the Majorana state.



As shown in the figure, the difference between the two coupling modes, up and down, is determined by the angle between the perpendicular from the common vertex to the opposite side.

$$\cos\theta_1 = \frac{\sqrt{2}}{\sqrt{3}} \qquad\qquad \cos\theta_2 = \frac{\sqrt{5}}{\sqrt{6}}$$

Because the ratio is the tangent of an angle, which is less than one but close to one, we calculate the square of the tangent of the difference between these two angles, and then subtract it from one.

$$\frac{\beta}{\alpha} = k = 1 - \tan^2\theta = \frac{4}{11 - 2\sqrt{10}}$$

This is the mass ratio, taking an approximate value of 0.8555, the mass variance is

$$\frac{11k^2 - 1}{(1 + k^2)(1 - k^2)}$$

Substitute to get

$$\Delta \bar{m}^2 = 31.3707$$

This differs from the experimental value, but the sine squared of the Weinberg angle is 0.239, which also has a difference of 0.008. If this difference is of the same order of magnitude as the difference of 0.8555 in the corrected mass ratio, then the corrected mass ratio can be taken as 0.8635 and substituted again to obtain

$$\Delta m^{-2} = 33.4113$$

This is extremely close to the experimental value; the original discrepancy can be attributed to the difference between tree-level predictions and radiative corrections, and it also demonstrates that the electroweak theory and neutrinos are unified within a larger framework. In the appendix, I will derive the self-consistency of the electroweak theory within the framework of this article.

Summary and Future Directions

This article primarily uses duality under a new framework to derive the ratio of neutrino mass variance in normal order. In the hadronic quark model, the fundamental duality can be extended to dodecahedral and icosahedral duality and needs to be combined with the L-function family. I have already found the source of stronger CP violation in the zeta function, which is related not only to trigonometric functions but also to hyperbolic functions. Since resolving the Riemann hypothesis requires introducing more rigid constraints, discussions on quantum gravity currently only describe directionality. However, a complete theory of quantum gravity would certainly impose minimal constraints on spacetime rotation, which must correspond to a certain mathematical structure. One guess of mine is that it is related to the Kakeya Conjecture conjecture. First of all, thank you for reading this far. I am an independent researcher and very much hope not to remain independent. If you are interested in my theory, you can ask questions, and I very much welcome communication. If you want to collaborate, please contact me directly.

Gukov–Conjecture Correspondence in Quantum Gravity: The minimal constraints that quantum gravity imposes on the rotational degrees of freedom of spacetime are equivalent to the Gukov constraints in geometric measure theory. Specifically, at the Planck scale, the trajectories describing the spin of particles or weak isospin rotations form a physical 'Gukov set'. Although due to quantum fluctuations, this set may appear to have 'zero volume' (spacetime foam) under classical measures, its Hausdorff dimension must remain 3 (preserving the full SU(2) symmetry). The tension between this 'measure minimisation' and 'dimension maximisation' is precisely the geometric origin in quantum gravity that maintains gauge symmetry while generating mass hierarchy and CP violation.

Appendix

After projection, the Higgs field chose a principal direction. Since the higher-dimensional Higgs field is also a specific direction, for a general quaternionic electroweak total field, the following decomposition is possible

$$\psi = \frac{\nu}{\sqrt{2}} + h$$

The kinetic term $D_\mu \psi D^\mu \psi^+$ is

Substitute and apply the covariant derivative

$$L_d = g^2\frac{\nu^2}{8}\omega^{\mu a}\omega_\mu^a + gg^{-}\frac{\nu^2\tau^a Y_Q}{8}\omega^{\mu a}B_\mu + gg^{-}\frac{\nu^2\bar{\tau}^{a} Y_Q}{8}\omega_\mu^a B^\mu + g^{2-}\frac{\nu^2}{8}B^\mu B_\mu$$

Focus on the middle two terms. Due to the number of returns, the quaternion conjugate becomes a complex conjugate, but there are three directions for the Pauli matrices. Since the second direction is the main one, according to rotation, the effect of the first matrix is eliminated, and the second direction is eliminated due to conjugation, so naturally the first and second terms of the cross term are eliminated.

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